

## **Solving the traffic assignment problem using real data for a segment of Medellín's transportation network**

### **Solución al problema de asignación del tránsito para un segmento de la red vial de Medellín a partir de datos reales**

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#### **Abstract**

This paper is based on data obtained from most recent transportation studies done in the Metropolitan Area of Valle de Aburrá, city of Medellín and other 9 municipalities. The studies were based on an Origin/Destination Survey (2005), Analysis of bus routes (2006), and Mobility Master Plan (2006). This paper explains the process of writing a software application for a given network (Network of Medellín) that solves the deterministic user equilibrium problem. The software code was implemented in Visual Basic .NET<sup>®</sup>, supported by some operations using Microsoft Excel<sup>®</sup>, and hardcoded for a segment of the Medellín network. The user equilibrium distribution of flow was found by using the *Frank-Wolfe* algorithm. The applied algorithm was analyzed in some aspects such as number of iterations, convergence patterns, response time, as well as changes in network demand. The traffic assignment models were analyzed by using the algorithm during the P.M. peak hour (hour of highest traffic congestion). The analysis was compared with the results from the traffic assignment procedure using TransCAD<sup>®</sup> (well-known and used transportation demand software) for the 2005 database and it was found that the software is somewhat faster than the algorithm, but the latter could be a good tool for practitioners and students for modeling small networks.

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----- *Keywords:* Traffic modeling, traffic assignment, Frank-Wolfe algorithm

### Resumen

Este artículo está basado en datos obtenidos en los más recientes estudios de transporte que se han realizado en el Área Metropolitana del Valle de Aburrá. (Medellín y otros 9 municipios). Estos estudios fueron la Encuesta Origen Destino (2005), análisis de las rutas de buses (2005) y el Plan Maestro de Movilidad (2006). En el artículo se explica el proceso utilizado para el desarrollo de una aplicación informática para resolver el problema determinístico de equilibrio de usuario en la red vial de Medellín. El código fue construido usando Visual Basic.NET ® y Microsoft Excel ® para la ejecución de algunas operaciones en un segmento de la red vial de Medellín. La distribución del flujo del equilibrio de usuario fue encontrada usando el algoritmo de Frank-Wolfe y fueron analizados algunos aspectos tales como número de iteraciones, patrones de convergencia, tiempo de respuesta y cambios en la demanda de viajes en la red. Los modelos de asignación del tránsito fueron analizados para las horas pico de la tarde. Se compararon los resultados de la asignación del tránsito del algoritmo desarrollado en este trabajo con los resultados de TransCAD ® para los datos del 2005 y fue encontrado que el software es un poco más rápido que el algoritmo, pero sin embargo éste último puede ser una buena herramienta para profesionales y estudiantes para la modelación de redes pequeñas.

----- *Palabras clave:* Modelación de tránsito, asignación del tránsito, algoritmo Frank-Wolfe

### Introduction

Medellín is the second largest city in Colombia and it is located in Valle de Aburrá; it has a population of 2.5 million people and has an area of 382 km<sup>2</sup>. Buses, taxis, and the “Metro de Medellín” (the only passenger municipal train system in Colombia) serve as public transportation services in the city. There are 4.8 million trips per day [1], where buses represent 34% of the trips, Metro de Medellín represents 10% of the trips, taxis and private automobiles represent 13% of the trips, and the other modes of transportation such as bicycles, walking, etc., represent 43% of the trips. Despite the variety of options, traffic in Medellín has become chaotic, as the number of vehicles has exceeded roadway capacity; the city has no further space for the construction

of new highways or roads. For this reason and other mobility aspects like accessibility, the city needs an optimal traffic assignment for the transportation network to plan and forecast the traffic demand for future scenarios.

The use of the convex combinations method (Frank-Wolfe algorithm) in conjunction with the label-correcting (shortest path) algorithm for the *direction finding* step, provides an easy and efficient approach to minimizing the equivalent User Equilibrium (UE) program. The convergence of the convex combinations method is asymptotic in nature; the marginal contribution of each additional iteration to the reduction in the value of the objective function is decreasing, the number of iterations required for convergence is primarily a function of the congestion over the

network, and the computational effort needed for each iteration is proportional to the number of origins and the size of the network [2]. Thus, the method can be implemented in a segment of the Medellin Network to obtain the traffic assignment by using the UE program and such is the purpose of this work.

