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An $[SU(3)]^4$ supersymmetric grand unified model

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Abstract

We present a grand unified model based on the supersymmetric $SU(3)_L \otimes SU(3)_{CL} \otimes SU(3)_{CR} \otimes SU(3)_R$ gauge group, which unifies in one single step the three gauge couplings of the standard model at an scale $M \sim 10^{18}$ GeV, and spontaneously breaks down to $SU(3)_c \otimes U(1)_{EM}$ using only fundamental representations of $SU(3)$. In this model the proton decay is highly suppressed and the doublet-triplet problem is lessened. The see-saw mechanism for the neutrinos is readily implemented with the use of an extra tiny mass sterile neutral particle for each generation which provides a natural explanation to the neutrino puzzle. © 1999 Published by Elsevier Science B.V. All rights reserved.

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1. Introduction

Strings provide us with a very compelling theory, giving a consistent framework which is finite and incorporates at the same time both, quantum gravity and chiral supersymmetric (SUSY) gauge theories. When one-loop effects are included in the perturbative heterotic string [1] they predict an unification of the gauge couplings at a scale $M_{\rm string} \sim 4 \times 10^{17}$ GeV.

On the other hand, the logarithmic running through the ‘‘desert’’ of the three gauge couplings $c_i \alpha_i^{-1}$ do merge together into a single point, only when the SUSY partners of the standard model (SM) elementary particles are included in the renormalization group equations (RGE), at a mass scale $M_{\rm susy} \sim 1$ TeV [2]. ($\alpha_i = g_i^2/4\pi$, $i=1,2,3$, and $\{c_1, c_2, c_3\} = \{\frac{3}{5},1,1\}$ are the gauge couplings and normalization constants of the SM factors $U(1)_Y, SU(2)_L$ and $SU(3)_c$, respectively.) This amazing result, which is not upset when higher order contributions are included in the RGE [3], has the inconvenience that the unification scale, 2×10^{16} GeV, is a factor of 20 smaller than the value $M_{\rm string}$.

Several efforts to reconcile these two perturbative scales have been made without success so far [4], producing always the theoretical result $M_{\rm string} > M_{\rm GUT}$, where $M_{\rm GUT}$ is the mass scale of the grand unified theory (GUT) under consideration.

In what follows we are going to study a new SUSY-GUT which has the property that $M_{\rm GUT} \sim M_{\rm string}$, without structure between $M_{\rm susy} \sim 1$ TeV and $M_{\rm GUT}$. The existence of this model can be inferred from Fig. 5 in Ref. [5]. This note is organized in the following way: In Section 2 we introduce the new model, implement the spontaneous symmetry breaking of the gauge group and calculate the mass spectrum of the fermion particles. In Section 3 we do the

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RGE analysis and set the two different mass scales in the model. Conclusions and remarks are presented in the last section.

2. The model

We propose a SUSY-GUT based on the gauge group $G_g \equiv [SU(3)]^4 \times Z_4$ which above $M_{\rm GUT}$ is just the SUSY chiral-color extension [6] of the trinification model of Georgi-Glashow-de Rújula [7]. The four $SU(3)$ factor groups are identified as $SU(3)_L$ which contains weak $SU(2)_L$, $SU(3)_{CL} \otimes SU(3)_{CR}$ which is the chiral color extension [6] of $SU(3)_c$, and $SU(3)_R$ which is the right-handed analog of $SU(3)_L$. The cyclic group Z_4 acting upon the four factor groups ensures that there is only one gauge coupling constant; more specifically, if (L,CL,CR,R) is a representation under $[SU(3)]^4$, the effect of Z_4 is to symmetrize it in the following way:

$$Z_4(L,CL,CR,R) = (L,CL,CR,R) \oplus (CL,CR,R,L) \oplus (CR,R,L,CL) \oplus (R,L,CL,CR).$$

The gauge bosons of G_g are assigned to the adjoint irreducible representation (irrep) $Z_4(8,1,1,1)$ which includes twelve light particles (gluons, photon, $W^{\pm},Z$), twenty superheavy, and their SUSY partners, which are all integrally charged.

