R parity from string compactification

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In this paper, we embed the $\mathbf{Z}_{4R}$ parity as a discrete subgroup of a global symmetry $\mathrm{U}(1)_\mathrm{R}$ obtained from $\mathbf{Z}_{12-I}$ compactification of a heterotic string $\mathrm{E}_8 \times \mathrm{E}_8'$. A part of $\mathrm{U}(1)_\mathrm{R}$ transformation is the shift of the anticommuting variable ϑ to $e^{i\alpha}\vartheta$, which necessarily incorporates the transformation of the internal space coordinate. Out of six internal spaces, we identify three U(1)'s whose charges are denoted as Q_{18}, Q_{20}, and Q_{22}. The $\mathrm{U}(1)_\mathrm{R}$ is defined as $\mathrm{U}(1)_\mathrm{EE} \times \mathrm{U}(1)_\mathrm{KK}$, where $\mathrm{U}(1)_\mathrm{EE}$ is the part from the $\mathrm{E}_8 \times \mathrm{E}_8'$ and $\mathrm{U}(1)_\mathrm{KK}$ is the part generated by Q_{18}, Q_{20}, and Q_{22}. We propose a method to define a $\mathrm{U}(1)_\mathrm{R}$ direction. The needed vacuum expectation values for breaking gauge U(1)'s except for $\mathrm{U}(1)_Y$ of the standard model carry a $\mathrm{U}(1)_\mathrm{R}$ charge 4 modulo 4 such that $\mathrm{U}(1)_\mathrm{R}$ is broken down to $\mathbf{Z}_{4R}$ at the grand unification scale. $\mathbf{Z}_{4R}$ is broken to $\mathbf{Z}_{2R}$ between the intermediate ($\sim 10^{11}$ GeV) and the electroweak scales (100 GeV $\sim$ 1 TeV). The conditions we impose are proton longevity, a large top quark mass, and acceptable magnitudes for the μ term and neutrino masses.

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I. INTRODUCTION

In supersymmetric (SUSY) extensions of the standard model (SM) and grand unified theories (GUTs), proton longevity invites additional symmetries. The most discussed one is the R parity [1,2].[1]

"How is the current allocation of flavors realized?" is the most urgent and also interesting question in the theoretical problems of the standard model of particle physics. Advocates of string theory for the heterotic string argue that string compactification is the most complete answer to this problem [4–9].

String compactifications aim at obtaining (i) large 3D space, (ii) standard-like models with three families, and (iii) no exotics at low energy (or vectorlike representations if they exist). Regarding a solution to item (i), the string landscape scenario is suggested [10], predicting about 10^{500} vacua for a reasonable cosmological constant (CC). Regarding item (ii), standard-like models from the heterotic string have been suggested from early days [11,12] until recently [13–31]. Model constructions are discussed in detail in Refs. [32–34]. It has been suggested that by exploring the entire string landscape, one might obtain statistical data which could lead to probabilistic experimental statements [35,36]. Yet the clearest statement to date is that standard-like models are exceedingly rare [37,38]. In addition, the flavor problem asks for a detailed model producing the observed Cabibbo-Kobayashi-Maskawa (CKM) [39,40] and Pontecorvo-Maki-Nakagawa-Sakada (PMNS) [41,42] matrices. In the future, a more refined statistical search, satisfying all the observed SM data, can be performed with the help of an artificial intelligence (AI) program. At present, an AI program is not available for this purpose, and hence we study this flavor problem analytically in the simplest orbifold compactification[2] based on $\mathbf{Z}_{12-I}$. Since the number of fields is over 100 in these standard-like models, we simplify further by choosing GUT models to ease the analytical study. Therefore, we require the following in addition to the above three items:

(iv) Supersymmetry is imposed at the grand unification (GUT) scale.

(v) We consider GUT-scale gauge groups as simple groups [43–46] or semisimple groups [47–49].

[1]For a systematic study of matter parity in addition to R parity, see Ref. [3], for example.

[2]Among the nine orbifolds of Ref. [5], we consider $\mathbf{Z}_{12-I}$ to be the simplest one, in the sense that it has only three fixed points.

SUSY models have been widely used to introduce a mechanism for generating a hierarchically small electroweak (EW) scale compared to the GUT scale. Somewhere above the EW scale, therefore, SUSY must be broken, since no superpartner has been observed up to a TeV scale [50]. In the model, a SUSY-breaking mechanism must be present. The well-known mechanism for SUSY breaking applicable to string compactification is the gaugino condensation [51–53]. Working in the SUSY-breaking models from compactification, we require the gauge group to be at the GUT scale as $G_{\rm GUT} \times G_{\rm cond}$. The most probable $G_{\rm cond}$ is $SU(4)'$, which can trigger SUSY breaking via gaugino condensation [54].

In SUSY models, R parity $P_{\rm R} = (-1)^{3(B-L)+2S}$ dictates proton stability, where B is the baryon number, L is the lepton number, and S is spin. For a conserved R parity, it is usually assigned to a subgroup of $B-L$. From string compactification, R parity has been calculated before in this framework [27,55,56]. Because of dangerous dimension-5 operators, leading to proton decay, $\mathbf{Z}_{4R}$ has been proposed in contrast to $\mathbf{Z}_{2R}$ [57–61]. In this paper, we will present a detailed study toward R parity from the continuous symmetry $U(1)_{\rm R}$. We will see that $U(1)_{\rm R}$ is also constraining some couplings, and hence helps to forbid some unwanted $\Delta B \neq 0$ operators.

GUTs from string compactification favor the flipped SU(5) semisimple GUTs [62–64] and anti-SU(7) [65]. For the simple group of GUTs, SU(5), SO(10), and E_6, we need an adjoint representation to break the GUT groups down to the SM gauge group, and it is impossible to obtain adjoint representation at level 1 [32]. [Note, however, that an adjoint representation of SO(10) was obtained in Ref. [66] at level 3]. So, for simple studies at level 1, anti-SU(N) GUTs are relevant for phenomenological studies.[3]

In Sec. II, after recapitulating the need for R parities toward proton longevity, we discuss the possibilities of embedding R parities in the global symmetry group $U(1)_{\rm R}$ from string compactification. A specific example is presented in the flipped SU(5) model of Ref. [64]. Here the details of U(1) quantum numbers are presented for all the spectra. In Sec. III, we define $U(1)_{\rm R}$ global symmetry, including the R-symmetry transformation of the anticommuting variable ϑ. The U(1) charges Q_{18}, Q_{20}, and Q_{22} are defined from three tori of the compactified six dimensional space. In Sec. IV, the neutral singlets which can obtain GUT-scale vacuum expectation values (VEVs) are discussed. In Sec. V, we discuss the resulting phenomenology. In Sec. VI, we discuss the vacuum structure, leading to the above VEVs, and Sec. VII is a conclusion. In the Appendix, we present some details for obtaining massless fields and their Q_{18}, Q_{20}, and Q_{22} quantum numbers.

[3]In Ref. [65], anti-SU(N) GUTs are defined as those for which the GUT breaking is achieved by the antisymmetric representations. In this definition, the flipped SU(5) is "anti-SU(5)."

II. R PARITIES

Beyond the SM (BSM), the baryon (B) and lepton (L) numbers are broken. The degree of B and L breaking depends on a BSM theory. The most widely discussed one that is also relevant in our paper is the B violation in SUSY extensions of the SM. Supersymmetric standard models (SSMs) can start with the gauge symmetry $SU(3)_{\rm C} \times SU(2)_{\rm W} \times U(1)_{\rm Y}$ with B- and L-conserving dimension-4 operators, which has led to the R-party conservation and predicted the lightest supersymmetric particle as a dark matter candidate. In standard-like models from string, a vacuum with R parity was explicitly shown to exist first in Ref. [27]. The standard R parity or $\mathbf{Z}_{2R}$, however, has been known to be dangerous for the proton longevity due to the dimension-5 operators [1,2]. Without R parity, a dangerous dimension-5 operator appears as shown in Fig. 1 [67].

Without R parity, forbidding dimension-5 B-violating operators involves considering all $SU(5)_{\rm flip}$ singlets which can obtain GUT scale VEVs in principle [68]. Therefore, it will be economic in the discussion if the model contains some kind of R parity. Dimension-5 B-violating operators and the μ term are required to be suppressed, but a dimension-5 L-violating Weinberg operator needs to be allowed [69], while the μ problem must be resolved [70,71]. Considering the anomaly coefficients in SUSY field theory, Lee *et al.* showed that non-R symmetries cannot be used to suppress the μ term [59]. Since we attempt to derive an R symmetry from string compactification that leads to consistent anomaly-free models, the consideration of anomaly coefficients from Lee *et al.*'s point of view is not necessary. Anyway, we adopt their conclusion on $\mathbf{Z}_{4R}$ that the needed R parity is a subgroup of a $U(1)_{\rm R}$ symmetry. So, let our $U(1)_{\rm R}$ be a linear combination of $U(1)_{\rm EE}$ and $U(1)_{\rm KK}$, where $U(1)_{\rm EE}$ is a U(1) from the gauge group $E_8 \times E_8'$ [72] and $U(1)_{\rm KK}$ is a U(1) from the internal space. In Fig. 2, gauge groups shown in 4D

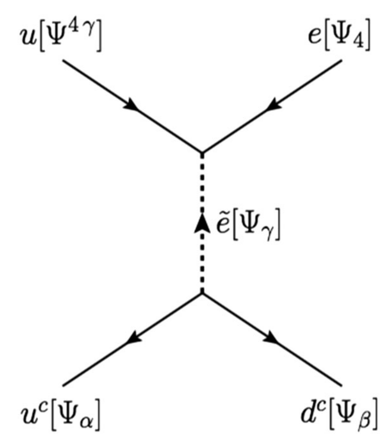

FIG. 1. A diagram for $\Delta B \neq 0$ without R parity. The cubic couplings in this diagram break R parity.

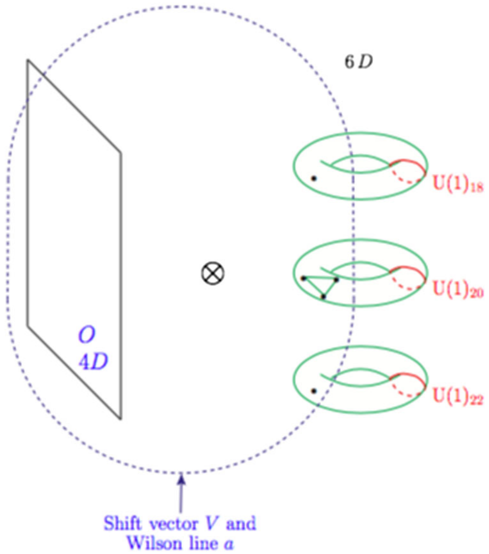

FIG. 2. Matching $\mathrm{U(1)_R}$ with the rotation of the anticommuting variable ϑ. Four-dimensional gauge groups are contained in the brane shown as the parallelogram, and three tori depict three $\mathrm{U(1)_{KK}}$'s, denoted as $\mathrm{U(1)_{18}}$, $\mathrm{U(1)_{20}}$, and $\mathrm{U(1)_{22}}$.

contain $\mathrm{U(1)_{EE}}$, and three tori depict three $\mathrm{U(1)_{KK}}$'s denoted as $\mathrm{U(1)_{18}}$, $\mathrm{U(1)_{20}}$, and $\mathrm{U(1)_{22}}$. For $\mathbf{Z}_3$ orbifolds, it was commented that a dimension-3 μ term is forbidden [73], but the intermediate scale M_I generates the EW scale as $\sim M_{\mathrm{I}}^3/M_{\mathrm{P}}^2$. It was known that a common scale for breaking the PQ symmetry and supergravity is needed [74]. Also, for a multiple appearance of Higgs pairs, the democratic mass matrix, by some kind of fine-tuning, always guarantees at least one massless pair of Higgs doublets [75]. So, we may consider the cases of discrete groups $\mathbf{Z}_4$, $\mathbf{Z}_6$, $\mathbf{Z}_8$, and $\mathbf{Z}_{12}$ of Ref. [59]. Illustration with $\mathbf{Z}_{4R}$ from $\mathrm{U(1)_{KK}}$ of the $\mathbf{Z}_{12-I}$ orbifold can be applicable to the other cases also.