### Background

The convex combination algorithm was originally suggested by Frank and Wolfe in 1956 as a procedure for solving quadratic programming problems with linear constraints and is known also as the Frank-Wolfe (FW) method [2]. In the traffic application, the linear program decomposes into a set of shortest path problems [3].

The Frank-Wolfe method [4] was first introduced in quadratic programming at once it proved very effective for the resolution of large scale flood problems, with particularly interesting assets, resting on the marvelous fusion of the mathematical programming and the graph theory, the Frank-Wolfe method is famous for its advantages: it is easy to implement and it performs well far from the optimal solution. Unfortunately, its convergence rate is not entirely satisfactory; this slowness is due mainly to the way in zigzag described by the points of the algorithm showing very slow asymptotic convergence. To remedy this disadvantage, much of attempts have been done, since the first "L. J. Leblanc" works until recent works of "Ziyu Gao and Al" [5]. The Frank-Wolfe method is one of the most widely used algorithms for solving routing problems in the telecom and traffic areas [6], and it is widely used to solve traffic equilibrium assignment problems [2]. It has the characteristics of simple implementation and modest memory requirement. However, it also faces some problems such as slow convergence, no providing path information, and so on [7], i.e. the FW algorithm converges very slowly when iterations are closing to the optimal solution [2].

Sheffi [2] shows that the application of Frank and Wolfe's convex combinations method to the

solution of transportation network equilibrium was first suggested by Bruynooghe in 1968 and applied by Murchland in 1969. Shortly thereafter it was used by LeBlanc in 1975, which coded and tested the algorithm for a small city. At the same time, Nguyen [8] suggested the use of the convex simplex method for solving the User Equilibrium equivalent minimization program.

Also Sheffi shows that Nguyen [8] also suggested the use of the reduced gradient method and a modified reduced gradient method for this purpose. In some side-by-side comparative experiments, Florian and Nguyen [9] found that even though the convex simplex method converges somewhat faster than the convex combinations method, it requires more computer memory. Consequently, the overall computational effort required by both methods is similar.

## Methodology

### Frank Wolfe Algorithm

In this study the traffic equilibrium is assumed to have additive path costs, users with perfect information, fixed demand, there is no link interaction in the network, and the cost functions are also monotonic, differentiable and continuous. The User Equilibrium (UE) objective function is given by equation (1):

$$\min z(x) = \sum_a \int_0^{x_a} t_a(w) dw \quad (1)$$

Subject to equation (2):

$$\sum_k f_k^{rs} = q_{rs} \quad f_k \geq 0 \quad (2)$$

Where  $x_a$  is the flow on link  $a$ ,  $t_a$  represents travel time on link  $a$ ,  $f_k^{rs}$  represents the flow on path  $k$  connecting origin  $r$  and destination  $s$ , and  $q_{rs}$  is the trip rate between origin  $r$  and destination  $s$  during the period of analysis.

The objective function is the sum of the integrals of the link performance functions. Applying the convex combinations algorithm for the

minimization of the UE program requires, at every iteration, a solution of the linear program (LP) that decomposes into a set of shortest path problems. The steps of the Frank-Wolfe Algorithm can be expressed as:

**Step 0: Initialization**

Perform all-or-nothing assignment based on  $t_a = t_a(0) \forall a$ . This yields  $\{x_a\}$ . Set counter  $n = 1$ .

**Step 1: Update**

Set  $t_a = t_a(x_a) \forall a$

**Step 2: Direction finding**

Perform all-or-nothing assignment based on  $\{t_a\}$ . This yields a set of (auxiliary) flows  $\{y_a\}$ .

**Step 3: Line search**

Find  $\alpha_n$  ( $0 \leq \alpha \leq 1$ ) that solves equation (3)

$$\min z(x) = \sum_a \int_0^{x_a + \alpha(y_a^n - x_a^n)} t_a(w) dw \quad (3)$$

**Step 4: Move**

Set  $x_a^{n+1} = x_a^n + \alpha_n(y_a^n - x_a^n), \forall a$

**Step 5: Convergence test**

If a convergence criterion is met, stop (the current solution,  $\{x_a^{n+1}\}$ , is the set of equilibrium link flows); otherwise, set  $n = n + 1$  and go to step 1.