Each family of fermions is assigned to $\psi_{36} = Z_4\psi(3^*,3,1,1)$ which under $[SU(3)_c,SU(2)_L,U(1)_Y]$ decompose as:

$$\psi(3^*,3,1,1) = (3,2,1/6) \oplus (3,1,-1/3),$$
$$\psi(3,1,1,3^*) = (1,2,1/2) \oplus 2(1,2,-1/2) \oplus (1,1,1) \oplus 2(1,1,0),$$
$$\psi(1,1,3^*,3) = (3^*,1,-2/3) \oplus 2(3^*,1,1/3),$$
$$\psi(1,3^*,3,1) = (8,1,0) \oplus (1,1,0),$$

where besides the 15 ordinary particles in each family, it contains the right-handed neutrino field ν^c (one of the $(1,1,0) \in \psi(3,1,1,3^*)$), one exotic down quark, one exotic field with electric charge one, three electrically neutral two component weyl spinors, the electrically neutrals spin 1/2 quaits (8,1,0), and the colorless quone $(1,1,0) \in \psi(1,3^*,3,1)$. For further reference let us introduce the following convenient notation for $\psi(3,1,1,3^*)$:

$$\psi(3,1,1,3^*) = \begin{pmatrix} N^0 & E^- & e^- \\ E^+ & N^{0c} & \nu \\ e^+ & \nu^c & M^0 \end{pmatrix}, \qquad (1)$$

where $e^{\pm}$, ν, and ν^c, stand for the electron, electron neutrino and right-handed electron neutrino fields, respectively.

At the unification scale, G_g breaks down spontaneously to the SUSY extension of the SM gauge group $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y = G_{SM}$ in one single step, with the particle content of the minimal supersymmetric standard model plus three new low energy elementary Higgs scalar doublets, needed to produce a realistic mass spectrum, as it is shown anon.

Indeed, the introduction of the following set of Higgs scalar fields $Z_4\phi(3^*,3,1,1)$ and $Z_4\chi(3,3^*,3,3^*)$ with vacuum expectation values (VEV) $\langle\phi(3^*,3,1,1)\rangle = \langle\phi(1,1,3^*,3)\rangle = 0$,

$$\langle\phi(1,3^*,3,1)\rangle = \begin{pmatrix} V & 0 & 0 \\ 0 & V & 0 \\ 0 & 0 & V \end{pmatrix},$$

$$\langle\phi(3,1,1,3^*)\rangle = \begin{pmatrix} \upsilon & 0 & 0 \\ 0 & \upsilon & 0 \\ 0 & 0 & V \end{pmatrix},$$

$$\langle\wp(3,3^*,3,3^*)\rangle = \begin{pmatrix} \upsilon & 0 & 0 \\ 0 & \upsilon' & 0 \\ 0 & 0 & V \end{pmatrix},$$

and

$$\langle\wp(3^*,3,3^*,3)\rangle = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & V \\ 0 & \upsilon & w \end{pmatrix};$$

where $\wp$ is the component of χ which points in the scalar quone direction, $V \sim M_{\rm GUT}$, and υ,υ' and w are related to the electroweak breaking scale.