Let us consider the following operators, relevant for the dimension-5 proton decay and neutrino mass operators:

$$\begin{aligned} W^{\Delta B} &\equiv \overline{\mathbf{10}}_m\overline{\mathbf{10}}_m\overline{\mathbf{10}}_m\mathbf{5}_m, \\ W^{\nu\mathrm{mass}} &\equiv \mathbf{5}_m\mathbf{5}_m\bar{\mathbf{5}}_{H_u}\bar{\mathbf{5}}_{H_u}, \end{aligned} \tag{1}$$

where the subscripts m and H denote matter fields and Higgs fields, respectively. If an operator is present in the superpotential, $\mathrm{U(1)_{KK}}$ transformations of the fields of an operator are canceled by the transformation of the anticommuting variable ϑ. Under a certain normalization, the superpotential is required to have $+2$ units of the $\mathrm{U(1)_{KK}}$ charge. Since the rotation angle of variable ϑ can be taken as the negative of the previous transformation, -2 units of the $\mathrm{U(1)_{KK}}$ charge must be allowed also, as illustrated in Fig. 3. So, we have a $\mathbf{Z}_4$ symmetry $-2 \equiv +2$; i.e., minimally we require $\mathbf{Z}_{4R}$ symmetry when we consider the global transformation of ϑ. The $\mathbf{Z}_{4R}$ quantum numbers

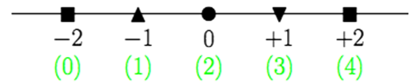

FIG. 3. $\mathbf{Z}_{4R}$ quantum numbers in the region $[-2, +2]$. Numbers in the region [0, 4] are shown in the brackets.

can be labeled as those in green, and the black number assignment is identical to those in green. Under $\mathbf{Z}_{4R}$, the superpotential W leading to the proton decay operator and the μ term is required to carry $+4 \equiv 0$ units, which is then forbidden by $\mathrm{U(1)_R}$, and the superpotential for the neutrino mass operator carries $+2$ units, which is allowed by $\mathrm{U(1)_R}$. These can be satisfied with the matter charge $+1$ and the Higgs charge 0, for example. In string compactification, the realization may be more complex, because one must take into account the sectors where these fields appear.

The R parity is a discrete subgroup of $\mathrm{U(1)_R}$,

$$\mathrm{U(1)_R} \subset \mathrm{U(1)_{EE}} \otimes \mathrm{U(1)_{KK}} \tag{2}$$

Superpotential carries $+2$ (modulo 4) units of $\mathrm{U(1)_R}$. On the other hand, the integrand under $d^2\vartheta d^2\bar{\vartheta}$ carries $+4$ (modulo 4) units of $\mathrm{U(1)_R}$.

A. Model

The shift vector V and Wilson line are

$$\begin{aligned} V &= \left(0,0,0,0,0;\frac{-1}{6},\frac{-1}{6},\frac{-1}{6}\right)\left(0,0,0,0,0;\frac{1}{4},\frac{1}{4},\frac{-2}{4}\right)', \\ a &= \left(\frac{2}{3},\frac{2}{3},\frac{2}{3},\frac{2}{3},\frac{2}{3};0,\frac{-2}{3},\frac{2}{3}\right)\left(\frac{2}{3},\frac{2}{3},\frac{2}{3},\frac{2}{3},0;\frac{-2}{3},0,0\right)', \end{aligned} \tag{3}$$

which gives the 4D gauge group $\mathrm{SU(5)}\times\mathrm{SU(5)'}\times\mathrm{SU(2)'}\times\mathrm{U(1)^7}$.

B. U(1) charges of $\mathbf{E_8 \times E_8'}$

The $\mathrm{U(1)_X}$ charge of $\mathrm{SU(5)_{flip}}$ is

$$X = (-2,-2,-2,-2,-2;0^3)(0^8)', \tag{4}$$

and

$$\begin{aligned} Q_1 &= (0^5;12,0,0)(0^8)', \\ Q_2 &= (0^5;0,12,0)(0^8)', \\ Q_3 &= (0^5;0,0,12)(0^8)', \\ Q_4 &= (0^8)(0^4,0;12,-12,0)', \\ Q_5 &= (0^8)(0^4,0;-6,-6,12)', \\ Q_6 &= (0^8)(-6,-6,-6,-6,18;0,0,6)'. \end{aligned} \tag{5}$$

Any combination of Q_i for $i = 1, 2, \ldots, 6$ can be used for $\mathrm{U(1)_{EE}}$.

C. $\mathbf{U(1)_{KK}}$

In this paper, the compactification of six internal dimensions (coordinate y) is specified as three two-tori. So, any effective field Φ can be a function of $\Phi(x, y)$. To an observer in the 4D x space, gauge symmetries in y are global symmetries. So, the $\mathrm{U(1)_R}$ symmetry we discuss must be a gauge symmetry in the y variable in the three tori. Let the radii of the three tori be $(\mathrm{radius})_1$, $(\mathrm{radius})_2$, and $(\mathrm{radius})_3$. Then, the six internal coordinates are parametrized by $(\mathrm{radius})_1 e^{-i\varphi_1}$, $(\mathrm{radius})_2 e^{-i\varphi_2}$, and $(\mathrm{radius})_3 e^{-i\varphi_3}$. The right-mover coordinates are given by[4]

$$(\oplus|+++),\quad (\oplus|+--),\quad (\oplus|-+-),\quad (\oplus|--+) \tag{6}$$

and

$$(\ominus|-++),\quad (\ominus|+-+),\quad (\ominus|++-),\quad (\ominus|---), \tag{7}$$

where Eq. (6) is called R-handed (with $\oplus$) and Eq. (7) is called L-handed (with $\ominus$). Gauge transformations in the y space rotate φ angles, and the generators for these rotations are called Q_{18}, Q_{20}, and Q_{22}, respectively, specifying the ranks of the total local group in addition to 16 of $\mathrm{E_8 \times E_8'}$. We normalize the charges as

$$\begin{aligned} Q_{18} &= \mathrm{diag.}(2, 0, 0), \\ Q_{20} &= \mathrm{diag.}(0, 2, 0), \\ Q_{22} &= \mathrm{diag.}(0, 0, 2). \end{aligned} \tag{8}$$

In the standard-like models in this compactification leading to $\mathrm{SU(3)_C \times SU(2)_W \times U(1)_Y} \times U(1)^n$, we have $n = 15$. In the flipped SU(5) of Ref. [64], $\mathrm{SU(5) \times SU(5)' \times SU(2)' \times U(1)}^n$, we have $n = 10$. To break all U(1)'s in the standard-like models, we need 16 independent vacuum expectation values (VEVs) of the Higgs fields.

To obtain superpotentials of massless superfields, the couplings must have $+2$ units of $\mathrm{U(1)_R}$ charge. An appropriate combination of three U(1)'s in Eq. (8) can be used for $\mathrm{U(1)_{KK}}$.

D. Multiplicity

We are interested in the multiplicity of massless states, $M^2 = M_L^2 + M_R^2 = 0$ for $M_L^2 = M_R^2 = 0$,

$$\begin{aligned} M_L^2 &= \frac{(P + kV_f)^2}{2} + \tilde{c}_k, \\ M_R^2 &= \frac{(s + k\phi')^2}{2} + c_k, \end{aligned} \tag{9}$$

where $\phi' = (0; \phi)$, $s = (\oplus \text{ or } \ominus; \tilde{s})$, and $2\tilde{c}_k$ and $2c_k$ are listed in the Appendix. With P's in Ref. [64], one can check that $M_L^2 = 0$ is satisfied. The $M_R^2 = 0$ condition is used to obtain the chirality.

For the $\mathbf{Z}_{12-I}$ model of Eq. (3), the multiplicity of the massless spectrum in the T_k sectors is

$$\mathcal{P}_k(f) = \frac{1}{12 \cdot 3} \sum_{l=0}^{11} \tilde{\chi}(\theta^k, \theta^l) e^{i2\pi l \Theta_k}, \tag{10}$$

where $f (= \{f_0, f_+, f_-\})$ denote twisted sectors associated with $kV_f = kV, k(V + a), k(V - a)$. The phase Θ_k is given by

$$\begin{aligned} \Theta_k = \sum_i (N_i^L - N_i^R)\hat{\phi}_i + (P + kV_f) \cdot V_f \\ - (s + k\phi) \cdot \phi - \frac{k}{2}(V^2 - \phi^2), \end{aligned} \tag{11}$$

where $\frac{1}{2}(V^2 - \phi^2) = \frac{2}{24}$, and $\hat{\phi}_j = \phi_j$ and $\hat{\phi}_{\bar{j}} = -\phi_j$. For $k = 0, 3, 6, 9$, $\mathcal{P}_k(f_0) = \mathcal{P}_k(f_+) = \mathcal{P}_k(f_-)$, and the overall coefficient in Eq. (10) is $\frac{1}{12}$ instead of $\frac{1}{36}$, and we require in addition

$$P \cdot a = 0 \quad \text{mod } Z \text{ in the } U, T_3, T_6, T_9 \text{ sectors.} \tag{12}$$

Note the four entry s's and the three entry $\tilde{s}$'s with the relation $s = (\oplus \text{ or } \ominus; \tilde{s})$ such that $\oplus$ or $\ominus$ is chosen to make the total number of minus signs even. For the subsector $f = 0$, i.e., for T_0^k, from the masslessness condition, $2\tilde{c}_k = \sum_i (N_i^L)\hat{\phi}_i + P \cdot V + \frac{k}{2}V^2$, of Table IV, we have

$$\begin{aligned} \Theta_k &= \sum_i (N_i^L - N_i^R)\hat{\phi}_i + P \cdot V - s \cdot \phi + \frac{k}{2}(V^2 - \phi^2) \\ &= \sum_i (N_i^L - N_i^R)\hat{\phi}_i + P \cdot V - s \cdot \phi + \frac{k}{12}. \end{aligned} \tag{13}$$

In the $\mathbf{Z}_N$ orbifold, the $k = \frac{N}{2}$ sector is self-conjugate in a sense. The reason is the following: In the prime orbifolds, there is no sector $T_{N/2}$. In the nonprime orbifolds, there is always a sector $T_{N/2}$. We do not consider the sector T_N in this notation. Instead, we consider the untwisted sector U.[5] Then, there are $N - 1$ twisted sectors where we consider only $k \leq N/2$. With $k < N/2$, effectively we encompass $2(N - 1)$ twisted sectors. The remaining two twisted sectors are in $T_{N/2}$.

E. Selection rule in $\mathbf{Z}_N$

One important selection rule of Yukawa couplings is to satisfy $\mathbf{Z}_N$ invariance for both for the L and R sectors. This

[4] $\pm$ are $\pm\frac{1}{2}$.

[5] The untwisted sector corresponds to $k = 0$ in Eq. (10). For the untwisted sector U_i for the torus $i (= 1, 2, 3)$, it is a closed string moving in the bulk of torus U_i.

amounts to satisfying the sum of phases $\sum_i \Theta_i = 0$ modulo 12 for dimension n superpotential, $W_n \propto \prod_{i=1}^n \Phi_i$.

III. U(1) CHARGES Q_{18}, Q_{20}, AND Q_{22}, AND THE SM FIELDS

In our $\mathbf{Z}_{12-I}$ model of Sec. II A, we use the following normalization:

$$Q_{18}: (2,0,0), \quad Q_{20}: (0,2,0), \quad Q_{22}: (0,0,2). \tag{14}$$

A. Untwisted sector

The multiplicity of the massless spectrum in the untwisted sector U_i occurs with $P \cdot V = \frac{n_i}{12}$, where $n_i = 5, 4, 1$ for the torus index $i = 1, 2, 3$. The U_3 fields in Table I have $P \cdot V = \frac{1}{12}$. This must be canceled by the phase of right movers. Note that $\tilde{\chi}(\theta^0, \theta^l)$ of Eq. (10) is 3, and

$$\Theta_{U_3} = \frac{1}{12} - \tilde{s}_i \cdot \phi. \tag{15}$$

The phase in the third torus, $\Theta_{U_3} = 0$, is achieved by $\tilde{s} = (\ominus; +-+)$ such that $\tilde{s}_i \cdot \phi = \frac{1}{12}$, where $\phi = (\frac{5}{12}, \frac{4}{12}, \frac{1}{12})$. It is L-handed, i.e., $\ominus$, in our definition of the handedness, and we obtain

$$Q_{18,20,22} = +1, -1, +1, \tag{16}$$

respectively, which are listed in Table I.

B. Twisted sectors

We will present the most twisted sector fields in detail in the Appendix, except for the Higgs fields needed in $\mathrm{SU}(5)_{\mathrm{flip}}$: $H_{u,d}$ and $\mathbf{10}_{+1H}$ and $\overline{\mathbf{10}}_{-1H}$. V_f^2 must be the numbers given in Table IV.

