Fall 1-24-2020

A Computational and Experimental Examination of the FCC Incentive Auction

Logan Gantner
Chapman University, gantner@chapman.edu

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A Dissertation by

Logan Gantner

Chapman University

Orange, CA

Schmid College of Science and Technology

Submitted in partial fulfillment of the requirements for the degree of

Doctor of Philosophy in Computational and Data Sciences

January 2020

Committee in charge:

David Porter, Ph.D., Committee Chair

Abel Winn, Ph.D.

Stephen Rassenti, Ph.D.

The Dissertation of Logan Gantner is approved.

David Porter, Ph.D., Committee Chair

Abel Winn, Ph.D.

Stephen Rassenti, Ph.D.

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Abstract

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In 2016, the Federal Communications Commission debuted a new auction mechanism, the Incentive Auction, with the intention of obtaining high frequency television broadcasting spectrum, repurposing it for cellular use, and reselling these licenses at profitable prices. In designing this process, the traditional mechanism used for spectrum auctions, the Simultaneous Multiple Round Auction (SMR), was modified in order to speed the process. This new mechanism, the Incentive Forward Auction (IFA), intended to reduce the number of rounds per auction by lumping similar spectrum licenses together. However, the IFA discourages straightforward bidding strategies and can result in bidders committing more in costs than their established budgets will allow.

These mechanisms are compared using two methods: a simulated environment using automated bidding strategies, and a lab environment using human subjects. In simulations, it was found that the IFA was successful in reducing the number of rounds per auction compared with the SMR. However, both simulation and experimental results found that using the IFA resulted in consistent losses to auction efficiency and revenue.

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Chapter 1

FCC Spectrum Allocation

Wireless data communication was an extremely valuable innovation in the modern era. Technologies in this field were instrumental in advancing scientific understanding and greatly expanded public exchange of information. Within the private sector alone, wireless communication is the foundation for the radio, television, cellular and paging industries. Today, the worldwide telecommunications sectors account for over $2.2 trillion in yearly revenue [23].

Wireless information is transmitted rapidly and over long distances through the use of the electromagnetic spectrum. However, the spectrum is an inherently finite resource—minimum bandwidths are required for specific applications, and establishing clear spectrum band licenses are necessary to establishment ownership rights and avoid interference between the signals of competing interests. To this end, a **spectrum license** is defined with respect to two pieces of information: the segment of spectrum to which the licensee has access, and a geographic location in which they are granted usage. In the United States, for example, television broadcasting stations are each assigned 6 MHz of bandwidth frequency. In each geographic location, the 54-60 MHz band is traditionally used for local channel 2 services.

In the past, the United States has been organized into many different geographic partition schemes for the purpose of spectrum allocation. These varied based on the spectrum location and its intended use. Schemes included Television Market Areas (TMAs), Regional Economic Areas (REAGs), and Major Economic Areas (MEAs) [10]. In 2014, a new scheme was created to repurpose television broadcasting spectrum for the cellular market using **partial economic areas (PEAs)**. Figure 1.1 shows an updated map of the 416 PEAs. In this work, we focus exclusively on PEAs as the geographic component of spectrum licenses.

Because spectrum is a finite resource with very high demand, there is a fundamental problem of allocation. The Federal Communications Commission (FCC) is tasked with

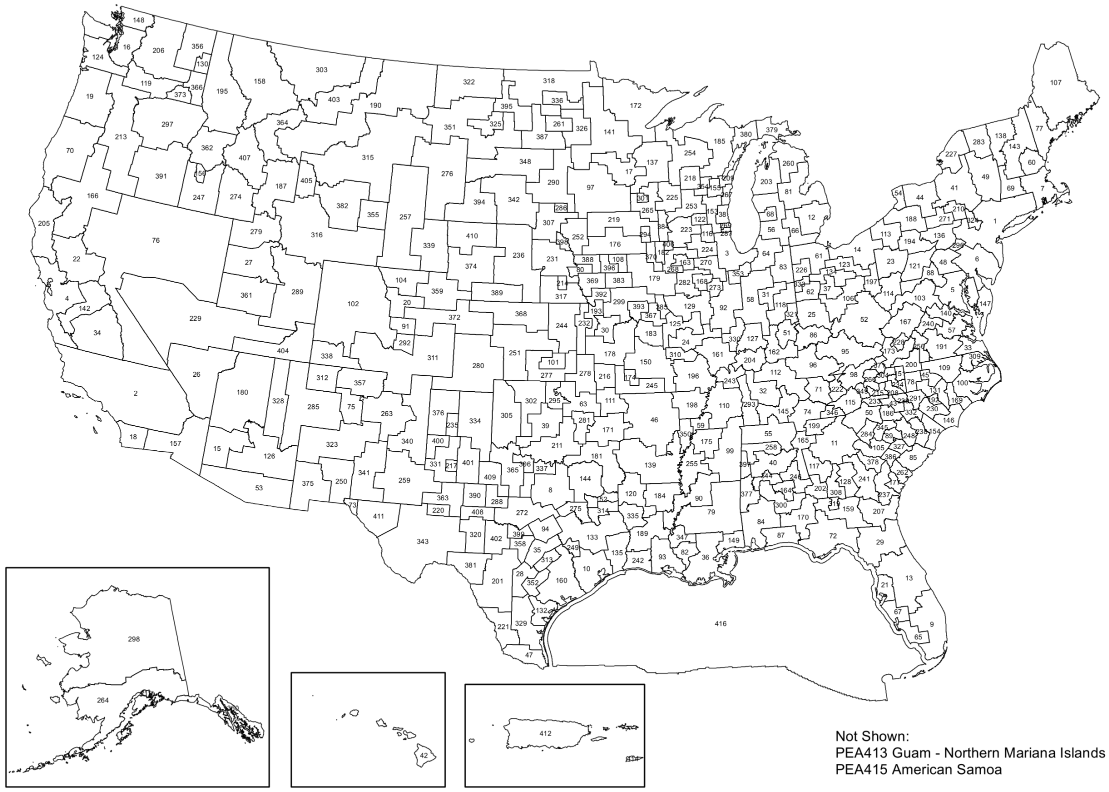


Figure 1.1: Partial economic areas used for spectrum licensing

assigning licenses to interested parties. For the past 25 years, they have adopted the use of auction mechanisms guided largely by market principles for this purpose. In this chapter, we discuss the events leading to a recently developed auction mechanism—the Incentive Auction—and and examine its rules and pitfalls. In Chapter 2, we propose an alternate mechanism—a variation of the Simultaneous Multiple Round Auction. We present several bidding strategies to be used for both strategies, and examine the result when applying them in a simulated auction environment. In Chapter 3, we do the same in an experiment environment with human subjects.

1.1 History of Spectrum Auctions

Since the introduction of the radio for public use in the early twentieth century, spectrum has been considered a public commodity. During the infancy of the radio industry, spectrum was considered plentiful and licenses were awarded by the Federal Radio Commission on a first-come-first-serve basis, with the primary intent to minimize interference between signals [13]. After two decades, demand had exceeded available supply of spectrum, and the Federal Communications Commission (FCC) was formally created through the Communications

Act of 1934. The agency was created "for the purpose of regulating interstate and foreign commerce in communication by wire and radio so as to make available, so far as possible to all the people of the United States a rapid, efficient, Nation-wide, and world-wide wire and radio communication service with adequate facilities at reasonable charges."

Since its inception, the FCC has attempted to tackle the license allocation problem in various ways. For 54 consecutive years (from 1927 to 1984), licensees were determined exclusively through a process called "comparative hearings", also known colloquially as "beauty contests". During these hearings, an official committee would hear arguments from prospective license holders and determine which party would use the spectrum to best serve the public interest [17].

In 1981, the FCC was tasked with awarding more than 1400 cellular licenses. Due to concerns over the political and slow nature of assignment committees, a different mechanism was required for this market. As a result, the FCC was authorized to implement a lottery system from 1984 to 1989, assigning licenses to "qualified telephone companies" entirely at random [22]. Despite this restriction, over $400,000$ applications were received—many stemming from companies created specifically for this purpose with the aid of consulting companies [14].

1.1.1 Early Usage of Spectrum Auctions

The use of spectrum auctions as a solution to the wireless license allocation problem is quite modern, being first utilized in the US in the 1990s. However, the concept of spectrum auctions preceded their application for decades. Ronald Coase recommended their use in 1959, arguing that "it is not clear why we should have to rely on the Federal Communications Commission rather than the ordinary pricing mechanism to decide whether a particular frequency should be used by [various interests]. Indeed, the multiplicity of these varied uses would suggest that the advantages to be derived from relying on the pricing mechanism would be especially great [2]." Coase references Leo Herzel making similar arguments as early as 1951.

What followed was a systematic, multi-faceted campaign to introduce a market-based mechanism for spectrum licenses. Although the social, efficiency and monetary costs of beauty contests and random allocations were significant and known, the FCC held out against adopting auctions in any form for 40 years. There have been many intersecting explanations for the government's lagging pace on the issue. These included anti-monopoly concerns, interfering special interests of regulators, and issues with the agency of central planning [12].

Ultimately, the winning argument would not be one of theory or merit, but of finances. Faced with public scrutiny over the rising national debt, Congress passed the Omnibus Budget and Reconciliation Act of 1993. In addition to changes in many other agencies and sectors, this gave the FCC the long-awaited authority "to grant [a spectrum] license or permit to a qualified applicant through the use of a system of competitive bidding." This was subject to priorities ensuring that the process was fair and economically efficient, while yielding to the government "a portion of the value of the public spectrum resource made available for commercial use [21]."

Although drawing public revenues was a key stipulation of the act, the FCC was forbidden from designing auction mechanisms with the purpose of maximizing revenue. Consistent with the FCC's roots, additional requirements were outlined calling for the protection of small business and minority opportunities [5]. We discuss the reserve auction split (a specific manner in which this expectation manifested itself) in Section 1.2.5, which produced unique issues for participating bidders.

With expectations set, the FCC went on to design and conduct a total of nine spectrum auctions from 1994 to 1996. During this period, a combination of government, private market and academic forces collaborated in developing what became the standard in spectrum allocation mechanisms—the **simultaneous multiple round auction (SMRA)**. The SMRA is a combinatorial clock auction—a number of spectrum licenses are available to bid on simultaneously over the course of a variable number of rounds, with a maximum cap on the bidding price of each license ascending from round to round in response to bidder demand. Each round, the bidder with the highest bid price per license was assigned to be the provisional winner, potentially displacing the previous provisional winner. The auction ended when a round produced no new bids. In this way, the SMRA intends to gradually reveal information to the bidders while providing them the flexibility to react to this information [4].

These preliminary auctions were a clear financial success, netting over $20 billion in revenue. More challenging to assess is the question of efficiency, which requires judging whether the licenses were optimally (or at least decently) assigned. While it is generally accepted that revenue and efficiency are closely tied, these auctions were considered successful based on additional indicators. Similar licenses consistently sold for similiar, if not identical, prices, while bidders were successful in forming license aggregations. And unlike with random allocations, there was very little post-auction resale activity [3].

SMR auctions continue to be used with regularity. From 1994 to 2009, there were a total

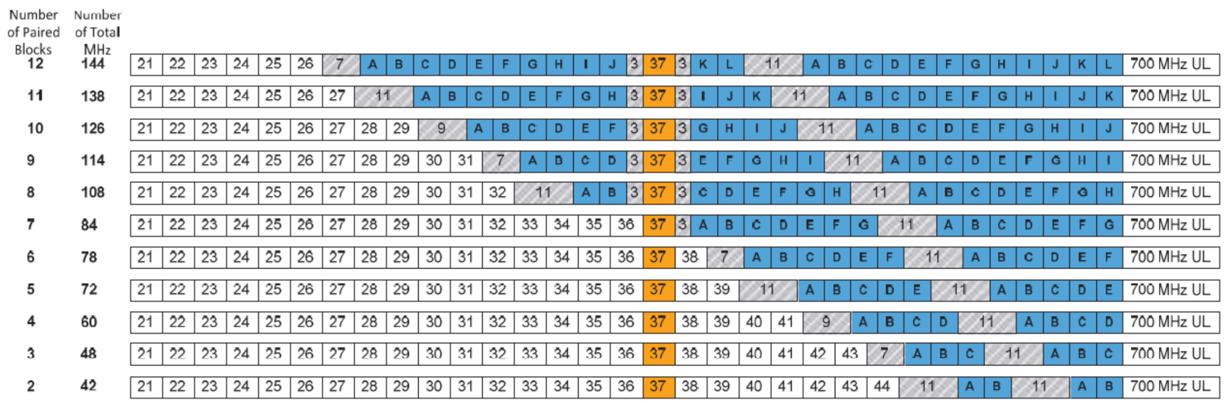


Figure 1.2: Incentive Auction band plan scenarios

of 85 auctions, almost exclusively making use of the SMR mechanism. These auctions sold a combined 27,484 licenses for a variety of purposes, netting the government $52.6 in earnings [12].

1.1.2 The Spectrum Act and 2016 Broadcast Incentive Auction

The SMR mechanism was the exclusive choice of FCC spectrum auctions through the first decade of the 20th century. But by 2012, the FCC had begun to explore alternate mechanisms. This was spurred in large part by the aggressive expansion of cellular market. As demand for high quality cellular transmission continued to rise, video compression technology and the insurgence of Digital Terrestrial Television effectively reduced supply needed in broadcasting. Responding to this, the World Radiocommunication Conference in 2007 authorized spectrum in the upper television broadcasting band to be used for telecommunication purposes [11].

Following suit, the United States Congress passed the Middle Class Tax Relief and Job Creation Act of 2012. Title VI of this act, referred to separately as the Spectrum Act, tasked the FCC with designing a new mechanism to repurpose ultra high frequency (UHF) television stations in the 600 MHz band for commercial mobile use [18]. This mechanism, known as the **Incentive Auction (IA)**, was outlined to perform two primary tasks: creating incentives for broadcasters to forfeit their rights to high-frequency television spectrum licenses, and in turn selling those licenses to firms at prices that would cover all costs (to the sellers, as well as administrative costs) [20]. These tasks were handled with independent reverse and forward clock auctions, respectively.

In designing the IA mechanism, the FCC was faced with a needling problem. There was a desire to free up as much of the broadcast band as possible for telecommunications, but only so far as to maintain profitability in the process. The choice of spectrum for repurposing was referred to as the band plan, which concerned itself with the number and band location of mobile licenses to be sold. Figure 1.2 shows the band plan scenarios anticipated by the FCC, up to a maximum of 12 allocated licenses per region. Repurposing began at channel 51 (698 MHz), where it would link up with existing 700 MHz mobile licenses. New licenses required sufficient guard bands to act as buffers, separating uplink and downlink components from the remaining broadcasting spectrum, channel 37 (reserved for radio astronomy), and one another [7].

To simultaneously handle the contrasting goals of the forward and reverse auctions, the IA mechanism opted to stagger the two. An initial band plan was assumed with an associated repurposing bandwidth (the “clearing target”). The reverse auction would be conducted to determine the sellers, licenses and necessary costs, and the forward auction would proceed. After an event of slowed activity was triggered, the auction would determine whether the “final stage rule” was satisfied (see Section 1.2.4 for details). This rule primarily assessed whether the ensuing revenue was sufficient to reimburse sellers and cover additional costs. If this rule was not met, the mechanism would return to the reverse auction, assume the next largest band plan (with supply reduced by one unit and the clearing target reduced accordingly), and the process would continue anew. In this manner, an incentive auction proceeds in a series of stages [15].

The Spectrum Act outlined a need for forward and reverse auctions within the mechanism, but it was not specific in how these components must be conducted. It was possible that the FCC may have opted for the standard SMR as a choice for each of the forward auction stages. However, because of this passing-of-the-baton relationship between buying and selling parties, there was some concern while designing the IA that the traditional SMR may be too slow, and that a speedier sub-mechanism would be preferred [19]. The mechanism ultimately chosen is the **Incentive Forward Auction (IFA)**.

Like its SMR counterpart, the IFA is a combinatorial clock auction. It departs from the SMR in two significant ways: (1) Rather than auctioning all licenses individually, similar licenses (those having similar impairment levels within the same PEA) were treated as identical units, called “blocks”. Bidders would bid on blocks instead of licenses, which would generally have a supply in excess of one. (2) The auction would no longer assign provisional winners. Instead, all valid bids to increase demand would be processed with certainty. Bidders who wished to reduce their demand were required to submit bids specifying this request. In this way, the price for many related licenses can ascend simultaneously. These rules are more thoroughly discussed in the next section.

Because buyers were no longer bidding on licenses directly, an additional phase was required within the Incentive Auction. This is the Assignment Phase, and consisted of a series of individual forward auctions for each group of blocks. These auctions determined which block winners would receive which licenses, and the final price that they would pay. Figure 1.3 summarizes the IA process, where the process of switching between the forward and reverse auctions is referred to as the Clock Phase.

The Clock Phase of the Broadcast Incentive Auction took place from August of 2016 until February of 2017. During this time, bidders spent a total of 30 (noncontinuous) days participating in the IFA. The initial clearing target was set at 128 MHz, and was ultimately reduced to 84 MHz over the course of four stages and 87 rounds. The auction brought in a gross revenue of $19.8 billion, pulling roughly $12 billion in government profits after costs [9].

1.2 The Incentive Forward Auction Structure and Rules

In this section, we examine the specifics of the IFA process at length. The bulk of the dissertation concerns this forward auction component, its issues, and its viable alternatives. Information in this section is based off education published by the FCC, found at [1].

Recall that the standard forward auction mechanism, the SMR, involves buyers bidding on licenses directly. Each round, a single provisional winner is selected for each license that received at least one bid. The IFA, by contrast, has buyers bidding for units of **product**. A product consists of two pieces of information–a PEA, and one of two primary categories (C1 and C2). The category specifies a product's maximum possible **impairment**.

Because the FCC was not able to perfectly repackage UHF broadcast television stations in every region, and because these stations require a buffer to protect them cellular service interference, a number of licenses were only granted a partial coverage of their corresponding PEA. This coverage is described as a percentage of the PEA's population by the license's impairment [8]. C1 products were allowed up to 15% impairment, while C2 products could have anywhere from 15% to 50% impairment. Impairment fundamentally reduces the value of a license, and so was automatically discounted during the Assignment Phase, as we discuss in Sections 1.2.3 and 1.2.4.

The generic units of product are called blocks, which are assigned to winning bidders as licenses during the Assignment Phase once the IFA has concluded.

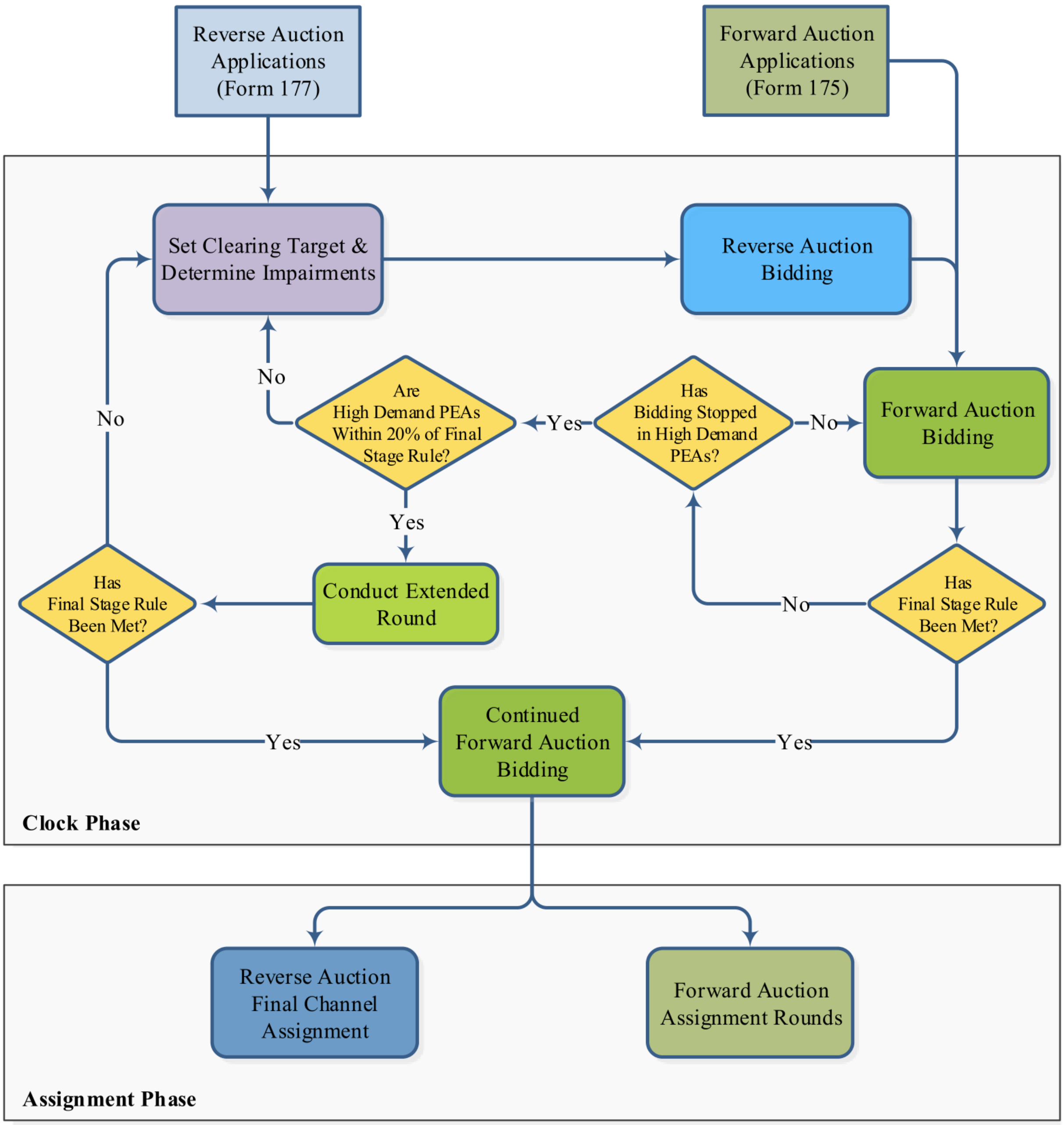


Figure 1.3: Flow of the Incentive Auction process

1.2.1 Bidding and Pricing

The forward auction of the clock phase takes place over multiple rounds and consists of a separate and independent clock auction for each product being sold. Each round, bidders observe the results from the previous round and submit a set of sealed bids spanning one or more products. Products have two prices associated with them each round–their (lower bound) **posted price** and (upper bound) **clock price**. All bids consist of both a quantity q and price p—the maximum amount of money that the bidder is willing to commit for the specified quantity of an item during that round. Bid prices may take any value at fixed, specified intervals between (and including) the posted and clock prices.

Further, bidders may submit multiple bids for the same product during the same round, so long as the bids contain different quantities and prices. Doing so allows bidders to specify non-constant demand curves between the posted and clock prices. Bidsets for the same product must obey **one-directionality**: when a set of bids are submitted for a product, each bid must demand a strictly lower quantity than any of the bids requesting lower prices. If a bidset does not satisfy one-directionality, the auction will reject those bids and none of them will be processed for the next round. This is one of several ways in which bids may be rejected.

A product's **supply** S is the number of blocks (or licenses) for sale within that PEA and category. Because of the two-sided nature of the Incentive Auction, product supply is subject to decrease during a stage transition. At the end of round t, the **demand** D_t for a product is equal to the total number of valid bids that are *submitted* for that item, whereas the **processed demand** PD_t is the number of successfully *processed* bids. When processed demand exceeds supply for a product, we say that it has **excess demand** ED_t, defined simply as

$$ED_t = \max\{0, PD_t - S\} \tag{1.1}$$

The processed demand for products is not provided as information for the bidders, but excess demand is public information.

From round to round, the posted and clock prices of products may increase. The extent of this depends on the relative values of a product's supply and demand. If demand is strictly less than supply, both the posted and clock prices will remain unchanged for the following round. If the auction concludes successfully during such a round, the products will be "sold" at the posted price to all bidders who submitted processed bids, and the assignment phase will begin. If, however, demand matches or exceeds supply, the posted price will increase to the largest possible value such that the resulting processed demand would not not fall below

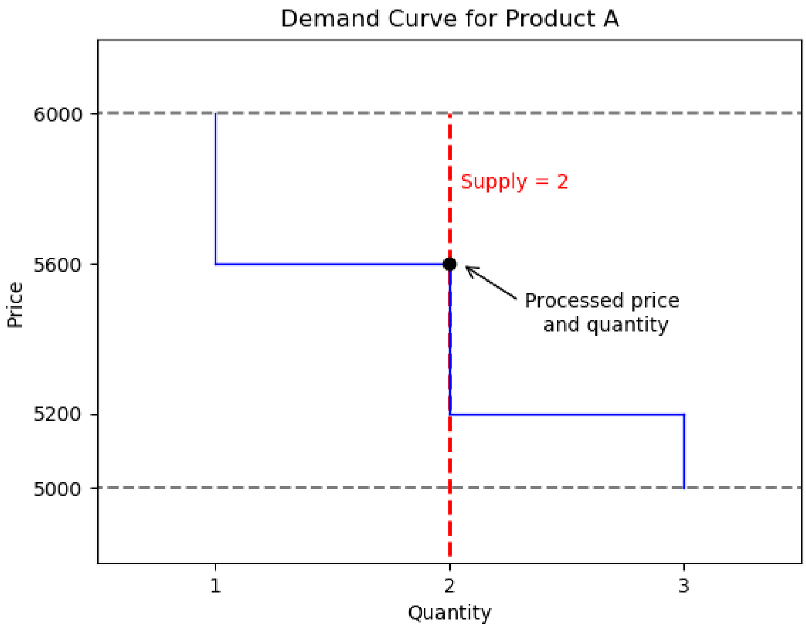


Figure 1.4: Demand curve from Example 1.1

supply.

In the following two examples, we will assume for simplicity that bids are expressing the maximum price that a bidder is willing to pay for the specified quantity. In reality, bids do not work this way in the IFA. We discuss the specifics of how bidders communicate preferences via bidding in Section 1.2.6.

Example 1.1. Suppose we have the following product and associated bids for a round:

Product	Supply	Posted Price	Clock Price
A	2	\$5000	\$6000

Bidder	Product	Price	Quantity
1	A	\$5200	1
2	A	\$5600	1
3	A	\$6000	1

Figure 1.4 shows the demand curve for Product A resulting from the round's bids. Because total demand for the item would drop from 2 to 1 if the price surpassed \$5600, the posted price is set to \$5600 for the following round, with $D_t = 3$, $PD_t = 2$, and $ED_t = 0$. The clock price is always maintained at a set percentage above the posted price (in this case, 20%), so

the new clock price would increase to \$6720. If all three bidders had submitted at the clock price in this example, then the posted price would have instead been set to the clock price (\$6000). All three bids would be processed, with $PD_t = 3$ and $ED_t = 1$.

It is of high priority that as many products of value are sold as possible. Because of this, the auction always maintains a monotonicity of demand—once a quantity of product is bid on, the auction enforces that buyers must continue bidding at this quantity until the conclusion of the auction, up to its supply. This manifests itself in the manner that prices increment, but also in the ability of buyers to revoke bids made during previous rounds. When a buyer is no longer interested in maintaining her demand for a product, she may communicate this by submitting bids with quantities strictly below her processed demand—or no bids at all. The auction is generally willing to increase a bidder's processed demand for an item, but it will only grant a decrease to processed demand if there is sufficient excess demand to accommodate the request. When multiple bidders attempt to reduce demand in the same round, their requests are prioritized from lowest to highest submitted prices. In the case of equal prices, the auction randomly determines which bidders will have their processed demand reduced. When processed demand falls short of supply, requests to further reduce demand will never be honored.

Example 1.2. Suppose we have the following product and associated bids for round t:

	Round t		
Product	Supply	Posted Price	Clock Price
A	2	\$5000	\$6000

		Round t			Round $t+1$	
Bidder	Product	PD_t	Bid Price	Bid Quantity	PD_{t+1}	Posted Price
1	A	1	\$5000	1	1	\$5000
2	A	1	\$5000	1	0	\$5000
3	A	1	\$6000	1	1	\$5000

Figure 1.5 shows the demand curve for Product A resulting from the round's bids. All three bidders have a processed demand of 1 for Product A. By submitting their bids at the posted price, Bidders 1 and 2 are signaling an intent to drop the product. However, both cannot

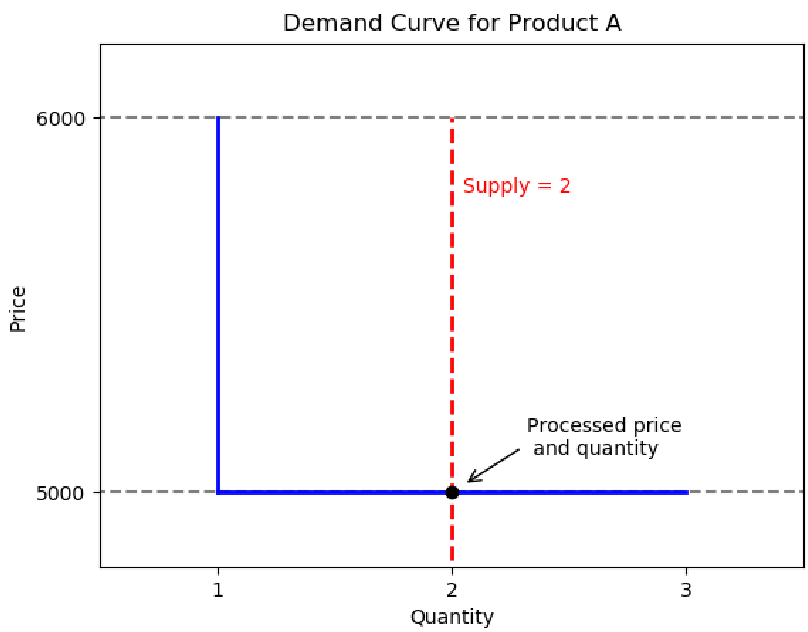


Figure 1.5: Demand curve from Example 1.2

drop without total demand falling below supply. In this example, Bidder 2 is randomly selected to have her processed demand reduced, and the posted price remains fixed.

For this reason, bidding on products can come with some inherent risk. Once bid on, a buyer can never be certain that she will be capable of dropping a product (and freeing the associated amount from her budget). The random means by which bidders are selected to reduce demand also introduces some unpredictability into many round outcomes. These issues are discussed in detail in Section 1.3.

1.2.2 Participation Requirements

Bidding has a sticky nature in the IFA—it is easy to increase demand for a product, but decreasing that demand is never guaranteed. The SMR was similar in this respect. Because reducing processed demand can be difficult, bidders may be incentivized to observe the activity within the auction without committing any actions themselves, even at the risk of the auction concluding early. For the FCC, low revenues are undesirable and can lead to negative press. To minimize this behavior, the Incentive Auction (as well as many earlier mechanisms) punishes buyers who fail to maintain a sufficient volume of processed bids.

In addition to the posted and clock prices, each product r has a (distinct) quantity associated with it called **bidding units** (b_r). Unlike prices, the number of bidding units per product is set and fixed for the entirety of the auction, and is roughly proportional to the population of the associated PEA. Buyers generally cannot bid on every available product at the same time. During round t, bidder i must budget a quantity unique to themselves called their **eligibility** ($E_{i,t}$). The auction will not accept a set of bids from a buyer unless the sum of bidding units across all requested products at any price is less than or equal to their eligibility.

Although a product's bidding units are static, a bidder's eligibility is not. At the end of each round, the auction determines the sum total of bidding units across all processed bids for each bidder. This quantity is called the bidder's **activity** ($A_{i,t}$) for that round, and determines what the eligibility will be for the following round. When a bidder's activity exceeds a set percentage of their eligibility, called the **activity requirement** (AR), their eligibility will be unchanged during the following round. Otherwise, eligibility for the following round will be reduced proportional to the activity as a percent of eligibility. Specifically,

$$E_{i,t+1} = \begin{cases} E_{i,t} & \text{if} \quad A_{i,t} \geq AR \cdot E_{i,t} \\ \frac{1}{AR} \cdot A_{i,t} & \text{otherwise} \end{cases} \tag{1.2}$$

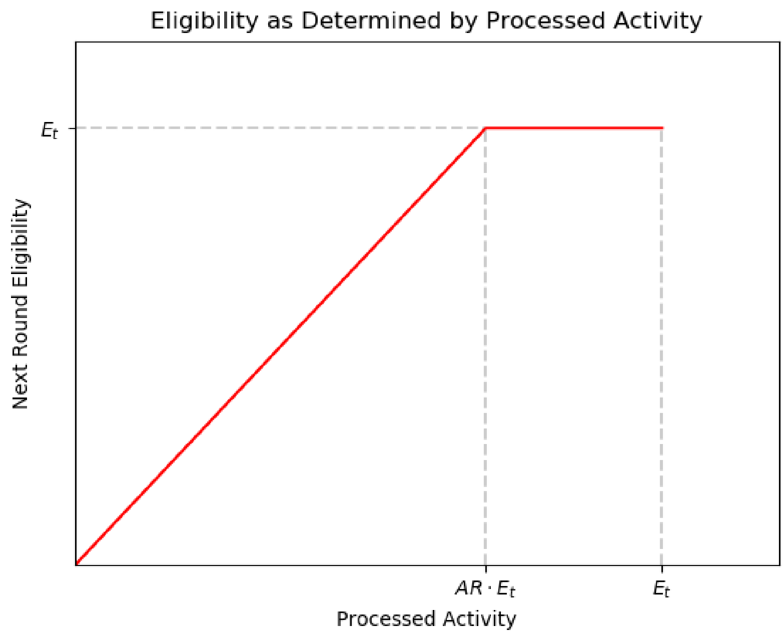
Example 1.3. An auction has activity requirement $AR = 90\%$. Consider a bidder's eligibility and activity over the course of several rounds:

Round	Eligibility	Processed Activity
t	500	470
$t+1$	500	360
$t+2$	400	—

During round t, bidder i starts with eligibility $E_{i,t} = 500$ and submits a bidset resulting in 470 units of processed activity. The activity requirement specifies that they must only process 450 units of activity in order to maintain their eligibility, so $E_{i,t+1} = E_{i,t} = 500$. However, they are only processed for 360 units of activity during round $t+1$ (80% of the activity requirement), so their eligibility is reduced by 80% to 400 bidding units at the start of round $t+2$.

Figure 1.6 shows the relationship between eligibility and activity. Note that eligibility is non-increasing in time (i.e. $E_{t+1} \leq E_t$), and strictly decreases when the activity requirement is not met. In this sense, activity can be viewed as the level of participation of a buyer, and eligibility as the ability for them to bid on many products. When the buyer is insufficiently active, they are penalized with reduced bidding power for the remainder of the auction, all but enforcing active participation.

Figure 1.6: Next round eligibility
Possible values for next round eligibility (E_{t+1}) as a function of current round eligibility, the activity requirement, and processed activity.



1.2.3 Discounts and Credits

The price that a bidder commits during the clock phase forward auction is, in reality, the *maximum* cost that they could expect to incur from purchasing that product. There are two types of discounts that the FCC applies to the purchasers of spectrum licenses.

The first of these discounts is the impairment discount. Although licenses are split roughly by quality, there is still a degree of within-category variance that may be of significance to buyers. This is adjusted for automatically. Suppose that at the conclusion of the forward auction, a license l from product r has a base price p_r and an impairment I_l (a license with 25% impairment would have $I_l = 0.25$). The price of this license after adjusting for impairment would be

$$p_l^{adj} = (1 - I_l) \cdot p_r \tag{1.3}$$

The impairment discount applies to all bidders, and is ultimately determined and applied during the assignment phase. The second of these discounts, referred to as bidding credits, applies only to special interests. Bidding credits are applied to varying degrees (as the FCC deems appropriate) to rural interests and small businesses. Before the start of the auction, bidder i may be assigned a bidding credit percentage, BC_i. The rural bidding credit percentage applies equally to all available products. However, these bidders may only receive a maximum of ten million dollars in credits, after which they will be charged full price for all items.

For small business bidders, products are segmented into two categories. Products are considered to be "small" if the partial economic area covers a population of 500,000 or fewer individuals. Small businesses may receive up to 150 million dollars in bidding credits. However, only a maximum of ten million of these may be applied to small market items. Thus, if bidder i has committed a total cost C_i^S across all small products and C_i^L across all remaining products, then her expected small business bidding credit would be

$$\text{Credit} = \min\Big\{\$150 \text{ million}\,, BC_i \cdot C_i^L + \min\{\$10 \text{ million}, BC_i \cdot C_i^S\}\Big\} \tag{1.4}$$

Depending on the amount of credit applied at the conclusion of the forward auction, a rural or small business bidder will have an effective bidding credit percentage. That is,

$$BC_i^{\mathit{eff}} = \text{Credit}/C_i \tag{1.5}$$

At the end of the assignment phase, all discounts are applied in series. With all taken into account, the adjusted price of an assigned license from product r is

$$p_l^{adj} = (1 - I_l) \cdot (1 - BC_i^{\mathit{eff}}) \cdot p_r \tag{1.6}$$

1.2.4 Closing Rules

At the end of each round of the forward auction, a check is performed to determine its current state. This check assesses whether the **Final Stage Rule (FSR)** has been met. Recall from the previous section that the final stage rule is the driving condition for whether the auction is nearing its conclusion or needs to be modified in order to satisfy FCC targets (i.e. reducing the clearing target and invoking a new stage). The objectives of the FSR are two-fold:

1. To ensure that bidders are not purchasing licenses significantly below competitive spectrum pricing, and

2. To ensure that revenue accounts for the total costs of the auction and reallocation process.

Objective 1: This first goal is somewhat complicated by the fact that the number of licenses to be reallocated (and the resulting "market value") is not known at the start of the auction. Specifically, mobile spectrum value may be expected to decrease in response to a large infusion to supply. To handle this, the FCC has split this objective into two components, only one of which must be satisfied in order to appease the final stage rule.

