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A note on vectorial AdS_5/CFT_4 duality for spin-j boundary theory

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Abstract: The vectorial holographic correspondences between higher-spin theories in AdS_5 and free vector models on the boundary are extended to the cases where the latter is described by free massless spin-j field. The dual higher-spin theory in the bulk does not include gravity and can only be defined on rigid AdS_5 background with S^4 boundary. We discuss various properties of these rather special higher-spin theories and calculate their one-loop free energies. We show that the result is proportional to the same quantity for spin-j doubleton treated as if it is a AdS_5 field. Finally, we consider even more special case where the boundary theory itself is given by an infinite tower of massless higher-spin fields.

Keywords: AdS-CFT Correspondence, Higher Spin Gravity, Higher Spin Symmetry

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1 Introduction

The holographic dualities where the boundary theories carry vector representations of $U(N)$ or $O(N)$ have been actively studied in recent years. These vector models are conjectured to be dual to Vasiliev's higher spin gravity [1, 2] in the bulk. The proposal was originally made for AdS dimensions four or higher [3–7] and then extended to AdS_3/CFT_2 in [8]. We refer the reader to the reviews [9, 10] for further details and progresses in this area. The vectorial dualities has simple classical bulk spectra which can be exactly identified. This special feature makes it possible to compute the one-loop partition function of the bulk theory by simply collecting the quantities for individual fields [11–14] and led to possible insights into the existence of multiparticle symmetries of the theory [15, 16]. In general dimensions, the one-loop partition function has been computed for the type A Vasiliev theory dual to free scalar on the boundary in [17–19]. In addition to the free scalar/type A duality, two other dualities can be considered in AdS_5/CFT_4 whose boundary theories are described by free fermion and Maxwell fields, respectively. The corresponding five-dimensional bulk theories are refered to as type B and C higher-spin theories. The spectific bulk description is known only for type-A case: it is described by the any-d Vasiliev equation [20]. About the construction of type-B higher-spin theory, see [21]. Even though we do not know exact formulations of type B and C theories, their perturbative field

contents can be identified from the correspondence, and the one-loop partition function was computed in [22]. See [23–29] for other discussions and more recent developments on the vectorial dualities, and [30, 31] for the extension of the one-loop partition function computation to free scalar and Yang-Mills adjoint models.

Motivated by these developments, in this paper, we consider a straightforward but in a sense rather exotic generalization of type A, B and C dualities in AdS_5/CFT_4. The generalization is based on the fact that the short representations of four-dimensional conformal symmetry $so(2,4)$ are the spin-j massless particles where j is any half-integer number. Clearly the cases of $j = 0, \frac{1}{2}$ and 1 give the type A, B and C models, whereas the other cases with $j \geq \frac{3}{2}$ have not been studied much and there are good reasons for that. See the appendix C of [28] for previous discussions on the point in the literature. Massless fields with spin $j \geq \frac{3}{2}$ can be defined in a conformally flat and Einstein background [32, 33]. Hence, one can consider S^4 as a consistent background, but another typical background $S^1 \times S^3$ is not compatible. Even though the spin-j theory on S^4 admits global conformal symmetry — its Hilbert space carry $so(2,4)$ unitary representation — it does not have a local energy momentum tensor. Moreover, the symmetry cannot be realized at the level of gauge potential but only curvature [34], hence for instance, the conformal symmetry cannot be realized in the standard Fronsdal formulation at least locally. Despite of these aspects, as we shall show, single-trace operators constructed from Fronsdal fields do carry proper representations of conformal algebra. This encourages us to consider the holographic dualities based on the vector models of massless spin-j field, whose bulk dual theory we shall refer to as 'type-j' higher-spin theory in analogy to type A, B and C. The type-j theories have another exotic property that they do not involve any massless spin-two field. Hence, these theories are non-gravitational higher-spin gauge theories in rigid AdS_5 background. This generalized version of AdS/CFT duality have been already considered in different contexts [35, 36]. We also analyze an even more exotic but interesting model where the boundary theory involves all massless higher-spin fields. Effectively the boundary theory is the free limit of four-dimensional Vasiliev theory defined now on the S^4.

The rest of the present paper is organized as follows. In section 2, we briefly review the spin-j doubleton representations, their character formulae and the single-trace operators. In section 3, we move to the AdS_5 side and discuss classical aspects of the bulk duals of the spin-j vector models, including their field content, cubic interactions and higher-spin symmetries. In section 4, we calculate the one-loop vacuum energy of type-j higher-spin theory and the bulk theory dual to all massless integral spins. We also comment on the Casimir energy. The final section 5 contains discussions.

2 Vector model of spin-j CFT_4

2.1 Massless spin-j theory

In four dimensions, free massless fields of any spin carry representations of conformal symmetry $so(2,4)$ and they correspond to the UIR $\mathcal{D}(j+1,(j,\pm j))$, where $\mathcal{D}(\Delta,(\ell_1,\ell_2))$ is the UIR having lowest energy Δ and its eigenvector(or tensor) transforms as (ℓ_1,ℓ_2) Young diagram representation of $so(4)$. In the notation of $su(2,2)$ they are denoted by

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$\mathcal{D}(j+1,[j,0])$ or $\mathcal{D}(j+1,[0,j])$ where $[j_+,j_-]$ is the $(2\,j_+ + 1)\times(2\,j_- + 1)$ dimensional representation of $su(2)\oplus su(2)\subset su(2,2)$. These representations are often referred as to spin-j *singleton* or *doubleton*. The sign of ℓ_2 or asymmetry in j_+ and j_- reflects that representations are chiral or anti-chiral whose field theoretical realization would require complexification of fields. Here, we consider parity-invariant combination,

$$\mathcal{S}_j := \mathcal{S}_{[j,0]} \oplus \mathcal{S}_{[0,j]}\,, \tag{2.1}$$

with

$$\mathcal{S}_{[j,0]} = \mathcal{D}(j+1,[j,0]) = \mathcal{D}(j+1,(j,j))\,, \quad \mathcal{S}_{[0,j]} = \mathcal{D}(j+1,[0,j]) = \mathcal{D}(j+1,(j,-j))\,. \tag{2.2}$$

The character of the spin-j doubleton is given by

$$\begin{aligned}\chi_{\mathcal{S}_{[j,0]}} &= \chi_{j+1,[j,0]} - \chi_{j+2,[j-\frac{1}{2},\frac{1}{2}]} + \chi_{j+3,[j-1,0]}\,,\\ \chi_{\mathcal{S}_{[0,j]}} &= \chi_{j+1,[0,j]} - \chi_{j+2,[\frac{1}{2},j-\frac{1}{2}]} + \chi_{j+3,[0,j-1]}\,,\end{aligned} \tag{2.3}$$

in terms of the character $\chi_{\Delta,[j_+,j_-]}$ for the Verma module $\mathcal{V}(\Delta,[j_+,j_-])$. It is given by

$$\chi_{\Delta,[j_+,j_-]}(q,x_+,x_-) = q^\Delta\, P(q,x_+,x_-)\,\chi_{j_+}(x_+)\,\chi_{j_-}(x_-)\,, \tag{2.4}$$

where χ_j is the $su(2)$ character in $(2j+1)$-dimensional representation:

$$\chi_j(x) = \frac{x^{j+\frac{1}{2}} - x^{-j-\frac{1}{2}}}{x^{\frac{1}{2}} - x^{-\frac{1}{2}}} = \frac{\sin(j+\frac{1}{2})\alpha}{\sin\frac{\alpha}{2}} \qquad [x = e^{i\,\alpha}]\,, \tag{2.5}$$

and $P(q,x_+,x_-)$ takes the form,

$$P(q,x_+,x_-) = \frac{1}{\left(1-q\,x_+^{\frac{1}{2}}\,x_-^{\frac{1}{2}}\right)\left(1-q\,x_+^{-\frac{1}{2}}\,x_-^{\frac{1}{2}}\right)\left(1-q\,x_+^{\frac{1}{2}}\,x_-^{-\frac{1}{2}}\right)\left(1-q\,x_+^{-\frac{1}{2}}\,x_-^{-\frac{1}{2}}\right)}. \tag{2.6}$$

Therefore, the characters for $\chi_{\mathcal{S}_{[j,0]}}$ and $\chi_{\mathcal{S}_{[0,j]}}$ read

$$\begin{aligned}\chi_{\mathcal{S}_{[j,0]}}(q,x_+,x_-) &= q^{j+1}\,P(q,x_+,x_-)\left[\chi_j(x_+) - q\,\chi_{j-\frac{1}{2}}(x_+)\left(x_-^{\frac{1}{2}} + x_-^{-\frac{1}{2}}\right) + q^2\,\chi_{j-1}(x_+)\right]\\ &= e^{-j\,\beta}\,\frac{\left(\cosh\beta\,\cos\frac{\alpha_+}{2} - \cos\frac{\alpha_-}{2}\right)\csc\frac{\alpha_+}{2}\,\sin j\alpha_+ \,+\sinh\beta\,\cos j\,\alpha_+}{2\left(\cosh\beta - \cos\frac{\alpha_+ + \alpha_-}{2}\right)\left(\cosh\beta - \cos\frac{\alpha_+ - \alpha_-}{2}\right)}\,,\\ \chi_{\mathcal{S}_{[0,j]}}(q,x_+,x_-) &= \chi_{\mathcal{S}_{[j,0]}}(q,x_-,x_+)\,,\end{aligned} \tag{2.7}$$

where $q = e^{-\beta}$ and $x_\pm = e^{i\,\alpha_\pm}$. Let us remark that when $j=0$ and $j=1/2$ the above character successfully reproduces that of scalar and spinor doubleton thanks to the identities $\chi_{-1/2} = 0$ and $\chi_{-1} = -1$. Combining $\chi_{\mathcal{S}_{[j,0]}}$ and $\chi_{\mathcal{S}_{[0,j]}}$, we obtain the character of parity-invariant representation as

$$\chi_{\mathcal{S}_j}(\beta,\alpha_+,\alpha_-) = \chi_{\mathcal{S}_{[j,0]}}(\beta,\alpha_+,\alpha_-) + \chi_{\mathcal{S}_{[0,j]}}(\beta,\alpha_+,\alpha_-)\,. \tag{2.8}$$

This character will play a key role in the subsequent analysis of this paper.