Sector T_4^0: Here, two families $(\xi_{2,3}, \bar{\eta}_{2,3}, \mu^c, \tau^c)$ of Table I appear, which are calculated in detail in the Appendix.

Sector T_6: We locate the light Higgs doublets in this sector. From Eq. (10), multiplicities in the T_6 sector are calculated[6] with the following $\tilde{\chi}(\theta^6, \theta^j)$:

$$\tilde{\chi}(\theta^6, \theta^j) = \begin{cases} j = & 0,\ 1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7,\ 8,\ 9,\ 10,\ 11 \\ & 16,\ 1,\ 1,\ 4,\ 1,\ 1,\ 16,\ 1,\ 1,\ 4,\ 1,\ 1 \end{cases}. \tag{17}$$

In T^6, we have from Eq. (13)

$$\Theta_6 = \sum_i (N_i^L - N_i^R)\hat{\phi}_i - \tilde{s} \cdot \phi + P \cdot V + \frac{1}{2}. \tag{18}$$

[6] We use s in the mass relation and $\tilde{s}$ in the phase calculation.

For $H_{uL} = (\underline{+10000}; -100)(0^5; -10+1)'$, we have $P \cdot V = \frac{+5}{12}$. Since $6\phi = (\frac{1}{2}, 0, \frac{1}{2})$, the masslessness condition is satisfied for $s = (\oplus \text{ or } \ominus; -, \pm, -)$, and we obtain the following multiplicity:

$$\begin{array}{lccccc} s & N_i\hat{\phi}_i, & \tilde{s}\cdot\phi, & \Theta_6, & \text{Multiplicity} & \\ (\oplus|-+-): & 0, & \frac{-1}{12}, & 0, & 4 \cdot H_{uR} & \\ (\ominus|---): & 0, & \frac{-5}{12}, & \frac{+4}{12}, & 2 \cdot H_{uL} & \end{array} \tag{19}$$

Similarly, we obtain H_d's, and there result the following Higgs doublets from T_6:

$$2 \cdot H_{uL}(-1,-1,-1) + 2 \cdot H_{dL}(-1,-1,-1) + 4 \cdot H_{uR}(-1,+1,-1) + 4 \cdot H_{dR}(-1,+1,-1). \tag{20}$$

Since the R-handed fields of Eq. (20) do not contribute to the superpotential for Yukawa couplings, we list only the L-handed fields, whose $Q_{18,20,22}$ quantum numbers are listed in Table I.

IV. BSM FIELDS: NEUTRAL SINGLETS

The BSM fields must be neutral singlets and vectorlike representations under $\mathrm{SU}(3)_{\mathrm{C}} \times \mathrm{SU}(2)_{\mathrm{W}} \times U(1)_{\mathrm{Y}}$. The state vectors containing neutral singlets are presented in the second column of Table II. All these neutral singlets appear in the twisted sectors. Neutral singlets are divided into two classes: one contained in the $\mathrm{SU}(5)_{\mathrm{flip}}$ nonsinglets $\Sigma^* = \overline{\mathbf{10}}_{-1}$ and $\Sigma = \mathbf{10}_{+1}$, and the other in the $\mathrm{SU}(5)_{\mathrm{flip}}$ singlets σ's. The VEVs of neutral components in Σ^* and Σ are needed to break the $\mathrm{SU}(5)_{\mathrm{flip}}$ down to the SM gauge group. In this section, we present the details on Σ^* and Σ. $\mathrm{SU}(5)_{\mathrm{flip}}$ singlets will be discussed in the Appendix.

A. $\overline{\mathbf{10}}_{-1} + \mathbf{10}_{+1}$ needed for spontaneous breaking of $\mathrm{SU}(5)_{\mathrm{flip}}$

In T_3 and T_9, we have

$$\tilde{\chi}(\theta^3, \theta^j) = \begin{cases} j = & 0,\ 1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7,\ 8,\ 9,\ 10,\ 11 \\ & 4,\ 1,\ 1,\ 4,\ 1,\ 1,\ 4,\ 1,\ 1,\ 4,\ 1,\ 1 \end{cases}. \tag{21}$$

In T^3, we have from Eq. (13)

$$\Theta_3 = \sum_i (N_i^L - N_i^R)\hat{\phi}_i - \tilde{s} \cdot \phi + P \cdot V. \tag{22}$$

For Σ_1^* with $P = (\underline{+++--}; +++)(0^5; -1, -1, +2)'$ and for Σ_2 with $P = (\underline{++---}; ---)(0^5; +1, +1, -2)'$, $P \cdot V$ is $+\frac{1}{4}$ and $-\frac{1}{4}$, respectively. Without oscillators, the masslessness condition is not satisfied. For $s = (\oplus \text{or} \ominus; -, \pm, -)$, we obtain the following multiplicities for massless Σ_1^* and Σ_2:

$$\begin{array}{cccccc} s & (N_i^L - N_i^R)\hat{\phi}_i, & \tilde{s}\cdot\phi, & \Theta_3, & \text{Multiplicity}, & P\cdot V \\ (\oplus|-+-): & \frac{+5}{12}\,(\text{torus}=1), & \frac{-1}{12}, & 0, & 2, & \frac{+1}{4}\,(\Sigma_1^*), \\ (\ominus|---): & \frac{-1}{12}\,(\text{torus}=\bar{3}), & \frac{-5}{12}, & \frac{+4}{12}, & 1, & \frac{-1}{4}\,(\Sigma_2), \end{array} \tag{23}$$

$$\begin{array}{cccccc} s & (N_i^L - N_i^R)\hat{\phi}_i, & \tilde{s}\cdot\phi, & \Theta_3, & \text{Multiplicity}, & P\cdot V \\ (\oplus|-+-): & \frac{+1}{12}\,(\text{torus}=3), & \frac{-1}{12}, & \frac{+8}{12}, & 1, & \frac{+1}{4}\,(\Sigma_1^*), \\ (\ominus|---): & \frac{-5}{12}\,(\text{torus}=\bar{1}), & \frac{-5}{12}, & 0, & 2, & \frac{-1}{4}\,(\Sigma_2). \end{array} \tag{24}$$

We obtain $Q_{18,20,22} = (-1,+1,-1)$ and $(-1,-1,-1)$. The L-handed field multiplicities are from the oscillators of the first and third tori, $\hat{\phi}_i = \pm\frac{5}{12}, \pm\frac{1}{12}$, as shown in Eqs. (23) and (24). These are explicitly shown in Table II, including the respective multiplicities and charges $Q_{18,20,22}$.

B. Neutral $SU(5)_{flip}$ singlets

U(1) charges of all neutral $SU(5)_{flip}$ singlets σ's are listed in Table II. For a few neutral $SU(5)_{flip}$ singlets, we present the explicit calculation in the Appendix. In Table II, we list U(1) charges of σ-type singlets. Those appearing with oscillators form vectorlike representations, out of which we kept only L-handed fields, because R fields would appear with more mass suppression factors in the Yukawa couplings of the SM fields. We kept up to one oscillator allowed in Table III, presented in the Appendix. For $Q_{18,20,22}$ charges, we listed only those of L-handed fields. We need some singlets carrying

$$\Phi = -\frac{1}{4} \tag{25}$$

to cancel all possible phases in the superpotential. But Table II does not include such a field, and we must consider $[\sigma_i(\frac{\pm1}{4})]^*$ to make a phase-invariant combination by providing $\Phi = -\frac{1}{4}$, which will appear in Sec. V.

V. YUKAWA COUPLINGS

A. μ problem

We find that there remain two pairs of L-handed H_{uL} and H_{dL} in T_6. There exists a superpotential term via T_6T_6, where T_6 is a field appearing in the twisted sector T_6, if the condition in Sec. II E is satisfied. So, we expect a μ term at the GUT scale:

$$W = -\mu_{\rm GUT} H^i_{uL} H^j_{dL}, \quad \text{for } i,j = \{a,b\}. \tag{26}$$

With two H_{uL} and two H_{dL} of Eq. (19), the condition of Sec. II E is not satisfied, since the phase $\frac{4\pi}{3}$ of $H^i_{uL}H^j_{dL}$ does not allow Eq. (26). This is because it does not carry two units of 2π, needed for a superpotential term. The basic reason is that the orbifold contains fixed points divisible by 3. In this regard, we note that $\mathbf{Z}_3$ orbifold has 27 fixed points, forbidding a dimension-3 μ term as first shown in Ref. [73]. $\mathbf{Z}_{12-I}$ contains twisted sectors where the number of fixed points is 3. In $\mathbf{Z}_{12-I}$ and $\mathbf{Z}_{6-I}$, Higgs doublets in the sector $T_{N/2}(N=12, \text{or } 6)$ form vectorlike representations of massless L-handed fields. In $\mathbf{Z}_{12-I}$, these vectorlike representations do not form a μ term, because of the above comment on the $U(1)_R$ charge condition. $\mathbf{Z}_{6-II}$ has 12 fixed points, and forbidding the dimension-3 μ term may be possible here also.

Two pairs surviving from the dimension-3 couplings couple to GUT-scale VEVs by high-dimensional operators. In this case, since two pairs are just from the phase condition on $\Theta_6 = \frac{1}{3}$ of $H^i_u (i=1,2)$ and $H^j_d (j=1,2)$ in Eq. (20), these two are not distinguished. So, if couplings of the $2\times 2\,\mu$ matrix are democratic,

$$\begin{pmatrix} \mu_{\rm GUT} & \mu_{\rm GUT} \\ \mu_{\rm GUT} & \mu_{\rm GUT} \end{pmatrix}, \tag{27}$$

there remains only one light pair. The heavy pair obtains the μ term $2\mu_{\rm GUT}$. This is a remarkable result. A possible Yukawa coupling leading to Eq. (27) arises in a dimension-4 superpotential:

$$W_\mu \propto \frac{1}{\tilde{M}^2} H_u\left(T_6, \frac{1}{3}\right) H_d\left(T_6, \frac{1}{3}\right) \cdot \left[\Sigma_1^*(T_3, 0)\Sigma_2\left(T_3, \frac{1}{3}\right)\right] \cdot \{\sigma_i\}, \tag{28}$$

where $\tilde{M}$ is a string/GUT-scale mass parameter, and the multiplicity of Σ_1^* and Σ_2 with the phase 0 is 2, and the multiplicity with the phase $\frac{1}{3}$ is 1. A σ_i is attached to make the $U(1)_R$ charge 2 modulo 4. Since the VEV of $\sigma_i (i=2,3,4)$ breaks $\mathbf{Z}_{4R}$ symmetry, $\tilde{M}$ and $\mu_{\rm GUT}$ are constrained such that the dimension-5 proton decay operator is sufficiently suppressed. We note that $\mu_{\rm GUT}$ in Eq. (27) is of order $|\langle\Sigma_1^*\Sigma_2\sigma_2\rangle|/\tilde{M}^2$, where $\langle\Sigma_1^*\rangle = \langle\Sigma_2\rangle$ at a GUT scale is needed to break the $SU(5)_{flip}$ to the SM. If we take $|\langle\Sigma_2\rangle| \sim \tilde{M} \simeq 10^{17}$ GeV, the scale $\mu_{\rm GUT}$ is about $\langle\sigma_2\rangle$ where $\mathbf{Z}_{4R}$ is broken.

If we take the democratic form in Eq. (27), the massless component of two H_u's, H_{ua} and H_{ub}, is

$$(H_u)_{\rm SM} = \frac{1}{\sqrt{2}}(H_{ua} - H_{ub}). \tag{29}$$

B. Vectorlike exotics

$\mathbf{Z}_{4R}$ is broken at an intermediate scale by singlet VEVs of $Q_R = 2$ modulo 4, and all vectorlike exotics would obtain masses at the intermediate scale. The gauge coupling unification toward the low-energy value of $\sin^2\theta_W \simeq 0.231$ [76–78] can be studied for all the intermediate-scale masses of these vectorlike exotics.

C. Negative masses

$$2M_L^2 = (P + kV_f)^2 + 2\tilde{c}_k,$$
$$2M_R^2 = s_0^2 + (\tilde{s} + k\phi)^2 + 2c_k, \tag{30}$$

where $\tilde{c}_k$ and c_k are given in Tables IV and V, respectively.

