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Insider Trading With Different Risk Attitudes*

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Abstract

This paper examines the impact of different risk attitudes on the financial decisions of two insiders trading in the stock market. We consider a static version of the Kyle (1985) model with two insiders. Insider 1 is risk neutral while insider 2 is risk averse with negative exponential utility. First, we analytically prove the existence of a unique linear equilibrium. Second, we carry out a comparative static analysis with respect to the duopoly case of risk-neutral insiders (Tighe (1989)) and with respect to the duopoly case of risk-averse insiders (Holden and Subrahmanyam (1994)) models. Our findings reveal that the market depth and the information revelation are higher in Tighe (1989) than in our model. However, compared to Holden and Subrahmanyam (1994), we find that the market depth depends crucially on the degree of risk aversion. Finally, we show that regardless of the degree of risk aversion, the stock price reveals more information in our model than the stock price in Holden and Subrahmanyam (1994).

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1 Introduction

Investors' attitudes toward risk play a central role in their investments decisions. Most of the literature about investors' risk attitudes, considers two types of risk tolerance: risk-neutral investors and risk-averse investors. In a recent paper on risk preferences, Falk et al. (2018) conducts a study on global variation in economic preferences in 76 countries and finds that there is a substantial heterogeneity in preferences across countries, and that these preferences drive the individual decision making. Our paper allows for risk heterogeneity among the investors and studies the impact of such risk heterogeneity on their financial decisions in the presence of insider trading.

There is a large body of applied and theoretical studies of the insider trading problem and its impact on the financial markets. The importance of this problem is highlighted by the policies adopted by the Securities and Exchange Commission (SEC) and its push to investigate and prosecute those accused of insider trading. The celebrated Kyle 1985-model gave a framework for the study of insider trading and it has been extended in several directions. Most of these extensions assume either risk neutral insiders or risk averse insiders.[1]. This paper, is the first to our knowledge,[2] to investigate the effect of hybrid risk attitudes on the financial decisions of two insiders trading in the stock market.

[1]For a detailed reference, the reader can check O'Hara (1995) or Vives (2010). Recent extensions considered the one period model of Kyle and risk-neutrality of the insider, include Jain and Mirman (1999), Daher and Mirman (2006, 2007), Wang, Wang and Ren (2009), Zhou (2011), Kyle et al. (2011), Daher et al. (2012), Vitale (2012), Karam and Daher (2013), Daher et al. (2014, a), Liu et al. (2017), McLennan el al. (2017), Wang (2017) and Lambert (2018). Other extensions considered the dynamic settings of the Kyle model. Among others, we cite Holden and Subrahmanyam (1992), Caldentey and Stacchetti (2010), Daher el al. (2014, b) and Back et al. (2018).

[2]In a recent paper in the industrial organization literature, Mungan (2019) studied the welfare analysis in the presence of two producers, one risk averse and one risk neutral, sharing information regarding the cost of production.

There are few extensions of the Kyle (1985) model to the case of risk-averse insiders, we can cite for example Subrahmanyam (1991), Holden and Subrahmanyam (1994), Vitale (1995), Zhang (2004) and Baruch (2004). Except Subrahmanyam (1991), in most of the discrete models, the comparative analysis studies were found numerically and this was due to the introduction of the risk-aversion type of the insider which made the computational analysis quite complex. Subrahmanyam (1991) extended the Kyle (1985) model to the case of partially informed insider and proved analytically many of her comparative analysis with respect to the risk-neutral case. Holden and Subrahmanyam (1994) extended Kyle's (1985) multi-period auction model to include multiple risk-averse informed traders with long-lived information. Vitale (1995) generalized Kyle's (1985) model to the case of a risk-averse informed trader where the solution methods are based on LEQG dynamic programming problems. Tighe (1989) extended Kyle model to multiple informed traders, all risk-neutral. But none of these extensions, studied the case of multiple informed traders with different risk attitudes. In this paper, we investigate the effect of the risk attitudes of two insiders trading in the stock market. We follow a static version of the Kyle (1985) model with two insiders. Insider 1 is risk neutral while insider 2 is risk averse with negative exponential utility.

Our paper has two objectives. Similar to Subrahmanyam (1991), our first objective is to prove analytically the existence and uniqueness of the linear equilibrium of the model as well as carry out analytically all the comparative statics analysis with respect to the duopoly static model of Holden and Subrahmanyam (1994) and to the risk-neutral insiders duopoly case studied in Tighe (1989)[3]. It should be pointed out to the reader that in the risk-neutral case, the normality distribution of the exogenous variables together with the linear structure of the stock price, simplifies the existence and the characterization of the linear Bayesian equilibrium. Thus, the comparative statics analysis becomes straightforward. However, when the risk-aversion structure is introduced, the computation of the linear Bayesian equilibrium becomes less tractable and the analysis turns out to be more complex. In this paper, we overcome much of the involved complexities and provide exact results for the equilibrium outcomes as well as the comparative statics analysis.

Our second objective consists in a rigorous analysis of the impact of different

[3]Our paper belongs to a research direction which is interested in proving the existence (or not) and/or uniqueness of Kyle-type model equilibria. See for example, Nödeke and Tröger (2001,2006), Caldentey and Stacchetti (2010), Vitale (2012), Daher el al. (2014, b), McLennan el al. (2017), Lamebrt et al. (2018) and Back et al. (2018).

risk attitudes on the equilibrium outcomes, and mainly on market liquidity and market efficiency. Liquidity is an important aspect of financial markets for investors, researchers, and regulators. Vayanos and Wang (2013) surveyed theoretical and empirical work on market liquidity in the presence of six imperfections: participation costs, transaction costs, asymmetric information, imperfect competition, funding constraints, and search. They found that asymmetric information and imperfect competition increase the market depth measure λ with respect to the full information and perfect competition case. However, the relationship between market liquidity and risk aversion has been given relatively less attention by academicians. As pointed out by the European Central Bank (ECB) report in December 2007: *" the relationship between risk aversion and financial market liquidity is usually found to be negative-i.e. higher risk aversion is associated with lower market volatility-the interdependence between the two is quite complex"*[4]. Recent empirical results studied the impact of uncertainty on market liquidity. For example, Chung and Chuwonganant (2014) showed the market uncertainty measured by VIX, decreased the market liquidity in the US markets. Ma et al. (2019) showed that increased risk perception (measured by VIX) reduces liquidity around the world.

In this paper, we analytically characterize the relationship between the insiders's risk tolerance and market liquidity. First, we show that the level of risk-aversion affects the trading order of the risk-neutral insider. Indeed, the different types of risk attitudes, induce a non symmetric equilibrium trading orders and both insiders orders depend crucially on the level of the risk aversion. However, we show that regardless of the value of the risk aversion coefficient, the risk-neutral insider order is always greater (in absolute value) than the risk-averse insider order. Intuitively, this result is associated to the fact that the risk averse insider trades less aggressively than the risk-neutral one.

Second, our model reveals that similar to risk-averse Kyle type models (Subrahmanyam (1991) and Holden and Subrahmanyam (1994)) the market liquidity measure λ (defined by Kyle (1985) as the inverse of the market depth measure), depends on the the level of risk aversion. When compared to the Tighe (1989) model, i.e. when the two insiders are risk-neutral, we find that the market liquidity is reduced in our model than in Tighe (1989). Intuitively, the presence of the risk averse insider in our model, drives the insiders to trade less aggressively and thus reducing the market liquidity than in the

[4]For details, see [11]

presence of two risk-neutral insiders (Tighe 1989). Note that this result is consistent with the empirical work mentioned above.

However, the comparison between the market liquidity measures in our paper and in Holden and Subrahmanyam (1994) is not straightforward. We show how the risk-aversion coefficient is a fundamental determinant of this comparison as pointed out by the ECB report. Similarly to Subrahmanyam (1991), we show that the market liquidity measure in our paper is unimodal with respect to the level of risk-aversion, when it is compared to the case of two risk-averse insiders (Holden and Subrahmanyam (1994). In other words, we found a benchmark level of risk aversion, above which the market liquidity measure in our model is lower than the market liquidity measure in the case of two risk averse insider. On the other hand, the market liquidity measure in our model is greater than the market liquidity measure in the case of two risk averse insiders for a level of risk aversion which is less than the benchmark one.

Finally, we study the impact of different risk attitudes on price revelation. We show that the equilibrium price in our model reveals more (less) information than the stock price in Holden and Subrahmanyam (1994) (Tighe (1989))

The paper is organized as follows: In section 2, we describe the model and characterize the unique linear Bayesian equilibrium of the model. In section 3, we conduct a comparative statics analysis of the equilibrium outcomes with respect to Holden and Subrahmanyam (1994) and Tighe (1989). All proofs are relegated to the Appendix.