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When j is an integer, the boundary operator corresponding to $\mathcal{S}_j$ is the *curvature* tensor, $R_{a_1b_1,\ldots,a_jb_j}$, having (j,j) Young diagram representation. They are traceless — any contraction of two indices vanish — and subject to the Bianchi identity and the conservation condition,

$$\nabla_{[c}R_{a_1b_1],\ldots,a_\ell b_\ell} = 0\,, \qquad \nabla^{a_1}R_{a_1b_1,\ldots,a_\ell b_\ell} = 0\,, \tag{2.9}$$

where the background is given by a conformally flat Einstein metric. In the spinor index notation, the chiral and anti-chiral part of the traceless curvatures, namely Weyl tensors, read

$$C_{\alpha_1\cdots\alpha_{2j}}\,, \qquad C_{\dot\beta_1\cdots\dot\beta_{2j}}\,, \tag{2.10}$$

Here α_i's and $\dot\beta_i$'s take two values and they are fully symmetric. The Bianch identity (or conservation condition) (2.9) becomes the Bargmann-Wigner equations [37],

$$\nabla^{\alpha_1\dot\beta}C_{\alpha_1\cdots\alpha_{2j}} = 0\,, \qquad \nabla^{\alpha\dot\beta_1}C_{\dot\beta_1\cdots\beta_{2j}} = 0\,. \tag{2.11}$$

Even though the realization through the curvature tensor makes manifest the conformal symmetries, treating the curvature as fundamental fields is not compatible with conventional action principle.[1] In fact, the Bianchi identity can be solved in terms of gauge potential $\phi_{a_1\cdots a_j}$ as [39–41]. In flat space, the solution reads

$$R_{a_1b_1,\ldots,a_jb_j} = \partial_{[a_1}\cdots\partial_{[a_j}\,\phi_{b_1]\cdots b_j]}\,, \tag{2.12}$$

with gauge symmetry,

$$\delta\,\phi_{a_1\cdots a_j} = \partial_{(a_1}\,\xi_{a_2\cdots a_j)}\,. \tag{2.13}$$

This makes link the curvature formulation to the Fronsdal's one [42] having two-derivative equation,

$$F_{a_1\cdots a_j} = \Box\,\phi_{a_1\cdots a_j} - j\,\partial_{(a_1}\partial^b\,\phi_{a_2\cdots a_j)b} + \frac{j(j-1)}{2}\,\partial_{(a_1}\partial_{a_2}\,\phi_{a_3\cdots a_j)b}{}^b = 0\,. \tag{2.14}$$

For the consistency, the gauge parameter and fields have to satisfy traceless and double-traceless condition: $\xi^{a_1}{}_{a_1\cdots a_{j-1}} = 0$ and $\phi^{a_1a_2}{}_{a_1\cdots a_j} = 0$. The action giving (2.14) reads

$$S_{\rm Fronsdal} = \int d^4x\,\phi^{a_1\cdots a_j}(x)\,G_{a_1\cdots a_j}(x)\,, \tag{2.15}$$

where $G_{a_1\cdots a_j} = F_{a_1\cdots a_j} - \frac{1}{4}\,s(s-1)\,\eta_{(a_1a_2}\,F_{a_3\cdots a_j)}$ is the spin-j generalization of 'Einstein tensor'. The Fronsdal's formulation reduces to that of Maxwell and Fierz-Pauli for the spin one and two cases. For half-integer j, there exists a similar formulation making use of rank-j tensor spinor [43]. The Fronsdal formulation has many advantages such as more standard form of equation and existence of conventional action principle. However, on the contrary to the curvature formulation, it does not admit any local form of conformal transformation at the level of gauge potential $\phi_{a_1\cdots a_j}$ (see [34] and references therein for more details). Moreover, for spin $j \geq 3/2$, it can be defined only on a conformally flat and Einstein background [32, 33].

[1] In principle, there may exist non-conventional but manifestly conformal invariant actions involving curvature-like fields. See e.g. [38] for an attempt of relaxing locality to get a curvature action.

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2.2 Single-trace operators of the vector model CFT

U(N) vector model. The spin-j U(N) model is described by N copies of complex massless spin-j fields. In Fronsdal formulation, the spin-j action is given by

$$S_{\rm CFT} = \int d^4x \sum_{i=1}^{N} \bar{\phi}_i^{a_1\cdots a_j} G_{i\,a_1\cdots a_j} \,. \tag{2.16}$$

All U(N)-invariant single-trace operators are given by bilinear in the Weyl curvatures $C_{i\,\alpha_1\cdots\alpha_{2j}}$ and $C_{i\,\beta_1\cdots\beta_{2j}}$, and their complex conjugates. The operator content can be identified group theoretically and divided into three parts as

$$\mathcal{H}_{j,{\rm U}(N)} = \mathcal{H}^{\rm Sym}_{j,{\rm U}(N)} \oplus \mathcal{H}^{\rm MixSym}_{j,{\rm U}(N)} \oplus \mathcal{H}^{\rm Massive}_{j,{\rm U}(N)} \,. \tag{2.17}$$

The first part corresponds the cross product (in considering the fact that $\mathcal{S}_j = \mathcal{S}_{[j,0]} \oplus \mathcal{S}_{[0,j]}$),

$$\mathcal{H}^{\rm Sym}_{j,{\rm U}(N)} = (\mathcal{S}_{[j,0]} \otimes \mathcal{S}_{[0,j]}) \oplus (\mathcal{S}_{[0,j]} \otimes \mathcal{S}_{[j,0]}) = 2 \bigoplus_{s=2j}^{\infty} \mathcal{D}\left(s+2, \left[\frac{s}{2}, \frac{s}{2}\right]\right) \,, \tag{2.18}$$

and the second and third parts come from the sum of 'square',

$$\mathcal{H}^{\rm MixSym}_{j,{\rm U}(N)} \oplus \mathcal{H}^{\rm Massive}_{j,{\rm U}(N)} = (\mathcal{S}_{[j,0]} \otimes \mathcal{S}_{[j,0]}) \oplus (\mathcal{S}_{[0,j]} \otimes \mathcal{S}_{[0,j]}) \,, \tag{2.19}$$

with

$$\mathcal{H}^{\rm MixSym}_{j,{\rm U}(N)} = \bigoplus_{s=2j+1}^{\infty} \mathcal{D}\left(s+2, \left[\frac{s}{2}+j, \frac{s}{2}-j\right]_{\rm PI}\right) \,, \tag{2.20}$$

and

$$\mathcal{H}^{\rm Massive}_{j,{\rm U}(N)} = 2\,\mathcal{D}(2j+2,[0,0]) \oplus \bigoplus_{r=1}^{2j} \mathcal{D}(2j+2,[r,0]_{\rm PI}) \,. \tag{2.21}$$

Here, the subscript PI means the parity-invariant combination,

$$[j_+, j_-]_{\rm PI} = [j_+, j_-] \oplus [j_-, j_+] \,. \tag{2.22}$$

The UIRs in $\mathcal{H}^{\rm Massive}_{j,{\rm U}(N)}$ (2.19) are long and correspond to non-conserved currents (dual to massive fields in AdS). Their explicit forms are given by

$$\begin{aligned} \mathcal{O}_{\alpha_1\cdots\alpha_{2r}} &= C_{(\alpha_1\cdots\alpha_r|\gamma_1\cdots\gamma_{2j-r}} \bar{C}_{\alpha_{r+1}\cdots\alpha_{2r})}{}^{\gamma_1\cdots\gamma_{2j-r}} \,, \\ \mathcal{O}_{\dot\beta_1\cdots\dot\beta_{2r}} &= C_{(\dot\beta_1\cdots\dot\beta_r|\dot\delta_1\cdots\dot\delta_{2j-r}} \bar{C}_{\dot\beta_{r+1}\cdots\dot\beta_{2r})}{}^{\dot\delta_1\cdots\dot\delta_{2j-r}} \,, \end{aligned} \tag{2.23}$$

Here, the summation over the internal U(N) index should be understood. The UIRs of $\mathcal{H}^{\rm Sym}_{j,{\rm U}(N)}$ (2.18) are the symmetric rank $s \geq 2j$ conserved currents which are nothing but the traceless Bel-Robinson current [44, 45],

$$\begin{aligned} J_{\alpha_1\cdots\alpha_s}{}^{\dot\beta_1\cdots\dot\beta_s} = &\sum_{n=0}^{s-2j} (-1)^n \binom{s}{n} \binom{s}{2j+n} \times \\ &\times \partial^{(\dot\beta_1}_{(\alpha_1} \cdots \partial^{\dot\beta_n}_{\alpha_n} C_{\alpha_{n+1}\cdots\alpha_{n+2j}} \partial^{\dot\beta_{n+1}}_{\alpha_{2j+n+1}} \cdots \partial^{\dot\beta_{s-2j}}_{\alpha_s)} \bar{C}^{\dot\beta_{s-2j+1}\cdots\dot\beta_s)} \,. \end{aligned} \tag{2.24}$$

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The other symmetric conserved currents $J'_{\alpha_1\cdots\alpha_s\dot\beta_1\cdots\dot\beta_s}$ have the same form as above but C and $\bar C$ interchanged. Lastly, the UIRs in $\mathcal{H}^{\rm Sym}_{j,{\rm U}(N)}$ (2.20) are the mixed-symmetry conserved currents having the form,

$$J_{\alpha_1\cdots\alpha_{s+2j}}{}^{\dot\beta_1\cdots\dot\beta_{s-2j}} = \sum_{n=0}^{s-2j} A^{s,j}_n \times \\ \times \partial^{(\dot\beta_1}_{(\alpha_1}\cdots\partial^{\dot\beta_n}_{\alpha_n} C_{\alpha_{n+1}\cdots\alpha_{n+2j}} \partial^{\dot\beta_{n+1}}_{\alpha_{2j+n+1}}\cdots\partial^{\dot\beta_{s-2j})}_{\alpha_s} \bar C_{\alpha_{s+1}\cdots\alpha_{s+2j})}\,. \tag{2.25}$$

with $s\geq 2j+1$ and the coefficient $A^{s,j}_n$ to be determined by the conservation condition,

$$\partial^{\alpha_1\dot\beta_1} J_{\alpha_1\cdots\alpha_{s+2j}\dot\beta_1\cdots\dot\beta_{s-2j}} = 0\,. \tag{2.26}$$

The explicit derivation of these currents can be most conveniently done in the unfolded formulation. See [45–47] for the details.