A megahertz-population (MHz-pop) is used as the basic unit of value for a product. As a result, the expected value of bandwidth is weighted by the population of its respective partial economic area. Prior to the start of the Incentive Auction, the FCC sets a value for X, the price per MHz-pop benchmark. In the Broadcast Incentive Auction, the value of X was fixed at \$1.25/MHz-pop. At the start of each new stage, the licensed spectrum benchmark T is updated based on the new clearing target. Let N be the set of all available products, H the set of specifically high demand C1 products, p_r the end-of-round posted price for product r, and pop_r the population within the PEA associated with product r. With this, objective 1 is satisfied if either of the two conditions are met:

- The average price per MHz-pop of high-demand items exceeds the price benchmark:

$$\frac{\sum_{r \in H} p_r \cdot q_r}{\sum_{r \in H} pop_r \cdot 10 S_r} \geq X \tag{1.7}$$

- The total expected revenue exceeds a stage-sensitive benchmark:

$$\sum_{r \in N} p_r \cdot q_r \geq X \cdot T \cdot \sum_{r \in H} pop_r \tag{1.8}$$

The first condition is the intuitive goal: to avoid a potentially embarrassing situation in which bidders manage to systemically win licenses at far below what the FCC considers to be their worth. The second condition loosens this constraint slightly, setting a revenue threshold that is achievable for higher clearing targets (and thus, higher supply) without necessarily meeting the specified average price.

Objective 2: Covering the minimum costs of the auction should not be a surprising objective, considering the origins of spectrum auctions. By far the most significant of these costs are clearing costs—payments owed to broadcasters per the results of the reverse auction. Whenever a new stage is triggered, supply is reduced and the reverse auction is resumed, reducing this cost at times very significantly. The clearing cost during the first stage of the Broadcast Incentive Auction was roughly $86 billion. By the final stage this threshold had fallen to just over $10 billion (still over 80% of total estimated costs). Apart from clearing costs, revenue must also account for administrative costs and the estimated cost of relocating broadcasters.

The cost requirement must be handled carefully: recall from Section 1.2.3 that discounts will be awarded during the assignment phase. It is easy enough to factor in rural and small business credits, which are static to the bidder. However, credits make the impact from impairment discounts harder to predict. Furthermore, when demand falls below supply there is no way to know a priori which licenses will fail to be awarded (and paid for).

Example 1.4. Suppose that a C1 product with a supply of 3 clears the Clock Phase of the Incentive Auction at a price of $1000. The licenses associated with this product have the following impairments:

License	Impairment	Effective Price
A	0%	$1000
B	5%	$950
C	12%	$880

The effective price shows the price of the auction reduced according to the impairment discount. Only two units of this product were won during the Clock Phase: One unit to Bidder 1, with an effective credit percentage of 10%, and one unit to Bidder 2, who qualified for no bidding credits.

We would like to determine the least amount of revenue that the auction can receive from this product during the assignment phase. This occurs when there is no competition between bidders for any specific license (which would drive up the prices). The lowest revenue is achieved when license B is assigned to Bidder 1, license C is assigned to bidders 2, and license A is assigned to the FCC (i.e. unpurchased), yielding

$$\text{Revenue} = (1 - 0.10) \cdot 950 + 880 = \$1735$$

The FCC cannot know with certainty what revenue to expect prior to the Assignment Phase. Even so, the FSR must guarantee that all costs will be covered before the Assignment Phase begins. To handle this, each round the auction determines δ_r, the least possible amount of revenue that may be generated per product. To calculate this, the total S_r licenses within product r are ordered from least to greatest impairment. If there is excess supply, it is assumed that the most valuable (least impaired) licenses will be assigned to the FCC, after which low impairment licenses are assumed purchased by the bidders with the greatest effective bidding credit percentages. If C is the estimated total cost of running the auction, then this worst-case revenue estimate is required to exceed C:

$$\sum_{r \in N} \delta_r \geq C \tag{1.9}$$

The FSR is checked at the end of every IFA round. If there is no excess demand in the 40 high demand PEAs and the FSR is still not satisfied, this signals to the FCC that activity is winding down and the benchmarks are unlikely to be met. In this case, an extended round, a new stage, or both will be triggered. If the FSR is at any point satisfied after a forward auction round, the auction will immediately transition to the final stage. During this stage, the reserve split will occur and bidding will continue until no excess demand remains for any product.

1.2.5 Reserve Eligible Bidders and the Reserve Split

At the start of the Incentive Auction, a number of bidders are classified as **reserve eligible**. These bidders are given exclusive access to a number of blocks during the final stage, per the FCC's mission statement to protect smaller and special interests. When the final stage rule is satisfied, the auction performs a check to see if any products have excess demand. If not, the forward auction concludes immediately and the assignment phase can begin. Otherwise, the auction performs the **reserve split** and the final stage begins.

To perform the reserve split, the auction selects a number of items from high-supply C1 products and re-categorizes them to the C1R category. C1R products are identical to C1 products once the assignment phase begins, but may only be bid on by reserve eligible bidders during the forward auction. Let S_j^1 and S_j^2 denote the supply for C1 and C2 products respectively within the same PEA. The number of reserve items S_j^R generated is limited in the following ways:

- S_j^R will not exceed the C1 supply within that PEA: $S_j^R \leq S_j^1$
- S_j^R will not exceed the total processed demand D_j from reserve-eligible bidders for the C1 product: $S_j^R \leq D_j$
- If there is exactly one reserve-eligible bidder with processed demand, then $S_j^R \leq 2$.

- In addition, S_j^R is limited by the quantity M_j, which depends on the amount of supply within the associated PEA. During the first stage, M_j is determined by the aggregate supply across both categories:

$$M_j = \begin{cases} 3 & \text{if } S_j^1 + S_j^2 \geq 7 \\ 2 & \text{if } S_j^1 + S_j^2 = 6 \\ 1 & \text{if } 4 \leq S_j^1 + S_j^2 \leq 5 \\ 0 & \text{if } S_j^1 + S_j^2 \leq 3 \end{cases} \tag{1.10}$$

  If the FSR is not triggered during the first stage, M_j for each product can be lowered contingent on the evolving supply and activity of reserve-eligible bidders.

To summarize, if k distinct reserve-eligible bidders have processed demand for product r when the FSR is satisfied, then the amount of C1 products converted into C1R products during the reserve split is given by

$$S_j^R = \begin{cases} \min(S_j^1, D_j, M_j) & \text{if } k > 1 \\ \min(S_j^1, D_j, M_j, 2) & \text{if } k = 1 \end{cases} \tag{1.11}$$

When the S_j^R reserve products are created, the auction automatically moves that number of reserve-eligible bidders into this new product. If there was more reserve demand than S_j^R, then the reserve-eligible bids will be randomly moved into C1R until supply matches demand. This is done by cycling through all reserve-eligible bidders in a random order and assigning one unit of demand to the reserve product at a time, ensuring as balanced a redistribution as possible.

1.2.6 Types of Bids

In order to give bidders as much control as possible over their preferred demand functions between the posted and clock prices for a given item, they are equipped with several varieties of bids that they may employ. These bids are the simple bid, the all-or-nothing bid, and the switch bid. Before we can distinguish between these bids and their resulting outcomes, we must touch on how bids can fail, in full or in part.

Fully and Partially Acceptable Bids: Recall that a submitted bidset may not always be honored, if it conflicts with fundamental auction conditions. If, for example, a bidset attempts to utilize more activity than a bidder's current eligibility (assuming posted prices for all products), then the bidset will be rejected in total and the round will proceed as if no bids were submitted whatsoever. This is an entirely avoidable situation—the amount of requested activity is never made unruly by the auction mechanism. There two main reasons that a bid may unexpectedly be less than fully applied:

1. In order to preserve monotonicity of demand: That is, to ensure that excess supply for a product never increases from round to round.

2. To prevent a bidder's activity from accidentally exceeding their eligibility.

Suppose that a bidder wishes to reduce their demand for product A from 2 units to 0 units. If there is sufficient excess demand for this product, then the bid will be be fully accepted and applied. However, if there is not at least 2 units of excess demand, or if multiple bidders attempt to reduce their demand during the same round, then the bid may not be applied fully, or at all. If there were no excess demand, then demand would not be reduced at all. If there was exactly one unit of excess demand, and no additional bidders attempted to increase or reduce their demand for the round, then the bid would be considered partially acceptable and may be applied in part, by reducing processed demand from 2 units to 1 unit.

Now suppose that a bidder is attempting to reduce demand in product A and increase demand in the product B during the same round. They are depending on freeing activity from product A in order to be eligible to hold more units of B. For the reasons just discussed, the amount of processed demand reduced for product A may be fully or partially applied. The auction will attempt to apply the bidder's request to increase demand for product B to the maximum extent possible, pending their success in shedding demand for product A. In this way, bids either to increase or decrease demand may be considered fully or partially acceptable, and applied appropriately.

Bid Collisions: Recall that bidsets for each product are required to be one-directional. In particular, bidders are not allowed to submitting "colliding" bids—multiple bids associated with the same product at the same price. This restriction is intuitive: a bidder should never need to request both 2 and 4 units of a product at the same price, and it is unclear how the auction should interpret such a request. Even so, it is important to emphasize that this rule applies simultaneously to simple bids, switch bids and all-or-nothing bids. A bidder may not submit two bids at the same price for a product even if the type of bid differs between them. This limitation poses problems for both switch and all-or-nothing bids, as we will see.

Type 1 (Simple Bids): Up to this point, all bids used in examples have been simple bids. A simple bid is a means of requesting up to q units of a product with an associated price p, expressed in the form (p, q). Recall that a bidder can specify multiple bids, $(p_1, q_1), \ldots, (p_k, q_k)$, for the same product in a given round. Suppose that during a round, a product has posted and clock prices given by p_r and P_r, respectively, and that the bids are ordered such that

$$p_r \leq p_1 < p_2 < \cdots < p_k \leq P_r$$

As a result of the one-directional requirement of bids, this implies that

$$q_1 > q_2 > \cdots > q_k$$

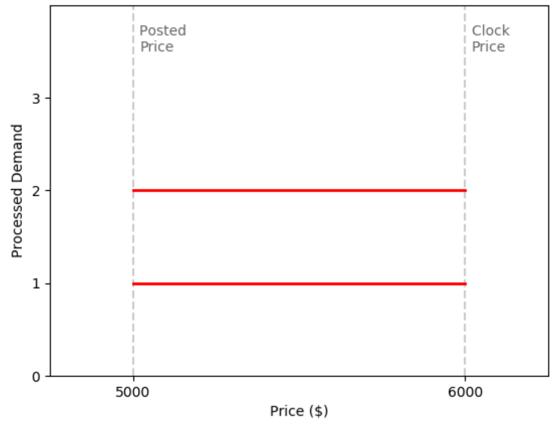
If simple bid (p_k, q_k) is the bidder's highest priced bid submitted a product, it is interpreted as a request for a maximum of q_k units, applicable for any price (up to the current clock price). If simple bid (p_i, q_i) is not the highest priced bid for the product, then the bidder is requesting a maximum of q_i units, applicable for prices not exceeding p_{i+1}, the next ordered price in their bidset. If a bidder has processed demand for a product and submits no bids for the round, the auction assumes a single simple bid $(p, q) = (p_r, 0)$. The next few examples illustrate the different possible outcomes from submitting simple bids.

Example 1.5. Product r has posted price $p_r = \$5000$ and clock price $P_r = \$6000$. A bidder with processed demand $D_r = 1$ submits a single simple bid, for $(p, q) = (\$5500, 2)$. Figure 1.7(a) shows the possible price/processed demand outcomes that may result from this bid. It is possible for the bidder to be processed at either 1 or 2 units of this product for any price between the posted and clock price. The auction will always process a request to increase demand, unless the request is impossible due to a lack of eligibility. This can occur if a request to reduce demand for a separate product is unsuccessful, failing to free up bidding units.

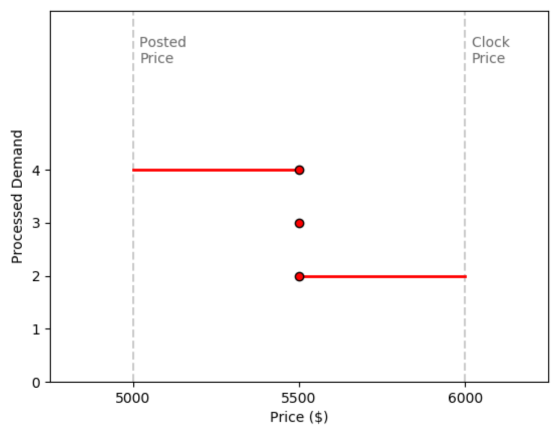
In this case, the price of the bid is arbitrary—any choice between the posted and clock prices would have the same effect.

Example 1.6. Product r has posted price $p_r = \$5000$ and clock price $P_r = \$6000$. A bidder with processed demand $D_r = 4$ submits a single simple bid, for $(p, q) = (\$5500, 2)$. Figure 1.7(b) shows the possible price/processed demand outcomes that may result from this bid. The only difference between this and the previous example is the bidder's current processed demand. As a result, their simple bid is now a request to decrease demand for prices at or exceeding \$5500.

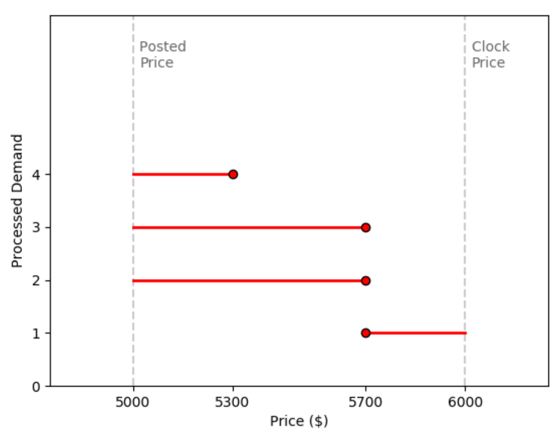
Figure 1.7: Potential processed demands using simple bids



(a) Example 1.5



(b) Example 1.6



(c) Example 1.7

As a result, the auction will not reduce demand for prices below \$5500, and will process exactly 2 units for prices exceeding \$5500. However, if the final price is exactly \$5500, one of three outcomes may transpire. If there is sufficient excess demand, the auction will fully process the request, and demand will be reduced to 2 units for the next round. If there is not sufficient excess demand, the auction will reduce demand by either 1 unit or no units. In the case that final demand is equal to 3, we say that the bid has been partially processed. These types of outcomes can be avoided by submitting all-or-nothing bids, as we discuss below.

Example 1.7. Product r has posted price $p_r = \$5000$ and clock price $P_r = \$6000$. A bidder with processed demand $D_r = 2$ submits simple bids

$$(p, q) \in \{(\$5000, 4), (\$5300, 3), (\$5700, 1)\}$$

Figure 1.7(c) shows the possible price/processed demand outcomes that may result from this set of bids. The bidder is requesting a maximum of 4 units for prices up to \$5300, 3 units up to \$5700, and no more than one unit up to the clock price. Because they have two processed units and their request to increase demand may fail, they may maintain as few as two units for prices between \$5000 and \$5700. However, if the price exceeds this range, the bidder must be processed for 1 and only 1 unit.

Type 2 (Switch bids): As a result of the partially acceptable nature of bids, the outcome of a submitted bidset can be unpredictable. A bidder can end up with more demand for products than expected. This is particularly vexing for products within the same PEA, where a buyer may not value licenses beyond a specific quantity. To help mitigate this, the Incentive Auction allows bidders to submit switch bids when attempting to shift demand between categories within an area. A bid of this type is expressed in the form (p, q, r_{from}, r_{to}), indicating that for prices at or exceeding p, the bidder would like to reduce demand within product r_{from} (to a minimum of q) and increase demand within r_{to} by the exact amount that demand in r_{from} is decreased. Using switch bids guarantees that the amount of processed demand within a specific PEA will not be altered in net even when shifting between categories.

A switch bid behaves very similarly to a simple bid requesting to reduce demand in the from-product. It must be applied in total if the round price exceeds the bid price, but it may be applied partially if the round price is equal to the bid price. No matter the outcome, the sum of C1 and C1R demands will remain unchanged between the current and following rounds.

Type 3 (All-or-Nothing Bids): In examples 1.6 and 1.7, we saw that requests to increase or decrease demand can be partially applied. This may not be acceptable for some bidders. A cellular provider, for example, may only wish to provide coverage for PEAs in which they possess two or more licenses. A single block is of no value to such bidders.

The incentive auction allows bidders to avoid this problem through the issuing of **all-or-nothing (AoN) bids**. These bids are similar to simple bids—they consist of a price and quantity expressing a cap on the number of units a bidder wishes to hold beyond the specified price—but they cannot be partially processed. When the bid attempts to increase demand, the interpretation is straightforward. When the bid attempts to decrease demand, the bidder is communicating a desire to reduce demand, but a willingness to maintain current demand for higher prices if this is not possible in its entirety.

Example 1.8. Product r has posted price $p_r = \$5000$ and clock price $P_r = \$6000$. A bidder with processed demand $D_r = 1$ submits a single AoN bid: $(p, q) = (\$5000, 3)$. Figure 1.8(a) shows the possible price/processed demand outcomes that may result from this bid.

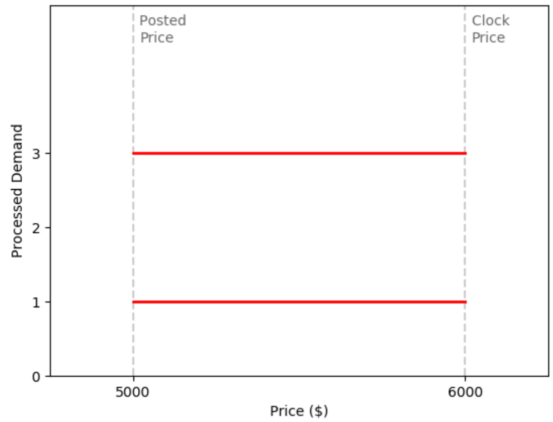
If the bidder has sufficient eligibility, they will be processed for 3 units at any price. If there is not sufficient eligibility, the bidder may continue to be processed for their original single unit, but they will never hold 2 units of processed demand.

Example 1.9. Product r has posted price $p_r = \$5000$ and clock price $P_r = \$6000$. A bidder with processed demand $D_r = 2$ submits a single AoN bid: $(p, q) = (\$5300, 0)$. Figure 1.8(b) shows the possible price/processed demand outcomes that may result from this bid.

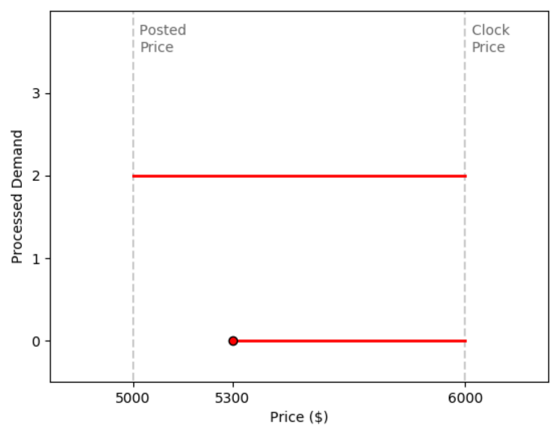
The bidder is requesting to reduce processed demand to zero for prices exceeding \$5300. If there is sufficient excess demand, this will be granted, and the bidder can be certain that they will never be processed for exactly 1 unit of demand. However, if there is exactly one unit of excess demand, the auction will continue to raise the price without reducing the bidder's demand (potentially up to the clock price).

The last example illustrates an undesirable property of AoN bids. When a bidder wishes to reduce all of their units at once, they have little say in where the price should stop. The auction includes one extra feature to counter this, called the **backstop**. If exactly one AoN bid to reduce demand is issued by a bidder, they may optionally include a backstop price. The backstop functionally converts the AoN bid into a simple bid for the same quantity at the specified price, breaking the rigid behavior of the request in order to halt the price.

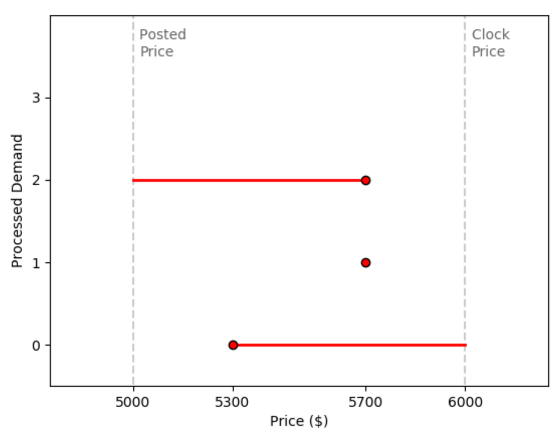
Figure 1.8: Potential processed demands using AoN bids



(a) Example 1.8



(b) Example 1.9



(c) Example 1.10

Example 1.10. Product r has posted price $p_r = \$5000$ and clock price $P_r = \$6000$. A bidder with processed demand $D_r = 2$ submits a single AoN bid: $(p, q) = (\$5300, 0)$ with a backstop at \$5700. Figure 1.8(c) shows the possible price/processed demand outcomes that may result from this bid.

We may imagine this as the same bidder from Example 1.9. They do not wish to win a single unit from this product, but they cannot afford both units at the clock price. As a compromise, they allow the possibility of processing a single unit at \$5700, if there is excess demand and the full reduction is not possible at earlier prices. The bidder can now guarantee that they will have zero demand for prices above this amount.

AoN bids were not frequently utilized during the 2016 Broadcast Incentive Auction. Of the nearly 140,000 total bids, only 121 of these (less than 0.1%) were some form of AoN bid. A grand total of 8 backstops were issued.

1.3 Problems Arising From the Incentive Auction

As we have been foreshadowing, the IFA will at times process bids in undesirable ways from the perspective of the bidders. There are many circumstances in which this may happen. In this section, we will cover several examples outlining common bidding requests that may result in inadvertently unpleasant incidents. These examples are not intended to exhaust all possible issues associated with the IFA mechanism.

Example 1.11. A bidder attempts to drop demand in one product while maintaining demand in another, resulting in going over budget. This is the simplest case in which problems naturally arise. Considering a bidder with a budget of \$420 in the following state:

Product	Posted Price	Clock Price	Profit at CP	Previous Demand	Current Demand
A	\$300	\$330	\$150	1	1
B	\$100	\$110	\$30	1	0

In this and the following examples, "quantity processed" indicate the bidder's holding from the previous round, and "quantity bid" indicates the bids submitted by the bidder for the *following* round. In this example, the bidder is currently committing to \$400 in costs. His budget cannot support the commitment for these products if their prices increment next round, so he elects to bid on the more profitable product A while dropping product B. This occurs in the following:

Product	Posted Price	Clock Price	Profit at CP	Previous Demand	Bid Status
A	$330	$360	$120	1	Success
B	$100	$110	$30	1	Failure

The auction blocked the bidder from reducing his quantity in product B due to a lack of demand. As is guaranteed in such circumstances, the price for product B not not increase. However, the price for product A *did* increase, and as a result, the bidder is now committed to $430 in costs, bringing him over budget.

This is a lose-lose situation for the bidder. If he chooses not to actively bid on product A (his most valuable product) to avoid this issue, he risks losing it, as well as its associated activity.

Example 1.12. A bidder attempts to merge demand from several products, resulting in going over budget. Consider a bidder with an eligibility of 100 and a budget of $250 who submits bids for the following round:

Product	Posted Price	Clock Price	Activity units	Quantity Processed	Quantity Bid
A	$200	$220	80	0	1
B	$100	$110	70	1	0
C	$100	$110	20	1	0

The bidder is attempting to "merge" their holdings from products B and C into a single unit of item A, resulting in the following outcome:

Product	Posted Price	Clock Price	Activity units	Quantity Processed	Bid Status
A	$220	$240	80	1	Success
B	$100	$110	70	0	Success
C	$100	$110	20	1	Failure

The auction granted the bidder's request to drop product B, but not product C. And because there was sufficient eligibility to hold product A, her bid to increase demand was still processed. The bidder is now committed to $320 in costs compared with her $250 budget. Under slightly different circumstances, a request like this can turn out quite differently, as we see in the next example.

Example 1.13. A bidder attempts to merge demand from several products, resulting in a large eligibility loss. Consider the identical scenario fro the previous example, but with an eligibility of 90 instead of 100:

Product	Posted Price	Clock Price	Activity units	Quantity Processed	Quantity Bid
A	$200	$220	80	0	1
B	$100	$110	70	1	0
C	$100	$110	20	1	0

Once again, the bidder is successful in reducing demand for product B but not C. Because of their reduced eligibility, this results in the following:

Product	Posted Price	Clock Price	Activity units	Quantity Processed	Bid Status
A	$220	$240	80	0	Failure
B	$100	$110	70	0	Success
C	$100	$110	20	1	Failure

Product C remains processed, leaving insufficient eligibility to hold product A. As a result, only 20 units of activity were processed for the round, which will mean a steep loss in eligibility for the bidder in all future rounds.

Example 1.14. A bidder attempts to move quantity between a reserve and non-reserve product, resulting in holding a valueless unit. For this example, a bidder with an eligibility of 140 has value for up to two pooled units of product A and its reserve equivalent, and up to one unit each of products B and C:

Product	Activity units	Quantity Processed	Quantity Bid
A	40	2	0
A_R	40	0	1
B	60	1	0
C	80	0	1

This bidder is trying to accomplish two goals at once. First, they would like to switch one of their units of product A to the reserve market. Second, they are attempting to merge units of product A and B into a single unit of product C. This results in the following:

Product	Activity units	Quantity Processed	Bid Status
A	40	2	Failure
A_R	40	1	Success
B	60	0	Success
C	80	0	Failure

The auction allowed demand to be reduced for product B, but not for product A. As a result, the bidder has only freed up 60 units of activity. Between the two bids to increase demand, only product A_R can be granted. The bidder is now holding three units of product A/A_R. Depending on the cost of the product, this error can lead to a net loss in profit for the bidder.

The auction design attempts to avoid this type of situation by allowing bidders to submit relatively low-risk switch bids. However, switch bids require a one-to-one exchange between categories: you may only issue a switch bid when you wish to move all of your bids from one category to another. Because the bidder in this example wished to simultaneously reduce demand for product A, switch bids were not an available option.

In the next chapter, we introduce an alternate mechanism that addresses these issues. We then examine the how the two mechanisms compare on a range of issues when simulated in a number of environments.

Chapter 2

Simulating the Auction Environment

In order to further assess our impressions concerning the Incentive Auction mechanism (as well as gaining additional insights), we made use of two approaches: a computational approach and an experimental approach. In this chapter, we discuss the computation approach, in which the auction environment was simulated on the computer and executed using automated bidders.

An analysis of our simulation results would be lacking without a point of comparison. We therefore take some time during this chapter to introduce an alternate mechanism in the form of a modified SMR. After this, we discuss the various bidding algorithms implemented within the two mechanisms, as well as the choice of parameters that composed the various auction environments. Finally, we discuss the findings of these simulations with respect to four major benchmarks—auction surplus, revenue, the number of rounds per auction, and the rate of bidder bankruptcy.

2.1 Introduction

Prior to designing and implementing an experiment with human subjects, many simulations of the FCC Incentive Auction clock phase were run over a variety of parameters. The primary goals of these simulations were two-fold:

1. To assess the effectiveness and efficiency of the Incentive forward auction mechanism with respect to some reasonable alternative, and
2. To explore the benefits and drawbacks of various bidding strategies.

The two goals are not unrelated. The performance of simpler bidding strategies exposed complications elicited from the structure of the auction itself, which in turn led to more informed, complex strategies.

A number of simplifications were made in translating the Incentive Auction into code. For one, we are interested in isolating the effects of the clock phase, and specifically the forward auction component, so we do not run a reverse auction to determine supply and opening prices, nor an assignment phase. A simulation takes place over only a single stage (plus the final stage where reserve items are split). There are no high-demand items; instead, the auction will always run until no excess demand remains in any product, recording at the end whether the FSR was satisfied. Because of this, there are no extended rounds or stage transitions apart from the reserve split.

Although the assignment phase does not occur, licenses are still expected to have some level of impairment (and an associated impairment discount). To handle this, each product is assigned an expected impairment rate that is common knowledge to all bidders. When choosing between items, bidders automatically factor the expected impairment discount into the current posted and clock prices.

At the start of a simulation, the opening prices for a predetermined set of items is drawn from a distribution. The values that bidders have for these items, as well as the number of each item to which they assign value, are also drawn from distributions. We discuss these distributions in more detail in Section 2.4.

We also make the following assumptions about the behavior of bidders participating in the auction, regardless of their strategy:

- Bidders will never bid strictly between the posted and clock prices, although the auction is able to handle such an occurrence. As a result, bidders never submit more than one bid per product per round, and prices will only ever increment from the posted price to the clock price (or not at all).

- When a bidder has value for multiple blocks of a single product, their value for each block is never greater than their value for the previous block. A bidder will never value two blocks of an item more than one block of that same item, for example. This largely eliminates any incentive to make use of all-or-nothing bids.

- Although they share the same PEA, The values that bidders have for C1 and C2 items are treated separately. If a bidder has value for one each of a C1 and C2 product, then purchasing two C1 blocks will not satisfy the need for the C2 product, and the second purchased block will be treated as valueless. However, the values for C1 and C1R products are pooled. Thus, if a bidder had value for two blocks of a C1 product, then they could maximize their value with two blocks of the C1 product, or two blocks of the corresponding C1R product, or one block of each.

We will proceed by presenting the alternative forward auction mechanism that will be used to contrast the Incentive Auction, before outlining several bidding strategies that were tested against both mechanisms. Finally, we present the results of the simulations and some implications that can be drawn from them.

2.2 An Alternative Treatment: The Simultaneous Multiple Round Auction

When transitioning from previous spectrum auction mechanisms to the Broadcast Incentive Auction, the primary goal of Congress via the FCC was the creation of a centralized two-sided market that would maximize market supply of licenses while guaranteeing that gross revenue would cover auction and administrative costs [17]. However, because of the linked, relay-like relationship between the seller and buyer markets, there was a desire to increase the speed of the forward auction. In debating the design of the eventual IFA mechanism, Paul Milgrom et al argued that deviations from the SMR "should permit the auction to be completed in a fraction of the time that would be required by a traditional SMR auction, with no loss of efficiency or added difficulty for bidders [19]." We will touch on all of these points when analyzing our simulations further on.

Recall from Section 1.1.1 that the previously used mechanism, the Simultaneous Multiple Round Auction, considered all licenses within a single PEA distinctly and independently for bidding. Such a volume of (very similar) products has the potential to create a very slow convergence of activity. In 1996, for example, the FCC auctioned roughly 1500 Broadband Personal Communications Services licenses. The process lasted from August of 1996 until January of 1997, requiring 276 rounds and 85 days of bidding in what ended up being the lengthiest auction to date [6]. Grouping spectrum licenses within the same PEA and creating the assignment phase was meant to mitigate this lengthiness. The 2016 Broadcast Incentive Auction repurposed roughly double the number of licenses, lasting for 87 rounds over the course of 30 bidding days (as well as lengthy waiting times between each of the 4 periods) [9].

These changes likely contributed to a speedier format. However, we contend that some of the changes made resulted in unnecessary adverse consequences for bidders and value-revealing incentives. In particular, auction eligibility as a concept attempts to coerce bidder activity each round, while the precarious nature of bid processing undermines this incentive. The results for bidders are unexpected losses to eligibility while risking committing more than intended (as discussed in Section 1.3). We propose a mechanism to address these issues and reduce the need for complex bidding strategies.

The Incentive Auction diverged significantly from the SMRA in how it handled provisional bidders for products at the end of each round. The SMRA had exactly one supply for each license, and only one bidder was selected to hold the item at the end of each round. This bidder either submitted the highest priced bid for the round, or was randomly chosen in the event of multiple equal bid amounts. When a bidder attempted to bid on a product and was not selected as the provisional bidder, activity was still processed toward their total eligibility. The Incentive Auction, on the other hand, allows any number of provisional winners in excess of a product's supply, and penalizes eligibility when such bids are not processed.

We wish to extend the behavior of the SMRA to handle the situation where products have multiple units of supply, while maintaining the large scale framework of the Incentive Forward Auction (multiple stages consisting of reverse auctions and extended rounds, a reserve split, and an assignment phase). The behavior of such a hybrid mechanism is outlined below.

2.2.1 Bidding and Provisional Winners

Unlike the Incentive auction, pricing is handled discretely rather than continuously. Every round, each product r will have only one (posted) price p_r associated with it, and a bidder i will only submit the quantity $q_{i,r}$ that they are willing to commit for the product. There are no switch bids and no all-or-nothing bids within this mechanism. When a buyer submits a bid (quantity), the auction treats the request as $q_{i,r}$ individual unit bids, a number of which may be processed for the round as **provisional bids**. The number of provisional bids will never exceed the supply of the product and, similar to the Incentive Auction, will never decrease in number from the previous round.

There are two types of provisional bids: **low priority provisional (LPP) bids** L_r and **high priority provisional (HPP) bids** H_r. We treat these variables as sets of bids, with $|L_r|$ and $|H_r|$ representing the number of low and high priority provisional bids, respectively. When a bidder submits a bid for a round that is successfully processed, that bid is initially a HPP bid for as long as the price holds constant. When there is positive excess demand for a product, the price increments by a set factor (similar to the relationship between the posted and clock prices in the Incentive Auction), and all HPP bids are reclassified as LPP bids. As a result, LPP bids for a product/round are associated with a **low priority provisional price** p^l_r. This price has no consequence for future bids, and is simply a record of the price committed for legacy bids as the auction progresses.

A bidder's total number of provisional bids for a given product is their processed demand $PD_{i,r}$ for the product. When a bidder with processed demand submits a bid $q_{i,r}$, they are not necessarily committing every one of these units at the current price. The first $PD_{i,r}$ units communicate only that they would prefer to hold their provisional bids at their corresponding prices (always applying to HPP bids before LPP bids). Any additional submitted quantity is committed at the current price.

As an example, suppose that a product has a LPP price $p^l_r = \$5000$, a posted price $p_r = \$5500$, and that bidder i has processed demand $PD_{i,r} = 3$, with one of their bids in H_r and the other two in L_r. Regardless of the number of bids submitted, they have a commitment satisfying

$$\text{Commitment} \geq \$5500 + 2 \cdot \$5000 = \$15{,}500$$

to account for the three items currently held. If no bids are submitted, then the bidder would like to be prioritized in freeing their activity from this product during bid processing. If they submit $q_{i,r} = 1$, then they are requesting that two bid be freed with high priority (the LPP bids, in this case), but are not allowing their one HPP bid to be freed. With $q_{i,r} = 2$, They are again blocking their HPP bid from being freed, and are now setting one LPP bid to be freed with high priority and the other with low priority. With $q_{i,r} = 4$, all provisional bids are set to be freed at low/no priority, and they request an additional HPP bid at the posted price. Thus, their requested commitment for the round is

$$\text{Committment} = \$5500 + 2 \cdot \$5000 + \$5500 = \$21{,}000$$

When a bidder has HPP bids, they are able to guarantee that up to that number of bids remain in that product by the end of the round. However, LPP bids are always susceptible to being displaced, and are guaranteed to be given a sufficient amount of excess demand for the round.

2.2.2 Processing the Bids

As with the Incentive Auction, once the bids are submitted the SMRA must assess their validity, determine provisional winners for the round, and increment prices where needed. Apart from attempting to submit bidsets with excess activity, bids will never be invalid under this mechanism, so this check can be performed as soon as bids are submitted. The bid processing procedure for each product r flows as follows:

1. Submitted bid quantities are itemized (split into individual unit bids) and separated into two categories: maintenance bids and additive bids. Maintenance bids specify that a bidder would like to maintain their current provisional bids, while additive bids request to further increase their processed demand. If a bidder with processed demand $PD_{i,r} = 2$ submits a bid $q_{i,r} = 5$, this bid will be split into two maintenance bids and three additive bids. If the same bidder had instead submitted a quantity of two or less, then all itemized bids would be categorized as maintenance bids.

2. The auction then determines the excess demand ED_r for the product. If PD_r is the total number of provisional bids from the previous round and AB_r is the total number of additive bids, then for a supply S_r the excess demand is calculated as

$$ED_r = \max\{0, PD_r + AB_r - S_r\} \tag{2.1}$$

   In the event that processed demand has reached capacity, the formula for excess demand for all future rounds reduces to the number of additive bids. Observe that provisional bids are counted toward processed demand even when bidders do not request to maintain their bids.

3. If $ED_r = 0$, all additive bids (if any) are immediately added to the HPP bid set, H_r. No existing provisional bids are freed in this case.

4. If $ED_r > 0$, the auction does the following:

   (a) Randomly sort all additive bids into a queue. If $PD_r < S_r$, immediately add the first $(S_r - PD_r)$ bids from the queue into H_r.