For the SM masses, we need some $\mathrm{SU}(5)_{\rm flip}$ singlets developing GUT/string-scale VEVs. There are no $\mathrm{SU}(5)_{\rm flip}$ singlets in the untwisted sector U. So, some tachyonic scarars may be needed in the twisted sectors; i.e., at the origin some scalar mass must be negative. This condition for the right movers in the kth twisted sector is accompanied by the condition $M_L^2 = M_R^2$:

$$\text{Right mover}: 2M_R^2 = (\tilde{s} + k\phi)^2 < -2c_k - \frac{1}{4}$$
$$= \begin{cases} \frac{5}{24} & \text{for } k = 1, \\ \frac{1}{4} & \text{for } k = 2, \\ \frac{3}{8} & \text{for } k = 3, \\ \frac{1}{12} & \text{for } k = 4, \\ \frac{5}{24} & \text{for } k = 5, \\ \frac{1}{4} & \text{for } k = 6. \end{cases} \tag{31}$$

For L-handed fields, we have

$$\begin{array}{cc|c|l} s & \tilde{s}\cdot\phi, & (\tilde{s}+k\phi)^2, & \text{Check} M^2 < 0 \quad \text{for } k = \\ \hline (\ominus|---): & \frac{-5}{12}, & \frac{5}{24}, \frac{1}{4}, \frac{3}{8}, \frac{12}{144}, \frac{5}{24}, \frac{1}{4} & 1(\times), 2(\times), 3(\times), 4(\times), 5(\times), 6(\times) \\ (\ominus|-++): & 0, & & 1(\times), 2(\times), 3(\times), 4(\times), 5(\times), 6(\times) \\ (\ominus|+-+): & \frac{+1}{12}, & & 1(\times), 2(\times), 3(\times), 4(\times), 5(\times), 6(\times) \\ (\ominus|--+): & \frac{+4}{12}, & & 1(\times), 2(\times), 3(\times), 4(\times), 5(\times), 6(\times) \end{array}. \tag{32}$$

We checked the first row to see whether some mass is negative, but there are no negative-mass states, which is symbolically shown with $\times$. The next three rows have larger values of $(\tilde{s} + k\phi)^2$, and again there are no negative-mass states. Overall, there are no negative-mass states from string compactification. The needed VEVs must arise with appropriate Yukawa couplings.

D. GUT breaking and $\mathbf{Z}_{4R}$

Let us define U_R charges such that matter fields carry $+1$ unit in the following way:

$$Q_R = \frac{1}{2}(Q_1 + Q_2 + Q_3) + \frac{1}{6}(Q_5 + Q_6) + 2Q_{20}, \tag{33}$$

which is listed in Tables I and II. By giving VEVs to $Q_R = 4$ $\mathrm{SU}(5)_{\rm flip}$ field(s), we obtain the discrete symmetry $\mathbf{Z}_4$. This is possible with the GUT-breaking VEVs $\langle\Sigma_1^*\rangle = \langle\Sigma_2\rangle$. If any other σ singlet, carrying $Q_R \neq 4$ modulo 4, develops a VEV, then it will break $\mathbf{Z}_4$.

E. Top quark mass

The top quark is in the U sector. The selection rule of Sec. II E requires the following coupling:

$$\sim \frac{1}{\tilde{M}^2} t(U_3, 0) t^c(U_3, 0) H_u\left(T_6, \frac{1}{3}\right) \Sigma_1^*\left(T_3, \frac{2}{3}\right) \Sigma_2(T_3, 0), \tag{34}$$

where $\tilde{M}$ is some string/GUT scale, and the second numbers in the brackets are Θ_i's given in Tables I and II. The Q_R's of T_3 fields [necessarily developing GUT-scale VEVs, as required for breaking $\mathrm{SU}(5)_{\rm flip}$] add up to 0 modulo 4. Thus, the total Q_R of (34) is $+2$, which is the required $\mathrm{U}(1)_{\rm R}$ charge in the superpotential. Then, the top quark mass is

$$m_{tt} \sim \langle H_{uL}\rangle \frac{M_{10}^2}{\tilde{M}^2}, \tag{35}$$

where $M_{10} = |\langle\Sigma_1^*\rangle| = |\langle\Sigma_2\rangle|$. The bottom quark mass arises similarly from

$$\propto b(U_3, 0) b^c(U_3, 0) H_d\left(T_6, \frac{1}{3}\right) \Sigma_1^*\left(T_3, \frac{2}{3}\right) \Sigma_2(T_3, 0). \tag{36}$$

F. Proton decay problem

The most dangerous operator for proton decay is the dimension-5 operator composed of matter fields, u, d, s, c, $b, t, e, \mu, \tau, \nu_e, \nu_\mu, \nu_\tau$ in Table I:

$$q^I q^J q^K \ell^L, \tag{37}$$

where I, J, K, and L are family indices. As shown in Table I, the $\mathbf{Z}_{4R}$ quantum numbers are -1 for the SM fermions. A superpotential term is allowed if $Q_R = 2$ modulo 4. Thus, the dimension-5 proton decay operator from the superpotential (37) is not allowed.

G. Neutrino masses

The neutrino mass operator is

$$\begin{aligned}
M_{33}^\nu &\propto \frac{1}{\tilde{M}_3^3}\int d^2\vartheta \bar{\eta}_3(U_3,0)\bar{\eta}_3(U_3,0)H_{uL}\left(T_6,\frac{1}{3}\right)\\
&\quad\times H_{uL}\left(T_6,\frac{1}{3}\right)\Sigma_1^*\left(T_3,\frac{2}{3}\right)\Sigma_1^*\left(T_3,\frac{2}{3}\right),\\
M_{22}^\nu &\propto \frac{1}{\tilde{M}_2^4}\int d^2\vartheta \bar{\eta}_2\left(T_4^0,\frac{1}{4}\right)\bar{\eta}_2\left(T_4^0,\frac{1}{4}\right)H_{uL}\left(T_6,\frac{1}{3}\right)\\
&\quad\times H_{uL}\left(T_6,\frac{1}{3}\right)\Sigma_1^*\left(T_3,\frac{2}{3}\right)\Sigma_1^*\left(T_3,\frac{2}{3}\right)\sigma_5\left(T_6,\frac{1}{2}\right),
\end{aligned} \tag{38}$$

where $\tilde{M}_{3,2}$ are some GUT-scale mass parameters, and we have satisfied $Q_R = 2$ above for $d^2\vartheta$ integration. Then, the above masses are estimated as

$$M_{33}^\nu \sim \frac{v_{\rm EW}^2}{\tilde{M}_3}\frac{M_{10}^2}{\tilde{M}_3^2}, \qquad M_{22}^\nu \sim \frac{v_{\rm EW}^2}{\tilde{M}_2}\frac{M_{10}^2|\langle\sigma_5\rangle|}{\tilde{M}_2^3}. \tag{39}$$

Then, the above neutrino masses are of order $v_{\rm EW}^2/\tilde{M}$, since the SM singlet VEVs, M_{10}, and $|\langle\sigma_5\rangle|$ can be at the GUT scale without breaking $\mathbf{Z}_{4R}$.

To obtain mixing between U_3 and T_4^0 neutrinos, we need $d^2\vartheta d^2\bar{\vartheta}$ integration; i.e., we require $Q_R = 0$ modulo 4 for $d^2\vartheta d^2\bar{\vartheta}$ integration:

$$\begin{aligned}
M_{32}^\nu, M_{31}^\nu &\propto \frac{1}{\tilde{M}_m^5}\int d^2\vartheta d^2\bar{\vartheta}\bar{\eta}_3(U_3,0)\bar{\eta}_{2,1}\left(T_4^0,\frac{1}{4}\right)\\
&\quad\times H_{uL}\left(T_6,\frac{1}{3}\right)H_{uL}\left(T_6,\frac{1}{3}\right)\Sigma_1^*\left(T_3,\frac{2}{3}\right)\\
&\quad\times \Sigma_1^*(T_3,0)\cdot\sigma_1\left(T_4^0,\frac{1}{2}\right)^*.
\end{aligned} \tag{40}$$

Then, the above mass mixing is estimated as

$$M_{13,23}^\nu \sim \frac{v_{\rm EW}^2}{\tilde{M}_m}\frac{M_{10}^2|\langle\sigma_1\rangle|}{\tilde{M}_m^3}. \tag{41}$$

Σ_1^* and Σ_2 can have the GUT-scale VEVs because all of them carry $Q_R = 4$, but $|\sigma_1| \ll \tilde{M}_m$ because it breaks $\mathbf{Z}_{4R}$. Depending on the ratio $M_{10}^2|\langle\sigma_1\rangle|/\tilde{M}_m^3$, the mixing masses can be tuned.

Comparing $M_{11,22,31,32}^\nu$ and M_{33}^ν,

$$\frac{M_{11}^\nu, M_{22}^\nu}{M_{33}^\nu} \approx \left|\frac{\sigma_5}{\tilde{M}}\right|, \qquad \frac{M_{31}^\nu, M_{32}^\nu}{M_{33}^\nu} \approx \left|\frac{\sigma_1}{\tilde{M}}\right|, \tag{42}$$

we note that the neutrino mass hierarchy favors the normal hierarchy (in the sense that ν_τ is the heaviest) if the VEVs of σ singlets are comparably small, $|\sigma_1|, |\sigma_5| \ll \tilde{M}$.

H. Mass matrices, and CKM and PMNS mixing angles

Mass matrices obtained in the weak basis are diagonalized to give the CKM matrix in the quark sector and the PMNS matrix in the lepton sector. The Yukawa couplings allowed by the $\mathbf{Z}_{4R}$ quantum numbers, shown in Tables I and II, dictate the forms of mass matrices in the weak basis. Fitting to the observed CKM angles in some detail is presented in a separate paper [79]. For the PMNS matrix, the observed data are not accurate enough to analyze it now.

VI. THE VACUUM STRUCTURE

In this section, our main interest is how the vacuum at the GUT scale, leading to the $\mathbf{Z}_{4R}$ discrete symmetry [59], is realized in our scheme. The following $\mathrm{U(1)_R}$ quantum numbers are determined if Σ_1^*, Σ_2, σ_5, σ_6, σ_7, and σ_8 develop GUT-scale VEVs. All the other fields are not required to have a GUT-scale VEV. Then, there remains a degeneracy which we remove by requiring a simple form for Q_R. Let us start by parametrizing the $\mathrm{U}(1)_R$ charge, without including the anomaly-free Q_4, as

$$\begin{aligned}
Q_R &= x_1 Q_1 + x_2(Q_2 + a_3 Q_3) + x_5 Q_5 + x_6 Q_6\\
&\quad + x_{20}(k_{18}Q_{18} + Q_{20} + k_{22}Q_{22}).
\end{aligned} \tag{43}$$

To break the $\mathrm{SU(5)_{flip}}$ down to a supersymmetric SM, Σ_1^* and Σ_2 must develop the same magnitude of VEV. Therefore, the contributions from the KK sector must be mutually exactly opposite (by considering effective D terms) for Σ_1^* and Σ_2. This condition is on the gauge charges and hence, toward SUSY below the GUT scale, we must require $k_{18} = k_{22} = 0$. Nonzero VEVs of σ_5 and σ_6, leading to the same discrete charge of σ_5 and σ_6 (for σ_7 and σ_8 also), give a possibility $a_3 = 1$. If $x_1 = x_2$, we have

$$Q_R = x_1(Q_1 + Q_2 + Q_3) + x_5 Q_5 + x_6 Q_6 + x_{20} Q_{20}.$$

As an illustration, let us try $x_{20} = 2$. To have a $\mathbf{Z}_4$ subgroup from the VEVs of Σ_1^* and Σ_2, from Q_5, Q_6, and Q_{20} charges in Table II, we have $x_5 Q_5 + x_6 Q_6 = \pm 2$ for Σ_1^* and Σ_2, respectively. Then, note that $x_5 Q_5 + x_6 Q_6 = \pm 2$ or 0 in Tables I and II. For the matter fields of Table I to have an odd

Q_R, we fix $x_1 = x_2 = \frac{1}{2}$. Still, x_5 and x_6 are not determined. For an illustration, we can choose $x_5 = x_6 = \frac{1}{6}$ such that

$$Q_R = \frac{1}{2}(Q_1 + Q_2 + Q_3) + \frac{1}{6}(Q_5 + Q_6) + 2Q_{20}. \tag{44}$$

In Tables II and III, the Q_R's of Eq. (44) are presented. This illustration is realized if the following conditions are met for the vacuum from gauge symmetries:

(a) SUSY is realized below the GUT-breaking scale.
(b) There exist VEVs for σ_5 and σ_6, and also for σ_7 and σ_8. Also, there exists a VEV of σ_1.
(c) $x_1 = x_2 = \frac{1}{4}x_{20}$.
(d) Realization of $\mathbf{Z}_{4R}$.