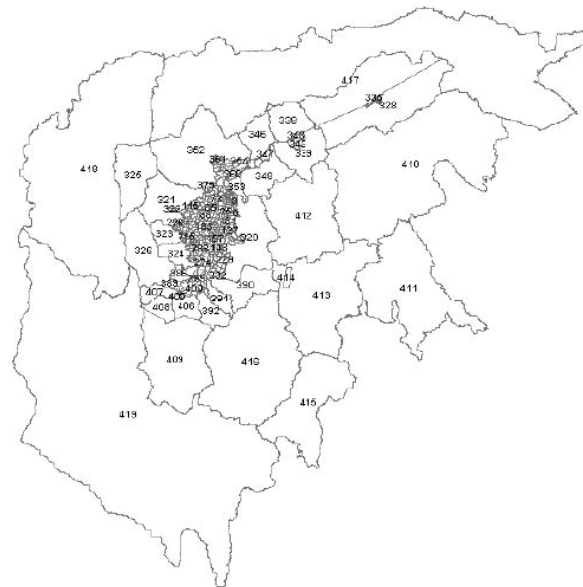
**Case study**

This study takes place in the Metropolitan Area of the Aburrá Valley (Medellín city and other 9 municipalities) and it is based on data obtained on most recent studies of transportation. Those studies were Origin/Destination Survey (2005), Analysis of transit routes (2005), and Mobility Master Plan (2006) [1]. For this study, the traffic assignment was done taking into account the

demand in PM peak hours (17:00 – 19:00) because is the time of the day with more congestion.

**Zoning and Origin-Destination (OD) Matrix**

The traffic assignment models take as input flows between all origins and destinations. The OD Matrix in the study area is defined by 419 zones that are distributed as depicted in figure 1 [1].



**Figure 1** Zoning of the study area

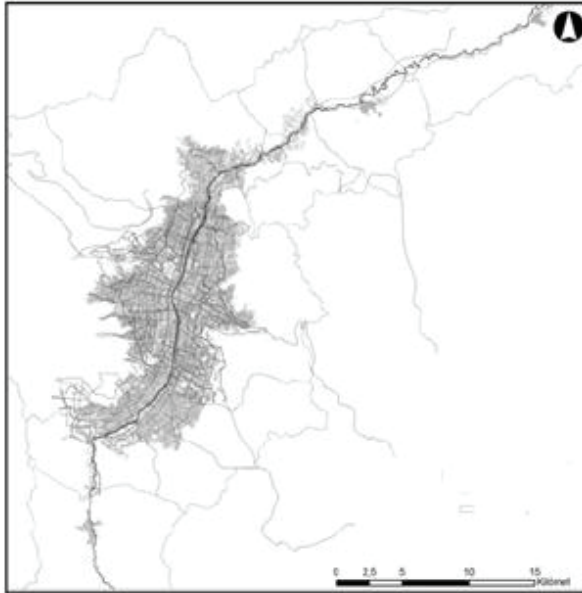
Taking into account the information above and using the software TransCAD®, figure 2 shows the OD matrix of all trips in all zones.

