

Clockwork graviton contributions to muon $g - 2$

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The clockwork mechanism for gravity introduces a tower of massive graviton modes, *clockwork gravitons*, with a very compressed mass spectrum, whose interaction strengths are much stronger than those of massless gravitons. In this work, we compute the lowest order contributions of the clockwork gravitons to the anomalous magnetic moment, $g - 2$, of muon in the context of an extra dimensional model with a five-dimensional Planck mass, M_5 . We find that the total contributions are rather insensitive to the detailed model parameters and are determined mostly by the value of M_5 . To account for the current muon $g - 2$ anomaly, M_5 should be around 0.2 TeV, and the size of the extra dimension has to be quite large, $l_5 \gtrsim 10^{-7}$ m. For $M_5 \gtrsim 1$ TeV, the clockwork graviton contributions are too small to explain the current muon $g - 2$ anomaly. We also compare the clockwork graviton contributions with other extra dimensional models such as Randall-Sundrum models or large extra dimensional models. We find that the leading contributions in the small curvature limit are universal, but the cutoff-independent subleading contributions vary for different background geometries and the clockwork geometry gives the smallest subleading contributions.

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I. INTRODUCTION

After the Higgs boson was discovered to complete the standard model (SM) of particle physics, there has been an intense search for new particles at the Large Hadron Collider (LHC) that probes TeV energy scales. At LHC Run 2 the mass limit of new particles has been pushed up above 1 TeV [1–3], putting most models for physics beyond SM in great tension with their naturalness criterion, advocated by 't Hooft [4].

Recently an interesting mechanism, called clockwork (CW), was proposed to generate naturally an exponential hierarchy for a given theory with multicomponents of fields [5–7]. Giudice and McCullough then proposed a clockwork solution to the electroweak hierarchy problem [8], which exhibits rather rich structure and phenomenology [9]. The clockwork scenario addresses, similar to other extra dimensional scenarios, the hierarchy problem by assuming that the fundamental scale of the theory is not much higher than

the electroweak scale. The simplest clockwork model can be constructed with a set of four-dimensional (4D) theories at $N + 1$ sites in a theory space with asymmetric couplings of link fields between nearby sites so that the zero mode of link fields is highly localized at a single site, while all SM particles reside at a site that has the least overlap with the zero mode to suppress its coupling to SM particles. If one identifies this zero mode as the 4D massless graviton,¹ the clockwork setup solves the naturalness problem associated with the weak scale and becomes in the large N limit the linear dilaton model [12,13] of five-dimensional (5D) little string theory [14,15]. The clockwork theory then predicts an infinite tower of massive Kaluza-Klein (KK) gravitons, with a unique spectrum, that couple to SM particles in a specific way. Especially the low-lying states of clockwork KK gravitons exhibit rather interesting signatures at colliders, compared to other models of extra dimensions, as studied in detail [9]. In this paper we study the contributions of the clockwork gravitons to the anomalous magnetic moments of muon and constrain the parameters of the clockwork gravitons, which will be complementary to collider searches. We find that the

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¹The clockwork symmetry between two neighboring sites is shown to be respected only for the Abelian case [10]. But, here we take the effective field theory approach, as emphasized in [11], for the clockwork gravity that is nothing but the discretized linear-dilaton model [12,13].

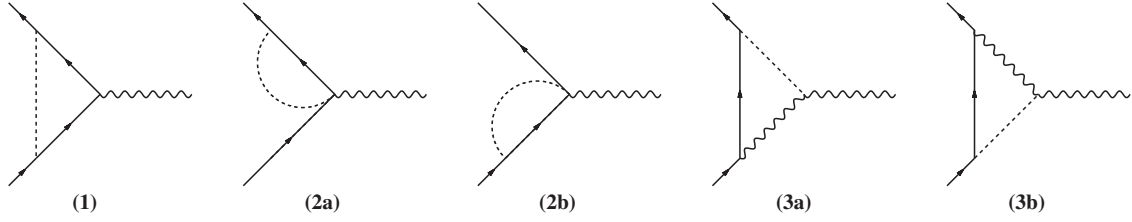


FIG. 1. Graviton contributions at one loop to the anomalous magnetic moment of muon. The dashed lines denote gravitons and the curly lines are photons.

intrinsic scale, M_5 , of the clockwork graviton has to be around 0.2 TeV or higher to be compatible with the current muon $g-2$ anomaly.