O(N) vector model. When the massless spin-j fields are real, the model becomes $O(N)$, and its single-trace operator spectrum is given by

$$\mathcal{H}_{j,O(N)} = \mathcal{S}_j \otimes_{\rm cyc} \mathcal{S}_j = \mathcal{H}^{\rm Sym}_{j,O(N)} \oplus \mathcal{H}^{\rm MixSym}_{j,O(N)} \oplus \mathcal{H}^{\rm Massive}_{j,O(N)}\,, \tag{2.27}$$

where $\otimes_{\rm cyc}$ is the (anti-)symmetric tensor product for (half-)integer j and

$$\begin{aligned}
\mathcal{H}^{\rm Sym}_{j,O(N)} &= \bigoplus_{s=2j}^{\infty} \mathcal{D}\left(s+2, \left[\frac{s}{2}, \frac{s}{2}\right]\right)\,, \\
\mathcal{H}^{\rm MixSym}_{j,O(N)} &= \bigoplus_{\text{even } s\geq 2j+1} \mathcal{D}\left(s+2, \left[\frac{s}{2}+j, \frac{s}{2}-j\right]_{\rm PI}\right)\,, \\
\mathcal{H}^{\rm Massive}_{j,O(N)} &= 2\,\mathcal{D}(2j+2,[0,0]) \oplus \bigoplus_{2\leq\text{even } r\leq 2j} \mathcal{D}(2j+2,[r,0]_{\rm PI})\,.
\end{aligned} \tag{2.28}$$

Correspondingly, the operators $\mathcal{O}_{\alpha_1\cdots\alpha_{2r}}$ with odd r and the symmetric currents $J'_{\alpha_1\cdots\alpha_s\dot\beta_1\cdots\dot\beta_s}$ are projected out. Let us note that in both of U(N) and $O(N)$ model, they do not admit any conserved rank-two tensor if $j\geq 3/2$. Hence, there is no energy-momentum tensor in these models, implying that the AdS dual theory — type-j higher-spin theory — is non-gravitational.

3 Classical type-j higher spin theory in AdS$_5$

In this section, we consider the AdS$_5$ dual of the spin-j vector model CFT in four dimensions. They will be referred here as to type-j higher-spin theory. Starting from the discussion of their field content, we discuss about its underlying higher-spin algebra.

3.1 Field content

The field content of type-j higher-spin theories are defined to coincide with the single-trace operator spectrum of the dual theory, which have been reviewed in the previous section.

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Depending on whether the boundary theory is U(N) or $O(N)$, the bulk theory differs: non-minimal theory dual to the former and minimal one dual to the latter.

There appear three types of fields. The first one is the massive fields

$$\Pi_{\mu_1\cdots\mu_r,\nu_1\cdots\nu_r} \quad [r=0,(1),\ldots,(2j-1),2j]\,, \qquad \Pi'\,, \tag{3.1}$$

where minimal theory has only even r and there are two scalar fields Π and Π' with the same mass. The above fields are dual to the operator $\mathcal{O}_{\alpha_1\cdots\alpha_{2r}}$ and its conjugate (2.23), hence carrying $\mathcal{D}(2j+2,(r,r)_{\rm PI})$ representation. Their mass-squared value is given by

$$m^2 = (2j+2)(2j-2) - 2r\,. \tag{3.2}$$

These fields are the generalization of the scalar field in the spectrum of type A theory. Next, we have two types of massless fields: the symmetric one and the mixed-symmetry one

$$\begin{aligned} &\Phi_{\mu_1\cdots\mu_s}\,, \quad (\Phi'_{\mu_1\cdots\mu_s}) \quad [s=2j,2j+1,\ldots]\,,\\ &\Psi_{\mu_1\cdots\mu_s,\nu_1\cdots\nu_{2j}} \quad [s=(2j+1),2j+2,\ldots]\,. \end{aligned} \tag{3.3}$$

In the minimal theory, we have only one copy of symmetric fields and even s mixed symmetry fields. The above fields are all gauge fields, hence the theory has infinite amount of gauge symmetries analogously to the type A, B, C models. The gauge symmetry takes the form of

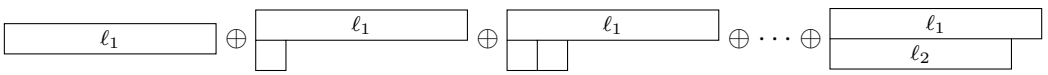
$$\begin{aligned} \delta\,\Phi_{\mu_1\cdots\mu_s} &= \nabla_{(\mu_1}\varepsilon_{\mu_2\cdots\mu_s)} + \mathcal{O}(g)\,,\\ \delta\,\Psi_{\mu_1\cdots\mu_s,\nu_1\cdots\nu_{2j}} &= \nabla_{(\mu_1}\xi_{\mu_2\cdots\mu_s),\mu_1\cdots\mu_{2j}} + \mathcal{O}(g)\,,\\ \delta\,\Pi_{\mu_1\cdots\mu_r,\nu_1\cdots\nu_r} &= \mathcal{O}(g)\,, \end{aligned} \tag{3.4}$$

where ∇_μ is the AdS covariant derivative and g is the coupling constant of the bulk theory, hence $\mathcal{O}(g)$ contains the field-dependent terms.

Apart from the type A theory, in all cases of type-j, the theory involves mixed-symmetry gauge fields $(s,2j)_{\rm PI}$. The degrees of freedom (DoF) of mixed-symmetry fields with definite parity — $(s,2j)$ or $(s,-2j)$ — can be easily counted as

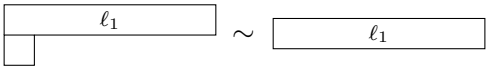
$$\dim\left(\pi^{O(4)}_{(s,\pm 2j)}\right) - \dim\left(\pi^{O(4)}_{(s-1,\pm 2j)}\right) = 2\,s+1\,, \tag{3.5}$$

where $\pi^{O(4)}_{(\ell_1,\ell_2)}$ indicates the $O(4)$ tensor representation corresponding to the Young diagram (ℓ_1,ℓ_2). Let us note that the DoF do not depend on the value of j (of type-j theory). Hence, all the mixed-symmetry gauge fields of definite parity have exactly same number of DoF, and in particular coincides with the totally-symmetric field. In terms of parity-invariant fields carrying $\mathcal{D}(s+2,(s,2j)_{\rm PI})$, the mixed-symmetry fields with $s \geq 2j+1$ have twice many DoF as the symmetric field. In the flat limit [48–52], the mixed-symmetry gauge field of (ℓ_1,ℓ_2) type decomposes into the massless helicity modes corresponding to the $O(3)$ — massless little group in five dimensions — Young diagram,



. (3.6)

Besides the first two helicity modes, all the rest having more than one boxes in the second row vanish identically hence do not propagate in five dimensions.[2] The first helicity mode is that of totally symmetric field and the second one can be dualized to give again totally symmetric DoF:



(3.7)

This accounts the twice many DoF of the mixed-symmetry gauge fields in AdS_5. Even though their DoF are related to those of the totally-symmetric field and they reduce to the latter in the flat limit, these mixed-symmetry gauge fields are genuinely different representations in AdS_5.

3.2 Cubic interactions

Let us consider the interaction nature of the type-j theory. Like the field content, the interaction structures are to match with the correlation functions of boundary operators through the Witten diagram. We shall mostly focus on the cubic interactions of these higher spin theories which are dual to the three-point correlators on the boundary. Cubic interactions of a gauge theory already tells many characteristic properties of the theory. The appendix C.1 of [28] contains several discussions on the structure of cubic interactions. Here, we provide additional discussions which, we hope, help to better understand the theory.

To make more clear our discussion, let us begin with a few comments on the generality. When a gauge theory has cubic interactions, then the field-dependent part of the gauge transformation (the $\mathcal{O}(g)$ part of (3.4)) should have terms linear in the field, so that the latter compensate the linearized gauge transformation of cubic vertices. Cubic interactions compatible with gauge symmetries can be classified into three different groups. The first group is what is called non-Abelian interactions, which is the part encoding the information of the underlying global symmetry which the theory is gauging. In other words, the bracket of the global symmetry generators are determined by these cubic vertices hence in the absence of these interactions, the generators would commute hence the algebra would remain Abelian. Typical such interactions are the self-couplings of Einstein gravity and Yang-Mills theory. Abelian cubic interactions are split again into two groups. The first one is 'deforming' interactions which requires that the gauge transformation of the relevant fields have linear term in field. Typical examples are the minimal coupling to the matter. Due to this coupling, matter field transforms under the gauge symmetry. The rest of the couplings are 'non-deforming' interactions whose presence does not necessitate any linear term in the gauge transformations. Non-minimal curvature interactions are the typical examples.

Higher spin algebra

Now considering back the type-j theory, all its non-Abelian interactions are in fact to be dictated by global symmetry, that is, higher spin algebra. With additional input that the boundary dual theory is the massless spin-j, we can identify the relevant higher spin algebra. It has two types of generators, which are respectively the solutions of the Killing

[2] However, they may become relevant for the would-be gauge modes surviving on the boundary.

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equations,

$$\nabla_{(\mu_1}\varepsilon_{\mu_2\cdots\mu_s)} = 0\,, \qquad \nabla_{(\mu_1}\xi_{\mu_2\cdots\mu_s),\mu_1\cdots\mu_{2j}} = 0\,. \tag{3.8}$$

As $O(2,4)$ tensors, they are characterized by

$$\begin{array}{|c|}\hline s-1 \\ \hline s-1 \\ \hline \end{array} \quad [s \geq 2j]\,, \qquad \begin{array}{|c|}\hline s-1 \\ \hline s-1 \\ \hline \end{array}\!\!\begin{array}{|c|}\hline 2\,j \\ \hline \end{array} \quad [s \geq 2j+1]\,, \tag{3.9}$$

and again depending on whether the theory is minimal (or not), the algebra contains one (or two) copy of $(s-1,s-1)$ generators and even s (and odd s) generators of mixed-symmetry type $(s-1,s-1,2j)$.