	1	2	3	4	5	6	7	8	9	10	Sum	
1	172	632	264	51	56	24	68	97	--	--	1997	
2	599	865	820	399	364	52	283	661	--	6	5241	
3	255	835	573	288	185	86	161	278	--	--	4126	
4	56	397	283	283	57	58	76	51	--	--	2275	
5	22	262	144	17	85	61	61	28	--	--	1121	
6	62	177	143	118	83	76	46	105	--	--	1585	
7	52	183	150	65	72	29	98	72	--	--	906	
8	86	682	386	51	52	51	112	76	--	6	1935	
9	--	--	--	--	--	--	--	--	2824	1389	12580	
10	--	6	--	--	--	--	--	6	1387	8548	38450	
11	--	--	--	--	--	--	--	--	--	32	4633	
12	--	--	--	--	--	--	--	--	--	--	1210	
13	--	--	--	26	--	--	--	--	--	--	68	6953
14	--	--	--	--	--	--	--	--	--	--	609	
15	--	--	--	--	6	64	6	6	174	828	5956	
16	--	66	--	--	--	--	22	182	621	13678	13678	
17	--	--	--	32	--	--	--	--	--	189	14625	
18	6	--	--	57	6	--	--	6	--	266	14784	
19	--	--	--	39	--	--	--	39	39	255	38297	
20	13	--	--	17	67	--	6	--	689	2973	52885	
21	--	6	6	55	11	--	--	6	87	575	25580	
22	--	--	6	70	--	--	--	--	25	125	21829	
23	--	6	--	--	--	33	--	6	22	373	11320	
24	--	--	--	--	--	--	--	--	--	285	24325	
25	17	--	6	6	6	--	6	6	--	538	8503	
26	--	--	--	--	--	--	--	--	--	52	2275	
Sum	1836	5327	4071	2361	1636	740	1178	1830	12301	39004	4784597	

**Figure 2** Daily OD Matrix

**The Network**

Medellin's network is composed by 1516 nodes and 4502 links connecting all nodes. In a common day, there are in Medellín 4.8 million trips [1], and those trips need a traffic network for the traffic assignment. Figure 3 shows the study network. Using TransCAD®, a network was built with links and nodes that include all 419 zones in the study area.



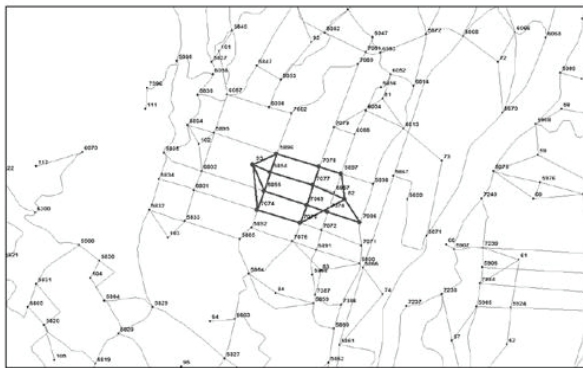
**Figure 3** Traffic Network and Network's structure

The network contains data for links and nodes (See table 1) including travel time, velocity, length, direction, etc. [1].

**Table 1** Network's data

ID	Length	Dir	From ID	To ID	Vel-Fl (Km/h)	Time-FL (min)
3,683	4.17	0	7,838	6,081	30	8.33
3,682	5.15	0	7,837	6,084	30	10.29
3,681	3.35	0	7,836	7,022	30	6.70
3,679	0.26	0	7,826	5,784	30	0.53
3,678	0.30	0	7,761	7,413	30	0.61
3,677	0.37	0	7,755	6,012	30	0.73
3,676	0.22	0	7,737	7,099	30	0.45
3,670	0.35	0	7,668	5,930	30	0.70
3,669	0.34	0	7,668	5,975	30	0.69
3,668	0.28	0	7,641	5,710	30	0.56
3,667	0.46	0	7,639	5,710	30	0.92
3,680	3.24	0	7,836	7,027	30	6.48
3,665	0.18	0	7,594	5,763	30	0.36
3,663	0.17	0	7,559	5,656	30	0.35
3,662	0.52	0	7,532	5,606	30	1.04
3,661	0.30	0	7,530	7,115	30	0.59
3,660	0.34	0	7,526	7,351	30	0.69
3,658	0.38	0	7,487	6,631	30	0.77
3,650	0.10	0	7,835	6,085	30	0.25
3,649	1.29	0	7,834	6,222	30	2.61

For the purpose of this study there is a sample of a segment (selection) of Medellín's network and its structure (See figure 4). This network segment is composed by 13 nodes and 21 links connecting all nodes (See figure 5). The demand between origin 93 and destination 82 was 1,433 trips in the PM peak hour.