2 The Model

We consider a static version of the Kyle (1985) model with two insiders. The economy consists of one financial asset. The underlying value of the asset is denoted by $\tilde{z}$. The prior distribution of $\tilde{z}$ is normal with mean $\bar{z}$ (assumed to be positive) and variance σ_z^2. The two insiders exhibit different attitudes toward risk. We assume that insider 1 is risk neutral while insider 2 is risk averse with negative exponential utility and risk-aversion coefficient A expressed as:

$$U(x) = -e^{-Ax}.$$

The two insiders trade in the stock market based on their inside information. There are three types of agents. First, there are two rational insiders, each of whom knows the realization z of $\tilde{z}$. Second, there are the (nonrational) noise traders, representing small investors with no information on z. The aggregate noise trade is assumed to be a random variable $\tilde{u}$, which is normally distributed with mean zero and variance σ_u^2. Finally, there are $K(K \geq 2)$ risk-neutral market makers who act like Bertrand competitors. We assume, as in Kyle (1985), that the market makers observe the total order flow signal. We assume that $\tilde{z}$ and $\tilde{u}$ are independent.

Following Kyle (1985), the trading mechanism is organized in two steps. In step one, a linear pricing rule and optimal order rules are determined by the market makers and the insiders, respectively, as a Bayesian Nash equilibrium. The market makers determine a (linear) pricing rule p, based on their a priori beliefs, where p is a measurable function $p : \mathbb{R} \longrightarrow \mathbb{R}$. Each insider chooses a stock trade function $\tilde{x}_i = x_i(\tilde{z})$, where $x_i : \mathbb{R} \longrightarrow \mathbb{R}$ is a measurable function. In the second step, the insiders observe the realization z of $\tilde{z}$ (random variables are denoted with a tilde. Realized values lack the tilde. The mean of the random variable is denoted with a bar) and submit their stock orders to the market makers based on the equilibrium stock trade functions. The market makers also receive orders from the noise traders, all these orders arrive as a total order flow signal $\tilde{y} = x_1(\tilde{z}) + x_2(\tilde{z}) + \tilde{u} = x(\tilde{z}) + \tilde{u}$. The order flow signal is used by the market makers to set the price $\tilde{p} = p(\tilde{y})$, based on the equilibrium price function, to clear the market. The insiders know only the value of $\tilde{z}$ and do not know the values of $\tilde{u}$ and $\tilde{y}$ before their order flow decisions are made. Moreover, each market maker does not know the realization z of $\tilde{z}$ but only knows its distribution. Finally, the market makers cannot observe either x_1, x_2 or u.

The profits for each of the two rational traders are given, respectively, by

$$\Pi_1 := (\tilde{z} - p) \cdot \tilde{x}_1 \text{ and } \Pi_2 := (\tilde{z} - p) \cdot \tilde{x}_2$$

This is a game of incomplete information because the market makers unlike the insiders do not know the realization of $\tilde{z}$. Hence, we seek for a Bayesian-Nash equilibrium defined as follows,

Definition 1 *A Bayesian-Nash equilibrium is a vector of three functions* $[x_1(.), x_2(.), p(.)]$ *such that:*

(a) Profit maximization of the risk neutral insider, i.e. insider 1*,*

$$\begin{aligned} &E[(\tilde{z} - p(x_1(\tilde{z}) + x_2(\tilde{z}) + \tilde{u}))x_1(\tilde{z})] \\ &\qquad \geq E[(\tilde{z} - p(x_1'(\tilde{z}) + x_2(z) + \tilde{u}))x_1'(\tilde{z})] \end{aligned} \tag{1}$$

for any alternative trading strategy $x_1'(\tilde{z})$*;*

(b) *Profit maximization of the risk-averse insider, i.e. insider* 2,

$$E[-e^{-A(\tilde{z} - p(x_1(\tilde{z}) + x_2(\tilde{z}) + \tilde{u}))x_2(\tilde{z})}]$$
$$\geq E[-e^{-A(\tilde{z} - p(x_1(\tilde{z}) + x_2'(\tilde{z}) + \tilde{u}))x_2'(\tilde{z})}] \quad (2)$$

for any alternative trading strategy $x_2'(\tilde{z})$*;*

(c) *Semi-Strong Market Efficiency: The pricing rule* $p(.)$ *satisfies,*

$$p(\tilde{y}) = E[\tilde{z}|\tilde{y}] \quad (3)$$

An equilibrium is linear if there exists constants μ and λ, such that,

$$\forall\ y, \quad p(y) = \mu + \lambda y. \quad (4)$$

Note that conditions (1) and (2) define optimal strategies of the two insiders while condition (3) guarantees the zero expected profits for the market makers. The stock price, set by the market makers, is equal to the conditional expectation of the asset value given their information. We restrict our study to linear equilibrium. The normal distributions of the exogenous random variables, enable us to derive and to prove the existence of a unique linear equilibrium.

In the following Proposition, we characterize the unique linear equilibrium of the model.

Proposition 1 *In the presence of one risk neutral and one risk averse informed traders, a linear equilibrium exists and it is unique. It is characterized by,*

(i) $x_1(\tilde{z}) = \dfrac{(\tilde{z} - \mu)(1 + A\lambda^*\sigma_u^2)}{\lambda^*(3 + 2A\lambda^*\sigma_u^2)}$ and $x_2(\tilde{z}) = \dfrac{(\tilde{z} - \mu)}{\lambda^*(3 + 2A\lambda^*\sigma_u^2)}$

(ii) $p(\tilde{y}) = \mu + \lambda^*\tilde{y}$*, where* $\mu = \bar{z}$*, and* λ^* *is the unique strictly positive root of the following quartic equation:*

$$4A^2\sigma_u^4\lambda^{\mathbf{4}} + 12A\sigma_u^2\lambda^{\mathbf{3}} + (9 - A^2\sigma_u^2\sigma_z^2)\lambda^{\mathbf{2}} - 3A\sigma_z^2\lambda - 2\frac{\sigma_z^2}{\sigma_u^2} = 0. \quad (5)$$

Proof: See Appendix A.

Discussion of the equilibrium: First, note that the relationship between this paper, Holden and Subrahmanyam (1994) and Tighe (1989) should be clear. Indeed, Holden and Subrahmanyam (1994) considered the case of two risk-averse insiders. Moreover, Tighe (1989) considered the Kyle (1985) model with two risk neutral insiders. Hence, our model can be seen as a hybrid-model in comparison with Holden and Subrahmanyam (1994) and with Tighe (1989). Consequently, in this paper we will be able to show the effect of different risk-attitudes on equilibrium outcomes.

Second, it should be pointed out that our hybrid risk attitudes structure affects the equilibrium trading orders. Proposition 1 shows that they are not symmetric as in Holden and Subrahmanyam (1994) and in Tighe (1989). Specifically, the risk neutral insider (insider 1) trades more (in absolute value) than the risk-averse insider (insider 2). Although both insiders are fully informed about the realization z of the risky asset, we notice that the risk neutrality dominates the risk-aversion in term of trading. Intuitively, this result is associated with the fact that the risk averse insider trades less aggressively than the risk-neutral one.

Third, Proposition 1 reveals the impact of the imperfect competition on the equilibrium trading orders. Indeed, the order of the risk-neutral insider (insider 1) depends on the risk-aversion coefficient A, i.e. insider 1 takes into account all the possible decisions of insider 2 in her maximization problem.

Fourth, as in Holden and Subrahmanyam (1994), our model reduces to the case of two risk-neutral insiders, i.e. it converges to the duopoly competition model of Kyle (1985), when the risk-aversion coefficient A converges to zero.[5] This is due to the fact, that in this case, the risk averse insider will behave like the risk neutral insider and thus their trades become symmetric and equal to the ones in the two insiders case of the Kyle (1985) model.[6]

On the other hand, when insider 2 is too risk-averse (A is large), Proposition 1 shows that the risk-averse insider (insider 2) has no incentive to trade ($x_2(\tilde{z}) = 0$) and thus our model converges to the one risk-neutral Kyle (1985) model in which the order of insider 1 is equal to the one in Kyle (1985).[7] To better understand this result, one should clarify the interdependence be-

[5] The risk aversion utility function adopted in our model converges to risk neutral one when the risk aversion coefficient converges to zero (see Marks (2014)).

[6] See Tighe (1989) or Holden and Subrahmanyam (1992).

[7] As suggested by one referee, the results of this case will be stated in Lemma 1 and its proof is relegated in the Appendix B.

tween the risk aversion coefficient A, the liquidity market measure λ^*, the risk neutrality and the market imperfection. Indeed, λ^* can obviously be seen as a function of A (see equation 5). Moreover, the expression of x_2 in Proposition 1, together with the fact that λ^* is asymptotically independent [8] from A show that the risk-averse trades will converge to zero. Consequently, the expression of x_1 in Proposition 1 will tend to be equal to the single insider trades as in Kyle (1985).

