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Electronic current in a nano-mechanical kicked electron shuttle

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A B S T R A C T

We have studied the phase-space dynamics and rectified current of a nano-electromechanical shuttle with two types of time-periodic potentials. By applying only sinusoidal voltage with frequency ω at one lead, regular shuttle motion constructs Arnold's tongues in parameter space, where the motion in each tongue has a period of π/ω or $2\pi/\omega$. The rectified current in each tongue is finite for the $2\pi/\omega$ period but zero for the π/ω period. We then apply discrete kicks, the periods of which modify the period of motion in the tongues. We find that it is important to the rectified current whether the integer of the lowest common multiple between the two periods is even or odd. Specifically, the rectified current is finite in the tongues when the lowest common multiple integer is even, while the current is zero when the integer is odd.

1. Introduction

Nano-mechanical electron shuttles are prototype nano-electromechanical systems (NEMS) that have attracted a great deal of attention due to their fundamental properties in new electronic transport [1–10]. An electron shuttle consists of a nano-mechanical resonator and two electrodes, with bias voltage and vibration between the electrodes. An integer number of electrons is loaded or unloaded with exponential dependence on the position of the shuttle when one electrode is closed by a dc bias voltage. The dc bias voltage, originating from the symmetry-breaking of the potential, transfers a discrete number of electrons from one side to the other. Responses of nano-electromechanical shuttles to ac fields have been reported in both asymmetric [11–13] and symmetric systems [14]; asymmetric systems show rectified current because of the imbalanced left and right flows, even though ac bias voltage respects the symmetry under every time period [11], and symmetric systems also show rectified current that in this case originates from spontaneous symmetry-breaking with increasing degrees of freedom [14].

A kicked rotor model provides us with interesting phenomena for describing discrete driven systems as distinguished from continuously driven systems [15,16]. In this model, system characteristics are determined by the period and strength of periodic pulsed forces (or kicks) as Dirac delta trains. In a classical sense, the model is a good route to chaos, showing diffusive momentum growth with increasing kick strength and period [17]. In the quantum case though, two extraordinary phenomena arise: dynamic localization [16,18–20] and quantum resonance [21–23] in momentum space, which correspond to whether the driving period is an irrational or rational multiple of 2π, respectively. The motion of a nano-electromechanical shuttle behaves classically with electrons tunneling between electrodes on the nanoscale, which can be treated semiclassically. We find then that the interplay between a semiclassical electron shuttle and discrete kicks exhibits interesting electronic transport and mechanical dynamics, the behaviors of which will help us to better understand NEMS and to design more useful nanoscale electron transport systems.

In this paper, we study an electron shuttle with two kinds of periodic voltage applied, sinusoidal and periodic kicks, and find significant mechanical dynamics and electronic current in the parameter space of the resonating shuttle that rely on kick period and strength. Essentially, a driving force induces a non-zero electronic current in the parameter space and results in the shuttle becoming highly efficient in transferring electrons. Moreover, strong high-frequency kicks train the system to be more robust against continuous perturbation, such as a sinusoidal force, thereby guaranteeing more stable electronic systems that can benefit NEMS applications.

2. Nano-mechanical electron shuttle

Let us consider a nano-mechanical electron shuttle (hereafter referred to as a nano shuttle) as a movable quantum dot weakly connected between two electric leads by a tunneling process, with the system under a dissipative force. In this system, we neglect nonequilibrium contributions to current-induced force and pumping current [24–26]. The two electrodes play different roles under a finite bias, namely as a "source" and a "drain". A time-dependent bias voltage is applied to the source electrode, and the drain electrode is directly connected to the ground; the applied force is thus confined along the direction of

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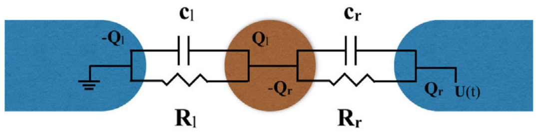

