# Mass scales and stability of the proton in $[\mathrm{SU}(6)]^{\mathbf{3}} \times \boldsymbol{Z}_{3}$ 

Juan B. Flórez<br>Departamento de Física, Centro de Investigación y Estudios Avanzados del I.P.N., Apartado Postal 14-740, 07000 México D.F., México and Departamento de Física, Universidad de Nariño, A.A. 1175 San Juan de Pasto, Colombia<br>William A. Ponce<br>Departamento de Física, Centro de Investigación y Estudios Avanzados del I.P.N., Apartado Postal 14-740, 07000 México D.F., México and Departamento de Física, Universidad de Antioquia, A.A. 1226, Medellín, Colombia<br>Arnulfo Zepeda<br>Departamento de Física, Centro de Investigación y Estudios Avanzados del I.P.N., Apartado Postal 14-740, 07000 México D.F., México

(Received 25 May 1993)


#### Abstract

We prove that the proton is stable in the gauge model $[\mathrm{SU}(6)]^{3} \times Z_{3}$ which unifies nongravitational forces with flavors, broken spontaneously by a minimal set of Higgs fields and vacuum expectation values down to $\mathrm{SU}(3)_{C} \otimes \mathrm{U}(1)_{\mathrm{EM}}$. We also compute the evolution of the gauge coupling constants and show how agreement with precision data can be obtained.


PACS number(s): $12.10 . \mathrm{Dm}, 12.15 . \mathrm{Ff}, 13.30 . \mathrm{Ce}$

Recently we proposed [1] a grand unification model of forces and flavors based on the gauge group $G=[\mathrm{SU}(6)]^{3} \times$ $Z_{3}$. Our aim has been to provide some clues for the explanation of the intriguing fermion mass spectrum and mixing parameters.

The fermion content of our model includes in a single irreducible representation of $G$ the three families of known fermions, each family being defined by the dynamics of the $\mathrm{SU}(3)_{C} \otimes \mathrm{SU}(2)_{L} \otimes \mathrm{SU}(2)_{R} \otimes \mathrm{U}(1)_{Y_{(B-L)}}$ gauge group. This last group is the left-right symmetric (LRS) extension of the $\mathrm{SU}(3)_{C} \otimes \mathrm{SU}(2)_{L} \otimes \mathrm{U}(1)_{Y}$ standard model (SM).
Explicitly, $G \equiv \mathrm{SU}(6)_{L} \otimes \mathrm{SU}(6)_{C} \otimes \mathrm{SU}(6)_{R} \times Z_{3}$ [1], where $\operatorname{SU}(6)_{C}$ is a vectorlike group which includes three hadronic and three leptonic colors. $\mathrm{SU}(6)_{C}$ includes as a subgroup the $\mathrm{SU}(3)_{C} \otimes \mathrm{U}(1)_{Y_{(B-L)}}$ of the LRS model. $\mathrm{SU}(6)_{L} \otimes \mathrm{SU}(6)_{R}$ is the left-right symmetric flavor group which includes the $\mathrm{SU}(2)_{L} \otimes \mathrm{SU}(2)_{R}$ gauge group of the LRS model.

The 105 gauge fields (GF's) and the 108 Weyl fermions fields in $G$ are explicitly depicted in Ref. [1]. Let us de-
scribe here some of them: The 105 GF's can be divided in two sets: 70 of them belonging to $\mathrm{SU}(6)_{L} \otimes \mathrm{SU}(6)_{R}$ and 35 associated with $\mathrm{SU}(6)_{C}$. The first set includes $W_{L}^{ \pm}$and $Z_{L}^{0}$, the GF's of the known weak interactions, plus the GF's associated with the postulated right weak interaction, plus the GF's of the horizontal interactions, etc. All of them are $\mathrm{SU}(3)_{C}$ singlets and have electrical charges 0 or $\pm 1$. The second set includes the eight gluon fields of $\mathrm{SU}(3)_{C}$; nine leptoquark GF's, $X_{i}, Y_{i}$, and $Z_{i}$, $i=1,2,3$ with electrical charges $-2 / 3,1 / 3$, and $-2 / 3$, respectively, another nine leptoquark GF's charge conjugated to the previous ones, six diquark GF's, $P_{a}^{ \pm}, P^{0}$, and $\tilde{P}^{0}, a=1,2$, with electrical charges as indicated, and three GF's associated with diagonal generators in $\mathrm{SU}(6)_{C}$, where $B_{Y_{(B-L)}}$, the GF's associated with the $\mathrm{U}(1)_{Y_{(B-L)}}$ factor in the LRS model, is one of them.