   (b) Based on the maintenance bids, group provisional bids into three randomly ordered queues: H_r bids *without* associated maintenance bids are added to the undesired HPP queue, L_r bids with associated maintenance bids are added to the desired LPP queue, and L_r bids without associated maintenance bids are added to the undesired LPP queue.

(c) Provisional bids are sequentially removed in the order specified by the provisional queues as bids from the additive queue are added to H_r. Provisional bids from the undesired LPP queue are removed until the undesired LPP queue is exhausted, followed by the desired LPP queue, and finally the HPP undesired queue (desired HPP bids are never removed). This process continues until either the additive queue or all provisional queues are exhausted.

(d) If any additive bids remain and $PD_r < S_r$, add the next $(S_r - PD_r)$ additive bids from the queue into H_r.

5. If $|H_r| = S_r$, this means that all blocks are requested at the posted price. In this case, transfer all bids from H_r into L_r, set $p_r^l = p_r$, and increment the posted price.

We will run through several examples to illustrate this process. For all the following examples, we assume that product r has a price increment of 10% with $p_r = \$5500$ and $p_r^l = \$5000$ (or no LPP price, if processed demand is below supply).

Example 2.1. A product has previously been bid on, but has not yet had its price incremented. Its state is

$$\text{Initial State:} \begin{cases} S_r = 5 \\ L_r = \{\} \\ H_r = \{1,1\} \end{cases}$$

Because all bids are unit bids, they are represented by an integer indicating the index of the issuing bidder. In the case, there are no LPP processed bids and two HPP processed bids, both held by Bidder 1. The bids submitted for the round (after being itemized) are separated for processing as follows:

$$\text{Submitted bids} = \{1,1,1,2\} \quad \Rightarrow \quad \begin{array}{rcl} \text{Maintenance bids} & = & \{1,1\} \\ \text{Additive bids} & = & \{1,2\} \end{array}$$

In other words, Bidder 1 submitted $q_{1,r} = 3$ and Bidder 2 submitted $q_{2,r} = 1$. Because Bidder 1 already had a processed demand of 2, his first two itemized bids are processed as requests to maintain this demand, and only the third unit is considered additive. Bidder 2 is a newcomer, so his one and only bid is additive. From here we calculate

$$ED_r = \max\{0\,,\,(|L_r| + |H_r|) + |\text{Additive bids}| - S_r\} = \max\{0\,,\,(0+2)+2-5\} = 0$$

There is no excess demand, so all additive bids can immediately be processed, leaving us with

$$\text{Final State:} \begin{cases} L_r = \{\} & p_r^l \text{ unassigned} \\ H_r = \{1,1,1,2\} & p_r = \$5500 \end{cases}$$

Example 2.2. We begin with the same state as the previous example:

$$\text{Initial State:} \begin{cases} S_r = 5 \\ L_r = \{\} \\ H_r = \{1,1\} \end{cases}$$

This time, more bids are submitted:

$$\text{Submitted bids} = \{1,2,2,3,4,4\} \quad \Rightarrow \quad \begin{array}{rcl} \text{Maintenance bids} & = & \{1\} \\ \text{Additive bids} & = & \{2,2,3,4,4\} \end{array}$$

Enough new bids were submitted that the product has positive excess demand:

$$ED_r = \max\{0\,,\,(0+2)+5-5\} = 2$$

Because $ED_r > 0$, the auction forms two random queues: an additive bid queue, which is a simple rearrangement of the additive bids, and the provisional queue, which lists and randomly orders all provisional bids that may be displaced by the additive bids. Because of the random nature of this process, the queues that follow are just one of many possible outcomes that may have occurred in this instance.

$$\text{Additive Bid Queue} = \leftarrow \boxed{4\,|\,2\,|\,4\,|\,3\,|\,2} \leftarrow$$
$$\text{Provisional Queue} = \leftarrow \boxed{1} \leftarrow$$

In this case, because Bidder 1 submitted $q_{1,r} = 1$ but had $PD_{1,r} = 2$, she is requesting to drop one of her provisional units. This undesired bid is represented in the provisional queue. One of Bidder 4's bids replaces one of Bidder 1's bids in H_r. At this point, the provisional queue is empty, but there are three units of the product available. Thus, the next three bids from the additive queue (corresponding to Bidders 2, 4, and 3) are added to H_r. Bidder 2's second bid is chosen as a provisional bid during this round. Finally, we find that $|H_r| = S_r$, so the bids are moved into L_r and the price increments, leading to the following:

$$\text{Final State:} \begin{cases} L_r = \{1,2,3,4,4\} & p_r^l = \$5500 \\ H_r = \{\} & p_r = \$6050 \end{cases}$$

Example 2.3. For the last example, we examine an item whose price has been incremented at some point already:

$$\text{Initial State:} \begin{cases} S_r = 8 \\ L_r = \{1,1,2\} \\ H_r = \{1,3,3,4,5\} \end{cases}$$

Again, a variety of additive and maintenance bids are submitted. Because we now have both LPP and HPP bids to keep track of, we will distinguish between these with l and h superscripts where needed.

$$\text{Submitted bids} = \{1,1,2,2,3,4,4\} \;\Rightarrow\; \begin{aligned} \text{Maintenance bids} &= \{1^h, 1^l, 2^l, 3^h, 4^h\} \\ \text{Additive bids} &= \{2,4\} \end{aligned}$$

Let's examine Bidder 1's bids in detail. Because she has $q_{1,r} = 2$ and $PD_{1,r} = 3$, all of her bids are for maintenance. However, a portion of her processed demand is invested in both L_r and H_r. Recall that when attempting to maintain bids, that maintenance will always prioritize HPP bids over LPP bids when possible. This means that her first maintenance bid is processed to maintain her only HPP unit, and only then does her second maintenance bid select one of her LPP bids to maintain, leaving the final LPP bid to be discarded with high priority. Because the price has incremented in the past, the excess demand is simply

$$ED_r = |\text{Additive bids}| = 2$$

There is positive excess demand, so the random queues are formed. Again, the following is one of multiple possible realizations from this step:

$$\text{Additive Bid Queue} = \leftarrow \boxed{4 \mid 2} \leftarrow$$
$$\text{Provisional Queue} = \leftarrow \boxed{1^l} \leftarrow \boxed{2^l \mid 1^l} \leftarrow \boxed{3^h \mid 5^h} \leftarrow$$

This time around, the provisional queue contains bids from all three possible categories. The left-most component contains the one LPP bid that Bidder 1 no longer wishes to maintain. The middle component contains the remaining LPP bids from Bidders 1 and 2 that are attempting to be maintained. Finally, the right-most component contains the HPP bids that are no longer desired. Bidder 3 has two bids in H_r but submitted only a single maintainence bid. Bidder 5 has one but submitted no bids at all.

Because the additive queue is shorter than the provisional queue, all additive bids are added to H_r, displacing bids 1^l and 2^l and resulting in the following:

$$\text{Final State:} \begin{cases} L_r = \{1\} & p_r^l = \$5000 \\ H_r = \{1,2,3,3,4,4,5\} & p_r = \$5500 \end{cases}$$

Although there was positive excess demand, the price did not increment this round. The price will only increment when the amount of excess demand matches or exceeds $|L_r|$.

2.2.3 Eligibility and Activity

The last change which needs addressing is how the auction processes bidder activity each round. We propose a more conservative treatment to avoid unexpected losses to eligibility, made possible by the discrete nature of pricing. Recall that the Incentive Auction would grant activity only for products that were successfully processed, which in turn depended on both the pricing of the product and the success in dropping (or failure in adding) alternative products. Instead, the SMR sums activity per bidder from two different sources, regardless of the round results within each market:

1. All additive bid requests
2. All provisional bids

Additive Bids: Whenever a bidder requests to increase their processed demand for a product, each requested unit (limited to the total supply) will count toward total activity. This leniency is necessary for the nature of provisional bidding in this mechanism—frequently, the auction will select only a subset of requests to be processed into provisional bids. Punishing unsuccessful bidders with a loss of eligibility would be detrimental to the bottom line of all parties involved.

Provisional Bids: As with the Incentive Auction, bidding on products has a sticky tendency. Once held, products cannot always be readily dropped, and when they are dropped it may occur unexpectedly after many unsuccessful attempts. To offset this negative trait, all provisional bids from previous rounds count toward a bidder's activity total. This is true whether or not the bidder has requested to maintain these bids. This allows a bidder to consistently signal that they wish to be removed from a market, without risking the loss of activity associated with being dropped.

Viewed from a different perspective, the activity from a bidder's provisional bids are automatically deducted from their eligibility at the start of a given round. This effectively reduces the decision space of a bidder, oftentimes drastically. If, for example, every requested bid is processed from the previous round, then 100% of the bidder's eligibility becomes tied in provisional bids, and they are limited to the submission of maintenance bids only. In this way, maintenance bids are very cheap to submit—they cost no additional activity, and will not increase in price.

Example 2.4. A bidder has an eligibility of $E_i = 1300$ and is considering bidding on the following products:

Product	Bidding Units	Processed Demand
A	350	2
B	200	1
C	400	0
D	150	0

Factoring in the amount of activity monopolized from their processed demand, the bidder only has

$$1300 - 2 \cdot 350 - 1 \cdot 200 = 400$$

units of activity to invest in additive bi ds. Suppose they submit a bidset consisting of one unit of product A, two units of product B, and one unit of product D. Their additive bids consist of one unit each of product B and D. Thus, their total activity for the round is

$$\begin{aligned}\text{Total Actvity} &= (\text{Provisional Activity}) + (\text{Additive Activity})\\ &= (2 \cdot 350 + 200) + (200 + 150) = 1250\end{aligned}$$

2.3 Auction Bidding Strategies

A number of strategies were implemented for the IFA and SMR auction mechanism. Strategies were deployed homogeneously (with all bidders making use of identical strategies) and heterogeneously (with different strategies mixed together competitively). The first few sections focus on the implementation of strategies within the IFA mechanism before discussing

SMR strategies, which only vary slightly. Before we dive in to these, we need a bit more notation. We assume that each bidder i will operate under a strict budget, B_i. Recall also that bidders have differing (and non-increasing) marginal values for each unit of product. That is, given a product r with a supply of S_r, bidder i will have **marginal values** $V_{i,r,1}$, $V_{i,r,2}$, ..., V_{i,r,S_r} satisfying

$$V_{i,r,1} \geq V_{i,r,2} \geq \cdots \geq V_{i,r,k} \geq \cdots \geq V_{i,r,S_r} \geq 0 \tag{2.2}$$

Finally, recall that products will come with a number of discounts depending on the impairment of the products and the status of the bidder. For each product, a bidder assumes for each product the discount associated with its expected impairment applied to the price, as well as applicable special interest credits. We choose $C_{i,r}$ to represent the expected cost *after discounts* (always using the current round clock price as the base value, unless otherwise specified). A bidder's **marginal expected profit** for the k^{th} unit of product r is then given by

$$P_{i,r,k} = V_{i,r,k} - C_{i,r} \tag{2.3}$$

Because these quantities assume the clock price for each product, they are a conservative estimate of profit, and are the basis for all bidder strategies that we consider.

2.3.1 The IFA Lexicographic Strategy

The lexicographic strategy, in spite of its name, is the simplest type of strategy simulated. The term implies a total mathematical ordering of a finite set [16] (in this case, licenses in accordance with their profit). The concept is straightforward: a bidset is formed by including as many licenses as can be afforded (limited by both budget and eligibility), prioritizing licenses worth the greatest expected profit first. One might imagine a very naive or inexperienced participant to employ the lexicographic strategy as their first attempt at optimization. We must be a bit careful with the details, as simulated bidders pool values between C1 and C1R products. With this in mind, the strategy makes use of the following algorithm:

1. Expected profits $P_{i,r,k}$ are calculated for each marginal license.

2. A subset of the marginal licenses are filtered out from consideration. Specifically, if r' is a C1 product with associated C1R product r'', then only one of these two will be included as a license for each marginal unit, based on which has greater expected

profit:

$$\text{Selected license} = \underset{r\in\{r',r''\}}{\text{argmax}}\left(P_{i,r,k}\right),\ k = 1, 2, \ldots, S_{r'} \tag{2.4}$$

This prevents the strategy from recommending redundant licenses, and also favoring the more valuable of the redundancies. When r is a C2 product, its associated licenses are all included for consideration (as we assume its value is not shared across other products).

3. Marginal licenses with negative expected profit are filtered out of consideration.

4. Remaining licenses are sorted from greatest to least profit.

5. Licenses that can be afforded are added one at a time from the sorted list into the bidder's bidset for the round. A license is considered affordable if, after factoring the licenses already added to the bidset, bidding on this new license will not commit the bidder to going over their eligibility or budget limits (assuming clock prices).

Step 5 continues until there are no marginal licenses remaining to consider (or either of the bidder's budget or eligibility is exhausted by the updated bidset).

Example 2.5. Suppose that bidder i has a budget of \$4000, an eligibility of 230, and has already determined the expected profit of all marginal licenses based on results from the previous round. This information is summarized in the table below:

PEA	Category	Supply	Bidding Units	Clock Price (\$)	$P_{i,r,1}$ (\$)	$P_{i,r,2}$ (\$)	$P_{i,r,3}$ (\$)
A	$C1$	3	40	1000	800	600	200
A	$C1R$	2	40	850	950	750	—
A	$C2$	1	40	600	400	—	—
B	$C1$	3	65	300	800	−100	−300
C	$C1$	1	150	1300	1200	—	—

We start by determining which licenses will be considered within the A region. Since the reserve products have higher marginal profits, we discard the first two marginal licenses within the A|C1 product. We do not end up comparing (or discarding) the third license between these two, since the reserve product becomes limited by its supply. We also remove the second and third licenses from product B|C1, since these are worth negative profit.

Finally, the remaining licenses are ranked and processed. We express marginal licenses in the form

(PEA | Category | Unit number)

For example, (A|C1R|2) is worth \$750 in profit. The processing occurs in the table below, where the costs of affordable items are deducted from the remaining budget and eligibility.

License	Remaining Budget	Remaining Eligibility	Can Afford?
(C|C1|1)	\$4000	230	Yes
(A|C1R|1)	\$2700	80	Yes
(B|C1|1)	\$1850	40	No
(A|C1R|2)	\$1850	40	Yes
(A|C2|1)	\$1000	0	No
(A|C1|3)	\$1000	0	No

The bidder first adds the two most valuable licenses, (C|C1|1) and (A|C1R|1), to their bidset. At this point, they only have 40 units of activity free from their eligibility, so they cannot afford the next most valuable license, (B|C1|1), which requires 65 units of activity. Finally, they add (A|C1R|2), which exhausts their remaining activity. In total, their bidset requests one unit of C|C1 and two units of A|C1R.

Advantages The most obvious advantage to the lexicographic strategy is its simplicity. It requires no software or mathematical knowledge, and may realistically be employed by a relatively unsavvy or small firm. A less obvious advantage is that the strategy will always find a suggested bidset. Later strategies have the chance to fail under poor conditions, requiring the implementation of backup strategies. In this way, the lexicographic strategy has the most predictable behavior.

Disadvantages The lexicographic strategy has at least two distinct disadvantages. Most significantly, the strategy invests no attention in addressing the activity costs of th products. Products with lower activity costs are never deliberately prioritized. In the example, the most profitable item is prioritized first despite costing significantly more eligibility than any other item and greatly limits the ability to hold other items. The strategy also makes no attempt to meet the activity requirement and maintain eligibility for future rounds.

In addition to this, the strategy pays no regard to the likelihood that bids will fail to be processed. As we have seen in Section 1.3, budget-related issues are liable to occur when frequently attempting to switch processed demand between products. The lexicographic strategy is inclined to recommend persistent "bid hopping" when the profits between multiple products are very close to one another. When the difference in expected profits is not very large, or when excess demand is low or turbulent, it could be argued that maintaining current processed demand is superior to significant movement in the market.

2.3.2 The IFA Straightforward Strategy

Ignoring bidding issues associated with the incentive auction, the lexicographic strategy fails in the sense that it does not optimize expected profit. The straightforward (SF) strategy assumes bids will always be processed and submits bidsets that maximize profits, thus revealing preferences between products. The structure of this problem—maximizing utility by selecting between items under two independent budget constraints—bears resemblance to the classic knapsack problem, as described in [24]. We therefore approach this using linear programming.

Some notation is required before proceeding. For the sake of brevity we will not include the bidder's index for this or the following strategy.

- $\delta_{r,k}$ is the **decision variable**—a binary variable specifying whether the bidder will request the k^{th} unit of product r as part of their bidset. When $\delta_{r,k} = 1$, the bidder requests the associated marginal license; if $\delta_{r,k} = 0$, they do not. When r is a C1 product, we will at times denote $\delta_{r,k}^{C1R}$ to be the decision variable for the associated C1R license, when such a license exists.
- A_r is the activity cost of product r.
- E is the eligibility of the bidder.
- C_r is the clock price of the product (after applying discounts).
- B is the budget of the bidder.
- $V_{r,k}$ is the value held for the k^{th} unit of product r.
- N_r is the number of licenses for product r to which the bidder assigns value.
- $1_P(V)$ is the negative profit indicator variable for the value of a license:

$$1_P(V_{r,k}) = \begin{cases} 1 & \text{if } V_{r,k} < C_r \\ 0 & \text{otherwise} \end{cases} \tag{2.5}$$

Finally, $\sum_r$ implies a sum over all available products, r. With this, the revelation optimization takes the following form:

$$\begin{aligned} \text{Maximize} \quad & \sum_r \sum_{k=1}^{N_r} \delta_{r,k} \cdot P_{r,k} \\ \text{subject to the} \quad & \text{Budget Constraint} \\ & \text{Maximum Eligibility Constraint} \\ & \text{Profitable Constraints} \\ & \text{Supply Constraints} \\ & \text{Pooled Value Constraints} \end{aligned} \tag{2.6}$$

We will address each of these named constraints in detail. It should be noted that, as in the previous section, profit $P_{r,k}$ is always assumed at the clock price and is therefore a conservative estimate.

The Budget Constraint: Presumably, a bidder would prefer not to commit a greater cost than their established budget, B. When this type of bidder requests units of a product, they conservatively assume that they will pay the (maximum) clock price C_r for the product, even if the product has never exhibited positive excess demand. The constraint takes the form

$$\sum_r \sum_{k=1}^{N_r} \delta_{r,k} \cdot C_r \leq B \tag{2.7}$$

That is, the maximum requested commitment shall not exceed the budget.

The Maximum Eligibility Constraint: In parallel with the budget constraint is the maximum eligibility constraint, which specifies that the total activity of a bidset may not exceed the bidder's eligibility. There are two distinctions worth mentioning: first, this is a hard condition, since any bidset not satisfying this property would immediately be rejected by the auction. Second, this condition is less conservative than the budget constraint, since the activity of successful bids is known a priori and need not be estimated. The constraint takes the form

$$\sum_r \sum_{k=1}^{N_r} \delta_{r,k} \cdot A_r \leq E \tag{2.8}$$

The Profitable Constraint: It is preferred that bidders never opt for unprofitable products, even if it would allow them the maintain a higher eligibility for the future. Recall that $1_P(V_{r,k})$ indicates whether a license is unprofitable. Then the constraint takes the form

$$\sum_r \sum_{k=1}^{N_r} \delta_{r,k} \cdot 1_P(V_{r,k}) \leq 0 \tag{2.9}$$

The Supply Constraints: A bidder is limited to submitting bids for a maximum of N_r licenses per product. At times it is possible for N_r to exceed S_r—particularly, after the reserve split has occurred. To prevent this, a series of constraints limit the number of bids submitted by the supply of each product. These constraints take the form

$$\sum_{k=1}^{N_r} \delta_{r,k} \leq S_r \quad \text{for each product, } r \tag{2.10}$$

The Pooled Value Constraints: Finally, the pooled value constraint ensures, as with the lexicographic strategy, that bidders will not mistakenly submit bids for the same unit of C1 and corresponding C1R products. This is accomplished with

$$\delta_{r,k} + \delta_{r,k}^{C1R} \leq 1 \quad \text{for each C1 product } r \text{ and unit, } k \tag{2.11}$$

Linear Program Summary: Substituting the constraints in their mathematical form, the revelation strategy optimizes the following:

$$\begin{aligned}
\text{Maximize} \quad & \sum_r \sum_{k=1}^{N_r} \delta_{r,k} \cdot P_{r,k} \\
\text{subject to} \quad & \sum_r \sum_{k=1}^{N_r} \delta_{r,k} \cdot C_r \leq B \\
& \sum_r \sum_{k=1}^{N_r} \delta_{r,k} \cdot A_r \leq E \\
& \sum_r \sum_{k=1}^{N_r} \delta_{r,k} \cdot 1_P(V_{r,k}) \leq 0 \\
& \sum_{k=1}^{N_r} \delta_{r,k} \leq S_r \quad \text{for each product, } r \\
& \delta_{r,k} + \delta_{r,k}^{C1R} \leq 1 \quad \text{for each C1 product } r \text{ and unit, } k
\end{aligned} \tag{2.12}$$

It should be noted that the profitable and supply constraints were not strictly needed as part of the linear program—they could also be handled by processing the auction data prior to optimization.

Advantages: Compared with the lexicographic strategy, the SF strategy makes some significant improvements. It properly balances the weights of activity and monetary costs, putting forward bidsets that will (theoretically) draw greater profits. Like the lexicographic strategy, it is also guaranteed that a solution to the optimization exists, since each of the 5 constraints are upper bound constraints. Thus, submitting no bids at all automatically satisfies the constraints, guaranteeing that the solution space will never be empty. When all bids are successful, the straightforward strategy results in properly maximizing profits for the current round of an auction.

Disadvantages: Although it is in many ways an improvement, the SF strategy suffers some of the same pitfalls of the lexicographic strategy. Activity and eligibility are at least acknowledged in the optimization, but the activity requirement is not. An important decision that all bidders must make during the auction is the extent to which they wish to compromise between opting for high profit products versus preserving their eligibility and subsequent bidding power. Revelation bidders do not bother with this at all. Optimizing profit sometimes means shedding large amounts of eligibility. In this sense, the revelation strategy is a very short-sighted optimizer.

Furthermore, no attempt is made to anticipate failed bids. One might be misled to believe that the existence of a hard budget constraint will prevent the bidder from committing to greater than their budget. However, we know from Section 1.3 that this is not the case. There is no clear reason to expect that the SF strategy should be any less prone to these issues than the lexicographic strategy, and we will see that both suffer budget-related problems under unfavorable circumstances.

Finally, both this and the lexicographic strategies are simplistically conservative in their product cost estimates. Realistically, a human bidder may observe consistently low-demand markets and make an educated guess about whether prices are likely to increment. This can improve the profitability of bidsets over those submitted by the revelation strategy, even when all bids are successfully processed.

2.3.3 The IFA Complex Optimization Strategy

The revelation strategy is correct in its attempt to optimize profit, but it is too simplistic and naive for this mechanism. The complex strategy attempts to build upon the revelation strategy by adding the following behavior:

1. The bidder will attempt to maintain eligibility when possible and reasonable

2. The bidder will make educated guesses about the price of a product, rather than assuming that the price will always increase to the clock price.

3. The bidder will not submit bids to reduce demand beyond what is likely to succeed.

These behaviors are open-ended and there are many means through which they may be interpreted and handled. As a first pass, a complex bidder exhibits behavior 1 by requiring the total activity of their bidset to satisfy the activity requirement. Behavior 2 dictates both the objective function and cost-related constraints by making use of each product's excess demand. The assumptions made here are very closely tied to those of behavior 3, where the bidder again makes use of excess demand in assessing the likelihood of success in reducing demand.

We require a bit more notation before the optimization can be tackled.

- C_r is the clock price of product r, and c_r the posted price (after applying discounts).

- ϵ_r is the **price increment variable**—a non-decision variable for product r determining the bidder's expectation for the pending price of that product, with

$$\text{Expected posted price next round} = \begin{cases} C_r, & \text{if } \epsilon_r = 0 \\ c_r, & \text{if } \epsilon_r = 1 \end{cases} \tag{2.13}$$

- γ_r is the **posted price variable**—a non-decision variable for product r specifying the number of bids expected to be processed at the posted price, of those submitted by the bidder this round.

- ED_r is the excess demand for product r from the previous round's processed bids.

- PD_r is the bidder's processed demand from the previous round for product r.

- AR is the activity requirement for the auction.

It may seem redundant for us to define both price increment and posted price variables. After all, when $\gamma_r > 0$ we can automatically infer that $\epsilon_r = 1$. And in this case, we know that γ_r must equal the total number of bids submitted for the product, since licenses within a product share the same cost. Even so, both sets of variable are required to preserve linearity for both the objective function and constraints. With this, the optimization takes the following form:

$$
\begin{aligned}
\text{Maximize} \quad & \sum_r \sum_{k=1}^{N_r} \delta_{r,k} \cdot P_{r,k} + \sum_r \gamma_r \cdot (C_r - c_r) \\
\text{subject to the} \quad & \text{Budget Constraint} \\
& \text{Maximum Eligibility Constraint} \\
& \text{Supply Constraints} \\
& \text{Pooled Value Constraints} \\
& \textbf{Profitable Constraints}^* \\
& \textbf{Minimum Eligibility Constraint} \\
& \textbf{Minimum Demand Constraints} \\
& \textbf{Product Consolidation Constraint} \\
& \textbf{Price Increment Constraints} \\
& \textbf{Dummy Constraints}
\end{aligned}
\tag{2.14}
$$

The objective once again sums the profits at the clock price for submitted bids. The second sum in the objective adds back the difference in cost between posted and clock prices for products that are expected not to increment in price. In effect, this is equivalent to summing the expected profit across all bids.

The first four constraints are identical to those from the revelation strategy. The last six bolded constraints add our desired functionality to the linear program. The (asterisked) profitable constraints are a special case: the straightforward optimization also contained a profitable constraint, but with the introduction of dummy variables the constraint is now handled differently. As before, we address these constraints one at a time.

Profitable Constraints: In the SF optimization, the single profitable constraint dictated that a bidder never opt for a product whose value did not exceed the current clock price.

We now formulate this constraint with respect to the bidder's expectation of the product's likelihood to increment during the following round. The constraint is broken into multiple subconstraints of the form

$$\Big\{\delta_{r,k}(V_{r,k} - C_r) + \epsilon_r(C_r - c_r) \geq 0\Big\} \text{ for each unit } k \text{ of product } r \tag{2.15}$$

The leftmost term in parentheses represents profit for unit k of product r, assuming the clock price. In the event that a price increment for product r is anticipated next round ($\epsilon_r = 0$), the constraint forces the decision variable to be zero if profit at the clock price is negative. When we do not expect an increment ($\epsilon_r = 1$), the constraint is loosened by adding the difference in the clock price and posted price to the profit term, effectively assuming posted price profit.

Minimum Eligibility Constraint: Akin to the maximum eligibility constraint barring requested activity above eligibility, the minimum eligibility constraint requires requested activity to reach the activity requirement. However, this constraint can be a bit narrow, if eligibility falls too low. If, for example, a bidder with a 95% activity requirement drops to 200 eligibility, then maintaining their current eligibility requires them to submit a bidset totaling at least 190 units of activity—a 10 unit target. To mitigate this, the minimum eligibility constraint is loosened to allow at least a 50 unit target at all times. The constraint has the form

$$\sum_r \sum_{k=1}^{N_r} \delta_{r,k} \cdot A_r \geq \min(E \cdot AR, E - 50) \tag{2.16}$$

Minimum Demand Constraints: It is a common occurrence for the lexicographic and revelation bidders to attempt to reduce demand in markets with no excess demand. When this fails, they will frequently attempt to repeat this request for dozens of consecutive rounds, maximizing the likelihood of an unanticipated outcome. This is not a particularly realistic response—we might expect a human bidder to give up after one or two failed attempts, or to prioritize shifting demand in markets with positive excess demand while the opportunity lasts.

This is the main idea behind the minimum demand constraint. It prohibits the bidder from reducing their demand beyond what is likely to succeed. We define D_r^S, the minimum demand required to maintain aggregate demand at or above supply for product r (assuming no shift in demand from other bidders):

$$D_r^S = \max(0, PD_r - ED_r) \tag{2.17}$$

If, for example, a bidder has a processed demand of 3 in a market with an excess demand of 2, then $D_r^S = 1$. The bidder might reasonably expect a request to reduce their demand below D_r^S to fail. Observe that when there is no excess demand, D_r^S is always equal to processed demand. Further, when processed demand is zero, $D_r^S = 0$ as well.

The constraint should be a bit more subtle than simply requiring demand to match D_r^S. Consider the following example: Bidder j is holding one processed license in a market with zero excess demand. The license is profitable to the bidder at the posted price, but not at the clock price. Because there is no excess demand, the bidder does not attempt to reduce their demand. However, some new bidders enter the market and the price increments, resulting in Bidder j holding an unprofitable license.

We would prefer to loosen the constraint in the event that licenses are unprofitable (at either the posted or clock prices). Define D_r^{PP} to be the number of licenses that would be profitable at the current posted price, and D_r^{CP} to be the number of licenses that would be profitable at the current clock price. Then the constraint takes the form

$$\left\{ \sum_k \delta_{r,k} \;\geq\; \epsilon_r \cdot \min(D_r^S, D_r^{PP}) + (1 - \epsilon_r) \cdot \min(D_r^S, D_r^{CP}) \right\} \text{ for each product } r \tag{2.18}$$

In the case that $D_r^{CP} > D_r^S$, the value of ϵ_r does not matter and the constraint breaks down into the requirement that round demand matches or exceeds D_r^S. Otherwise, the value of ϵ_r determines whether we expect a posted or clock price, and reduces the minimum demand to avoid potentially unprofitable holdings.

Product Consolidation Constraint: One potential source of problems when bidding in the IFA arises when attempting to consolidate products—that is, when attempting to drop several sources of activity while adding a single unit of high-activity product. This sort of action is problematic because it requires the success of multiple independent requests. It is easy to imagine a situation in which some, but not all, of the requests to reduce demand are processed, freeing insufficient activity for the new bid to add demand and resulting in an unnecessary loss of eligibility. Even if there were sufficient activity to pick up the new product, failing to drop some may result in going over budget. This behavior is limited with the following constraint:

$$\sum_{r} \sum_{k=1}^{N_r} \delta_{r,k} \geq \sum_{r} PD_r - d_{max} \tag{2.19}$$

In other words, the total number of bids requested for the current round may not fall below a buffer of the number of currently processed bids. The buffer, d_{max}, is a bidding strategy parameter that we address further in Section 2.4.

Price Increment Constraints: This and the following constraint are not concerned with the stated behavioral goals of the bidder. Rather, they exist to ensure that ϵ_r and γ_r operate logically. The price increment constraints dictate the largest number of bids that can be submitted during the current round before the price is expected to increment, and is given by

$$\left\{\gamma_r \leq D_r^S\right\} \text{ for each product } r \tag{2.20}$$

This follows directly from the definitions for γ_r and D_r^S.

Dummy Constraints: The last set of constraints consists of the following three families:

$$\left\{\begin{array}{ll} \text{(i)} & \gamma_r \leq 100\epsilon_r \\ \text{(ii)} & \gamma_r \leq \sum_{k=1}^{N_r} \delta_{r,k} \\ \text{(iii)} & \gamma_r \geq \sum_{k=1}^{N_r} \delta_{r,k} - 100(1-\epsilon_r) \end{array}\right\} \text{ for each product } r \tag{2.21}$$

The value 100 where it appears is effectively infinite, since the number of bids for any product will always fall far short of this. Rather than interpret each constraint individually, it is easier to look at how they break down for each value of ϵ_r. When $\epsilon_r = 0$, we have the following:

$$\left\{\begin{array}{lll} \text{(i)} & \gamma_r = 0 & \\ \text{(ii)} & \sum_{k=1}^{N_r} \delta_{r,k} \geq 0 & \text{(nonconstraining)} \\ \text{(iii)} & \sum_{k=1}^{N_r} \delta_{r,k} \leq 100 & \text{(nonconstraining)} \end{array}\right\} \text{ for each product } r \tag{2.22}$$

In other words, when $\epsilon_r = 0$ (signifying that a price increment is anticipated), the number of bids expected to be awarded at the posted price is held at zero, and no additional restrictions are placed on the number of bids that may be submitted per product. We might have expected that the number of bids should be held above the count required to trigger a price increment. However, the utility of the bidder is always improved when the price increment variable is equal to one, so this is handled automatically. When $\epsilon_r = 1$, we have the following:

$$\left.\begin{cases} \text{(i)} & \gamma_r \leq 100 \qquad \text{(nonconstraining)} \\ \text{(ii)} & \displaystyle\sum_{k=1}^{N_r} \delta_{r,k} \geq \gamma_r \\ \text{(iii)} & \displaystyle\sum_{k=1}^{N_r} \delta_{r,k} \leq \gamma_r \end{cases}\right\} \text{ for each product } r \tag{2.23}$$

The only effective constraint here is that the number of bids submitted must equal γ_r. This makes sense: if no price increment is expected, then the number of bids submitted must equal the number of bids we expect to be processed at the posted price.

The First Backup Optimization: Unlike the SF strategy, The complex optimization is not guaranteed to have any feasible solutions. As a simple example, imagine that only two items are available, each costing 70 units of activity to bid. We have an eligibility of 130 and the minimum eligibility constraint demands that a bidset must have a combined activity total of at least 80 units. There is no way to accomplish this. Other constraints can further exacerbate the situation. In such situations, the complex bidders solve a new, looser optimization:

$$\begin{aligned}
\text{Maximize} \quad & \sum_r \sum_{k=1}^{N_r} \delta_{r,k} \cdot P_{r,k} + \sum_r \gamma_r \cdot (C_r - c_r) \\
\text{subject to the} \quad & \text{Budget Constraint} \\
& \text{Maximum Eligibility Constraint} \\
& \text{Supply Constraints} \\
& \text{Pooled Value Constraints} \\
& \text{Minimum Demand Constraints} \\
& \text{Price Increment Constraints} \\
& \text{Dummy Constraints}
\end{aligned} \tag{2.24}$$

The objective function and constraints for this linear program are identical to those of the primary complex optimization, with the exception that the minimum eligibility, product consolidation, and profitable constraints have been dropped. This backup linear program is very similar to the one used by the SF strategy. Four of the first five constraints are identical to those used by the SF strategy (with the profitable constraint replaced by the related minimum demand constraint). The last two constraints act only to shift assumed cost between the posted and clock prices.

The Second Backup Optimization: Even this backup linear program is not guaranteed to have a feasible solution. If the bidder's processed commitment is within their budget, a viable bidset will always exist by bidding the least amount possible as specified by the minimum demand constraints. However, if a bidder enters a round over budget, and their holdings have particularly low excess demand, then there will be no way to satisfy the budget and minimum demand constraints simultaneously. In such cases, the complex strategy defaults to the second (final) backup optimization. This is not a linear program at all. Instead, it makes use of the following algorithm:

1. Begin with a bidset containing the products and quantities currently processed from the previous round. Assume a total cost using posted prices for each product.

2. Partition each unit of product from the bidset into two groups, based on whether the associated product has positive excess demand from the previous round.

3. Sort the units within each group by ascending profit, assuming the clock price.

4. While the total cost of the bidset exceeds the budget:

   (i) Iterate through the sorted list of units having positive excess demand. For each unit, deduct the bidset's quantity for the associated product by one, and reduce the total cost by the posted price of the product.

   (ii) Repeat for the list of units having no excess demand.

5. Submit the resulting bidset.

Because this optimization will only be triggered in the event that the bidder is already over budget, it grossly limits the actions that may be taken. The bidder may not target any new products; they may only attempt to drop products that they are currently holding. Furthermore, the strategy acts to target the products that are most likely to be dropped successfully (those with positive excess demand), prioritized by potential profit. This optimization is very similar to the lexicographic strategy, but restricted to products that the bidder has already processed and ordered loosely by excess demand. Needless to say, this strategy will always submit a bidset.

Advantages: This strategy makes some significant improvements on either the lexicographic or SF strategies. First, it is the only strategy that makes weighted decisions about pricing. By assuming the clock price for all products, the first two strategies exhibit unnecessarily conservative behavior that can be harmful to eligibility preservation and accurate comparisons of products. The strategy more directly attempts to manage eligibility by requiring, when possible, that total activity comes close to the activity requirement. In addition, the strategy blocks attempts to reduce demand where it is unlikely to succeed. Because of the tiered design of the strategy, it is equipped to handle different circumstances in a targeted fashion (particularly, what to do when the bidder finds themselves over budget).

Disadvantages: The naming of the complex strategy suggests a significant disadvantage: it can be challenging to determine the reasoning behind the resulting bidset. From the perspective of running simulations, the complex strategy is also computationally expensive to perform. While the primary optimization contains many features for clever bidding, this linear program is rarely used in cases where the bidder is forced to shed eligibility. The minimum eligibility constraint is particularly strict, requiring the bidder at times to bid for activity points on low cost products that are otherwise of very little value. The minimum demand constraint, for all of its benefits, prohibits bidders from attempting to reduce demand in problematic markets unless there is excess demand. This means that others bidders actively attempting to leave the market will receive priority in the event that new bidders enter the market or increase demand. Fortunately, this is offset by the limitation that negative profit products cannot be requested at all. Finally (and perhaps most significantly), the complex strategy encourages buyers to bid strategically instead of revealing their preferences.