Fig. 1. Schematic of an electron shuttle system.

the two electrodes as an effective 1D system. The equation of motion for such a shuttle can be written as follows,

$$m\ddot{x} + m\gamma\dot{x} = F_s + F_k + F_h, \tag{1}$$

where m is the effective mass of the shuttle and γ is a damping parameter. The time-dependent bias voltage applied to the source electrode of our system is made up of two different voltages, $U(t) = U_s(t) + U_\xi$, where the first part is normal sinusoidal voltage $U_s(t) = \alpha \sin\omega t$ in which α is the amplitude, and the second part is pulsed driven voltage $U_\xi = \xi D_\epsilon(t/T_k)$ in which $D_\epsilon(t/T_k)$ is a discrete function. $D_\epsilon(t/T_k) = 0$ when $t/T_k \in [n-1, n-\epsilon)$, and $D_\epsilon(t/T_k) = 1$ when $t/T_k \in [n-\epsilon, n)$ with integer number n and pulsed period T_k.

An ac voltage induces a normal continuous force F_s, while a pulsed voltage induces a discretized kicked force F_k. The setup of our system is illustrated by the effective circuit with two electrodes in Fig. 1. The position-dependent tunneling process can be represented by a parallel circuit with constant capacitance $c_{l/r}$ and a resistance that exponentially depends on position $R_{l/r} = R_0 e^{\pm x/\lambda}$, where λ is a characteristic length. The continuous force is $F_s = -cU_s^2(t)\mathcal{F}(x)/\lambda$, and the kicked force is $F_k = -c\left[2\xi U_s(t) D_\epsilon(t/T_k) + \xi^2 D_\epsilon^2(t/T_k)\right]\mathcal{F}(x)/\lambda$, where $c = c_l = c_r$ is the effective capacitance of the symmetric circuit, and $\mathcal{F}(x) = 2\left(e^{x/\lambda} + \sinh x/\lambda\right)/\cosh^3(x/\lambda)$ as derived in Appendix. F_h is the harmonic potential of the nano shuttle.

Discrete electronic current results from transferring an integer number of electrons from the source electrode to the drain electrode (and vice versa) through the oscillation of the shuttle. The dynamics of the resonating shuttle in classical phase space dramatically changes the electronic current behavior.

2.1. Equations of motion for the shuttle

In order to analyze the dynamics of the nano shuttle in phase space, we can consider Hamiltonian equations that read $\dot{p} = -\partial H/\partial x$ and $\dot{x} = \partial H/\partial p$. The coupled equations are reduced as

$$\ddot{x} = \frac{\partial^2 H}{\partial p \partial t} + \left(\frac{\partial^2 H}{\partial p \partial x}\right)\left(\frac{\partial H}{\partial p}\right) - \left(\frac{\partial^2 H}{\partial p^2}\right)\left(\frac{\partial H}{\partial x}\right), \tag{2}$$

which is the same equation as Newton's second law $m\ddot{x} = F$, where F is a force applied to an object. We can obtain a Hamiltonian coupled with a bath environment by applying force with damping $F = -m\gamma\dot{x} - \partial V(x,t)/\partial x$ to the shuttle, where $V(x,t)$ is potential. The modified Hamiltonian equation with damping then reads

$$\frac{\partial^2 H}{\partial p \partial t} + \left(\frac{\partial^2 H}{\partial p \partial x} + \gamma\right)\left(\frac{\partial H}{\partial p}\right) - \left(\frac{\partial^2 H}{\partial p^2}\right)\left(\frac{\partial H}{\partial x}\right) = -\frac{\partial V(x,t)}{m\partial x}. \tag{3}$$

We can re-write the Hamiltonian equations as follows,

$$\dot{p} = -\frac{\partial V(x,t)}{\partial x} e^{\gamma t}, \tag{4}$$

$$\dot{x} = \frac{p}{m} e^{-\gamma t}, \tag{5}$$

where the modified force is $F = -\partial V(x,t)/\partial x = F_s + F_k + F_h$, which is considered in a new Hamiltonian with damping,

$$H = \frac{p^2}{2m} e^{-\gamma t} + V(x,t) e^{\gamma t}. \tag{6}$$

Given one initial point, we can calculate the trajectory of the resonator and reveal the corresponding motion properties using Eqs. (4) and (5).