One of the simplest way to understand the type-j higher spin algebra, which we shall refer as to HS_j, is to view it as the maximal symmetry of the boundary theory, that is, the endomorphism algebra of the spin-j doubleton Hilbert space:

$$HS_j = \mathrm{End}(\mathcal{S}_j)\,. \tag{3.10}$$

Since $\mathcal{S}_j$ can be decomposed into $\mathcal{S}_j = \mathcal{S}_{[j,0]} \oplus \mathcal{S}_{[0,j]}$ where $\mathcal{S}_{[j,0]} = \mathcal{D}(j+1,[j,0])$, the type-$j$ higher spin algebra also splits into

$$HS_j = \mathrm{End}(\mathcal{S}_{[j,0]}) \oplus \mathrm{End}(\mathcal{S}_{[0,j]}) \oplus \mathrm{Hom}(\mathcal{S}_{[j,0]},\mathcal{S}_{[0,j]}) \oplus \mathrm{Hom}(\mathcal{S}_{[0,j]},\mathcal{S}_{[j,0]})\,. \tag{3.11}$$

The first two parts $\mathrm{End}(\mathcal{S}_{[j,0]}) \oplus \mathrm{End}(\mathcal{S}_{[0,j]})$ form an subalgebra, and they are isomorphic to each other,

$$hs_j = \mathrm{End}(\mathcal{S}_{[j,0]}) \simeq \mathrm{End}(\mathcal{S}_{[0,j]})\,. \tag{3.12}$$

In the minimal theory case, we have only one copy of hs_j as subalgebra. hs_j contains only the generators of symmetric spin $s \geq 2j$ and can be obtained from the universal enveloping algebra of $su(2,2)$ by quotienting it with the annihilator of spin-j module.[3] At the level of Lie algebra, one can consider j as a continuous parameter, say λ, then hs_j can be enhanced to $hs_\lambda(su(2,2))$ containing all the generators of symmetric spin $s \geq 1$. More explicitly, it is given as the coset of the tensor algebra generated by $su(2,2)$ generators by the equivalence relation [55],

$$L^{[a}{}_{b} \otimes L^{c]}{}_{d} + \delta^{[a}_{(b}\, L^{c]}{}_{d)} + \lambda\, \delta^{[a}_{[b}\, L^{c]}{}_{d]} + \frac{\lambda^2-1}{4}\, \delta^{[a}_{[b}\, \delta^{c]}_{d]} \sim 0\,, \tag{3.13}$$

where $L^a{}_b$ are the $su(2,2)$ generators. When the parameter λ takes an half-integer values j (so that the Lie-algebra representation uplifts to that of Lie-group), the algebra $hs_j(su(2,2))$ develops the ideal algebra hs_j (3.12). The coset algebra $hs_j(su(2,2))/hs_j$ is isomorphic to

$$u\left(\frac{2}{3}\,j\left(j^2+\frac{1}{2}\right), \frac{2}{3}\,j\left(j^2-1\right)\right) \quad \text{or} \quad u\left(\frac{2}{3}\,j\left(j^2-\frac{1}{4}\right), \frac{2}{3}\,j\left(j^2-\frac{1}{4}\right)\right), \tag{3.14}$$

[3]The annihilator of spin-j module is the maximal ideal of the universal enveloping algebra of $su(2,2)$, namely Joseph ideal [53]. The representations underlying Joseph ideal are called minimal representations, so the spin-j doubletons are the minimal representations of $su(2,2)$. Joseph ideals and minimal representations are extensively studied in mathematics community, and its discussion in the context of higher spin algebra can be found in [54].

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depending on whether $2j$ is even or odd.[4] The one-parameter family algebra has been first considered in higher spin context in [56], and more recent discussions can be found in [54, 57–60].

The type-j higher spin algebra HS_j can be realized by oscillators with commutation relation,

$$[\, Y^\alpha_A \,,\, Y^\beta_B \,]_\star = \eta_{AB}\, \epsilon^{\alpha\beta}\,, \qquad \{\, \theta^i_A \,,\, \theta^j_B \,\}_\star = \eta_{AB}\, \delta^{ij}\,, \tag{3.15}$$

where A, B, α, β and i, j are respectively the $O(2,4)$, $\mathrm{Sp}(2)$ and $O(2j)$ fundamental indices. Noticing that the $so(2,4)$, realized in these oscillators as

$$M_{AB} = Y^\alpha_A\, Y_{B\beta} + \theta^i_A\, \theta_{Bj}\,, \tag{3.16}$$

commutes with the $osp(2j|2)$,

$$K^{\alpha\beta} = Y^\alpha_A\, Y^{A\beta}\,, \qquad R^{ij} = \theta^i_A\, \theta^{Aj}\,, \qquad S^{\alpha i} = Y^\alpha_A\, \theta^{Ai}\,, \tag{3.17}$$

one can define a higher spin algebra from the Weyl-Clifford algebra $A_{6,12j}$ freely generated by Y^a_A and θ^i_A. More precisely, it is given by the quotient of the $osp(2j|2)$-centralizer by the $osp(2j|2)$ ideal. See [21, 61] for more details. This realization makes use of vector oscillators, but for $so(2,4)$ it is more convenient to use spinor oscillators based on $su(2,2)$ (see [3, 21, 62, 63] for related discussions) with commutation relations,

$$[\, y^a \,, \bar{y}_b \,]_\star = \delta^a_b\,, \qquad \{\, \theta^i_a \,,\, \bar{\theta}^b_j \,\}_\star = \delta^i_j\,. \tag{3.18}$$

The a, b are i, j now the $\mathrm{SU}(2,2)$ and $O(2j)$ fundamental indices. With these oscillators, the $su(2,2)$ generators are given by

$$L^a{}_b = y^a\, \bar{y}_b + \bar{\theta}^a_i\, \theta^i_b\,, \tag{3.19}$$

and they commute with $gl(2j|1)$ given by

$$K = y^a\, \bar{y}_b\,, \qquad R^i{}_j = \bar{\theta}^i_a\, \theta^a_i\,, \qquad S^i = y^a\, \theta^i_a \qquad \bar{S}_i = \bar{y}_a\, \bar{\theta}^a_i\,. \tag{3.20}$$

Hence, one can obtain a higher spin algebra as the $gl(2j|1)$-centralizer of Weyl-Clifford algebra $A_{4,8j}$ by the $gl(2j|1)$ ideal. However the higher spin algebras defined in these ways are not the type-j one, HS_j, because the former always include the $su(2,2)$ generators associated with the graviton in the bulk whereas the type-j theory does not have any symmetric

[4] In fact, the algebra $hs_\lambda(su(2,2))$ is just a particular case of sl_N higher spin algebra: in the same way one can define $hs_\lambda(sl_N)$ and show that the appearance of ideal and finite quotient algebra is a generic feature of this series of higher spin algebra: when $N(\lambda-1)/2$ takes an integer value M, $hs_\lambda(sl_N)$ develops an ideal consisting of generators with $r \geq M$ and the corresponding coset algebra becomes $gl_{\binom{N+M-1}{M}}$ consisting of the generators with $r = 0, 1, \ldots, M-1$. The most well-known example of this would be the $N = 2$ case where we get the half of the three dimensional higher spin algebra, often referred to as $hs(\lambda)$. Let us also remind the reader that the coset algebras appearing in this way starting from the right real form $su(1,1)$ and $su(2,2)$ respectively for three and six dimensional cases do not have the right reality structure to be interpreted as unitary theory of massless higher spin fields because they have alternating kinetic term sign. Instead the ideal parts have the right reality structure: its bilinear form has all same sign for each spin blocks (of course, in each spin block there are negative norm components, but what is important is the relative sign between different spins).

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spin fields lower than spin $2j$. Though not manifest in the oscillator form, these oscillator algebras are not semi-simple but contains an ideal corresponding to HS_j. By focusing on the subalgebra hs_j sector, the oscillator realization corresponds hence to $hs_j(su(2,2))$ rather than the ideal hs_j. See the forthcoming work [64] for the detailed analysis.

4 One-loop partition function of type-j theory

The partition function of a generic theory in AdS_5 can be expanded order by order in the number of loops as

$$\Gamma_{\mathcal{H}} = -\log\left[\int \prod_i \mathcal{D}\varphi_i \, \exp\left(-\frac{1}{g} S_{\mathcal{H}}[\varphi]\right)\right] = \frac{1}{g} S_{\mathcal{H}} + \Gamma^{(1)}_{\mathcal{H}} + \mathcal{O}(g)\,, \tag{4.1}$$

where the first term, $S_{\mathcal{H}}$, is the classical action evaluated with vacuum field configuration and the one-loop part $\Gamma^{(1)}_{\mathcal{H}}$ can be computed by evaluating the Gaussian path integral. Since $\Gamma^{(1)}_{\mathcal{H}}$ depends only on the field content $\mathcal{H}$, we can calculate the quantity for the individual fields and resum them for the total one-loop result. In [17, 18], this resummation has been carried out for Vasiliev theory on AdS_{d+1} with S^d boundary. The resummation requires additional regularization but it turns out that if we keep the UV regulator while resumming then the summation is convergent and the UV divergence also cancels in all the cases. Recently, in [30, 31], the authors of the current paper have introduced a method of doing this resummation with regulator turned on. In other words, it allows to calculate the spectral zeta function directly from the character underlying the field content of the theory. This method, referred to as Character Integral Representation of Zeta function (CIRZ), allows us to bypass the explicit identification of the field content and the subtlety arising in the resummation process.

4.1 One-loop vacuum energy in AdS_5 with S^4 boundary

The renormalized one-loop partition function (or free energy) will be referred to as *vacuum energy* and takes the form,

$$\Gamma^{(1)\,\mathrm{ren}}_{\mathcal{H}} = (-1)^F \gamma_{\mathcal{H}} \log R\,, \tag{4.2}$$

where R is the IR cut-off of the AdS_5 space, $\gamma_{\mathcal{H}}$ is a constant which depends on the theory and F is 0 for bosonic $\mathcal{H}$ and 1 for fermionic $\mathcal{H}$. Due to the $\log R$ term, for the entire AdS_5 with $R \to \infty$, the above quantity is divergent. Nevertheless, the way it diverges, namely the factor $\gamma_{\mathcal{H}}$, encodes the one-loop information of the theory. Using the CIRZ method [30, 31], we have shown that $\gamma_{\mathcal{H}}$ is given by the sum of three quantities,

$$\gamma_{\mathcal{H}} = \gamma_{\mathcal{H}|2} + \gamma_{\mathcal{H}|1} + \gamma_{\mathcal{H}|0}\,, \tag{4.3}$$

where each of $\gamma_{\mathcal{H}|n}$ is the contour integral,

$$\gamma_{\mathcal{H}|n} = -(-4)^n\, n! \oint \frac{d\beta}{2\pi i} \frac{f_{\mathcal{H}|n}(\beta)}{\beta^{2(n+1)}}\,, \tag{4.4}$$

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of a function given by the character of the theory:

$$
\begin{aligned}
f_{\mathcal{H}|2}(\beta) &= \frac{\sinh^4\frac{\beta}{2}}{2}\,\chi_{\mathcal{H}}\left(\beta,0,0\right),\\
f_{\mathcal{H}|1}(\beta) &= \sinh^2\frac{\beta}{2}\left[\frac{\sinh^2\frac{\beta}{2}}{3}-1-\sinh^2\frac{\beta}{2}\left(\partial_{\alpha_1}^2+\partial_{\alpha_2}^2\right)\right]\chi_{\mathcal{H}}\left(\beta,\alpha_+,\alpha_-\right)\Big|_{\alpha_\pm=0},\\
f_{\mathcal{H}|0}(\beta) &= \left[1+\frac{\sinh^2\frac{\beta}{2}\left(3-\sinh^2\frac{\beta}{2}\right)}{3}\left(\partial_{\alpha_1}^2+\partial_{\alpha_2}^2\right)\right.\\
&\qquad\left.-\frac{\sinh^4\frac{\beta}{2}}{3}\left(\partial_{\alpha_1}^4-12\,\partial_{\alpha_1}^2\partial_{\alpha_2}^2+\partial_{\alpha_2}^4\right)\right]\chi_{\mathcal{H}}\left(\beta,\alpha_+,\alpha_-\right)\Big|_{\alpha_\pm=0}.
\end{aligned}
\tag{4.5}
$$

In the AdS theories dual to vector models on the boundary, the functions $f_{\mathcal{H}|n}$ are analytic except for poles at the origin. In such cases, we can take the contour as the circle around the origin and use the residue theorem. In the end, the quantities $\gamma_{\mathcal{H}|n}$ are simply proportional to the β^{2n+1} Laurent coefficient of $f_{\mathcal{H}|n}$. Armed with the results (4.3 – 4.5), let us calculate the vacuum energy of the non-minimal and minimal type-j higher spin theory in AdS_5.

