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Article:

Jafarian, A and Jafari, R orcid.org/0000-0001-7298-2363 (2019) A new computational method for solving fully fuzzy nonlinear matrix equations. International Journal of Fuzzy Computation and Modelling, 2 (4). pp. 275-285. ISSN 2052-353X

https://doi.org/10.1504/IJFCM.2019.100317

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A new computational method for solving fully fuzzy nonlinear matrix equations

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Abstract

Since the uncertainty in parameters can be transformed into fuzzy set theory, fuzzy set and fuzzy system theory are good tools to deal with uncertainty systems. The popularity related to fuzzy nonlinear systems has always shown an upward trend, and also incorporated with wide spread applications in industries. The solutions of them are applied to analyze many engineering problems. Multi formulations and computational methodologies have been suggested to extract solution related to fuzzy nonlinear programming problems. However, In some cases the methods which have been utilized in order to find the solution of these problems involve greater complexity. On the basis of the mentioned reason, the current research work is intended towards introduction of a simple method for finding the fuzzy optimal solution related to fuzzy nonlinear issues. The main idea is on the basis of employing nonlinear system with equality constraints in order to find nonnegative fuzzy number matrixs $\widetilde{X}, \widetilde{X}^2, ..., \widetilde{X}^n$ which satisfies $\widetilde{A}\widetilde{X} + \widetilde{C}\widetilde{X}^2 + ... + \widetilde{E}\widetilde{X}^n = \widetilde{B}$ where $\widetilde{A}, \widetilde{C}, ..., \widetilde{E}$ are $n \times n$ arbitrary triangular fuzzy number matrices, $\widetilde{B}$ is a $n \times 1$ arbitrary triangular fuzzy number matrix. The proposed method is validated and is confirmed to be applicable by suggesting some demonstrated examples. The results confirm that the proposed method is so esay to understand and to apply for solving fully fuzzy nonlinear system (FFNS).

Keywords: Fuzzy solution; Fuzzy numbers; Fully fuzzy nonlinear system; Fully fuzzy matrix equations.

1 Introduction

One field of applied mathematics that has many applications in various areas of science is solving fuzzy nonlinear systems. Fuzzy nonlinear systems intervene in many guises in several problems of engineering, economics, biology, chemistry and physics [1, 2, 3, 4]. At the same time, the implication of numerical methods takes an important place to solve the fuzzy nonlinear systems.

Takagi and Sugeno [5] have presented first Numerical approach of fuzzy systems. Theoretical aspects of a fuzzy linear system were discussed by Dubois and Prade [6]. A general model for solving a fuzzy $n \times n$ linear system whose coefficient matrix is crisp and the right-hand side column is an arbitrary fuzzy number vector was first proposed by Friedman et al [7]. They utilized the parametric form of fuzzy numbers and replaced the original fuzzy $n \times n$ linear system by a crisp $2n \times 2n$ linear system and studied the duality in fuzzy linear systems $Ax = Bx+y$ where A and B are two real $n\times n$ matrices and the unknown x and the known y are two vectors whose components are n fuzzy numbers [8]. Allahviranloo [9] proposed solution of a fuzzy linear system by applying iterative method (Jacobi and Gauss Seidel methods), later on the same author proposed the solution of such system using successive over relaxation iterative method [10]. Abbasbandy et al [11] proposed the Conjugate gradient scheme, for solving fuzzy symmetric positive definite system of linear equation. Dehghan et al [12] proposed classic methods such as Cramers rule, Gaussian elimination method, LU decomposition scheme from linear algebra and linear programming for finding the approximated solution of a fully fuzzy linear systems of the form $\widetilde{A} \otimes \widetilde{x} = \widetilde{b}$ where $\widetilde{A}$ is a positive fuzzy matrix, $\widetilde{x}$ is an unknown and $\widetilde{b}$ is a known positive fuzzy vector respectively. Abbasbandy and Jafarian [13] used steepest descent method for approximation of the unique solution of fuzzy system of linear equation. Wang et al. [14] presented an iterative algorithm for solving dual linear systems of the form $x = Ax + u$, where A is a real $n \times n$ matrix, the unknown x and the constant u are all vectors whose components are fuzzy numbers. Abbasbandy et al [15] used LU decomposition method for solving fuzzy system of linear equation when the coefficient matrix is symmetric positive definite. Nasseri et al [16] used a certain decomposition methods of the coefficient matrix for solving fully fuzzy linear system of equations. The detailed introduction and survey of major results can be extracted from Refs. [17, 18, 19, 20]

In general, there exists no method based on matrices that yields fuzzy solutions for FFNS. Apart from the problems where the mathematical model is written in matrix form, nonlinear matrix equations also appear when one uses special techniques to solve scalar or vector problems.

