A SUB-NYQUIST CO-PRIME SAMPLING MUSIC SPECTRAL APPROACH FOR NATURAL FREQUENCY IDENTIFICATION OF WHITE-NOISE EXCITED STRUCTURES

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Motivated by practical needs to reduce data transmission payloads in wireless sensors for vibration-based monitoring of civil engineering structures, this paper proposes a novel approach for identifying resonant frequencies of white-noise excited structures using acceleration measurements acquired at rates significantly below the Nyquist rate. The approach adopts the deterministic co-prime sub-Nyquist sampling scheme, originally developed to facilitate telecommunication applications, to estimate the autocorrelation function of response acceleration time-histories of low-amplitude white-noise excited structures treated as realizations of a stationary stochastic process. This is achieved without posing any sparsity conditions to the signals. Next, the standard MUSIC algorithm is applied to the estimated autocorrelation function to derive a denoised super-resolution pseudo-spectrum in which natural frequencies are marked by prominent spikes. The accuracy and applicability of the proposed approach is numerically assessed using computer-generated noise-corrupted acceleration time-history data obtained by a simulation-based framework pertaining to a white-noise excited structural system with two closely-spaced modes of vibration carrying the same amount of energy, and a third isolated weakly excited vibrating mode. All three natural frequencies are accurately identified by sampling at as low as 78% below Nyquist rate for signal to noise ratio as low as 0dB (i.e., energy of additive white noise equal to the signal energy), suggesting that the proposed approach is robust and noise-immune while it can reduce data transmission requirements in acceleration wireless sensors for natural frequency identification of engineering structures.

Keywords: Co-prime sampling, MUSIC pseudo-spectrum, compressive sensing, spectral estimation, system identification, closely-spaced modes.

1 Introduction

Linear system identification using solely response acceleration measurements is widely considered in practice to extract modal dynamic properties of civil engineering structures exposed to service and/or ambient unmonitored low-amplitude broadband/white excitations within the so-called operational modal analysis (OMA) setting [1]. These properties, such as natural frequencies and modes of vibration, are subsequently used for design verification of newly constructed structures, updating/calibrating computational models of existing structures, as well as

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structural condition assessment in the aftermath of natural disasters and continuous health monitoring through the lifetime of structures. Given the practical importance of these tasks, the potential of using wireless sensors/accelerometers in OMA deployments has been heavily explored over the last decades, as they provide low up-front cost and rapid deployments compared to arrays of tethered sensors [2,3]. Nonetheless, wireless sensors are constrained by frequent battery replacement requirements leading to increase maintenance costs while their bandwidth limitations pose restrictions to the amount of data that can be reliably transmitted. It has been established that the above disadvantages may be alleviated by considering system identification techniques using measurements sampled at low rates, significantly below the nominal application-dependent Nyquist rate [4-9].

Most of these techniques rely on the compressive sensing (CS) paradigm in which response acceleration time-histories are randomly sampled in time at sub-Nyquist rates at the front-end (sensor level) and, then, sparse recovery algorithms are applied to the compressed measurements at the back-end (base-station level) to retrieve the acceleration time-series [4,7] or, directly, modal data [5,6]. In CS-based techniques, the achieved level of data compression (sub-Nyquist rate) for faithful time-series recovery and/or modal properties extraction depends on the acceleration signals sparsity, i.e., non-zero signal coefficients on a given basis. Alternatively, the authors developed a power spectrum blind sampling (PSBS) approach [9] which relies on sub-Nyquist non-uniform in time deterministic multi-coset data acquisition to estimate the power spectral density (PSD) matrix of response acceleration signals treated as realizations of a multi-dimensional stationary stochastic process without imposing any sparsity conditions. Whilst the latter approach does not return the acceleration time-series, it achieves quality mode shape estimation via standard frequency domain OMA techniques at lower (sub-Nyquist) sampling rates compared to standard CS techniques even for noisy signals [8].