It is well known that the standard model estimation of the anomalous magnetic moment of muon has quite a significant deviation from the experiments, which thus provides interesting constraints for new physics, if it were to explain the deviation.² The current deviation between the experimental value, obtained at the Brookhaven National Laboratory [16], and the SM estimate, based on e^+e^- hadronic cross sections [17], is found to be

$$\Delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{th}} = (290 \pm 90) \times 10^{-11}, \quad (1)$$

where $a_\mu \equiv (g-2)/2$ and g is the gyromagnetic ratio of the magnetic moment of muon. The current deviation corresponds to about 3.2σ . An improved muon ($g-2$) experiment at the Fermilab is about to take data, aiming to achieve a precision of 0.14 ppm [18,19], which will then move the current deviation, if persistent, to more than 5σ .

II. MASSIVE GRAVITON CONTRIBUTIONS TO MUON $g-2$

While a massless spin-2 particle that couples to the energy-momentum tensor necessarily leads at low energy to Einstein's general relativity, a consistent description of massive gravitons, respecting the general coordinate invariance, has been found only recently [20]. To describe a (massive) graviton in a flat spacetime, we write the metric as

$$g_{\mu\nu} = \eta_{\mu\nu} + 2\kappa h_{\mu\nu}, \quad (2)$$

where $\eta_{\mu\nu}$ is the Minkowski metric for the flat spacetime and $h_{\mu\nu}$ is the graviton field with coupling $\kappa \equiv \sqrt{8\pi G}$ for Einstein gravity with Newton's gravitational constant G . Under a general coordinate transformation, $x^\mu \mapsto x^\mu + \xi^\mu(x)$, the graviton field transforms as

$$h_{\mu\nu} \mapsto h_{\mu\nu} - \frac{1}{2\kappa} (\partial_\mu \xi_\nu + \partial_\nu \xi_\mu). \quad (3)$$

²The muon $g-2$ would also provide interesting constraints on new physics even if there were not any deviation.

Fixing the above gauge degrees of freedom, the massive graviton propagator of mass M in D -dimensional spacetime becomes

$$\int_x e^{ip \cdot x} \langle 0 | T \{ h^{\mu\nu}(x) h^{\alpha\beta}(0) \} | 0 \rangle = \frac{i \tilde{\eta}^{\mu\alpha} \tilde{\eta}^{\nu\beta} + \tilde{\eta}^{\mu\beta} \tilde{\eta}^{\nu\alpha} - \beta \tilde{\eta}^{\mu\nu} \tilde{\eta}^{\alpha\beta}}{2(p^2 - M^2 + i\epsilon)}, \quad (4)$$

where $\tilde{\eta}^{\mu\nu} = \eta^{\mu\nu} - p^\mu p^\nu / M^2$ and $\beta = \frac{2}{D-1}$. (For massless gravitons, $\tilde{\eta}^{\mu\nu} = \eta^{\mu\nu} - p^\mu p^\nu / p^2$ and $\beta = \frac{2}{D-2}$. Here we consider $D=4$ only.) The gravitational interactions are given at the linear level as

$$\mathcal{L}_{\text{int}} = -\kappa h_{\mu\nu} T^{\mu\nu}, \quad (5)$$

where $T^{\mu\nu} \equiv \frac{-2}{\sqrt{-g}} \frac{\delta S}{\delta g_{\mu\nu}}$ with S being the SM action is a symmetric and conserved energy-momentum tensor of SM that sources (massive) gravitons.