The algebra shows that:

$$G_g \xrightarrow{V} G_{SM} \xrightarrow{\upsilon} SU(3)_c \otimes U(1)_{\rm EM}.$$

With the scalars ϕ and χ and their VEV as introduced above, the following trilinear invariants can be constructed:

1. $\psi(3,1,1,3^*)\psi(3,1,1,3^*)\langle\phi(3,1,1,3^*)\rangle$ which gives rise to a mass term of the form: $\upsilon(N^0M^0 + N^{0c}M^0 - \nu^c\nu - e^-e^+) + V(N^0N^{0c} - E^-E^+) + {\rm h.c.}$

2. $\psi(3^*,3,1,1)\psi(1,1,3^*,3)\langle\chi(3,3^*,3,3^*)\rangle$ which gives rise to masses of order v, v', and V to the up, down and exotic down quarks, respectively.
3. $\psi(1,3^*,3,1)\psi(1,3^*,3,1)\langle\phi(1,3^*,3,1)\rangle$ which gives rise to masses of order V to the eight spin $1/2$ quaits and to the quone.
4. $\psi(3,1,1,3^*)\psi(1,3^*,3,1)\langle\chi(3^*,3,3^*,3)\rangle$ which gives rise to a mass term of the form $\sqrt{3}D^0(v\nu + V\nu^c + wM) + \text{h.c.}$, where D^0 is the spin $1/2$ quone $(1,1,0)\in\psi(1,3^*,3,1)$.

From the former results, the six electrically neutral spin $1/2$ color singlets in one generation mix in the following way (in the basis given by $\{D^0,\nu^c,\nu,N^0,N^{0c},M^0\}$):

$$\begin{pmatrix} 2V & \sqrt{3}V & \sqrt{3}v & 0 & 0 & w \\ \sqrt{3}V & 0 & -v & 0 & 0 & 0 \\ \sqrt{3}v & -v & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & V & v \\ 0 & 0 & 0 & V & 0 & v \\ w & 0 & 0 & v & v & 0 \end{pmatrix}, \tag{2}$$

which for the particular case $w=0$ (which does not alter the symmetry breaking pattern) has four eigenvalues of order V and two seesaw eigenvalues, $-2v^2/V$ and $8v^2/3V$, corresponding to the mixing of M with N^0 and N^{0c}, and of ν with ν^c and D^0, respectively (when $w \le v$, the eigenvalues are of the same order, but a more general mixing occurs).

Notice that the number of low energy ($\sim v(v')$) Higgs doublet scalar fields introduced in the former expressions is five, independent of the value for w which is the VEV of a scalar field which is a singlet under the SM quantum numbers.

3. The mass scales

The two loop RGE predictions for the gauge couplings in the SUSY standard model (ignoring Yukawa couplings) can be written as:

$$\alpha_i^{-1}(m_Z) = \frac{\alpha^{-1}}{c_i} - b_i^0 \ln\left(\frac{M}{m_Z}\right) + \sum_{j=1}^{3}\frac{b_{ij}^1}{b_j^0}\ln\left(\frac{c_j\alpha}{\alpha_j(m_Z)}\right) + \Delta_i, \tag{3}$$

where M is the GUT scale, $\alpha = g^2/4\pi$ is the gauge coupling for G_g, $\{c_1,c_2,c_3\} = \{\frac{3}{5},1,\frac{1}{2}\}$, and b_i^0, b_{ij}^1, $i,j=1,2,3$ are the one loop and two loops SUSY beta functions, respectively. In the former expression we have lumped together into Δ_i ($i=1,2,3$) the $\overline{\text{MS}}$ to $\overline{\text{DR}}$ [8] renormalization scheme conversion factor $(C_2(G_i)/12\pi)$, the SUSY thresholds, and other effects as for example possible (small) contributions from extra dimensions, contributions of possible nonrenormalizable operators, etc.

Starting our analysis with the one loop calculations we set $\Delta_i = b_{ij} = 0$, and use the one loop SUSY beta functions [9]:

$$2\pi\begin{pmatrix} b_1^0 \\ b_2^0 \\ b_3^0 \end{pmatrix} = \begin{pmatrix} 0 \\ 6 \\ 9 \end{pmatrix} - \begin{pmatrix} 10/3 \\ 2 \\ 2 \end{pmatrix} F - \begin{pmatrix} 1/2 \\ 1/2 \\ 0 \end{pmatrix} H, \tag{4}$$

where $F=3$ is the number of SUSY families and $H=5$ is the number of light $SU(2)_L$ scalar doublets present in the model.