In this paper, we describe the technique for solving the fully fuzzy nonlinear matrix equations (FFNME) such as $\widetilde{A}\widetilde{X} + \widetilde{C}\widetilde{X}^2 + ... + \widetilde{E}\widetilde{X}^n = \widetilde{B}$, where $\widetilde{A}, \widetilde{C}, ..., \widetilde{E}$ are $n\times n$ arbitrary triangular fuzzy number matrices, $\widetilde{B}$ is a $n\times 1$ arbitrary triangular fuzzy number matrix and the unknown $\widetilde{X}, \widetilde{X}^2, ..., \widetilde{X}^n$ are matrices consisting of n positive fuzzy numbers. We define the fuzzy matrices $\widetilde{X}^2, ..., \widetilde{X}^n$ with following elements:

$$If\ \widetilde{X} = \begin{bmatrix} \widetilde{x}_{1,1} \\ \widetilde{x}_{2,1} \\ \vdots \\ \widetilde{x}_{n,1} \end{bmatrix},\ then\ \widetilde{X}^2 = \begin{bmatrix} \widetilde{x}^2_{1,1} \\ \widetilde{x}^2_{2,1} \\ \vdots \\ \widetilde{x}^2_{n,1} \end{bmatrix}, ...,\ \widetilde{X}^n = \begin{bmatrix} \widetilde{x}^n_{1,1} \\ \widetilde{x}^n_{2,1} \\ \vdots \\ \widetilde{x}^n_{n,1} \end{bmatrix}.$$

For this purpose, we employ a nonlinear system with equality constraints to find nonnegative fuzzy number matrixs $\widetilde{X}, \widetilde{X}^2, ..., \widetilde{X}^n$ which satisfies $\widetilde{A}\widetilde{X} + \widetilde{C}\widetilde{X}^2 + ... + \widetilde{E}\widetilde{X}^n = \widetilde{B}$.

This paper organized as follows: Some basic definitions are reviewed in Section 2. In Section 3, a new method for solving FFNS is introduced, explained and verified with numerical examples. Section 4 ends the paper with Concluding remarks.

2 Basic definitions and notations

This section introduces the basic notations used in fuzzy operations. We start by defining the fuzzy number.

Definition 1. A fuzzy number is a fuzzy set $\widetilde{u} : \mathbb{R}^1 \to I = [0, 1]$ such that

i $\widetilde{u}$ is upper semi-continuous.

ii $\widetilde{u}(x) = 0$ outside some interval $[a, d]$.

iii There are real numbers b and c, $a \leq b \leq c \leq d$, for which

1. $\widetilde{u}(x)$ is monotonically increasing on $[a, b]$,
2. $\widetilde{u}(x)$ is monotonically decreasing on $[c, d]$,
3. $\widetilde{u}(x) = 1, b \leq x \leq c$.

The set of all fuzzy numbers is denoted by E^1 [21, 22].

Definition 2. A popular fuzzy number is the triangular fuzzy number $\widetilde{v} = (m - \alpha, m, m + \beta) = (v_m, v_l, v_u)$ with membership function as follows:

$$\mu_{\widetilde{v}}(x) = \begin{cases} \frac{x-m}{\alpha} + 1, & m - \alpha \leq x \leq m, \\ \frac{m-x}{\beta} + 1, & m \leq x \leq m + \beta, \\ 0, & otherwise, \end{cases}$$

for $\alpha, \beta > 0$ where $v_m = m - \alpha$, $v_l = m$ and $v_u = m + \beta$.
Triangular fuzzy numbers are fuzzy numbers in LR representation where the reference functions L and R are linear [23].

Definition 3. A triangular fuzzy number $\widetilde{v} = (v_m, v_l, v_u)$ is said to be non-negative if $v_m \geq 0$ [24].