Items (a) and (d) are what we want. Item (a) is automatically fulfilled in our construction because we obtained a SUSY-flipped SU(5) from the orbifold construction [5,6]. Item (d) follows if items (b) and (c) are fulfilled. We can see it by choosing $x_{20} = 2$, for which we obtain odd numbers for matter fields of Table I, and there are no fractional numbers in Tables I and II. Thus, $\mathbf{Z}_{4R}$ is realized. In the remainder of this section, therefore, we discuss the points related to items (b) and (c).

Item (b) requires showing that σ_5 and σ_6 and also σ_7 and σ_8 develop VEVs. The BSM fields in Table II can have the following ϑ-dependent gauge-invariant terms in the superpotential:

$$\Sigma_1^*\left(T_3,\frac{2}{3}\right)\Sigma_2\left(T_3,\frac{1}{3}\right), \qquad \Sigma_1^*(T_3,0)\Sigma_2\left(T_3,\frac{1}{3},0\right),$$
$$\sigma_5\left(T_6,\frac{1}{2}\right)\bar{\sigma}_7\left(T_6,-\frac{1}{2}\right),$$
$$\sigma_6\left(T_6,\frac{1}{2}\right)\bar{\sigma}_8\left(T_6,-\frac{1}{2}\right),$$
$$\sigma_5\left(T_6,\frac{1}{2}\right)\sigma_6\left(T_6,\frac{1}{2}\right)\bar{\sigma}_7\left(T_6,-\frac{1}{2}\right)\bar{\sigma}_8\left(T_6,-\frac{1}{2}\right),$$

plus any combinations of gauge-invariant field products in Table II, having $\sum_i \Theta_i = 0$. Consider a superpotential constructed with the above quadratic combinations:

$$\begin{aligned} W = &-m_1\Sigma_1^*\Sigma_2 - m_2\sigma_5\bar{\sigma}_7 - m_2'\sigma_6\bar{\sigma}_8 + \frac{\lambda_1}{2M}(\Sigma_1^*\Sigma_2)^2 \\ &+ \frac{\lambda_2}{2M}(\sigma_5\bar{\sigma}_7)^2 + \frac{\lambda_2'}{2M}(\sigma_6\bar{\sigma}_8)^2 + \frac{\lambda_2''}{M}\sigma_5\sigma_6\bar{\sigma}_7\bar{\sigma}_8 \\ &+ \frac{\lambda_3}{M}\Sigma_1^*\Sigma_2\sigma_5\bar{\sigma}_7 + \frac{\lambda_3'}{M}\Sigma_1^*\Sigma_2\sigma_6\bar{\sigma}_8, \end{aligned} \tag{45}$$

where, for simplicity, we consider only one combination of $\Sigma_1^*\Sigma_2$. Since σ_5, σ_6, $\bar{\sigma}_7$, and $\bar{\sigma}_8$ appear in the same twisted sector, T_6, later we set for simplicity $m_2 = m_2'$, $\lambda_2 = \lambda_2' = \lambda_2''$, and $\lambda_3 = \lambda_3'$. The SUSY conditions require

$$\begin{aligned} \delta\Sigma_1^*&: \; -m_1\Sigma_2 + \frac{\lambda_1}{M}\Sigma_2\Sigma_1^*\Sigma_2 + \frac{\lambda_5}{M}\Sigma_2\sigma_5\bar{\sigma}_7 + \frac{\lambda_6}{M}\Sigma_2\sigma_6\bar{\sigma}_8 = 0, \\ \delta\Sigma_2&: \; -m_1\Sigma_1^* + \frac{\lambda_1}{M}\Sigma_1^*\Sigma_2\Sigma_1^* + \frac{\lambda_5}{M}\Sigma_1^*\sigma_5\bar{\sigma}_7 + \frac{\lambda_6}{M}\Sigma_1^*\sigma_6\bar{\sigma}_8 = 0, \\ \delta\sigma_5&: \; -m_2\bar{\sigma}_7 + \frac{\lambda_2}{M}\bar{\sigma}_7\sigma_5\bar{\sigma}_7 + \frac{\lambda_4}{M}\sigma_6\bar{\sigma}_7\bar{\sigma}_8 + \frac{\lambda_5}{M}\Sigma_1^*\Sigma_2\bar{\sigma}_7 = 0, \\ \delta\sigma_6&: \; -m_3\bar{\sigma}_8 + \frac{\lambda_3}{M}\bar{\sigma}_8\sigma_6\bar{\sigma}_8 + \frac{\lambda_4}{M}\sigma_5\bar{\sigma}_7\bar{\sigma}_8 + \frac{\lambda_6}{M}\Sigma_1^*\Sigma_2\bar{\sigma}_8 = 0, \\ \delta\bar{\sigma}_7&: \; -m_2\sigma_5 + \frac{\lambda_2}{M}\sigma_5\bar{\sigma}_7\sigma_5 + \frac{\lambda_4}{M}\sigma_5\sigma_6\bar{\sigma}_8 + \frac{\lambda_5}{M}\Sigma_1^*\Sigma_2\sigma_5 = 0, \\ \delta\bar{\sigma}_8&: \; -m_3\sigma_6 + \frac{\lambda_3}{M}\sigma_6\bar{\sigma}_8\sigma_6 + \frac{\lambda_4}{M}\sigma_5\sigma_6\bar{\sigma}_7 + \frac{\lambda_6}{M}\Sigma_1^*\Sigma_2\sigma_6 = 0. \end{aligned} \tag{46}$$

We choose the vacuum where all of the above fields develop VEVs, $V_i = \langle\sigma_i\rangle$ and $V_{10} = \langle\Sigma_1^*\rangle = \langle\Sigma_2\rangle$. In terms of $m_{1,2}$ and $\lambda_{1,2,3}$, we obtain two independent relations,

$$\begin{aligned} -m_1 + \frac{\lambda_1}{M}V_{10}^2 + \frac{\lambda_3}{M}(V_5V_7 + V_6V_8) = 0, \\ -m_2 + \frac{\lambda_3}{M}V_{10}^2 + \frac{\lambda_2}{M}(V_5V_7 + V_6V_8) = 0, \end{aligned} \tag{47}$$

from which we conclude that the singlets Σ_1^*, Σ_2, and $\sigma_i(i = 5,6,7,8)$ develop GUT-scale VEVs:

$$\begin{aligned} V_{10}^2 &= \frac{(\lambda_2 m_1 - \lambda_3 m_2)}{\lambda_1\lambda_2 - \lambda_3^2}M, \\ V_5V_7 + V_6V_8 &= \frac{(\lambda_3 m_1 - \lambda_1 m_2)}{\lambda_3^2 - \lambda_1\lambda_2}M. \end{aligned} \tag{48}$$

We also need $\sigma_i(i = 1,2,3,4)$ VEVs which are much smaller than the GUT scale, such that the B-violating dimension-5 operators are sufficiently suppressed because $\langle\sigma_i(i = 1,2,3,4)\rangle$ would break $\mathbf{Z}_4$ down to $\mathbf{Z}_2$. These VEVs are considered to be a perturbation to the VEVs of Eq. (48), and $\langle\sigma_i(i = 1,2,3,4)\rangle$ would be close to 0. Note that the σ's in Table II are not moduli, and there is no σ_i with all gauge charges vanishing. Therefore, in order not to produce runaway solutions of σ_i, the mass parameters in the numerator of renormalizable terms and in the denominator of nonrenormalizable terms, leading to $\langle\sigma_i(i = 1,2,3,4)\rangle$, are required to be of sub-GUT scale. Consider the following gauge and Θ invariant D terms containing $\sigma_i(i = 1,2,3,4)$ for m_I, $M_I \ll \tilde{M}$:

$$-\frac{1}{2}m_I\sum_{i=1}^{4}\int d^2\vartheta d^2\bar{\vartheta}\sigma_i\sigma_i^*, \qquad \frac{\lambda_I}{2M_I^8}\int d^2\vartheta d^2\bar{\vartheta}\sigma_1^2\sigma_2^2\sigma_3^2\sigma_7^2\sigma_8^2, \tag{49}$$

where σ_i carry ϑ, and $\sigma_{7,8}$ and σ_i^* carry $\bar{\vartheta}$. One among σ_7 and σ_8 carries two KK windings and the others carry one KK winding, such that $\mathrm{U}(1)_{20}$ invariance is satisfied. Then, V contains the following:

$$V \ni -\frac{1}{2}m_I^2\sum_{i=1}^{4}|\sigma_i|^2 + \frac{\lambda_I}{2M_I^8}[\sigma_1(\sigma_1^*)^2\sigma_2^2\sigma_3^2\sigma_7^2\sigma_8(\sigma_8^*)^2 + \cdots] + \left| -m_2\sigma_6 + \frac{\lambda_2}{M}\sigma_6\bar{\sigma}_8\sigma_6 + \frac{\lambda_2}{M}\sigma_5\sigma_6\bar{\sigma}_7 + \frac{\lambda_3}{M}\Sigma_1^*\Sigma_2\sigma_6\right|^2, \tag{50}$$

$$\frac{dV}{d\sigma_1^*} = 0 \to \frac{\lambda_I}{M_I^8}[\sigma_1\sigma_1^*\sigma_2^2\sigma_3^2\sigma_7^2\sigma_8(\sigma_8^*)^2 + \cdots] = m_I^2\sigma_1, \tag{51}$$

or

$$\sigma_1^* = \frac{m_I^2 M_I^8}{\lambda_I(\sigma_2^2\sigma_3^2\sigma_7^2\sigma_8(\sigma_8^*)^2 + \cdots)}. \tag{52}$$

For $\sigma_5 = \sigma_6 = \sigma_8 = \bar{\sigma}_7 = V_5$ and $\sigma_1 = \sigma_2 = \sigma_3 = \sigma_4 = V_I$,

$$V_I = \left(\frac{m_I^2 M_I^3}{\lambda_I}\right)^{1/5}\frac{M_I}{V_5}. \tag{53}$$

where we neglect $\cdots$ in Eq. (52). If $\sqrt{m_I M_I} \approx 10^{-3}V_5$, then we obtain an intermediate scale $V_I \approx (\lambda_I)^{1/5}10^{-6}V_5$. This can be a kind of model realizing the scale of the "very light" axion in supergravity models [74]. So, we conclude that the vacuum, satisfying item (b), can be realized. In addition, if the solution for the μ term is realized *à la* Ref. [70], then the electroweak scale may be obtained along our vacuum direction.

Item (c) requires showing that the quantum numbers of x_2 are $\frac{3}{4}$ times $(3x_5 + x_6)$. Referring to the six U(1) gauge quantum numbers of Tables I and II, integers are possible if x_1, x_2, x_2a_3, x_5, x_6, and x_{20} are integer multiples of $\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{3}, \frac{1}{3}$, and 1, respectively. Item (a) requires the relation

$$3x_5 + x_6 = \frac{x_{20}}{3}. \tag{54}$$

If we choose $x_{20} = 2x$, we obtain $x_2 = \frac{1}{2}x$. So far, there remain two degrees of freedom, i.e., arbitrary x_1 and $x_6 (= \frac{x_{20}}{3} - 3x_5)$. Requiring a VEV for σ_1, we note that $\langle\sigma_1\rangle$ breaks one U(1) gauge symmetry: $\mathrm{U}(1)_{\sigma_1}$. The breaking direction of $\mathrm{U}(1)_{\sigma_1}$ should not affect other gauge symmetries. Thus, we require $x_1 = x_2 = x_2a_3$ in Eq. (43) because the Q_1, Q_2, and Q_3 quantum numbers of σ_1 in Table II are the same. This proves item (c).

There remains one free parameter x_6. We cannot determine this parameter by VEVs of scalars. To give a smaller number for coefficients, we choose $x_5 = x_6$, and the result is given by Eq. (44) and Tables I and II.

VII. CONCLUSION

We obtained an R parity as a discrete subgroup of $\mathrm{U}(1)_\mathrm{R}$ global symmetry of $\mathrm{U}(1)_\mathrm{EE} \times \mathrm{U}(1)_\mathrm{KK}$, where $\mathrm{U}(1)_\mathrm{EE}$ is the part from $\mathrm{E}_8 \times \mathrm{E}_8'$ and $\mathrm{U}(1)_\mathrm{KK}$ is the part generated by Q_{18}, Q_{20}, and Q_{22}. We checked that the needed VEVs toward flavor mixing, the μ term, the neutrino mass operators, and forbidding dangerous dimension-5 $\Delta B \neq 0$ operators, are consistent with the $\mathrm{U}(1)_\mathrm{R}$ direction. This has been possible because the number of $Q_R = 4$ (modulo 4) fields of Table II is enough to render the needed operators. One more interesting feature is that the SM quarks and leptons of Table I carrying $Q_R = \pm 1$ (modulo 4) are not enough by themselves to cancel the unwanted dimension-5 $\Delta B \neq 0$ operators. But the oringinal $\mathrm{U}(1)_\mathrm{R}$ charge helps to forbid these unwanted terms because the origin of $\mathbf{Z}_{4R}$ symmetry in the ultraviolet completed theory is the global $\mathrm{U}(1)_\mathrm{R}$ which forbids the unwanted dimension-5 $\Delta B \neq 0$ operators.