Our approach is the known one [3] of using the experimental inputs [10] for $\alpha_1^{-1}(m_Z) = 98.330 \pm 0.091$ and $\alpha_2^{-1} = 29.517 \pm 0.043$ in Eqs. (3) for $i=1,2$ in order to calculate values for M and α, and then use those results in the other Eq. (3) ($i=3$) in order to predict a value for $\alpha_3(m_Z)$. When the algebra is done we get $M \sim 1.5\times 10^{18}$ GeV and $\alpha^{-1} = 14.86$ which in turn implies $\alpha_3(m_Z) = 0.083$ which is about 30% off the experimental value [10] $\alpha_3^{\exp}(m_Z) = 0.119 \pm 0.017$.

Next let us look for solution to Eqs. (3) including the second order effects. We then use $\Delta_i = \delta_i/12\pi$, ($\delta_i = 0,2,3$ for $i=1,2,3$, respectively, the $\overline{\text{MS}}$ to $\overline{\text{DR}}$ renormalization scheme conversion factor), the two loop beta functions [9]:

$$8\pi^2\begin{pmatrix} b_{11}^1 & b_{12}^1 & b_{13}^1 \\ b_{21}^1 & b_{22}^1 & b_{23}^1 \\ b_{31}^1 & b_{32}^1 & b_{33}^1 \end{pmatrix} = \begin{pmatrix} -\frac{190}{27}F - \frac{1}{2}H & -2F - \frac{3}{2}H & -\frac{88}{9}F \\ -\frac{2}{3}F - \frac{1}{2}H & 24 - 14F - \frac{7}{2}H & -8F \\ -\frac{11}{9}F & -3F & 54 - \frac{68}{3}F \end{pmatrix} \tag{5}$$

($F=3$ and $H=5$ as before), and introduce the SUSY partners of the known particles in the SM at the weak scale m_Z in order to take into account low energy threshold effects [3]. When the algebra is done we get $M \sim 3 \times 10^{18}$ GeV, $\alpha^{-1} = 16.76$ and $\alpha_3(m_Z) = 0.128$, this last value within the experimental limits allowed by $\alpha_3^{\rm exp}(m_Z)$. [The solution quoted has a strong dependence on the H value; as a matter of fact, $H = 4$ produces a very small value for $\alpha_3(m_Z)$].

This amazing result suffers from the flaw that the GUT scale predicted is almost one order of magnitude greater than $M_{\rm string}$, where gravity becomes at least as important as the other interactions and can not be ignored. Now, if we claim that $M_{\rm string}$ is not 4×10^{17} GeV, but a smaller value (something in between 1 TeV and 10^{11} GeV) coming from the nonperturbative effects of the string [11], then the entire idea of a GUT must be reconsidered. A more reasonable approach is to assume that even the non perturbative effects in the string are at most of the same order of the perturbative ones (which are small at this scale as we will see next). If this is the case then we may argue that other effects as for example contributions from Kaluza-Klein (KK) modes, or extra dimensions, are tractable and may slightly change the perturbative GUT scale and the value for $\alpha_3(m_Z)$. Lets us see this with the following example: if we use for Δ_i the expression [12]

$$\Delta_i = \frac{\delta_i}{12\pi} + \tilde{b}_i \left\{ \frac{1}{2\pi} \left[\left(\frac{M}{M_{\rm string}} \right) - 1 - \ln\left(\frac{M}{M_{\rm string}} \right) \right] \right\}, \tag{6}$$

where $\tilde{b}_i$ are the beta functions for the KK modes, and assume that the only KK modes present are the gauge bosons and an $SU(2)_L$ doublet of scalar fields, then we get for solution to the new set of equations $M \sim 1.28 \times 10^{18}$ GeV and $\alpha_3(m_Z) = 0.114$; so the net effect of this KK modes is to lower a little the GUT scale and to bring $\alpha_3(m_Z)$ closer to its experimental value. Other KK modes may do the opposite, but the net effect will be small because $M \sim M_{\rm string}$.