Definition 4. Two triangular fuzzy number $\widetilde{v} = (v_m, v_l, v_u)$ and $\widetilde{u} = (u_m, u_l, u_u)$ are said to be equal if and only if

$$v_m = u_m, v_l = u_l, v_u = u_u.$$

Definition 5. A matrix $\widetilde{A} = (\widetilde{a}_{ij})$ is called a fuzzy number matrix, if each element of $\widetilde{A}$ is a fuzzy number. $\widetilde{A}$ will be a positive (negative) fuzzy matrix and denoted by $\widetilde{A} > 0$ ($\widetilde{A} < 0$) if each element of $\widetilde{A}$ is positive (negative). $\widetilde{A}$ will be non-positive (non-negative) and denoted by $\widetilde{A} \leq 0$ ($\widetilde{A} \geq 0$) if each element of $\widetilde{A}$ is non-positive (non-negative).

Definition 6. Let $\widetilde{v} = (v_m, v_l, v_u)$ and $\widetilde{u} = (u_m, u_l, u_u)$ be two triangular fuzzy numbers. Then [24]:

1. $\widetilde{v} \oplus \widetilde{u} = (v_m + u_m, v_l + u_l, v_u + u_u)$,
2. $-\widetilde{v} = (-v_u, -v_l, -v_m)$,
3. $\widetilde{v} \ominus \widetilde{u} = (v_m - u_u, v_l - u_l, v_u - u_m)$.

Considering the fuzzy multiplication, some computational expense problems can be investigated. The result of a fuzzy multiplication is a fuzzy number in LR representation, but it is difficult to compute the new functions L and R because they are not necessarily linear. We approximate this fuzzy multiplication such that it computes a triangular fuzzy number too. This fuzzy multiplication is denoted by $\hat{*}$ [25].
This fuzzy multiplication is based on the extension principle but is a bit different from the classical fuzzy multiplication. This operation is performed via the following equation:

$$\widetilde{v} \hat{*} \widetilde{u} = (q_m, q_l, q_u),$$

with

$$q_l = v_l.u_l,$$

$$q_m = min(v_m.u_m, v_m.u_u, v_u.u_m, v_u.u_u),$$

$$q_u = max(v_m.u_m, v_m.u_u, v_u.u_m, v_u.u_u).$$

If $\widetilde{v}$ is any triangular fuzzy number and $\widetilde{u}$ is a non-negative one, then we have:

$$\widetilde{v} \hat{*} \widetilde{u} = \begin{cases} (v_m.u_m, v_l.u_l, v_u.u_u), & v_m \geq 0, \\ (v_m.u_u, v_l.u_l, v_u.u_u), & v_m < 0, \ v_u \geq 0, \\ (v_m.u_m, v_l.u_l, v_u.u_m), & v_m < 0, \ v_u < 0. \end{cases}$$

3 Fully fuzzy nonlinear matrix equation

We are interested in solving the FFNME such as:

$$\begin{bmatrix} \widetilde{a}_{11} & \widetilde{a}_{12} & \dots & \widetilde{a}_{1n} \\ \widetilde{a}_{21} & \widetilde{a}_{22} & \dots & \widetilde{a}_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \widetilde{a}_{n1} & \widetilde{a}_{n2} & \dots & \widetilde{a}_{nn} \end{bmatrix} \begin{bmatrix} \widetilde{x}_{11} \\ \widetilde{x}_{21} \\ \vdots \\ \widetilde{x}_{n1} \end{bmatrix} + \begin{bmatrix} \widetilde{c}_{11} & \widetilde{c}_{12} & \dots & \widetilde{c}_{1n} \\ \widetilde{c}_{21} & \widetilde{c}_{22} & \dots & \widetilde{c}_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \widetilde{c}_{n1} & \widetilde{c}_{n2} & \dots & \widetilde{c}_{nn} \end{bmatrix} \begin{bmatrix} \widetilde{x}^2_{11} \\ \widetilde{x}^2_{21} \\ \vdots \\ \widetilde{x}^2_{n1} \end{bmatrix} + ...$$