Note that our result (in the case of large values for A) differs fundamentally from the case of 2 risk-averse insiders (Holden and Subrahmanyam (1994)). Indeed, in their case, the trading orders of the two insiders are equal and both converge to zero for large values of A.[9] The intuition for this result is that when the insiders are too risk-averse, they tend to trade less aggressively making higher the market liquidity. Consequently, as response to a such increase, they have the incentive not to trade. However, the different risk-attitudes of the insiders in our model, have different effects on their corresponding trades. The risk-averse insider prefers not to trade in that case. Taking into account of his/her decision, the risk-neutral insider exploits his/her private information and trades like the single insider of the Kyle (1985) model. In the following Lemma (its proof is stated in Appendix B), we summarize the effects of large values of the risk aversion coefficient on the insiders trading orders.

Lemma 1 *When the risk aversion coefficient A is too large, the risk averse insider (insider 2) has no incentive to trade while the risk neutral insider (insider 1) trading order is equal to the monopolistic insider trading of the Kyle (1985) model.*

Fifth, Proposition 1 reveals very interesting results about the risk tolerance effect on the insiders trades, when the liquidity trades are too noisy (σ_u^2 is large). Similar to the 2-risk averse insiders model and to the 2 risk-neutral model, the insiders'trades in our model are increasing with respect to an increase in the liquidity trading noise. However, Proposition 1 shows that the risk-attitudes and the market imperfection structure in our model, drive the risk-neutral trading order to be linearly dependent on the risk-averse trading order. Indeed, considering the ratio of the two insiders in Proposition 1, we obtain

$$\frac{x_1(\tilde{z})}{x_2(\tilde{z})} = 1 + A\lambda^*\sigma_u^2 \tag{6}$$

[8] Lemma 3 shows that λ^* is asymptotically convergent to the λ of Kyle (1985).

[9] We invite the reader to check Proposition 3 and Lemma 4 in the Appendix.

Note that the relation between the insiders' trades depend crucially on the relation between the liquidity trading noise and the liquidity measure λ^* through the expression $\lambda^*\sigma_u^2$. We show in the appendix that $\lambda^*\sigma_u^2$ is a linear function of σ_u. Thus, an increase in the noise trades induces a non symmetric increase in both insiders trades. Intuitively, in this case, we notice that the risk neutral exploits his/her informational power to trade more aggressively than the risk averse insider.

Finally, it should be pointed out that the risk-averse property has a direct effect on the computation of the equilibrium outcomes and more specifically on the market depth parameter λ. Indeed, considering the static version of Kyle (1985) and its extensions when the insiders are risk neutral[10] we noticed that the market depth parameter is directly computed. On the other hand, when insiders are risk-averse, the market depth λ is the solution of a quartic equation.[11]

In the next section we develop the comparative static analysis. We will analyze the market depth and the stock price informativeness compared to their corresponding expressions in Holden and Subrahmanyam (1994) and Tighe (1989).

3 Comparative Statics

3.1 market depth parameter λ

In this section, we compare our market depth parameter λ to the ones in Holden and Subrahmanyam (1994) and Tighe (1989). Lemma 2 shows that market depth parameter in Tighe (1989) model is higher than the market depth parameter in our model. However, the relation with respect to Holden and Subrahmanyam (1994) is not straightforward. Indeed,

Lemma 2 *The market depth parameter λ^* in our model is greater than the market depth in Tighe (1989). However, the relation with respect to Holden and Subrahmanyam (1994) depends on the risk aversion degree and given as*

[10]See for example Tighe (1989), Jain and Mirman (1999), Daher and Mirman (2007), Wang et al. (2009), Daher et al. (2012) and Karam and Daher (2013). In Daher et al. (2014, a), the computation of the market depth parameter is quite complex (but not a solution of a quartic equation) due to the real market structure.

[11]See for example Vitale (1995), Holden and Subrahmanyam (1994), Subrahmanyam(1991), Zhang (2004).

follows.

(a)

$$\lambda^T \leq \lambda^*$$

(b) There exists a risk-aversion coefficient level A^* such that,

$$\begin{cases} \lambda^* > \lambda^{HS} \text{ forall } A < A^* \\ \lambda^* < \lambda^{HS} \text{ forall } A > A^* \end{cases} \tag{7}$$

where λ^T and λ^{HS} refer to the market depth parameters in Tighe (1989) and Holden and Subrahmanyam (1994) respectively.

Proof: See Appendix C.

Note that Lemma 2 plays a central key in providing almost all the results in this paper. All the articles which extended the static Kyle (1985) models with risk neutral insiders, found that the parameter λ was explicitly characterized as a function of the exogenous variables of these models. However, in the presence of risk aversion, the market depth parameter is implicitly characterized as a solution of a quartic equation.[12] Thus, most of the comparative results in the literature, were made numerically. But, in our paper, the results of Lemma 2 are proved analytically and it will be used as foundation to subsequent results of this paper.

Lemma 2 shows the impact of different risk attitudes on the market depth parameter λ. In part a) of Lemma 2, we highlight the impact of risk-aversion on the market liquidity measure in our paper when compared to the two risk-neutral insiders case studied in Tighe (1989). Indeed, Lemma 2 shows that the market depth parameter is greater in our model than in Tighe (1989). In other words, when the two insiders are risk neutral (Tighe 1989), the market is deeper (as defined by Kyle (1985) to be $\frac{1}{\lambda}$) than the market in the presence of two insiders with one of them is risk-averse (our model).

[12]See Subrahmanyam (1991), Holden and Subrahmanyam (1994), Vitale (1995) and many related papers.

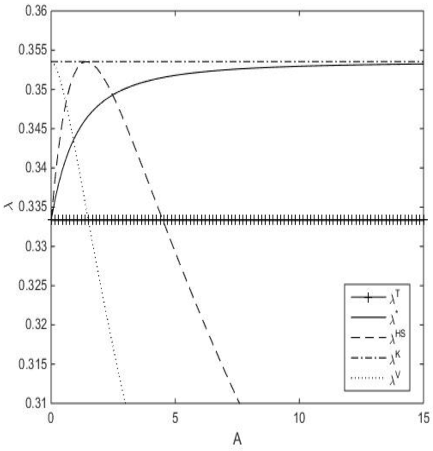

Figure 1: The graph of λ as function of the risk aversion coefficient A when $\sigma_z^2 = 1$ and $\sigma_u^2 = 2$

In Figure 1, we plot the market depth parameter λ as a function of the risk aversion coefficient A. Obviously, the depth parameter in Kyle (1985) denoted by λ^K and the depth parameter in Tighe (1989) denoted by λ^T correspond to the case when $A = 0$ and they are both represented by horizontal lines. We notice that the depth parameter λ^* in our model lies between λ^K (the risk neutral monopoly insider model) and λ^T (the risk neutral duopoly insider model). Note that in the risk-neutral case, an increase in the number of the informed traders, decreases the market depth parameter ($\lambda^T \leq \lambda^K$), i.e. increases the market depth. Lemma 2 part a), reveals that this result also holds when one of the insiders is risk-averse. In other words, we notice the Cournot competition among the insiders will also increase the market depth when we go from the monopoly case (risk neutral insider) to the duopoly case (one of the two insiders is at least risk neutral).

However, Lemma 2 part a) shows that Cournot competition among the risk neutral insiders (Tighe 1989) increases the market depth than in the presence of Cournot competition among insiders with the hybrid risk tolerance (our model). Intuitively, this is due to the fact the risk-averse insider trades less aggressively than the risk neural one and thus decreasing the market depth when compared to the two risk neutral insiders case.

We turn now to discuss the comparison between the market depth parameters in our paper and in Holden and Subrahmanyam (1994) model. Indeed, the comparison seems to be ambiguous and not intuitive. Specifically, following the analysis of part a) of Lemma 2, one expects that the presence of the risk-neutral insider in our model will increase the market liquidity with respect to the two risk averse insiders model of Holden and Subrahmanyam (1994). But, the second part of Lemma 2 shows that this observation does not hold. Instead, it shows the existence of a unique risk aversion coefficient, A^*, before which the market depth parameter in our model is less than the market depth parameter in the presence of two risk averse insiders (Holden and Subrahmanyam (1994) model). Moreover, for the values of risk aversion coefficients greater than A^*, the market depth parameter in our model is greater than the market depth parameter in the presence of two risk averse insiders. Thus, the risk aversion degree is a principal determinant of market liquidity.

To better understand the effect of risk aversion on the market depth parameter, we decide to plot in Figure 1, additional to λ^K, λ^T and λ^*, λ^V which corresponds to market depth parameter in the case of a single risk-averse insider (Vitale 1995) and λ^{HS} that corresponds to market depth parameter in the case of two risk-averse insiders both knowing the underlying value of the risky asset $\tilde{z}$ (Holden and Subrahmanyam (1994)).

Note that when A converges to 0, λ^V corresponds to λ^K in Kyle (1985). Similarly the market depth parameter in our model and in Holden and Subrahmanyam (1994) converges to λ^T (Tighe 1989) when $A = 0$.