The one-loop contributions of massless graviton to muon anomalous magnetic moment was calculated by Berends and Gastmans [21] and that of massive graviton was calculated in [22]. Both are found to be finite. We briefly discuss the single graviton contributions first. There are five diagrams that contribute at one-loop to muon $g-2$, shown in Fig. 1. All five diagrams are ultraviolet (UV) divergent and hence need to be regularized. But, the sum of all five diagrams for muon $g-2$ turns out to be finite. For massless graviton, one finds

$$a_\mu^{\text{gr}} = \frac{7}{32\pi^2} \kappa^2 m^2, \quad (6)$$

where m is the mass of muon [21]. For the one-loop contribution of massive gravitons of mass M to muon $g-2$, we find

$$a_\mu^{\text{massive}} = \frac{5}{16\pi^2} \kappa^2 m^2 f\left(\frac{m}{M}\right), \quad (7)$$

where $f(x)$ is a monotonically decreasing function from 1 to $2/3$ as x increases from 0 to ∞ (see Fig. 2), which agrees with the previous result in the integral form in [22].³ We

³There is a minor typo in Eq. (16) of [22] that the function in the integrand should be R instead of R/L .

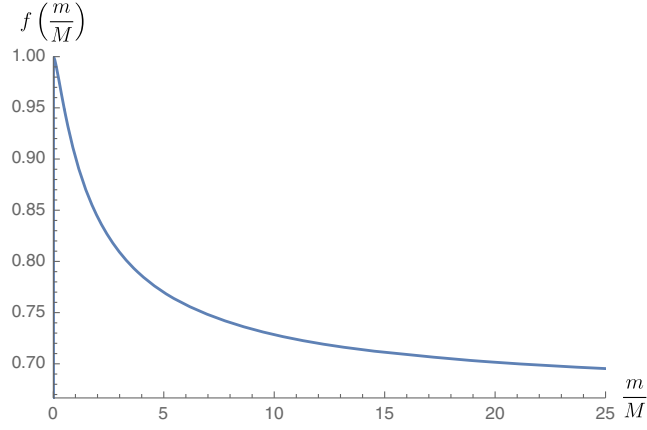


FIG. 2. The massive graviton contributions in the unit of $a_\mu^{\text{massive}}(M \rightarrow \infty)$.

note that the massive graviton does not decouple in the loop corrections to muon $g-2$, as its mass goes to infinity, $M/m \rightarrow \infty$, since the gravity is nonrenormalizable.⁴ In the large mass limit, $M \gg m$,

$$a_\mu^{\text{massive}} = \frac{5}{16\pi^2} \kappa^2 m^2 \left[1 + \left\{ \frac{1}{3} \ln\left(\frac{m}{M}\right) + \frac{11}{72} \right\} \frac{m^2}{M^2} + \dots \right], \quad (8)$$

where the ellipsis denotes the higher order terms in m/M . The massless limit of massive graviton is known to be discontinuous due to the nondecoupling of the longitudinal mode [24]. For the muon anomalous magnetic moment one finds the following discontinuity [22]:

$$a_\mu^{\text{massive}}(M \rightarrow 0) = a_\mu^{\text{gr}} + \frac{\kappa^2 m^2}{48\pi^2} \left(1 - \frac{D-1}{D-2} \right), \quad (9)$$

where $D = 4$ is the dimension of the spacetime in which massive gravitons propagate.

III. CLOCKWORK GRAVITONS AND OTHER EXTRA DIMENSION MODELS

If gravitons propagate in extra dimensions as well as in 4D spacetime where SM particles reside, either in a continuous extra dimension, such as the large extra dimensional (LED) model [25] and the Randall-Sundrum (RS) model [26], or in a discrete extra dimension as the deconstructed gravity [23] and the CW gravity [8], there will be a (infinite) tower of massive gravitons, $h_{\mu\nu}^{(n)}$, that interact with 4D SM particles with specific couplings,

⁴The graviton coupling to muon grows in energy as $\kappa^2 E^2$, and the massive graviton contribution to the muon $g-2$ is thus not suppressed. The unitarity is also expected to be violated at energy $E \sim \kappa^{-1} \equiv M_P (= 2.4 \times 10^{18} \text{ GeV})$. But, because the gravity is nonlinear, the theory necessarily has a UV cutoff below the Planck scale, M_P [23].

$$\mathcal{L}_{\text{int}} = - \sum_{n=1} \frac{1}{\Lambda_n} h_{\mu\nu}^{(n)} T^{\mu\nu}. \quad (10)$$

The mass of the n th graviton $M_{(n)}$ and the coupling $1/\Lambda_n$ are the function of the model parameters and its intrinsic scale M_5 that describes the extra dimension, discussed below. In this paper, we are mostly interested in the clockwork case. However, it turns out that the leading contribution to the muon $g-2$ is quite independent of detailed model parameters and is only the function of m and M_5 . Let us start from the CW case first, and we will discuss later the cases of RS and LED backgrounds.