2.3.4 SMR Bidding Strategies

For the SMR mechanism, we employ slight variations on the IFA lexicographic and straightforward strategies. The algorithms used by both versions of the strategies are identical, but optimize over a subset of products in the SMR case. This works as follows:

1. For each unit of product processed from the previous round, reduce the bidder's current eligibility by the associated activity cost, and their budget by the associated posted price.

2. Remove each processed unit from the pool of units under consideration, starting with the most valuable products. If, for example, bidder i has (nonincreasing) values

$$V_{i,r,1} \geq V_{i,r,2} \geq V_{i,r,3} \geq \cdots \geq V_{i,r,S_r}$$

   associated with product r and is holding two units of processed demand, then they will only optimize over values $V_{i,r,3}, \ldots, V_{i,r,S_r}$ (assuming there is any remaining supply).

3. Perform either the lexicographic or SF strategy over the reduced selection of products, using the reduced values for budget and eligibility.

4. For each bid created by the strategy, if there is any processed quantity from the previous round, add this to the bid.

5. Submit the bidset.

Effectively, this modification works by removing all processed product units from consideration when optimizing. In fact, neither strategy attempts to submit maintenance bids unless they wish to submit additive bids for a product with processed demand—at which point maintenance bids become necessary. The complex strategy is not implemented for the SMR mechanism, since most of the issues within the IFA mechanism that this strategy attempts to handle have been eliminated.

Advantages: The relative advantages between the two strategies are largely the same as those of the IFA mechanism. With respect to mechanism, the IFA strategies have many benefits over their SMR counterparts. There is no longer a need for price speculation, since all additive bids can only be processed at the clock price. This means that bidsets will never result in unexpected bankruptcies. The strategies do not need to concern themselves with the outcome of unsuccessful bids, since these will never result in a loss of eligibility.

Disadvantages: The relative disadvantages between the two strategies are largely the same as those of the IFA mechanism. Although the SF strategy accounts for activity costs in its optimization, neither strategy attempts to manage eligibility by meeting the activity requirement. Furthermore, both strategies suffer a disadvantage specific to the mechanism: a failure to submit maintenance bids to properly express holding preferences. In reality, a bidder may wish to prioritize maintaining demand for high-value products, including in situations where they are not attempting to increase demand. This is a minor disadvantage, and including this feature would require a significant overhaul of both strategies.

2.4 Simulation Parameters

There are quite a few parameters to consider within the incentive auction. Some of these parameters are held fixed across al simulations, while others are varied to observe their effects. For the purpose of organization, we categorize these parameters as auction, bidder and product parameters. In the following three sections, we briefly discuss each of these parameters. In section 2.5, we briefly summarize the significant, non-static parameters that will be the focus of our analysis.

2.4.1 Product Parameters

Location: The location includes both the geographical region and locality for a product. This information is used in determining which products certain bidders may value, as discussed in the next section.

Market Scale: The market scale is a high-level label used for organizing products into groups. The scale is a treatment effect that varies across simulations. The three scales are:

- The small scale, consisting of only a single region and a handful of localities.

- The biregional scale, consisting of two full-sized regions
- The national scale, consisting a six full-sized regions. The national scale was designed to emulate the market for the 2015 wireless spectrum forward auction.

All product-related parameters that follow hold constant values within each scale.[1]

Population: The population residing within the associated PEA. This information is used to determining closing conditions.

High Demand Flag: A label used for the purposes of determining timing for the reserve split and closing rules.

Basic Product Information: This includes a product's supply, bidding units, and opening price.

Expected Value to Bidders: The value that each bidder holds for a product is drawn from a distribution with an expected value inherent to the product itself. The process by which bidders determine value for products is outlined in the next section.

Expected Impairment: In reality, specific licenses for products each have distinct impairment values which will manifest during the post-forward auction allocation phase. To simplify matters, each product within a simulation is assigned an expected impairment value, which bidders and auction assume for all impairment-related calculations.

Impairment Value Discount Distribution: Bidders generally discount the value of products based on their impairment. However, this impairment discount rate differs from bidder to bidder. The discount rate that a bidder applies is randomly drawn from a uniform distribution with mean and standard deviation fixed for each product.

2.4.2 Bidder Parameters

Strategy: The algorithm employed by the bidder in making their bidding decisions. This algorithm is either the lexicographic, straightforward, or (in the case of the IFA mechanism) complex strategy. These strategies are outlined in Section 2.3.

[1] Complete scale information can be found at:
`https://github.com/logantner/FCC-Incentive-Dissertation/`

Type: Each bidder's interaction with the market is largely dictated by their type, which loosely specifies their resources and breadth of value across all products. These types are as follows:

- National bidders: These have the largest budgets of all the bidder types, and have value for products across all regions and localities.
- Regional bidders: These have smaller budgets than national bidders, and have value for products across all localities within a specific region.
- Local bidders: These have the smallest budgets, and their value for products is limited to a small number of localities within a specific region.
- Opportunistic bidders: These are very similar to national bidders, having value for products spanning all regions. However, opportunistic bidders have only a fraction of the budget of their national counterparts. They are analogous to smaller firms looking to participate in the auction for investment or speculative purposes.

The bidder type is a higher level attribute, influencing values of lower level attributes such as budget and coverage distributions. In this sense, it is largely an organizational tool.

Coverage Distribution: Bidder type broadly determines regions and localities bidders can have value. The value coverage distribution determines how much products from these allowed sectors will be valued. For example, if a regional bidder has a value coverage of 60%, then they will be assigned (roughly) 60% of the products from their assigned region to have positive value. The coverage itself is drawn from a uniform distribution whose limits depend on both the bidder type and market scale. These values can be found in Table 2.1.

In the case of national, opportunistic and regional bidders, the value coverage is drawn and used to determine the quantity of valued products. These products are then drawn from the relevant item pool (all items for national and opportunistic, the assigned region for regional) using a weighted distribution with respect to the expected value of each item. Local bidders are a special case. Instead of sampling a region, they sample one or more localities from an assigned region as their item pool. Their coverage is always uniformly drawn between 0% and 100%, with a minimum item count of 1, after which they sample from their item pool like normal.

Maximum Localities (local types only): As mentioned when discussing coverage, local bidder types sample one or more localities in determining value draws. The number of localities is drawn uniformly between one and the maximum number of localities, which is itself a function of the market scale. Because local bidders always randomize coverage uniformly between 0% and 100% of their item pool, Table 2.1 presents the locality range for local bidders instead of their coverage range.

Maximum Valued Licenses: In addition to being assigned products of value, bidders will additionally be assigned a number of valued licenses within these products. This value is drawn uniformly between a single unit and a maximum number of licenses. This maximum number depends on the bidder type and the market scale, but is also subject to a binary treatment effect. Each row of table 2.1 displays the two values that a bidder type within a scale may assume, depending on the treatment. For example, opportunistic bidders in a national market will have value for up to two licenses of each product in the lower demand treatment, and up to four licenses of value in the higher demand treatment. The maximum number of licenses is always capped at the supply of a product, where supply would be a limiting factor.

Value Distribution: Once licenses of value are determined, bidders are assigned random value draws for each of these licenses. A value, V, is drawn from a uniform distribution as

$$V \sim U(\mu - \sqrt{3}\sigma, \mu + \sqrt{3}\sigma)$$

The expected value, μ, is fixed to the product.[2] The distribution's width, on the other hand, depends on σ and is a binary treatment effect across simulations. Its value will either be one third of the product's mean value, or 2% of the mean value. The intention is to create environments which differ significantly in competitive engagement.

When a bidder has value for more than one license within a product, these values are drawn independently from the same distribution. Afterward, they are sorted from highest to lowest value to guarantee monotonicity of values. These are base values, and have not yet factored details such as expected impairment.

[2] Complete product information can be found at:
https://github.com/logantner/FCC-Incentive-Dissertation/

Table 2.1: Bidder information by type and market scale.
Each cell under max valued licenses contains two values representing the two possible valued license counts based on the treatment implemented. Local bidders always drew coverage between 0% and 100%, so coverage indicates the maximum number of localities that local bidders may select from.

Scale	**Bidder Type**	**Min Budget (millions)**	**Max Budget (millions)**	**Coverage**	**Max Valued Licenses**
Small	National	10	20	80 - 100 %	2 / 2
	Opportunistic	1	2	80 - 100 %	1 / 2
	Regional	3	12	80 - 100 %	1 / 2
	Local	0.5	2	1 locality	1 / 2
Biregional	National	108	3046	20 - 90 %	3 / 3
	Opportunistic	10.8	304.6	20 - 90 %	1 / 2
	Regional	11	174	25 - 75 %	1 / 2
	Local	22	109	4 localities	1 / 2
National	National	326	9136	20 - 90 %	5 / 5
	Opportunistic	32.6	913.6	20 - 90 %	2 / 4
	Regional	11	174	25 - 75 %	2 / 4
	Local	22	109	4 localities	2 / 4

Impairment Value Discount: The value that a bidder is assigned for each value assumes that the bidder applies no discount to the product based on its impairment. In reality potential buyers may respond to impairment very differently based on their needs, and so are assigned unique impairment discount rates that are applied to their product valuations:

$$\text{Bidder Value} = (\text{Base Value}) \cdot (\text{Expected impairment}) \cdot (\text{Impairment discount})$$

The impairment discount rate is randomly drawn from a uniform distribution specific to the product, as discussed in the previous section.

Budget and Budget Distribution: Each bidder is assigned a budget when generated. This budget is drawn from a uniform distribution, whose limits are a function of bidder type and market scale. These values can be found in Table 2.1.

Starting Eligibility: Finally, each bidder is assigned a starting eligibility as a function of their budget. Bidders receive a unit of starting eligibility for each $2500 of budget, rounded down. This applies to all bidders in all market scales.

Reserve Eligibility Status: This includes an indicator of whether the bidder is reserve eligible and may therefore bid on reserve products after the reserve split. National bidders are never reserve eligible, while opportunistic, regional and local bidders are always reserve eligible. Reserve eligible bidders treat the value of reserve products identically to their non-reserve counterparts.

2.4.3 Auction Parameters

Mechanism: Either the IFA or SMR auction mechanism. This is the most significant of all treatment effects.

Clock Increment: The amount that the price increments in the event of sufficient demand, as a percentage of the posted price. This is a binary treatment effect, with increments of either 5% or 10%.

Scale: The simulation scale is a higher-level version of the product scale which incorporates bidder information as well as product information. Like the market scale, the simulation scale is split into small, biregional, and national categories, and always uses product sets of the same name. We will simply refer to simulation scale as "scale" for future reference.

Bidder Count: The total number of bidders participating in an auction. This value depends on both the scale of the simulation, as well as a binary treatment effect. Table 2.2 contains specific bidder population counts. Each cell contains two values which represent the lower and higher population treatment effects, respectively.

Bidder Type Partition: The number of bidders assigned to each bidder type (national, opportunistic, regional, and local). Like total bidder count, this depends on both the scale

Table 2.2: Bidder type distributions within each simulation scale.
Each cell contains two values representing the two possible bidder counts based on the population treatment implemented.

Scale	Total Bidders	National Bidders	Opportunistic Bidders	Regional Bidders	Local Bidders
Small	16 / 32	2 / 4	2 / 4	4 / 8	8 / 16
Biregional	30 / 45	6 / 9	4 / 6	10 / 15	10 / 15
National	50 / 70	10 / 13	6 / 9	17 / 24	17 / 24

and bidder population treatment effect. Table 2.2 details this information. Each cell contains two values which represent the lower and higher population treatment effects, respectively.

Bidding Strategy Partition: Similar to the bidder type partition, this is the number of bidders assigned to each strategy outlined in Section 2.3. For each simulation, an auction is performed using a pure strategy set (i.e. all bidders use either the lexicographic, SF, or complex strategies), as well as a mixed strategy set, in which bidders are split equally across available strategies for the given mechanism.

Activity Requirement: The minimum percentage of eligibility usage required to maintain the current amount for future rounds. The activity requirement is fixed for all simulations at 95%.

Parameters for Closing Rules: Recall in Section 1.2.4 the rules used by the FCC for assessing the conclusion of the forward auction. The parameters concerning this procedure differ only by simulation scale, as follows:

- A price per MHz-pop benchmark $X =$ \$0.05/MHz-pop for small scale simulations, and $X =$ \$0.5/MHz-pop for biregional and large scales.
- A clearing target of 20 for the small scale, and 50 for the biregional and large scales.
- A licensed spectrum benchmark $T = 70$ for all scales.
- A minimum revenue requirement of one million dollars for the small scale, and one hundred million dollars for the biregional and large scales. These values are affiliated with Objective 2 of Section 1.2.4.

2.5 Simulation Results

For the purpose of this work, we are limiting the results and analysis of simulations to four major measurements: The amount of revenue earned by an auction, the total surplus of the auction, the number of rounds that the auction took to complete, and the percentage of bidders committing in excess of their established budget at the conclusion of an auction (i.e. the bankruptcy rate).

The revenue, surplus, and number of rounds are auction-level metrics and demonstrate a broad sense of success (or lack thereof) of a mechanism within an environment. The surplus here is defined as the sum of bidder-held values across all products based on their final allocations. For both the revenue and surplus, we must pay special consideration to the case where a bidder is bankrupt at the end of the auction. Do we assume that the items awarded to these individuals should not apply to surplus, and that the costs should not be included in the revenue? We offer several metrics for measuring these quantities, discussed further in Sections 2.5.5 and 2.5.4. Making changes to auction mechanisms for the purpose of optimizing revenue is generally avoided, as improving surplus is the primary goal and is considered strongly tied to revenue [5]. Even so, we include the impact on revenue as part of a complete analysis.

Because timeliness was a factor in the motivation for designing the Incentive Forward Auction [19], the number of rounds is particularly important when comparing auction mechanisms. Finally, because accidental bankruptcy has been established as an issue with the Incentive Auction, we would like to assess how this issue is affected by various factors, and whether the SMR is successful in eliminating it.

For the sake of brevity, the following shorthand is used throughout the analysis to represent the various treatment effects discussed in Section 2.4:

- **ValVar:** At the start of an auction simulation, each bidder draws their base value for a given product from a common normal distribution. When ValVar = 0, the standard deviation of this distribution is 2% of the mean value; otherwise, it is 30% of the mean.
- **nB:** The number of bidders participating. Recall that a value of 0 indicates fewer bidders, with the exact number varying by scale as follows:
  - The small scale varies between 16 and 24 bidders.
  - The bi-regional scale varies between 30 and 45 bidders.
  - The national scale varies between 50 and 70 bidders.
- **Dem:** The maximum quantity that regional, local and opportunistic bidders may demand for each product. A value of 0 indicates less maximum demand, with the exact number varying by scale as follows:
  - In the small scale, national bidders always have a maximum demand of 2 licenses, with regional, local and opportunistic bidders varying between a cap of 1 and 2 licenses.

- In the bi-regional scale, national bidders always have a maximum demand of 3 licenses, with regional, local and opportunistic bidders varying between a cap of 1 and 2 licenses.
- In the national scale, national bidders always have a maximum demand of 5 licenses, with regional, local and opportunistic bidders varying between a cap of 2 and 4 licenses.

- **CInc:** The clock increment for each product. When CInc = 0, prices increment by 5% of their previous value; otherwise, they increment at 10%.
- **SMR:** The mechanism employed. When SMR = 0, the Incentive Forward Auction mechanism is used; otherwise, the SMR is used.
- **Lex:** A dummy variable determining whether the lexicographic strategy was purely implemented.
- **SF:** A dummy variable determining whether the straightforward strategy was purely implemented.
- **Comp:** A dummy variable determining whether the complex strategy was purely implemented.

Note that the Lex, SF and Comp variables indicate auctions in which their respective strategies were purely implemented. When all three variables take a value of 0 it is implied that an even mixture of strategies was applied. Recall from Section 2.4 that simulations were performed within three different environments: small, bi-regional and national scales. Within the small and bi-regional environments, a total of 10,000 simulations were collected for each possible permutation of control variables. Within the national environment, 1000 such simulations were collected.

The overlapping nature of the control variables necessitates a study of the interactions between them when addressing any of the performance metrics. Since Lex, SF and Comp are all dummies representing a single variable (strategy), a linear regression may include a maximum of six-way interaction terms. This can be undesirable for a number of reasons, particularly when attempting to interpret the meaning of regression coefficients. When assessing the efficacy of higher-order interaction terms, we must balance two points:

1. Is the corresponding interaction coefficient significant in the regression?

2. Does including the interaction term (significantly) improve the model's prediction quality?

Assuming all variables are included with maximum-order interaction effects, there are 112 coefficients to consider per model. In most cases, more than half of these coefficients were found to be significant in the regression, with an emphasis on the lower-order effects. Rather than focus on this, we will assess point (2), taking into consideration the AIC, BIC and adjusted R^2 values as batches of higher-order terms are introduced into the model. The results parallel one another across all three measurements, with the AIC values summarized in Table A.1.

For all scales, the AIC and BIC decreased substantially when transitioning from a direct effects model to a two-way interaction model. In most cases, the AIC and BIC continue to decrease for three-way and higher complexity models. However, the difference between these and the two-way models differ very narrowly—in all cases AIC is reduced by less than a single percentage point, and typically only a fraction of this. Similarly, the adjusted R^2 is relatively improved by increasing from direct effects to include two-way interactions. Increasing to third-way or higher interactions does not notably increase R^2 further. Considering the increased difficulty to interpret more complex interaction effects, we choose to limit our analysis to the results of direct and two-way interaction models.

We find the following, discussed in the following sections:

- Auctions using the SMR mechanism take substantially longer than those using the IFA mechanism, especially for larger scales.
- Doubling the clock increment is expected to reduce the number of rounds by roughly 50%, at little cost to either revenue or surplus.
- SMR bidders never go bankrupt, and complex bidders greatly reduce the likelihood to do so.
- The SMR mechanism and complex IFA bidders substantially increase revenue and surplus compared with other IFA strategy mixtures.
- Both revenue and surplus are independent of the choice of strategy when using the SMR mechanism.

2.5.1 The Data and Models

We present the data in three formats, found in the Appendix. The first set of tables in Section A.2 consists of averaged dependent variables for each implemented treatment, as well as pooled averages. Section A.3 contains raw and scaled coefficients for fixed effects models, while Section A.4 contains coefficients for two-way interaction models. Section A.4 contains the range of marginal effects predicted by these interaction models.

It is prudent to discuss the meaning of the values within the direct and two-way interaction tables. As an example, the small scale direct effects model of Table A.27 contains raw coefficients extracted from the regression

$$\text{NumRounds} = 1.23 - 0.55 \cdot \text{ValVar} - 0.38 \cdot \text{nB} + 0.03 \cdot \text{Dem} + \cdots - 0.73 \cdot \text{Comp}$$

In both the direct effects and interaction models, the constant value represents the expected value for the base case. The base case represents a simulation with a choice of zero for all relevant parameters. In the event that all variables are included in a model, this is a simulation with a narrow bidder value distribution, fewer bidders, a lower amount of demand per bidder, a lower (5%) clock increment, and a mixed set of bidder strategies. All non-constant coefficients represent deviations from this base state.

In the direct effect tables, the raw values represent absolute deviations, while the scaled coefficients represent deviations as a percent of the constant coefficient. The (small scale) two-way interaction model from Table A.35 is an extension of the scaled direct effect models, with the primary row/column containing direct effects and the remaining elements representing two-way effects. These values are extracted from the regression

$$\begin{aligned}\text{NumRounds} = 35.6 + 0.8 \cdot \text{ValVar} - 1.1 \cdot \text{nB} + \cdots - 1.1 \cdot \text{Comp} \\ + 2.3 \cdot \text{ValVar} \cdot \text{nB} + 0.4 \cdot \text{ValVar} \cdot \text{Dem} + \ldots 0.1 \cdot \text{SMR} \cdot \text{SF}\end{aligned}$$

For example, the term $(2.3 \cdot \text{ValVar} \cdot \text{nB})$ is represented in the cell corresponding to the ValVar row and nB column, as 6.6% of the constant coefficient (i.e. $2.3/35.6 = 0.066$). We can interpret this coefficient as a component of the effect of switching from a narrow to a wide value variance, given that we will be including a greater number of bidders in the simulation ($\text{nB} = 1$). Equivalently, it can be interpreted as a component of the effect of increasing the number of bidders in the simulation, given that bidders are drawing from a wide distribution ($\text{ValVar} = 1$). We will refer to these as conditional effects.

In addition to this, we can determine the joint effect from applying a wide value distribution from a large number of bidders by including both direct effects. In this case, we would find the joint effect from using ValVar = 1 and nB = 1 to be

$$\text{Joint effect} = 2.3\% - 3.2\% + 6.6\% = 5.7\%$$

The two-way interaction models are most valuable in their ability to describe the effects of dependent variables as a function of the value of other dependent variables. Because of the binary nature of the independent variables, we can easily extract the entire range of effects. For example, the (scaled) value variance effect is given by

$$\begin{aligned}\text{ValVar Effect} = 2.3 + 6.6 \cdot \text{nB} - 1.2 \cdot \text{Dem} - 4.0 \cdot \text{CInc} + 7.0 \cdot \text{SMR}\\ - 1.4 \cdot \text{Lex} + 0.8 \cdot \text{SF} - 0.4 \cdot \text{Comp}\end{aligned}$$

In the case of the small scale number of rounds per auction, the value variance will have a maximum (most positive) effect when nB = Dem = SMR = SF = 1, and a minimum (most negative) effect when CInc = Lex = 1. These effects are given by

$$\begin{aligned}\text{Maxmimum ValVar Effect} &= 2.3 + 6.6 + 1.2 + 7.0\\ &= 17.1\%\end{aligned}$$

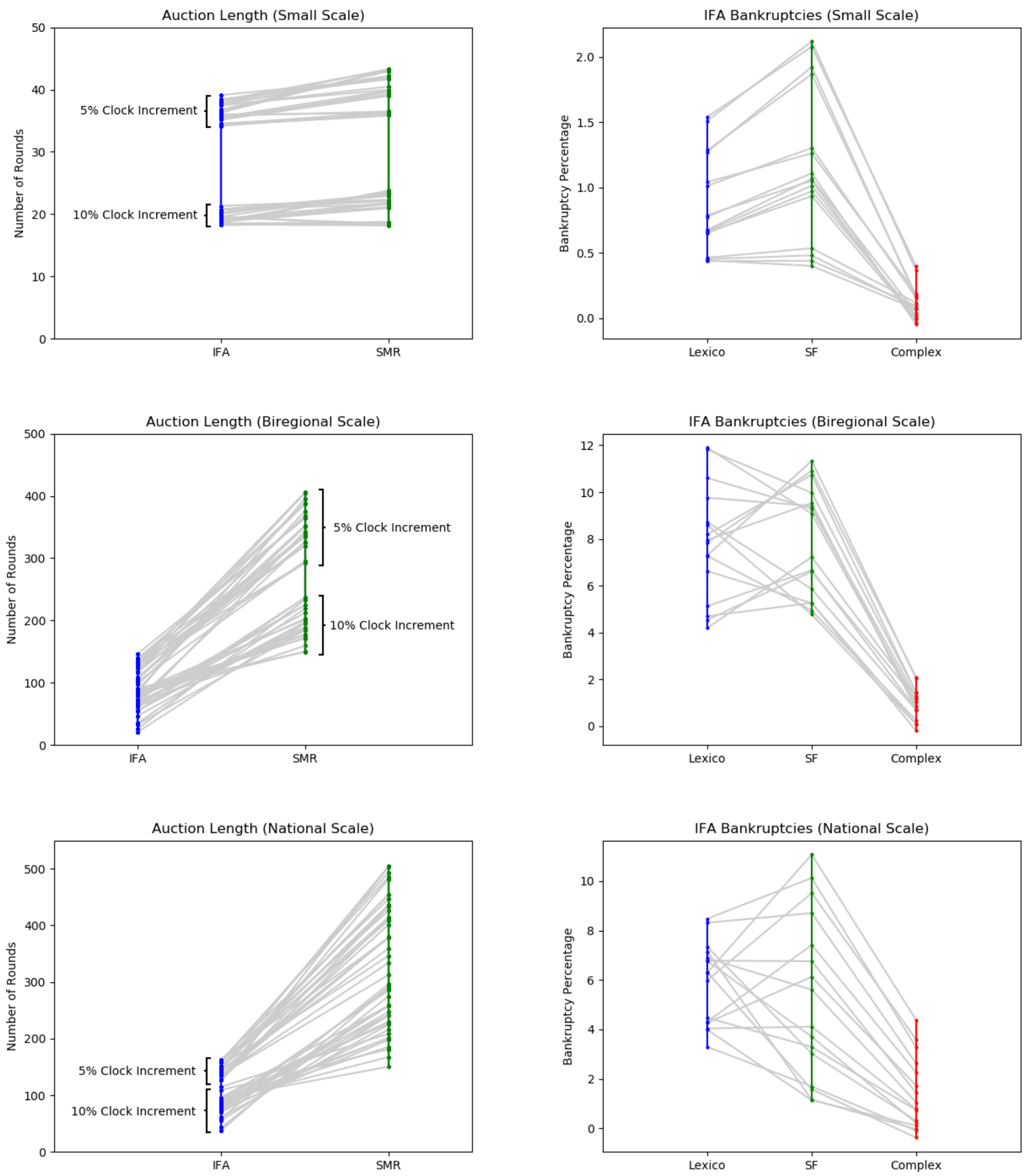
$$\begin{aligned}\text{Minimum ValVar Effect} &= 2.3 - 4.0 - 1.4\\ &= -3.1\%\end{aligned}$$

It is important to remember that we cannot choose Lex = 1 and Comp = 1 simultaneously, since these are mutually exclusive dummy variables representing the strategy of the bidders. From the minimum and maximum effects, we can see that broadening the value distribution can have the effect of increasing or decreasing the expected number of rounds compared with the base value, contingent on the state of the simulation.

2.5.2 Number of Rounds Per Auction

Simulation results strongly corroborate the hypothesis that SMR auctions take a greater number of rounds to complete compared with the IFA mechanism. This is supported in the averages found in Tables A.5, A.6 and A.7, the direct effects models from Table A.27, and

Figure 2.1: Simulation results of auction length and bankruptcies
Strategy comparisons of auction length (as measured by the total number of rounds) and bankruptcy rates. Because SMR bidders never went over budget, bankruptcies are limited to IFA treatments. Each point represents the expected value of a treatment from the corresponding two-way interaction model, with equivalent treatments connected.



the two-way interaction models from Table A.35. Figure 2.1 shows the expected number of rounds per treatment as estimated by the two-way models. Within this figure, light gray lines connect equivalent treatments applied to IFA and SMR auctions. These plots make two points immediately clear—that the SMR mechanism is expected to increase the number of rounds regardless of treatment, and that the disparity between treatments is much more dramatic at larger scales.

When averaging across treatments, small scale IFA auctions took about 28 rounds, compared with 31 rounds for SMR auctions—an 11% increase. Many treatments have little to no intra-mechanism change. By comparison, switching from the IFA to SMR mechanism increased the average length of the auction from 86 to 271 rounds on the biregional scale, and from 102 to 325 rounds on the national scale. In both cases, this exceeded a 200% expected increase in length. Even in the least extreme case on the national scale (a large number of SF bidders with a wide value distribution, higher demand and a 10% clock increment), the model still anticipates that using the SMR will add about 60 rounds to the auction—an increase of roughly 40%.

Figure 2.1 does not present auctions exclusively using the complex optimization strategy, since a version of this was not implemented for the SMR mechanism. However, on the national scale this strategy assortment resulted in auctions completing the fastest. Below we find the range of marginal effects (on number of rounds) in switching from each IFA strategy prescription to the pure complex strategy set:

	Marginal Effect	
Strategy	Minimum	Maximum
Lexico	−76.3	12.5
SF	−74.1	−25.2
Mixed	−64.5	−27.0

Only the lexicographic strategy set is expected to perform slightly faster under any circumstances (specifically, with 10% clock increments), and all strategy sets are expected to improve by $60 - 80$ rounds in the extreme. We might expect this round reduction to have negative impacts on the auction surplus or revenue when bidders make use of the complex strategy. However, we see in sections 2.5.4 and 2.5.5 that the reverse of this is true, thanks in large part to bankruptcy losses.

Finally, as we would expect, Figure 2.1 shows that treatments are distinctly clustered based on the clock increment, with auctions that employ larger (10%) clock increments taking fewer rounds to complete. We discuss the implications of this further in Section 2.5.6.

Summary of results:

- Switching from IFA to SMR mechanisms on the small scale increased auction length by an average of 11%
- On the biregional and national scales, switching to the SMR mechanism more than tripled the expected number of rounds.

2.5.3 Bankruptcy Rates

Simulations confirm that the SMR mechanism satisfies one of its primary intentions—neither SMR bidding strategy has a single instance of going over budget. The bid processing rules are such that a bidder cannot go over budget without deliberately requesting a higher commitment than their budget. For this reason, this section examines only IFA data and its intra-mechanism effects. Averages of these values can be found in Tables A.2, A.3 and A.4, while direct effects and two-way interaction models can be found in Tables A.26 and A.34, respectively.

The right column of Figure 2.1 shows the expected bankruptcy rate for each IFA treatment as determined by the two-way models. While there is not a clear advantage between the lexicographic and SF strategies, the complex strategy is effective at reducing the bankruptcy rate in all cases. The average bankruptcy frequency is reduced by a factor of roughly 8 on the small and biregional scales, and by a factor of 4 on the national scale. In a vast majority of circumstances, complex bidders go over budget fewer than one in a hundred times. While these results are encouraging, issues prevail on the national scale where bankruptcies exceed 4% given wide value distributions and less crowded markets.

Wide value distributions were a problem for all strategies on the national scale—more than doubling the average bankruptcy rate. Decreasing the number of bidders had a similar effect, implying that greater market competition was linked to reduced budget issues on this scale.

Bankruptcies play an important role in how we measure both revenue and surplus. For this reason, results from this and the following two sections are meaningfully correlated.

Summary of results:

- SMR bidders never went over budget
- IFA complex strategy bidders significantly reduced bankruptcy rates compared with other IFA strategies in all cases.
- Bankruptcies were more of an issue when bidders had wide value distributions

2.5.4 Auction Surplus

Before we begin to analyze auction surplus, we need to determine how it is to be measured. When all bidders are able to afford their commitments, the calculation is straightforward. Do we assume that bidders who bankrupt themselves by the final round and are unable to pay for their bids, are ultimately allocated their licenses anyway? We will consider both extremes to this question.

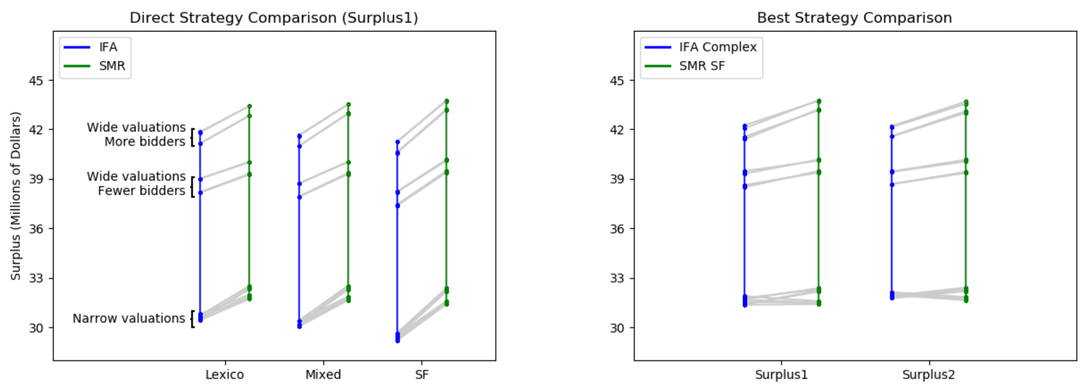
- We denote the **pessimistic auction surplus** (surplus1 as shorthand) to be surplus in which bankrupt bidders are assigned none of the products they would have otherwise been awarded. Bankrupt bidders contribute zero to total surplus.
- We denote the **opimistic auction surplus** (surplus2 as shorthand) to be surplus in which bidders are assigned all of their products regardless of bankrupt status.

We focus on the pessimistic interpretation, since this is how the FCC has chosen to conduct the Incentive Auction in the past. Tables A.8 – A.13 display surplus averages. Tables A.28 and A.29 contain direct effects models, while Tables A.36 and A.37 contain two-way interaction models. Figure 2.2 displays the results of the interaction models in two forms. The left column shows direct comparisons of pessimistic surplus across mechanisms, holding all factors (including strategy) constant. The right column shows how the "best" strategies from each mechanism (complex for IFA; SF for SMR) compare with respect to both surplus metrics.

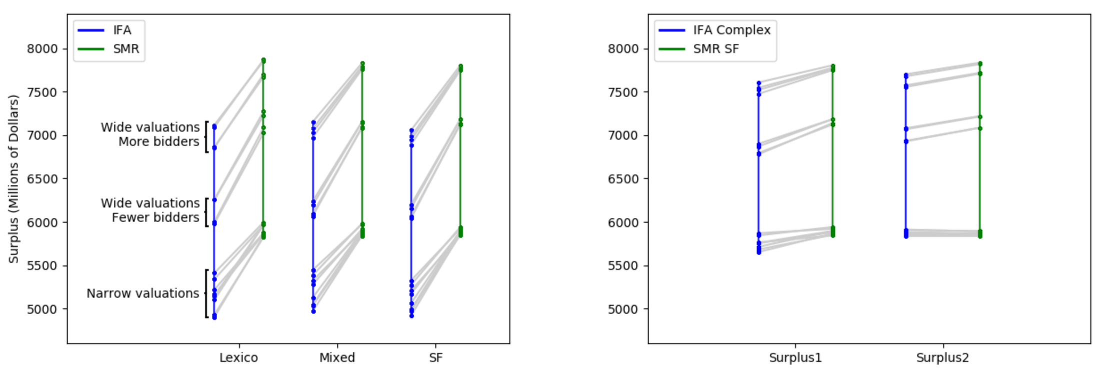
From this, it is clear that the SMR mechanism is expected to improve pessimistic surplus compared with every combination of IFA strategies. The table below shows the percentage increase to surplus when comparing lumped SMR and IFA treatments:

Figure 2.2: Simulation results of auction surplus

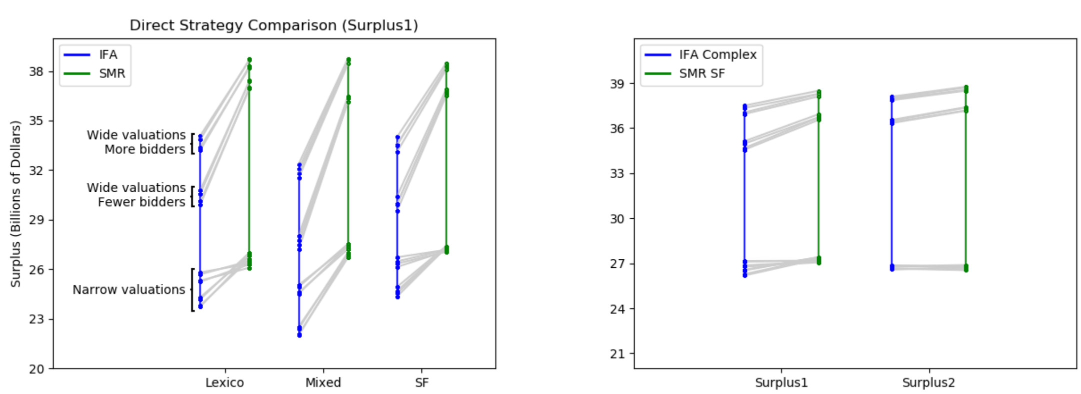
Strategy comparisons of surplus. Each point represents the expected value of a treatment from the corresponding two-way interaction model, with equivalent treatments connected.



(a) Small scale



(b) Biregional scale



(c) National scale

Scale	Surplus1	Surplus2
Small	4.3%	1.0%
Biregional	9.9%	1.6%
National	10.9%	3.5%

The benefit to surplus of using the SMR increases with scale. Much of this increase can be explained by the elimination of bankruptcies. Including both pessimistic and optimistic surplus allows us to see the exact extent of this. Even if all bidders over budget were still awarded their items, the SMR mechanism would still improve surplus by an expected 3.5% on the national scale.

Of all IFA strategies, the complex strategy performs the best. This is not surprising, given its substantially reduced bankruptcy rates. The direct effects models of Table A.28 suggest an increase of roughly 3% on the small scale, up to double digit increases on the larger scales. It is fair to question whether the SMR can outperform a group of IFA complex bidders, given these circumstances. When limiting to the SMR SF and IFA complex strategies and averaging across treatments, the SMR improves surplus over the IFA as follows:

Scale	Surplus1	Surplus2
Small	2.2%	1.5%
Biregional	2.9%	1.1%
National	3.1%	1.0%

Even when assuming the ideal IFA strategy, the SMR maintains a respectable improvement to both measurements of surplus across all scales. There were instances where the two performed roughly equal to one another—specifically, when bidders had narrow value distributions. And while the strategy distribution was very important within the IFA, it was virtually arbitrary within the SMR. There was no treatment within any scale where the choice of SMR strategy made more than a 1% impact on surplus. This robustness to SMR strategy was common to many dependent variables, and underscores the power of this mechanism.