In the type A, B and C cases, the vacuum energies were found to be proportional to those of spin 0,1/2 and 1 doubletons treated as if they are bulk fields. In fact, these quantities also happen to coincide with the genuine one-loop partition function of boundary theory on S^4 once the standard UV/IR correspondence of AdS/CFT is used. Having in mind that this might generalize to the type-j theories, we first calculate the vacuum energy of spin-j doubleton treated as if it is AdS_5 field. Eventually, the latter quantity will be related to the vacuum energy of type-j theory.

Vacuum energy of spin-j doubleton. We first calculate the AdS_5 vacuum energy of spin-j doubleton even though it can be better described as massless spin-j field on S^4. In principle, we would need a AdS_5 description of spin-j doubleton to compute such a quantity, but the CIRZ method allows us to avoid this step. Practically, it is sufficient to know the character $\chi_{\mathcal{S}_j}$ of spin-j doubleton representation, which is already given in (2.8). We may therefore calculate first the functions $f_{\mathcal{S}_j|n}$ following the recipe (4.5) to obtain

$$
\begin{aligned}
f_{\mathcal{S}_j|2}(\beta) &= \frac{1}{8}\,e^{-j\beta}\left[2\,j\,(\cosh\beta-1)+\sinh\beta\right],\\
f_{\mathcal{S}_j|1}(\beta) &= \frac{1}{12}\,e^{-j\beta}\left[4\left(j^3+j\right)(\cosh\beta-1)+\left(6\,j^2+1\right)\sinh\beta\right],\\
f_{\mathcal{S}_j|0}(\beta) &= \frac{1}{6}\,e^{-j\beta}\,j\left[2\,j^2(j^2+2)\cosh\beta+(5\,j^3+j)\,\sinh\beta-2(j^2-1)^2\right].
\end{aligned}
\tag{4.6}
$$

We then simply Laurent expand the above to extract the relevant coefficient to get

$$
\gamma_{\mathcal{S}_j|2}=\frac{15\,j^4-1}{30}\,,\qquad \gamma_{\mathcal{S}_j|1}=\frac{6\,j^4-3\,j^2+1}{18}\,,\qquad \gamma_{\mathcal{S}_j|0}=\frac{j^4-j^2}{2}\,. \tag{4.7}
$$

Finally, summing these three numbers, we obtain the vacuum energy as

$$
\Gamma_{\mathcal{S}_j}^{(1)\,\mathrm{ren}}=(-1)^{2j}\,\frac{60\,j^4-30\,j^2+1}{45}\,\log R\,, \tag{4.8}
$$

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where the overall $(-1)^{2j}$ accounts for fermionic statistics of particles with half-integer j. We readily see that these expressions reduce to their previously computed counterparts for the special case of $j = 0, 1/2$ and 1:

$$\Gamma^{(1)\,\mathrm{ren}}_{\mathcal{S}_0} = \frac{1}{45}\,\log R\,, \qquad \Gamma^{(1)\,\mathrm{ren}}_{\mathcal{S}_{\frac{1}{2}}} = \frac{11}{180}\,\log R\,, \qquad \Gamma^{(1)\,\mathrm{ren}}_{\mathcal{S}_1} = \frac{31}{45}\,\log R\,. \tag{4.9}$$

As an side, the vacuum energies of the chiral and anti-chiral singletons $\mathcal{S}_{[j,0]}$ and $\mathcal{S}_{[0,j]}$ are the half of the vacuum energy of $\mathcal{S}_j$. We now turn to the holographic duals of vector models built from such boundary fields.

Non-minimal type-j theory. This theory is the AdS_5 dual of the U(N) vector model built from the complex spin-j doubletons. Again, to compute the one-loop vacuum energy by the CIRZ method, it is sufficient to identify the underlying character. For the non-minimal case, it is given by

$$\chi_{j,\text{non-min}}(\beta, \alpha_1, \alpha_2) = \chi_{\mathcal{S}_j}(\beta, \alpha_1, \alpha_2)^2\,. \tag{4.10}$$

We begin with the case of vector models built from the the parity invariant singleton. The expressions for the $f_{j,\text{non-min}|n}$ are quite lengthy and are hence omitted here. We directly write the expressions for the $\gamma_{j,\text{non-min}|n}$ defined in (4.4) as

$$\begin{aligned}
\gamma_{j,\text{non-min}|2} &= \frac{2}{105}\,n_j\,\left(72\,j^4 - 24\,j^2 + 1\right),\\
\gamma_{j,\text{non-min}|1} &= \frac{4}{315}\,n_j\,\left(60\,j^4 - 27\,j^2 + 2\right),\\
\gamma_{j,\text{non-min}|0} &= \frac{8}{15}\,n_j\,\left(j^4 - j^2\right),
\end{aligned} \tag{4.11}$$

where n_j is an integer given by

$$n_j = \frac{(2\,j-1)\,2\,j\,(2\,j+1)}{6}\,. \tag{4.12}$$

These may in turn be summed up to give

$$\Gamma^{(1)\,\mathrm{ren}}_{j,\text{non-min}} = \frac{2}{45}\,n_j\,\left(60\,j^4 - 30\,j^2 + 1\right)\log R = (-1)^{2j}\;n_j\,2\,\Gamma^{(1)\,\mathrm{ren}}_{\mathcal{S}_j}\,, \tag{4.13}$$

where we have used the expression (4.8) in the second step. One can see that the vacuum energy vanishes for $j = 0, 1/2$ reproducing the result of type A and B [18]. Further, for $j = 1$ we obtain the relation,

$$\Gamma^{(1)\,\mathrm{ren}}_{j=1,\text{non-min}} = 2\,\Gamma^{(1)\,\mathrm{ren}}_{\mathcal{S}_1}, \tag{4.14}$$

which reproduces the result of [22].

Let us note that the vacuum energy of the non-minimal theory can be split into the contribution of symmetric fields in $\mathcal{H}^{\mathrm{Sym}}_{j,\text{non-min}}$ (2.18) and that of the massless and massive mixed-symmetry fields in $\mathcal{H}^{\mathrm{MixSym}}_{j,\text{non-min}}$ (2.20) as

$$\Gamma^{(1)\,\mathrm{ren}}_{j,\text{non-min}|\mathrm{Sym}} = n_j\,\frac{48\,j^4 - 18\,j^2 + 1}{45}\,\log R\,, \tag{4.15}$$

$$\Gamma^{(1)\,\mathrm{ren}}_{j,\text{non-min}|\mathrm{MixSym}} + \Gamma^{(1)\,\mathrm{ren}}_{j,\text{non-min}|\mathrm{Massive}} = n_j\,\frac{72\,j^4 - 42\,j^2 + 1}{45}\,\log R\,. \tag{4.16}$$

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In principle, we can also consider the case where the boundary theory is given by only positive (or negative) helicity j, hence $\mathcal{S}_{[j,0]}$ (or $\mathcal{S}_{[0,j]}$). Then, the unitarity contrains the boundary fields to be complex: it can have U(N) symmetry but not $O(N)$. If we apply the vectorial duality to this chiral model, then the field content of the bulk (non-minimal) theory would have the spectrum

$$\mathcal{H}_{[j,0],\text{non-min}} = \mathcal{S}_{[j,0]} \otimes \mathcal{S}_{[0,j]} = \frac{1}{2}\,\mathcal{H}_{j,\text{non-min}|\text{Sym}}\,, \tag{4.17}$$

where $\mathcal{S}_{[0,j]}$ is for the complex conjugate of the chiral fields in $\mathcal{S}_{[j,0]}$. Therefore, its vacuum energy will be given by

$$\Gamma^{(1)\,\text{ren}}_{[j,0],\text{non-min}} = \frac{1}{2}\,\Gamma^{(1)\,\text{ren}}_{j,\text{non-min}|\text{Sym}} = n_j\,\frac{48\,j^4 - 18\,j^2 + 1}{90}\,\log R\,, \tag{4.18}$$

In contrast to the parity invariant model, the above vacuum energy is not proportional to that of $\mathcal{S}_{[j,0]}$,

$$\Gamma^{(1)\,\text{ren}}_{\mathcal{S}_{[j,0]}} = (-1)^{2j}\,\frac{60\,j^4 - 30\,j^2 + 1}{90}\,\log R\,, \tag{4.19}$$

with an integer factor except for the trivial cases of $j = 0$ and $\frac{1}{2}$ where the prefactor n_j itself vanishes.

Minimal type-j theory. We now turn to the minimal type-j theory, which is the putative dual of the $O(N)$ vector model built from spin-j doubleton. The character of minimal type-j theory is given by

$$\chi_{j,\text{min}}(\beta,\alpha_+,\alpha_-) = \frac{\chi_{\mathcal{S}_j}(\beta,\alpha_+,\alpha_-)^2 + (-1)^{2j}\,\chi_{\mathcal{S}_j}(2\,\beta,2\,\alpha_+,2\,\alpha_-)}{2}, \tag{4.20}$$

where $\mathcal{S}_j$ is the parity invariant spin-j singleton. The contribution to the vacuum energy from the first term is the half of the non-minimal theory, already computed in (4.13). We will work with the second term, $\chi_h(\beta,\alpha_+,\alpha_-) = (-1)^{2j}\,\frac{1}{2}\,\chi_{\mathcal{S}_j}(2\,\beta,2\,\alpha_+,2\,\alpha_-)$. Also, for brevity we directly write the expressions for the $\gamma_{h|n}$'s:

$$\begin{aligned}
\gamma_{h|2} &= \frac{(-1)^{2j}}{960}\left(480\,j^4 - 240\,j^2 + 13\right),\\
\gamma_{h|1} &= \frac{(-1)^{2j}}{576}\left(192\,j^4 - 96\,j^2 + 5\right),\\
\gamma_{h|0} &= (-1)^{2j}\,\frac{2\,j^4 - j^2}{4}\,.
\end{aligned} \tag{4.21}$$

These may in turn be summed to obtain the contribution of this term to the one-loop vacuum energy,

$$\Gamma^{(1)\,\text{ren}}_{h} = (-1)^{2j}\,\frac{1}{45}\left(60\,j^4 - 30\,j^2 + 1\right)\log R\,. \tag{4.22}$$

We observe that this exactly matches with the corresponding answer for $\mathcal{S}_j$. Finally, the one-loop vacuum energy of the minimal Type-j model may be expressed as

$$\Gamma^{(1)\,\text{ren}}_{j,\text{min}} = \frac{1}{2}\,\Gamma^{(1)\,\text{ren}}_{j,\text{non-min}} + \Gamma^{(1)\,\text{ren}}_{h} = \left[(-1)^{2j}\;n_j + 1\right]\Gamma^{(1)\,\text{ren}}_{\mathcal{S}_j}\,. \tag{4.23}$$

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Compared to the non-minimal theory case (4.13), we find that the relation between the vacuum energy of type-j theory and that of spin-j doubleton has an additional term '+1'. The latter contribution is responsible a shift of bulk coupling constant proposed in the lower j cases.