First, it should be pointed out that Figure 1 shows that in the presence of one risk-averse insider (Vitale 1995), the market depth parameter λ^V is always less than the market depth parameter in the case of risk-neutral insider (Kyle 1985), λ^K. This result is reversed when we compare our model's market depth parameter λ^* to λ^T (part a) of Lemma 2). Hence, the effect of risk-neutrality on the market depth dominates the effect of the risk-aversion on the market depth, when we add another risk neutral insider to Kyle model (Tighe 1989) and to Vitale model (our model)

Second, part b) of Lemma 2 shows how the effect of risk-aversion on the market depth parameter is crucial when we compare our hybrid duopoly model to the risk-aversion duopoly model of Holden and Subrahmanyam (1994). Note that going from the risk-aversion monopoly case (Vitale 1995) to the risk-aversion duopoly case (Holden and Subrahmanyam (1994)), Figure 1 shows that the market depth parameter in both models is not monotonic (in contrast to the risk neutral case when we compare the market depth param-

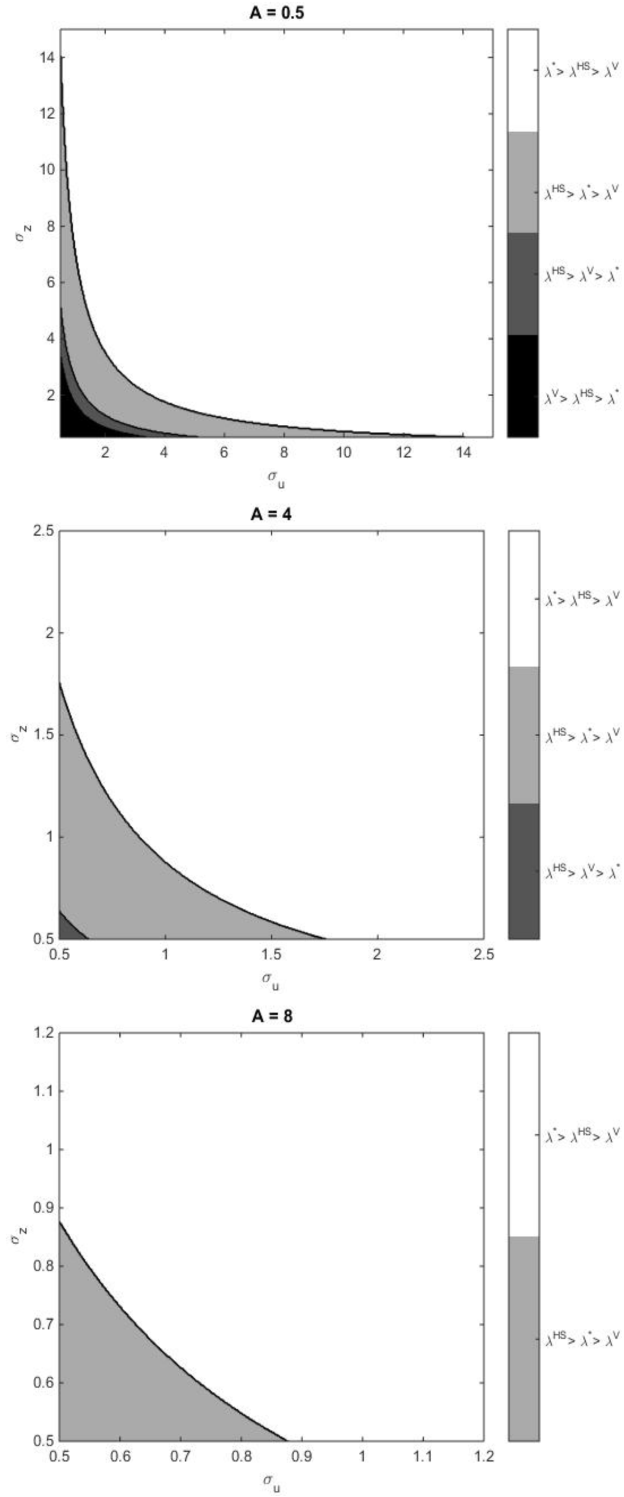

Figure 2: Regions of comparison between λ^V, λ^{HS} and λ^* as functions of σ_u and σ_z, for $A = 0.5,\ A = 4$ and $A = 8$.

eters in the monopoly case of Kyle 1985 and in the duopoly case of Tighe 1989) and more specifically, it is unimodal with respect to the risk-aversion coefficient. Thus the impact of risk-aversion on the market depth is the key point of this relation.

For this purpose, we derive in Figure 2, for three values of A ($A = 0.5,\ A = 4$ and $A = 8$), the regions of comparison between λ^*, λ^V and λ^{HS}. Figure 2 reveals interesting results regarding the impact of risk tolerance on the market liquidity. First note that an increase in the risk aversion, reduces the comparison regions among the market depth parameters of the three corresponding models. Such result, on one hand, reflects the unimodal property of the market depth parameters in the presence of only the risk aversion structure, i.e. the monopoly case, λ^V, and the duopoly case, λ^{HS}. On the other hand, it reflects the monotonic property of the market depth parameter, λ^*, in our model, as highlighted by the white area in the upper side of the each of the cases'graphs .

Second, when $A = 0.5$, the dark black area representing the case in which $\lambda^V > \lambda^{HS} > \lambda^*$ and the light black area representing the case in which $\lambda^{HS} > \lambda^V > \lambda^*$ show that the market depth parameter in the risk averse monopoly case, λ^V, is either greater than the market depth parameters in the case of duopoly competition among the insiders (the dark black area) or greater than the market depth parameter in the case of duopoly competition among the insiders with the hybrid risk tolerance structure (the light black area). Intuitively, when A decreases the risk averse monopolist model tends more closely to the risk neutral monopolist model. Hence, this will induce an increase in the trading aggressiveness of the insider which in turn justifies the increase in the market liquidity parameter λ^V. Moreover, remark that when A increases, the dark black area and the light black area disappear which reflects the effect of the risk aversion degree on the market liquidity.

It should be pointed out that Holden and Subrahmanyam (1994) focused on the effect of the dynamic structure of trading and compared their results to the Kyle model. However, in the static case with 2 insiders, the relation between λ^{HS} and λ^T is quite similar to the relation between λ^{HS} and λ^*. In sum, the relation between the market depth parameters in the case of monopoly (Vitale 1994 and Kyle 1985) is monotonic but it does not hold when we consider the duopoly case (Tighe 1989 and Holden and Subrahmanyam 1994)(See Figure 1) .[13]

Consequently, we show that the market liquidity is directly affected not only by the number of trading rounds (Holden and Subrahmanyam 1994) or the number of informed traders (Subrahmanyam (1991)), but also by the risk attitudes of the insiders.

3.2 Information Revelation

In this section we discuss the information revelation in our model and compare it to the results obtained in Tighe (1989) and Holden and Subrahmanyam (1994). By adopting the same measure of information, i.e. the conditional variance of the liquidation asset value given the total order flow, we obtain the following result.

Proposition 2 *The equilibrium price reveals less information in the presence of 1 risk-neutral and 1 risk-averse insiders (our model) than in the presence of two risk-neutral insiders (Tighe 1989). However, with two risk-averse insiders (Holden and Subrahmanyam 1994), the equilibrium price is less revealing than in our model.*

Proposition 2 highlights the impact of risk attitude on price informativeness. First, it should be noted that the hybrid structure of the risk attitudes of the insiders does not alter the relation of the price informativeness with respect to the risk neutral case. Indeed, Proposition 2 shows that the presence of two risk neutral insiders increases price revelation of information with respect to our model in which one of the two insiders is risk averse. This holds in Vitale (1995) model which considered the risk-averse monopolistic case and compared the price revelation to the result in Kyle (1985). For the duopoly case, Holden and Subrahmanyam (1994) also obtained the same result as well. Intuitively, this is due to the fact that risk-averse insider trades less aggressively than the risk neutral insider. Thus, pushing the price to be less informative than the price in the risk neutral case.

The second part of Proposition 2 states that the stock price conveys more information in our model than in Holden and Subrahmanyam (1994) model. The intuition behind this result is not straightforward since the relation between the trading aggressiveness of the insiders in both models, reflected by

[13] Subrahmanyam (1991) found the same result with finite number of partially informed traders and analyzed the impact of the number of the insiders on the market depth parameter.

the market liquidity parameters, λ^{HS} and λ^{*}, is not monotonic (See Lemma 2). To better understand the origin of the relation between the information revelation in our model and in Holden and Subrahmanyam (1994), we compare the information revelation results in Vitale (1995) and in Holden and Subrahmanyam (1994). Recall that similar to our case, the relation between λ^{HS} and λ^{V} is not monotonic (See Figure 1) and thus, checking their price information results is quite helpful. Computing the conditional variance in Vitale (1995) and in Holden and Subrahmanyam (1994), we found that the stock price in Vitale (1995) reveals more information than in the duopoly case studied in Holden and Subrahmanyam (1994) regardless of the risk-aversion level. This result shows that an increase in the number of risk-averse insiders decreases the informativeness of the stock price regardless of the trading aggressiveness of the market.