$$+ \begin{bmatrix} \widetilde{e}_{11} & \widetilde{e}_{12} & \dots & \widetilde{e}_{1n} \\ \widetilde{e}_{21} & \widetilde{e}_{22} & \dots & \widetilde{e}_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \widetilde{e}_{n1} & \widetilde{e}_{n2} & \dots & \widetilde{e}_{nn} \end{bmatrix} \begin{bmatrix} \widetilde{x}^n_{11} \\ \widetilde{x}^n_{21} \\ \vdots \\ \widetilde{x}^n_{n1} \end{bmatrix} = \begin{bmatrix} \widetilde{b}_{11} \\ \widetilde{b}_{21} \\ \vdots \\ \widetilde{b}_{n1} \end{bmatrix},$$

where $\widetilde{a}_{ij}$, $\widetilde{c}_{ij}$ and $\widetilde{e}_{ij}$ $(for\ 1 \leq i,j \leq n)$, are arbitrary triangular fuzzy numbers, the elements $\widetilde{b}_{i1}$ in the right-hand matrix and the unknown elements $\widetilde{x}_{i1}$ are nonnegative fuzzy numbers. Using matrix notation, we have

$$\widetilde{A}\hat{*}\widetilde{X} + \widetilde{C}\hat{*}\widetilde{X}^2 + ... + \widetilde{E}\hat{*}\widetilde{X}^n = \widetilde{B}. \tag{1}$$

The fuzzy number matrices $\widetilde{X} = (\widetilde{x}_1, \widetilde{x}_2, ..., \widetilde{x}_n)^T, \widetilde{X}^2 = (\widetilde{x}_1^2, \widetilde{x}_2^2, ..., \widetilde{x}_n^2)^T, ...,$ $\widetilde{X}^n = (\widetilde{x}_1^n, \widetilde{x}_2^n, ..., \widetilde{x}_n^n)^T$ given by $\widetilde{x}_i = (\widetilde{y}_{i1}, \widetilde{x}_{i1}, \widetilde{z}_{i1}), \widetilde{x}_i^2 = (\widetilde{y}_{i1}^2, \widetilde{x}_{i1}^2, \widetilde{z}_{i1}^2), ..., \widetilde{x}_i^n = (\widetilde{y}_{i1}^n, \widetilde{x}_{i1}^n, \widetilde{z}_{i1}^n)$, $(for\ 1 \leq i \leq n)$, are the solutions of the fuzzy matrix system Eq. (1) if

$$\widetilde{a}_i\hat{*}\widetilde{X} + \widetilde{c}_i\hat{*}\widetilde{X}^2 + ... + \widetilde{e}_i\hat{*}\widetilde{X}^n = \widetilde{b}_i, \quad 1 \leq i \leq n, \tag{2}$$

where

$$\widetilde{b}_i = (\widetilde{d}_{i1}, \widetilde{b}_{i1}, \widetilde{f}_{i1}),$$

$$\widetilde{a}_i = ((\widetilde{g}_{i1}, \widetilde{a}_{i1}, \widetilde{h}_{i1}), (\widetilde{g}_{i2}, \widetilde{a}_{i2}, \widetilde{h}_{i2}), ..., (\widetilde{g}_{in}, \widetilde{a}_{in}, \widetilde{h}_{in})),$$
$$\widetilde{c}_i = ((\widetilde{k}_{i1}, \widetilde{c}_{i1}, \widetilde{p}_{i1}), (\widetilde{k}_{i2}, \widetilde{c}_{i2}, \widetilde{p}_{i2}), ..., (\widetilde{k}_{in}, \widetilde{c}_{in}, \widetilde{p}_{in})),$$
$$\widetilde{e}_i = ((\widetilde{q}_{i1}, \widetilde{e}_{i1}, \widetilde{u}_{i1}), (\widetilde{q}_{i2}, \widetilde{e}_{i2}, \widetilde{u}_{i2}), ..., (\widetilde{q}_{in}, \widetilde{e}_{in}, \widetilde{u}_{in})).$$

If in the FFNME Eq. (1), each element of $\widetilde{A}, \widetilde{C}, ..., \widetilde{E}, \widetilde{X}, \widetilde{X}^2, ..., \widetilde{X}^n$ and $\widetilde{B}$ is a nonnegative fuzzy number, then we call the system (1) a nonnegative FFNME.