From Eq. (5), we also find that momentum p and $m\dot{x}$ are not equivalent due to the dissipative force. To be exact, their relation is given by $p = m\dot{x}e^{\gamma t}$.

The dimensionless variables here are $\tilde{x} = x/\lambda$, $\tilde{p} = pe^{-\gamma t}/\lambda\omega_0 m$, $\tilde{t} = \omega_0 t$, $\tilde{\omega} = \omega/\omega_0$, $\tilde{\gamma} = \gamma/\omega_0$, $\tilde{T_k} = T_k/\omega_0$, $\tilde{\alpha} = \alpha\sqrt{c/m}/\lambda\omega_0$, and $\tilde{\xi} = \xi\sqrt{c/m}/\lambda\omega_0$. The Hamiltonian equations are modified as

$$\dot{p} = -\left[\alpha \sin(\omega t) + \xi D_\epsilon(t/T_k)\right]^2 \mathcal{F}(x) - x - \gamma p, \tag{7}$$

$$\dot{x} = p, \tag{8}$$

where we omit the tildes (~). These Hamiltonian equations with rescaled quantities give us the shuttle dynamics in phase space based on x and $\dot{x}$.

2.2. Electronic current

Charge accumulation on the left side of the shuttle system is determined by Kirchhoff's law in a circuit (Fig. 1) with bias voltage $U(t)$ as a function of the time-dependent position of the shuttle (x), as $Q_l(x,t) = cU(t)R_l(x)/[R_l(x) + R_r(x)]$ where $R_{l/r}(x)$ is resistance at the left/right junction. From the definition of current $I(t) = \frac{Q_l(x)}{cR_l(x)}$, the instantaneous current of the system is

$$I(t) = \frac{U(t)}{2R_0 \cosh(x)}, \tag{9}$$

where the resistance depends on position $R_{l,r}(x) = R_0 e^{\eta x}$ and η is ± 1 depending on left/right. When we apply a sinusoidal bias voltage with frequency ω and discrete kicks, the instantaneous current reads

$$I(t) = \frac{1}{2R_0} \frac{\alpha \sin\omega t + \xi D_\epsilon(t/T_k)}{\cosh(x)}. \tag{10}$$

Rectified current can then be defined by the average of the instantaneous current during the corresponding time period, as

$$I_{dc} = \frac{1}{t}\int_0^t I(\tau) d\tau = \frac{1}{2tR_0}\int_0^t \frac{\alpha \sin\omega\tau + \xi D_\epsilon(\tau/T_k)}{\cosh(x_\tau)} d\tau. \tag{11}$$

If the duration ϵ and strength ξ of the kicks satisfy $\epsilon\xi \ll 1$, and the oscillation amplitude is meaningfully large $(\cosh(x_\tau) \geq 1)$, then we can omit the contribution of the kicks. The rectified current can be simplified with period T_I, which is the period of instantaneous current, as follows,

$$I_{dc} = \frac{\alpha}{2T_I R_0}\int_0^{T_I} \frac{\sin\omega\tau}{\cosh(x_\tau)} d\tau. \tag{12}$$

In this case, for regular shuttle motion, the period of motion matches the period of the time-dependent current. However, when the motion of the shuttle is irregular, the period cannot be determined; under this situation, we set $t = t_c$ for the calculation of rectified current, and $t_c = 1/\gamma$ is the characteristic time for the motion of the shuttle resonator.