Summary of results:

- The SMR has a positive impact on surplus with respect to nearly every treatment
- When holding bidding strategies fixed, SMR bidders on the national scale increase Surplus1 by an average of 10.9%, and Surplus2 by an average of 3.5%

- When comparing SMR treatments to complex IFA treatments, Surplus1 increases by an average of 3.1% and Surplus2 by an average of 1.0%
- Surplus is independent of strategy when using the SMR mechanism

2.5.5 Auction Revenue

Similar to surplus, auction revenue may be calculated differently depending on how we wish to handle bidder bankruptcies. We will consider four metrics in our analysis:

- We denote the **pessimistic auction revenue** (revenue1 as shorthand) to be revenue in which bankrupt bidders contribute nothing to auction revenue.
- We denote the **realistic auction revenue** (revenue2 as shorthand) to be revenue in which bankrupt bidders contribute their upfront payment to auction revenue.
- We denote the **generous auction revenue** (revenue3 as shorthand) to be revenue in which bankrupt bidders contribute their entire budgets to auction revenue.
- We denote the **optimistic auction revenue** (revenue4 as shorthand) to be revenue in which all bidders, including bankrupt bidders, somehow contribute 100% of their final round commitments as revenue.

We have chosen the shorthand in such a way that the revenue is necessarily ordered:

$$\text{Revenue1} \;\leq\; \text{Revenue2} \;\leq\; \text{Revenue3} \;\leq\; \text{Revenue4} \tag{2.25}$$

Because realistic revenue measures how the Broadcast Incentive Auction collected revenue, this will generally be the focus of our analysis. Tables A.14 – A.25 display revenue averages. Tables A.30 – A.33 contain direct effects models, while Tables A.38 – A.41 contain two-way interaction models. Figure 2.3 displays the results of the interaction models in two forms. The left column shows direct comparisons of realistic revenue across mechanisms, holding all factors (including strategy) constant. The right column shows how the "best" strategies from each mechanism (complex for IFA; SF for SMR) compare with respect to all four revenue metrics.

We see, as with auction surplus, that revenue is aided to some extent by running in the SMR mechanism compared with IFA strategies. The table below shows the average percentage increase to revenue when comparing lumped SMR and IFA treatments:

Scale	Revenue1	Revenue2	Revenue3	Revenue4
Small	9.0%	7.5%	5.8%	5.6%
Biregional	20.3%	16.1%	14.0%	13.3%
National	19.9%	15.7%	13.9%	13.4%

These numbers are not trivial: even Revenue4, which ignores bankruptcies entirely, is expected to increase by more than 10% for the larger scales. However, Figure 2.3 highlights that much of this improvement comes from the relatively low revenue accumulated by purely lexicographic IFA bidders. While this is an interesting result in itself, it is not a particularly likely scenario in reality. Omitting pure groups of lexicographic bidders from consideration, the SMR improves on the IFA as follows:

Scale	Revenue1	Revenue2	Revenue3	Revenue4
Small	7.9%	6.3%	4.4%	4.2%
Biregional	14.1%	10.1%	7.6%	6.9%
National	10.1%	4.9%	2.6%	2.0%

While these results are much more tempered (especially on the national scale), the expectation of an improvement to revenue continues to hold for all metrics. And when it comes to the question of which IFA strategy performs best for revenue, there is not a clear answer between SF and complex. While the SF strategy significantly outperforms the complex strategy for Revenue1 and Revenue2 on all scales, the SF strategy actually picks up a percentage point over complex for Revenue4 on the national scale. Even so, we choose to compare SMR SF bidders against purely complex IFA bidders, since they are consistently the most competitive with respect to realistic revenue. In this matchup, the SMR improves over the SMR by the following average percentages:

Scale	Revenue1	Revenue2	Revenue3	Revenue4
Small	6.2%	5.9%	5.5%	5.5%
Biregional	4.3%	3.5%	2.6%	2.6%
National	2.2%	0.7%	0.3%	0.2%

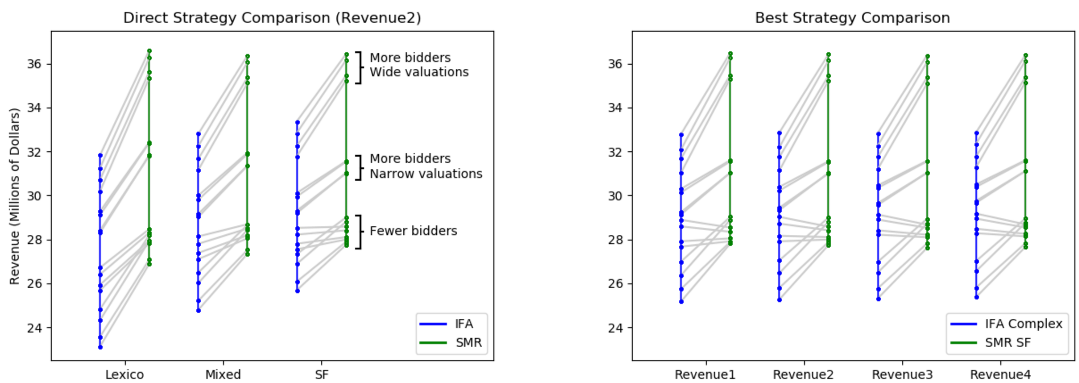
As was the case with surplus, matching up best strategies reduces the expected benefit of using the SMR mechanism, but the expectation is consistently an improvement. This is not entirely the case when we break things into individual treatments. On the national scale, our two-way model anticipates a slight advantage to the IFA given narrow bidder value distributions. This expected discrepancy is very slight—less than 1% difference.

Finally, as with revenue, it is necessary to point out that while the best IFA strategy combinations are competitive with the SMR, they are not robust in the way that the SMR is. When comparing strategies within the SMR mechanism while holding the treatment fixed, no revenue metric was expected to deviate by more than 5%. Within the IFA, revenue could be cut by as much as 50% if bidders behaved lexicographically. The reduced revenue of mixed strategies suggests that revenue is negatively impacted if even a fraction of bidders behave in such a manner.

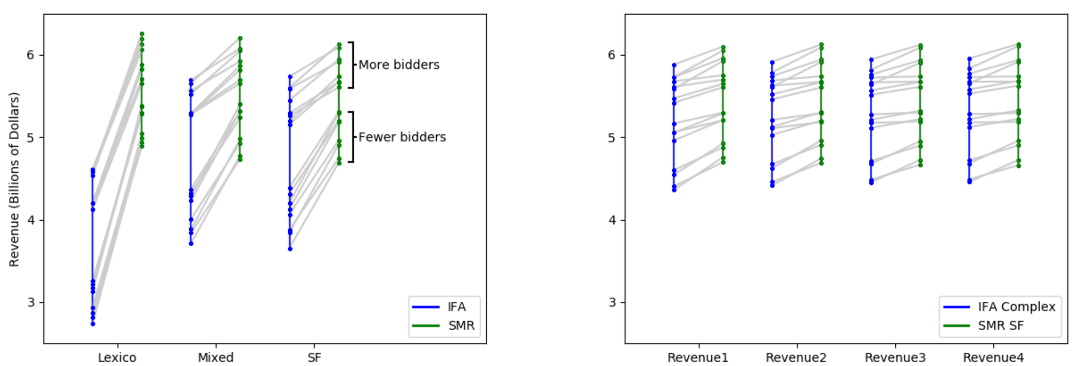
Summary of results:

- The SMR has a positive impact on revenue with respect to nearly every treatment
- When holding bidding strategies fixed, SMR bidders on the national scale increase revenue by an average of 13.4% – 19.9%, depending on metric. When excluding lexicographic bidders, the increase is 2.0% – 10.1%
- When comparing SMR treatments to complex IFA treatments, national scale surplus increases by an average of 0.2% – 2.2%
- Revenue is independent of strategy when using the SMR mechanism

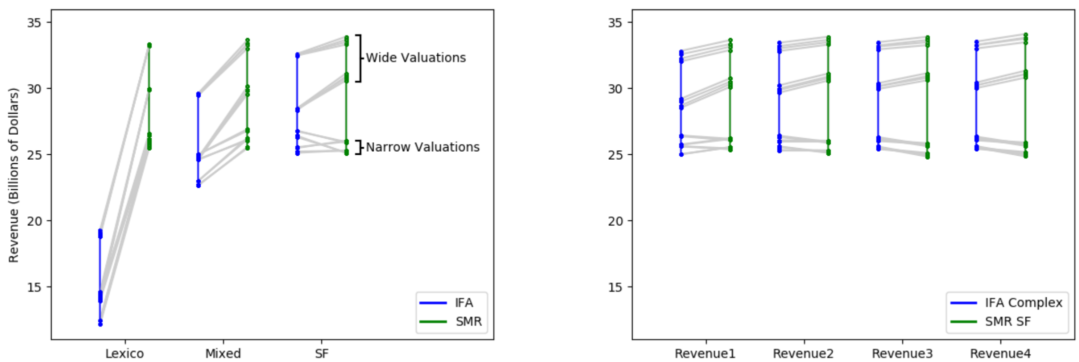
Figure 2.3: Simulation results of auction revenue
Strategy comparisons of revenue. Each point represents the expected value of a treatment from the corresponding two-way interaction model, with equivalent treatments connected.



(a) Small scale



(b) Biregional scale



(c) National scale

2.5.6 The Clock Increment

Although the clock increment was a control variable, it is significant enough to merit an isolated discussion. Recall from Section 2.5.2 that the clock increment was observed to have a significant impact on the expected number of rounds per auction. Figure 2.1 shows that the clock increment splits treatments into discrete clusters. We outline some point statistics on auction length data below:

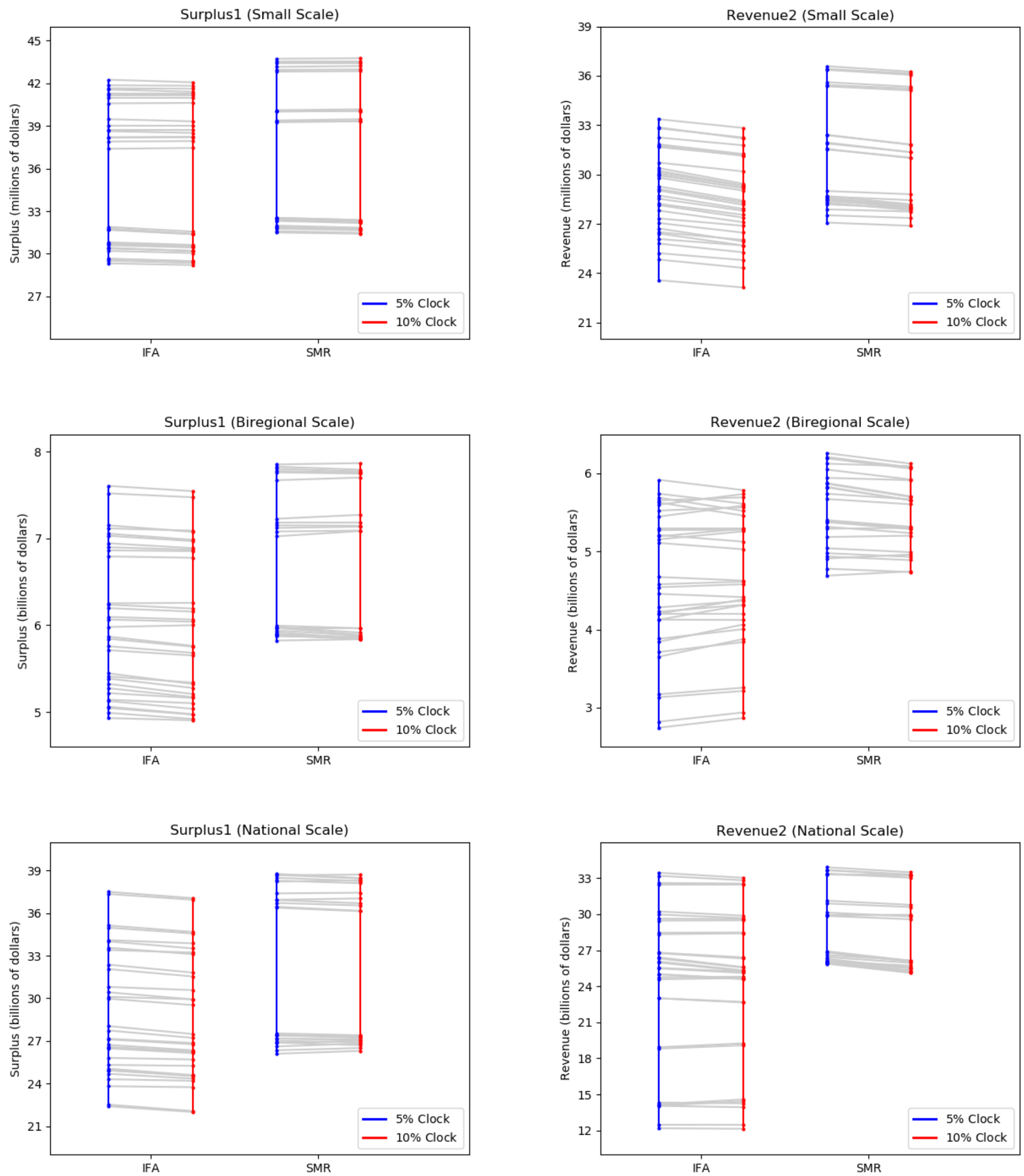
Mechanism/Scale		Clock Increment 5%	10%	Reduction
IFA,	Small	36	19	46.5%
SMR,	Small	40	22	46.4%
IFA,	Biregional	110	62	43.8%
SMR,	Biregional	348	194	44.1%
IFA,	National	131	72	44.8%
SMR,	National	419	232	44.6%

While the expected number of rounds changes dramatically with scale, the impact from the clock increment does not. When averaging across treatments, doubling the clock increment to 10% is expected to reduce the number of rounds by roughly 45% for every mechanism and scale combination. Although intuitive, this confirms that increasing the clock increment can be an effective tool for cutting down on auction length with great predictability. This begs the question: does increasing the clock increment negatively impact either surplus or revenue, and if so, to what extent?

Figure 2.4 shows the effect of increasing the clock increment on both pessimistic surplus (surplus1) and realistic revenue (revenue2) per treatment, as determined by our two-way interaction models. The model suggests very little impact on surplus, for either mechanism. Averaged across treatments, increasing the clock is expected to reduce pessimistic surplus by less than a percentage point within each scale, although there were national scale IFA treatments where the reduction was as high as 5%. The response was even more muted within the SMR, where no treatment increased or decreased surplus by more than 1%.

Revenue was somewhat more responsive to the clock increment. Increasing to 10% had the effect of reducing revenue within the SMR mechanism by roughly 1.5% on average for all scales. Revenue was also expected to decrease slightly on the small and national scales for IFA auctions, but on the biregional scale many treatments resulted in a slight increase to revenue. No single treatment resulted in a loss of more than 3% on any scale. Thus, while

Figure 2.4: Simulation effects of clock increment
Comparisons of surplus and revenue with contrasting clock increments. Each point represents the expected value of a treatment from the corresponding two-way interaction model, with equivalent treatments connected.



there is some expectation of loss as a result of speeding up the auction, it is small compared with expected changes from switching mechanisms.

Summary of results:

- Increasing the clock increment from 5% to 10% consistently reduced the number of rounds per auction by roughly 45%.
- Increasing the clock increment reduced surplus by less than 1% on average.
- Increasing the clock increment reduced revenue by 1.5% on average.

Chapter 3

An Experimental Approach

3.1 Motivation

The simulated auction environment is very helpful in several ways. It inexpensively allows the production of trials within many different and complex auction environments, as well as testing the efficacy and effects of bidder strategies within these environments. However, simulations come with significant implicit assumptions. The behavior of simulated bidders is necessarily rigid, and there is no way of knowing that any combination of bidding strategies captures realistic human behavior. As a results, we have an interest in producing a laboratory auction environment with human subjects as bidders.

This experimental data provides much more fluid, organic and authentic input. It removes the need to presume bidder behavior, and can help confirm observations originating from simulated data. However, it comes with its own set of notable limitations. Experimental data is much more expensive to obtain, requiring extensive time, physical resources, and incentivized participants. As a result, we are able to obtain only a small fraction of the number of data points produced via simulation. Of these data points, far fewer bidders and products are utilized than was the case for the FCC Broadcast Incentive Auction. Finally, because subjects must be familiarized with the auction mechanism in a relatively short amount of time, the mechanism must be extensively simplified. Despite this, experimental data provides critical insights to analysis.

In Section 3.2 we discuss in greater detail the experimental design and execution. In Section 3.3 we discuss the results and analysis of the experiment.

Table 3.1: The number of sessions conducted within each treatment category. The "Display 1" treatment specifies that the lexicographic strategy was bound to the first option button, with "Display 2" specifying that the lexicographic strategy was bound to the second option button.

		Display Option		
		Display 1	Display 2	No Display
Mechanism	IFA	5	5	3
	SMR	5	5	3

3.2 Structure and Design

3.2.1 Overview

A session of the FCC experiment consisted of a static group of subjects performing a series of independent auctions. A group consisted of six subjects who remained together until the conclusion of a session. For this reason, we use "group" and "session" interchangeably. After receiving instructions, subjects completed a single practice auction followed by ten regular auctions. Auctions consisted of a variable number of rounds, with the final round of regular auctions determined by a lack of excess demand within all products simultaneously. The practice auction was distinct in that it was halted at the end of the fifth round. At the end of each session, one of the ten regular auctions was randomly selected as the payout auction, with payouts determined based on the total net profits of each bidder's winnings. Bidders using the IFA mechanism were paid one dollar for each 330 awarded session dollars, while bidders using the SMR mechanism were paid one dollar for each 250 awarded session dollars. Across all sessions, subjects were awarded an average of $28.87.

Each round, subjects were presented with a spreadsheet of information about the auction's products. As part of the effort to keep the auctions simple to explain and comprehend, every product had a supply of exactly one for each auction (a side effect of this decision was a lack of reserve split). Thus, the only decisions subjects needed to make each round was whether to select and bid on each available product (at the labeled clock price). In addition, some groups were provided with "bidding options"—two buttons which, when selected, would recommend items to bid. These options determined their recommendations using the same lexicographic and SF bidding strategies utilized for the simulations. At the end of each round, subjects were presented with a summary of their processed and unprocessed bids, or winnings and total profits if the auction had concluded.

Sessions were conducted with a 2×3 treatment design. The first treatment variable was the auction mechanism, the second was the display treatment. As with the simulations, both the IFA and SMR mechanisms were implemented. In addition, groups varied by how the automatic bidding options were presented. For some groups, options were not provided at all. For others, options were generically labeled as "Option 1" and "Option 2". Some groups had the lexicographic strategy bound to Option 1, with others binding the lexicographic strategy to Option 2. A total of 26 experiment observations were collected, with group sample sizes for each treatment summarized in Table 3.1.

3.2.2 Environment

The Products: Each auction consisted of 15 products. These products had static names and activity costs which carried over from auction to auction. However, the values that bidders could have for a product changed each auction, with the range of possible values differing from product to product. Table 3.2 summarizes the activities and possible bidder values for each product. The initial posted price was \$2000 for each product at the start of each new auction. The clock increment was set at 10% of the current posted price, rounded to the nearest hundred dollars. Thus, each product's price incremented as follows:

$$\$2000 \rightarrow \$2200 \rightarrow \$2400 \rightarrow \$2600 \rightarrow \$2900 \rightarrow \$3200 \rightarrow \ldots$$

Table 3.2: Activity costs and possible nonzero bidder values per product.

Product	Activity Cost	Value Range
A	100	9,000 – 11,000
B	30	9,000 – 11,000
C	80	7,000 – 9,000
D	70	7,000 – 9,000
E	60	7,000 – 9,000
F	70	5,500 – 6,500
G	70	5,500 – 6,500
H	50	5,500 – 6,500
I	50	5,500 – 6,500
J	40	3,500 – 4,500
K	40	3,500 – 4,500
L	40	3,500 – 4,500
M	20	3,500 – 4,500
N	20	3,500 – 4,500
O	20	3,500 – 4,500

The Bidders: Throughout each session of the experiment, the same six subjects competed with one another over the course of all eleven auctions. Each auction, bidders had nonzero value for exactly eight of the fifteen available products. Each auction, bidders were assigned a new budget in the range of $\$5000 - \$10000$. At the start of the first round of each auction, bidders had an eligibility of 500—an amount that allows them to bid on most or all of their valued products. This means that bidders are always budget-constrained during the first round of bidding each period. This decision allowed subjects more autonomy in determining which strategy they wished to pursue.

The Interface: Figure 3.1 contains a sample screenshot of the interface used by subjects for making informed bidding decisions. Most of the information about products and round results was contained within the spreadsheet in the upper left quadrant of the interface. This table contained the following columns:

- **Selection:** A checkbox which, when selected, specified that the subject was willing to commit to the associated product at the next ("clock") price.
- **Item:** The name of the product described by the associated row.
- **Activity Units:** The activity cost of bidding on this product.
- **Current Price:** The product's posted price, which all current holders (if any) were committing to at present.
- **Next Price:** The product's clock price, which would be the following round's current price in the event that there was positive excess demand. This is also the price that bidding subjects were committing to for the next round.
- **Profit at Next Price:** The amount of profit (in experiment currency) that the bidder stood to earn should they win the product at its next price.
- **Excess Demand:** The amount of excess demand for the product from the previous round, defined here as

$$\text{Excess Demand} = \begin{cases} 0\,, & \text{if demand} = 0, \\ \text{demand} - 1\,, & \text{otherwise.} \end{cases}$$

- **Profit Per Activity Unit:** The amount of profit that the product would net at the next price for each unit of activity invested into it.

- **Profit Per Dollar Spent:** The amount of profit that the product would net at the next price for each unit of experiment currency invested into it at the next price.

The final two columns contained information that could be deduced from predecessor columns, and were included as potential optimization tools for the subjects. The table also included aggregate information in the bottom row concerning activity, prices and profits. These aggregates were a function of the currently checked selections. This information was also communicated within the Expected Cost and Activity meters below the table, which were updated in real time as bidders checked and unchecked their selections. The behavior of these meters differed by mechanism, as we discuss shortly. Finally, table rows were highlighted in green to indicate that a bidder was currently holding that product.

When subjects were finished preparing their bidding selections, they pressed the green “Submit” button to finalize their decisions. After this point, subjects could take no more actions for the round. There was a limited amount of time in which to submit bids, displayed at the top of the interface. If subjects did not submit their selections before the timer reached zero, the auction would behave as if they pressed the submit button with no products selected. The amount of time granted was three minutes for the first round, and decreased by 30 seconds for each subsequent round until reaching a minimum of 30 seconds for round 6 and beyond. Finally, if all six subjects submitted their bids before the clock reached zero, the round immediately concluded and round results were displayed before the next round commenced.

In the example displayed, the bidding options are visible and presented above the green submission button. Subjects were informed of the strategies utilized by each of these options during the instructions. In addition, tool tips reminded the subjects of the optimizations used by each option. When a subject clicked an option, the ensuing strategy calculateed the recommended bidset and checked all of the associated products. However, the option did not submit these bids for the round, and the bidder could continue to make selections as long as time permitted.

Finally, the graphs to the right gave an account of the subject’s bidding history for that auction. The top plot recorded the evolution of the subject’s eligibility (referred to simply as activity units), while the bottom plot recorded the bidder’s (processed) commitment per round. Commitments that exceeded the bidder’s budget and would have resulted in a bankruptcy status were represented by a red bar in this graph.

Between each round, bidders were given updates on the success or failure of bidding attempts. Figure 3.2 shows an example of this summary. These round updates included

- Any failure to increase demand for a product due to lack of activity.
- And failure to reduce demand for a product due to insufficient excess demand (IFA).
- Any failure to provisionally win a product that was successfully bid on (SMR).
- An indication that all requests to increase and reduce demand were successful.

At the end of each auction, a summary of the subject's auction winnings were displayed, as in Figure 3.3. When a bidder had gone over budget for that particular auction, the products won were still shown, but an additional message indicated that total profits for that period would consequently be equal to zero. Finally, at the end of the experiment, one of the ten regular auctions was selected for payment, and a summary displayed the selected round, as well as potential earnings for all completed auctions, as in Figure 3.4.

Differences Between Mechanisms: The rules under which the auction performed bid processing within the IFA and SMR mechanisms were identical to those used within the simulated auction environment, with the caveat that reserve splits did not occur in the experiment. There were two ways in which the bidder experience with the interface differed with respect to mechanism: in the behavior of the cost bars, and the behavior of the option buttons.

The activity and expected cost bars ignored provisional holdings in the IFA mechanism. As subjects clicked the boxes associated with their selections, the bars filled up in amounts proportional to the cost of their selections. In both mechanisms, the cost bar assumed the next price for each item—the upper bound on how much the bidder may have been committing with their selections. However, in the case of the SMR mechanism, the activity and expected cost bars always included representations for current holdings. The expected cost for an unchecked held product was equal to its current price. If the bidder chose to select a held product, the expected cost increased slightly—from the current price to its next price—and the activity remained unchanged. This is because the SMR mechanism automatically "charged" the activity cost of held products against the bidder's eligibility, even if they did not bid on the product again. In both mechanisms, the auction blocked the subject from submitting if their activity total exceeded their eligibility, and displayed a warning if their expected costs exceeded their budget.

Finally, the options differed slightly between the two mechanisms in their selections. The IFA options continued to ignore current holdings, suggesting optimizations over the entire range of products. The SMR options, on the other hand, would never recommend bidding on currently held products, as they optimized over the space of unheld products under an accordingly reduced budget and eligibility. In particular, a bidder who provisionally won all of the products that they bid on (or held) from the previous round would be advised not to bid on any products whatsoever, since they would have no available activity for unheld items under this circumstance.

Figure 3.1: The bidding interface for the FCC experiment

Figure 3.2: Experiment results summary between two rounds of an auction

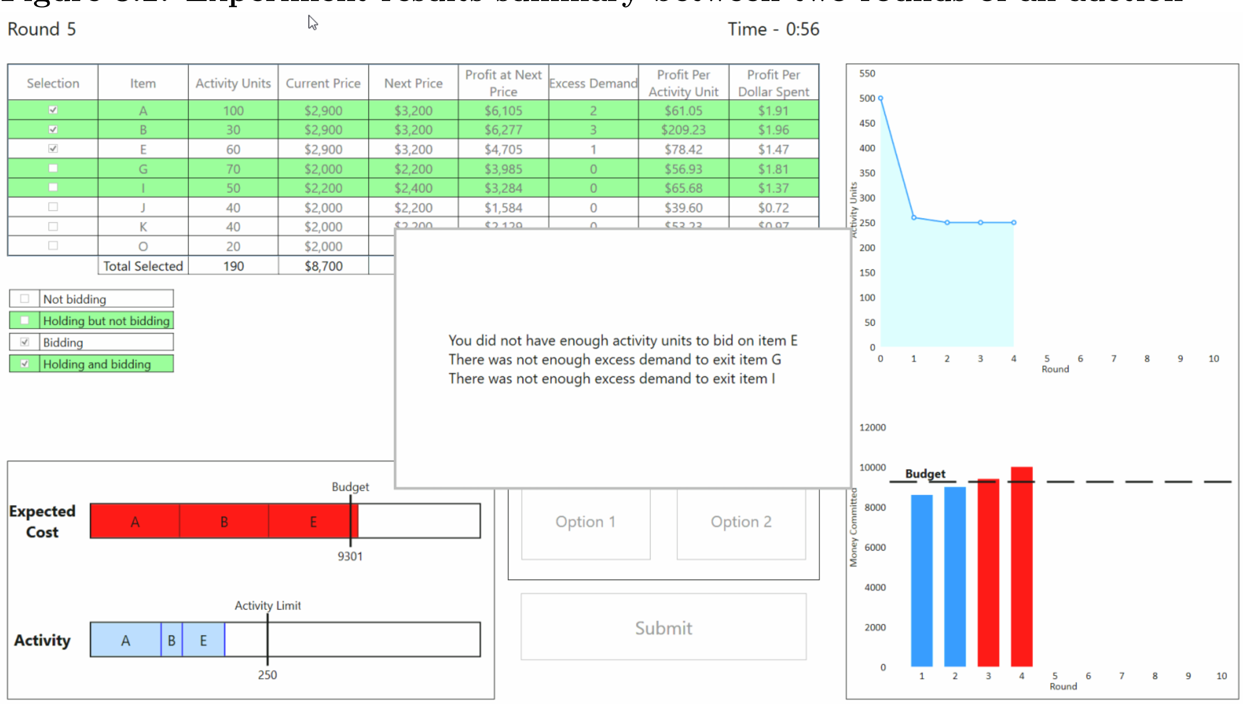


Figure 3.3: Results displayed at the conclusion of a mid-session auction

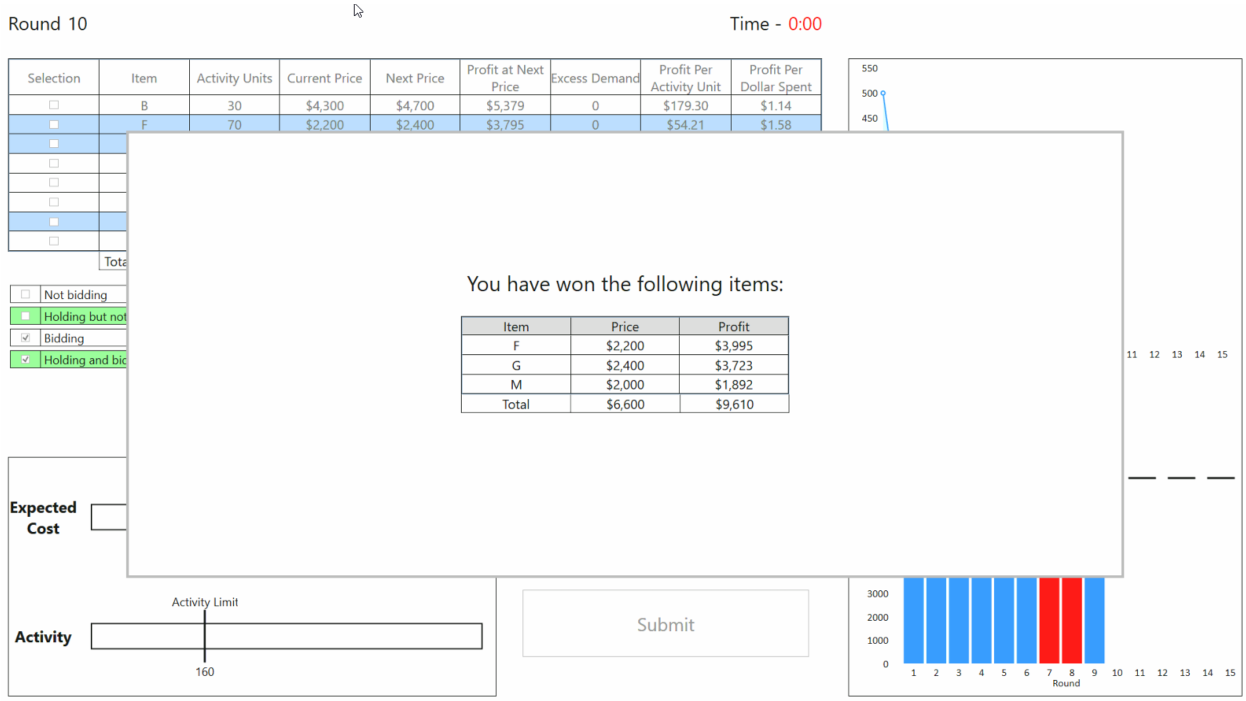
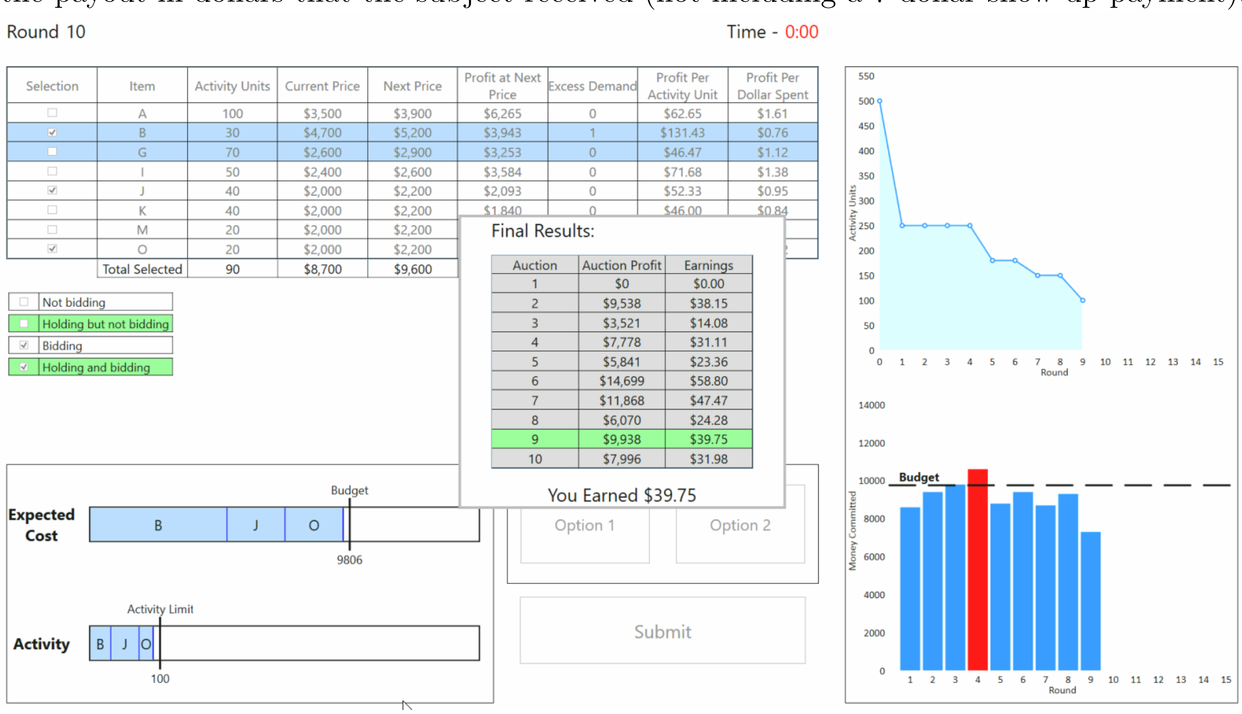


Figure 3.4: Results displayed at the conclusion of an experiment session

The highlighted row indicates which of the ten auctions was chosen at random and includes the payout in dollars that the subject received (not including a 7 dollar show-up payment).



3.3 Results

For the purpose of analysis, we are interested in examining the number of rounds per auction, the rate at which bidders went bankrupt, the auction surplus, and the auction revenue. We also briefly discuss auction length and its implications.

We are interested in the rate at which bidders held commitments exceeding their established budgets at the conclusion of an auction. We discuss in Section 1.3 how the nature of the incentive auction elicits unintentional overbidding in a variety of circumstances. When simulating bidding behavior, we find that it is possible to eliminate bankruptcies altogether. However, it is unclear whether human bidders would display this same aversion. We therefore wish to compare the rate at which bidders go over budget with respect to each mechanism, and to assess the reasoning for doing so.

Finally, auction surplus and revenue are standard auction performance metrics that we wish to examine. We expect that reducing uncertainty of outcome in the auction design (when transitioning from the IFA to the SMR) may encourage bidders to bid with greater alignment to their preferences, improving surplus (and potentially revenue). There are multiple ways that we may choose to measure both surplus and revenue, discussed in greater detail in their respective sections.

A total of four control variables are needed to describe the experimental treatments. These controls are as follows:

- **SMR:** Recall "SMR" from Section 2.5, which takes a value of 1 when the SMR mechanism is implemented, and a value of 0 when the IFA mechanism is implemented.

- **Auction:** The current (regular) auction within a group, treated as a cardinal variable. The value of Auction ranges from 1 to 10, and does not include the practice auction.

- **Option1:** A dummy variable representing the presentation of options in the interface. A value of 1 indicates that the lexicographic strategy was displayed to the left of the straightforward strategy, labeled as "Option 1".

- **Option2:** A dummy variable representing the presentation of options in the interface. A value of 1 indicates that the lexicographic strategy was displayed to the right of the straightforward strategy, labeled as "Option 2". When both Option1 and Option2 are equal to 0, this indicates that no automated strategy options were supplied to subjects.

Because each group of subjects experiences auctions in a necessarily sequential order, auction results within groups are treated as a time series, which necessitates panel data analysis. In

Table 3.3: Random effects (experiment) model results.
Coefficients for bankruptcies represent proportions of subjects going bankrupt. Coefficients for surplus and revenue models represent thousands of bidding experiment dollars. Bold values indicate that the coefficient is significant at the 5% level.