Recognizing that all current sub-Nyquist sampling techniques for OMA focus primarily on mode shape estimation, herein, a novel sub-Nyquist approach is put forth to estimate accurately natural frequencies of existing linearly vibrating structures widely considered for structural damage detection [10,11] and for assessing the performance of existing slender structures as well as for structures controlled by passive dynamic vibration absorbers [12]. The considered approach couples the deterministic sub-Nyquist co-prime sampling scheme originally introduced in [13] with the multiple signal classification (MUSIC) algorithm for spectral estimation (e.g., [14]) – a fusion that was originally developed in radar and telecommunication applications [15]. At first instance, it benefits from the inherent super-resolution and denoising capabilities of the MUSIC algorithm yielding a pseudo-spectrum which is found to outperform conventional Fourier transform-based spectral estimators for extracting natural frequencies in vibration-based system identification applications using ordinary Nyquist-sampled data (e.g., [16,17]). Further, similar to the multi-coset PSBS method [9], the proposed approach does not rely on any signal sparsity conditions treating the acquired signals as wide-sense stationary stochastic processes in alignment with the OMA framework that assumes stochastic (white noise) excitation and linear structural response [1]. In this context, it is a signal reconstruction-free compressive power spectral estimation approach aiming to estimate the auto-correlation function of stochastic structural response processes directly from noisy co-prime sampled measurements. The latter sampling scheme utilizes two conventional uniform in time sampling units (analog-to-discrete converters) per sensor operating at different a priori selected sub-Nyquist rates and accumulating, collectively in time, a much smaller number of measurements than a single sensor operating at Nyquist rate.

In the numerical part of the work, different co-prime sampling schemes achieving different data compression (sub-Nyquist sampling rates) are considered to sample computer-generated response acceleration signals from a three degree-of-freedom (DOF) white noise excited structure with two closely-spaced modes and one weakly-excited vibration mode having known natural frequencies. The signals are contaminated by different levels of additive white noise and attention is focused on assessing the potential of the co-prime MUSIC approach to resolve the natural frequencies for various sub-Nyquist sampling rates and signal-to-noise ratios.

In the remainder of this paper, Section 2 outlines the adopted co-prime sampling for autocorrelation estimation of stationary stochastic processes and reviews the mathematical details of the MUSIC algorithm. Section 3 appraises the usefulness of the adopted sub-Nyquist pseudo-spectral estimation method in OMA applications and numerically attests its efficiency in resolving closely-spaced natural frequencies from computer-generated compressed data sampled at different sub-Nyquist rates and contaminated with various levels of additive white noise. Lastly, Section 4 summarizes concluding remarks.

2 Mathematical Background

2.1 *Co-prime sampling and auto-correlation estimation of stationary stochastic processes*

Let $x(t)$ be a real-valued wide-sense stationary band-limited stochastic process assuming a spectral representation by a superposition of R sinusoidal functions with frequencies f_r, real amplitudes B_r, and uncorrelated random phases θ_r uniformly distributed in the interval [0, 2π]. That is,

$$x(t) = \sum_{r=1}^{R} B_r \cos(2\pi f_r t + \theta_r) \qquad (1)$$

Co-prime sampling (e.g., [13,15]) assumes that the process $x(t)$ is simultaneously acquired by two sampling devices, operating at different (sub-Nyquist) sampling rates, $1/(N_1 T_s)$ and $1/(N_2 T_s)$, where N_1, N_2 are co-prime numbers ($N_1 < N_2$), and $1/T_s = 2f_{\max}$ is the Nyquist sampling rate with fmax being the highest frequency component in Eq.(1). The process $x(t)$ is then divided in time blocks of $(2N_1\text{-}1) N_2 T_s$ duration and, within each such block, only $2N_1 + N_2 - 1$ samples are retained from a total number of floor$\{2(N_1+N_2)-1- N_2/N_1\}$ acquired measurements. The thus retained samples of $x(t)$ from the two different samplers are