3. Nano-mechanical electron shuttle without kicked voltage

Let us focus on the dynamics of a nano-electromechanical shuttle without kicks, which is therefore a simple grounded shuttle with time-periodic bias voltage. Such asymmetry induces instabilities and current rectification in the shuttle system.

3.1. Dynamics in phase space

The dynamics of the nano shuttle in phase space is well defined by the integration of Hamiltonian equations (7) and (8), as

$$p(t) = \int_0^t \left[-\alpha^2 \sin^2 \omega\tau \mathcal{F}(x) - x - \gamma p\right] d\tau + p(0), \tag{13}$$

$$x(t) = \int_0^t p d\tau + x(0). \tag{14}$$

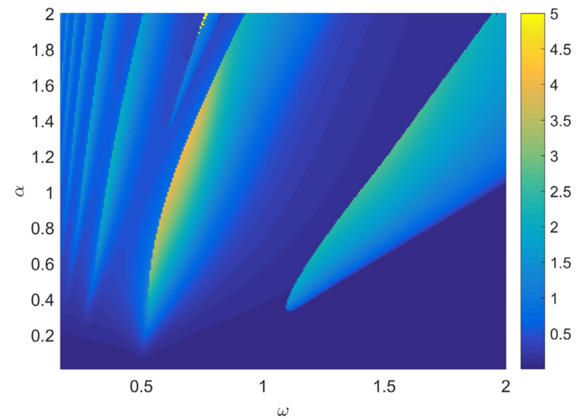

Fig. 2. Magnitude of shuttle displacement Δx as a function of α and ω with $\gamma = 0.1$. The initial state is $(x_0, p_0) = (-0.1, -0.1)$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

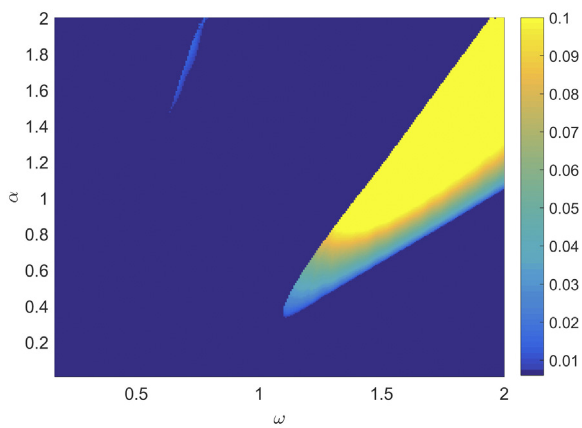

Fig. 3. Rectified current I_{dc} as a function of α and ω with the same parameters as Fig. 2.

Given an initial condition (x_0, p_0), we can obtain a single trajectory in phase space. In order to characterize trajectory size, we should first define the magnitude of shuttle displacement, which is given as $\Delta x = x_{max} - x_{min}$, where x_{max}/x_{min} is the maximum/minimum position of the trajectory. The displacement of the shuttle distinguishes the α–ω parameter space and can be nonzero with some parameters when pumping energy by external voltage is larger than dissipation energy. Fig. 2 shows the stability of Δx in phase space as a function of α and ω with dissipation parameter $\gamma = 0.1$ and initial state $(x_0, p_0) = (-0.1, -0.1)$. The deep blue regions indicate zero motion, while the cyan and yellow tongues are regular motion trajectories with finite amplitudes and different periods that converge to a fixed point. In Fig. 2, for instance, the periods of the (from right to left) first, second, and third tongues are $2\pi/\omega$, π/ω, and $2\pi/\omega$, respectively.