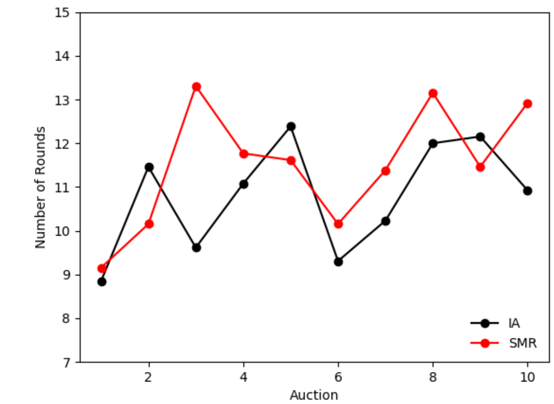
		Const	SMR	Auction	Option1	Option2
NumRounds	**coeff**	**9.137**	0.708	**0.211**	0.780	0.530
	p-val	0.000	0.160	0.003	0.240	0.425
Bankruptcies	**coeff**	**0.043**	-0.009	0.001	0.002	0.012
	p-val	0.006	0.396	0.704	0.905	0.402
Surplus1	**coeff**	**74.4**	**5.5**	0.4	-1.8	-2.3
	p-val	0.000	0.000	0.074	0.324	0.213
Surplus2	**coeff**	**79.8**	**4.6**	**0.4**	-1.5	-1.0
	p-val	0.000	0.000	0.000	0.173	0.349
Revenue1	**coeff**	**27.1**	**2.3**	**0.4**	0.1	-0.3
	p-val	0.000	0.000	0.000	0.896	0.745
Revenue3	**coeff**	**28.7**	**1.9**	**0.4**	0.3	0.3
	p-val	0.000	0.000	0.000	0.622	0.671
Revenue4	**coeff**	**29.1**	**1.9**	**0.4**	0.2	0.3
	p-val	0.000	0.000	0.000	0.730	0.638

the case of each dependent variable, the Hausman test strongly prefers a random effects model over a fixed effects model. Table 3.3 summarizes the results of these random effects models, which are discussed in greater detail throughout the next four sections.

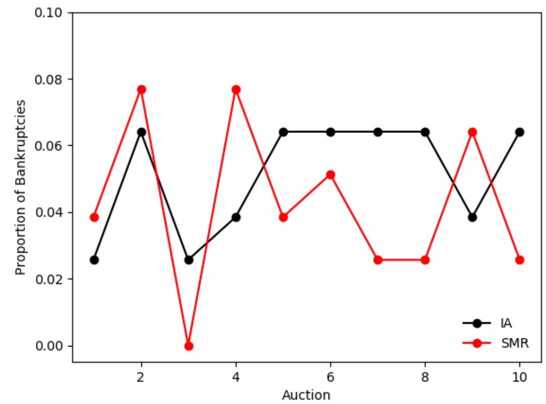
3.3.1 Number of Rounds Per Auction

Consistent with small scale simulations, subjects within the SMR mechanism took slightly more rounds on average compared with their IFA counterparts. However, the results of Table 3.3 suggest that there is no evidence suggesting that the auction mechanism played any significant role in the auction duration. This is not a surprising result, considering that the primary benefit of using the IFA–merging licenses into products with non-unit supply—effectively does not exist in this experiment. However, duration is expected to increase slightly as more auctions are performed. This behavior can be seen in Figure 3.5, where averages are accumulated within each mechanism and auction.

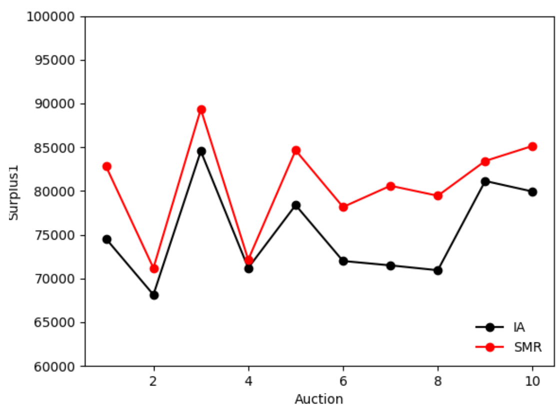
Figure 3.5: Within-auction averages



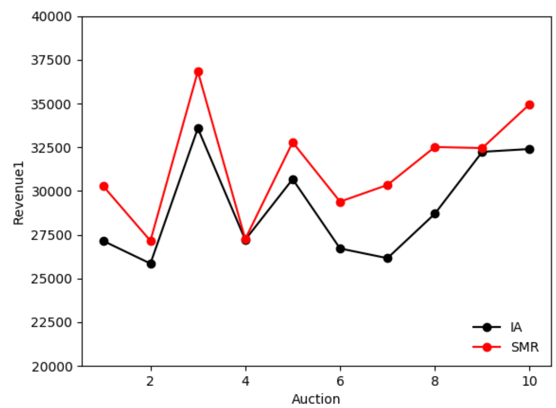
(a) Number of rounds per auction



(b) Auction bankruptcies



(c) Auction surplus



(d) Auction revenue

3.3.2 Bankruptcy Rates

Perhaps unexpectedly, neither the mechanism nor any other control factor contributed significantly to the incidence of bankruptcies. Within both mechanisms, approximately 4% of bidders went bankrupt per auction. This low incidence combined with a relatively low sample size calls the entire bankruptcy model into question. The random effects model has a Wald χ^2 value of 1.83 with an associated p-value of 0.77.

The most significant takeaway is that SMR bidders went over budget at all. This suggests that, although the mechanism at no point forces bidders into a position where their commitment exceeds their budgets, subjects took the initiative to do so regardless in potentially strategic ways. Roughly equal bankruptcy rates also simplifies the across-mechanism comparisons of surplus and revenue.

All of this raises the question of why SMR bidders would ever go over budget. If we take as a given that the mechanism itself is not forcing bidders into bankruptcy, then it is likely that subjects may be strategically bidding beyond their means. This strategic behavior would account for many of the IFA bankruptcies as well. One motivation for bidding over budget is a desire to preserve eligibility. To measure this for a given bidder, we define **excess eligibility** each round as the difference between the eligibility that would have resulted from the straightforward strategy bidset and the subject's final eligibility for the round. Thus, excess eligibility is positive when a bidder provisionally wins products summing higher activity than their optimal bidset for the round.

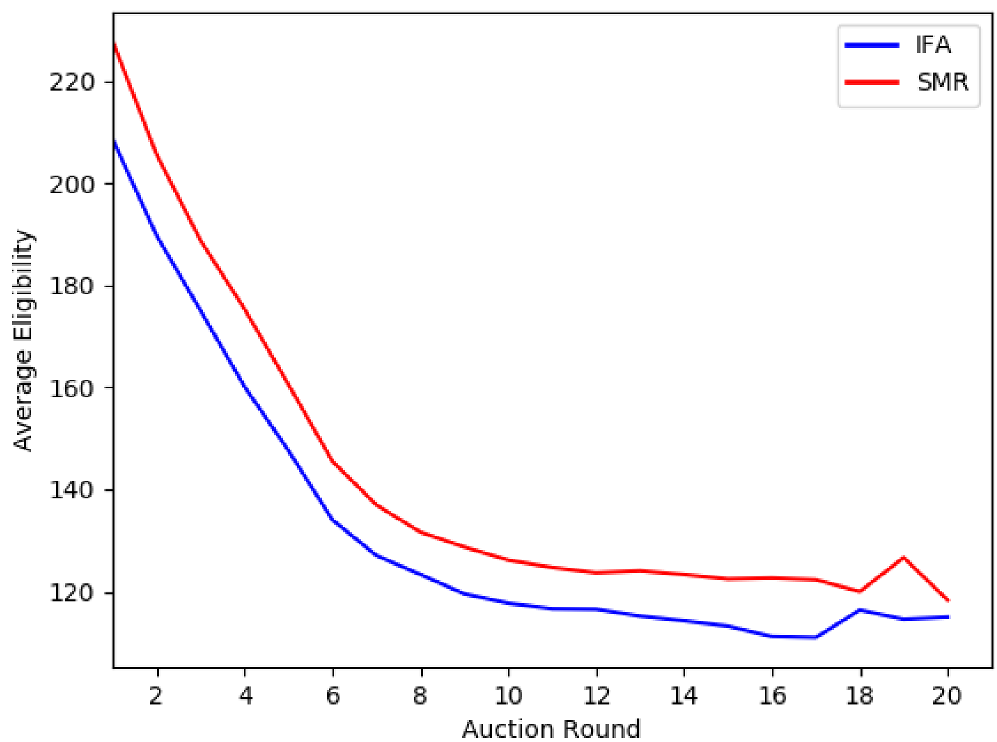
Choosing a unified measurement for excess eligibility across both mechanisms is difficult, since the two treat holdings differently. SMR bidders are not required to bid on held products, since these count toward processed activity automatically. Should a held SMR product automatically count toward requested activity and the associated excess eligibility calculation? This would significantly bias the calculation. An IFA bidder requesting to drop a bid would receive a negative contribution to their excess eligibility, while the same SMR bidder would receive no contribution whatsoever in waiting to drop a held product. Considering that bidders may attempt to drop the same product for many consecutive rounds, this issue compounds on itself.

However, the two are perfectly comparable during the first round, when holdings have yet to become a factor in the bidding process. IFA bidders submitted bids resulting in an average of 6.5 excess eligibility points during the first round, compared with an average of 25.6 excess eligibility from SMR bidders. At the end of round 1, IFA bidders had positive excess eligibility 34.1% of the time, compared with a 54.9% incidence rate from SMR bidders. These statistics suggest that SMR bidders are willing to manage their eligibilities more aggressively than IFA bidders, both in frequency and amount. And they are successful in doing so. Figure 3.6 shows the average amount of eligibility per auction round maintained by IFA and SMR bidders. SMR bidders consistently hold an eligibility advantage over IFA bidders throughout the duration of an auction, averaging 10.4 additional points.

Summary of results:

- Bankruptcies were independent of mechanism, occurring at an average rate of roughly 4%
- SMR bidders averaged 25.6 points of excess eligibility in the first round, compared

Figure 3.6: Average eligibility of subjects
Averages are split by auction mechanism and accumulated by round number.



with 6.5 points from IFA bidders.

- SMR bidders maintained an average of 10 additional points of eligibility over IFA bidders from round to round.

3.3.3 Auction Surplus

As in Section 2.5, we examined two metrics for surplus, denoted Surplus1 and Surplus2. Surplus1 is the calculation of surplus assuming that products corresponding to bankrupt bidders are left unassigned. This version of surplus was implemented within the experiment. Surplus2 is the calculation of surplus assuming that all products are assigned to winning bidders regardless of bankrupt status. Although unimplemented within the experiment, Surplus2 shows the effects of mechanism on surplus in the absence of bankruptcy effects.

Including both allows us to examine upper and lower bound estimates of surplus.

Table 3.3 supports the claim that switching from the IFA mechanism to SMR significantly benefits auction surplus. When assuming that bankrupt bidders do not contribute to surplus, the SMR mechanism is expected to increase surplus by $5,500 relative to a $74,400 baseline (an increase of 7.4%). Even in the case where bidders would be assigned their products regardless, SMR increases surplus by an expected $4,600 relative to a $79,800 baseline (an increase of 5.8%). As a disclaimer, it should be mentioned that subject behavior would most likely have shifted in the event that products were assigned to them regardless of bankruptcy status.

Summary of results:

- Auction surplus increased by more than 5% when comparing SMR auctions to IFA auctions, even when ignoring the effects of bankruptcy.

3.3.4 Auction Revenue

Recall the four metrics for measuring revenue defined in Section 2.5.5:

- **Revenue1:** The calculation of revenue assuming that bankrupt bidders contribute nothing to total revenue.
- **Revenue2:** The calculation of revenue assuming that bankrupt bidders contribute their upfront payments to total revenue.
- **Revenue3:** The calculation of revenue assuming that bankrupt bidders contribute their budgets to total revenue.
- **Revenue4:** The calculation of revenue assuming that all bidders, including bankrupt bidders, somehow contribute their entire commitment to total revenue.

Notably, Revenue2 is missing from Table 3.3 and our discussion. Recall that Revenue2 is calculated by assuming that bidders forfeit their upfront payments in the event of bankruptcy. Upfront payments were in turn a function of the starting eligibility of each bidder. In the experiment, however, all bidders began with the same eligibility at the start of each auction in such a way that they were not immediately activity-constrained. This means that upfront payments would always equal their budgets, and there would be no difference between Revenue2 and Revenue3.

The results of 3.3 demonstrate a robustness of the effects of mechanism across the variants of revenue. The amount that the SMR mechanism is expected to increase auction revenue ranges from 6.5% (Revenue4) to 8.4% (Revenue1). More strikingly, because the bankruptcy rate is statistically equal between the two mechanisms, this means almost all of the increase in revenue (and surplus) is a result of bidder strategy and behavior. Furthermore, all forms of revenue are expected to increase as bidders become more familiar with the auction environment (i.e. as Auction increases).

Summary of results:

- Auction revenue increased by more than 6% when comparing SMR auctions to IFA auctions, even when ignoring the effects of bankruptcy.
- All forms of revenue increased slightly as bidders become more familiar with the auction.

Chapter 4

Conclusions

The Incentive Auction was a novel approach in centralizing a private two-sided market. It balanced the goals of maximizing the supply of repurposed cellular spectrum licenses while achieving a profit for the FCC. However, the decision to redesign the SMR into the IFA mechanism for the forward auction component was an optional decision in this scheme which resulted in positive and negative repercussions. As argued by Milgrom in the process of designing the mechanism, these changes "should permit the auction to be completed in a fraction of the time that would be required by a traditional SMR auction, with no loss of efficiency or added difficulty for bidders [19]." Indeed, simulations confirmed that SMR auctions took far more rounds than IFA auctions to complete—as many as three times the number of rounds, under some circumstances.

However, this increase in speed did not come without its costs. Simulations showed that using the IFA caused bidders employing any strategy to unintentionally go over budget, implying that the risk of doing so was inherent to the mechanism an unavoidable. The incidence of bankruptcies is very serious to both the bidder and the FCC, since failure to pay for winnings results in a forfeiture of the down payment for the bidder and a failure to allocate the license(s). Simulations using the SMR, by contrast, did not result in bankruptcies for any bidding strategy, calling into question the argument that the IFA should cause "no added difficulty for bidders."

Experimental results showed that there was no discernible difference in bankruptcy rates between mechanisms. However, bidders participating in SMR auctions were able to maintain consistently greater eligibilities than their IFA counterparts. This was a likely factor in explaining the additional 7% surplus and 8.4% revenue enjoyed by these SMR auctions. These numbers were consistent with simulation results, where increases to surplus ranged from $2 - 11\%$ and increases to (realistic) revenue ranged from $1 - 16\%$. It is interesting to note that much of the benefit in simulations can be attributed to losses from bankruptcies, whereas the experimental mechanisms were largely independent of these effects.

Finally, it should be argued that the benefits of reduced auction length do not outweigh the downsides of the IFA mechanism. While using the IFA has the potential to reduce auction length by hundreds of rounds, this effect can be replicated in part with simple to changes to auction parameters such as the clock increment. Simulations found that doing so had minimal effects on either revenue or surplus, and would continue to encourage straightforward bidding strategies. Further, the benefit of auction reduction is not very significant. During the 2016 Broadcast Incentive Auction, later rounds were conducted at a rate of 6 per day, and likely could have been performed more frequently. The FCC should consider carefully whether reducing auction length by several bidding days is worth the potential cost of billions to surplus and revenue elicited by an unpredictable mechanism.

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Appendices

Appendix A

Simulation Data Tables

A.1 Interaction Model AICs and BICs

Table A.1: AIC for each model as a percentage of its corresponding (left-most) direct-effects model.

		Max N-Way Interaction					
		1	**2**	**3**	**4**	**5**	**6**
Small Scale	**NumRounds**	100	96.6	96.1	96.1	96.1	96.1
	Revenue1	100	99.6	99.6	99.6	99.6	99.6
	Revenue2	100	99.5	99.4	99.4	99.4	99.4
	Revenue3	100	99.3	99.2	99.2	99.2	99.2
	Revenue4	100	99.3	99.2	99.2	99.2	99.2
	Surplus1	100	99.9	99.8	99.8	99.8	99.8
	Surplus2	100	99.6	99.6	99.6	99.6	99.6
Bi-Reg Scale	**NumRounds**	100	98.8	98.7	98.7	98.7	98.7
	Revenue1	100	99.1	99.1	99.1	99.1	99.1
	Revenue2	100	98.9	98.8	98.8	98.8	98.8
	Revenue3	100	98.7	98.6	98.6	98.6	98.6
	Revenue4	100	98.7	98.6	98.6	98.6	98.6
	Surplus1	100	99.7	99.7	99.7	99.7	99.7
	Surplus2	100	98.9	98.8	98.8	98.8	98.8
National Scale	**NumRounds**	100	97.5	97.1	97.1	97.1	97.1
	Revenue1	100	98.2	98.1	98.1	98.1	98.1
	Revenue2	100	96.9	96.8	96.8	96.8	96.8
	Revenue3	100	96.0	95.8	95.8	95.8	95.8
	Revenue4	100	95.9	95.6	95.6	95.6	95.6
	Surplus1	100	99.4	99.3	99.3	99.3	99.3
	Surplus2	100	98.0	97.8	97.8	97.8	97.8

A.2 Averages

Table A.2: Average small-scale bankruptcy percentages. The final row represents averages across all parameter sets.

				IFA Strategies			
ValVar	nB	Dem	CInc	Lexico	SF	Complex	Mixed
0	0	0	0	1.3	1.9	0.1	1.3
0	0	0	1	1.5	2.2	0.3	1.5
0	0	1	0	1.2	2.0	0.1	1.3
0	0	1	1	1.4	2.2	0.3	1.5
0	1	0	0	0.7	1.0	0.1	0.7
0	1	0	1	1.1	1.2	0.2	0.9
0	1	1	0	0.8	1.0	0.1	0.8
0	1	1	1	1.1	1.2	0.2	1.0
1	0	0	0	0.7	0.9	0.0	0.6
1	0	0	1	0.7	0.8	0.1	0.6
1	0	1	0	0.7	1.0	0.0	0.7
1	0	1	1	0.7	0.9	0.1	0.7
1	1	0	0	0.4	0.5	0.0	0.4
1	1	0	1	0.4	0.5	0.0	0.4
1	1	1	0	0.5	0.6	0.0	0.5
1	1	1	1	0.4	0.5	0.1	0.4
				0.9	1.2	0.1	0.8

Table A.3: Average biregional-scale bankruptcy percentages. The final row represents averages across all parameter sets. SMR strategies were not included because no bankruptcies occurred within this mechanism.

				IFA Strategies			
ValVar	nB	Dem	CInc	Lexico	SF	Complex	Mixed
0	0	0	0	12.8	8.8	0.9	7.4
0	0	0	1	12.1	9.9	1.2	8.8
0	0	1	0	7.1	9.2	1.2	7.3
0	0	1	1	7.3	11.1	1.6	9.0
0	1	0	0	7.8	4.9	0.2	3.7
0	1	0	1	9.0	6.3	0.3	4.9
0	1	1	0	5.2	5.1	0.3	3.6
0	1	1	1	5.7	6.6	0.3	4.6
1	0	0	0	11.2	9.5	0.8	7.7
1	0	0	1	9.7	8.7	1.2	7.7
1	0	1	0	7.9	11.5	1.7	8.4
1	0	1	1	7.3	11.6	2.3	9.1
1	1	0	0	6.5	5.1	0.5	4.5
1	1	0	1	6.1	5.4	0.6	4.8
1	1	1	0	4.8	6.3	0.8	5.0
1	1	1	1	4.6	6.9	1.0	5.8
				7.8	7.9	0.9	6.4

Table A.4: Average national-scale bankruptcy percentages. The final row represents averages across all parameter sets. SMR strategies were not included because no bankruptcies occurred within this mechanism.

				IFA Strategies			
ValVar	nB	Dem	CInc	Lexico	SF	Complex	Mixed
0	0	0	0	7.1	3.1	0.2	5.1
0	0	0	1	7.2	3.2	0.3	5.9
0	0	1	0	4.1	3.5	0.4	6.8
0	0	1	1	4.3	3.8	0.4	7.1
0	1	0	0	6.5	1.3	0.1	1.7
0	1	0	1	6.3	1.5	0.1	1.9
0	1	1	0	4.0	1.6	0.1	2.2
0	1	1	1	3.9	1.7	0.2	2.0
1	0	0	0	9.2	9.0	2.5	9.8
1	0	0	1	8.4	10.3	3.2	11.5
1	0	1	0	6.0	9.0	4.2	11.6
1	0	1	1	5.5	11.7	4.7	14.0
1	1	0	0	7.0	5.5	1.0	6.1
1	1	0	1	6.4	6.9	1.2	7.5
1	1	1	0	4.6	5.6	1.7	7.4
1	1	1	1	4.3	7.5	1.8	8.9
				5.9	5.3	1.4	6.8

Table A.5: Average small-scale number of rounds per auction. The final row represents averages across all parameter sets.

				Number of Rounds Per Auction						
				Lexicographic		SF		Comp	Mixed	
ValVar	nB	Dem	CInc	IFA	SMR	IFA	SMR	IFA	IFA	SMR
0	0	0	0	38	41	35	41	34	35	41
0	0	0	1	21	22	18	22	18	19	22
0	0	1	0	39	40	34	40	34	35	40
0	0	1	1	21	21	18	21	18	19	21
0	1	0	0	36	35	35	35	35	35	35
0	1	0	1	19	19	18	19	18	19	19
0	1	1	0	36	35	35	35	35	35	35
0	1	1	1	19	19	18	19	18	19	19
1	0	0	0	37	43	36	43	35	36	43
1	0	0	1	20	23	19	23	19	19	23
1	0	1	0	38	43	37	43	35	37	43
1	0	1	1	20	23	20	23	19	20	23
1	1	0	0	38	42	38	42	37	38	42
1	1	0	1	20	23	20	23	20	20	23
1	1	1	0	39	42	38	42	37	38	42
1	1	1	1	21	23	20	23	20	20	23
				29	31	28	31	27	28	31

Table A.6: Average biregional-scale number of rounds per auction. The final row represents averages across all parameter sets.

				Number of Rounds Per Auction						
				Lexicographic		SF		Comp	Mixed	
ValVar	nB	Dem	CInc	IFA	SMR	IFA	SMR	IFA	IFA	SMR
0	0	0	0	65	412	125	410	100	114	411
0	0	0	1	35	224	70	224	54	63	223
0	0	1	0	65	400	133	397	100	122	399
0	0	1	1	35	220	76	220	55	67	220
0	1	0	0	123	367	142	370	106	138	368
0	1	0	1	70	206	81	209	60	78	207
0	1	1	0	122	321	145	318	99	137	319
0	1	1	1	67	179	83	179	57	78	180
1	0	0	0	62	344	112	344	88	99	344
1	0	0	1	34	190	63	190	48	55	190
1	0	1	0	60	333	121	334	89	108	333
1	0	1	1	32	187	69	188	50	60	188
1	1	0	0	115	321	133	322	97	126	321
1	1	0	1	64	183	77	184	56	72	183
1	1	1	0	116	285	139	284	91	129	285
1	1	1	1	64	164	81	164	53	74	163
				71	271	103	271	75	95	271

Table A.7: Average national-scale number of rounds per auction. The final row represents averages across all parameter sets.

				Number of Rounds Per Auction						
				Lexicographic		SF		Comp	Mixed	
ValVar	nB	Dem	CInc	IFA	SMR	IFA	SMR	IFA	IFA	SMR
0	0	0	0	118	506	149	474	96	158	496
0	0	0	1	63	277	84	262	53	85	273
0	0	1	0	119	467	149	429	93	158	457
0	0	1	1	63	255	85	235	52	84	249
0	1	0	0	141	419	149	396	88	149	414
0	1	0	1	80	235	85	222	51	82	233
0	1	1	0	149	379	149	351	86	147	372
0	1	1	1	84	215	86	200	49	81	211
1	0	0	0	112	546	156	370	90	148	452
1	0	0	1	60	290	84	209	51	81	248
1	0	1	0	121	561	157	338	87	146	437
1	0	1	1	63	292	86	191	50	79	236
1	1	0	0	145	430	159	311	87	146	370
1	1	0	1	77	235	87	183	50	82	212
1	1	1	0	153	446	160	287	86	143	349
1	1	1	1	80	244	88	164	49	80	201
				102	362	120	289	70	116	326

Table A.8: Average small-scale pessimistic auction surplus (surplus1), in millions of dollars. The final row represents averages across all parameter sets.

				Average Pessimistic Auction Surplus (Surplus1)						
				Lexicographic		SF		Comp	Mixed	
ValVar	nB	Dem	CInc	IFA	SMR	IFA	SMR	IFA	IFA	SMR
0	0	0	0	30.7	31.9	28.9	31.9	31.6	30.1	31.9
0	0	0	1	30.6	31.9	28.8	31.9	31.1	29.9	31.9
0	0	1	0	30.8	32.0	29.0	32.0	31.7	30.2	32.0
0	0	1	1	30.6	31.9	28.8	31.9	31.4	30.0	31.9
0	1	0	0	30.9	32.2	29.7	32.2	31.8	30.6	32.2
0	1	0	1	30.5	32.0	29.6	32.0	31.4	30.4	32.0
0	1	1	0	31.0	32.2	29.9	32.2	32.0	30.6	32.2
0	1	1	1	30.5	32.1	29.6	32.1	31.6	30.3	32.1
1	0	0	0	38.1	39.1	37.7	39.1	38.7	38.0	39.1
1	0	0	1	38.1	39.1	37.8	39.0	38.6	38.0	39.0
1	0	1	0	39.1	39.9	38.6	39.9	39.6	38.9	39.9
1	0	1	1	39.1	39.8	38.5	39.8	39.4	38.8	39.8
1	1	0	0	40.8	43.3	40.4	43.3	41.4	40.7	43.3
1	1	0	1	40.9	43.2	40.6	43.2	41.3	40.7	43.2
1	1	1	0	41.5	43.7	41.2	43.8	42.0	41.3	43.8
1	1	1	1	41.6	43.7	41.2	43.7	41.8	41.4	43.7
				35.3	36.8	34.4	36.8	36.0	35.0	36.8

Table A.9: Average small-scale optimistic auction surplus (surplus2), in millions of dollars. The final row represents averages across all parameter sets.

				Average Optimistic Auction Surplus (Surplus2)						
				Lexicographic		SF		Comp	Mixed	
ValVar	nB	Dem	CInc	IFA	SMR	IFA	SMR	IFA	IFA	SMR
0	0	0	0	31.9	31.9	32.0	31.9	31.9	31.9	31.9
0	0	0	1	31.8	31.9	31.9	31.9	31.8	31.8	31.9
0	0	1	0	32.0	32.0	32.0	32.0	31.9	32.0	32.0
0	0	1	1	31.9	31.9	31.9	31.9	31.9	31.9	31.9
0	1	0	0	32.1	32.2	32.1	32.2	32.0	32.1	32.2
0	1	0	1	31.9	32.0	32.0	32.0	32.0	32.0	32.0
0	1	1	0	32.1	32.2	32.2	32.2	32.1	32.1	32.2
0	1	1	1	32.0	32.1	32.0	32.1	32.0	32.0	32.1
1	0	0	0	38.9	39.1	39.4	39.1	38.8	39.1	39.1
1	0	0	1	38.9	39.1	39.5	39.0	38.8	39.1	39.0
1	0	1	0	39.9	39.9	40.3	39.9	39.7	40.0	39.9
1	0	1	1	39.9	39.8	40.2	39.8	39.6	39.9	39.8
1	1	0	0	41.8	43.3	42.1	43.3	41.4	41.8	43.3
1	1	0	1	41.7	43.2	42.0	43.2	41.4	41.8	43.2
1	1	1	0	42.5	43.7	42.7	43.8	42.0	42.4	43.8
1	1	1	1	42.4	43.7	42.6	43.7	42.0	42.4	43.7
				36.4	36.8	36.6	36.8	36.2	36.4	36.8

Table A.10: Average biregional-scale pessimistic auction surplus (surplus1), in billions of dollars. The final row represents averages across all parameter sets.

				Average Pessimistic Auction Surplus (Surplus1)						
				Lexicographic		SF		Comp	Mixed	
ValVar	nB	Dem	CInc	IFA	SMR	IFA	SMR	IFA	IFA	SMR
0	0	0	0	4.90	5.87	5.01	5.87	5.71	5.04	5.87
0	0	0	1	4.93	5.86	4.92	5.86	5.65	4.94	5.86
0	0	1	0	5.24	5.89	5.08	5.89	5.73	5.09	5.89
0	0	1	1	5.20	5.87	4.94	5.87	5.66	4.97	5.87
0	1	0	0	5.17	5.93	5.28	5.93	5.84	5.45	5.93
0	1	0	1	5.04	5.89	5.11	5.89	5.80	5.32	5.89
0	1	1	0	5.44	5.93	5.32	5.94	5.85	5.47	5.93
0	1	1	1	5.37	5.90	5.14	5.90	5.80	5.35	5.90
1	0	0	0	5.83	7.09	6.09	7.06	6.83	6.12	7.07
1	0	0	1	5.96	7.09	6.15	7.06	6.76	6.09	7.07
1	0	1	0	6.25	7.22	6.13	7.19	6.94	6.22	7.20
1	0	1	1	6.30	7.20	6.14	7.18	6.86	6.15	7.19
1	1	0	0	6.84	7.74	6.99	7.71	7.49	7.04	7.73
1	1	0	1	6.88	7.73	6.94	7.71	7.44	6.99	7.72
1	1	1	0	7.11	7.83	7.03	7.81	7.61	7.13	7.82
1	1	1	1	7.13	7.82	6.95	7.79	7.54	7.03	7.81
				5.85	6.68	5.83	6.67	6.47	5.90	6.67

Table A.11: Average biregional-scale optimistic auction surplus (surplus2), in billions of dollars. The final row represents averages across all parameter sets.

				Average Optimistic Auction Surplus (Surplus2)						
				Lexicographic		SF		Comp	Mixed	
ValVar	nB	Dem	CInc	IFA	SMR	IFA	SMR	IFA	IFA	SMR
0	0	0	0	5.78	5.87	5.81	5.87	5.85	5.82	5.87
0	0	0	1	5.77	5.86	5.81	5.86	5.84	5.82	5.86
0	0	1	0	5.70	5.89	5.83	5.89	5.87	5.83	5.89
0	0	1	1	5.69	5.87	5.83	5.87	5.85	5.83	5.87
0	1	0	0	5.88	5.93	5.88	5.93	5.90	5.89	5.93
0	1	0	1	5.87	5.89	5.86	5.89	5.87	5.87	5.89
0	1	1	0	5.88	5.93	5.89	5.94	5.91	5.90	5.93
0	1	1	1	5.87	5.90	5.87	5.90	5.88	5.88	5.90
1	0	0	0	6.72	7.09	6.96	7.06	6.93	6.87	7.07
1	0	0	1	6.73	7.09	7.01	7.06	6.92	6.90	7.07
1	0	1	0	6.89	7.22	7.11	7.19	7.09	7.01	7.20
1	0	1	1	6.89	7.20	7.15	7.18	7.07	7.04	7.19
1	1	0	0	7.41	7.74	7.63	7.71	7.57	7.60	7.73
1	1	0	1	7.44	7.73	7.65	7.71	7.55	7.62	7.72
1	1	1	0	7.54	7.83	7.75	7.81	7.69	7.72	7.82
1	1	1	1	7.55	7.82	7.76	7.79	7.67	7.73	7.81
				6.48	6.68	6.61	6.67	6.59	6.58	6.67

Table A.12: Average national-scale pessimistic auction surplus (surplus1), in billions of dollars. The final row represents averages across all parameter sets.

				Average Pessimistic Auction Surplus (Surplus1)						
				Lexicographic		SF		Comp	Mixed	
ValVar	nB	Dem	CInc	IFA	SMR	IFA	SMR	IFA	IFA	SMR
0	0	0	0	23.5	26.9	25.3	26.9	26.7	22.1	26.9
0	0	0	1	23.3	26.8	25.1	26.8	26.3	21.9	26.8
0	0	1	0	24.1	26.9	25.3	26.9	26.6	21.9	26.9
0	0	1	1	24.0	26.8	25.0	26.8	26.4	21.7	26.8
0	1	0	0	24.6	27.0	26.4	27.0	26.8	25.4	27.0
0	1	0	1	24.5	26.9	26.0	26.9	26.6	25.2	26.9
0	1	1	0	25.2	27.0	26.4	27.0	26.9	25.4	27.0
0	1	1	1	25.2	26.9	26.1	26.9	26.6	25.3	26.9
1	0	0	0	30.4	36.4	29.2	36.8	34.9	27.6	36.7
1	0	0	1	30.8	36.4	28.9	36.8	34.2	26.8	36.7
1	0	1	0	31.5	36.6	30.2	37.0	34.8	28.1	36.8
1	0	1	1	31.6	36.6	29.2	37.0	34.4	26.6	36.8
1	1	0	0	33.1	38.1	33.6	38.3	37.5	32.3	38.2
1	1	0	1	33.4	38.1	33.0	38.2	37.1	31.3	38.2
1	1	1	0	34.1	38.3	34.0	38.5	37.5	32.4	38.4
1	1	1	1	34.2	38.2	33.3	38.5	37.2	31.5	38.4
				28.3	32.1	28.6	32.3	31.3	26.6	32.2

Table A.13: Average national-scale optimistic auction surplus (surplus2), in billions of dollars. The final row represents averages across all parameter sets.

				Average Optimistic Auction Surplus (Surplus2)						
				Lexicographic		SF		Comp	Mixed	
ValVar	nB	Dem	CInc	IFA	SMR	IFA	SMR	IFA	IFA	SMR
0	0	0	0	24.6	26.9	26.5	26.9	26.8	26.4	26.9
0	0	0	1	24.6	26.8	26.2	26.8	26.6	26.2	26.8
0	0	1	0	24.4	26.9	26.6	26.9	26.8	26.4	26.9
0	0	1	1	24.5	26.8	26.3	26.8	26.6	26.3	26.8
0	1	0	0	25.8	27.0	26.7	27.0	26.9	26.6	27.0
0	1	0	1	25.8	26.9	26.4	26.9	26.6	26.4	26.9
0	1	1	0	25.7	27.0	26.7	27.0	26.9	26.7	27.0
0	1	1	1	25.7	26.9	26.5	26.9	26.7	26.5	26.9
1	0	0	0	32.3	36.4	36.2	36.8	36.4	34.6	36.7
1	0	0	1	32.5	36.4	36.2	36.8	36.3	34.6	36.7
1	0	1	0	32.4	36.6	36.4	37.0	36.6	34.9	36.8
1	0	1	1	32.4	36.6	36.4	37.0	36.5	34.9	36.8
1	1	0	0	34.8	38.1	37.9	38.3	38.0	36.7	38.2
1	1	0	1	34.9	38.1	37.8	38.2	37.8	36.7	38.2
1	1	1	0	34.9	38.3	38.0	38.5	38.1	36.9	38.4
1	1	1	1	35.0	38.2	37.9	38.5	37.9	36.9	38.4
				29.4	32.1	31.8	32.3	32.0	31.1	32.2

Table A.14: Average small-scale pessimistic auction revenue (revenue1), in millions of dollars. The final row represents averages across all parameter sets.

				Average Pessimistic Auction Revenue (Revenue1)						
				Lexicographic		SF		Comp	Mixed	
ValVar	nB	Dem	CInc	IFA	SMR	IFA	SMR	IFA	IFA	SMR
0	0	0	0	25.5	28.8	26.8	28.8	28.4	26.9	28.8
0	0	0	1	24.8	28.3	26.2	28.3	27.2	26.1	28.3
0	0	1	0	25.5	29.0	26.9	29.0	28.6	27.2	29.0
0	0	1	1	24.8	28.5	26.3	28.5	27.6	26.4	28.5
0	1	0	0	29.5	31.5	28.7	31.5	30.5	29.4	31.5
0	1	0	1	28.4	30.9	28.1	30.9	29.4	28.6	30.9
0	1	1	0	29.5	31.6	29.0	31.6	30.6	29.5	31.6
0	1	1	1	28.5	31.0	28.1	31.0	29.6	28.6	31.0
1	0	0	0	23.3	27.0	26.1	27.0	26.0	25.2	27.0
1	0	0	1	22.8	26.9	25.8	26.9	25.6	24.8	26.9
1	0	1	0	24.4	28.2	27.4	28.2	27.3	26.5	28.2
1	0	1	1	23.9	27.9	26.9	27.9	26.8	26.0	27.9
1	1	0	0	30.1	35.9	31.4	35.9	31.3	30.9	35.9
1	1	0	1	29.6	35.7	31.0	35.7	30.8	30.4	35.7
1	1	1	0	31.3	36.8	32.5	36.8	32.4	32.0	36.8
1	1	1	1	30.7	36.5	32.0	36.5	31.8	31.5	36.6
				27.0	30.9	28.3	30.9	29.0	28.1	30.9

Table A.15: Average small-scale realistic auction revenue (revenue2), in millions of dollars. The final row represents averages across all parameter sets.

				Average Realistic Auction Revenue (Revenue2)						
				Lexicographic		SF		Comp	Mixed	
ValVar	nB	Dem	CInc	IFA	SMR	IFA	SMR	IFA	IFA	SMR
0	0	0	0	25.9	28.8	27.9	28.8	28.5	27.6	28.8
0	0	0	1	25.3	28.3	27.4	28.3	27.4	26.9	28.3
0	0	1	0	26.1	29.0	28.2	29.0	28.7	27.9	29.0
0	0	1	1	25.4	28.5	27.5	28.5	27.8	27.2	28.5
0	1	0	0	29.9	31.5	29.7	31.5	30.6	30.0	31.5
0	1	0	1	29.0	30.9	29.1	30.9	29.6	29.3	30.9
0	1	1	0	30.1	31.6	30.0	31.6	30.7	30.2	31.6
0	1	1	1	29.2	31.0	29.2	31.0	29.8	29.4	31.0
1	0	0	0	23.6	27.0	26.7	27.0	26.0	25.5	27.0
1	0	0	1	23.1	26.9	26.3	26.9	25.6	25.1	26.9
1	0	1	0	24.7	28.2	28.0	28.2	27.3	26.8	28.2
1	0	1	1	24.2	27.9	27.5	27.9	26.8	26.4	27.9
1	1	0	0	30.4	35.9	31.9	35.9	31.4	31.3	35.9
1	1	0	1	29.8	35.7	31.5	35.7	30.8	30.7	35.7
1	1	1	0	31.7	36.8	33.1	36.8	32.4	32.4	36.8
1	1	1	1	31.0	36.5	32.5	36.5	31.8	31.8	36.6
				27.5	30.9	29.2	30.9	29.1	28.7	30.9

Table A.16: Average small-scale generous auction revenue (revenue3), in millions of dollars. The final row represents averages across all parameter sets.