$$\begin{aligned} x_1[k] &= x(kN_1T) = \\ &= \sum_{r=1}^{R} B_r \cos(2\pi f_r k N_1 T_s + \theta_r) + \varepsilon_1[k], \\ x_2[\ell] &= x(\ell N_2 T) = \\ &= \sum_{r=1}^{R} B_r \cos(2\pi f_r \ell N_2 T + \theta_r) + \varepsilon_2[\ell], \end{aligned} \qquad (2)$$

where $k \in \{0,\ldots,N_2-1\}, \ell \in \{1,\ldots,2N_1-1\}$, and $\varepsilon_1[k]$ and $\varepsilon_2[\ell]$ are zero-mean uncorrelated Gaussian white noise sequences with common variance σ_ε^2 modelling errors/noise introduced during sampling. In this setting, N_2 samples are obtained from the first sampler operating at sampling rate $1/(N_1 T_s)$ and $(2N_1\text{-}1)$ samples are obtained from the second sampler operating at sampling rate $1/(N_2 T_s)$. This choice is not arbitrary; it was shown in [13] that the cross-difference set of numbers $\Omega = \{N_2\ell - N_1 k\}$ contains all possible integers within the range $[N_1N_2, N_1N_2]$, with some repetition. Thus, the cross-correlation function of the sequences $x_1[k],\ x_2[\ell]$, whose support involves all the time-lags included in the set Ω, can be estimated in the above range of interest once redundant indices are discarded.

To this aim, the sequences in Eq.(2) are first stacked in the vector $\mathbf{y}_n \in \mathbb{R}^{(2N_1+N_2-1)}$ written as

$$y_n = \left[x_1^T\left[2N_2 n + k\right] \;\; x_2^T\left[2N_1 n + \ell\right] \right]^T = \\ = \sum_{r=1}^{R} B_r\ e(f_r)\ \cos(2\pi f_r N_1 N_2 n T_s + \theta_r) + \varepsilon_n, \tag{3}$$

where the superscript "T" denotes vector/matrix transposition, $\boldsymbol{\varepsilon}_n \in \mathbb{R}^{(2N_1+N_2-1)}$ is the vector collecting the sampling noise terms, and $\mathbf{e}(f_r) \in \mathbb{R}^{(2N_1+N_2-1)}$ is given as

$$\mathbf{e}(f_r) = [1 \quad \cos(2\pi f_r\ N_1 T_s) \ \cdots \\ \cos(2\pi f_r (N_2 - 1) T_s) \\ \cos(2\pi f_r\ N_2 T_s) \ \cdots \\ \cos(2\pi f_r (2N_1 - 1) N_2 T_s)]^{\mathrm{T}}. \tag{4}$$

Importantly, in Eq.(3), the inclusion of the non-negative integer index $n \in \mathbb{Z}^*$ allows for arbitrarily placing the co-prime sampling block in time (e.g., for n=0 the time block starts at t=0 and corresponds to the block considered in Eq.(2)). Therefore, an arbitrary large number of blocks (and corresponding vectors $\mathbf{y}_n$) can be used for sampling the theoretically infinitely long stationary process $x(t)$. The position of each block in time depends on the adopted values of n. The autocorrelation matrix of $\mathbf{y}_n$ is given as [15]

$$\mathbf{R}_{\mathbf{yy}} = \mathrm{E}\left\{\mathbf{y}_n\ \mathbf{y}_n^{\mathrm{T}}\right\} \\ = \sum_{r=1}^{R} B_r^2\ \mathbf{e}(f_r)\ \mathbf{e}^{\mathrm{T}}(f_r) + \sigma_\varepsilon^2 \mathbf{I} \tag{5}$$

in which $\mathbf{I} \in \mathbb{R}^{(2N_1+N_2-1)\times(2N_1+N_2-1)}$ is the identity matrix, while the mathematical expectation operator $\mathrm{E}\{\cdot\}$ averages over n, i.e., the matrix Ryy in Eq.(5) is computed by averaging over all the time blocks considered within a Monte Carlo-based context.