3.2. Electronic current

Eq. (12) gives the rectified current of the shuttle with periodic force as a function of α and ω, as shown in Fig. 3 with the same parameters as Fig. 2. The current is determined by two time periods, one of which is the period of shuttle motion $T_x = n\pi/\omega$ by $sin^2(\omega t)$ where $n = 1$ or 2 [27], and the other is the oscillation of driving voltage $T_v = 2\pi/\omega$ by $sin(\omega t)$. The period T_I of instantaneous current $I(t)$ is the lowest common multiple of T_v and T_x. In the odd case, $n = 1$, the lowest common multiple is $2\pi/\omega$. The rectified current should thus be zero when we consider separating the current into two terms using half periods, as follows,

$$\begin{aligned} I_{dc} &= \frac{\alpha}{2T_I R_0}\left[\int_0^{\pi/\omega} \frac{\sin\omega\tau}{\cosh(x_\tau)} d\tau + \int_{\pi/\omega}^{2\pi/\omega} \frac{\sin\omega\tau}{\cosh(x_\tau)} d\tau\right] \\ &= \frac{\alpha}{2T_I R_0}\left[\int_0^{\pi/\omega} \frac{\sin\omega\tau}{\cosh(x_\tau)} d\tau - \int_0^{\pi/\omega} \frac{\sin\omega\tau}{\cosh(x_\tau)} d\tau\right] \\ &= 0. \end{aligned} \tag{15}$$

In the even case, $n = 2$, the lowest common multiple is again $2\pi/\omega$; however, in this case the second integral function of Eq. (15) cannot cancel the first term because the position coordinate x of the shuttle is asymmetric in the time region, $(0, \pi/\omega)$ and $(\pi/\omega, 2\pi/\omega)$, which means that the rectified current is finite. In this way, rectified current depends on the periodicity of the shuttle resonator.

4. Nano-mechanical electron shuttle with kicked voltage

Now let us move on to discuss the dynamics and electronic properties that result from applying discrete external force in the form of kicks to the nano-electromechanical shuttle. The additional kicked force is also a time-periodic function. In this case, nano shuttle motion is not only related to the period of the continuous force induced by the sinusoidal voltage but also depends on the period of the discrete force induced by the periodic kicks. The kicks therefore modulate the dynamics of the nano shuttle, where the rectified current is determined by the lowest common multiple of the continuous force and discrete force periods.

4.1. Dynamics in phase space

We previously discussed the classical dynamics of kicked systems in terms of the discrete kicked period. During a complete kick period, we can split the time into two parts as general continuous (kick-free) dynamics $t/T_k \in [(n-1), n-\varepsilon)$ and the dynamics including kicks $t/T_k \in [n-\varepsilon, n)$, both with a time interval, where ε is the time duration of the kick force. Within the kick-free interval, the Hamiltonian equations read

$$p(t) = \int_{(n-1)T_k}^{t} \left[-\alpha^2 \sin^2 \omega\tau \mathcal{F}(x) - x - \gamma p\right] d\tau + p[(n-1)T_k], \tag{16}$$

$$x(t) = \int_{(n-1)T_k}^{t} p d\tau + x[(n-1)T_k], \tag{17}$$

which are the same equations of motion as the driven nano shuttle without kicks. Within the kick interval, the Hamiltonian equations are changed into

$$p(nT_k) = \int_{nT_k-\varepsilon}^{nT_k} \left[-\mathcal{F}(x)\left(\alpha\sin\omega\tau + \xi\right)^2 - x - \gamma p\right] d\tau + p(nT_k - \varepsilon), \tag{18}$$

$$x(nT_k) = \int_{nT_k-\varepsilon}^{nT_k} p d\tau + x(nT_k - \varepsilon). \tag{19}$$

If ε is small enough, then we assume that the position of the nano shuttle and the sinusoidal function are unchanged as $x(t) \sim x(nT_k - \varepsilon)$ and $\sin(\omega t) \sim \sin\omega(nT_k - \varepsilon)$ during the time interval $[nT_k - \varepsilon, nT_k)$, respectively. The momentum shift is written by

$$\begin{aligned} p(nT_k) = &-\mathcal{F}[x(nT_k-\varepsilon)]\left(\alpha\sin\omega(nT_k-\varepsilon)+\xi\right)^2\varepsilon \\ &- x(nT_k-\varepsilon)\varepsilon - \int_{nT_k-\varepsilon}^{nT_k} \gamma p(\tau) d\tau \\ &+ p(nT_k-\varepsilon). \end{aligned} \tag{20}$$