4.2 One-loop Casimir energy in AdS_5 with $S^1 \times S^3$ boundary

When the background is the thermal AdS_5 with $S^1 \times S^3$ boundary, the one-loop partition function has two contributions,

$$\Gamma_{\mathcal{H}}^{(1)\,\mathrm{ren}}(\beta) = \beta\,\mathcal{E}_{\mathcal{H}} + \hat{\mathcal{F}}_{\mathcal{H}}(\beta)\,, \tag{4.24}$$

where β is the radius of S^1 and the Casimir energy $\mathcal{E}_{\mathcal{H}}$ is given by a contour integral of the character,

$$\mathcal{E}_{\mathcal{H}} = -(-1)^F\,\frac{1}{2}\oint \frac{d\beta}{2\,\pi\,i\,\beta^2}\,\chi_{\mathcal{H}}(\beta,0,0)\,, \tag{4.25}$$

where F is the fermion number for $\mathcal{H}$. The contour can be taken as a circle around the origin and the residue theorem can be applied [31]. Thus, $\mathcal{E}_{\mathcal{H}}$ is given by $-\frac{1}{2}$ times the coefficient of the β^1 term in the Laurent expansion of $\chi_{\mathcal{H}}(\beta)$ at $\beta = 0$.

As we mentioned in Introduction, the boundary $S^1 \times S^3$ is not a compatible space for massless fields with spin $j \geq 3/2$ as the latter can be defined only in a conformally flat and Einstein background [32, 33], whereas $S^1 \times S^3$ is not Einstein. Nevertheless, one can consider the field content of type-j theory in the thermal AdS_5 and calculate its Casimir energy. Even though we expect the type-j holography in thermal AdS to be inconsistent, we present the calculation of its Casimir energy for completeness. Indeed, one can see that the result shows an important difference from the S^4 boundary case. This calculation has been already presented in [28] (while the present work has been almost completed), and various problematic features of the theory are discussed. For completeness, we reproduce the result here.

For the computation of the Casimir energy, it is again sufficient to know the (blind) character of the system, which is nothing but the $\alpha_1 = \alpha_2 = 0$ case of the character (2.8) and given by

$$\chi_{\mathcal{S}_j}(\beta,0,0) = e^{-j\,\beta}\,\frac{2\,j\,(\cosh\beta - 1) + \sinh\beta}{(\cosh\beta - 1)^2}\,. \tag{4.26}$$

From the above, it is sufficient to extract its β-linear Laurent coefficient to get the residue. In this way, we get

$$\mathcal{E}_{\mathcal{S}_j} = (-1)^{2j}\,\frac{30\,j^4 - 20\,j^2 + 1}{120}\,. \tag{4.27}$$

In principle, this can be interpreted as the Casimir energy of a single massless spin-j if we ignore the fact that the latter is not well-defined around $S^1 \times S^3$. We can proceed to the non-minimal type-j theory, whose character is simply the square of (4.26) as in (4.10):

$$\chi_{j,\text{non-min}}(\beta,0,0) = e^{-2j\,\beta}\,\frac{[2\,j\,(\cosh\beta - 1) + \sinh\beta]^2}{(\cosh\beta - 1)^4}\,, \tag{4.28}$$

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and from its β-linear coefficient, we conclude that

$$\mathcal{E}_{j,\text{non-min}} = n_j \, \frac{288\, j^4 - 208\, j^2 - 3}{420} \,. \tag{4.29}$$

The blind character of the minimal theory is defined as in (4.20), from which one can extract the Casimir energy as

$$\mathcal{E}_{j,\text{min}} = n_j \, \frac{288\, j^4 - 208\, j^2 - 3}{840} + (-1)^{2j} \, \frac{30\, j^4 - 20\, j^2 + 1}{120} \,. \tag{4.30}$$

We see that apart from the familiar 'low spin' cases of $j = 0$, $j = \frac{1}{2}$ [19], and $j = 1$ [22], there is no value of j for which the Casimir energy $\mathcal{E}_{j,\text{non-min}}$ or $\mathcal{E}_{j,\text{min}}$ is an integer times of $\mathcal{E}_{\mathcal{S}_j}$. This is to be contrasted to the results (4.13) and (4.23) in the S^4 boundary case.

4.3 Extension to type AZ theory

We consider now a more speculative model where the boundary theory itself is given by an infinite number of massless higher spin fields. More precisely, we take it as the collection of free massless spin-j fields with $j = 0, 1, 2, \ldots, \infty$. This spectrum coincides with that of non-minimal Vasiliev theory but now it is placed on the boundary (see [65] for related discussions). The boundary theory carries the reducible representation,

$$\mathcal{S}_{\text{AZ}} = \mathcal{S}_{[0,0]} \oplus \bigoplus_{j=1}^{\infty} \left(\mathcal{S}_{[j,0]} \oplus \mathcal{S}_{[0,j]} \right) , \tag{4.31}$$

and the corresponding character can be obtained by summing the spin-j characters (2.8) over all integers j as

$$\begin{aligned} \chi_{\mathcal{S}_{\text{AZ}}}(\beta, \alpha_1, \alpha_2) &= \chi_{\mathcal{S}_{[0,0]}} + \sum_{j=1}^{\infty} \left(\chi_{\mathcal{S}_{[j,0]}} + \chi_{\mathcal{S}_{[0,j]}} \right) \\ &= \frac{1 + \cos\alpha_1 + \cos\alpha_2 + \cosh\beta}{1 + \cos(2\alpha_1) + \cos(2\alpha_2) + \cosh(2\beta) - 4\, \cos\alpha_1 \cos\alpha_2 \, \cosh\beta} . \end{aligned} \tag{4.32}$$

Here, one needs to be careful in treating the $j = 0$ contribution as it is already parity invariant. The field-theory description is nothing but the collection of Fronsdal fields on the boundary:

$$S_{\text{CFT}} = \int d^4x \sum_{i=1}^{N} \sum_{j=0}^{\infty} \bar{\phi}_i^{a_1 \cdots a_j} \, G_{i\, a_1 \cdots a_j} \,. \tag{4.33}$$

Again, we can put these fields either in U(N) or $O(N)$ vector representation. In the latter case, the Fronsdal fields become real.

As the boundary fields carry a vector representation, all single trace operators are given as bilinear gauge invariants. Again, the content of the latter operators can be easily identified by taking tensor products of the representation $\mathcal{R}$ carried by the boundary theory. This operator content corresponds to the field content of the bulk theory, which we shall

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refer to as type AZ theory. When the boundary theory carries U(N) vector representation, its single trace operator spectrum is given by

$$\mathcal{H}_{\mathrm{AZ,U}(N)} = \mathcal{S}_{\mathrm{AZ}} \otimes \mathcal{S}_{\mathrm{AZ}} \,. \tag{4.34}$$

Since each $\mathcal{S}_{\mathrm{AZ}}$ contains all spins, its product contains all possible products [22, 66]:

$$\begin{aligned} S_{[j,0]} \otimes S_{[j',0]} &= \bigoplus_{k=|j-j'|}^{j+j'-1} \mathcal{D}(j+j'+2,[k,0]) \oplus \bigoplus_{k=0}^{\infty} \mathcal{D}\left(j+j'+2+k;\left[j+j'+\frac{k}{2},\frac{k}{2}\right]\right) , \\ S_{[j,0]} \otimes S_{[0,j']} &= \bigoplus_{k=0}^{\infty} \mathcal{D}\left(j+j'+2+k;\left[j+\frac{k}{2},j'+\frac{k}{2}\right]\right) . \end{aligned} \tag{4.35}$$

Here, we again split the spectrum of $\mathcal{H}_{\mathrm{AZ,U}(N)}$ into three parts:

$$\mathcal{H}_{\mathrm{AZ,U}(N)} = \mathcal{H}^{\mathrm{Sym}}_{\mathrm{AZ,U}(N)} \oplus \mathcal{H}^{\mathrm{MixSym}}_{\mathrm{AZ,U}(N)} \oplus \mathcal{H}^{\mathrm{Massive}}_{\mathrm{AZ,U}(N)} \,. \tag{4.36}$$

The first part contains symmetric conserved currents,

$$\mathcal{H}^{\mathrm{Sym}}_{\mathrm{AZ,U}(N)} = \bigoplus_{s=0}^{\infty} (2[s/2]+1)\, \mathcal{D}\left(s+2,\left[\frac{s}{2},\frac{s}{2}\right]\right) , \tag{4.37}$$

which are dual to massless symmetric fields in AdS_5. One can see that there are growing number of fields as the spin increases. The second and third parts contain the mixed-symmetry conserved currents dual to massless mixed symmetry fields and the long operators dual to non-gauge two-row fields, respectively. The precise multiplicities of these fields in the type AZ theory can be identified from (4.35). One can obtain analogously the single-trace operator content for $O(N)$ model which is dual to minimal type AZ theory in the bulk.