The known fermion fields are included in $\psi(108)_{L}=$ $Z_{3} \psi(6,1, \overline{6}) \equiv \psi(6,1, \overline{6})_{L}+\psi(1, \overline{6}, 6)_{L}+\psi(\overline{6}, 6,1)_{L}$, with quantum numbers with respect to $\left(\mathrm{SU}(3)_{C}, \mathrm{SU}(2)_{L}\right.$, $\left.\mathrm{U}(1)_{Y}\right)$ given by

$$
\begin{aligned}
& \psi(\overline{6}, 6,1) \equiv \psi_{a}^{\alpha}: 3(3,2,1 / 3) \oplus 6(1,2,-1) \oplus 3(1,2,1) \\
& \psi(1, \overline{6}, 6) \equiv \psi_{\alpha}^{A}: 3(\overline{3}, 1,-4 / 3) \oplus 3(\overline{3}, 1,2 / 3) \oplus 6(1,1,2) \oplus 9(1,1,0) \oplus 3(1,1,-2) \\
& \psi(6,1, \overline{6}) \equiv \psi_{A}^{a}: 9(1,2,1) \oplus 9(1,2,-1)
\end{aligned}
$$

As is clear, $a, b, \ldots ; A, B, \ldots, \alpha, \beta, \ldots=1, \ldots, 6$ label $\mathrm{SU}(6)_{L}$, $\mathrm{SU}(6)_{R}$, and $\mathrm{SU}(6)_{C}$ tensor indices, respectively.

The analysis done in Ref. [1] shows that the most economical set of Higgs fields (HF's) and vacuum expectation values (VEV's) which breaks the symmetry from $G$ down to $\mathrm{SU}(3)_{C} \otimes \mathrm{U}(1)_{\mathrm{EM}}$ and at the same time pro-
duces what we called the modified horizontal survival hypothesis is formed by

$$
\begin{equation*}
\phi_{1}=\phi(675)=\phi_{1,[a, b]}^{[A, B]}+\phi_{1,[A, B]}^{[\alpha, \beta]}+\phi_{1,[\alpha, \beta]}^{[a, b]} \tag{1}
\end{equation*}
$$

with VEV's in the directions $[a, b]=[1,6]=-[2,5]=$
$-[3,4],[A, B]$ similar to $[a, b]$ and $[\alpha, \beta]=[5,6]$,

$$
\begin{equation*}
\phi_{2}=\phi(1323)=\phi_{2,\{a, b\}}^{\{A, B\}}+\phi_{2,\{A, B\}}^{\{\alpha, \beta\}}+\phi_{2,\{\alpha, \beta\}}^{\{a, b\}} \tag{2}
\end{equation*}
$$

with VEV's in the directions $\{a, b\}=\{1,4\}=-\{2,3\}$, $\{A, B\}$ similar to $\{a, b\}$ and $\{\alpha, \beta\}=\{4,5\}$,

$$
\begin{equation*}
\phi_{3}=\phi^{\prime}(675)=\phi_{3,[a, b]}^{[A, B]}+\phi_{3,[A, B]}^{[\alpha, \beta]}+\phi_{3,[\alpha, \beta]}^{[\alpha, b]} \tag{3}
\end{equation*}
$$