Back to our model, we noticed that the presence of risk neutral insider has a direct impact on the price information revelation when compared to the two risk aversion duopoly case studied by Holden and Subrahmanyam (1994). Moreover, similar to Subrahmanyam (1991), we found two common results related to price efficiency. First, note that an increase in the variance of liquidity trading decreases the price efficiency. This similarity shows that the presence of the risk-averse insider has more effect on the price efficiency than the risk neutral insider does. Second, it should be pointed out that price efficiency is decreasing in the risk-aversion coefficient. In other words, when insider 2 becomes more risk-averse (increasing the risk-aversion coefficient), her trades are less aggressive. This effect has a direct increase on the market depth parameter λ (Lemma 2, part b)) and thus reducing the price efficiency.

Finally, it is worth noting that our model highlights the impact of the risk attitudes on the price efficiency. Indeed, although one of the insiders is risk neutral, the price efficiency is directly affected by the risk-averse insider's behavior.

4 Conclusion

In this paper, we analytically proved the existence of a unique linear equilibrium and examined the impact of different risk attitudes on the financial decisions of two insiders (insider 1 is risk neutral while insider 2 is risk averse) in a static version of the Kyle (1985) model. Modeling risk-aversion in Kyle type models is quite complex and several assumptions are needed for tractability results. For example we assume that the market maker receives one signal of information for, namely the noisy order flow. This assumption is less appropriate for the case of security analysts, who are likely to possess more diverse signals. We leave this as open question for future research.

From a regulatory perspective, our analysis shows how the impact the of risk aversion on the insiders'decisions and equilibrium variables in a one shot game is quite important while the dynamic models (see Holden and Subrahmanyam (1994)) studied the impact of risk aversion on the equilibrium outcomes over time. Such observation opens the doors to look more closely on understanding the role of risk tolerance in the world of insider trading.

Appendices

Appendix A: proof of Proposition 1

We begin by the maximization problem of the risk neutral insider, i.e. insider 1. The decision rule of insider 1 is the function $x_1(\tilde{z})$. The expected profits after plugging the linear pricing function, become,

$$E[(\tilde{z} - p(x_1(\tilde{z}) + x_2(\tilde{z}) + \tilde{u}))x_1(\tilde{z})] = E[(\tilde{z} - \mu - \lambda(x_1(\tilde{z}) + x_2(\tilde{z}) + \tilde{u}))x_1(\tilde{z})]$$

The first and second order conditions are

$$x_1(\tilde{z}) = \frac{\tilde{z} - \mu - \lambda x_2(\tilde{z})}{2\lambda} \qquad \text{and} \qquad \lambda > 0. \tag{8}$$

We move now to solve the maximization of the risk averse insider, i.e. insider 2. The decision rule of insider 2 is the function $x_2(\tilde{z}))$. The expected profits after plugging the linear pricing function, become,

$$E[-e^{-A(\tilde{z}-p(x_1(\tilde{z})+x_2(\tilde{z})+\tilde{u}))x_2(\tilde{z})}|\tilde{z}] = E[-e^{-A(\tilde{z}-\mu-\lambda(x_1(\tilde{z})+x_2(\tilde{z})+\tilde{u}))x_2(\tilde{z})}|\tilde{z}]$$

Using the normality and the independency of the noise traders orders $\tilde{u}$, the first and the second order conditions are

$$x_2(\tilde{z}) = \frac{\tilde{z} - \mu - \lambda x_1(\tilde{z})}{\lambda(2 + A\lambda\sigma_u^2)} \qquad \text{and} \qquad \lambda(2 + A\lambda\sigma_u^2) > 0. \tag{9}$$

Combining equations 8 and 9, we obtain

$$x_1(\tilde{z}) = \frac{(\tilde{z} - \mu)(1 + A\lambda\sigma_u^2)}{\lambda(3 + 2A\lambda\sigma_u^2)} \qquad \text{and} \qquad x_2(\tilde{z}) = \frac{(\tilde{z} - \mu)}{\lambda(3 + 2A\lambda\sigma_u^2)} \tag{10}$$

Regarding the price function coefficients, μ and λ, first note that the semi-strong market efficiency together with linear price function assumption lead to,

$$\mu + \lambda\tilde{r} = E[\tilde{z}|\tilde{r}] \tag{11}$$

Evaluating the expectation on both sides of equation 11 and then applying the law of iterated expectations, we obtain

$$\mu + \lambda\bar{r} = \bar{z} \tag{12}$$

where $\bar{r} = \bar{x}_1 + \bar{x}_2 + \bar{u} = \bar{x}_1 + \bar{x}_2$. Using equation 10 to find the expression of $\bar{r}$ and plugging the result in equation 12, we obtain

$$\mu = \bar{z} \tag{13}$$

To complete the proof, it remains to find a unique value of the price function slope λ. Indeed, note that the linear expressions of the insiders strategies decisions, $\tilde{x}_1$ and $\tilde{x}_2$, induce the normality distribution of the total order flow $\tilde{r}$. Thus, by applying the projection theorem to equation 11, we have

$$\lambda = \frac{cov(\tilde{z}, \tilde{r})}{var(\tilde{r})} \tag{14}$$

Evaluating the right-hand side of equation 14 and after certain arrangement we find that λ is a root of the following quadric equation

$$4A^2\sigma_u^4\lambda^{\mathbf{4}} + 12A\sigma_u^2\lambda^{\mathbf{3}} + (9 - A^2\sigma_u^2\sigma_z^2)\lambda^{\mathbf{2}} - 3A\sigma_z^2\lambda - 2\frac{\sigma_z^2}{\sigma_u^2} = 0. \tag{15}$$

By Descartes' rule of signs,[14] there is only one positive root satisfying the second order condition which ends the proof. ■

Appendix B: proof of Lemma 1

We first prove the results of Lemma 3 which will be used in many of the subsequent proofs. It should be pointed out that Lemma 3 provides a quantitative characterization for λ as function of A.

Lemma 3 *The following statements hold*

[14] **(Theorem: Descartes'rule of signs)** If the terms of a single variable polynomial with real coefficients are ordered by descending variable exponent, then the number of **positive** roots of the polynomial is either equal to the number of sign differences between consecutive nonzero coefficients, or less than it by a multiple of two.

i) $\lambda = \dfrac{1}{2}\dfrac{\sigma_z}{\sigma_u}$ *is an asymptote in the* (A, λ) *plane. Moreover, the curve defined by equation 5, will not cross its asymptote.*

ii) λ *is increasing in* A.

Proof of i). To find the asymptote, we view the quartic (equation 5)

$$f(\lambda, A) = 4A^2\sigma_u^4\lambda^4 + 12A\sigma_u^2\lambda^3 + \left(9 - A^2\sigma_u^2\sigma_z^2\right)\lambda^2 - 3A\sigma_z^2\lambda - 2\frac{\sigma_z^2}{\sigma_u^2} = 0, \quad (16)$$

as a plane algebraic curve.

Note that most of the curves represented by $f(\lambda, A; \sigma_z, \sigma_u)$ are irreducible. We work under this assumption. The projective curve corresponding to the affine curve $f = 0$ is

$$F(\lambda, A, Z) = 4A^2\sigma_u^4\lambda^4 + 12A\sigma_u^2\lambda^3 Z^2 + \left(9Z^4 - A^2\sigma_u^2\sigma_z^2 Z^2\right)\lambda^2 - 3A\sigma_z^2\lambda Z^4 - 2\frac{\sigma_z^2}{\sigma_u^2}Z^6 = 0. \quad (17)$$

It is not difficult to see that this projective curve has the singular points $(0:1:0)$ and $(1:0:0)$. Now, we consider the affine view $Z = 1$. Put $Z = 0$ into the equation (17) to get $A^2\lambda^4 = 0$. So the points $(1:0:0)$ and $(0:1:0)$ are at infinity. The second of these points is on the A-axis, and as noted above it is singular. Set $A = 1$ in the equation of the projective curve (17), we thus obtain the affine curve

$$4\sigma_u^4\lambda^4 + 12\sigma_u^2\lambda^3 Z^2 + \left(9Z^4 - \sigma_u^2\sigma_z^2 Z^2\right)\lambda^2 - 3\sigma_z^2\lambda Z^4 - 2\frac{\sigma_z^2}{\sigma_u^2}Z^6 = 0.$$

The lower order terms $4\sigma_u^4\lambda^4 - \sigma_u^2\sigma_z^2 Z^2\lambda^2$ give us the distinct tangents $\lambda = 0, \lambda = \frac{1}{2}\sigma_z\frac{Z}{\sigma_u}, \lambda = -\frac{1}{2}\sigma_z\frac{Z}{\sigma_u}$. Now dehomogenize to obtain, for $\lambda > 0$, the affine asymptote $\lambda = \frac{\sigma_z}{2\sigma_u}$.