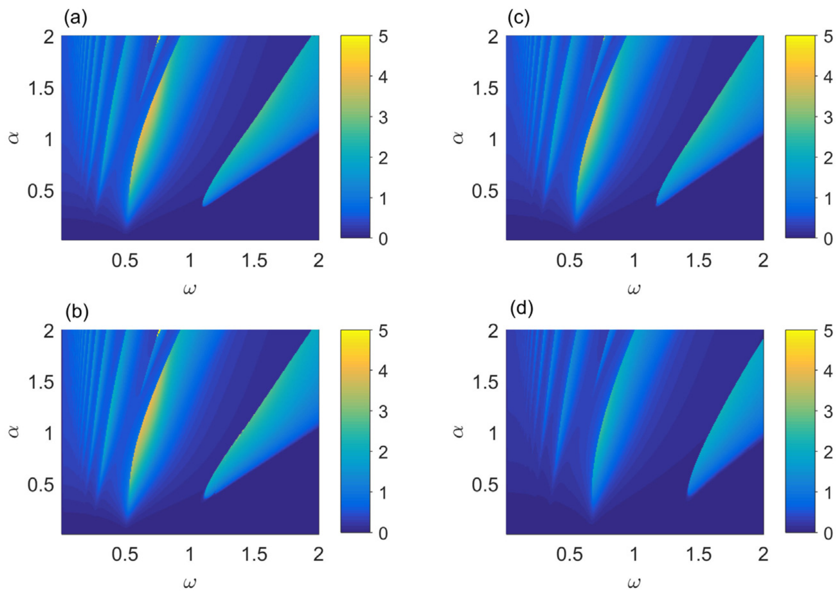

Fig. 4. Magnitude of shuttle displacement Δx as a function of α and ω with different T_k and ξ. The parameters are (a) $T_k = \pi/11$ and $\xi = 10$, (b) $T_k = \pi/59$ and $\xi = 10$, (c) $T_k = \pi/11$ and $\xi = 100$, and (d) $T_k = \pi/59$ and $\xi = 100$. Other parameters are $\gamma = 0.1$, $x_0 = -0.1$, $p_0 = -0.1$, and $\varepsilon = 10^{-6}$.

With the assumption that the kick strength is stronger than the sinusoidal force, $\xi \ll \alpha$, the momentum after one pulse is as follows,

$$p(nT_k) = -\xi_0 \mathcal{F}[x(nT_k - \varepsilon)] + p(nT_k - \varepsilon), \quad (21)$$

where $\xi_0 = \xi^2\varepsilon$. This is the same framework as the discrete kick period. After one complete kick, the final position and momentum are the same as the initial condition of the next period; thus, given the initial state, we can obtain the whole trajectory in phase space.

As shown in Fig. 4(a)–(b), the stability of the displacement for a small kick strength is similar to the stability of the shuttle without kicks (Fig. 2), even with the assumption $\xi \gg \alpha$. In other words, the small kicks do not affect the dynamics of the nano shuttle when the kick period is short ($T_k\xi < 1$). With increasing kick strength, the shorter and shorter periods shift the Arnold's tongues to higher and higher frequencies, and the magnitude of displacement is reduced, as shown in Fig. 4(c)–(d). This means that the influence of sinusoidally driven α weakens according to larger strengths and shorter periods of the kicks; i.e., kicked voltage makes the system robust against sinusoidal perturbation as defined by the α parameter.