Next, following the spatial smoothing technique in [14], the autocorrelation matrix in Eq.(5) is first stacked in a column vector, $\mathbf{r}_{\mathbf{y}} = \mathrm{vec}\left(\mathbf{R}_{\mathbf{yy}}\right)$, with $\mathbf{r}_{\mathbf{y}} \in \mathbb{R}^{(2N_1+N_2-1)^2\times 1}$. Then, the elements of $\mathbf{r}_y$ are sorted and truncated within the range $[-N_1N_2,\ N_1N_2]$, while repeated terms are eliminated, so that the integer indices of the exponential terms in Eq.(4) are given in increasing order with no repetition. The thus generated reduced autocorrelation vector $\hat{\mathbf{r}}_{\mathbf{y}}$ (i.e. sorted and truncated), is subsequently divided into $i=\{1,2,\ldots,\ N_1N_2+1\}$ overlapping subarrays, $\hat{\mathbf{r}}_{\mathbf{y}_i}$, each consisting of (N_1N_2+1) elements, which are averaged as in

$$\mathbf{R}_{ss} = \frac{1}{N_1 N_2 + 1} \sum_{i=1}^{N_1N_2+1} \hat{\mathbf{r}}_{\mathbf{y}_i} \hat{\mathbf{r}}_{\mathbf{y}_i}^{\mathrm{T}}, \tag{6}$$

to generate the spatially smoothed matrix $R_{ss} \in \mathbb{R}^{(N_1N_2+1)\times(N_1N_2+1)}$. In the following section, this matrix is used as input to the MUSIC super-resolution spectral estimator to detect the R frequencies f_r, (r= 1,2,…,R), of the considered stochastic process $x(t)$.

2.2 *Multiple signal classification (MUSIC) algorithm for natural frequencies estimation*

The Multiple Signal Classification (MUSIC) algorithm is a super-resolution pseudo-spectrum estimation method, which relies on the eigenvalue decomposition of autocorrelation matrices estimated by field measurements (e.g., [14]). Herein, the MUSIC algorithm is applied to the autocorrelation matrix Rss in Eq.(6), which is decomposed as in

$$\mathbf{R}_{ss} = \sum_{i=1}^{R} (\lambda_i + \sigma_\varepsilon^2) \mathbf{v}_i \mathbf{v}_i^{\mathrm{T}} + \sum_{i=R+1}^{N_1 N_2 + 1} \sigma_\varepsilon^2 \mathbf{v}_i \mathbf{v}_i^{\mathrm{T}} \qquad (7)$$

where the eigenvectors $\mathbf{v}_i$ are orthonormal, i.e. $\mathbf{v}_i \mathbf{v}_j^{\mathrm{T}} = 0$ for $i \neq j$. The first term in Eq.(7) represents the signal sub-space with R eigenvalues $(\lambda_i + \sigma_\varepsilon^2)$, i=1,…,R, and R principal eigenvectors spanning the same subspace with the signal vector in Eq.(4). The second term corresponds to the noise sub-space with (N_1N_2-R) identical eigenvalues σ_ε^2, and (N_1N_2-R) eigenvectors. Then, the unbiased MUSIC pseudo-spectrum estimator is defined as

$$G_{MUSIC}(f) = \frac{1}{\mathbf{e}^{\mathrm{T}}(f) \left(\sum_{i=R+1}^{N_1 N_2 + 1} \mathbf{v}_i \mathbf{v}_i^{\mathrm{T}} \right) \mathbf{e}(f)} \qquad (8)$$

The above estimator becomes theoretically infinite at f=f_r, that is, at the location of the structural natural frequencies (i.e., the frequencies in the stochastic process of Eq.(1)). Numerical implementation, though, involves errors in approximating the solution of the eigenvalue problem in Eq.(7) and, therefore, Eq.(8) takes on finite values observing sharp peaks at each f_r, resulting in a spectrum-like shape (pseudo-spectrum). Limitations of the MUSIC algorithm are the *a priori* knowledge on the number of R signal components and the increased computational cost with R. Nonetheless, the significance of the proposed approach lies on its capability to capture up to $R \leq N_1N_2$ natural frequencies in noisy signals, at the high frequency resolution of 1/($N_1N_2T_s$) (in Hz), outperforming conventional approaches at Nyquist rate that can only retrieve up to (2N_1+N_2-2) frequencies (see also [15]).