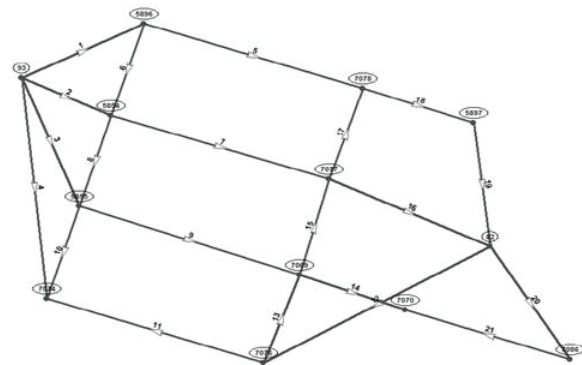
**Figure 4** Studied network

The cost function used for the links in the network is given by Bureau of Public Roads (BPR) function equation (4):

$$t(x_a) = t_o \left[ 1 + \alpha \left( \frac{x_a}{c} \right)^\beta \right] \quad (4)$$

Where:

- $t(x_a)$  = cost of using link a (given by time)
- $t_o$  = free flow time
- $x_a$  = flow on link a
- $c$  = capacity of the link
- $\alpha, \beta$  = given parameters of the cost function. It is assumed that  $\alpha = 0.15$  and  $\beta = 4$



**Figure 5** Zoom-in of the studied network

### The Code

The code was implemented in Visual Basic .NET® and it was supported for some operations using Microsoft Excel. The first step of this process was to think in a way to make the program to understand the configuration of the network. The basic configuration of the code allows finding the shortest path using *Dijkstra's algorithm* [10] between an origin and a destination taking into account the costs (time) in all paths that connect this pair O-D. The algorithm essentially scans the network nodes in an iterative manner. At each iteration the algorithm tries to find a path from the root to the node being scanned that is better (shorter) than the current path. The data used for the algorithm is presented in table 2.

**Table 2** Used data for traffic assignment

From Node	To Node	Link	Travel Time (min)	Capacity (vph)	Xa (veh)	Time BPR (min)
93	5,896	1	0.37	1,800	0	0.37
93	5,854	2	0.27	1,800	0	0.27
93	5,855	3	0.41	1,800	0	0.41
93	7,074	4	0.66	1,800	0	0.66
5,854	5,896	6	0.17	1,800	0	0.17
5,854	5,855	8	0.17	1,800	0	0.17
5,855	7,069	9	0.38	1,800	0	0.38
5,855	7,074	10	0.17	1,800	0	0.17
5,896	7,078	5	0.42	1,800	0	0.42
5,897	82	19	0.37	1,800	0	0.37

Table 2 continued

From Node	To Node	Link	Travel Time (min)	Capacity (vph)	Xa (veh)	Time BPR (min)
7,069	7,070	14	0.19	1,800	0	0.19
7,069	7,077	15	0.18	1,800	0	0.18
7,070	7,086	21	0.36	1,800	0	0.36
7,074	7,076	11	0.47	1,800	0	0.47
7,076	82	12	0.71	1,800	0	0.71
7,076	7,069	13	0.17	1,800	0	0.17
7,077	5,854	7	0.42	1,800	0	0.42
7,077	82	16	0.49	1,800	0	0.49
7,077	7,078	17	0.17	1,800	0	0.17
7,078	5,897	18	0.21	1,800	0	0.21
7,086	82	20	0.4	1,800	0	0.40

The O-D pairs were introduced into the algorithm and the name of each vertex was modified with a new name called Element x, thus in instance, Node 93 (origin) is Element 0 and Node 82 (destination) is Element 12, and those values were organized as start element, end element and weight (variable cost). At the end, the Algorithm shows the previous vertex name.

Excel was helpful in order to introduce the data and find some values such as step size and others. The results of the traffic assignment using Frank Wolfe algorithm (convergence criteria of 0.01 and using the shortest path with all or nothing assignment) are presented in table 3.

The shortest path was presented as shown in figure 6, where it presented the first shortest path using all or nothing assignment. The Shortest Path from origin 93 to destination 82 is given by: 93-->5,854-->7,077-->82.