4.2. Electronic current

As aforementioned, the period of the whole system is determined by the lowest common multiple of T_s and T_k, where $T_s = \pi/\omega$ and T_k is the time period of driving forces F_s and F_k. Whether these two periods are commensurate or not gives us a new degree of freedom in the time period to control the shuttle dynamics.

Let us first consider that T_s and T_k are commensurate as $T_k = mT_s/l$, where m, l are arbitrary integer numbers. The lowest common multiple of T_s and T_k is mT_s, and then nmT_s ($n = 1$ or 2) can be the regular motion period of the shuttle resonator. When the integer m of the lowest common multiple is odd or even, the rectified current can be zero or not according to the periodicity of the shuttle, respectively; as shown in Section 3.2, the rectified current is zero for the odd $n = 1$ and nonzero for the even $n = 2$. In Fig. 5, (a)–(b) show the displacement and (c)–(d) show the rectified current of the shuttle with a commensurate kick period and $n = 1$. We can compare with rectified currents with different periods, of which Fig. 5(c) and (d) are $T_k = \pi/11$ and $T_k = 2\pi/11$, respectively. These two plots obviously indicate that kicks with an even m period induce the finite rectified current but kicks with an odd m period do not. For even m, the period of shuttle motion is always an even number, and the rectified current for all of the tongues is finite. On the other hand, when T_s and T_k are incommensurate, the motion of the shuttle resonator is quasiperiodic because there is no lowest common multiple of T_s and T_k. Thus, the rectified current is finite and irregular for any available parameters under this consideration due to the infinite periodicity.

5. Conclusion

In conclusion, we studied rectified current through a nanoelectromechanical shuttle with sinusoidal and kicked bias voltage, as well as the shuttle's mechanical motion in phase space. Shuttle motion from only sinusoidal voltage formed tongue structures in the α–ω parameter space, with different periods presented on each tongue. The corresponding rectified currents were finite for tongues with a period of $2T_s$ but zero for tongues with a period of T_s. We found that the periodicity of motion on each tongue can be modified by applying kicked bias voltage; variation of the kick period changes the period of the regular motion of the nano shuttle from T_s to an even multiple of T_s. When the lowest common multiple integer is odd, then the rectified current is zero and finite depending on whether the motion of the shuttle is odd or even, respectively. Meanwhile, when the lowest common multiple integer is even, then the nano shuttle transfers finite rectified current on every tongue. This operation renders the electron shuttle system more efficient in terms of electron transfer.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgments

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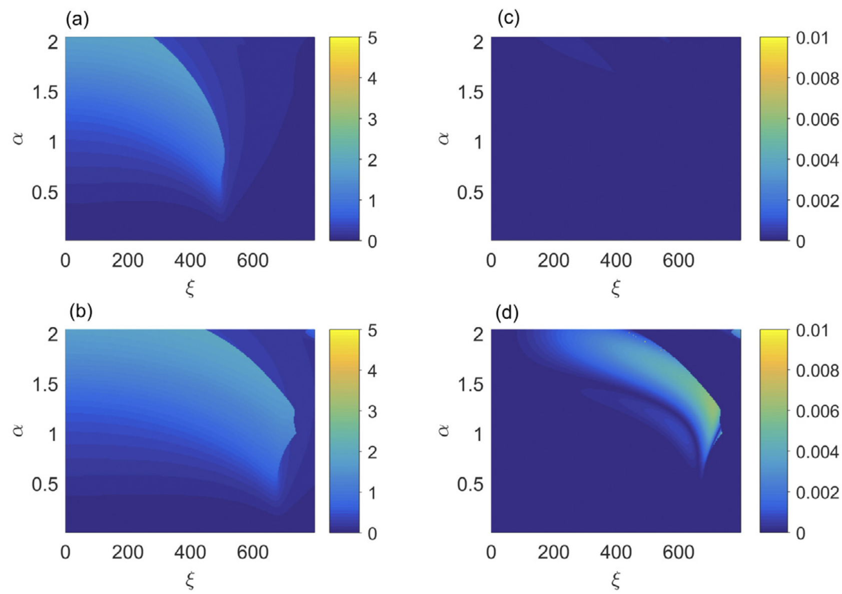