3 Numerical Application

3.1 *Simulation framework for Nyquist-sampled response acceleration signals generation*

Consider a viscously-damped linear multi-degree-of-freedom (MDOF) structural system with R modes of vibration, excited by zero-mean Gaussian white-noise band-limited to ωmax with unit amplitude power spectral density (PSD). Let $x(t)$ be the real-valued response/output acceleration process along a single monitored DOF of the MDOF system. The PSD of the latter process can be written as [18].

$$G_x(\omega) = \sum_{r,s=1}^{R} \left[\frac{A_{rs}}{[(\omega_r{}^2 - \omega^2)^2 + (2\omega\omega_r\zeta_r)^2]} \frac{(\omega_r{}^2 - \omega^2)(\omega_s{}^2 - \omega^2) + 4\omega^2\omega_r\omega_s\zeta_r\zeta_s}{[(\omega_s{}^2 - \omega^2)^2 + (2\omega\omega_s\zeta_s)^2]} \right] \omega^4 \qquad (9)$$

in which ω_r (ω_s) is the resonant frequency and ζ_r (ζ_s) the damping ratio of the r-th (s-th) mode of vibration of the MDOF system, while the amplitude A_{rs} is a parameter associated with the mode shapes and the modal participation factors whose value depends on the monitored DOF one line space before and after the figure caption.

In numerically assessing the potential of the proposed approach to estimate the resonant frequencies ω_r for any given white-noise excited MDOF structural system, a simulation-based framework is adopted to generate the required time-histories. The framework first defines the continuous-time (analog) PSD in Eq.(9) corresponding to the acceleration response process of a white noise excited MDOF structural system with known modal properties (ω_r, A_r, ζ_r). This represents the "target" PSD which is, then, replaced by a surrogate discrete-time auto-regressive moving average (ARMA) filter of order (p, q) given by the transfer function

$$H(e^{i\omega T_s}) = \frac{\sum_{\ell=0}^{q} c_\ell e^{-\ell i\omega T_s}}{1+\sum_{k=1}^{p} b_k e^{-ki\omega T_s}} \tag{10}$$

where T_s=π/$\omega_{\max}$ is the time-discretization step used in defining the ARMA filter to avoid aliasing and b_k , k=(1,2,…,p), and c_ℓ, $\ell=(0,1,...,q)$ are the ARMA filter coefficients. The latter coefficients are derived by using the auto/cross-spectrum correlation matching method [19], widely used for spectrum compatible simulation applications (e.g., [20,21]).
Next, the thus defined ARMA filter is excited by Gaussian white-noise sequences to generate a discrete-time Nyquist-sampled realization of an underlying stochastic process representing the acceleration response of the original MDOF structural system with properties (ω_r,A_r,ζ_r). Finally, the above realization is contaminated by additive white noise to model ambient/environmental noise observed in field recorded response acceleration signals in OMA applications and co-prime sampled as detailed in section 2.1.

3.2 *Adopted structural system and simulated noisy acceleration responses*

A continuous MDOF structural system with R=3 DOFs is taken with two equally-excited closely-spaced modes of vibration and a third higher weaker excited mode attaining a low spectral amplitude. Specifically, the first two resonant frequencies are at f_1=66 Hz and f_2=70Hz having, thus, a percentage difference of 6%. The third natural frequency is at f_3=120Hz. The critical damping of ζ_r =5% (r=1,2,3) is assumed for all vibrating modes and coefficients A_{11}= A_{13}= A_{31}=A_{22}= A_{23}= A_{32}=1, A_{12}=A_{21}=2, A_{33}=0.25 are taken in Eq.(9).