In the clockwork theory the massive gravitons, $h_{\mu\nu}^{(n)}$, have mass, given as

$$M_{(n)}^2 = k^2 + \frac{n^2}{R^2} + \mathcal{O}\left(\frac{1}{N}\right), \quad n = 1, \dots, N, \quad (11)$$

where k and R correspond, respectively, to the warped factor or the clockwork spring and the radius of the extra dimension, orbifolded by Z_2 . In the continuum limit ($N \rightarrow \infty$) the clockwork geometry becomes, having the SM particles localized at the $y = 0$ brane,

$$ds^2 = e^{\frac{4k|y|}{3}} (dx_\mu dx^\mu - dy^2), \quad (12)$$

which is nothing but the linear dilaton model studied in [12]. Being a 5D theory, the continuum clockwork theory has an intrinsic 5D Planck scale M_5 , which is an additional parameter of the clockwork theory, in addition to the radius R and the warped factor k or the 5D cosmological constant $-2k^2$. Upon the KK reduction of the 5D clockwork gravity, one finds the clockwork gravitons' couplings to the SM particles at the linear level [8], given as the inverse of

$$\Lambda_n = \sqrt{M_5^3 \pi R \left(1 + \frac{k^2 R^2}{n^2} \right)}, \quad (13)$$

and the effective 4D Planck mass M_P or the effective coupling of the massless graviton, $\kappa = 1/M_P$, is defined as

$$M_P^2 = \frac{M_5^3}{k} (e^{2k\pi R} - 1). \quad (14)$$

The n th clockwork graviton contribution to the muon $g-2$ at one loop is from Eqs. (8) and (10),

$$a_\mu^{(n)} = \frac{5}{16\pi^2} \frac{m^2}{\Lambda_n^2} \left[1 + \mathcal{O}\left(\frac{m^2}{M_n^2}\right) \right]. \quad (15)$$

Since massive gravitons do not decouple to the muon anomalous magnetic moment in the large mass limit ($M_{(n)} \gg m$), one might assume all towers of gravitons

do contribute to the muon $g-2$. However, since the effective description of massive gravitons in 4D will break down at very short distances, smaller than the UV cutoff, Λ_{cut} , of the clockwork theory, only a finite number of gravitons are relevant in the 4D effective theory. The relevant gravitons should have therefore mass $M_{(n)} \lesssim \Lambda_{\text{cut}}$ or the highest level n_c that massive gravitons can be excited in the 4D effective theory should be from Eq. (11),

$$n_c = \sqrt{(\Lambda_{\text{cut}}R)^2 - (kR)^2}. \quad (16)$$

Since the choice of the cutoff Λ_{cut} depends on the regularization scheme of the 5D effective theory [27], we parametrize the regularization scheme dependence by a constant α as

$$\Lambda_{\text{cut}} = \alpha M_5, \quad (17)$$

where α should be positive and expected to be $\mathcal{O}(1)$ by the naive dimensional analysis but not predictable in the 5D effective theory. We will keep this parameter in our discussion in order to understand its physical meaning, and then take $\alpha = 1$ just to estimate some numerical values. The 5D Planck mass M_5 of the clockwork theory also sets the upper bound of the curvature of 5D clockwork geometry. For the clockwork to work the warped factor should be smaller than the intrinsic scale, $k \ll M_5$, and we have $n_c \gg 1$ for a given kR . Furthermore, since we are considering only the linear terms for the graviton interactions, our approximation will break down when massive gravitons couple strongly, where the effects of UV completed quantum gravity are important. Therefore, there should be an upper bound for the highest level, $n_c \leq n_*$, the massive gravitons of the 4D effective theory can reach. Namely at the upper bound $n = n_*$, the graviton mass is of the order of its effective Planck scale, $\Lambda_{n_*} \approx M_{(n_*)}$. Approximately we then have

$$M_5^3 \pi R \left(1 + \frac{k^2 R^2}{n_*^2}\right) = k^2 + \frac{n_*^2}{R^2}. \quad (18)$$