with VEV's such that $\left\langle\phi_{3,[a, b]}^{[A, B]}\right\rangle=\left\langle\phi_{3,[\alpha, \beta]}^{[a, b]}\right\rangle=0$, and $\left\langle\phi_{3,[A, B]=[4,6]}^{[\alpha, \beta]=[4,6]}\right\rangle \equiv M_{R}$,

$$
\begin{equation*}
\phi_{4}=\phi(108)=\phi_{4, \alpha}^{A}+\phi_{4, a}^{\alpha}+\phi_{4, A}^{a} \tag{4}
\end{equation*}
$$

with VEV's such that $\left\langle\phi_{\alpha}^{A}\right\rangle=\left\langle\phi_{a}^{\alpha}\right\rangle=0$ and $\left\langle\phi_{A}^{a}\right\rangle \equiv$ $M_{Z}$, with values different from zero only in the directions $\left\langle\phi_{2}^{2}\right\rangle=\left\langle\phi_{4}^{2}\right\rangle=\left\langle\phi_{6}^{2}\right\rangle=\left\langle\phi_{2}^{4}\right\rangle=\left\langle\phi_{4}^{4}\right\rangle=\left\langle\phi_{6}^{4}\right\rangle=\left\langle\phi_{2}^{6}\right\rangle=$ $\left\langle\phi_{4}^{6}\right\rangle=\left\langle\phi_{6}^{6}\right\rangle=M_{Z} \sim 10^{2} \mathrm{GeV}$.

In Eqs. (1)-(3), the symbols $\{.,$.$\} and [.,.] indicate$ symmetrization and antisymmetrization, respectively, of the indices inside the brackets. The mass hierarchy suggested in Ref. [1] is $\left\langle\phi_{3}\right\rangle>\left\langle\phi_{1}\right\rangle \simeq\left\langle\phi_{2}\right\rangle \gg M_{Z} \sim 10^{2}$ GeV .

For the renormalization group equation (RGE) analysis which follows, we adopt the working conditions known as "the survival hypothesis" [2] and "the extended survival hypothesis" [3]. The survival hypothesis claims that [2] at each energy scale, the only fermion fields which are relevant are those belonging to chiral representations of the unbroken symmetries. The extended survival hypothesis claims that [3] at each energy scale the only scalars which are relevant are those that develop VEV's at that scale and at lower mass scales. Both hypothesis are satisfied if a particular selection of scalar fields and VEV's is made, and appropriate terms in the scalar potential and Yukawa Lagrangian are included.

When the symmetry is broken in two steps, $G \xrightarrow{M}$ $\mathrm{SU}(3)_{C} \otimes \mathrm{SU}(2)_{L} \otimes \mathrm{U}(1)_{Y} \xrightarrow{M_{\mathrm{z}}} \mathrm{SU}(3)_{C} \otimes \mathrm{U}(1)_{\mathrm{EM}}, \quad$ where $M=\left\langle\phi_{1}\right\rangle+\left\langle\phi_{2}\right\rangle+\left\langle\phi_{3}\right\rangle$ and $M_{Z}=\left\langle\phi_{4}\right\rangle$, the one loop running coupling constants of the standard model satisfy

$$
\begin{equation*}
\alpha_{i}^{-1}\left(M_{Z}\right)=\alpha_{i}^{-1}(M)-b_{i}^{0} \ln \left(M / M_{Z}\right) \tag{5}
\end{equation*}
$$

where $\alpha_{i}=g_{i}^{2} / 4 \pi, i=1,2,3$ refers to $\mathrm{U}(1)_{Y}, \mathrm{SU}(2)_{L}$, and $\mathrm{SU}(3)_{C}$, respectively, and

$$
\begin{align*}
b_{i}^{\kappa}=\{ & \left\{\frac{11}{3} C_{i}^{\kappa}(\text { vectors })-\frac{2}{3} C_{i}^{\kappa}(\text { Weyl fermions })\right. \\
& \left.-\frac{1}{6} C_{i}^{\kappa}(\text { scalars })\right\} / 4 \pi \tag{6}
\end{align*}
$$