				Average Generous Auction Revenue (Revenue3)						
				Lexicographic		SF		Comp	Mixed	
ValVar	nB	Dem	CInc	IFA	SMR	IFA	SMR	IFA	IFA	SMR
0	0	0	0	26.5	28.8	29.5	28.8	28.6	28.5	28.8
0	0	0	1	25.8	28.3	28.9	28.3	27.9	27.8	28.3
0	0	1	0	26.5	29.0	29.6	29.0	28.8	28.7	29.0
0	0	1	1	25.9	28.5	29.0	28.5	28.1	28.1	28.5
0	1	0	0	30.6	31.5	31.0	31.5	30.7	30.8	31.5
0	1	0	1	29.7	30.9	30.3	30.9	29.9	30.1	30.9
0	1	1	0	30.6	31.6	31.1	31.6	30.7	30.9	31.6
0	1	1	1	29.8	31.0	30.3	31.0	30.0	30.2	31.0
1	0	0	0	23.8	27.0	27.3	27.0	26.1	25.9	27.0
1	0	0	1	23.3	26.9	26.9	26.9	25.7	25.5	26.9
1	0	1	0	25.0	28.2	28.6	28.2	27.4	27.2	28.2
1	0	1	1	24.4	27.9	28.1	27.9	26.9	26.7	27.9
1	1	0	0	30.8	35.9	32.6	35.9	31.4	31.7	35.9
1	1	0	1	30.1	35.7	32.1	35.7	30.9	31.2	35.7
1	1	1	0	32.0	36.8	33.7	36.8	32.5	32.9	36.8
1	1	1	1	31.3	36.5	33.1	36.5	31.9	32.2	36.6
				27.9	30.9	30.1	30.9	29.2	29.3	30.9

Table A.17: Average small-scale optimistic auction revenue (revenue4), in millions of dollars. The final row represents averages across all parameter sets.

				Average Optimistic Auction Revenue (Revenue4)						
				Lexicographic		SF		Comp	Mixed	
0	0	0	0	26.6	28.8	29.6	28.8	28.6	28.6	28.8
0	0	0	1	25.9	28.3	29.0	28.3	27.9	27.9	28.3
0	0	1	0	26.6	29.0	29.8	29.0	28.8	28.8	29.0
0	0	1	1	26.0	28.5	29.1	28.5	28.1	28.2	28.5
0	1	0	0	30.6	31.5	31.1	31.5	30.7	30.9	31.5
0	1	0	1	29.8	30.9	30.4	30.9	29.9	30.2	30.9
0	1	1	0	30.7	31.6	31.2	31.6	30.7	31.0	31.6
0	1	1	1	29.9	31.0	30.5	31.0	30.0	30.3	31.0
1	0	0	0	23.9	27.0	27.4	27.0	26.1	25.9	27.0
1	0	0	1	23.4	26.9	27.0	26.9	25.7	25.5	26.9
1	0	1	0	25.0	28.2	28.7	28.2	27.4	27.3	28.2
1	0	1	1	24.5	27.9	28.2	27.9	26.9	26.8	27.9
1	1	0	0	30.8	35.9	32.7	35.9	31.4	31.8	35.9
1	1	0	1	30.2	35.7	32.1	35.7	30.9	31.2	35.7
1	1	1	0	32.1	36.8	33.8	36.8	32.5	32.9	36.8
1	1	1	1	31.3	36.5	33.2	36.5	31.9	32.3	36.6
				28.0	30.9	30.2	30.9	29.2	29.3	30.9

Table A.18: Average biregional-scale pessimistic auction revenue (revenue1), in billions of dollars. The final row represents averages across all parameter sets.

				Average Pessimistic Auction Revenue (Revenue1)						
				Lexicographic		SF		Comp	Mixed	
ValVar	nB	Dem	CInc	IFA	SMR	IFA	SMR	IFA	IFA	SMR
0	0	0	0	2.74	5.43	3.59	5.33	5.12	3.89	5.39
0	0	0	1	2.77	5.33	4.02	5.24	4.97	3.97	5.29
0	0	1	0	2.87	5.48	3.69	5.39	5.19	3.95	5.44
0	0	1	1	2.86	5.38	4.09	5.31	5.05	4.04	5.34
0	1	0	0	4.02	5.73	4.89	5.72	5.60	5.19	5.73
0	1	0	1	4.03	5.59	4.83	5.58	5.43	5.01	5.59
0	1	1	0	3.94	5.75	4.94	5.74	5.62	5.22	5.74
0	1	1	1	3.96	5.60	4.87	5.59	5.46	5.05	5.60
1	0	0	0	2.62	4.85	3.44	4.62	4.38	3.52	4.74
1	0	0	1	2.68	4.81	3.70	4.62	4.33	3.60	4.71
1	0	1	0	2.73	5.02	3.52	4.80	4.60	3.62	4.91
1	0	1	1	2.75	4.98	3.76	4.79	4.53	3.68	4.89
1	1	0	0	4.25	6.16	5.13	5.99	5.69	5.21	6.09
1	1	0	1	4.29	6.12	5.17	5.98	5.61	5.20	6.05
1	1	1	0	4.19	6.31	5.27	6.15	5.90	5.37	6.24
1	1	1	1	4.24	6.26	5.27	6.13	5.79	5.30	6.20
				3.43	5.55	4.39	5.44	5.20	4.49	5.50

Table A.19: Average biregional-scale realistic auction revenue (revenue2), in billions of dollars. The final row represents averages across all parameter sets.

				Average Realistic Auction Revenue (Revenue2)						
				Lexicographic		SF		Comp	Mixed	
ValVar	nB	Dem	CInc	IFA	SMR	IFA	SMR	IFA	IFA	SMR
0	0	0	0	3.08	5.43	3.88	5.33	5.18	4.16	5.39
0	0	0	1	3.09	5.33	4.38	5.24	5.03	4.29	5.29
0	0	1	0	3.06	5.48	4.00	5.39	5.25	4.23	5.44
0	0	1	1	3.06	5.38	4.52	5.31	5.12	4.38	5.34
0	1	0	0	4.34	5.73	5.16	5.72	5.62	5.40	5.73
0	1	0	1	4.43	5.59	5.19	5.58	5.45	5.27	5.59
0	1	1	0	4.19	5.75	5.25	5.74	5.64	5.45	5.74
0	1	1	1	4.25	5.60	5.29	5.59	5.48	5.33	5.60
1	0	0	0	2.91	4.85	3.74	4.62	4.42	3.76	4.74
1	0	0	1	2.93	4.81	4.01	4.62	4.38	3.86	4.71
1	0	1	0	2.94	5.02	3.92	4.80	4.67	3.91	4.91
1	0	1	1	2.95	4.98	4.18	4.79	4.61	4.01	4.89
1	1	0	0	4.50	6.16	5.41	5.99	5.72	5.43	6.09
1	1	0	1	4.53	6.12	5.48	5.98	5.65	5.45	6.05
1	1	1	0	4.39	6.31	5.64	6.15	5.95	5.64	6.24
1	1	1	1	4.44	6.26	5.68	6.13	5.84	5.63	6.20
				3.69	5.55	4.73	5.44	5.25	4.76	5.50

Table A.20: Average biregional-scale generous auction revenue (revenue3), in billions of dollars. The final row represents averages across all parameter sets.

				Average Generous Auction Revenue (Revenue3)						
				Lexicographic		SF		Comp	Mixed	
ValVar	nB	Dem	CInc	IFA	SMR	IFA	SMR	IFA	IFA	SMR
0	0	0	0	3.14	5.43	4.11	5.33	5.25	4.42	5.39
0	0	0	1	3.16	5.33	4.68	5.24	5.12	4.59	5.29
0	0	1	0	3.08	5.48	4.18	5.39	5.31	4.46	5.44
0	0	1	1	3.09	5.38	4.76	5.31	5.20	4.64	5.34
0	1	0	0	4.50	5.73	5.39	5.72	5.65	5.57	5.73
0	1	0	1	4.62	5.59	5.46	5.58	5.50	5.47	5.59
0	1	1	0	4.22	5.75	5.42	5.74	5.68	5.58	5.74
0	1	1	1	4.30	5.60	5.49	5.59	5.52	5.48	5.60
1	0	0	0	2.97	4.85	3.88	4.62	4.44	3.89	4.74
1	0	0	1	2.98	4.81	4.16	4.62	4.42	4.01	4.71
1	0	1	0	2.96	5.02	4.02	4.80	4.70	4.02	4.91
1	0	1	1	2.97	4.98	4.31	4.79	4.66	4.14	4.89
1	1	0	0	4.58	6.16	5.56	5.99	5.75	5.58	6.09
1	1	0	1	4.60	6.12	5.65	5.98	5.69	5.62	6.05
1	1	1	0	4.42	6.31	5.77	6.15	5.97	5.76	6.24
1	1	1	1	4.47	6.26	5.82	6.13	5.88	5.76	6.20
				3.76	5.55	4.92	5.44	5.30	4.94	5.50

Table A.21: Average biregional-scale optimistic auction revenue (revenue4), in billions of dollars. The final row represents averages across all parameter sets.

				Average Optimistic Auction Revenue (Revenue4)						
				Lexicographic		SF		Comp	Mixed	
0	0	0	0	3.19	5.43	4.16	5.33	5.25	4.48	5.39
0	0	0	1	3.21	5.33	4.76	5.24	5.13	4.68	5.29
0	0	1	0	3.11	5.48	4.23	5.39	5.32	4.52	5.44
0	0	1	1	3.12	5.38	4.83	5.31	5.21	4.73	5.34
0	1	0	0	4.54	5.73	5.44	5.72	5.65	5.61	5.73
0	1	0	1	4.69	5.59	5.54	5.58	5.50	5.53	5.59
0	1	1	0	4.25	5.75	5.47	5.74	5.68	5.63	5.74
0	1	1	1	4.34	5.60	5.56	5.59	5.52	5.55	5.60
1	0	0	0	3.01	4.85	3.92	4.62	4.44	3.93	4.74
1	0	0	1	3.02	4.81	4.20	4.62	4.42	4.06	4.71
1	0	1	0	2.99	5.02	4.06	4.80	4.70	4.06	4.91
1	0	1	1	3.00	4.98	4.36	4.79	4.66	4.20	4.89
1	1	0	0	4.60	6.16	5.60	5.99	5.75	5.62	6.09
1	1	0	1	4.63	6.12	5.70	5.98	5.69	5.67	6.05
1	1	1	0	4.44	6.31	5.80	6.15	5.97	5.80	6.24
1	1	1	1	4.49	6.26	5.88	6.13	5.89	5.82	6.20
				3.79	5.55	4.97	5.44	5.30	4.99	5.50

Table A.22: Average national-scale pessimistic auction revenue (revenue1), in billions of dollars. The final row represents averages across all parameter sets.

				Average Pessimistic Auction Revenue (Revenue1)						
				Lexicographic		SF		Comp	Mixed	
ValVar	nB	Dem	CInc	IFA	SMR	IFA	SMR	IFA	IFA	SMR
0	0	0	0	11.8	26.1	25.0	26.1	26.0	21.3	26.1
0	0	0	1	12.0	25.5	24.3	25.5	25.2	20.9	25.5
0	0	1	0	11.4	26.1	24.9	26.1	26.0	21.1	26.1
0	0	1	1	11.7	25.6	24.2	25.5	25.3	20.8	25.6
0	1	0	0	14.0	26.3	26.0	26.3	26.2	24.9	26.3
0	1	0	1	14.5	25.6	25.3	25.6	25.5	24.3	25.6
0	1	1	0	13.9	26.4	26.0	26.3	26.3	24.9	26.4
0	1	1	1	14.5	25.7	25.3	25.7	25.5	24.4	25.7
1	0	0	0	13.7	30.0	24.9	30.2	28.9	21.0	30.1
1	0	0	1	14.1	29.9	24.9	30.1	28.3	20.6	30.0
1	0	1	0	13.9	30.1	25.7	30.3	29.0	21.5	30.2
1	0	1	1	14.1	29.9	25.1	30.2	28.6	20.5	30.1
1	1	0	0	17.8	33.5	30.0	33.5	32.9	26.8	33.5
1	1	0	1	18.2	33.2	29.5	33.2	32.4	26.2	33.2
1	1	1	0	18.1	33.6	30.3	33.7	33.0	26.9	33.7
1	1	1	1	18.4	33.3	29.7	33.4	32.5	26.3	33.4
				14.5	28.8	26.3	28.9	28.2	23.3	28.8

Table A.23: Average national-scale realistic auction revenue (revenue2), in billions of dollars. The final row represents averages across all parameter sets.

				Average Optimistic Auction Revenue (Revenue2)						
				Lexicographic		SF		Comp	Mixed	
ValVar	nB	Dem	CInc	IFA	SMR	IFA	SMR	IFA	IFA	SMR
0	0	0	0	12.3	26.1	25.6	26.1	26.0	22.8	26.1
0	0	0	1	12.5	25.5	25.0	25.5	25.3	22.7	25.5
0	0	1	0	11.6	26.1	25.6	26.1	26.1	23.2	26.1
0	0	1	1	11.9	25.6	24.9	25.5	25.4	22.6	25.6
0	1	0	0	14.5	26.3	26.2	26.3	26.2	25.5	26.3
0	1	0	1	15.1	25.6	25.5	25.6	25.5	24.9	25.6
0	1	1	0	14.1	26.4	26.2	26.3	26.3	25.6	26.4
0	1	1	1	14.7	25.7	25.5	25.7	25.6	25.0	25.7
1	0	0	0	14.4	30.0	28.5	30.2	29.8	24.2	30.1
1	0	0	1	14.7	29.9	28.9	30.1	29.6	24.2	30.0
1	0	1	0	14.3	30.1	28.9	30.3	30.2	24.7	30.2
1	0	1	1	14.4	29.9	29.0	30.2	29.9	24.4	30.1
1	1	0	0	18.5	33.5	32.7	33.5	33.2	29.2	33.5
1	1	0	1	19.0	33.2	32.7	33.2	32.8	29.3	33.2
1	1	1	0	18.4	33.6	32.8	33.7	33.4	29.4	33.7
1	1	1	1	18.8	33.3	32.8	33.4	33.0	29.4	33.4
				15.0	28.8	28.2	28.9	28.6	25.5	28.8

Table A.24: Average national-scale generous auction revenue (revenue3), in billions of dollars. The final row represents averages across all parameter sets.

				Average Generous Auction Revenue (Revenue3)						
				Lexicographic		SF		Comp	Mixed	
ValVar	nB	Dem	CInc	IFA	SMR	IFA	SMR	IFA	IFA	SMR
0	0	0	0	12.3	26.1	26.0	26.1	26.1	24.8	26.1
0	0	0	1	12.6	25.5	25.4	25.5	25.4	24.5	25.5
0	0	1	0	11.6	26.1	26.0	26.1	26.1	24.8	26.1
0	0	1	1	11.9	25.6	25.4	25.5	25.5	24.5	25.6
0	1	0	0	14.6	26.3	26.3	26.3	26.3	26.0	26.3
0	1	0	1	15.2	25.6	25.6	25.6	25.6	25.3	25.6
0	1	1	0	14.1	26.4	26.3	26.3	26.3	26.0	26.4
0	1	1	1	14.7	25.7	25.6	25.7	25.6	25.4	25.7
1	0	0	0	14.5	30.0	30.2	30.2	30.1	25.7	30.1
1	0	0	1	14.8	29.9	30.4	30.1	30.0	25.9	30.0
1	0	1	0	14.3	30.1	30.4	30.3	30.5	26.1	30.2
1	0	1	1	14.4	29.9	30.5	30.2	30.2	26.1	30.1
1	1	0	0	18.6	33.5	33.5	33.5	33.3	30.2	33.5
1	1	0	1	19.0	33.2	33.4	33.2	33.0	30.2	33.2
1	1	1	0	18.4	33.6	33.5	33.7	33.5	30.3	33.7
1	1	1	1	18.8	33.3	33.4	33.4	33.2	30.4	33.4
				15.0	28.8	28.9	28.9	28.8	26.6	28.8

Table A.25: Average national-scale optimistic auction revenue (revenue4), in billions of dollars. The final row represents averages across all parameter sets.

				Average Optimistic Auction Revenue (Revenue4)						
				Lexicographic		SF		Comp	Mixed	
ValVar	nB	Dem	CInc	IFA	SMR	IFA	SMR	IFA	IFA	SMR
0	0	0	0	12.3	26.1	26.1	26.1	26.1	25.3	26.1
0	0	0	1	12.7	25.5	25.5	25.5	25.4	25.0	25.5
0	0	1	0	11.6	26.1	26.2	26.1	26.1	25.3	26.1
0	0	1	1	11.9	25.6	25.5	25.5	25.5	25.0	25.6
0	1	0	0	14.6	26.3	26.3	26.3	26.3	26.1	26.3
0	1	0	1	15.3	25.6	25.6	25.6	25.6	25.5	25.6
0	1	1	0	14.2	26.4	26.3	26.3	26.3	26.1	26.4
0	1	1	1	14.8	25.7	25.6	25.7	25.6	25.5	25.7
1	0	0	0	14.5	30.0	30.7	30.2	30.1	26.1	30.1
1	0	0	1	14.8	29.9	31.0	30.1	30.0	26.4	30.0
1	0	1	0	14.3	30.1	30.9	30.3	30.5	26.4	30.2
1	0	1	1	14.4	29.9	31.1	30.2	30.3	26.6	30.1
1	1	0	0	18.6	33.5	33.8	33.5	33.3	30.4	33.5
1	1	0	1	19.1	33.2	33.7	33.2	33.0	30.6	33.2
1	1	1	0	18.5	33.6	33.8	33.7	33.5	30.5	33.7
1	1	1	1	18.9	33.3	33.8	33.4	33.2	30.7	33.4
				15.0	28.8	29.1	28.9	28.8	26.9	28.8

A.3 Direct Effects Models

Table A.26: Direct effects predicting the bidder bankruptcy percentage. All coefficients were significant at the 1% level. Scaled coefficients are percentages of the base value.

	Base	ValVar	nB	Dem	CInc	Lex	SF	Comp
Raw	1.23	−0.55	−0.38	0.03	0.09	0.02	0.33	−0.73
Scaled	—	−44.3	−30.4	2.1	7.5	1.7	26.4	−58.7

(a) Small scale

	Base	ValVar	nB	Dem	CInc	Lex	SF	Comp
Raw	7.78	0.03	−2.95	−0.28	0.43	1.43	1.54	−5.46
Scaled	—	0.4	−38.0	−3.6	5.5	18.4	19.8	−70.2

(b) Bi-regional scale

	Base	ValVar	nB	Dem	CInc	Lex	SF	Comp
Raw	5.98	3.63	−2.33	−0.07	0.49	−0.93	−1.53	−5.47
Scaled	—	60.7	−38.9	−1.2	8.2	−15.5	−25.5	−91.5

(c) National scale

Table A.27: Direct effects predicting the number of rounds per auction. All coefficients were significant at the 1% level. Scaled coefficients are percentages of the base value.

	Base	ValVar	nB	Dem	CInc	SMR	Lex	SF	Comp
Raw	35.8	2.5	−0.7	0.0	−17.6	2.9	0.6	−0.1	−0.8
Scaled	—	6.9	−2.0	0.1	−49.3	8.0	1.7	−0.2	−2.2

(a) Small scale

	Base	ValVar	nB	Dem	CInc	SMR	Lex	SF	Comp
Raw	155.0	−21.2	−3.6	−7.7	−93.2	181.5	−12.1	4.3	−16.9
Scaled	—	−13.7	−2.3	−5.0	−60.1	117.1	−7.8	2.8	−10.9

(b) Bi-regional scale

	Base	ValVar	nB	Dem	CInc	SMR	Lex	SF	Comp
Raw	190.9	−7.9	−23.6	−8.6	−113.6	213.3	11.4	−16.4	−44.2
Scaled	—-	−4.2	−12.4	−4.5	−59.5	111.7	6.0	−8.6	−23.1

(c) National scale

Table A.28: Direct effects predicting pessimistic auction surplus (Surplus1). All coefficients were significant at the 1% level. Scaled coefficients are percentages of the base value.

	Base	ValVar	nB	Dem	CInc	SMR	Lex	SF	Comp
Raw	29.22	9.43	1.76	0.39	−0.12	1.85	0.15	−0.30	1.01
Scaled	—	32.3	6.0	1.3	−0.4	6.3	0.5	−1.0	3.5

(a) Small scale (in millions of dollars)

	Base	ValVar	nB	Dem	CInc	SMR	Lex	SF	Comp
Raw	4892	1475	449	90	−40	814	−22	−40	591
Scaled	—	30.1	9.2	1.8	−0.8	16.6	−0.5	−0.8	12.1

(b) Bi-regional scale (in millions of dollars)

	Base	ValVar	nB	Dem	CInc	SMR	Lex	SF	Comp
Raw	22.00	8.69	1.78	0.20	−0.23	4.37	0.82	1.02	4.07
Scaled	—	39.5	8.1	0.9	−1.0	19.9	3.7	4.6	18.5

(c) National scale (in billions of dollars)

Table A.29: Direct effects predicting optimistic auction surplus (Surplus2). Bold values indicate that the coefficient is significant at the 1% level. Scaled coefficients are percentages of the base value.

	Base	ValVar	nB	Dem	CInc	SMR	Lex	SF	Comp
Raw	**30.87**	**9.08**	**1.68**	**0.38**	**−0.07**	**0.32**	−0.01	**0.09**	**−0.20**
Scaled	—	**29.4**	**5.4**	**1.2**	**−0.2**	**1.0**	−0.0	**0.3**	**−0.7**

(a) Small scale (in millions of dollars)

	Base	ValVar	nB	Dem	CInc	SMR	Lex	SF	Comp
Raw	**5619**	**1492**	**354**	**64**	**−8**	**115**	**−50**	**12**	**22**
Scaled	—	**26.6**	**6.3**	**1.1**	**−0.1**	**2.0**	**−0.9**	**0.2**	**0.4**

(b) Bi-regional scale (in millions of dollars)

	Base	ValVar	nB	Dem	CInc	SMR	Lex	SF	Comp
Raw	**25.35**	**10.11**	**1.01**	**0.09**	**−0.08**	**1.44**	**−0.90**	**0.37**	**1.02**
Scaled	—	**39.9**	**4.0**	**0.4**	**−0.3**	**5.7**	**−3.5**	**1.5**	**4.0**

(c) National scale (in billions of dollars)

Table A.30: Direct effects predicting pessimistic auction revenue (Revenue1). All coefficients were significant at the 1% level. Scaled coefficients are percentages of the base value.

	Base	ValVar	nB	Dem	CInc	SMR	Lex	SF	Comp
Raw	**25.0**	**1.1**	**4.8**	**0.6**	**−0.5**	**3.1**	**−0.5**	**0.1**	**1.0**
Scaled	—	**4.5**	**19.2**	**2.4**	**−2.2**	**12.3**	**−2.2**	**0.4**	**4.1**

(a) Small scale (in millions of dollars)

	Base	ValVar	nB	Dem	CInc	SMR	Lex	SF	Comp
Raw	**3760**	**−43**	**1057**	**85**	**−25**	**1392**	**−500**	**−80**	**909**
Scaled	—	**−1.2**	**28.1**	**2.2**	**−0.7**	**37.0**	**−13.3**	**−2.1**	**24.2**

(b) Bi-regional scale (in millions of dollars)

	Base	ValVar	nB	Dem	CInc	SMR	Lex	SF	Comp
Raw	**19.1**	**4.1**	**2.6**	**0.1**	**−0.4**	**7.5**	**−4.4**	**1.5**	**5.9**
Scaled	—	**21.6**	**13.7**	**0.4**	**−1.9**	**39.1**	**−23.1**	**8.0**	**30.9**

(c) National scale (in billions of dollars)

Table A.31: Direct effects predicting realistic auction revenue (Revenue2). Bold values indicate that the coefficient is significant at the 1% level. Scaled coefficients are percentages of the base.

	Base	ValVar	nB	Dem	CInc	SMR	Lex	SF	Comp
Raw	**25.6**	**0.9**	**4.8**	**0.6**	**−0.5**	**2.5**	**−0.6**	**0.3**	**0.5**
Scaled	—	**3.7**	**18.6**	**2.5**	**−2.1**	**9.7**	**−2.3**	**1.0**	**2.1**

(a) Small scale (in millions of dollars)

	Base	ValVar	nB	Dem	CInc	SMR	Lex	SF	Comp
Raw	**4038**	**−49**	**1049**	**91**	**−7**	**1099**	**−507**	**−44**	**670**
Scaled	—	**−1.2**	**26.0**	**2.2**	**−0.2**	**27.2**	**−12.6**	**−1.1**	**16.6**

(b) Bi-regional scale (in millions of dollars)

	Base	ValVar	nB	Dem	CInc	SMR	Lex	SF	Comp
Raw	**20.6**	**4.9**	**2.3**	0.0	**−0.3**	**6.0**	**−5.3**	**1.4**	**4.5**
Scaled	—	**23.9**	**11.3**	0.2	**−1.4**	**28.9**	**−25.5**	**6.6**	**21.7**

(c) National scale (in billions of dollars)

Table A.32: Direct effects predicting generous auction revenue (Revenue3). Bold values indicate that the coefficient is significant at the 1% level. Scaled coefficients are percentages of the base.

	Base	ValVar	nB	Dem	CInc	SMR	Lex	SF	Comp
Raw	**26.4**	**0.7**	**4.8**	**0.6**	**−0.5**	**1.8**	**−0.7**	**0.4**	0.0
Scaled	—	**2.7**	**18.0**	**2.3**	**−1.9**	**6.9**	**−2.6**	**1.6**	0.1

(a) Small scale (in millions of dollars)

	Base	ValVar	nB	Dem	CInc	SMR	Lex	SF	Comp
Raw	**4217**	**−81**	**1045**	**71**	**4**	**959**	**−563**	**−39**	**560**
Scaled	—	**−1.9**	**24.8**	**1.7**	**0.1**	**22.8**	**−13.4**	**−0.9**	**13.3**

(b) Bi-regional scale (in millions of dollars)

	Base	ValVar	nB	Dem	CInc	SMR	Lex	SF	Comp
Raw	**21.6**	**5.1**	**2.1**	0.0	**−0.3**	**5.3**	**−5.9**	**1.1**	**3.7**
Scaled	—	**23.6**	**9.7**	0.1	**−1.2**	**24.7**	**−27.1**	**5.2**	**17.2**

(c) National scale (in billions of dollars)

Table A.33: Direct effects predicting optimistic auction revenue (Revenue4). Bold values indicate that the coefficient is significant at the 1% level. Scaled coefficients are percentages of the base.

	Base	ValVar	nB	Dem	CInc	SMR	Lex	SF	Comp
Raw	**26.5**	**0.7**	**4.7**	**0.6**	**−0.5**	**1.7**	**−0.7**	**0.4**	**−0.0**
Scaled	—	**2.6**	**17.9**	**2.3**	**−1.9**	**6.6**	**−2.6**	**1.7**	**−0.2**

(a) Small scale (in millions of dollars)

	Base	ValVar	nB	Dem	CInc	SMR	Lex	SF	Comp
Raw	**4272**	**−88**	**1043**	**69**	**11**	**911**	**−574**	**−41**	**510**
Scaled	—	**−2.1**	**24.4**	**1.6**	**0.2**	**21.3**	**−13.4**	**−1.0**	**11.9**

(b) Bi-regional scale (in millions of dollars)

	Base	ValVar	nB	Dem	CInc	SMR	Lex	SF	Comp
Raw	**21.9**	**5.2**	**2.0**	0.0	**−0.2**	**5.1**	**−6.0**	**1.1**	**3.5**
Scaled	—	**23.6**	**9.3**	0.1	**−1.1**	**23.5**	**−27.4**	**5.0**	**15.8**

(c) National scale (in billions of dollars)

A.4 Two-Way Interaction Models

Table A.34: Two-way interaction effects predicting the bidder bankruptcy percentage. Non-constant coefficients are represented as a percent of the constant coefficient. Bold values indicate that the coefficient is significant at the 1% level.

	Base	ValVar	nB	Dem	CInc	Lex	SF	Comp
Base	**1.30**	−49.9	**−42.5**	2.0	**16.3**	−1.1	**43.7**	−86.8
ValVar	−49.9	—	**21.8**	**3.0**	−19.1	1.0	−19.1	**37.9**
nB	−42.5	**21.8**	—	0.1	−0.1	**4.2**	**−20.3**	**26.6**
Dem	2.0	**3.0**	0.1	—	−1.2	**−3.1**	2.3	**−3.3**
CInc	**16.3**	−19.1	−0.1	−1.2	—	**3.3**	−0.1	0.9
Lex	−1.1	1.0	**4.2**	**−3.1**	**3.3**	—	—	—
SF	**43.7**	−19.1	**−20.3**	2.3	−0.1	—	—	—
Comp	−86.8	**37.9**	**26.6**	**−3.3**	0.9	—	—	—

(a) Small scale

	Base	ValVar	nB	Dem	CInc	Lex	SF	Comp
Base	**7.51**	**3.8**	−48.7	−4.5	**13.5**	**58.3**	**20.8**	**−86.3**
ValVar	**3.8**	—	−0.5	**15.5**	**−10.3**	**−20.9**	−1.0	**−1.0**
nB	−48.7	−0.5	—	0.5	**2.6**	**4.9**	**−8.6**	**36.1**
Dem	−4.5	**15.5**	0.5	—	**4.1**	**−47.9**	**10.6**	0.3
CInc	**13.5**	**−10.3**	**2.6**	**4.1**	—	**−14.5**	**−1.6**	**−8.0**
Lex	**58.3**	**−20.9**	**4.9**	**−47.9**	**−14.5**	—	—	—
SF	**20.8**	−1.0	**−8.6**	**10.6**	**−1.6**	—	—	—
Comp	**−86.3**	**−1.0**	**36.1**	0.3	**−8.0**	—	—	—

(b) Bi-regional scale

	Base	ValVar	nB	Dem	CInc	Lex	SF	Comp
Base	**4.97**	**110.0**	**−67.6**	**22.2**	**14.6**	**47.7**	**−39.2**	**−93.9**
ValVar	**110.0**	—	**−24.9**	**10.9**	**14.8**	**−90.4**	**4.4**	**−63.4**
nB	**−67.6**	**−24.9**	—	**−5.8**	**−5.3**	**63.7**	**30.1**	**60.9**
Dem	**22.2**	**10.9**	**−5.8**	—	**2.8**	**−79.9**	**−16.8**	**−13.6**
CInc	**14.6**	**14.8**	**−5.3**	**2.8**	—	**−26.1**	−0.9	**−16.3**
Lex	**47.7**	**−90.4**	**63.7**	**−79.9**	**−26.1**	—	—	—
SF	**−39.2**	**4.4**	**30.1**	**−16.8**	−0.9	—	—	—
Comp	**−93.9**	**−63.4**	**60.9**	**−13.6**	**−16.3**	—	—	—

(c) National scale

Table A.35: Two-way interaction effects predicting the number of rounds per auction. Non-constant coefficients are represented as a percent of the constant coefficient. Bold values indicate that the coefficient is significant at the 1% level.

	Base	ValVar	nB	Dem	CInc	SMR	Lex	SF	Comp
Base	**35.6**	**2.3**	**−3.2**	**−0.2**	**−46.0**	**11.9**	**5.2**	**−1.1**	**−3.2**
ValVar	**2.3**	—	**6.6**	**1.2**	**−4.0**	**7.0**	**−1.4**	**0.8**	**−0.4**
nB	**−3.2**	**6.6**	—	**0.5**	**1.1**	**−6.5**	**−1.5**	**0.3**	**1.8**
Dem	**−0.2**	**1.2**	**0.5**	—	**−0.1**	**−1.6**	**0.4**	0.1	0.1
CInc	**−46.0**	**−4.0**	**1.1**	**−0.1**	—	**−4.6**	**−1.0**	0.1	**1.4**
SMR	**11.9**	**7.0**	**−6.5**	**−1.6**	**−4.6**	—	**−3.4**	**0.4**	—
Lex	**5.2**	**−1.4**	**−1.5**	**0.4**	**−1.0**	**−3.4**	—	—	—
SF	**−1.1**	**0.8**	**0.3**	0.1	0.1	**0.4**	—	—	—
Comp	**−3.2**	**−0.4**	**1.8**	0.1	**1.4**	—	—	—	—

(a) Small scale

	Base	ValVar	nB	Dem	CInc	SMR	Lex	SF	Comp
Base	**118.6**	**−18.0**	**17.3**	**4.9**	**−54.6**	**241.0**	**−28.4**	**7.6**	**−11.3**
ValVar	**−18.0**	—	**7.8**	**2.7**	**12.0**	**−27.7**	**1.7**	0.7	0.6
nB	**17.3**	**7.8**	—	**−11.2**	**5.2**	**−50.9**	**10.8**	**−1.4**	**−14.6**
Dem	**4.9**	**2.7**	**−11.2**	—	**4.7**	**−19.7**	**−1.8**	0.3	**−4.7**
CInc	**−54.6**	**12.0**	**5.2**	**4.7**	—	**−87.0**	**4.9**	**−1.1**	**8.0**
SMR	**241.0**	**−27.7**	**−50.9**	**−19.7**	**−87.0**	—	**20.7**	**−6.5**	—
Lex	**−28.4**	**1.7**	**10.8**	**−1.8**	**4.9**	**20.7**	—	—	—
SF	**7.6**	0.7	**−1.4**	0.3	**−1.1**	**−6.5**	—	—	—
Comp	**−11.3**	0.6	**−14.6**	**−4.7**	**8.0**	—	—	—	—

(b) Bi-regional scale

	Base	ValVar	nB	Dem	CInc	SMR	Lex	SF	Comp
Base	**161.7**	**−9.0**	**−5.6**	**−5.7**	**−49.4**	**202.4**	**−20.4**	0.8	**−36.2**
ValVar	**−9.0**	—	**1.9**	**6.2**	**2.2**	**−11.1**	**20.0**	**−8.8**	**2.6**
nB	**−5.6**	**1.9**	—	0.2	**12.0**	**−43.8**	**4.3**	**5.3**	**−3.5**
Dem	**−5.7**	**6.2**	0.2	—	**3.0**	**−13.5**	**6.0**	−1.0	−0.2
CInc	**−49.4**	**2.2**	**12.0**	**3.0**	—	**−76.0**	**−6.6**	**7.7**	**16.9**
SMR	**202.4**	**−11.1**	**−43.8**	**−13.5**	**−76.0**	—	**31.4**	**−25.2**	—
Lex	**−20.4**	**20.0**	**4.3**	**6.0**	**−6.6**	**31.4**	—	—	—
SF	0.8	**−8.8**	**5.3**	−1.0	**7.7**	**−25.2**	—	—	—
Comp	**−36.2**	**2.6**	**−3.5**	−0.2	**16.9**	—	—	—	—

(c) National scale

Table A.36: Two-way interaction effects predicting pessimistic auction surplus (Surplus1). Bold values indicate that the coefficient is significant at the 1% level.