Definition 7. In the nonnegative FFNME Eq. (1), with new notations $\widetilde{A} = (G, A, H), \widetilde{C} = (K, C, P), ..., \widetilde{E} = (Q, E, U)$ where $G, A, H, K, C, P, ..., Q, E, U$ are crisp matrices, we say that $\widetilde{X}, \widetilde{X}^2, ..., \widetilde{X}^n$ are the solutions if:

$$\begin{cases} GY + KY^2 + ... + QY^n = D, \\ AX + CX^2 + ... + EX^n = B, \\ HZ + PZ^2 + ... + UZ^n = F. \end{cases}$$

Moreover, if $Y \geq 0, X-Y \geq 0, Z-X \geq 0, X^2-Y^2 \geq 0, Z^2-X^2 \geq 0, ..., X^n-Y^n \geq 0, Z^n-X^n \geq 0$, then we say that $\widetilde{X}, \widetilde{X}^2, ..., \widetilde{X}^n$ are consistent solutions of the nonnegative FFNME.

3.1 The general method

In this subsection, a new method for finding fuzzy solutions of an FFNME is proposed. Consider the FFNME Eq. (2) where all the parameters $\widetilde{a}_{ij}, \widetilde{c}_{ij}, ..., \widetilde{e}_{ij}$, $\widetilde{x}_{i1}$ and $\widetilde{b}_{i1}$ are represented by triangular fuzzy numbers $(g_{ij}, a_{ij}, h_{ij}), (k_{ij}, c_{ij}, p_{ij})$, $..., (q_{ij}, e_{ij}, u_{ij})$, (y_{i1}, x_{i1}, z_{i1}) and (d_{i1}, b_{i1}, f_{i1}) respectively. Then this FFNME may be written as

$$(G, A, H)(Y, X, Z) + (K, C, P)(Y^2, X^2, Z^2) + ...$$
$$+(Q, E, U)(Y^n, X^n, Z^n) = (D, B, F), \tag{3}$$

Assuming $(g_{ik}, a_{ik}, h_{ik})\hat{*}(y_{k1}, x_{k1}, z_{k1}) + (k_{ik}, c_{ik}, p_{ik})\hat{*}(y^2_{k1}, x^2_{k1}, z^2_{k1}) + ... + (q_{ik}, e_{ik}, u_{ik})\hat{*}(y^n_{k1}, x^n_{k1}, z^n_{k1}) = (w^{(j)}_{k1}, q^{(j)}_{k1}, u^{(j)}_{k1})$, $1 \leq i, j, k \leq n$, where each (y_{k1}, x_{k1}, z_{k1}) is a nonnegative triangular fuzzy number the FFNME (2) may be written as:

$$\sum_{k=1}^{n}(w^{(j)}_{k1}, q^{(j)}_{k1}, u^{(j)}_{k1}) = (d_{i1}, b_{i1}, f_{i1}), \quad 1 \leq i \leq n. \tag{4}$$

Using arithmetic operations, defined in section 2, we have the following nonlinear programming. In which, we have added the artificial variables r_i, $i = 1, 2, ..., n^2$.
Minimize $r_1 + r_2 + ... + r_{n^2}$,

$$\textit{subject to} \begin{cases} \sum_{k=1}^{n} w^{(1)}_{k1} + r_1 = d_{11}, \\ \sum_{k=1}^{n} w^{(2)}_{k1} + r_2 = d_{21}, \\ \vdots \\ \sum_{k=1}^{n} w^{(n)}_{k1} + r_n = d_{n1}, \\ \sum_{k=1}^{n} q^{(1)}_{k1} + r_{n+1} = b_{11}, \\ \vdots \\ \sum_{k=1}^{n} u^{(n)}_{k1} + r_{3n} = f_{n1}. \end{cases}$$

There are various methods to eliminate these artificial variables. One of these methods consists of minimizing their sum, subject to the constraints Eq. (4) and $r_i \geq 0$, $i = 1, 2, ..., n^2$.

4 Numerical examples

To illustrate the technique proposed in this paper, consider the following examples.