Figure 1 plots the target PSD of Eq.(9), which is desired to be approximated by the proposed sub-Nyquist pseudo-spectral estimation method (i.e., MUSIC algorithm fused with co-prime sampling). To this aim, the target PSD is first replaced by a surrogate discrete-time ARMA filter of order (120, 12) which is then subjected to a clipped white-noise excitation of 20s duration, sampled at a Nyquist rate of F_s=1/T_s=500Hz (i.e., T_s =0.002s) following the simulation framework of section 3.1.

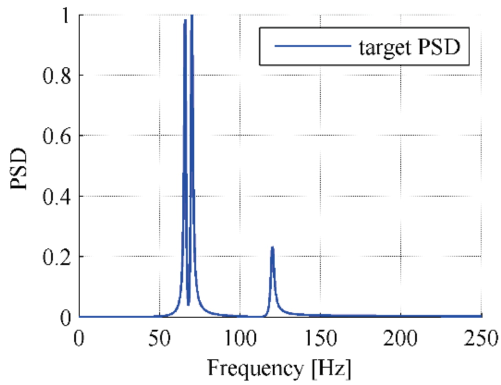

Figure 1. Normalized target PSD for the adopted 3DOF system.

To assess the efficacy of the proposed system identification approach at different additive noise levels, the generated discrete-time ARMA response signal is corrupted by additive white noise at five different signal-to-noise ratios (SNRs) ranging within [0, 20] dB.

3.3 *Sub-Nyquist pseudo-spectral estimation*

The noisy discrete-time acceleration response signals generated as previously described are co-prime sampled as detailed in section 2 using 4 different sampling settings listed in Table 1. In particular, the four considered pairs (N_1, N_2) of co-prime numbers are reported together with the pertinent average sub-Nyquist sampling rates $1/(N_1T_s)+1/(N_2T_s)$, spectral resolutions $1/(N_1N_2T_s)$, and sizes $(N_1N_2+1) \times (N_1N_2+1)$ of the smoothed autocorrelation matrix Rss in Eq.(6). To illustrate the calculations involved, consider the sampling case with co-prime $N1=_7$ and $N_2=11$. The underlying assumption is that two samplers are deployed per recording location to acquire uniform samples of the same acceleration response signal (in time), with sampling rates equal to $1/(7\ T_s)$ and $1/(11\ T_s)$, respectively. The two samplers accumulate measurements at an average rate of $1/(7\ T_s) + 1/(11\ T_s)$ samples per second, which is about 76.6% lower than the Nyquist rate. Further, the assumed co-prime numbers define the cross-difference set $\Omega = \{11\ell - 7k,\ k \in [0,10], \ell \in [1,13]\}$, which includes all discrete time lags within the support [-77, 77] of the cross-correlation function between the measurements of the two sensors (see also section 2). It is further assumed that the measured acceleration signal is divided in K non-overlapping time-blocks that are used for the computation of the autocorrelation matrix in Eq.(5). Each block contains $(2\ N_1\text{-}1)\times\ N_2=143$ Nyquist samples from which only $2\ N_1+\ N_2\text{-}1=24$ samples are taken to populate the $\mathbf{R}_{yy} \in \mathbb{R}^{24\times24}$ matrix in Eq.(5). Next, the spatial smoothing technique is employed to generate the semi-positive correlation matrix $\mathbf{R}_{ss} \in \mathbb{R}^{78\times78}$ in Eq.(6) directly from the coprime-sampled (compressed) measurements. Finally, the MUSIC estimator in Eq.(8) is evaluated, based on the assumption of R=3 degrees of freedom being present in the measured acceleration response signals. For the other sub-Nyquist sampling cases in Table 1 the pertinent co-prime parameters are defined in a similar manner as above.