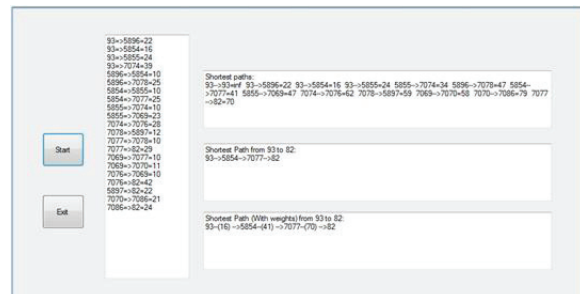


Figure 6 Application window showing the Shortest Path for the algorithm

Table 3 Traffic assignment results

From Node	To Node	Link	Travel Time (sec)	Xa (Veh)	Time BPR (Sec)	Ya (Veh)	Function	Flow (Veh)
93	5896	1	22	0	22.20	1,433	7,889.525	355.384
93	5,854	2	16	1,433	17.18	0	17,457.420	1,077.616
93	5,855	3	25	0	24.60	0	0.000	0.000
93	7,074	4	40	0	39.60	0	0.000	0.000
5,854	5,896	6	10	0	10.20	0	0.000	0.000
5,854	5,855	8	10	0	10.20	0	0.000	0.000

Table 3 continued

<b>From Node</b>	<b>To Node</b>	<b>Link</b>	<b>Travel Time (sec)</b>	<b>Xa (Veh)</b>	<b>Time BPR (Sec)</b>	<b>Ya (Veh)</b>	<b>Function</b>	<b>Flow (Veh)</b>
5,855	7,069	9	23	0	22.80	0	0.000	0.000
5,855	7,074	10	10	0	10.20	0	0.000	0.000
5,896	7,078	5	25	0	25.20	1,433	8,955.677	355.384
5,897	82	19	22	0	22.20	1,433	7,889.525	355.384
7,069	7,070	14	11	0	11.40	0	0.000	0.000
7,069	7,077	15	11	0	10.80	0	0.000	0.000
7,070	7,086	21	22	0	21.60	0	0.000	0.000
7,074	7,076	11	28	0	28.20	0	0.000	0.000
7,076	82	12	43	0	42.60	0	0.000	0.000
7,076	7,069	13	10	0	10.20	0	0.000	0.000
7,077	5,854	7	25	1,433	26.72	0	27,155.980	1,077.616
7,077	82	16	29	1,433	31.17	0	31,681.980	1,077.616
7,077	7,078	17	10	0	10.20	0	0.000	0.000
7,078	5,897	18	13	0	12.60	1,433	4,477.839	355.384
7,086	82	20	24	0	24.00	0	0.000	0.000
Step ( $\alpha_n$ )	0.248							

**Traffic Assignment with TransCAD ®**

Using TransCAD ® and taking into account the data above was done the traffic assignment with the User Equilibrium method and convergence criteria of 0.01. The results are presented in table 4 and in table 5. The traffic assignment is depicted in figure 7

**Table 4** TransCAD® results for the traffic assignment

<b>Link</b>	<b>From Node</b>	<b>To Node</b>	<b>TOT_Flow (Veh)</b>
1	93	5896	355.911
2	93	5,854	1,077.089
3	93	5,855	0.000
4	93	7,074	0.000
5	5,896	7,078	355.911
6	5,854	5,896	0.000
7	7,077	5,854	1,077.089

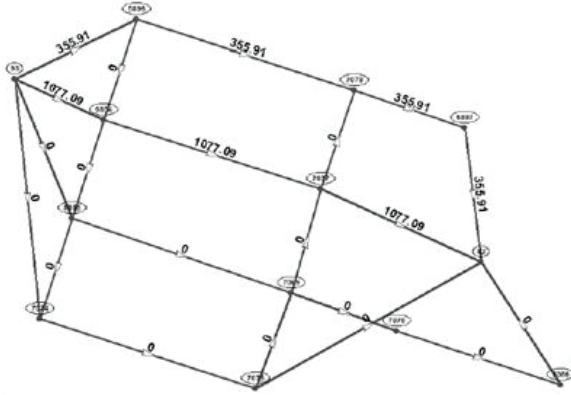
Table 4 continued

<b>Link</b>	<b>From Node</b>	<b>To Node</b>	<b>TOT_Flow (Veh)</b>
8	5,854	5,855	0.000
9	5,855	7,069	0.000
10	5,855	7,074	0.000
11	7,074	7,076	0.000
12	7,076	82	0.000
13	7,076	7,069	0.000
14	7,069	7,070	0.000
15	7,069	7,077	0.000
16	7,077	82	1,077.089
17	7,077	7,078	0.000
18	7,078	5,897	355.911
19	5,897	82	355.911
20	7,086	82	0.000
21	7,070	7,086	0.000