Next we show that the curve will not cross its asymptote. The equation of the affine curve above (16) can be written as

$$\left(4\sigma_u^2\lambda^2 - \sigma_z^2\right)\left(\left(\sigma_u^2\lambda A + 3\right)\lambda A + \frac{2}{\sigma_u^2}\right) = -\lambda^2 < 0.$$

This then implies that $4\sigma_u^2\lambda^2 - \sigma_z^2 < 0$, and so $\lambda < \frac{\sigma_z}{2\sigma_u}$ for $\lambda > 0$.
■

Proof of ii) In this part of the proof, we show that λ is increasing with respect to A where σ_u and σ_z are treated as parameters. By implicit differentiation, we obtain

$$\frac{d\lambda}{dA} = -\frac{\frac{\partial f}{\partial A}}{\frac{\partial f}{\partial \lambda}},$$

where $\frac{\partial f}{\partial \lambda}$ must not be zero. It is then simple to find that

$$\frac{d\lambda}{dA} = \frac{-8A\sigma_u^4\lambda^4 - 12\sigma_u^2\lambda^3 + 2A\sigma_u^2\sigma_z^2\lambda^2 + 3\sigma_z^2\lambda}{16A^2\sigma_u^4\lambda^3 + 36A\sigma_u^2\lambda^2 + 2\left(9 - A^2\sigma_u^2\sigma_z^2\right)\lambda - 3A\sigma_z^2}.$$

Note that the numerator has one positive root - by Descartes' rule (since we are interested in $A \geq 0$). Now factor the expression, and so we have

$$\begin{aligned} \frac{d\lambda}{dA} &= \frac{-\lambda\left(2\sigma_u\lambda - \sigma_z\right)\left(2\sigma_u\lambda + \sigma_z\right)\left(2\lambda A\sigma_u^2 + 3\right)}{\left(2\lambda A\sigma_u^2 + 3\right)\left(8A\sigma_u^2\lambda^2 + 6\lambda - A\sigma_z^2\right)} \\ &= -\lambda\left(2\sigma_u\lambda - \sigma_z\right)\frac{2\sigma_u\lambda + \sigma_z}{8A\sigma_u^2\lambda^2 + 6\lambda - A\sigma_z^2} \end{aligned}$$

Clearly, $\frac{d\lambda}{dA} > 0$ if $2\sigma_u\lambda - \sigma_z < 0$, or $\lambda < \frac{\sigma_z}{2\sigma_u}$ and provided that $8A\sigma_u^2\lambda^2 + 6\lambda - A\sigma_z^2 > 0$.

Consider the upper bound (from part i)) to see what is the value of $f(\lambda, A)$ for this value of λ. The evaluation gives that,

$$4A^2\sigma_u^4\left(\frac{\sigma_z}{2\sigma_u}\right)^4 + 12A\sigma_u^2\left(\frac{\sigma_z}{2\sigma_u}\right)^3 + \left(9 - A^2\sigma_u^2\sigma_z^2\right)\left(\frac{\sigma_z}{2\sigma_u}\right)^2 - 3A\sigma_z^2\left(\frac{\sigma_z}{2\sigma_u}\right) - 2\frac{\sigma_z^2}{\sigma_u^2} = \frac{1}{4}\frac{\sigma_z^2}{\sigma_u^2} > 0.$$

On the other hand, for $\lambda = \dfrac{\sqrt{2}\sigma_z}{3\sigma_u}$, we have

$$\begin{aligned} & 4A^2\sigma_u^4\left(\frac{\sqrt{2}\sigma_z}{3\sigma_u}\right)^4 + 12A\sigma_u^2\left(\frac{\sqrt{2}\sigma_z}{3\sigma_u}\right)^3 + \left(9 - A^2\sigma_u^2\sigma_z^2\right)\left(\frac{\sqrt{2}\sigma_z}{3\sigma_u}\right)^2 - 3A\sigma_z^2\left(\frac{\sqrt{2}\sigma_z}{3\sigma_u}\right) - 2\frac{\sigma_z^2}{\sigma_u^2} \\ = & -\frac{1}{81}\sigma_z^3\frac{A}{\sigma_u}\left(2A\sigma_z\sigma_u + 9\sqrt{2}\right) < 0. \end{aligned}$$

This shows that the unique root λ^* is bracketed between $\frac{\sqrt{2}\sigma_z}{3\sigma_u}$ and $\frac{\sigma_z}{2\sigma_u}$ (by the Intermediate Value Theorem).

It remains to show that $8A\sigma_u^2\lambda^2 + 6\lambda - A\sigma_z^2 > 0$. Solve

$$8A\sigma_u^2\lambda^2 + 6\lambda - A\sigma_z^2 = 0$$

for A, we get $A = -6\frac{\lambda}{8\sigma_u^2\lambda^2 - \sigma_z^2}$. As we are interested in the part of the domain where $\lambda > 0$ and $A > 0$, then we should have $8\sigma_u^2\lambda^2 - \sigma_z^2 < 0$. That is $\lambda^2 < \frac{\sigma_z^2}{8\sigma_u^2} = \frac{1}{2}\left(\frac{\sigma_z}{2\sigma_u}\right)^2$. But this is lower than the square of the lower bound as $\left(\frac{\sqrt{2}\sigma_z}{3\sigma_u}\right)^2 - \frac{1}{2}\left(\frac{\sigma_z}{2\sigma_u}\right)^2 = \frac{7}{72}\frac{\sigma_z^2}{\sigma_u^2}$. Now as $\lambda > 0$, it is clear that

$$8A\sigma_u^2\lambda^2 + 6\lambda - A\sigma_z^2 > \left(8\sigma_u^2\lambda^2 - \sigma_z^2\right) A.$$

As the lower bound on λ^* is $\frac{\sqrt{2}\sigma_z}{3\sigma_u}$, then $8\sigma_u^2\left(\frac{\sqrt{2}\sigma_z}{3\sigma_u}\right)^2 - \sigma_z^2 = \frac{7}{9}\sigma_z^2 > 0$. This means $r(\lambda, A) = 8A\sigma_u^2\lambda^2 + 6\lambda - A\sigma_z^2$ is positive on the domain of interest. ■

We turn now to prove the results in Lemma 1. Lemma 3 shows that λ^* is increasing with to respect to A and converges to $\frac{\sigma_z}{2\sigma_u}$ as A getting large. Note that $\frac{\sigma_z}{2\sigma_u}$ is independent from A and thus it can be treated as constant in our analysis. Hence, one can see that $x_2(\tilde{z}) = \frac{(\tilde{z}-\mu)}{\lambda^*(3+2A\lambda^*\sigma_u^2)}$ converges to zero as A getting large. We show now that the trading order of the risk neutral insider will converge to the one of Kyle (1985) model. Recall the risk neutral insider order in Kyle model and in our model are respectively given by

$$x(\tilde{z}) = \frac{(\tilde{z}-\mu)\sigma_u}{\sigma_z} \qquad \text{and} \qquad x_1(\tilde{z}) = \frac{(\tilde{z}-\mu)(1 + A\lambda^*\sigma_u^2)}{\lambda^*(3 + 2A\lambda^*\sigma_u^2)}$$

When A is getting large and taking into account that λ^* is treated as constant (by Lemma 3), one can notice that $x_1(\tilde{z})$ converges to $\frac{(\tilde{z}-\mu)}{2\lambda^*}$. Since λ^* will converge to $\frac{\sigma_z}{2\sigma_u}$, replacing its expression in $x_1(\tilde{z})$, we obtain the same expression as in Kyle (1985).

Appendix C: proof of Lemma 2

Recall that the market depth parameter in Tighe (1989) is given by $\lambda^T = \frac{\sqrt{2}}{3}\frac{\sigma_z}{\sigma_u}$ which corresponds to our market depth parameter when A converges to 0. Since we showed in Lemma 3 that λ^* is increasing in A , the relation in part a) holds.

We turn now to prove part b) of Lemma 2. Recall the one-shot game equilibrium of the Holden-Subrahmanyam (1994) model.