Example 4.1. Consider the following FFNME:

$$\begin{bmatrix} (2,3,5) & (2,4,5) \\ \\ (1,2,3) & (3,4,6) \end{bmatrix} \begin{bmatrix} \widetilde{x}_{11} \\ \\ \widetilde{x}_{21} \end{bmatrix} + \begin{bmatrix} (1,2,3) & (3,5,6) \\ \\ (3,4,5) & (1,3,4) \end{bmatrix} \begin{bmatrix} \widetilde{x}^2_{11} \\ \\ \widetilde{x}^2_{21} \end{bmatrix} = \begin{bmatrix} (19,140,467) \\ \\ (14,136,436) \end{bmatrix},$$

where $\widetilde{x}_{11}, \widetilde{x}_{21}, \widetilde{x}^2_{11}, \widetilde{x}^2_{21}$, are triangular fuzzy numbers.
Assuming $\widetilde{x}_{11} = (y_{11}, x_{11}, z_{11})$, $\widetilde{x}_{21} = (y_{21}, x_{21}, z_{21})$, $\widetilde{x}^2_{11} = (y^2_{11}, x^2_{11}, z^2_{11})$ and $\widetilde{x}^2_{21} = (y^2_{21}, x^2_{21}, z^2_{21})$. The given FFNME is written as follows:

$$\begin{cases} (2,3,5)\hat{*}(y_{11}, x_{11}, z_{11}) + (2,4,5)\hat{*}(y_{21}, x_{21}, z_{21}) + (1,2,3)(y^2_{11}, x^2_{11}, z^2_{11}) \\ +(3,5,6)(y^2_{21}, x^2_{21}, z^2_{21}) = (19,140,467), \\ \\ (1,2,3)\hat{*}(y_{11}, x_{11}, z_{11}) + (3,4,6)\hat{*}(y_{21}, x_{21}, z_{21}) + (3,4,5)(y^2_{11}, x^2_{11}, z^2_{11}) \\ +(1,3,4)(y^2_{21}, x^2_{21}, z^2_{21}) = (14,136,436). \end{cases}$$

Wherein

$$\begin{cases} (2y_{11}+2y_{21}+y_{11}^2+3y_{21}^2, 3x_{11}+4x_{21}+2x_{11}^2+5x_{21}^2, 5z_{11}+5z_{21}+3z_{11}^2 \\ +6z_{21}^2) = (19, 140, 467), \\ \\ (y_{11}+3y_{21}+3y_{11}^2+y_{21}^2, 2x_{11}+4x_{21}+4x_{11}^2+3x_{21}^2, 3z_{11}+6z_{21}+5z_{11}^2 \\ +4z_{21}^2) = (14, 136, 436). \end{cases}$$

Applying the proposed technique, the above FFNME is converted into the following crisp system:

$$\begin{cases} 2y_{11}+2y_{21}+y_{11}^2+3y_{21}^2 = 19, \\ 3x_{11}+4x_{21}+2x_{11}^2+5x_{21}^2 = 140, \\ 5z_{11}+5z_{21}+3z_{11}^2+6z_{21}^2 = 467, \\ y_{11}+3y_{21}+3y_{11}^2+y_{21}^2 = 14, \\ 2x_{11}+4x_{21}+4x_{11}^2+3x_{21}^2 = 136, \\ 3z_{11}+6z_{21}+5z_{11}^2+4z_{21}^2 = 436. \end{cases}$$

Minimize $\quad r_1+r_2+...+r_6$

$$\begin{cases} 2y_{11}+2y_{21}+y_{11}^2+3y_{21}^2+r_1 = 19, \\ 3x_{11}+4x_{21}+2x_{11}^2+5x_{21}^2+r_2 = 140, \\ 5z_{11}+5z_{21}+3z_{11}^2+6z_{21}^2+r_3 = 467, \\ y_{11}+3y_{21}+3y_{11}^2+y_{21}^2+r_4 = 14, \\ 2x_{11}+4x_{21}+4x_{11}^2+3x_{21}^2+r_5 = 136, \\ 3z_{11}+6z_{21}+5z_{11}^2+4z_{21}^2+r_6 = 436, \end{cases}$$

where $r_1+r_2+...+r_6 \geq 0$. The optimal solution is $y_{11}=1$, $y_{21}=2$, $y_{11}^2=1$, $y_{21}^2=4$, $x_{11}=4$, $x_{21}=4$, $x_{11}^2=16$, $x_{21}^2=16$, $z_{11}=6$, $z_{21}=7$, $z_{11}^2=36$, $z_{21}^2=49$. Hence the fuzzy solution is given by $\widetilde{x}_{11}=(1,4,6)$, $\widetilde{x}_{21}=(2,4,7)$, $\widetilde{x}_{11}^2=(1,16,36)$ and $\widetilde{x}_{21}^2=(4,16,49)$.