with $C_{i}^{\kappa}(\cdots)$ the index of the representation to which the $(\cdots)$ particles are assigned. For a complex field the value of $C_{i}^{\kappa}$ (scalars) should be doubled. With the normalization of the generators of $G$ such that $\alpha_{1}(M)=\alpha_{2}(M)=$ $\alpha_{3}(M)$, the relationship

$$
\begin{equation*}
\alpha_{\mathrm{EM}}=\frac{1}{3} \alpha_{2} \sin ^{2} \theta_{W}=\frac{3}{14} \alpha_{1} \cos ^{2} \theta_{W} \tag{7}
\end{equation*}
$$

where $\theta_{W}$ is the weak mixing angle, is valid at all energy scales. This last equation implies also that

$$
\begin{equation*}
3 \alpha_{\mathrm{EM}}^{-1}=14 \alpha_{1}^{-1}+9 \alpha_{2}^{-1} \tag{8}
\end{equation*}
$$

Equations (5)-(8) give straightforwardly
$\frac{3}{23} \alpha_{E M}^{-1}\left(M_{Z}\right)=\alpha_{3}^{-1}\left(M_{Z}\right)+\left(b_{3}^{0}-\frac{14}{23} b_{1}^{0}-\frac{9}{23} b_{2}^{0}\right) \ln \left(M / M_{Z}\right)$
and

$$
\begin{align*}
\sin ^{2} \theta_{W}\left(M_{Z}\right)= & 3 \alpha_{\mathrm{EM}}\left(M_{Z}\right)\left[\alpha_{3}^{-1}\left(M_{Z}\right)\right. \\
& \left.+\left(b_{3}^{0}-b_{2}^{0}\right) \ln \left(M / M_{Z}\right)\right] \tag{10}
\end{align*}
$$

where $b_{3}^{0}=(11-4) / 2 \pi, b_{2}^{0}=\left[\frac{22}{9}-\frac{4}{3}\left(3-n_{2}^{0}\right)-\frac{N_{H}}{18}\right] / 2 \pi$, $b_{1}^{0}=-\left[\frac{4}{3}\left(3-n_{1}^{0}\right)+\frac{N_{H}}{28}\right] / 2 \pi, \quad N_{H}=9$ is the number of low-energy Higgs fields doublets in $\left\langle\phi_{4}\right\rangle$, and $n_{2}^{0}=2, n_{1}^{0}=27 / 14$ are related to the number of fermion fields which decouple from $\psi(108)_{L}$ according to the survival hypothesis and the Appelquist-Carrazone theorem [4] $\left[n_{1}^{0}=n_{2}^{0}=0\right.$ when all the fermion fields in $\psi(108)_{L}$ contribute to $\left.b_{1,2}^{0}\right]$.

Substituting in the last two equations the experimental values [5] $\sin ^{2} \theta_{W}\left(M_{Z}\right)=0.233, \alpha_{E M}^{-1}\left(M_{Z}\right)=127.9$, and $\alpha_{3}\left(M_{Z}\right)=0.122$ we get from Eq. (9) $\ln \left(M / M_{Z}\right)=6.3$, while from Eq. (10) $\ln \left(M / M_{Z}\right)=1.1$ which are widely incompatible solutions. Therefore the model with only two mass scales is excluded.

When the symmetry is broken in three steps: $G \xrightarrow{M}$ $G_{L} \otimes G_{C} \otimes G_{R} \otimes \cdots \xrightarrow{M_{L}} \mathrm{SU}(3)_{C} \otimes \mathrm{SU}(2)_{L} \otimes \mathrm{U}(1)_{Y}$ $\xrightarrow{M_{Z}} \mathrm{SU}(3)_{C} \otimes \mathrm{U}(1)_{\mathrm{EM}}$ where $M \gg M_{I} \gg M_{Z}=\left\langle\phi_{4}\right\rangle$, the one loop runming coupling constants of the SM satisfy

$$
\begin{equation*}
\alpha_{i}^{-1}\left(M_{Z}\right)=\alpha_{i}^{-1}(M)-b_{i}^{0} \ln \left(M_{I} / M_{Z}\right)-b_{i}^{1} \ln \left(M / M_{I}\right) \tag{11}
\end{equation*}
$$