Proposition 3 *(H.S. 1994) In the presence of two risk averse informed traders, a linear equilibrium exists and it is unique. It is characterized by,*

(i)

$$x_1(\tilde{z}) = x_2(\tilde{z}) = \frac{(\tilde{z} - \mu)}{\lambda^{HS}(3 + A\lambda^{HS}\sigma_u^2)} \tag{18}$$

(ii) $p(\tilde{y}) = \mu + \lambda^{HS}\tilde{y}$, *where* $\mu = \bar{z}$, *and* λ^{HS} *is the unique strictly positive root of the following quadric equation:*

$$A^2\sigma_u^4\lambda^{\mathbf{4}} + 6A\sigma_u^2\lambda^{\mathbf{3}} + 9\lambda^{\mathbf{2}} - 2A\sigma_z^2\lambda - 2\frac{\sigma_z^2}{\sigma_u^2} = 0. \tag{19}$$

We move now to prove the relation between λ^* and λ^{HS}. Note that (19) can also be considered as an algebraic curve $g(A, \lambda; \sigma_u, \sigma_z) = 0$, where σ_u and σ_z as viewed as parameters. By implicit differentiation, we obtain

$$\frac{d\lambda}{dA} = -\frac{\frac{\partial g}{\partial A}}{\frac{\partial g}{\partial \lambda}},$$

where $\frac{\partial g}{\partial \lambda}$ must not be zero. Thus, we have

$$\frac{d\lambda}{dA} = \frac{-\left(2A\sigma_u^4\lambda^4 + 6\sigma_u^2\lambda^3 - 2\sigma_z^2\lambda\right)}{4A^2\sigma_u^4\lambda^3 + 18A\sigma_u^2\lambda^2 + 18\lambda - 2A\sigma_z^2}.$$

Note that the numerator has one positive root - by Descartes' rule (since we are interested in $A \geq 0$); and we can write this equation as

$$\frac{d\lambda}{dA} = \frac{-\lambda\left(\lambda^2\sigma_u^2\left(\lambda A\sigma_u^2 + 3\right) - \sigma_z^2\right)}{\lambda\left(\lambda A\sigma_u^2 + 3\right)\left(2\lambda A\sigma_u^2 + 3\right) - A\sigma_z^2}.$$

Observe that $\frac{d\lambda}{dA} = 0$, implies that

$$\lambda\left(\lambda^2\sigma_u^2\left(\lambda A\sigma_u^2 + 3\right) - \sigma_z^2\right) = 0.$$

Solving this equation for A, gives $A = -\frac{1}{\lambda^3}\frac{3\lambda^2\sigma_u^2 - \sigma_z^2}{\sigma_u^4}$. Put this expression for A into the quartic to find λ, we then obtain

$\lambda = \frac{1}{2}\frac{\sigma_z}{\sigma_u}$ or $\lambda = -\frac{1}{2}\frac{\sigma_z}{\sigma_u}$. Since $\lambda > 0$, we see that $g(A, \lambda) = 0$, for

$(A, \lambda) = \left(-\frac{1}{\lambda^3}\frac{3\lambda^2\sigma_u^2 - \sigma_z^2}{\sigma_u^4}, \frac{1}{2}\frac{\sigma_z}{\sigma_u}\right) = \left(-\frac{1}{\left(\frac{1}{2}\frac{\sigma_z}{\sigma_u}\right)^3}\frac{3\left(\frac{1}{2}\frac{\sigma_z}{\sigma_u}\right)^2\sigma_u^2 - \sigma_z^2}{\sigma_u^4}, \frac{1}{2}\frac{\sigma_z}{\sigma_u}\right) = \left(\frac{2}{\sigma_z\sigma_u}, \frac{1}{2}\frac{\sigma_z}{\sigma_u}\right).$

On the other hand, solve (19) for A to obtain $A = \frac{1}{2\lambda^3\sigma_u^4}\left(-6\lambda^2\sigma_u^2 + 2\sigma_z^2 \pm 2\sqrt{\left(-4\lambda^2\sigma_u^2\sigma_z^2 + \sigma_z^4\right)}\right).$

In order to have real roots, we need $-4\lambda^2\sigma_u^2\sigma_z^2+\sigma_z^4 \geq 0$, that is $-4\lambda^2\sigma_u^2\sigma_z^2+\sigma_z^4$ $= -\sigma_z^2\left(2\lambda\sigma_u - \sigma_z\right)\left(2\lambda\sigma_u + \sigma_z\right) \geq 0$, or equivalently, $\left(2\lambda\sigma_u - \sigma_z\right)\left(2\lambda\sigma_u + \sigma_z\right) \leq 0$. This means that $\left(2\lambda\sigma_u - \sigma_z\right) \leq 0$, that is $\lambda \leq \frac{1}{2}\frac{\sigma_z}{\sigma_u}$.
So this bound is the maximal value of λ. By the analysis above, this happens when $A = \frac{2}{\sigma_z\sigma_u}$. We now study the sign of $\frac{d\lambda}{dA}$. If we evaluate $\frac{d\lambda}{dA}$ at $A=0$, we get

$$\frac{d\lambda}{dA}|_{A=0} = \frac{-\left(6\sigma_u^2\lambda^3 - 2\sigma_z^2\lambda\right)}{18\lambda} = -\frac{1}{3}\lambda^2\sigma_u^2 + \frac{1}{9}\sigma_z^2.$$

Now when $A=0$, $g\left(0,\lambda\right) = 9\lambda^2 - 2\frac{\sigma_z^2}{\sigma_u^2} = 0$, which has the solutions $\lambda = \pm\frac{1}{3}\sqrt{2}\frac{\sigma_z}{\sigma_u}$. Since $\lambda > 0$, we get that

$$\frac{d\lambda}{dA}|_{A=0} = \frac{-\left(6\sigma_u^2\lambda^3 - 2\sigma_z^2\lambda\right)}{18\lambda} = -\frac{1}{3}\left(\frac{1}{3}\sqrt{2}\frac{\sigma_z}{\sigma_u}\right)^2\sigma_u^2 + \frac{1}{9}\sigma_z^2 = \frac{1}{27}\sigma_z^2 > 0.$$

On the other hand, one can show that $\frac{d\lambda}{dA}|_{A=\frac{3}{\sigma_z\sigma_u}} < 0$. Thus, the derivative at $A=0$ is positive, and to the right of A at $A = \frac{3}{\sigma_z\sigma_u}$ is negative. This shows that the plane curve described by $g\left(A,\lambda\right) = 0$, has a max given by $\left(A,\lambda\right) = \left(\frac{2}{\sigma_z\sigma_u}, \frac{1}{2}\frac{\sigma_z}{\sigma_u}\right)$. So we can conclude that the algebraic curve $g=0$ is unimodal. In fact, this can be verified by computing the second derivative of lambda with respect to A, which evaluates at $\left(\frac{2}{\sigma_u\sigma_z}, \frac{\sigma_z}{2\sigma_u}\right)$, is equal to $-\frac{1}{128}(\sigma_z)^3(\sigma_u) < 0$. This shows that it is concave down on the region containing the maximum point.

Now, computing the first derivative of λ^* with respect to A when $A=0$, we obtain

$$\begin{aligned}\frac{d\lambda^*}{dA}|_{A=0} &= -\frac{1}{6}\left(2\lambda\sigma_u - \sigma_z\right)\left(2\lambda\sigma_u + \sigma_z\right)\\ &= -\frac{1}{6}\left(2\left(\frac{1}{3}\sqrt{2}\frac{\sigma_z}{\sigma_u}\right)\sigma_u - \sigma_z\right)\left(2\left(\frac{1}{3}\sqrt{2}\frac{\sigma_z}{\sigma_u}\right)\sigma_u + \sigma_z\right)\\ &= -\frac{1}{54}\sigma_z^2\left(2\sqrt{2}-3\right)\left(2\sqrt{2}+3\right) = \frac{\sigma_z^2}{54} > 0\end{aligned}$$

This shows that the initial slope in Holden and Subrahmanyam(1994) case is exactly twice that of obtained in our case. Consider now the difference $h = g - f$, we get

$$h\left(A,\lambda\right) = \lambda A\left(-3A\lambda^3\sigma_u^4 - 6\lambda^2\sigma_u^2 + \sigma_z^2 + A\lambda\sigma_u^2\sigma_z^2\right)$$

So to find the intersection of f and g, h must vanish. Then either $\lambda A = 0$ or $-3A\lambda^3\sigma_u^4 - 6\lambda^2\sigma_u^2 + \sigma_z^2 + A\lambda\sigma_u^2\sigma_z^2 = 0$. The cubic polynomial has one sign change, and therefore, it has one positive root by Descarte's sign rule.

We show in the table below that at $A = \frac{3}{\sigma_z\sigma_u}$, $f < g$, but that at $A = \frac{4}{\sigma_z\sigma_u}$, $f > g$. It also shows the intersection point .

$\lambda\backslash A$	$\frac{3}{\sigma_z\sigma_u}$	$\frac{4}{\sigma_z\sigma_u}$	$\frac{3.5}{\sigma_z\sigma_u}$
λ from $f = 0$	$.49291\frac{\sigma_z}{\sigma_u}$	$.49481\frac{\sigma_z}{\sigma_u}$	$.49397\frac{\sigma_z}{\sigma_u}$
λ from $g = 0$	$.497\frac{\sigma_z}{\sigma_u}$	$.49051\frac{\sigma_z}{\sigma_u}$	$.49402\frac{\sigma_z}{\sigma_u}$

Note that the intersection point is roughly $\left(\tilde{A}, \tilde{\lambda}\right) = \left(\frac{3.5}{\sigma_z\sigma_u}, .494\,\frac{\sigma_z}{\sigma_u}\right)$.
Summarizing, we have that

1) the initial slope of g is bigger than the initial slope of f;

2) g is unimodal, it has one critical point which is a maximum occuring at $A = \frac{2}{\sigma_u\sigma_z}$;

3) the intersection point of f and g occurs to the right of the maximum of g, at roughly $A^* \simeq \frac{3.5}{\sigma_u\sigma_z}$;

4) $h = 0$ at two points.