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Note added in the proof.—Recently, a dynamical breaking in SU(5) is suggested [80].

APPENDIX: $\mathrm{U}(1)_\mathrm{KK}$ CHARGES OF NEUTRAL SINGLETS

The definition of shift ϕ and the number of fixed points χ in $\mathbf{Z}_N$ orbifolds are presented in Table III together with the allowed oscillating modes. The smallest number of fixed points is 3, which are possible in $\mathbf{Z}_{12-I}$ and $\mathbf{Z}_{6-I}$. Among these, we studied $\mathbf{Z}_{12-I}$, which allows more possibilities of Yukawa couplings.

For the massless modes, the phase determining the multiplicity is given in Eq. (11). The massless modes relevant for the Higgs mechanism in our model are for just V; i.e., we do not use the fields from Wilson line added shifts. So, we set $V_f = V$ in Eq. (11):

$$\begin{aligned}\Theta_k^0 &= \sum_i (N_i^L - N_i^R)\hat{\phi}_i + P\cdot V - s\cdot\phi + \frac{k}{2}(V^2 - \phi^2)\\ &= \sum_i (N_i^L - N_i^R)\hat{\phi}_i + P\cdot V - s\cdot\phi + \frac{k}{12},\end{aligned} \tag{A1}$$

$$V^2 = \frac{11}{24}, \qquad \phi^2 = \frac{7}{24}, \qquad \frac{V^2 - \phi^2}{2} = \frac{1}{12}. \tag{A2}$$

In the main text, we illustrated the U(1) charge calculation for $\mathrm{SU}(5)_\mathrm{flip}$ nonsinglets. In this Appendix, we illustrate the calculational methods of massless spectrum and U(1) charges explicitly. The left-mover and right-mover massless states satisfy

TABLE I. U(1) charges of matter fields in the SM. ξ_i and $\bar{\eta}_i$ contain the left-handed quark and lepton doublets, respectively, in the ith family.

	State$(P+kV_0)$	Θ_i	$\mathbf{R}_X$ (Sect.)	Q_R	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_{18}	Q_{20}	Q_{22}
ξ_3	$(\underline{+++--};--+)(0^8)'$	0	$\overline{\mathbf{10}}_{-1}(U_3)$	−5	−6	−6	+6	0	0	0	+1	−1	+1
$\bar{\eta}_3$	$(\underline{+----};+--)(0^8)'$	0	$\mathbf{5}_{+3}(U_3)$	−5	+6	−6	−6	0	0	0	+1	−1	+1
τ^c	$(+++++;-+-)(0^8)'$	0	$\mathbf{1}_{-5}(U_3)$	−5	−6	+6	−6	0	0	0	+1	−1	+1
ξ_2	$(\underline{+++--};-\frac{1}{6},-\frac{1}{6},-\frac{1}{6})(0^8)'$	$\frac{+1}{4}$	$\overline{\mathbf{10}}_{-1}(T_4^0)$	−5	−2	−2	−2	0	0	0	−1	−1	−1
$\bar{\eta}_2$	$(\underline{+----};-\frac{1}{6},-\frac{1}{6},-\frac{1}{6})(0^8)'$	$\frac{+1}{4}$	$\mathbf{5}_{+3}(T_4^0)$	−5	−2	−2	−2	0	0	0	−1	−1	−1
μ^c	$(+++++;-\frac{1}{6},-\frac{1}{6},-\frac{1}{6})(0^8)'$	$\frac{+1}{4}$	$\mathbf{1}_{-5}(T_4^0)$	−5	−2	−2	−2	0	0	0	−1	−1	−1
ξ_1	$(\underline{+++--};-\frac{1}{6},-\frac{1}{6},-\frac{1}{6})(0^8)'$	$\frac{+1}{4}$	$\overline{\mathbf{10}}_{-1}(T_4^0)$	−5	−2	−2	−2	0	0	0	−1	−1	−1
$\bar{\eta}_1$	$(\underline{+----};-\frac{1}{6},-\frac{1}{6},-\frac{1}{6})(0^8)'$	$\frac{+1}{4}$	$\mathbf{5}_{+3}(T_4^0)$	−5	−2	−2	−2	0	0	0	−1	−1	−1
e^c	$(+++++;-\frac{1}{6},-\frac{1}{6},-\frac{1}{6})(0^8)'$	$\frac{+1}{4}$	$\mathbf{1}_{-5}(T_4^0)$	−5	−2	−2	−2	0	0	0	−1	−1	−1
H_{uL}	$(\underline{+10000};000)(0^5;\frac{-1}{2}\frac{+1}{2}0)'$	$\frac{+1}{3}$	$2\cdot\mathbf{5}_{-2}(T_6)$	−2	0	0	0	−12	0	0	−1	−1	−1
H_{dL}	$(\underline{-10000};000)(0^5;\frac{+1}{2}\frac{-1}{2}0)'$	$\frac{+1}{3}$	$2\cdot\bar{\mathbf{5}}_{+2}(T_6)$	−2	0	0	0	+12	0	0	−1	−1	−1

$$\text{Left mover}:\ M_L^2=\frac{(P+kV_f)^2}{2}+\tilde{c}_k=0,$$

$$\text{Right mover}:\ M_R^2=\frac{(s+k\phi)^2}{2}+c_k=0,$$

$$s=(\oplus\quad\text{or}\quad\ominus;\tilde{s}).\tag{A3}$$

In Eq. (3), $\oplus$ or $\ominus$ is chosen such that the total number of minus signs is even.

1. Two families from T_4^0

For matter fields in T_4^0 without oscillators, we insert $P\cdot V=-\frac{1}{4}$ for $k=4$ in Eq. (1):

$$\Theta_4^0(\text{matter})=-\tilde{s}\cdot\phi+\frac{1}{12}.\tag{A4}$$

Note that 4ϕ is $(\frac{20}{12},\frac{16}{12},\frac{4}{12})\to(\frac{2}{3},\frac{1}{3},\frac{1}{3})$, where we must use the entries in the region [0, 1). It is like the shift in $\mathbf{Z}_3$ orbifold. With this 4ϕ, the masslessness condition is $s_0^2+(\tilde{s}+k\phi)^2=-2c=\frac{1}{3}$; i.e., $(\tilde{s}+k\phi)^2=\frac{1}{12}$, which is satisfied by $\tilde{s}=(---)$. So, we choose $s=(\ominus;\tilde{s})$, i.e., it is L-handed, and we obtain $\Theta_4^0(\text{matter})=\frac{1}{2}$. With the multiplicity contribution

$$\tilde{\chi}(\theta^4,\theta^j)=\begin{cases} j= & 0,\ 1,\ 2,\ 3,\ \ 4,\ 5,\ 6,\ 7,\ \ 8,\ 9,\ 10,\ 11\\ & 27,\ 3,\ 3,\ 3,\ 27,\ 3,\ 3,\ 3,\ 27,\ 3,\ \ 3,\ \ 3\end{cases},\tag{A5}$$

we obtain $\mathcal{P}=2$, and the charges Q_{18}, Q_{20}, and Q_{22} of matter from T_4^0 are $-1,-1$, and -1, respectively, which are listed in Table I.

For Higgs fields H_u with $P=(\underline{10000};111)(0^8)'$ in T_4^0, we have $P\cdot V=-\frac{1}{2}$. For $k=4$ in Eq. (1),

$$\Theta_4^0(\text{Higgs})=-\tilde{s}\cdot\phi+\frac{-2}{12}.\tag{A6}$$

Since 4ϕ is $(\frac{2}{3},\frac{1}{3},\frac{1}{3})$, the masslessness condition is the same as above, $s=(\ominus;---)$; i.e., it is left handed (L-handed), and we obtain $\Theta_4^0(\text{Higgs})=\frac{1}{4}$. Again, we obtain $\mathcal{P}=2$ using Eq. (A5), and the charges Q_{18}, Q_{20}, and Q_{22} of Higgs from T_4^0 are $-1,-1$, and -1, respectively. These were used for the Higgs fields in Ref. [68], but here we will not use these for the Higgs fields for breaking the SM.

2. $\sigma_{1\text{–}4}$ from T_4^0

For T_4^0, we have calculated above the chirality as $s=(\ominus;---)$. With $P=(0^8)(0^5;-1,-1,0)'$ for σ_1, we have $P\cdot V=\frac{-1}{2}$, $\Theta_4=\frac{+3}{12}$, and we obtain $\mathcal{P}=2$ without an oscillator. With $P=(0^5;0,+1,+1)(0^8)'$ for σ_2, we have $P\cdot V=\frac{-4}{12}$ and $\Theta_4=\frac{+5}{12}$. With the oscillator $1_{\bar{1}}$, we obtain $\mathcal{P}=3$. For $\sigma_{1,2,3,4}$, we have

$$Q_{18,20,22}=(-1,-1,-1).\tag{A7}$$

3. $\sigma_{5\text{–}8}$ from T_6

For T_6, the multiplicity factor and the phase are given in Eqs. (17) and (18). The allowed chiralities are $s=(\oplus|-+-)$ and $s=(\ominus|---)$ for $6\phi=(\frac{1}{2},0,\frac{1}{2})$. For σ_5, we use $P=(0^5;+1,+2,+1)(0^5;0,-1,+1)'$ and obtain $P\cdot V=\frac{-5}{12}$, and massless fields arise without oscillators:

$$\begin{array}{lccccc} s & N_i, & \tilde{s}\cdot\phi, & \Theta_6, & \text{Multiplicity} & \\ (\oplus|-+-): & 0, & \frac{-1}{12}, & \frac{+2}{12}, & 0\cdot\sigma_{5,6} & . \\ (\ominus|---): & 0, & \frac{-5}{12}, & \frac{+6}{12}, & 2\cdot\sigma_{5,6} & \end{array}\tag{A8}$$

TABLE II. U(1) charges of L-handed neutral scalars (but $\sigma_{7,8}$ for R-handed). We kept up to one oscillator represented in $(N^L)_j$, meaning *Number of resulting fields* ([number of oscillating mode]$_{\text{torus of oscillating mode}}$). For example, $n(1_{\bar{1}})$ means that there result n multiplicities with one oscillator with phase $\frac{-5}{12}$. For $Q_{18,20,22}$ charges, here we listed only those of L-handed fields, participating in the Yukawa couplings.