Example 4.2. Consider the following FFNME:

$$\begin{bmatrix} (3,4,5) & (1,2,4) \\ \\ (-3,-2,-1) & (-4,-2,-1) \end{bmatrix} \begin{bmatrix} \widetilde{x}_{11} \\ \\ \widetilde{x}_{21} \end{bmatrix} + \begin{bmatrix} (-5,-4,-2) & (2,4,5) \\ \\ (1,2,3) & (-4,-3,-1) \end{bmatrix} \begin{bmatrix} \widetilde{x}_{11}^2 \\ \\ \widetilde{x}_{21}^2 \end{bmatrix}$$

$$+ \begin{bmatrix} (2,3,5) & (-3,-2,-1) \\ \\ (-5,-3,-2) & (3,4,5) \end{bmatrix} \begin{bmatrix} \widetilde{x}_{11}^3 \\ \\ \widetilde{x}_{21}^3 \end{bmatrix} = \begin{bmatrix} (-299,132,1180) \\ \\ (-1145,-93,365) \end{bmatrix},$$

where $\widetilde{x}_{11}, \widetilde{x}_{21}, \widetilde{x}_{11}^2, \widetilde{x}_{21}^2, \widetilde{x}_{11}^3, \widetilde{x}_{21}^3$, are triangular fuzzy numbers.
Assuming $\widetilde{x}_{11}=(y_{11},x_{11},z_{11})$, $\widetilde{x}_{21}=(y_{21},x_{21},z_{21})$, $\widetilde{x}_{11}^2=(y_{11}^2,x_{11}^2,z_{11}^2)$, $\widetilde{x}_{21}^2=(y_{21}^2,x_{21}^2,z_{21}^2)$,$\widetilde{x}_{11}^3=(y_{11}^3,x_{11}^3,z_{11}^3)$ and $\widetilde{x}_{21}^3=(y_{21}^3,x_{21}^3,z_{21}^3)$. The given

FFNME is written as follows:

$$\begin{cases} (3,4,5)\hat{*}(y_{11},x_{11},z_{11})+(1,2,4)\hat{*}(y_{21},x_{21},z_{21}) \\ +(-5,-4,-2)(y_{11}^2,x_{11}^2,z_{11}^2)+(2,4,5)(y_{21}^2,x_{21}^2,z_{21}^2) \\ +(2,3,5)(y_{11}^3,x_{11}^3,z_{11}^3)+(-3,-2,-1)(y_{21}^3,x_{21}^3,z_{21}^3) \\ =(-299,132,1180), \\ \\ (-3,-2,-1)\hat{*}(y_{11},x_{11},z_{11})+(-4,-2,-1)\hat{*}(y_{21},x_{21},z_{21}) \\ +(1,2,3)(y_{11}^2,x_{11}^2,z_{11}^2)+(-4,-3,-1)(y_{21}^2,x_{21}^2,z_{21}^2) \\ +(-5,-3,-2)(y_{11}^3,x_{11}^3,z_{11}^3)+(3,4,5)(y_{21}^3,x_{21}^3,z_{21}^3) \\ =(-1145,-93,365). \end{cases}$$

Wherein

$$\begin{cases} (3y_{11}+y_{21}-5y_{11}^2+2y_{21}^2+2y_{11}^3-3y_{21}^3, 4x_{11}+2x_{21}-4x_{11}^2+4x_{21}^2+3x_{11}^3 \\ -2x_{21}^3, 5z_{11}+4z_{21}-2z_{11}^2+5z_{21}^2+5z_{11}^3-1z_{21}^3)=(-299,132,1180), \\ \\ (-3y_{11}-4y_{21}+y_{11}^2-4y_{21}^2-5y_{11}^3+3y_{21}^3, -2x_{11}-2x_{21}+2x_{11}^2-3x_{21}^2 \\ -3x_{11}^3+4x_{21}^3, -z_{11}-z_{21}+3z_{11}^2-z_{21}^2-2z_{11}^3+5z_{21}^3)=(-1145,-93,365), \end{cases}$$