**Table 5** TransCAD ® Report

<i>Iteration</i>	<i>Step</i>	<i>Relative Gap</i>	<i>Max. Flow Change</i>	<i>RMSE</i>	<i>% RMSE</i>
1	0.248368	0.194931	355.911002	14.04	1,469.85
2	0.000000	0.000004	0.000000	0.00	0.00



**Figure 7** Traffic assignment in the network segment using TransCAD®

## Results and discussion

The results of the traffic assignment in the segment of Medellín's Network using the algorithm and TransCAD ® are presented in table 6.

Comparing the two result sets using TransCAD® and the algorithm, the flow is very similar (except for some decimals), using convergence criteria of 0.01 it converged in both cases after two iterations, the step size using TransCAD ® for the first iteration was 0.248368 and for the second iteration was 0.000000. For the other source the step size was 0.248000 for the first iteration and for the second one was 0.000000

**Table 6** Results of the traffic assignment using TransCAD® and the Algorithm

<i>From Node</i>	<i>To Node</i>	<i>Link</i>	<i>Flow Using TransCAD</i>	<i>Flow Using Algorithm</i>	<i>Cost (time)</i>
93	5,896	1	355.91	355.38	22
93	5,854	2	1,077.09	1,077.62	17
93	5,855	3	0.00	0.00	25
93	7,074	4	0.00	0.00	40
5,854	5,896	6	0.00	0.00	10
5,854	5,855	8	0.00	0.00	10
5,855	7,069	9	0.00	0.00	23
5,855	7,074	10	0.00	0.00	10
5,896	7,078	5	355.91	355.38	25
5,897	82	19	355.91	355.38	22
7,069	7,070	14	0.00	0.00	11
7,069	7,077	15	0.00	0.00	11
7,070	7,086	21	0.00	0.00	22
7,074	7,076	11	0.00	0.00	28
7,076	82	12	0.00	0.00	43
7,076	7,069	13	0.00	0.00	10
7,077	5,854	7	1077.09	1077.62	27

Table 6 continued

<i>From Node</i>	<i>To Node</i>	<i>Link</i>	<i>Flow Using TransCAD</i>	<i>Flow Using Algorithm</i>	<i>Cost (time)</i>
7,077	82	16	1077.09	1077.62	31
7,077	7,078	17	0.00	0.00	10
7,078	5,897	18	355.91	355.38	13
7,086	82	20	0.00	0.00	24

**Traffic Assignment varying of demand, alpha, beta and convergence criteria**

One important issue in transportation modeling is the forecasting process. For that reason in this paper we assumed a different value of demand between nodes 93 and 82. It is assumed that there

will be 1,691 trips in 2015 (increase of 18%) instead of 1,433 trips. The traffic assignment was done using the User Equilibrium method and convergence criteria of 0.0001. Using the algorithm, the step size is  $\alpha_n = 0.3460$  and using TransCAD® is 0.3457. The result set of the traffic assignment in the segment of Medellín's Network is presented in table 7

**Table 7** Results of the traffic assignment for 2015 varying demand and convergence criteria

<i>From Node</i>	<i>To Node</i>	<i>Link</i>	<i>Flow Using TransCAD</i>	<i>Flow Using Algorithm</i>	<i>Cost (time)</i>
93	5,896	1	584.607	585.086	22
93	5,854	2	1,106.393	1,105.914	18
93	5,855	3	0.000	0.000	25
93	7,074	4	0.000	0.000	40
5,854	5,896	6	0.000	0.000	10
5,854	5,855	8	0.000	0.000	10
5,855	7,069	9	0.000	0.000	23
5,855	7,074	10	0.000	0.000	10
5,896	7,078	5	584.607	585.086	25
5,897	82	19	584.607	585.086	22
7,069	7,070	14	0.000	0.000	11
7,069	7,077	15	0.000	0.000	11
7,070	7,086	21	0.000	0.000	22
7,074	7,076	11	0.000	0.000	28
7,076	82	12	0.000	0.000	43
7076	7,069	13	0.000	0.000	10
7077	5,854	7	1,106.393	1,105.914	28
7077	82	16	1,106.393	1,105.914	33
7077	7,078	17	0.000	0.000	10
7078	5,897	18	584.607	585.086	13
7086	82	20	0.000	0.000	24