It is easy then to show that

$$
\begin{align*}
\frac{3}{23} \alpha_{\mathrm{EM}}^{-1}\left(M_{Z}\right)= & \alpha_{3}^{-1}\left(M_{Z}\right)+\left(b_{3}^{0}-\frac{14}{23} b_{1}^{0}-\frac{9}{23} b_{2}^{0}\right) \ln \left(\frac{M_{I}}{M_{Z}}\right) \\
& +\left(b_{C}^{1}-\frac{14}{23} b_{Y}^{1}-\frac{9}{23} b_{L}^{1}\right) \ln \left(\frac{M}{M_{I}}\right) \tag{12}
\end{align*}
$$

and
$\sin ^{2} \theta_{W}\left(M_{Z}\right)$

$$
\begin{gather*}
=3 \alpha_{\mathrm{EM}}\left(M_{Z}\right)\left[\alpha_{3}^{-1}\left(M_{Z}\right)+\left(b_{3}^{0}-b_{2}^{0}\right) \ln \left(\frac{M_{I}}{M_{Z}}\right)\right. \\
\left.+\left(b_{C}^{1}-b_{L}^{1}\right) \ln \left(\frac{M}{M_{I}}\right)\right] \tag{13}
\end{gather*}
$$

where $b_{i}^{0}, i=1,2,3$ are the same as above, but $b_{i}^{1}, i=$ $C, Y, L$ depend upon the structure of the subgroup $G_{L} \otimes$ $G_{C} \otimes G_{R} \otimes \cdots \equiv G_{I}$.

Equation (13) indicates that $G_{I}$ cannot be $\mathrm{SU}(3)_{C} \otimes \mathrm{SU}(2)_{L} \otimes \mathrm{SU}(2)_{R} \otimes \mathrm{U}(1)_{Y(B-L)}$ because if that were the case $b_{C}^{1}=b_{3}^{0}$ and $b_{L}^{1}=b_{2}^{0}$, and since the first entry of the right-hand side of Eq. (13) has an experimental value of $0.192, M_{I} \simeq 3 M_{Z}$ is required to be satisfied and, therefore, there is a very small value for the masses of the gauge bosons associated with $\mathrm{SU}(2)_{R}$. With the minimal set of Higgs fields $G_{I}$ contains therefore flavor-changing
neutral currents and thus $M_{I}$ has to be greater than 100 TeV . The first two terms of the right-hand side of Eq. (13) have then a lower bound of 0.36 , and the experimental value for $\sin ^{2} \theta_{W}\left(M_{Z}\right)$ requires that $\left(b_{C}^{1}-b_{L}^{1}\right)<0$ which is not satisfied by the minimal set of Higgs fields and VEV's [1].

We are therefore led to consider introducing a minimum change in the set of HF's and/or VEV's such that the new set properly breaks the symmetry, guarantees the survival hypothesis, produces appropriate values for $\sin ^{2} \theta_{W}\left(M_{Z}\right)$, and satisfies the mass hierarchy $M \gg M_{I} \gg M_{Z} \sim 10^{\mathbf{2}} \mathrm{GeV}$. An analysis of Table I shows that a symmetry-breaking pattern in three steps with $G_{I}=\mathrm{SU}(6)_{L} \otimes \mathrm{SU}(4)_{C} \otimes \mathrm{U}(1)_{Y} \otimes \cdots$ produces consistent results provided we introduce the following two changes. First, add a new set of Higgs fields

$$
\begin{equation*}
\phi_{2}^{\prime}=\phi^{\prime}(1323)=\phi_{2,\{a, b\}}^{\prime,\{A, B\}}+\phi_{2,\{A, B\}}^{\prime,\{\alpha, \beta\}}+\phi_{2,\{\alpha, \beta\}}^{\prime,\{a, b\}} \tag{14}
\end{equation*}
$$