	Base	ValVar	nB	Dem	CInc	SMR	Lex	SF	Comp
Base	**30.2**	**25.5**	**0.6**	**0.6**	**−0.5**	**5.2**	**1.3**	**−2.9**	**4.9**
ValVar	**25.5**	—	**9.6**	**2.1**	**0.6**	**−0.6**	**−0.4**	**1.2**	**−2.5**
nB	**0.6**	**9.6**	—	**−0.5**	−0.1	**1.9**	**−0.3**	**0.4**	**−0.5**
Dem	**0.6**	**2.1**	**−0.5**	—	**−0.1**	**−0.3**	0.1	0.0	0.1
CInc	**−0.5**	**0.6**	−0.1	**−0.1**	—	0.1	0.0	0.1	**−0.5**
SMR	**5.2**	**−0.6**	**1.9**	**−0.3**	0.1	—	**−1.0**	**2.0**	—
Lex	**1.3**	**−0.4**	**−0.3**	0.1	0.0	**−1.0**	—	—	—
SF	**−2.9**	**1.2**	**0.4**	0.0	0.1	**2.0**	—	—	—
Comp	**4.9**	**−2.5**	**−0.5**	0.1	**−0.5**	—	—	—	—

(a) Small scale (in millions of dollars)

	Base	ValVar	nB	Dem	CInc	SMR	Lex	SF	Comp
Base	**5046**	**20.8**	**6.7**	**1.6**	**−1.5**	**16.5**	**−2.3**	**−1.1**	**13.2**
ValVar	**20.8**	—	**11.8**	**1.2**	**0.9**	**3.0**	0.0	**0.5**	**0.6**
nB	**6.7**	**11.8**	—	**−0.4**	**−0.6**	**−4.7**	**−1.0**	**−1.1**	**−4.1**
Dem	**1.6**	**1.2**	**−0.4**	—	**−0.3**	**−1.4**	**2.6**	**−0.2**	**−0.7**
CInc	**−1.5**	**0.9**	**−0.6**	**−0.3**	—	**0.8**	**1.0**	**0.1**	**0.3**
SMR	**16.5**	**3.0**	**−4.7**	**−1.4**	**0.8**	—	**1.2**	**1.4**	—
Lex	**−2.3**	0.0	**−1.0**	**2.6**	**1.0**	**1.2**	—	—	—
SF	**−1.1**	**0.5**	**−1.1**	**−0.2**	**0.1**	**1.4**	—	—	—
Comp	**13.2**	**0.6**	**−4.1**	**−0.7**	**0.3**	—	—	—	—

(b) Bi-regional scale (in millions of dollars)

	Base	ValVar	nB	Dem	CInc	SMR	Lex	SF	Comp
Base	**22.4**	**23.8**	**11.3**	**0.5**	**−1.8**	**20.6**	**6.3**	**10.2**	**18.6**
ValVar	**23.8**	—	**8.0**	**0.9**	**−0.5**	**18.0**	**4.3**	−0.2	**13.7**
nB	**11.3**	**8.0**	—	0.0	0.0	**−9.0**	**−4.6**	**−3.3**	**−8.7**
Dem	**0.5**	**0.9**	0.0	—	−0.2	**−1.1**	**1.7**	**0.6**	**−0.7**
CInc	**−1.8**	**−0.5**	0.0	−0.2	—	**1.2**	**1.5**	0.3	0.5
SMR	**20.6**	**18.0**	**−9.0**	**−1.1**	**1.2**	—	**−8.1**	**−8.5**	—
Lex	**6.3**	**4.3**	**−4.6**	**1.7**	**1.5**	**−8.1**	—	—	—
SF	**10.2**	−0.2	**−3.3**	**0.6**	0.3	**−8.5**	—	—	—
Comp	**18.6**	**13.7**	**−8.7**	**−0.7**	0.5	—	—	—	—

(c) National scale (in billions of dollars)

Table A.37: Two-way interaction effects predicting optimistic auction surplus (Surplus2). Bold values indicate that the coefficient is significant at the 1% level.

	Base	ValVar	nB	Dem	CInc	SMR	Lex	SF	Comp
Base	**32.0**	**21.9**	**−0.1**	**0.5**	**−0.2**	**−0.7**	**−0.2**	**0.4**	0.0
ValVar	**21.9**	—	**9.4**	**2.0**	**0.1**	**1.7**	−0.0	**0.4**	**−1.0**
nB	**−0.1**	**9.4**	—	**−0.5**	**−0.1**	**2.3**	0.1	−0.1	**−0.2**
Dem	**0.5**	**2.0**	**−0.5**	—	−0.1	**−0.2**	**0.1**	−0.0	−0.1
CInc	**−0.2**	**0.1**	**−0.1**	−0.1	—	−0.1	−0.0	−0.0	0.1
SMR	**−0.7**	**1.7**	**2.3**	**−0.2**	−0.1	—	**0.1**	**−0.5**	—
Lex	**−0.2**	−0.0	0.1	**0.1**	−0.0	**0.1**	—	—	—
SF	**0.4**	**0.4**	−0.1	−0.0	−0.0	**−0.5**	—	—	—
Comp	0.0	**−1.0**	**−0.2**	−0.1	0.1	—	—	—	—

(a) Small scale (in millions of dollars)

	Base	ValVar	nB	Dem	CInc	SMR	Lex	SF	Comp
Base	**5806**	**18.7**	**1.8**	**0.3**	0.0	**1.1**	**−1.4**	**0.3**	**0.9**
ValVar	**18.7**	—	**10.2**	**2.1**	**0.3**	**2.0**	**−0.9**	**0.5**	**−0.2**
nB	**1.8**	**10.2**	—	**−0.3**	**−0.2**	**−0.8**	**0.2**	**−0.3**	**−1.0**
Dem	**0.3**	**2.1**	**−0.3**	—	**−0.1**	**−0.1**	**−0.1**	0.0	**0.1**
CInc	0.0	**0.3**	**−0.2**	**−0.1**	—	**−0.4**	0.0	**0.1**	**−0.4**
SMR	**1.1**	**2.0**	**−0.8**	**−0.1**	**−0.4**	—	**2.0**	**−0.6**	—
Lex	**−1.4**	**−0.9**	**0.2**	**−0.1**	0.0	**2.0**	—	—	—
SF	**0.3**	**0.5**	**−0.3**	0.0	**0.1**	**−0.6**	—	—	—
Comp	**0.9**	**−0.2**	**−1.0**	**0.1**	**−0.4**	—	—	—	—

(b) Bi-regional scale (in millions of dollars)

	Base	ValVar	nB	Dem	CInc	SMR	Lex	SF	Comp
Base	**26.2**	**32.7**	**1.9**	**0.1**	**−0.4**	**3.0**	**−6.3**	**1.7**	**2.2**
ValVar	**32.7**	—	**5.6**	**0.6**	**0.5**	**4.2**	**−1.9**	**2.6**	**4.0**
nB	**1.9**	**5.6**	—	−0.0	**−0.1**	**−1.7**	**1.5**	**−0.6**	**−1.6**
Dem	**0.1**	**0.6**	−0.0	—	−0.0	**0.1**	**−0.3**	−0.0	−0.1
CInc	**−0.4**	**0.5**	**−0.1**	−0.0	—	−0.0	**0.2**	**−0.2**	**−0.3**
SMR	**3.0**	**4.2**	**−1.7**	**0.1**	−0.0	—	**6.2**	**−2.3**	—
Lex	**−6.3**	**−1.9**	**1.5**	**−0.3**	**0.2**	**6.2**	—	—	—
SF	**1.7**	**2.6**	**−0.6**	−0.0	**−0.2**	**−2.3**	—	—	—
Comp	**2.2**	**4.0**	**−1.6**	−0.1	**−0.3**	—	—	—	—

(c) National scale (in billions of dollars)

Table A.38: Two-way interaction effects predicting pessimistic auction revenue (Revenue1). Non-constant coefficients are represented as a percent of the constant coefficient. Bold values indicate that the coefficient is significant at the 1% level

	Base	**ValVar**	**nB**	**Dem**	**CInc**	**SMR**	**Lex**	**SF**	**Comp**
Base	**27.1**	**−8.2**	**7.6**	**1.0**	**−2.8**	**5.2**	**−4.6**	0.1	**5.5**
ValVar	**−8.2**	—	**16.2**	**3.5**	**1.3**	**4.5**	**−1.1**	**2.0**	**−2.3**
nB	**7.6**	**16.2**	—	**−0.5**	**−0.3**	**5.0**	**2.4**	**−0.8**	**−1.9**
Dem	**1.0**	**3.5**	**−0.5**	—	**−0.2**	**−0.3**	−0.1	−0.0	0.1
CInc	**−2.8**	**1.3**	**−0.3**	**−0.2**	—	**1.0**	−0.1	**0.2**	**−0.6**
SMR	**5.2**	**4.5**	**5.0**	**−0.3**	**1.0**	—	**4.0**	**−0.7**	—
Lex	**−4.6**	**−1.1**	**2.4**	−0.1	−0.1	**4.0**	—	—	—
SF	0.1	**2.0**	**−0.8**	−0.0	**0.2**	**−0.7**	—	—	—
Comp	**5.5**	**−2.3**	**−1.9**	0.1	**−0.6**	—	—	—	—

(a) Small scale (in millions of dollars)

	Base	**ValVar**	**nB**	**Dem**	**CInc**	**SMR**	**Lex**	**SF**	**Comp**
Base	**3968.0**	**−12.8**	**27.0**	**1.3**	**1.1**	**34.0**	**−27.3**	**−3.9**	**27.5**
ValVar	**−12.8**	—	**19.4**	**2.2**	**1.4**	**−1.0**	**3.0**	**1.0**	**−3.7**
nB	**27.0**	**19.4**	—	**−1.0**	**−2.4**	**−13.9**	**−1.1**	**−0.7**	**−13.1**
Dem	**1.3**	**2.2**	**−1.0**	—	**−0.3**	**1.0**	**−0.8**	0.0	**1.4**
CInc	**1.1**	**1.4**	**−2.4**	**−0.3**	—	**−3.3**	**0.4**	**2.3**	**−3.6**
SMR	**34.0**	**−1.0**	**−13.9**	**1.0**	**−3.3**	—	**28.0**	**1.1**	—
Lex	**−27.3**	**3.0**	**−1.1**	**−0.8**	**0.4**	**28.0**	—	—	—
SF	**−3.9**	**1.0**	**−0.7**	0.0	**2.3**	**1.1**	—	—	—
Comp	**27.5**	**−3.7**	**−13.1**	**1.4**	**−3.6**	—	—	—	—

(b) Bi-regional scale (in millions of dollars)

	Base	**ValVar**	**nB**	**Dem**	**CInc**	**SMR**	**Lex**	**SF**	**Comp**
Base	**21.5**	**−0.9**	**12.3**	**−0.5**	**−1.9**	**21.9**	**−43.0**	**14.2**	**19.9**
ValVar	**−0.9**	—	**13.8**	**0.8**	**1.1**	**17.4**	**4.7**	**3.6**	**16.0**
nB	**12.3**	**13.8**	—	0.2	**−0.4**	**−8.7**	**−2.8**	**−4.0**	**−9.1**
Dem	**−0.5**	**0.8**	0.2	—	−0.1	0.2	0.1	**0.5**	0.5
CInc	**−1.9**	**1.1**	**−0.4**	−0.1	—	**−1.0**	**1.9**	−0.2	**−1.3**
SMR	**21.9**	**17.4**	**−8.7**	0.2	**−1.0**	—	**40.7**	**−14.1**	—
Lex	**−43.0**	**4.7**	**−2.8**	0.1	**1.9**	**40.7**	—	—	—
SF	**14.2**	**3.6**	**−4.0**	**0.5**	−0.2	**−14.1**	—	—	—
Comp	**19.9**	**16.0**	**−9.1**	0.5	**−1.3**	—	—	—	—

(c) National scale (in billions of dollars)

Table A.39: Two-way interaction effects predicting realistic auction revenue (Revenue2). Non-constant coefficients are represented as a percent of the constant coefficient. Bold values indicate that the coefficient is significant at the 1% level

	Base	ValVar	nB	Dem	CInc	SMR	Lex	SF	Comp
Base	**27.8**	**−9.3**	**7.2**	**1.2**	**−2.5**	**2.5**	**−5.0**	**1.5**	**3.3**
ValVar	**−9.3**	—	**16.0**	**3.4**	**1.0**	**5.8**	**−0.9**	**1.6**	**−1.2**
nB	**7.2**	**16.0**	—	**−0.5**	**−0.3**	**5.0**	**2.5**	**−1.0**	**−1.8**
Dem	**1.2**	**3.4**	**−0.5**	—	**−0.2**	**−0.5**	−0.1	−0.1	−0.1
CInc	**−2.5**	**1.0**	**−0.3**	**−0.2**	—	**0.9**	−0.1	0.1	**−0.4**
SMR	**2.5**	**5.8**	**5.0**	**−0.5**	**0.9**	—	**4.3**	**−1.8**	—
Lex	**−5.0**	**−0.9**	**2.5**	−0.1	−0.1	**4.3**	—	—	—
SF	**1.5**	**1.6**	**−1.0**	−0.1	0.1	**−1.8**	—	—	—
Comp	**3.3**	**−1.2**	**−1.8**	−0.1	**−0.4**	—	—	—	—

(a) Small scale (in millions of dollars)

	Base	ValVar	nB	Dem	CInc	SMR	Lex	SF	Comp
Base	**4229.0**	**−12.2**	**24.7**	**1.3**	**2.1**	**25.7**	**−25.0**	**−2.4**	**20.8**
ValVar	**−12.2**	—	**18.1**	**2.7**	**0.9**	**−0.5**	**2.1**	**1.0**	**−3.2**
nB	**24.7**	**18.1**	—	**−0.9**	**−2.0**	**−12.8**	**−0.3**	**−0.4**	**−12.5**
Dem	**1.3**	**2.7**	**−0.9**	—	−0.1	**0.7**	**−2.2**	**0.5**	**1.1**
CInc	**2.1**	**0.9**	**−2.0**	−0.1	—	**−4.0**	−0.1	**2.3**	**−4.0**
SMR	**25.7**	**−0.5**	**−12.8**	**0.7**	**−4.0**	—	**26.6**	**−0.7**	—
Lex	**−25.0**	**2.1**	**−0.3**	**−2.2**	−0.1	**26.6**	—	—	—
SF	**−2.4**	**1.0**	**−0.4**	**0.5**	**2.3**	**−0.7**	—	—	—
Comp	**20.8**	**−3.2**	**−12.5**	**1.1**	**−4.0**	—	—	—	—

(b) Bi-regional scale (in millions of dollars)

	Base	ValVar	nB	Dem	CInc	SMR	Lex	SF	Comp
Base	**23.1**	**6.5**	**8.3**	**−0.4**	**−1.2**	**13.2**	**−46.0**	**10.3**	**12.2**
ValVar	**6.5**	—	**12.6**	**0.7**	**1.8**	**9.3**	**0.8**	**5.4**	**10.6**
nB	**8.3**	**12.6**	—	0.2	**−0.4**	**−5.8**	**−0.3**	**−2.9**	**−6.9**
Dem	**−0.4**	**0.7**	0.2	—	−0.0	**0.7**	**−1.0**	0.0	**0.7**
CInc	**−1.2**	**1.8**	**−0.4**	−0.0	—	**−1.6**	**1.3**	−0.2	**−1.8**
SMR	**13.2**	**9.3**	**−5.8**	**0.7**	**−1.6**	—	**45.4**	**−11.4**	—
Lex	**−46.0**	**0.8**	**−0.3**	**−1.0**	**1.3**	**45.4**	—	—	—
SF	**10.3**	**5.4**	**−2.9**	0.0	−0.2	**−11.4**	—	—	—
Comp	**12.2**	**10.6**	**−6.9**	**0.7**	**−1.8**	—	—	—	—

(c) National scale (in billions of dollars)

Table A.40: Two-way interaction effects predicting generous auction revenue (Revenue3). Non-constant coefficients are represented as a percent of the constant coefficient. Bold values indicate that the coefficient is significant at the 1% level

	Base	ValVar	nB	Dem	CInc	SMR	Lex	SF	Comp
Base	**28.7**	**−10.7**	**6.7**	**0.9**	**−2.4**	**−0.7**	**−5.7**	**3.1**	**0.7**
ValVar	**−10.7**	—	**15.8**	**3.4**	**0.8**	**7.2**	**−0.6**	**1.0**	**−0.2**
nB	**6.7**	**15.8**	—	**−0.4**	**−0.3**	**5.0**	**2.6**	**−1.2**	**−1.7**
Dem	**0.9**	**3.4**	**−0.4**	—	**−0.1**	**−0.3**	**−0.1**	−0.1	−0.1
CInc	**−2.4**	**0.8**	**−0.3**	**−0.1**	—	**0.9**	−0.1	0.0	0.0
SMR	**−0.7**	**7.2**	**5.0**	**−0.3**	**0.9**	—	**4.8**	**−3.0**	—
Lex	**−5.7**	**−0.6**	**2.6**	**−0.1**	−0.1	**4.8**	—	—	—
SF	**3.1**	**1.0**	**−1.2**	−0.1	0.0	**−3.0**	—	—	—
Comp	**0.7**	**−0.2**	**−1.7**	−0.1	0.0	—	—	—	—

(a) Small scale (in millions of dollars)

	Base	ValVar	nB	Dem	CInc	SMR	Lex	SF	Comp
Base	**4460.0**	**−13.5**	**22.8**	**0.4**	**2.6**	**19.4**	**−26.6**	**−2.2**	**16.0**
ValVar	**−13.5**	—	**17.6**	**2.8**	**0.7**	**0.9**	**2.5**	**0.8**	**−2.0**
nB	**22.8**	**17.6**	—	**−1.0**	**−2.0**	**−12.1**	**0.5**	0.1	**−11.8**
Dem	**0.4**	**2.8**	**−1.0**	—	**−0.1**	**1.7**	**−2.5**	**0.3**	**1.9**
CInc	**2.6**	**0.7**	**−2.0**	**−0.1**	—	**−4.3**	**−0.3**	**2.3**	**−4.0**
SMR	**19.4**	**0.9**	**−12.1**	**1.7**	**−4.3**	—	**27.7**	**−0.8**	—
Lex	**−26.6**	**2.5**	**0.5**	**−2.5**	**−0.3**	**27.7**	—	—	—
SF	**−2.2**	**0.8**	0.1	**0.3**	**2.3**	**−0.8**	—	—	—
Comp	**16.0**	**−2.0**	**−11.8**	**1.9**	**−4.0**	—	—	—	—

(b) Bi-regional scale (in millions of dollars)

	Base	ValVar	nB	Dem	CInc	SMR	Lex	SF	Comp
Base	**24.4**	**7.5**	**5.1**	**−0.3**	**−1.0**	**7.7**	**−48.7**	**7.2**	**6.7**
ValVar	**7.5**	—	**11.8**	**0.5**	**1.7**	**7.2**	**0.2**	**6.5**	**9.5**
nB	**5.1**	**11.8**	—	0.1	**−0.4**	**−3.5**	**1.5**	**−2.2**	**−4.3**
Dem	**−0.3**	**0.5**	0.1	—	−0.0	**0.7**	**−1.0**	−0.0	**0.6**
CInc	**−1.0**	**1.7**	**−0.4**	−0.0	—	**−1.6**	**1.1**	**−0.3**	**−1.5**
SMR	**7.7**	**7.2**	**−3.5**	**0.7**	**−1.6**	—	**47.5**	**−9.1**	—
Lex	**−48.7**	**0.2**	**1.5**	**−1.0**	**1.1**	**47.5**	—	—	—
SF	**7.2**	**6.5**	**−2.2**	−0.0	**−0.3**	**−9.1**	—	—	—
Comp	**6.7**	**9.5**	**−4.3**	**0.6**	**−1.5**	—	—	—	—

(c) National scale (in billions of dollars)

Table A.41: Two-way interaction effects predicting optimistic auction revenue (Revenue4). Non-constant coefficients are represented as a percent of the constant coefficient. Bold values indicate that the coefficient is significant at the 1% level

	Base	ValVar	nB	Dem	CInc	SMR	Lex	SF	Comp
Base	**28.8**	**−10.8**	**6.7**	**0.9**	**−2.3**	**−1.0**	**−5.8**	**3.2**	**0.5**
ValVar	**−10.8**	—	**15.7**	**3.4**	**0.8**	**7.3**	**−0.6**	**1.0**	−0.1
nB	**6.7**	**15.7**	—	**−0.4**	**−0.2**	**5.0**	**2.7**	**−1.2**	**−1.7**
Dem	**0.9**	**3.4**	**−0.4**	—	**−0.1**	**−0.3**	**−0.1**	−0.1	−0.1
CInc	**−2.3**	**0.8**	**−0.2**	**−0.1**	—	**0.9**	−0.1	0.0	0.0
SMR	**−1.0**	**7.3**	**5.0**	**−0.3**	**0.9**	—	**4.9**	**−3.1**	—
Lex	**−5.8**	**−0.6**	**2.7**	**−0.1**	−0.1	**4.9**	—	—	—
SF	**3.2**	**1.0**	**−1.2**	−0.1	0.0	**−3.1**	—	—	—
Comp	**0.5**	−0.1	**−1.7**	−0.1	0.0	—	—	—	—

(a) Small scale (in millions of dollars)

	Base	ValVar	nB	Dem	CInc	SMR	Lex	SF	Comp
Base	**4524.0**	**−13.7**	**22.3**	**0.3**	**3.0**	**17.7**	**−26.7**	**−2.4**	**14.5**
ValVar	**−13.7**	—	**17.4**	**2.9**	**0.6**	**1.3**	**2.5**	**0.8**	**−1.6**
nB	**22.3**	**17.4**	—	**−1.0**	**−1.9**	**−11.8**	**0.7**	**0.3**	**−11.4**
Dem	**0.3**	**2.9**	**−1.0**	—	**−0.1**	**1.8**	**−2.6**	**0.3**	**1.9**
CInc	**3.0**	**0.6**	**−1.9**	**−0.1**	—	**−4.5**	**−0.4**	**2.3**	**−4.3**
SMR	**17.7**	**1.3**	**−11.8**	**1.8**	**−4.5**	—	**27.8**	**−0.7**	—
Lex	**−26.7**	**2.5**	**0.7**	**−2.6**	**−0.4**	**27.8**	—	—	—
SF	**−2.4**	**0.8**	**0.3**	**0.3**	**2.3**	**−0.7**	—	—	—
Comp	**14.5**	**−1.6**	**−11.4**	**1.9**	**−4.3**	—	—	—	—

(b) Bi-regional scale (in millions of dollars)

	Base	ValVar	nB	Dem	CInc	SMR	Lex	SF	Comp
Base	**24.8**	**7.7**	**4.3**	**−0.4**	**−0.8**	**6.4**	**−49.1**	**6.4**	**5.3**
ValVar	**7.7**	—	**11.7**	**0.5**	**1.8**	**6.6**	0.1	**7.0**	**9.0**
nB	**4.3**	**11.7**	—	0.1	**−0.4**	**−2.9**	**2.0**	**−2.0**	**−3.5**
Dem	**−0.4**	**0.5**	0.1	—	−0.0	**0.8**	**−1.1**	−0.1	**0.6**
CInc	**−0.8**	**1.8**	**−0.4**	−0.0	—	**−1.8**	**0.9**	**−0.4**	**−1.7**
SMR	**6.4**	**6.6**	**−2.9**	**0.8**	**−1.8**	—	**47.9**	**−8.7**	—
Lex	**−49.1**	0.1	**2.0**	**−1.1**	**0.9**	**47.9**	—	—	—
SF	**6.4**	**7.0**	**−2.0**	−0.1	**−0.4**	**−8.7**	—	—	—
Comp	**5.3**	**9.0**	**−3.5**	**0.6**	**−1.7**	—	—	—	—

(c) National scale (in billions of dollars)

A.5 Interaction Model Marginal Effects

Table A.42: The range of (within-strategy) marginal effects with respect to bankruptcy rate, as a percentage of the corresponding constant coefficient.

		ValVar		nB		Dem		CInc	
Scale	Strategy	Min	Max	Min	Max	Min	Max	Min	Max
Small	Lexico	-68.0	-24.1	-38.3	-16.5	1.9	-1.1	0.5	22.9
	SF	-88.1	-44.2	-62.8	-41.0	7.3	4.3	-2.9	19.5
	Comp	-31.1	12.8	-15.9	5.9	1.7	-1.3	-1.9	20.5
BiReg	Lexico	-27.4	-1.6	-43.8	-41.2	-52.4	-32.8	-11.3	5.7
	SF	-7.5	18.3	-57.3	-54.7	6.1	25.7	1.6	18.6
	Comp	-7.5	18.3	-12.6	-10.0	-4.2	15.4	-4.8	12.2
Nat	Lexico	-5.3	45.3	-39.9	-3.9	-63.5	-44.0	-16.8	6.1
	SF	89.5	140.1	-73.5	-37.5	-0.4	19.1	8.4	31.3
	Comp	21.7	72.3	-42.7	-6.7	2.8	22.3	-7.0	15.9

Table A.43: The range of (strategy-specific) marginal effects with respect to bankruptcy rate, as a percentage of the corresponding constant coefficient.

	Small		BiReg		National	
Strategy	Min	Max	Min	Max	Min	Max
Lexico	-4.2	6.4	-25.0	63.2	-148.7	111.4
SF	4.3	43.7	10.6	31.4	-56.0	-4.7
Comp	-90.1	-22.3	-95.3	-50.2	-187.2	-33.0

Table A.44: The range of (within-strategy) marginal effects with respect to the number of rounds per auction, as a percentage of the corresponding constant coefficient.

Scale	Strategy	ValVar Min	ValVar Max	nB Min	nB Max	Dem Min	Dem Max	CInc Min	CInc Max	SMR Min	SMR Max
Small	Lexico	-3.1	15.7	-11.2	3.5	-1.5	1.9	-51.1	-45.9	-4.2	15.5
	SF	-0.9	17.9	-9.4	5.3	-1.8	1.6	-50.0	-44.8	-0.4	19.3
	Comp	-2.1	9.7	-1.4	6.8	-0.2	1.6	-44.1	-43.5	—	—
BiReg	Lexico	-44.0	6.2	-34.0	41.1	-27.8	10.5	-136.7	-27.8	76.4	261.7
	SF	-45.0	5.2	-46.2	28.9	-25.7	12.6	-142.7	-33.8	49.2	234.5
	Comp	-17.4	5.1	-8.5	15.7	-11.0	7.6	-46.6	-24.7	—	—
Nat	Lexico	1.0	21.1	-43.4	12.0	-13.2	9.4	-131.1	-39.5	90.6	232.7
	SF	-28.4	-8.3	-42.7	12.7	-20.3	2.3	-116.9	-25.3	33.9	176.0
	Comp	-5.9	3.4	-8.7	4.2	-5.9	3.2	-32.0	-15.7	—	—

Table A.45: The range of (strategy-specific) marginal effects with respect to the number of rounds per auction, as a percentage of the corresponding constant coefficient.

Strategy	Small Min	Small Max	BiReg Min	BiReg Max	National Min	National Max
Lexico	-2.1	5.6	-30.2	9.7	-27.4	41.5
SF	-1.1	0.4	-1.4	7.6	-33.3	13.9
Comp	-3.6	0.0	-30.6	-3.3	-39.8	-17.0

Table A.46: The range of (within-strategy) marginal effects with respect to pessimistic surplus (surplus1), as a percentage of the corresponding constant coefficient.

Scale	Strategy	ValVar Min	ValVar Max	nB Min	nB Max	Dem Min	Dem Max	CInc Min	CInc Max	SMR Min	SMR Max
Small	Lexico	24.5	37.4	-0.2	11.8	-0.2	2.8	-0.6	0.1	3.3	6.1
	SF	26.1	39.0	0.5	12.5	-0.3	2.7	-0.5	0.2	6.3	9.1
	Comp	23.0	35.3	-0.4	9.7	0.1	2.8	-1.1	-0.4	—	—
BiReg	Lexico	20.8	37.7	0.0	17.5	2.1	5.4	-1.4	1.2	11.6	21.5
	SF	21.3	38.2	-0.1	17.4	-0.7	2.6	-2.3	0.3	11.8	21.7
	Comp	21.4	35.3	1.6	14.4	0.2	2.1	-2.1	-0.3	—	—
Nat	Lexico	27.6	55.0	-2.3	14.7	1.1	3.1	-0.8	0.9	2.4	31.7
	SF	23.1	50.5	-1.0	16.0	0.0	2.0	-2.0	-0.3	2.0	31.3
	Comp	37.0	46.4	2.6	10.6	-0.2	0.7	-1.8	-1.3	—	—

Table A.47: The range of (within-strategy) marginal effects with respect to optimistic surplus (surplus2), as a percentage of the corresponding constant coefficient.

		ValVar		nB		Dem		CInc		SMR	
Scale	Strategy	Min	Max	Min	Max	Min	Max	Min	Max	Min	Max
Small	Lexico	21.9	35.1	-0.6	11.7	-0.1	2.6	-0.3	-0.1	-0.8	3.4
	SF	22.3	35.5	-0.8	11.5	-0.2	2.5	-0.3	-0.1	-1.4	2.8
	Comp	20.9	32.4	-0.9	9.1	-0.1	2.4	-0.2	0.0	—	—
BiReg	Lexico	17.8	32.4	0.7	12.2	-0.3	2.3	-0.7	0.3	1.8	5.1
	SF	19.2	33.8	0.2	11.7	-0.2	2.4	-0.6	0.4	-0.8	2.5
	Comp	18.5	31.1	0.3	11.0	0.0	2.5	-0.7	-0.1	—	—
Nat	Lexico	30.8	41.7	1.6	9.0	-0.2	0.5	-0.3	0.3	7.5	13.5
	SF	35.3	46.2	-0.5	6.9	0.1	0.8	-0.7	-0.1	-1.0	5.0
	Comp	36.7	43.4	0.2	5.9	0.0	0.6	-0.8	-0.2	—	—

Table A.48: The range of (strategy-specific) marginal effects with respect to pessimistic surplus (surplus1), as a percentage of the corresponding constant coefficient.

	Small		BiReg		National	
Strategy	Min	Max	Min	Max	Min	Max
Lexico	-0.4	1.3	-3.3	2.5	-6.4	13.8
SF	-2.9	0.8	-2.4	0.9	-1.6	10.8
Comp	1.4	4.9	8.4	14.1	9.2	32.3

Table A.49: The range of (strategy-specific) marginal effects with respect to optimistic surplus (surplus2), as a percentage of the corresponding constant coefficient.

	Small		BiReg		National	
Strategy	Min	Max	Min	Max	Min	Max
Lexico	-0.2	0.0	-2.4	0.8	-8.5	1.6
SF	-0.1	0.8	-0.6	0.9	-1.4	4.3
Comp	-1.2	0.0	-0.7	1.0	0.3	6.2

Table A.50: The range of marginal effects with respect to surplus when switching from pure IFA complex strategies to pure SMR SF strategies, as a percentage of the corresponding constant coefficient.

	Surplus1		Surplus2	
Scale	Min	Max	Min	Max
Small	-1.0	6.0	-1.1	4.7
BiReg	1.0	7.1	-0.4	2.7
Nat	0.1	9.0	-0.5	3.3

Table A.51: The range of (within-strategy) marginal effects with respect to pessimistic auction revenue (Revenue1), as a percentage of the corresponding constant coefficient.

		ValVar		nB		Dem		CInc		SMR	
Scale	Strategy	Min	Max	Min	Max	Min	Max	Min	Max	Min	Max
Small	Lexico	-9.3	16.2	9.2	31.2	-0.1	4.4	-3.4	-0.6	8.9	19.7
	SF	-6.2	19.3	6.0	28.0	0.0	4.5	-3.1	-0.3	4.2	15.0
	Comp	-10.5	10.5	4.9	21.9	0.4	4.6	-3.9	-2.1	—	—
BiReg	Lexico	-10.8	13.2	8.6	45.3	-0.8	3.7	-4.5	2.9	43.8	63.0
	SF	-12.8	11.2	9.0	45.7	0.0	4.5	-2.6	4.8	16.9	36.1
	Comp	-16.5	6.5	10.5	33.3	1.4	4.9	-5.2	-1.1	—	—
Nat	Lexico	4.0	37.1	0.5	23.2	-0.3	0.6	-1.1	1.1	53.2	80.1
	SF	2.9	36.0	-0.7	22.0	0.2	1.1	-3.2	-1.0	-1.6	25.3
	Comp	15.3	31.0	2.7	16.7	0.1	1.0	-3.6	-2.2	—	—

Table A.52: The range of (within-strategy) marginal effects with respect to realistic auction revenue (Revenue2), as a percentage of the corresponding constant coefficient.

		ValVar		nB		Dem		CInc		SMR	
Scale	Strategy	Min	Max	Min	Max	Min	Max	Min	Max	Min	Max
Small	Lexico	-10.2	16.0	8.9	30.7	-0.1	4.5	-3.1	-0.7	6.3	18.5
	SF	-7.7	18.5	5.4	27.2	-0.1	4.5	-2.9	-0.5	0.2	12.4
	Comp	-10.5	9.9	4.6	21.4	0.4	4.5	-3.4	-2.0	—	—
BiReg	Lexico	-10.6	11.6	8.7	42.5	-1.8	2.5	-4.0	2.9	35.0	53.0
	SF	-11.7	10.5	8.6	42.4	0.9	5.2	-1.6	5.3	7.7	25.7
	Comp	-15.4	6.3	9.3	30.3	1.5	5.1	-3.9	-1.0	—	—
Nat	Lexico	7.5	31.6	1.9	20.6	-1.2	0.0	-1.9	1.7	51.7	68.9
	SF	12.2	36.3	-0.7	18.0	-0.2	1.0	-3.3	0.3	-5.6	11.6
	Comp	17.4	32.3	1.2	14.0	0.4	1.1	-3.3	-1.3	—	—

Table A.53: The range of (within-strategy) marginal effects with respect to generous auction revenue (Revenue3), as a percentage of the corresponding constant coefficient.

		ValVar		nB		Dem		CInc		SMR	
Scale	Strategy	Min	Max	Min	Max	Min	Max	Min	Max	Min	Max
Small	Lexico	-11.3	15.9	8.6	30.1	0.0	4.2	-2.9	-0.8	3.8	17.2
	SF	-9.7	17.5	4.8	26.3	0.0	4.2	-2.8	-0.7	-4.0	9.4
	Comp	-10.9	9.1	4.3	20.8	0.3	4.2	-2.8	-1.6	—	—
BiReg	Lexico	-11.0	11.0	8.2	40.9	-3.2	2.4	-4.1	3.0	30.7	49.7
	SF	-12.7	9.3	7.8	40.5	-0.4	5.2	-1.5	5.6	2.2	21.2
	Comp	-15.5	5.6	8.0	28.6	1.2	5.1	-3.5	-0.7	—	—
Nat	Lexico	7.7	28.9	2.8	18.3	-1.3	-0.1	-1.7	1.7	50.2	62.9
	SF	13.9	35.1	-0.9	14.6	-0.3	0.9	-3.2	0.2	-6.4	6.3
	Comp	17.0	31.0	0.5	12.6	0.3	0.9	-2.8	-0.9	—	—

Table A.54: The range of (within-strategy) marginal effects with respect to optimistic auction revenue (Revenue4), as a percentage of the corresponding constant coefficient.

		ValVar		nB		Dem		CInc		SMR	
Scale	Strategy	Min	Max	Min	Max	Min	Max	Min	Max	Min	Max
Small	Lexico	-11.4	15.8	8.8	30.1	0.0	4.2	-2.7	-0.7	3.6	17.1
	SF	-9.8	17.4	4.9	26.2	0.0	4.2	-2.6	-0.6	-4.4	9.1
	Comp	-10.9	9.0	4.4	20.7	0.3	4.2	-2.6	-1.5	—	—
BiReg	Lexico	-11.2	11.0	8.3	40.4	-3.4	2.4	-3.9	3.2	29.2	48.6
	SF	-12.9	9.3	7.9	40.0	-0.5	5.3	-1.2	5.9	0.7	20.1
	Comp	-15.3	5.6	8.0	28.3	1.1	5.1	-3.3	-0.7	—	—
Nat	Lexico	7.9	28.6	3.0	17.8	-1.4	-0.1	-1.9	1.9	49.7	61.5
	SF	14.7	35.4	-0.9	13.9	-0.3	1.0	-3.2	0.6	-6.9	4.9
	Comp	16.7	30.8	0.4	12.4	0.3	0.9	-2.9	-0.8	—	—

Table A.55: The range of (strategy-specific) marginal effects with respect to pessimistic auction revenue (Revenue1), as a percentage of the corresponding constant coefficient.

	Small		BiReg		National	
Strategy	Min	Max	Min	Max	Min	Max
Lexico	-5.7	1.8	-29.2	4.1	-45.7	4.7
SF	-1.4	2.3	-4.6	0.5	-3.9	18.3
Comp	0.7	5.5	7.1	28.9	9.6	36.2

Table A.56: The range of (strategy-specific) marginal effects with respect to realistic auction revenue (Revenue2), as a percentage of the corresponding constant coefficient.

	Small		BiReg		National	
Strategy	Min	Max	Min	Max	Min	Max
Lexico	-5.9	1.8	-27.5	3.7	-47.6	1.6
SF	-1.3	3.1	-3.5	1.4	-4.0	16.3
Comp	-0.1	3.3	1.1	21.9	4.0	23.8

Table A.57: The range of (strategy-specific) marginal effects with respect to generous auction revenue (Revenue3), as a percentage of the corresponding constant coefficient.

	Small		BiReg		National	
Strategy	Min	Max	Min	Max	Min	Max
Lexico	-6.4	1.7	-29.4	4.1	-49.8	1.8
SF	-1.1	4.1	-3.0	1.2	-4.5	13.6
Comp	-1.2	0.7	-1.8	17.9	1.0	16.7

Table A.58: The range of (strategy-specific) marginal effects with respect to optimistic auction revenue (Revenue4), as a percentage of the corresponding constant coefficient.

	Small		BiReg		National	
Strategy	Min	Max	Min	Max	Min	Max
Lexico	-6.5	1.8	-29.7	4.3	-50.3	1.8
SF	-1.1	4.2	-3.1	1.3	-4.5	13.5
Comp	-1.2	0.5	-2.8	16.4	0.0	14.7

Table A.59: The range of marginal effects with respect to revenue when switching from pure IFA complex strategies to pure SMR SF strategies, as a percentage of the corresponding constant coefficient.

	Revenue1		Revenue2		Revenue3		Revenue4	
Scale	Min	Max	Min	Max	Min	Max	Min	Max
Small	-1.3	15.8	-1.6	14.7	-1.6	13.5	-1.7	13.4
BiReg	1.8	10.0	1.1	7.9	0.2	6.2	0.0	6.1
Nat	-1.4	7.6	-2.0	4.0	-2.8	3.1	-2.7	3.6