The maximum upper bound for graviton mass is therefore given by $n_* = M_5 R \sqrt{M_5 \pi R}$, as three parameters of the clockwork theory k , R , and M_5 are related to each other by Eq. (14). The number of allowed KK levels, n_c , or the hierarchies among $M_{(1)} \simeq k$, $1/R$, and M_5 are very sensitive to the value of kR , as shown in Tables I and II, which leads to very different collider phenomenology. However, the muon $g-2$ is rather insensitive to such model parameters for $k \ll M_5$. Since the more the allowed KK levels are, the weaker their couplings, two effects cancel with each other. Summing up the contributions of the clockwork gravitons up to the n_c th level, we get

TABLE I. The maximum Kaluza-Klein graviton level n_c in the clockwork geometry with $k = 10/R$, n_* in the 4D effective theory, and the contribution to the muon $g-2$ for different M_5 with $\alpha = 1$.

M_5 [TeV]	$M_5 R$	n_c	n_*	$a_\mu^{\text{CW}} \times 10^{10}$
0.5	1.2×10^5	1.2×10^5	7.4×10^7	4.5
1	3.1×10^4	3.0×10^4	9×10^6	1.1
5	1.2×10^3	1.2×10^3	7.3×10^4	0.045
10	306	305	9.5×10^3	0.011
50	12	7	74	4.0×10^{-5}

TABLE II. Same as Table I, but fixing k as $k = 0.1M_5$.

M_5 [TeV]	kR	n_c	n_*	$a_\mu^{\text{CW}} \times 10^{10}$
0.5	11.1	111	2.1×10^3	3.9
1	10.9	109	2.1×10^3	1.0
5	10.4	102	1.9×10^3	0.038
10	10.2	99	1.8×10^3	0.010
50	9.7	93	1.7×10^3	3.9×10^{-4}

$$\begin{aligned} a_\mu^{\text{CW}} &\simeq \sum_{n=1}^{n_c} a_\mu^{(n)} \\ &= \frac{5}{16\pi^3} \left(\frac{m}{M_5}\right)^2 \left[\alpha \sqrt{1 - \left(\frac{k}{\alpha M_5}\right)^2} - \frac{k}{M_5} \sum_{n=1}^{n_c} \frac{kR}{n^2 + (kR)^2} \right], \end{aligned} \quad (19)$$

which becomes for $n_c \gg 1$ or $M_5 \gg k$

$$\begin{aligned} a_\mu^{\text{CW}} &\simeq \frac{5}{16\pi^3} \left(\frac{m}{M_5}\right)^2 \\ &\times \left[\alpha - \frac{k}{M_5} \left\{ -\frac{1}{2kR} + \frac{\pi}{2} \coth(\pi kR) \right\} + \mathcal{O}\left(\frac{k^2}{M_5^2}\right) \right]. \end{aligned} \quad (20)$$

We note that the contribution of clockwork gravitons could be divided by the regularization scheme-dependent part,

$$\Delta_{(1)} a_\mu^{\text{CW}} \simeq \frac{5\alpha}{16\pi^3} \left(\frac{m}{M_5}\right)^2, \quad (21)$$

which is independent of detailed graviton spectrum, and the scheme-independent part

$$\Delta_{(2)} a_\mu^{\text{CW}} \simeq -\frac{5}{32\pi^2} \frac{k}{M_5} \left(\frac{m}{M_5}\right)^2, \quad (22)$$

which depends on the 5D geometry. The fact that $\Delta_{(1)}a_\mu^{\text{CW}}$ is independent of background geometry is easy to understand. In a 5D theory, this term represents a linear divergence that is dominated by large graviton momentum, p_5 , along the fifth dimension. Such a graviton of large momentum would not see the background geometry. Also this term could be absorbed by a local counterterm for the graviton-loop contribution such as

$$\mathcal{L}_{\text{c.t.}} \sim \frac{\lambda}{M_5^2} (\bar{\psi} \gamma^{\mu\nu} \gamma^\rho D_\rho \psi) F_{\mu\nu}, \quad (23)$$