	State$(P+kV_0)$	Θ_i	$(N^L)_j$	$\mathcal{P}\cdot\mathbf{R}_X$ (Sect.)	Q_R	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_{18}	Q_{20}	Q_{22}
Σ_1^*	$(\underline{+++--};0^3)(0^5;\frac{-1}{4}\frac{-1}{4}\frac{+2}{4})'$	0	$2(1_1)$	$2\overline{\mathbf{10}}_{-1}(T_3)_L$	+4	0	0	0	0	+9	+3	−1	+1	−1
Σ_1^*	$(\underline{+++--};0^3)(0^5;\frac{-1}{4}\frac{-1}{4}\frac{+2}{4})'$	$\frac{+2}{3}$	$1(1_3)$	$1\overline{\mathbf{10}}_{-1}(T_3)_L$	+4	0	0	0	0	+9	+3	−1	+1	−1
Σ_2	$(\underline{++---};0^3)(0^5;\frac{+1}{4}\frac{+1}{4}\frac{-2}{4})'$	0	$2(1_{\bar{1}})$	$2\mathbf{10}_{+1}(T_3)_L$	−4	0	0	0	0	−9	−3	−1	−1	−1
Σ_2	$(\underline{++---};0^3)(0^5;\frac{+1}{4}\frac{+1}{4}\frac{-2}{4})'$	$\frac{+1}{3}$	$1(1_{\bar{3}})$	$1\mathbf{10}_{+1}(T_3)_L$	−4	0	0	0	0	−9	−3	−1	−1	−1
σ_1	$(0^5;\frac{-2}{3}\frac{-2}{3}\frac{-2}{3})(0^8)'$	$\frac{+1}{4}$	0	$2\cdot\mathbf{1}_0(T_4^0)$	−14	−8	−8	−8	0	0	0	−1	−1	−1
σ_2	$(0^5;\frac{-2}{3}\frac{+1}{3}\frac{+1}{3})(0^8)'$	0	$3(1_{\bar{1}})$	$3\cdot\mathbf{1}_0(T_4^0)$	−2	−8	+4	+4	0	0	0	−1	−1	−1
σ_3	$(0^5;\frac{1}{3}\frac{-2}{3}\frac{1}{3})(0^8)'$	0	$3(1_{\bar{1}})$	$3\cdot\mathbf{1}_0(T_4^0)$	−2	+4	−8	+4	0	0	0	−1	−1	−1
σ_4	$(0^5;\frac{1}{3}\frac{1}{3}\frac{-2}{3})(0^8)'$	0	$3(1_{\bar{1}})$	$3\cdot\mathbf{1}_0(T_4^0)$	−2	+4	+4	−8	0	0	0	−1	−1	−1
σ_5	$(0^5;010)(0^5;\frac{1}{2}\frac{-1}{2}0)'$	$\frac{+1}{2}$	0	$2\cdot\mathbf{1}_0(T_6)$	+4	0	+12	0	+12	0	0	−1	−1	−1
σ_6	$(0^5;001)(0^5;\frac{-1}{2}\frac{1}{2}0)'$	$\frac{+1}{2}$	0	$2\cdot\mathbf{1}_0(T_6)$	+4	0	0	+12	−12	0	0	−1	−1	−1
σ_7	$(0^5;0-10)(0^5;\frac{-1}{2}\frac{1}{2}0)'$	$\frac{+1}{2}$	0	$2\cdot\mathbf{1}_0(T_6)_R$	+8	0	+12	0	+12	0	0	−1	+1	−1
σ_8	$(0^5;00-1)(0^5;\frac{1}{2}\frac{-1}{2}0)'$	$\frac{+1}{2}$	0	$2\cdot\mathbf{1}_0(T_6)_R$	+8	0	0	+12	−12	0	0	−1	+1	−1
σ_{11}	$(0^5;\frac{-1}{2}\frac{-1}{2}\frac{-1}{2})(0^5;\frac{3}{4}\frac{-1}{4}\frac{-1}{2})'$	$\frac{+2}{3}$	$2(1_1+1_3,1_{\bar{1}}+1_{\bar{3}})$	$2\cdot\mathbf{1}_0(T_3)$	−9	−6	−6	−6	+12	−9	−3	+1	+1	−1
σ'_{11}	$(0^5;\frac{-1}{2}\frac{-1}{2}\frac{-1}{2})(0^5;\frac{3}{4}\frac{-1}{4}\frac{-1}{2})'$	0	$4(1_1+1_3,1_{\bar{1}}+1_{\bar{3}})$	$4\cdot\mathbf{1}_0(T_3)$	−9	−6	−6	−6	+12	−9	−3	−1	+1	+1
σ_{12}	$(0^5;\frac{-1}{2}\frac{1}{2}\frac{1}{2})(0^5;\frac{3}{4}\frac{-1}{4}\frac{-1}{2})'$	$\frac{+1}{3}$	$2(1_1+1_3,1_{\bar{1}}+1_{\bar{3}})$	$2\cdot\mathbf{1}_0(T_3)$	+3	−6	+6	+6	+12	−9	−3	+1	+1	−1
σ'_{12}	$(0^5;\frac{-1}{2}\frac{1}{2}\frac{1}{2})(0^5;\frac{3}{4}\frac{-1}{4}\frac{-1}{2})'$	$\frac{+2}{3}$	$2(1_1+1_3,1_{\bar{1}}+1_{\bar{3}})$	$2\cdot\mathbf{1}_0(T_3)$	+3	−6	+6	+6	+12	−9	−3	−1	+1	+1
σ_{13}	$(0^5;\frac{1}{2}\frac{1}{2}\frac{-1}{2})(0^5;\frac{-1}{4}\frac{3}{4}\frac{-1}{2})'$	$\frac{+1}{3}$	$2(1_1+1_3,1_{\bar{1}}+1_{\bar{3}})$	$2\cdot\mathbf{1}_0(T_3)$	+3	+6	+6	−6	−12	−9	−3	+1	+1	−1
σ'_{13}	$(0^5;\frac{1}{2}\frac{1}{2}\frac{-1}{2})(0^5;\frac{-1}{4}\frac{3}{4}\frac{-1}{2})'$	$\frac{+2}{3}$	$2(1_1+1_3,1_{\bar{1}}+1_{\bar{3}})$	$2\cdot\mathbf{1}_0(T_3)$	+3	+6	+6	−6	−12	−9	−3	−1	+1	+1
σ_{14}	$(0^5;\frac{1}{2}\frac{1}{2}\frac{-1}{2})(0^5;\frac{-1}{4}\frac{-1}{4}\frac{1}{2})'$	$\frac{+2}{3}$	$2(1_{\bar{1}})+1(1_{\bar{3}})$	$3\cdot\mathbf{1}_0(T_3)$	+7	+6	+6	−6	0	+9	+3	−1	+1	+1
σ_{15}	$(0^5;\frac{-1}{2}\frac{-1}{2}\frac{-1}{2})(0^5;\frac{+3}{4}\frac{-1}{4}\frac{-1}{2})'$	$\frac{+2}{3}$	$2(1_1+1_3,1_{\bar{1}}+1_{\bar{3}})$	$2\cdot\mathbf{1}_0(T_3)$	−9	−6	−6	−6	+12	−9	−3	+1	+1	−1
σ'_{15}	$(0^5;\frac{-1}{2}\frac{-1}{2}\frac{-1}{2})(0^5;\frac{+3}{4}\frac{-1}{4}\frac{-1}{2})'$	0	$2(1_1+1_3,1_{\bar{1}}+1_{\bar{3}})$	$4\cdot\mathbf{1}_0(T_3)$	−9	−6	−6	−6	+12	−9	−3	−1	+1	+1
σ_{16}	$(0^5;\frac{-1}{2}\frac{+1}{2}\frac{+1}{2})(0^5;\frac{+3}{4}\frac{-1}{4}\frac{-1}{2})'$	$\frac{+1}{3}$	$2(1_1+1_3,1_{\bar{1}}+1_{\bar{3}})$	$2\cdot\mathbf{1}_0(T_3)$	+3	−6	+6	+6	+12	−9	−3	+1	+1	−1
σ'_{16}	$(0^5;\frac{-1}{2}\frac{+1}{2}\frac{+1}{2})(0^5;\frac{+3}{4}\frac{-1}{4}\frac{-1}{2})'$	$\frac{+2}{3}$	$2(1_1+1_3,1_{\bar{1}}+1_{\bar{3}})$	$2\cdot\mathbf{1}_0(T_3)$	+3	−6	+6	+6	+12	−9	−3	−1	+1	+1
σ_{17}	$(0^5;\frac{+1}{2}\frac{+1}{2}\frac{-1}{2})(0^5;\frac{-1}{4}\frac{+3}{4}\frac{-1}{2})'$	$\frac{+1}{3}$	$2(1_1+1_3,1_{\bar{1}}+1_{\bar{3}})$	$2\cdot\mathbf{1}_0(T_3)$	+3	+6	+6	−6	−12	−9	−3	+1	+1	−1
σ'_{17}	$(0^5;\frac{+1}{2}\frac{+1}{2}\frac{-1}{2})(0^5;\frac{-1}{4}\frac{+3}{4}\frac{-1}{2})'$	$\frac{+2}{3}$	$2(1_1+1_3,1_{\bar{1}}+1_{\bar{3}})$	$2\cdot\mathbf{1}_0(T_3)$	+3	+6	+6	−6	−12	−9	−3	−1	+1	+1
σ_{18}	$(0^5;\frac{1}{2}\frac{+1}{2}\frac{-1}{2})(0^5;\frac{+3}{4}\frac{-1}{4}\frac{-1}{2})'$	$\frac{+2}{3}$	$2(1_{\bar{1}})+1(1_{\bar{3}})$	$2\cdot\mathbf{1}_0(T_3)$	+7	+6	+6	−6	0	+9	+3	−1	+1	+1
σ_{21}	$(0^5;\frac{-1}{6}\frac{-1}{6}\frac{-1}{6})(0^5;\frac{1}{4}\frac{1}{4}\frac{-1}{2})'$	0	$1(1_{\bar{1}})$	$\mathbf{1}_0(T_1^0)$	−3	−2	−2	−2	0	+9	+3	−1	−1	−1
σ_{22}	$(0^5;\frac{-5}{6}\frac{1}{6}\frac{1}{6})(0^5;\frac{1}{4}\frac{1}{4}\frac{-1}{2})'$	0	$1(1_{\bar{1}}+1_3)$	$\mathbf{1}_0(T_5^0)$	+1	−10	+2	+2	0	+9	+3	−1	+1	+1
σ_{23}	$(0^5;\frac{1}{6}\frac{-5}{6}\frac{1}{6})(0^5;\frac{1}{4}\frac{1}{4}\frac{-1}{2})'$	0	$1(1_{\bar{1}}+1_3)$	$\mathbf{1}_0(T_5^0)$	+1	−10	+2	+2	0	+9	+3	−1	+1	+1
σ_{24}	$(0^5;\frac{1}{6}\frac{1}{6}\frac{-5}{6})(0^5;\frac{1}{4}\frac{1}{4}\frac{-1}{2})'$	0	$1(1_{\bar{1}}+1_3)$	$\mathbf{1}_0(T_5^0)$	+1	−10	+2	+2	0	+9	+3	−1	+1	+1

For σ_7, we use $P=(0^5;+1,0,+1)(0^5;0,-1,+1)'$ and obtain $P\cdot V=\frac{-1}{12}$, and massless fields arise without oscillators:

$$\begin{array}{ccccc} s & N_i, & \tilde{s}\cdot\phi, & \Theta_6, & \text{Multiplicity} \\ (\oplus|-+-): & 0, & \frac{-1}{12}, & \frac{+6}{12}, & 2\cdot\sigma_{7,8} \\ (\ominus|---): & 0, & \frac{-5}{12}, & \frac{+10}{12}, & 0\cdot\sigma_{7,8} \end{array}. \tag{A9}$$

Here, $Q_{18,20,22}$ charges are as $(-1,-1,-1)$ for L-handed fields $\sigma_{5,6}$ and $(-1,+1,-1)$ for R-handed fields $\sigma_{7,8}$.

4. $\sigma_{11\text{–}18}$ from T_3 and T_9

In T_3, the multiplicity factor is given as

$$\tilde{\chi}(\theta^3,\theta^j)=\begin{cases} j= & 0,\ 1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7,\ 8,\ 9,\ 10,\ 11 \\ & 4,\ 1,\ 1,\ 4,\ 1,\ 1,\ 4,\ 1,\ 1,\ 4,\ \ 1,\ \ 1 \end{cases}. \tag{A10}$$

Since $3\phi=(\frac{1}{4},0,\frac{1}{4})$, the allowed chiralities are $s=(\oplus;\pm,-,\mp)$, and $s=(\ominus;\pm,+,\mp)$. We have massless conditions for right movers as $\tilde{s}\cdot\phi=0,-\frac{1}{3}$ for $\oplus$ (R-handed fields) and as $\tilde{s}\cdot\phi=0,+\frac{1}{3}$ for $\ominus$ (L-handed fields).

TABLE III. Allowed mode N_i for calculating Θ.

$\mathbf{Z}_N$	ϕ	χ	Allowed oscillating mode N_i
$\mathbf{Z}_{12-I}$	$(\frac{5}{12},\frac{4}{12},\frac{1}{12})$	3	$1_1,\ 1_{\bar 1}, 1_3, 1_{\bar 3}$
$\mathbf{Z}_{12-II}$	$(\frac{6}{12},\frac{5}{12},\frac{1}{12})$	4	$1_2,\ 1_{\bar 2},1_3,1_{\bar 3},2_2,2_{\bar 2},2_3,2_{\bar 3},$ $3_2,3_{\bar 2},3_3,3_{\bar 3}$
$\mathbf{Z}_{8-I}$	$(\frac{3}{8},\frac{2}{8},\frac{1}{8})$	4	$1_2,\ 1_{\bar 2}, 1_{1+3}, 1_{\bar 1+\bar 3}$
$\mathbf{Z}_{8-II}$	$(\frac{4}{8},\frac{3}{8},\frac{1}{8})$	8	$1_1,\ 1_{\bar 1}, 1_{2+3}, 1_{\bar 2+\bar 3}$
$\mathbf{Z}_{7}$	$(\frac{3}{7},\frac{2}{7},\frac{1}{7})$	7	0
$\mathbf{Z}_{6-1}$	$(\frac{2}{6},\frac{1}{6},\frac{1}{6})$	3	0
$\mathbf{Z}_{6-II}$	$(\frac{3}{6},\frac{2}{6},\frac{1}{6})$	12	$1_1,\ 1_{\bar 1}$
$\mathbf{Z}_{4}$	$(\frac{2}{4},\frac{1}{4},\frac{1}{4})$	16	$1_1,\ 1_{\bar 1}$
$\mathbf{Z}_{3}$	$(\frac{2}{3},\frac{1}{3},\frac{1}{3})$	27	0

TABLE IV. Two times the right-mover vacuum energy $2\tilde c$ of Ref. [32]. Typos in $\mathbf{Z}_2(6D)$ of Ref. [32] are corrected here.