From a practical viewpoint, it is important to note that the K block length depends on the co-prime numbers and, thus, is different for each sampling scheme considered. To this end, in establishing a meaningful comparison among sampling schemes, the number of blocks K observed is set such that the total length of the observation window (latency) remains roughly the same for all four sampling schemes. Under this assumption, the last column of the table reports the total number of samples acquired by co-prime sampling: ultimately, this is the number of measurements that need to be transmitted by a (wireless) sensor in an actual monitoring deployment. It is seen that higher co-prime numbers improves both frequency resolution and data compression which, under the assumption of constant latency, results in fewer measurements. These benefits come at the expense of a larger eigenvalue problem to solve (size of matrix $\mathbf{R}_{ss}$) to obtain the MUSIC pseudo-spectrum which implies higher computational cost. Nevertheless, this operation is expected to be undertaken off-line,

Table 1. Co-prime sampling specifications

Co-prime numbers (N_1, N_2)	Average sampling rate below Nyquist	Resolution [Hz]	Size of matrix $\mathbf{R}_{ss}$	Block length $(2N_1\text{-}1)N_2$	Number of blocks K	Total number of co-prime measurements
(3,7)	52.4%	23.81	22x22	35	285	3420
(5,7)	65.7%	14.29	36x36	63	144	2304
(7,11)	76.6%	6.49	78x78	143	69	1656
(7, 13)	78.0%	5.49	92x92	169	59	1534

at a base-station and, therefore, does not compromise the efficiency of the sensors deployment.

3.4 *Identification of closely-spaced structural resonances from noisy data*

Figure 2 plots the obtained MUSIC pseudo-spectra for the adopted 3DOF structural system for all four co-prime sampling specifications of Table 1 and for all five *SNR* values. All spectra are normalized to unit amplitude to facilitate comparison and plotted with different colors along a horizontal axis labelled after the pertinent *SNR*s. From Figure 2, it is readily observed that the efficacy of the MUSIC pseudo-spectrum in extracting the two closely-spaced natural frequencies depends strongly on the frequency resolution achieved by the adopted co-prime sampling scheme.