We now turn to computing the one-loop vacuum energy of type AZ theory in AdS_5. Following the CIRZ method, it is sufficient to identify the underlying character of the theory. In the case of the non-minimal theory, dual to the U(N) vector model, the character is given by

$$\chi_{\mathrm{AZ,non\text{-}min}}(\beta,\alpha_1,\alpha_2) = \left[\chi_{\mathcal{S}_{\mathrm{AZ}}}(\beta,\alpha_1,\alpha_2)\right]^2 \,. \tag{4.38}$$

The next step is to calculate $f_{\mathrm{AZ,non\text{-}min}|n}$ using (4.5). Explicit expressions encountered in the derivation are fairly lengthy, but the final form taken by these quantities is compact, and given by

$$\begin{aligned} f_{\mathrm{AZ,non\text{-}min}|2}(\beta) &= \frac{1}{128}\,(\cosh\beta+3)^2\,\mathrm{csch}^4\frac{\beta}{2} \,, \\ f_{\mathrm{AZ,non\text{-}min}|1}(\beta) &= \frac{1}{768}\,(\cosh\beta+3)(64\,\cosh\beta+\cosh(2\beta)+79)\,\mathrm{csch}^6\frac{\beta}{2} \,, \\ f_{\mathrm{AZ,non\text{-}min}|0}(\beta) &= \frac{1}{64}\,(115\,\cosh\beta+21\,\cosh(2\beta)+\cosh(3\beta)+103)\,\mathrm{csch}^8\frac{\beta}{2} \,. \end{aligned} \tag{4.39}$$

Then by series expanding about $\beta = 0$ and extracting the appropriate Laurent coefficients, we find that the coefficients $\gamma_{\mathrm{non\text{-}min}|n}$ (4.3) all vanish:

$$\gamma_{\mathrm{AZ,non\text{-}min}|2} = \gamma_{\mathrm{AZ,non\text{-}min}|1} = \gamma_{\mathrm{AZ,non\text{-}min}|0} = 0 \,. \tag{4.40}$$

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Therefore, the full one-loop vacuum energy of non-minimal type AZ theory vanishes:

$$\Gamma^{(1)\,\mathrm{ren}}_{\mathrm{AZ,non\text{-}min}} = 0\,. \tag{4.41}$$

If we decompose the above full vacuum energy into the contributions from symmetric and massless and massive mixed-symmetry fields as (4.36), each one-loop vacuum energy does not vanish:

$$\Gamma^{(1)\,\mathrm{ren}}_{\mathrm{AZ,non\text{-}min|Sym}} = \frac{1033}{90720}\,\log R\,, \quad \Gamma^{(1)\,\mathrm{ren}}_{\mathrm{AZ,non\text{-}min|MixSym}} + \Gamma^{(1)\,\mathrm{ren}}_{\mathrm{AZ,non\text{-}min|Massive}} = -\frac{1033}{90720}\,\log R\,, \tag{4.42}$$

but they cancel each other to give (4.41).

One can repeat the computation for the minimal theory, dual to $O(N)$ vector model, by using the character,

$$\chi_{\mathrm{AZ,min}}(\beta,\alpha_1,\alpha_2) = \frac{[\chi_{\mathcal{S}_{\mathrm{AZ}}}(\beta,\alpha_1,\alpha_2)]^2 + \chi_{\mathcal{S}_{\mathrm{AZ}}}(2\,\beta,2\,\alpha_1,2\,\alpha_2)}{2}\,. \tag{4.43}$$

After straightfoward calculations, we find again that all the $\gamma_{\mathrm{AZ,min}|n}$ coefficients vanish, hence so does the full one-loop vacuum of minimal type AZ theory:

$$\gamma_{\mathrm{AZ,min}|2} = \gamma_{\mathrm{AZ,min}|1} = \gamma_{\mathrm{AZ,min}|0} = 0\,, \qquad \Gamma^{(1)\,\mathrm{ren}}_{\mathrm{AZ,min}} = 0\,. \tag{4.44}$$

In fact, there is a simple reason for that all these $\gamma_{\mathcal{H}|n}$ coefficients vanish (hence the vacuum energy). It is because the underlying character $\chi_{\mathcal{S}_{\mathrm{AZ}}}$ is even in β:

$$\chi_{\mathcal{S}_{\mathrm{AZ}}}(-\beta,\alpha_1,\alpha_2) = \chi_{\mathcal{S}_{\mathrm{AZ}}}(\beta,\alpha_1,\alpha_2)\,. \tag{4.45}$$

This property is preserved in making the characters for non-minimal (4.38) and minimal (4.43) theories. Moreover, if the character is invariant in $\beta \leftrightarrow -\beta$, then the corresponding $f_{\mathcal{H}|n}(\beta)$'s are even in β. Since $\gamma_{\mathcal{H}|n}$ correspond to Laurent coefficients of odd β powers, they simply vanish for any even function $f_{\mathcal{H}|n}(\beta)$, hence as a result, the one-loop vacuum energy vanishes.

A priori, the collection of all massless integer spins cannot be better defined on $S^1 \times S^3$ than its individual spin part. Nevertheless, we can compute the Casimir energy of type AZ theory by placing its field content in the thermal AdS_5. The computation of Casimier energy requires only the blind character, which has again very simple form,

$$\chi_{\mathcal{S}_{\mathrm{AZ}}}(\beta,0,0) = \frac{1}{8}\,(\cosh\beta + 3)\,\mathrm{csch}^4\,\frac{\beta}{2}\,. \tag{4.46}$$

Like the full character itself, the above blind character is even in β and this property guarantees that Casimir energies for boundary theory, and non-minimal and minimal bulk theories are zero:

$$\mathcal{E}_{\mathcal{S}_{\mathrm{AZ}}} = 0\,, \quad \mathcal{E}_{\mathrm{AZ,non\text{-}min}} = 0\,, \quad \mathcal{E}_{\mathrm{AZ,min}} = 0\,. \tag{4.47}$$

It is not clear though whether this result suggests that the type AZ theory has any better chance to be well-defined in the thermal AdS_5.

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5 Discussion

Let us recapitulate the type-j one-loop results we obtained in this paper. The AdS_5 vacuum energies are

$$\Gamma^{(1)\,\mathrm{ren}}_{j,\text{non-min}} = (-1)^{2j}\, n_j\, 2\,\Gamma^{(1)\,\mathrm{ren}}_{\mathcal{S}_j}\,, \qquad \Gamma^{(1)\,\mathrm{ren}}_{j,\text{min}} = \left[(-1)^{2j}\, n_j + 1\right] \Gamma^{(1)\,\mathrm{ren}}_{\mathcal{S}_j}\,, \tag{5.1}$$

where n_j is an integer and $\Gamma^{(1)\,\mathrm{ren}}_{\mathcal{S}_j}$ is the vacuum energy of spin-j doubleton treated as if it is a AdS_5 field:

$$n_j = \frac{(2\,j-1)\,2\,j\,(2\,j+1)}{6}\,, \qquad \Gamma^{(1)\,\mathrm{ren}}_{\mathcal{S}_j} = (-1)^{2j}\,\frac{60\,j^4 - 30\,j^2 + 1}{45}\,\log R\,. \tag{5.2}$$

Let us first emphasize that it is a non-trivial fact that the one-loop result of the bulk theory is related to that of doubleton in this special fashion: this does not happen neither in (anti-)chiral model case nor in the Casimir energy computations. In the type A, B, C models with $j = 0, \frac{1}{2}, 1$, the vacuum energy $\Gamma^{(1)\,\mathrm{ren}}_{\mathcal{S}_j}$,

$$\left(\Gamma^{(1)\,\mathrm{ren}}_{\mathcal{S}_0}\,,\, \Gamma^{(1)\,\mathrm{ren}}_{\mathcal{S}_{\frac{1}{2}}}\,,\, \Gamma^{(1)\,\mathrm{ren}}_{\mathcal{S}_1}\right) = \left(\frac{1}{90}\,,\, \frac{11}{180}\,,\, \frac{31}{45}\right)\log R\,, \tag{5.3}$$

happens to be related to the free energy of the boundary theory on S^4 (with $N = 1$),

$$\left(F_0\,,\, F_{\frac{1}{2}}\,,\, F_1\right) = \left(\frac{1}{90}\,,\, \frac{11}{180}\,,\, \frac{31}{45}\right)\log \Lambda_{\mathrm{CFT}}\,, \tag{5.4}$$

which are proportional to the a-anomaly coefficients. This result, upon the IR/UV identification, $\log R = \log \Lambda_{\mathrm{CFT}}$, suggests that the inverse coupling constant of the bulk theory be related to N with a certain integer shift:

$$g^{-1}_{\text{non-min}} = N - (-1)^{2j}\, n_j\,, \qquad g^{-1}_{\text{min}} = N - (-1)^{2j}\, n_j - 1 \qquad \left[j = 0, \frac{1}{2}, 1\right]\,, \tag{5.5}$$

with n_j as defined in (5.2). Now the question is whether the vacuum energy $\Gamma^{(1)\,\mathrm{ren}}_{\mathcal{S}_j}$ can be analogously related to the S^4 free energy F_j of massless spin-j for higher values of j. If so, we would expect the the dictionary (5.5) to continue to hold for these theories as well. In the following, we examine this possibility.

Let us now turn to the free energy of massless integer spin j field over S^4 of unit radius. It is given by

$$F_j = \frac{1}{2}\left[\log\det\left(-\Box - \left(j^2 - 2j - 2\right)\right)_{(j)} - \log\det\left(-\Box - \left(j^2 - 1\right)\right)_{(j-1)}\right], \tag{5.6}$$

where $-\Box$ is the Laplace operator on S^4 with positive definite eigenvalues and the subscript (j) denotes that the operator acts on a rank j symmetric transverse traceless (STT) tensor. The multiplicities $d_n(j)$ and the eigenvalues of the kinetic operators are given by $\lambda_n(\Delta, j)$ [67]:

$$\begin{aligned} d_n(j) &= \dim(n+j, j) = \frac{1}{6}\,(2j+1)\,(n+1)\,(2j+2+n)\,(2j+3+2n)\,, \\ \lambda_n(\Delta, j) &= (n+j+\Delta)(n+j+3-\Delta)\,, \quad n \geq 0. \end{aligned} \tag{5.7}$$

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Focussing on the logarithmically divergent part, the S^4 free energy is given by

$$F_j = -\left(\zeta_{j+1,j}(0) - \zeta_{j+2,j-1}(0) - \eta_j\right) \log \Lambda_{\rm CFT}\,, \tag{5.8}$$

where η_j is the contribution due to zero modes and the zeta function $\zeta_{\Delta,j}$ for an irreducible field labeled by (Δ, j), is given by

$$\zeta_{\Delta,j}(z) = \sum_{n=0} d_n(j)\, \lambda_n(\Delta, j)^{-z}\,, \tag{5.9}$$

Here, we are interested only in the logarithmically divergent part of F_j, so we need to idenify $\zeta_{j+1,j}(0) - \zeta_{j+2,j-1}(0)$ and η_j. The former can be readily obtained as

$$\zeta_{j+1,j}(0) - \zeta_{j+2,j-1}(0) = \frac{15\, j^2 - 1}{45}\,. \tag{5.10}$$

Therefore, if the zero-mode contribution η_j is the desired quantity $\eta_j^{\rm desired}$:

$$\eta_j^{\rm desired} = \frac{j^2\,(4\,j^2-1)}{3} = j\, n_j\,, \tag{5.11}$$

then the free energy F_j would coincide with $\Gamma^{(1)\,\rm ren}_{\mathcal{S}_j}$, and the dictionary (5.5) would make a sense for any j.