which is consistent with symmetries of theory and independent of the background geometry. On the other hand, the contribution $\Delta_{(2)}a_\mu^{\text{CW}}$ is UV finite and scheme independent. Therefore it can be considered as the contribution that represents a genuine feature of CW geometry. To see that $\Delta_{(2)}a_\mu^{\text{CW}}$ is scheme independent, we may regularize the KK sum by different methods. For example, using

$$\begin{aligned} \sum_{n=1}^{\infty} \frac{n^2}{n^2 + (kR)^2} &= \left[\frac{\partial^2}{\partial \epsilon^2} \sum_{n=1}^{\infty} \frac{e^{-\epsilon n}}{n^2 + (kR)^2} \right]_{\epsilon \rightarrow 0^+} \\ &= \frac{1}{\epsilon} + \frac{1}{2} + \frac{ikR}{2} \left[\frac{\Gamma'(ikR)}{\Gamma(ikR)} - \text{H.c.} \right] \\ &\approx \frac{1}{\epsilon} - \frac{\pi kR}{2} \quad \text{for } kR \gg 1, \end{aligned} \quad (24)$$

where H.c. denotes the Hermitian conjugate and $\Gamma(x)$ and $\Gamma'(x)$ the gamma function and its derivative, respectively, we get the same finite contribution for $\Delta_{(2)}a_\mu^{\text{CW}}$.

In the absence of a computation in a UV complete theory, however, it is still useful to calculate whole clockwork graviton contributions, taking $\alpha=1$, although a UV complete theory is needed to determine α . Taking $kR \simeq 10$, we get, respectively, for $M_5 = 0.5, 1, 10, 50$ TeV as

$$a_\mu^{\text{CW}} \times 10^{10} \simeq 4.5, 1.1, 0.045, 0.01, 4.5 \times 10^{-4}. \quad (25)$$

For $M_5 = 0.2$ TeV, we find that the CW theory does explain the muon $g-2$ anomaly for any value of k up to its maximum value $k_{\text{max}} \sim M_5$, which corresponds to $k_{\text{max}} R \sim 11$ or $n_c \sim \mathcal{O}(1)$ (see Fig. 3). The KK graviton contributions for generic values of k and M_5 are plotted in Fig. 4.⁵ In Fig. 4, we also show that there is a lower bound for the size of extra dimension l_5 , defined as

$$l_5 \equiv \int_0^{\pi R} dy \sqrt{-g_{55}} = \frac{3(e^{3k\pi R} - 1)}{2k}, \quad (26)$$

⁵Note that in this plot we did not impose the constraints on the low values of M_5 from other experiments just to see the parameter dependence of a_μ more clearly. However, the muon $g-2$ anomaly alone already excludes the clockwork theory with $M_5 < 0.2$ TeV, unless there are negative contributions from other new particles.

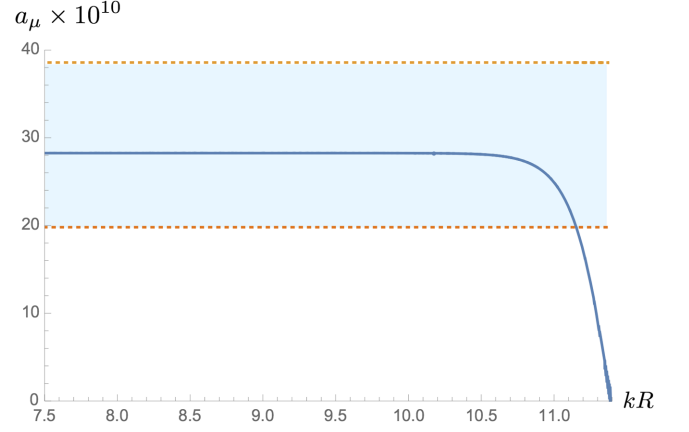


FIG. 3. The clockwork graviton contributions, shown in the solid line, to the muon $g-2$ for $M_5 = 0.2$ TeV and $kR \in [7.5, 11.3]$. The 1σ band of the muon $g-2$ anomaly lies in the shaded region between two dashed lines.

to explain the muon $g-2$ anomaly. If l_5 is smaller than 10^{-7} m, the CW gravitons' contribution to the muon $g-2$ is always smaller than the current muon $g-2$ anomaly (see the left panel of Fig. 4).