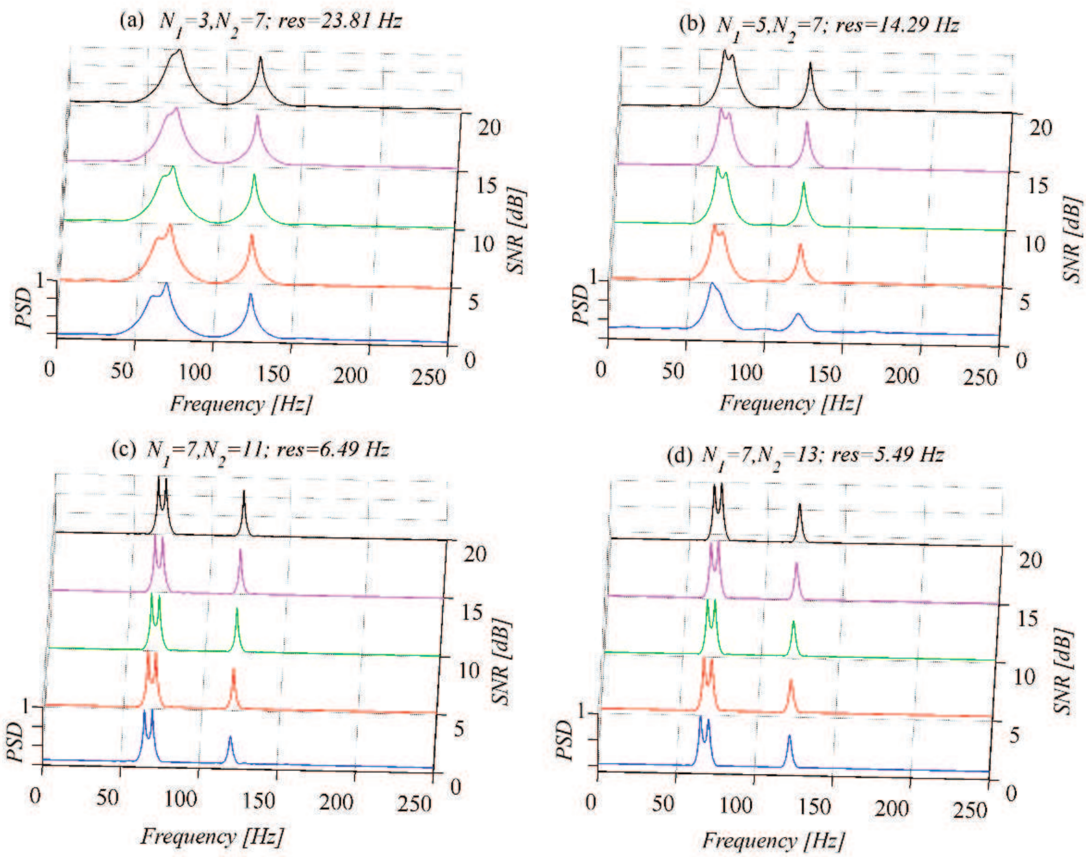

Figure 2. MUSIC pseudo-spectra for the target PSD spectrum of Figure 1 and for 5 different *SNR* values and co-prime sampling specifications of Table 1.

Specifically, the sampling scheme corresponding to the lowest frequency resolution cannot discriminate the closely-spaced frequencies in Figures 2(a); the closely-spaced resonant frequencies are merged yielding a single spectral peak at a frequency value of 70Hz. Still, the third high-frequency and least excited mode is retrieved at 120Hz. By increasing resolution, (N_1=5, N_2=17) scheme, the two peaks of the closely-spaced frequencies become discernible at least for SNR>5dB. As the co-prime numbers increase, achieving higher resolution, the proposed approach yields sharper spectral peaks, capable to clearly discriminate the two closely-spaced natural frequencies as well as resolve the weakly excited mode of vibration with high accuracy. More importantly, this particular example shows that the estimator is practically immune to additive noise suggesting that the choice of the pair of co-prime numbers will be based on the trade-off between computational cost required in obtaining the eigenvalue decomposition in Eq.(7) and the number of the acquired co-prime measurements as seen in Table 1.

4 Concluding Remarks

A novel natural frequency identification approach has been established utilizing response acceleration measurements of white-noise excited structures sampled at rates significantly below the Nyquist rate supporting reduced data transmission in wireless sensors for vibration-based structural monitoring. The approach relies on the standard MUSIC pseudo-spectrum applied to the autocorrelation function of response acceleration time-histories estimated from co-prime sampled sub-Nyquist measurements. In this context, acceleration time-histories are treated as realizations of a stationary stochastic process without posing any sparse structure requirements. Further, the considered approach benefits from the super-resolution and denoising capabilities of the MUSIC spectral estimator to achieve high-accuracy in structural natural frequency identification even for noise-corrupted measurements.

The potential of the proposed approach was numerically verified using computer-generated acceleration time-history data obtained by a simulation-based framework pertaining to a white-noise excited structural system with two closely-spaced modes of vibration carrying the same amount of energy, and a third isolated weakly excited vibrating mode. Parametric analysis was conducted using co-prime sampled measurements at four different sub-Nyquist rates contaminated by additive white noise at five different SNRs. All three natural frequencies have been successfully retrieved for co-prime sampling schemes achieving sampling rates up to 78% below Nyquist and for high level of additive noise with even equal signal and noise power (SNR=0dB). Overall, the herein furnished results demonstrate that the adopted co-prime sampling-based MUSIC pseudo-spectrum is a potent tool for accurate natural frequency identification in a noise-immune setting and thus it can be readily implementable in wireless sensors for cost-efficient (in terms of data sampling and wireless transmission rates) vibration-based structural monitoring.

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