In section 2 of [68], Tseytlin analyzed the zero modes for massless spin-j: they arise when we decompose a traceless rank-j tensor $\phi^{\rm T}_{(j)}$ into the traceless and transverse part $\phi^{\rm TT}_{(j)}$ and the rest:

$$\phi^{\rm T}_{\mu_1\cdots\mu_j} = \phi^{\rm TT}_{\mu_1\cdots\mu_j} + \Pi\left[\nabla_{(\mu_1}\, \xi^{\rm T}_{\mu_2\cdots\mu_j)}\right]. \tag{5.12}$$

The rest is the traceless part (Π is the traceless projector) of gradient of a traceless tensor $\xi^{\rm T}_{(j-1)}$. In order that this decomposition is one-to-one, $\xi^{\rm T}_{(j-1)}$ should not involve the solutions of $\Pi[\,\nabla\,\xi^{\rm T}_{(j-1)}\,] = 0$, namely zero modes. These precisely correspond to the spin-j conformal Killing tensors, whose number is $j^2(j+1)^2(2j+1)/12$. In the analysis of massless spin-j, we face this decomposition twice, once for the rank-j physical mode and the other time for the rank-$(j-1)$ gauge mode. Hence, according to [68], the total zero-mode contribution is

$$\eta_j^{\rm Tseytlin} = \frac{j^2\,(j+1)^2\,(2j+1)}{12} - \frac{(j-1)^2\,j^2\,(2j-1)}{12} = \frac{5\,j^4 + j^2}{6}\,, \tag{5.13}$$

and consequently, the free energy with the zero-mode contribution (5.13),

$$F_j = \frac{75j^4 - 15j^2 + 2}{90}\, \log \Lambda_{\rm CFT}\,, \tag{5.14}$$

differs from the vacuum energy $\Gamma^{(1)\,\rm ren}_{\mathcal{S}_j}$ (5.2). Let us make one curious observation: if we neglect the $\gamma_{\mathcal{H}|0}$ term (4.7) — which is non-vanishing only for two-row Young diagram tensors — in the $\Gamma^{(1)\,\rm ren}_{\mathcal{S}_j}$ computation, we would get the result (5.14).

Differently from the renormalized quantity, the logarithmically divergent part of free energy may depend on field-theoretical realizations [69]. Therefore, in principle, there

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might be other formulation of massless spin-j than the Fronsdal one with the free energy given by $\Gamma^{(1)\,\mathrm{ren}}_{\mathcal{S}_j}$. For instance, if one considers the Maxwell-like formulation [70–72] where the gauge field is traceless and the gauge parameter is traceless and traceverse, we need to consider the decomposition,

$$\phi^{\mathrm{T}}_{\mu_1\cdots\mu_j} = \phi^{\mathrm{TT}}_{\mu_1\cdots\mu_j} + \nabla_{(\mu_1}\,\xi^{\mathrm{TT}}_{\mu_2\cdots\mu_j)}\,, \tag{5.15}$$

only once, hence the zero modes only appear here. They are the solutions of $\nabla\,\xi^{\mathrm{TT}}_{(j-1)} = 0$, namely spin-$j$ Killing tensors. These also coincide with the zero modes of the gauge sector $(\Delta = j+2, s = j-1)$ in (5.7) and they correspond to $n = 0$ modes with the multiplicity,

$$d_0(j-1) = \frac{j\,(4\,j^2-1)}{3} = n_j\,. \tag{5.16}$$

Still this number is not sufficient to give the desired contribution (5.11), but misses a factor of j. Hopefully, there may be yet another formulation of massless spin-j field which gives the desired zero modes.

As a final note, we also point out that the above zero mode analysis may be affected by their non-trivial scalings, which have been pointed out in [73–78]. In the appendix, we discuss how the inclusion of such scalings may alter possibly the results of one-loop free energy computations on S^4 so as to match the result with $\Gamma^{(1)\,\mathrm{ren}}_{\mathcal{S}_j}$.

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A Free energy of Fronsdal fields on S^4

We begin with a discussion of a generality that has been reviewed many times in the context of computing quantum corrections in AdS spaces [73–80]. Given the one-loop determinant of a theory, for definiteness taken to be in four dimensions, we have

$$\mathcal{Z}_{1-\ell} = \int [\mathcal{D}\phi]\, e^{-\int d^4x\,\phi\,\mathcal{O}\,\phi} = \left(\det{}'\mathcal{O}\right)^{-1/2} \times \mathcal{Z}_{\mathrm{zero}}, \tag{A.1}$$

where by $\det'$ we mean the determinant evaluated over the non-zero modes of $\mathcal{O}$ and $\mathcal{Z}_{\mathrm{zero}}$ is the path integral over the zero modes of $\mathcal{O}$. Next, we assume that the theory is defined on a manifold with a length scale a and we are computing the part of the free energy $\ln \mathcal{Z}_{1-\mathrm{loop}}$ proportional to $\ln a$. It was observed in [73, 74] that the zero mode associated with a given field ϕ in the spectrum of the theory may scale with a differently compared

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to the non zero modes. In particular, suppose that the contribution of a single zero mode of the field ϕ scales as

$$\mathcal{Z}_{\text{zero}} \sim a^{\beta_\phi}\, \mathcal{Z}_o, \tag{A.2}$$

where $\mathcal{Z}_o$ does not scale with a. Then if the field ϕ has n_ϕ number of zero modes then

$$F = -\ln \mathcal{Z}_{1-\ell} = -\left(\tilde{\zeta}\,(0) + n_\phi\, \beta_\phi\right) \ln a + \dots, \tag{A.3}$$

where $\dots$ denote terms that are not proportional to $\ln a$, and $\tilde{\zeta}$ is the zeta function evaluated over the non-zero modes of the field ϕ. In practice, it is often easier to compute the zeta function over all possible modes, denoted by ζ including zero modes, and use the equivalent expression[5] [73, 74]

$$F = -\ln \mathcal{Z}_{1-\ell} = -\left(\zeta\,(0) + n_\phi\,(\beta_\phi - 1)\right) \ln a + \dots. \tag{A.4}$$

Note that generically, β_ϕ is not equal to one. We refer the reader to [74, 76, 78] for explicit examples in various dimensions.

The eigenvalues and degeneracies of the kinetic operators in (5.6) have been enumerated in (5.7). Using those expressions we see that for the expression (5.6) it is the ghost determinant acting over spin $j-1$ STT tensors which has a zero eigenvalue for the lowest quantum number $n = 0$. The number of such zero modes is given (5.16). Physically, since the zero modes belong to the ghost path integral, they should be interpreted as the part of the gauge symmetry $\delta\phi_{(j)} = \nabla\, \xi_{(j-1)}$ that remains even after fixing gauge [78, 81]. Next, we turn to the computation of the free energy from the one-loop determinants (5.6) and get (5.8) with

$$\eta_j = n_j\ (\beta_j - 1)\,, \tag{A.5}$$

where n_j is exhibited in (5.16). We now turn to the computation of β_j following the analysis of [73–78, 82] who worked in Anti-de Sitter space. We start with the normalization for the path integral for the ghost determinant for a spin-j Fronsdal field on S^4, i.e.

$$\int \left[\mathcal{D}\xi_{\mu_2\dots\mu_j}\right] \exp\left[-\int d^4x\, \sqrt{g}\, g^{\mu_2\nu_2}\ \dots\ g^{\mu_j\nu_j}\, \xi_{\mu_2\dots\mu_j}\, \xi_{\nu_2\dots\nu_j}\right] = 1. \tag{A.6}$$

Next, if the radius of the S^4 is a then the metric g scales as $g_{\mu\nu} = a^2\, g^{(0)}_{\mu\nu}$. As a result, the normalization becomes

$$\int \left[\mathcal{D}\xi_{\mu_2\dots\mu_j}\right] \exp\left[-a^{4-2(j-1)} \int d^4x\, \sqrt{g^{(0)}}\, g^{(0)\mu_2\nu_2}\ \dots\ g^{(0)\mu_j\nu_j}\, \xi_{\mu_2\dots\mu_j}\, \xi_{\nu_2\dots\nu_j}\right] = 1. \tag{A.7}$$

Hence the correctly normalized integration measure is given by

$$\left[\mathcal{D}\xi_{\mu_2\dots\mu_j}\right] = \prod_{x,(\mu_1\dots\mu_j)} d\left(a^{3-j}\xi_{\mu_2\dots\mu_j}\right). \tag{A.8}$$

As we commented above, the zero modes of the ghost operator are associated with gauge transformations of the Fronsdal field $\delta\phi_{\mu_1\dots\mu_j} = \nabla_{(\mu_1}\xi_{\mu_2\dots\mu_j)}$, and in particular are some

[5] In particular, $\tilde{\zeta}\,(0) = \zeta\,(0) - n_\phi$.

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specific rank $j-1$ STT tensors. It now remains to determine the scaling of the fields $\xi_{\mu_2\ldots\mu_j}$. The procedure is to determine the tensorial properties of the field ξ which generate an a-independent symmetry algebra, and take those tensors to scale as a^0. Then, by using the metric $g_{\mu\nu}$ we may infer the scaling of $\xi_{\mu_2\ldots\mu_j}$. For this we use the fact that these are linearized transformations of a non-linear symmetry algebra which contains the isometry of S^4 as a subalgebra, and that the a-independent S^4-isometry is generated by the ξ^μ. Hence, for the higher-spin algebra to be independent of a, we must generate it by $\xi^{\mu_2\cdots\mu_j}$, which we therefore take to scale as a^0. Then it follows that $\xi_{\mu_2\ldots\mu_j}$ scales as $a^{2(j-1)}$. Therefore, the scaling of each zero mode is given by $a^{2(j-1)}\times a^{3-j}=a^{j+1}$. That is, over a single zero mode

$$\mathcal{Z}_{\text{zero}}\sim a^{j+1}\mathcal{Z}_o. \tag{A.9}$$

We therefore see, on comparing with (A.2) that

$$\beta_j=j+1. \tag{A.10}$$

Plugging the above in (A.5), we get the desired zero mode contribution (5.11).

However we add a word of caution, though hyperboloids and spheres share many common features there are also potential subtle differences for example in the zero mode spectrum. For these reasons, the computations in this appendix might be regarded as encouraging, but still preliminary and are currently under further exploration in a related context [83].

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