Then it follows that there is one point $\left(\tilde{A}, \tilde{\lambda}\right)$, where $\lambda < \lambda_{HS}$ for $A < \tilde{A}$, and $\lambda > \lambda_{HS}$ for $A > \tilde{A}$. ■

Appendix D: proof of Proposition 2

Recall that the conditional variance in Tighe (1989) is given by

$$var(\tilde{z}|\tilde{y}) = \frac{1}{3}\sigma_z^2 \tag{20}$$

Computing the conditional variances in our model and in the Holden-Subrahmanyam (1994) model, we obtain respectively

$$var(\tilde{z}|\tilde{y}) = \frac{\left(3\left(1 + A\lambda^*\sigma_u^2\right)^2 + 2\left(1 + A\lambda^*\sigma_u^2\right)\right)\sigma_z^2 - (\lambda^*)^2\left(2\left(1 + A\lambda^*\sigma_u^2\right) + 1\right)^2\sigma_u^2}{\left(2\left(1 + A\lambda^*\sigma_u^2\right) + 1\right)^2}, \tag{21}$$

$$var(\tilde{z}|\tilde{y}) = \frac{[(3 + A\lambda^{HS}\sigma_u^2)^2 - 4]\sigma_z^2 - (\lambda^{HS})^2(3 + A\lambda^{HS}\sigma_u^2)^2\sigma_u^2}{(3 + A\lambda^{HS}\sigma_u^2)^2}. \tag{22}$$

We begin first by showing that (20) is less than (21). Indeed, combining (21) with (15) and after some simplifications, the problem becomes equivalent to show that

$$\frac{1}{3} < \frac{3 + 5A\lambda^* \sigma_u^2 + 2(\lambda^*)^2 A^2 \sigma_u^4}{\left(2\left(1 + A\lambda^* \sigma_u^2\right) + 1\right)^2}. \tag{23}$$

Plugging the lower bound, λ^T, (the upper bound, $\frac{\sigma_z}{2\sigma_u}$) of λ^* found in Lemma 3 in the numerator (denominator) of the right hand side of (23), we obtain,

$$\frac{3 + 5A\sigma_u^2 \left(\frac{\sqrt{2}}{3}\frac{\sigma_z}{\sigma_u}\right) + 2\left(\frac{\sqrt{2}}{3}\frac{\sigma_z}{\sigma_u}\right)^2 A^2 \sigma_u^4}{\left(2\left(1 + A\left(\frac{\sigma_z}{2\sigma_u}\right)\sigma_u^2\right) + 1\right)^2} < \frac{3 + 5A\lambda\sigma_u^2 + 2\lambda^2 A^2 \sigma_u^4}{\left(2\left(1 + A\lambda\sigma_u^2\right) + 1\right)^2}.$$

The left hand side simplifies to

$$\frac{1}{3} + \frac{\left(\left(15\sqrt{2} - 18\right) A\sigma_u\sigma_z + \sigma_z^2\sigma_u^2 A^2\right)}{9\left(3 + A\sigma_z\sigma_u\right)^2} > \frac{1}{3}.$$

This completes the first part of the proof. It remains to show that the conditional variance in our model is less than the conditional variance in Holden and Subrahmanyam model (1994). Indeed, combining (21) with (15) and combining (22) with (19), the problem reduces to showing that

$$\frac{1 + A\lambda^* \sigma_u^2}{3 + 2A\lambda^* \sigma_u^2} \leq \frac{A\lambda^{HS}\sigma_u^2 + 1}{A\lambda^{HS}\sigma_u^2 + 3}. \tag{24}$$

In order to complete the proof, we need to show first the following result.

Lemma 4 $\lambda^{HS} \in \left[\frac{\sigma_z}{4\sigma_u}, \frac{\sigma_z}{2\sigma_u}\right]$

Proof: Consider the equation

$$g(\lambda) = A^2\sigma_u^4\lambda^4 + 6A\sigma_u^2\lambda^3 + 9\lambda^2 - 2A\sigma_z^2\lambda - 2\frac{\sigma_z^2}{\sigma_u^2}, \tag{25}$$

such that $g(\lambda^{HS}) = 0$. Note that

$$\begin{aligned} g\left(\frac{\sigma_z}{4\sigma_u}\right) &= A^2\sigma_u^4\left(\frac{\sigma_z}{4\sigma_u}\right)^4 + 6A\sigma_u^2\left(\frac{\sigma_z}{4\sigma_u}\right)^3 + 9\left(\frac{\sigma_z}{4\sigma_u}\right)^2 - 2A\sigma_z^2\left(\frac{\sigma_z}{4\sigma_u}\right) - 2\frac{\sigma_z^2}{\sigma_u^2} \\ &= \frac{1}{256}A^2\sigma_z^4 - \frac{13}{32}\frac{A}{\sigma_u}\sigma_z^3 - \frac{23}{16}\frac{\sigma_z^2}{\sigma_u^2} = \frac{1}{256}\sigma_z^2\frac{A^2\sigma_z^2\sigma_u^2 - 104A\sigma_z\sigma_u - 368}{\sigma_u^2}. \end{aligned}$$

Consider the numerator $A^2\sigma_z^2\sigma_u^2 - 104A\sigma_z\sigma_u - 368 = 0$. It has the solutions $A = 4\frac{13+8\sqrt{3}}{\sigma_z\sigma_u} = \frac{107.43}{\sigma_z\sigma_u}$ or $A = 4\frac{13-8\sqrt{3}}{\sigma_z\sigma_u} = -\frac{3.4256}{\sigma_z\sigma_u}$. Now, the derivative

$$\frac{d\left(\frac{1}{256}\sigma_z^2\frac{A^2\sigma_z^2\sigma_u^2-104A\sigma_z\sigma_u-368}{\sigma_u^2}\right)}{dA} = \frac{1}{128}\frac{\sigma_z^3}{\sigma_u}\left(A\sigma_z\sigma_u - 52\right).$$

So for $0 \leq A < \frac{52}{\sigma_z\sigma_u}$, the derivative is negative and $g\left(\frac{\sigma_z}{4\sigma_u}\right)$ (at this value only) is decreasing in A for $0 \leq A < \frac{52}{\sigma_z\sigma_u}$. As also for $A = 0$, $g\left(\frac{\sigma_z}{4\sigma_u}\right) = 9\left(\frac{\sigma_z}{4\sigma_u}\right)^2 - 2\frac{\sigma_z^2}{\sigma_u^2} = -\frac{23}{16}\frac{\sigma_z^2}{\sigma_u^2}$ is negative, we can conclude that $\frac{\sigma_z}{4\sigma_u}$ is a lower bound for λ. Similarly, it follows from what we had before,

$$\begin{aligned} & A^2\sigma_u^4\left(\frac{\sigma_z}{2\sigma_u}\right)^4 + 6A\sigma_u^2\left(\frac{\sigma_z}{2\sigma_u}\right)^3 + 9\left(\frac{\sigma_z}{2\sigma_u}\right)^2 - 2A\sigma_z^2\left(\frac{\sigma_z}{2\sigma_u}\right) - 2\frac{\sigma_z^2}{\sigma_u^2} \\ = \; & \frac{1}{16}\sigma_z^2\frac{A^2\sigma_z^2\sigma_u^2 - 4A\sigma_z\sigma_u + 4}{\sigma_u^2} = \frac{1}{16}\sigma_z^2\frac{\left(A\sigma_z\sigma_u - 2\right)^2}{\sigma_u^2} > 0. \end{aligned}$$

Thus, by the Intermediate Value Theorem, the proof of Lemma 3 is complete. ■

We turn now to show that (24) holds. Since the expressions in (24) are both monotonically increasing in λ, plugging the lower bound of Lemma 4 in the right hand side of (24), and plugging the upper bound of Lemma 4 in the left hand side of (24), and then subtracting the resulting expressions from each others, we obtain

$$\frac{1}{2}\frac{2 + A\sigma_z\sigma_u}{3 + A\sigma_z\sigma_u} - \frac{A\sigma_z\sigma_u + 4}{A\sigma_z\sigma_u + 12} = -\frac{1}{2}\frac{\left(A\sigma_z\sigma_u\right)^2}{\left(3 + A\sigma_z\sigma_u\right)\left(A\sigma_z\sigma_u + 12\right)} < 0. \tag{26}$$

This completes the proof of Proposition 2. ■

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