In the large n_c limit or in the limit of small curvature of the extra dimension, $k \ll M_5$, the leading contribution, $\Delta_{(1)}a_\mu^{\text{CW}}$, to the muon $g-2$ is independent of the background geometry of extra dimension. Such a universal form of the leading contribution to the muon $g-2$ in the large n_c limit encourages us to calculate the value in other types of background geometry. Being a 5D theory in a warped geometry, the continuum clockwork theory is quite similar to the Randall-Sundrum model [26]. But, the differences lie in the mass spectrum and the graviton couplings, which lead to quite different features generically in low-energy physics such as the muon $g-2$ as well as in collider physics. The metric of RS models takes for $0 \leq y \leq \pi R$

$$ds^2 = e^{2ky} dx_\mu dx^\mu - dy^2, \quad (27)$$

but the 4D effective Planck mass is same as that of CW theory, Eq. (14). The RS graviton has mass given by the zeros of the Bessel function of first kind $J_1(j_n) = 0$ as

$$M_{(n)} = k j_n \approx \left(n + \frac{1}{4} \right) \pi k, \quad n = 1, 2, 3, \dots, \quad (28)$$

and its 4D effective gravitational inverse coupling

$$\Lambda_n \simeq \sqrt{\frac{M_5^3}{k}}. \quad (29)$$

Similar to the CW gravitons, the highest level that RS gravitons can be excited to is given by

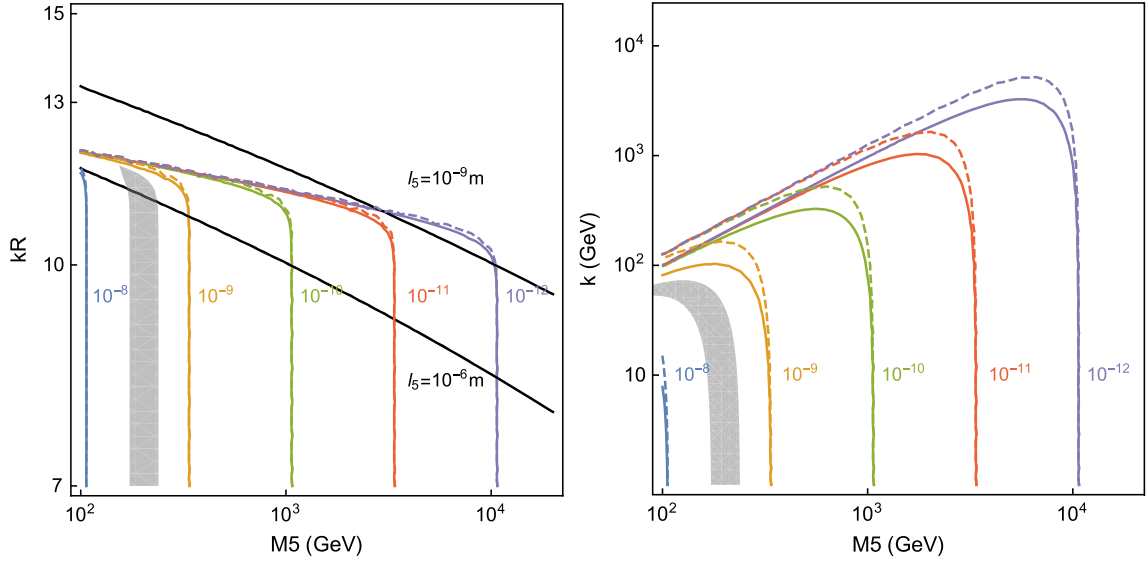


FIG. 4. The contribution of the KK gravitons to the muon $g-2$ for $\Delta a_\mu = 10^{-8}, \dots, 10^{-12}$ in the $M_5 - kR$ (left) and $M_5 - k$ (right) plots. The contributions from the clockwork gravitons are shown by the solid lines. Here we also show the contributions with the Randall-Sundrum background, Eq. (31), by dashed lines. The gray region is the 1σ band of the current muon $g-2$ anomaly [16]. In the left panel, the size of extra dimension, l_5 [Eq. (26)] is shown by black lines for $l_5 = 10^{-6}$ m and 10^{-9} m.

