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Ultrabroadband nonlinear optics in nanophotonic periodically poled lithium niobate waveguides: supplementary material

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This document provides supplementary information to "Ultrabroadband nonlinear optics in nanophotonic periodically poled lithium niobate waveguides," https://doi.org/10.1364/OPTICA.7.000040. In this supplemental, we discuss the calculation of the effective areas for nonlinear interactions in tightly confining waveguides. The detailed calculations presented here will help readers to reproduce the results shown in the main article.

1. EFFECTIVE AREA OF SECOND HARMONIC GENERATION

In this section, we derive the nonlinear coupling between two waveguide modes and define the effective area, A_{eff}, associated with these interactions. The effective area provides a measure of the strength of a nonlinear interaction due to the tight confinement of the waveguide; small effective areas correspond to large field intensities and large normalized efficiencies. We begin by reviewing the necessary components of linear optical waveguide theory to establish the notation used throughout this section. Then, we review the coupled wave equations for second harmonic generation (SHG) in a nonlinear waveguide. The treatment used here accounts for the fully-vectorial nature of the modes, with each field component of the waveguide mode coupled together by the full nonlinear tensor, d_{ijk}, of the media that comprise the waveguide. Remarkably, these equations have the same form as the coupled wave equations for SHG in much simpler contexts, such as SHG of plane waves and paraxial gaussian beams. The effective area arises naturally when calculating the normalized conversion efficiency of the power in the second harmonic, $P_{2\omega}/P_{\omega}$.

Waveguide modes arise as the solution to Maxwell's equations in the absence of a nonlinear polarization, with a dielectric constant that varies in two spatial dimensions,

$$\nabla \times H(x,y,z,\omega) = i\omega\bar{\epsilon}(x,y,\omega)E(x,y,z,\omega), \tag{S1a}$$
$$\nabla \times E(x,y,z,\omega) = -i\omega\mu_0 H(x,y,z,\omega). \tag{S1b}$$

We note that the media considered here are anisotropic, thus $\bar{\epsilon}(x,y,\omega)$ is a second order tensor. We expand the field in a series of guided modes

$$E(x,y,z,\omega) = \sum_{\mu} a_{\mu}(\omega)E_{\mu}(x,y,\omega)e^{-ik_{\mu}(\omega)z} \tag{S2a}$$

$$H(x,y,z,\omega) = \sum_{\mu} a_{\mu}(\omega)H_{\mu}(x,y,\omega)e^{-ik_{\mu}(\omega)z} \tag{S2b}$$

where a_{μ} represents the component of E contained in mode μ around frequency ω. The transverse mode profiles, E_{μ} and H_{μ}, and their associated propagation constant, k_{μ}, arise as solutions to an eigenvalue problem[1], and each pair of waveguide modes satisfies the orthogonality relation

$$\int_A \frac{1}{2}\text{Re}\left(\left[E_{\nu} \times H_{\mu}^{*}\right]\cdot\hat{z}\right)dxdy = P\delta_{\mu,\nu}. \tag{S3}$$

The fields, as defined, are normalized such that $P = 1W$, and therefore the power contained in mode μ is $P|a_{\mu}|^2$. For convenience, we define the transverse mode profiles using dimensionless functions $e(x,y)$ and $h(x,y)$

$$E_\mu(x,y) = \sqrt{\frac{2Z_0 P}{n_\mu A_{\text{mode},\mu}}} e_\mu(x,y), \tag{S4a}$$

$$H_\mu(x,y) = \sqrt{\frac{2n_\mu P}{Z_0 A_{\text{mode},\mu}}} h_\mu(x,y), \tag{S4b}$$

where n_μ is the effective index of mode μ, and Z_0 is the impedance of free space. As a consequence of Eqn. (S3), the effective area of mode μ is given by $A_{\text{mode},\mu} = \int (e_\mu \times h_\mu^*) \cdot \hat{z} dxdy$.

The presence of a nonlinear polarization at frequency ω gives rise to driving terms that cause a_μ to evolve in z. Under these conditions, it can be shown using the methods described in [2, 3] that a_μ evolves as

$$\partial_z a_\mu(z,\omega) = \frac{-i\omega}{4P} e^{ik_\mu} \int E_\mu^* \cdot P_{NL,\mu} dxdy. \tag{S5}$$

For second harmonic generation in the limit where one pair of modes is close to phasematching, we consider one mode for the fundamental at frequency ω and for the second harmonic at frequency 2ω without loss of generality. For the remainder of this article, the modes under consideration will be referred to as a_ω and $a_{2\omega}$ for the fundamental and second harmonic, respectively. In this case, the nonlinear polarization is given by

$$P_{NL,\omega} = 2\epsilon_0 d_{\text{eff}} a_{2\omega} a_\omega^* \sum_{jk} \bar{d}_{ijk} E_{j,2\omega} E_{k,\omega}^* e^{-i(k_{2\omega}-k_\omega)z} \tag{S6a}$$

$$P_{NL,2\omega} = \epsilon_0 d_{\text{eff}} a_\omega^2 \sum_{jk} \bar{d}_{ijk} E_{j,\omega} E_{k,\omega} e^{-2ik_\omega z} \tag{S6b}$$

where $i,j,k \in \{x,y,z\}$. $d_{\text{eff}} = \frac{2}{\pi} d_{33}$ is the effective nonlinear coefficient for a 50% duty cycle periodically poled waveguide, and $\bar{d}_{ijk}$ is the normalized $\chi^{(2)}$ tensor. For lithium niobate, this is expressed using contracted notation[4] in the coordinates of the crystal as

$$\bar{d}_{iJ} = \frac{1}{d_{33}} \begin{bmatrix} 0 & 0 & 0 & 0 & d_{15} & d_{16} \\ d_{16} & -d_{16} & 0 & d_{15} & 0 & 0 \\ d_{15} & d_{15} & d_{33} & 0 & 0 & 0 \end{bmatrix}$$

where $d_{15} = 3.67$ pm/V, $d_{16} = 1.78$ pm/V, and $d_{33} = 20.5$ pm/V for SHG of 2-μm light. These values are found using a least squares fit of Miller's delta scaling to the values reported in [5, 6], and have relative uncertainties of $\pm 5\%$. We therefore expect a relative uncertainty in any calculated normalized efficiency to be $\pm 10\%$.

We arrive at the coupled wave equations for SHG by substituting Eqns. (S6a-S6b) into Eqn. (S5) and defining $A_\omega = \sqrt{P} a_\omega$

$$\partial_z A_\omega = -i\kappa A_{2\omega} A_\omega^* e^{-i\Delta k} \tag{S7a}$$

$$\partial_z A_{2\omega} = -i\kappa A_\omega^2 e^{i\Delta k} \tag{S7b}$$

The coupling coefficient, κ, and the associated effective area are given by

$$\kappa = \frac{\sqrt{2Z_0}\omega d_{\text{eff}}}{cn_\omega \sqrt{A_{\text{eff}} n_{2\omega}}} \tag{S8a}$$

$$A_{\text{eff}} = \frac{A_{\text{mode},\omega}^2 A_{\text{mode},2\omega}}{\left| \int \sum_{i,j,k} \bar{d}_{ijk} e_{i,2\omega}^* e_{j,\omega} e_{k,\omega} dxdy \right|^2} \tag{S8b}$$

We conclude this section by noting that for the waveguide modes considered here the overlap integral in Eqn. (S8b) and the resulting κ are real. The general case, in which κ is complex, is beyond the scope of this supplemental.

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