4. Concluding remarks

In this note we have presented various aspects of a new SUSY-GUT which unifies, in one single step, the three gauge couplings of the SM at a mass scale 10^{19} GeV $> M_{\rm GUT} \geq M_{\rm string}$. We believe this model opens a door in the so called string-GUT problem [13], due to the fact that it uses only fundamental irreps (and their conjugates) for scalar and spinor fields. In addition, when we compare our normalization coefficients c_i with the Kac-Moody levels of the four dimensional string, we have that $\kappa_i = c_i^{-1}$, which for $c_2 = 1$ and $c_3 = \frac{1}{2}$ implies that only level one and two could be needed when the ten dimensional SUSY-string is compactified to four dimensions. From the literature [14] we know that it is simple to compactify at levels $\kappa = 1,2$ and produce at the same time massless states in the fundamental irreps of the gauge group.

Proton decay is highly suppressed in the context of this model: the gauge bosons are integrally charged and can not mediate proton decay, and there are no Higgs scalars multiplets of the form $Z_4 \phi(3^*,1,3,1)$ which are the only ones which couple to both, quarks and leptons at tree level.

By imposing the validity of the extended survival hypothesis [15], the doublet-triplet Higgs splitting problem, present in GUT $SU(5)$ and its extensions, is lessened in our model, since the representations containing $SU(3)_c$ Higgs field triplets which are $SU(2)_{L(R)}$ doublets do not develop VEV at all. also the chiral color Higgs fields are either quaits or quones of $SU(3)_c$, with only the quones developing VEV and existing at the low energy scale.

It is worth mentioning the peculiar way in which the seesaw mechanism for the neutrinos is implemented in the context of the model, via mixings with the right-handed neutrino field ν^c (coupled with $SU(2)_R$ scalar singlets instead of triplets as it is usually done), and with the peculiar sterile quone D^0 which is a SM singlet. Also, besides the usual tiny massive neutrinos, there is an extra light particle in each family, it is the sterile M^0 which mixes with ν when $w \neq 0$. Those particles which may contribute to the dark matter of the universe, but very little to nucleosynthesis [16], are the right ingredients needed to explain the neutrino puzzle [17]; that is, to explain the neutrino oscillations in the sun, in the atmosphere, and at the LSND [18] experiment in los Alamos [19].

The fact that $H = 5$ is used, instead of other value, is not arbitrary. Indeed, the suppression of any

Higgs field $SU(2)_L$ doublet with VEV of order $v(v')$ in our analysis, will imply either a zero mass for a known particle (up or down quark and electron), or a failure in the implementation of the see-saw mechanism. So, to take $H \geq 5$ is compulsory, but $H > 5$ is redundant.

What is the advantage of moving from $[SU(3)]^3$ to $[SU(3)]^4$? As it can be seen from the second paper in Ref. [7], it is very difficult to get a decent mass spectrum for the known particles in the trinification model (some particular assumptions on the radiative corrections of the model must be made). On the contrary, the mass spectrum in our model comes easily at tree level, for a reduce set of scalar fields.

Why SUSY $[SU(3)]^4$ rather than the non-SUSY version? Because the non-SUSY version of $[SU(3)]^4$ does not unify the gauge coupling constants, unless a very large amount of Higgs field doublets is introduced ($H = 27$).

Finally let us mention that the VEV structure of the Higgs scalars used is the minimum compatible with a consistent mass spectrum. To increase the number of possible VEV will produce tiny see-saw masses of order v^2/V to the electron or to the bottom quark. To reduce the number of possible VEV will produce zero masses to some known particles. It will be very nice if the pattern of VEV we used can be obtained from the minimum of the scalar potential, but such analysis is beyond the scope of the present work.

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