Comparing the two result sets using TransCAD® and the algorithm, the flow is very similar and the results are similar as in the previous case. Now, taking into account different convergence criteria of 0.05 and 0.001 and calibrated parameters from the network of alpha (0.68) and beta (2.2), the algorithm took 4 iterations for the convergence criteria of 0.05, and 10 iterations for the convergence criteria of 0.001. The results for the latter are presented in table 8

**Table 8** Convergence Criteria for the used algorithm

<i>Iteration</i>	<i>Step</i>
1	0.39448
2	0.20241
3	0.05952
4	0.02317
5	0.01640
6	0.00732
7	0.00565
8	0.00260
9	0.00213

The results for the traffic assignment in the network varying demand, parameters and convergence criteria for 2015 are presented in table 9

Comparing the traffic assignments with variable convergence criteria and with new values of alpha and beta obtained from the calibration process, we can see that the flow paths are different from previous cases because it depends of the value of the step size; and it can be observed that if the convergence criteria is lower, the number of iterations increases significantly (from 4 to 10), and the step size reduces in each iteration until get the appropriate value with the same demand.

### Conclusions

- This work introduces a software application in order to get the traffic assignment in a

real network, with real demand and costs using Visual Basic .NET® and Excel®. It was made for a segment of Medellín's Network and the results were compared with TransCAD® results finding that TransCAD® is a bit faster and shows graphically the path with the respective flows and costs, and has more accuracy in the results; but the algorithm can be used for modeling small networks and it is easy to use, besides of the cost of it (it is free). It could be useful for practitioners and students.

**Table 9** Traffic assignment for 2015 varying demand, parameters and convergence criteria

<i>From Node</i>	<i>To Node</i>	<i>Link</i>	<i>Flow (Veh)</i>
93	5,896	1	472.307
93	5,854	2	860.342
93	5,855	3	358.351
93	7,074	4	0.000
5,854	5,896	6	0.000
5,854	5,855	8	0.000
5,855	7,069	9	358.351
5,855	7,074	10	0.000
5,896	7,078	5	472.307
5,897	82	19	472.307
7,069	7,070	14	0.000
7,069	7,077	15	358.351
7,070	7,086	21	0.000
7,074	7,076	11	0.000
7,076	82	12	0.000
7,076	7,069	13	0.000
7,077	5,854	7	860.342
7,077	82	16	1,218.693
7,077	7,078	17	0.000
7,078	5,897	18	472.307
7,086	82	20	0.000

- All the flows found when the algorithm converged, satisfied the user equilibrium optimality condition. It explains that the network studied was in equilibrium and no user had any incentive to change their path choice because they would increase their travel costs in the network.
- The traffic assignment in this study took into account only travel time and capacity of the network. The values of Beta and Alpha were assumed. For best results it would be great to analyze more factors such as velocity, preloads, stops, etc. and use values of alpha and beta calibrated from the network. The purpose of this paper is to give an introduction to solve the traffic assignment in a real network but the code must be improved in order to get a better model of traffic assignment in any network.
- For different demands, the traffic assignment procedure used the same paths (the shortest path was always the same) in the network increasing only the flows in each link according to the traffic forecast. It could change it if is analyzed the entire network because the ratio of forecast of trips in all city is not the same.
- When there are various traffic assignments with variable convergence criteria the flow is different in both cases because it depends of the value of the step size; and if the convergence criteria is lower, the number of iterations increases significantly using the Frank-Wolfe Algorithm.
- Finally, it would be good to clarify that the algorithm implementation needs to improve the integration of Excel ® and Visual Basic. NET ® in order to get better results providing input mechanisms of all data required for the traffic assignment. Thus, it is recommended to continue actively working on the application user interface in order to facilitate the process of entering this information.

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