with VEV's in the directions $\{a, b\}=\{3,6\}=-\{4,5\}$, $\{A, B\}$ similar to $\{a, b\}$ and $\{\alpha, \beta\}=\{5,5\}$, and second, orient the VEV's such that $\left\langle\phi_{2,\{\alpha, \beta\}=\{5,5\}}^{\prime,\{a, b\}}\right\rangle=$ $\left\langle\phi_{2,\{a, b\}}^{\prime,\{A, B\}}\right\rangle=\left\langle\phi_{2,\{a, b\}}^{\{A, B\}}\right\rangle=0$. For this particular choice of VEV's we have that $b_{C}^{1}=\left(\frac{88}{3}-\frac{2 \times 12}{3}-\frac{148}{3}\right) / 4 \pi$, $b_{L}^{1}=\left(\frac{132}{3}-\frac{2 \times 12}{3}-\frac{107}{3}\right) / 4 \pi$ and $b_{Y}^{1}=-\left(\frac{2 \times 12}{3}+\frac{9}{14}\right) / 4 \pi$, where the extended survival hypothesis [3] was taken into account for the contribution of the HF's.

As can be seen, the Higgs fields play a fundamental role in Eqs. (12) and (13). Notice also that the symmetrybreaking pattern is achieved with $M=\left\langle\phi_{3}\right\rangle, M_{I}=$ $\left\langle\phi_{1}\right\rangle+\left\langle\phi_{2}\right\rangle+\left\langle\phi_{2}^{\prime}\right\rangle, M_{Z}=\left\langle\phi_{4}\right\rangle$, and that $\phi_{3}$ plays no role in the evolution of the gauge coupling constants.

Substituting now in Eqs. (12) and (13) the experimental values for $\sin ^{2} \theta_{W}\left(M_{Z}\right), \alpha_{E M}\left(M_{Z}\right), \alpha_{3}\left(M_{Z}\right)$ we obtain the equations

$$
\begin{aligned}
& 1.10=1.02 \ln \left(\frac{M_{I}}{M_{Z}}\right)-2.26 \ln \left(\frac{M}{M_{I}}\right), \\
& 7.83=1.25 \ln \left(\frac{M_{I}}{M_{Z}}\right)-1.82 \ln \left(\frac{M}{M_{I}}\right),
\end{aligned}
$$

which for $M_{Z}=91 \mathrm{GeV}$ have the solutions $M_{I} \sim 10^{9}$ GeV and $M \sim 10^{12} \mathrm{GeV}$. These results are in good agreement with those obtained from the analysis of the generational seesaw mechanism in this model [6].

## STABILITY OF THE PROTON

## A. Baryon number for the particles

The elementary particles in the model are the ones associated with the 105 GF's, the 108 Weyl fields in $\psi(108)_{L}$ and the 4104 HF's in $\phi_{i}, i=1-4$ and $\phi_{2}^{\prime}$. Now, all the elementary particles in our model have a well-defined Baryon Number $B$. Let us note the following.
(1) The GF. The 70 GF's associated with $\mathrm{SU}(6)_{L}$ $\otimes \operatorname{SU}(6)_{R}$ have $B=0$. For $\mathrm{SU}(6)_{C}$ we have that the 9 leptoquarks have $B$ equal to $1 / 3$, and the other 17 GF's have $B=0$ (including the 8 gluon fields).
(2) The Weyl fermion fields. The quark fields in $\psi(\overline{6}, 6,1)_{L}$ have $B=1 / 3$, the quark fields in $\psi(1, \overline{6}, 6)_{L}$ have $B=-1 / 3$ and all the other fields in $\psi(108)_{L}$ have $B=0$.
(3) The HF. B for the 4104 HF 's of the model is given in Table II.