$2\tilde c(k=)$	1	2	3	4	5	6
$\mathbf{Z}_{12-I}$	$-\frac{35}{24}$	$-\frac{3}{2}$	$-\frac{13}{8}$	$-\frac{4}{3}$	$-\frac{35}{24}$	$-\frac{3}{2}$
$\mathbf{Z}_{12-II}$	$-\frac{103}{72}$	$-\frac{31}{18}$	$-\frac{11}{8}$	$-\frac{14}{9}$	$-\frac{103}{72}$	$-\frac{3}{2}$
$\mathbf{Z}_{8-I}$	$-\frac{47}{32}$	$-\frac{11}{8}$	$-\frac{47}{32}$	$-\frac{3}{2}$		
$\mathbf{Z}_{8-II}$	$-\frac{45}{32}$	$-\frac{13}{8}$	$-\frac{45}{32}$	$-\frac{3}{2}$		
$\mathbf{Z}_{7}$	$-\frac{10}{7}$	$-\frac{10}{7}$	$-\frac{10}{7}$			
$\mathbf{Z}_{6-I}$	$-\frac{3}{2}$	$-\frac{4}{3}$	$-\frac{3}{2}$			
$\mathbf{Z}_{6-II}$	$-\frac{25}{18}$	$-\frac{28}{18}$	$-\frac{3}{2}$			
$\mathbf{Z}_{4}$	$-\frac{11}{8}$	$-\frac{3}{2}$				
$\mathbf{Z}_{3}$	$-\frac{4}{3}$					
$\mathbf{Z}_{2}(6D)$	$-\frac{3}{2}$					

$P=(0^8)(0^5,0,-1,+1)'$ of σ_{11} gives $P\cdot V=\frac{+1}{4}$, and we obtain

$$\begin{array}{ccccc} s & N_i, & \tilde s\cdot\phi, & \Theta_3, & \text{Multiplicity} \\ (\oplus|+--): & 0, & 0, & \frac{+6}{12}, & 0\cdot\sigma_{11} \\ (\oplus|--+): & 0, & \frac{-4}{12}, & \frac{+10}{12}, & 0\cdot\sigma_{11} \\ (\ominus|++-): & 0, & \frac{+4}{12}, & \frac{+2}{12}, & 0\cdot\sigma_{11} \\ (\ominus|-++): & 0, & 0, & \frac{+6}{12}, & 0\cdot\sigma_{11} \end{array}\ . \tag{A11}$$

To have massless modes, we need additional phases $\frac{\pm2}{12}$ and $\frac{\pm6}{12}$. But $\frac{\pm2}{12}$ cannot be used as shown in Table III, and we have the following:

$$\begin{array}{ccccc} s & N_i, & \tilde s\cdot\phi, & \Theta_3, & \text{Multiplicity} \\ (\oplus|+--): & \frac{\pm6}{12}, & 0, & \frac{+12,0}{12}, & [2(1_1+1_3),2(1_{\bar 1}+1_{\bar 3})]\cdot\sigma_{11,15} \\ (\oplus|--+): & \frac{\pm6}{12}, & \frac{-4}{12}, & \frac{+4,+4}{12}, & [1(1_1+1_3),1(1_{\bar 1}+1_{\bar 3})]\cdot\sigma_{11,15}\ . \\ (\ominus|++-): & \frac{\pm6}{12}, & \frac{+4}{12}, & \frac{+8,-4}{12}, & [1(1_1+1_3),1(1_{\bar 1}+1_{\bar 3})]\cdot\sigma_{11,15} \\ (\ominus|-++): & \frac{\pm6}{12}, & 0, & \frac{+12,0}{12}, & [2(1_1+1_3),2(1_{\bar 1}+1_{\bar 3})]\cdot\sigma_{11,15} \end{array} \tag{A12}$$

$P=(0^5;0,+1,+1)(0^5,0,-1,+1)'$ of σ_{12} gives $P\cdot V=\frac{-1}{12}$, and we obtain

$$\begin{array}{ccccc} s & N_i, & \tilde s\cdot\phi, & \Theta_3, & \text{Multiplicity} \\ (\oplus|+--): & \frac{\pm6}{12}, & 0, & \frac{+8,-4}{12}, & [1(1_1+1_3),1(1_{\bar 1}+1_{\bar 3})]\cdot\sigma_{12,13,16,17} \\ (\oplus|--+): & \frac{\pm6}{12}, & \frac{-4}{12}, & \frac{+12,0}{12}, & [1(1_1+1_3),2(1_{\bar 1}+1_{\bar 3})]\cdot\sigma_{12,13,16,17}\ . \\ (\ominus|++-): & \frac{\pm6}{12}, & \frac{+4}{12}, & \frac{+4,-8}{12}, & [1(1_1+1_3),1(1_{\bar 1}+1_{\bar 3})]\cdot\sigma_{12,13,16,17} \\ (\ominus|-++): & \frac{\pm6}{12}, & 0, & \frac{+8,-4}{12}, & [1(1_1+1_3),1(1_{\bar 1}+1_{\bar 3})]\cdot\sigma_{12,13,16,17} \end{array} \tag{A13}$$

$P=(0^5;+1,+1,0)(0^5,-1,-1,0)'$ of σ_{14} gives $P\cdot V=\frac{-5}{6}$ and $\Theta^0=P\cdot V+\frac{k}{12}=\frac{-7}{12}$. So, we have

TABLE V. Two times the right-mover vacuum energy $2c$ of Ref. [32]. Typos in $\mathbf{Z}_{6-II}$ of Ref. [32] are corrected here.

$2c(k=)$	1	2	3	4	5	6
$\mathbf{Z}_{12-I}$	$-\frac{11}{24}$	$-\frac{1}{2}$	$-\frac{5}{8}$	$-\frac{1}{3}$	$-\frac{11}{24}$	$-\frac{1}{2}$
$\mathbf{Z}_{12-II}$	$-\frac{31}{72}$	$-\frac{13}{18}$	$-\frac{3}{8}$	$-\frac{5}{9}$	$-\frac{31}{72}$	$-\frac{1}{2}$
$\mathbf{Z}_{8-I}$	$-\frac{15}{32}$	$-\frac{3}{8}$	$-\frac{15}{32}$	$-\frac{1}{2}$		
$\mathbf{Z}_{8-II}$	$-\frac{13}{32}$	$-\frac{5}{8}$	$-\frac{13}{32}$	$-\frac{1}{2}$		
$\mathbf{Z}_{7}$	$-\frac{3}{7}$	$-\frac{3}{7}$	$-\frac{3}{7}$			
$\mathbf{Z}_{6-I}$	$-\frac{1}{2}$	$-\frac{1}{3}$	$-\frac{1}{2}$			
$\mathbf{Z}_{6-II}$	$-\frac{7}{8}$	$-\frac{5}{9}$	$-\frac{1}{2}$			
$\mathbf{Z}_{4}$	$-\frac{3}{8}$	$-\frac{1}{2}$				
$\mathbf{Z}_{3}$	$-\frac{1}{3}$					
$\mathbf{Z}_{2}(6D)$	$-\frac{1}{2}$					

$$\begin{array}{ccccc} s & N_i, & \tilde{s}\cdot\phi, & \Theta_3, & \text{Multiplicity} \\ (\oplus|+--): & \frac{-5,-1}{12}, & 0, & \frac{-12,-8}{12}, & [2(1_{\bar{1}}),1(1_{\bar{3}})]\cdot\sigma_{14,18} \\ (\oplus|--+): & \frac{-1}{12}, & \frac{-4}{12}, & \frac{-12}{12}, & 2(1_{\bar{3}})\cdot\sigma_{14,18} \\ (\ominus|++-): & \times, & \frac{+4}{12}, & \times, & 0\cdot\sigma_{14,18} \\ (\ominus|-++): & \frac{-5,-1}{12}, & 0, & \frac{-12,-8}{12}, & [2(1_{\bar{1}}),1(1_{\bar{3}})]\cdot\sigma_{14,18} \end{array} . \tag{A14}$$

We list only L-handed fields in Table II.

5. $\sigma_{9–10}$ from T_2^0

We have

$$\tilde{\chi}(\theta^2,\theta^j) = \begin{cases} j= & 0,\ 1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7,\ 8,\ 9,\ 10,\ 11 \\ & 3,\ 3,\ 3,\ 3,\ 3,\ 3,\ 3,\ 3,\ 3,\ 3,\ \ 3,\ \ 3 \end{cases}. \tag{A15}$$

For T_2, the masslessness condition of the R sector is $(s+(2\phi))^2=\frac{1}{2}$, which is satisfied by $s=(\ominus;---)$. With $P=(0^8)(0^5;-1,0,+1)'$, we have $P\cdot V=\frac{1}{4}$ and $\Theta_2^0=\frac{-2}{12}$. We cannot make up $\frac{-3}{12}$ with the modes allowed in Table III.

6. $\sigma_{19–21}$ from T_1^0

For T_1, we have the following multiplicity factor:

$$\tilde{\chi}(\theta^1,\theta^j) = \begin{cases} j= & 0,\ 1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7,\ 8,\ 9,\ 10,\ 11 \\ & 3,\ 3,\ 3,\ 3,\ 3,\ 3,\ 3,\ 3,\ 3,\ 3,\ \ 3,\ \ 3 \end{cases}. \tag{A16}$$

For T_1^0, $(s+\phi)^2=\frac{11}{24}$ is satisfied by $s=(\ominus|---)$. With $P=(0^8)(0^5;-1,0,+1)'$ and $P\cdot V=\frac{1}{4}$, Eq. (1) gives

$$\begin{array}{ccccc} s & N_i, & \tilde{s}\cdot\phi, & \Theta_1^0, & \text{Multiplicity} \\ (\ominus|---): & 0, & \frac{-5}{12}, & \frac{+9}{12}, & 0\cdot\sigma_{19,20,21} \end{array}, \tag{A17}$$

and there is no massless field without oscillators. We cannot make up $\frac{+3}{12}$ with the modes allowed in Table III.

7. $\sigma_{22–24}$ from T_5^0

For T_5, we have the following multiplicity factor:

$$\tilde{\chi}(\theta^7,\theta^j) = \begin{cases} j= & 0,\ 1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7,\ 8,\ 9,\ 10,\ 11 \\ & 3,\ 3,\ 3,\ 3,\ 3,\ 3,\ 3,\ 3,\ 3,\ 3,\ \ 3,\ \ 3 \end{cases}. \tag{A18}$$

For T_5^0, $(s+5\phi)^2=\frac{11}{24}$ is satisfied by $s=(\oplus|+--)$ and $(\ominus|-++)$. With $P=(0^5;0,+1,+1)(0^5;-1,-1,0)'$, we have $P\cdot V=\frac{+2}{12}$. Equation (1) gives

$$\begin{array}{ccccc} s & N_i, & \tilde{s}\cdot(\phi), & \Theta_5^0, & \text{Multiplicity} \\ (\oplus|+--): & 0, & 0, & \frac{+4}{12}, & 0\cdot\sigma_{22,23,24}\ , \\ (\ominus|-++): & 0, & 0, & \frac{+4}{12}, & 0\cdot\sigma_{22,23,24} \end{array} \tag{A19}$$

and there is no massless field without oscillators. With the modes allowed in Table III, we obtain

$$\begin{array}{ccccc} s & N_i, & \tilde{s}\cdot(\phi), & \Theta_5^0, & \text{Multiplicity} \\ (\oplus|+--): & \frac{-4}{12}(1_{\bar{1}}+1_3), & 0, & 0, & 1\cdot\sigma_{22,23,24}\ , \\ (\ominus|-++): & \frac{-4}{12}(1_{\bar{1}}+1_3), & 0, & 0, & 1\cdot\sigma_{22,23,24} \end{array} \tag{A20}$$

and the charges of L-handed fields are

$$Q_{18,20,22}=(-1,+1,+1). \tag{A21}$$

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