Applying the proposed technique, the above FFNME is converted into the following crisp system:

$$\begin{cases} 3y_{11}+y_{21}-5y_{11}^2+2y_{21}^2+2y_{11}^3-3y_{21}^3=-299, \\ 4x_{11}+2x_{21}-4x_{11}^2+4x_{21}^2+3x_{11}^3-2x_{21}^3=132, \\ 5z_{11}+4z_{21}-2z_{11}^2+5z_{21}^2+5z_{11}^3-1z_{21}^3=1180, \\ -3y_{11}-4y_{21}+y_{11}^2-4y_{21}^2-5y_{11}^3+3y_{21}^3=-1145, \\ -2x_{11}-2x_{21}+2x_{11}^2-3x_{21}^2-3x_{11}^3+4x_{21}^3=-93, \\ -z_{11}-z_{21}+3z_{11}^2-z_{21}^2-2z_{11}^3+5z_{21}^3=365. \end{cases}$$

Minimize $\quad r_1+r_2+...+r_6$

$$\begin{cases} 3y_{11}+y_{21}-5y_{11}^2+2y_{21}^2+2y_{11}^3-3y_{21}^3+r_1=-299, \\ 4x_{11}+2x_{21}-4x_{11}^2+4x_{21}^2+3x_{11}^3-2x_{21}^3+r_2=132, \\ 5z_{11}+4z_{21}-2z_{11}^2+5z_{21}^2+5z_{11}^3-1z_{21}^3+r_3=1180, \\ -3y_{11}-4y_{21}+y_{11}^2-4y_{21}^2-5y_{11}^3+3y_{21}^3+r_4=-1145, \\ -2x_{11}-2x_{21}+2x_{11}^2-3x_{21}^2-3x_{11}^3+4x_{21}^3+r_5=-93, \\ -z_{11}-z_{21}+3z_{11}^2-z_{21}^2-2z_{11}^3+5z_{21}^3+r_6=365. \end{cases}$$

where $r_1+r_2+...+r_6 \geq 0$. The optimal solution is $y_{11}=3$, $y_{21}=2$, $y_{11}^2=9$, $y_{21}^2=4$, $y_{11}^3=27$, $y_{21}^3=8$, $x_{11}=4$, $x_{21}=3$, $x_{11}^2=16$, $x_{21}^2=9$, $x_{11}^3=64$, $x_{21}^3=27$, $z_{11}=6$, $z_{21}=4$, $z_{11}^2=36$, $z_{21}^2=16$, $z_{11}^3=216$, $z_{21}^3=64$. Hence the fuzzy solution is given by $\widetilde{x}_{11}=(3,4,6)$, $\widetilde{x}_{21}=(2,3,4)$, $\widetilde{x}_{11}^2=(9,16,36)$, $\widetilde{x}_{21}^2=(4,9,16)$, $\widetilde{x}_{11}^3=(27,64,216)$, and $\widetilde{x}_{21}^3=(8,27,64)$.

5 Concluding remarks

Through the last few decades, the fuzzy nonlinear systems have become increasingly important in numerical analysis, and they have proven to be a very useful tool from both theoretical and practical points of view. In this paper, a new method to obtain the nonnegative fuzzy optimal solutions of FFNME like $\widetilde{A}\widetilde{X} + \widetilde{C}\widetilde{X}^2 + ... + \widetilde{E}\widetilde{X}^n = \widetilde{B}$ is introduced, where $\widetilde{A}, \widetilde{C}, ..., \widetilde{E}$ are $n \times n$ arbitrary triangular fuzzy number matrices, $\widetilde{B}$ is a $n \times 1$ arbitrary triangular fuzzy number matrix and the unknown $\widetilde{X}, \widetilde{X}^2, ..., \widetilde{X}^n$ are matrices consisting of n positive fuzzy numbers. A nonlinear system with equality constraints to FFNME is used as a new method for solving FFNME and the validity of the proposed method is examined with numerical examples. The results reveal the efficiency of this method for solving these problems. The constructed method is efficient in determination of consistency of FFNS occurring in real life situations.

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