$$M_{(n_c)} \approx \alpha M_5 \quad \text{or} \quad n_c \approx \frac{\alpha M_5}{\pi k} - \frac{1}{4}, \quad (30)$$

which is always smaller than the maximum value of the highest level, $n_c < n_* = n_c^{3/2} \sqrt{\pi}$, at which $M_{(n_*)} \sim \Lambda_{n_*}$. The one-loop contribution of RS gravitons to the muon $g-2$ anomaly is summed up to its allowed highest level n_c to get⁶

$$a_\mu^{\text{RS}} = \sum_{n=1}^{n_c} a_\mu^{(n)} \approx \frac{5}{16\pi^3} \left(\frac{m}{M_5}\right)^2 \left[\alpha - \frac{\pi k}{4M_5} \right]. \quad (31)$$

We see that for large $n_c \gg 1$ or $M_5 \gg k$, both the clockwork and Randall-Sundrum models give at the leading order the same contribution of gravitons to the muon $g-2$, namely, $\Delta_{(1)} a_\mu^{\text{RS}} = \Delta_{(1)} a_\mu^{\text{CW}}$. On the other hand, the sub-leading but scheme-independent contribution of the RS background is bigger than that of the CW background. The CW contribution in the small curvature limit ($M_5 \gg k$) is therefore smaller than the RS contribution.

Finally we briefly consider the massive graviton contributions to the muon $g-2$ in the large extra dimensions, which has a flat 5D metric with the effective 4D Planck mass, given in terms of 5D intrinsic scale M_5 and the radius, R , of the fifth direction,

$$M_p^2 = M_5^3 \pi R. \quad (32)$$

Since the metric is flat and by the boundary condition of the graviton wave function, $\psi(y + \pi R) = \pm \psi(y)$, the Kaluza-Klein mass spectrum of the LED graviton takes with $n = 1, 2, 3, \dots$

$$M_{(n)} = \frac{n}{R}. \quad (33)$$

Taking the highest level of LED gravitons $n_c = \alpha M_5 R$, we find the total contribution of LED gravitons,

$$a_\mu^{\text{LED}} \approx \frac{5\alpha}{16\pi^3} \left(\frac{m}{M_5}\right)^2 = 4.5 \times 10^{-10} \alpha \left(\frac{0.5 \text{ TeV}}{M_5}\right)^2, \quad (34)$$

which is nothing but the lowest upper bound of all warped extra dimensions as α is independent of the background geometry. For the next order contributions, if $k \sim M_5$, only a few massive gravitons are allowed, $n_c \sim \mathcal{O}(10)$, and in this case RS gravitons always give bigger contributions to the muon $g-2$ than CW gravitons but smaller contributions, compared to LED gravitons, $a_\mu^{\text{CW}} \lesssim a_\mu^{\text{RS}} \lesssim a_\mu^{\text{LED}}$, though all the extra dimensional models give similar contributions for the same intrinsic scale, M_5 .

IV. RESULTS AND DISCUSSION

We have calculated in the clockwork theory the contributions of massive gravitons to the anomalous magnetic moment of muon. We find that the clockwork gravitons do explain the current muon $g-2$ anomaly or

⁶The muon $g-2$ in the Randall-Sundrum model has been calculated with a different cutoff, n_c , in [28,29]. The subleading part $\Delta_{(2)} a_\mu^{\text{RS}}$ could also be shown scheme independent by other regularization methods.

$\Delta a_\mu \approx 28.8 \times 10^{-10}$, if the intrinsic scale of the extra dimension $M_5 \sim 0.2$ TeV with $k \lesssim 0.1$ TeV, which corresponds to a quite large extra dimension, $l_5 \gtrsim 10^{-7}$ m. For generic clockwork models, however, with $M_5 \sim 1\text{--}100$ TeV and $M_5 > k$, the contributions of clockwork gravitons are too small to account for the current muon $g - 2$ anomaly. We also find that the lowest upper bound of the graviton contributions to the muon $g - 2$ is given by the large extra dimension models among all the extra dimension models with the same 5D intrinsic scale M_5 , if the UV sensitive contribution is the same, which might be useful in top-down model building. This property could be confirmed for extra dimension models with a more generic 5D metric background and KK graviton spectra that solve the gauge hierarchy problem [30].

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