Fig. 5. Magnitude of shuttle displacement Δx (left panels) and rectified current I_{dc} (right panels) vs. α and ξ with (a) and (c) $T_k = \pi/11$, and (b) and (d) $T_k = 2\pi/11$. Other parameters are $\gamma = 0.1$, $\omega = 1.0$, $x_0 = -0.1$, $p_0 = -0.1$, and $\epsilon = 10^{-6}$.

Appendix. Force in the electron shuttle

Circuit diagrams of the kicked nano shuttle are shown in Fig. 1. Based on this structure and Kirchhoff's law, we obtain the following equations:

$$\frac{Q_l(x)}{R_l(x)c_l} - \frac{Q_r(x)}{R_r(x)c_r} = 0, \tag{A.1}$$

$$\frac{Q_l(x)}{c_l} + \frac{Q_r(x)}{c_r} = U(t), \tag{A.2}$$

where x is the position of the shuttle (right is the positive direction), $l(r)$ indicates the left(right) junction, and $Q_{l,r}$, $R_{l,r}$, and $c_{r,l}$ are the accumulated charge, resistance, and capacitance at left(right), respectively. $U(t)$ is the voltage applied to the source electrode. Let us assume a symmetric junction, namely as $c_l = c_r = c$. From Eqs. (A.1) and (A.2), we can solve

$$Q_{l/r}(x,t) = \frac{cU(t)R_{l/r}(x)}{R_l(x) + R_r(x)}. \tag{A.3}$$

Now, the total charge on the shuttle is given by

$$Q_t(x,t) = cU(t)\frac{R_l(x) - R_r(x)}{R_l(x) + R_r(x)}. \tag{A.4}$$

The resistance can be given as $R_l = R_l^0 e^{(d+x)/\lambda}$ and $R_r = R_r^0 e^{(d-x)/\lambda}$, where λ is the characteristic length of the system, and d is the distance between an electrode and the center point between the two electrodes. Here, $R_l^0 e^{d/\lambda} = R_r^0 e^{d/\lambda} = R_0$ under the symmetric junction $R_l = R_r$ at $x = 0$. The resistances of the left and right electrodes are $R_l = R_0 e^{x/\lambda}$ and $R_r = R_0 e^{-x/\lambda}$. Substituting these resistance in Eq. (A.4), we can find the total charge through $Q_t(x,t) = cU(t)\tanh(x/\lambda)$. From the circuit, the voltage for the oscillator is given by $U(x,t) = Q_t(x,t)/c$, and thus, under such voltage, the shuttle has electronic energy as $E(x,t) = Q_t(x,t)U(x,t)$. Correspondingly, the force acting on the nano shuttle is

$$F = -\frac{\partial E(x,t)}{\partial x} = -cU^2(t)\frac{\partial}{\partial x}\left[\frac{2\tanh x/\lambda e^{x/\lambda}}{\cosh x/\lambda}\right],$$

$$= -\frac{cU^2(t)}{\lambda}\mathcal{F}(x), \tag{A.5}$$

where

$$\mathcal{F}(x) = \frac{2\left(e^{x/\lambda} + \sinh x/\lambda\right)}{\cosh^3(x/\lambda)}. \tag{A.6}$$

Considering $U(t) = U_s(t) + \xi D_\epsilon(t/T_k)$ and driven force $F = F_s + F_k$, we get

$$F_s = -\frac{c}{\lambda}U_s^2(t)\mathcal{F}(x), \tag{A.7}$$

$$F_k = -\frac{c}{\lambda}\left[2\xi U_s(t)D_\epsilon(t/T_k) + \xi^2 D_\epsilon^2(t/T_k)\right]\mathcal{F}(x). \tag{A.8}$$

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