## B. Baryon number as a symmetry of the model

In the subspace of the fundamental representation of $\mathrm{SU}(6)_{C} B$ can be associated with the $6 \times 6$ diagonal ma$\operatorname{trix} B=\operatorname{Diag}(1 / 3,1 / 3,1 / 3,0,0,0)$. This matrix does not correspond to a generator of $\mathrm{SU}(6)_{C}$ neither of $G$. Now, the full Lagrangian $\mathcal{L}=\mathcal{L}_{\text {gauge }}+\mathcal{L}_{\text {Higgs }}+\mathcal{L}_{\text {Yukawa }}$ has a $\mathrm{U}(1)_{\chi}$ global symmetry, where $\chi$ is a constant whose magnitude depends solely on the label of the $\mathrm{SU}(6)_{C}$ representation. For example $\chi=1$ for $\psi(\overline{6}, 6,1), \chi=0$ for $G(1,35,1), \chi=-2$ for $\phi(1, \overline{15}, 15)$, etc. Conveniently normalized, the $\mathrm{U}(1)_{\chi}$ generator may be written in the fundamental representation of $\mathrm{SU}(6)_{C}$ as $\chi=$ $\operatorname{Diag}(1,1,1,1,1,1) / \sqrt{12}$, which is not an element of the Lie algebra of $G$ either.

On the other hand, in the Lie algebra of $G$ there is a generator, an element of the $\mathrm{SU}(6)_{C}$ subalgebra, of the form

TABLE I. The index of the scalars for $\mathrm{SU}(4)_{C}$ and $\mathrm{SU}(6)_{L}$.

| $\langle\phi\rangle$ | $\mathrm{SU}(4)_{C}$ | $\mathrm{C}_{C}^{1}$ (scalars) | $\mathrm{SU}(6)_{L}$ | $\mathrm{C}_{L}^{1}(\mathrm{scalars})$ |
| :---: | :---: | :---: | :---: | :---: |
| $\left\langle\phi_{1,[a, b]}^{[A, B]}\right\rangle$ |  | 0 | fourteen 15's | 56 |
| $\left\langle\phi_{1,[A, B]}^{[\alpha, \beta]}\right\rangle$ | fourteen 4's | 14 |  | 0 |
| $\left\langle\phi_{1,[\alpha, \beta]}^{[a, b]}\right\rangle$ | fifteen 4's | 15 | four 15's | 16 |
| $\left\langle\phi_{2,\{a, b\}}^{\{A, B\}}\right\rangle$ |  | 0 | fourteen 21's | 112 |
| $\left\langle\phi_{2,\{A, B\}}^{\{\alpha, \beta\}=\{4,5\}}\right\rangle$ | fourteen 4's | 14 |  | 0 |
| $\left\langle\phi_{2,\{\alpha, \beta\}\{4,5\}}^{\{a, b\}}\right\rangle$ | twenty-one 4's | 21 | four 21's | fourteen 21's |
| $\left\langle\phi_{2,\{a, b\}}^{\prime\{A, B\}}\right\rangle$ |  | 0 |  | 112 |
| $\left\langle\phi_{2,\{A, B\}=\{5,5\}}^{\prime\{\alpha, \beta,}\right\rangle$ | fourteen 10's | 84 | ten 21's | 0 |
| $\left\langle\phi_{2,\{\alpha, \beta\}=\{5,5\}}^{\prime\{a, b\}}\right\rangle$ | twenty-one 10's | 126 | three 6's | 80 |
| $\left\langle\phi_{4, A}^{a}\right\rangle$ | 0 |  | 3 |  |

TABLE II. Baryon number of the 4104 Higgs fields.

| $\phi$ | $\alpha, \beta$ | $a, b$ | $A, B$ | B |
| :---: | :---: | :---: | :---: | :---: |
| $\overline{\phi_{1(3),[a, b]}^{[A, B]}}$ |  | $a, b=1, \ldots, 6$ | $A, B=1, \ldots, 6$ | 0 |
| $\phi_{1(3),[A, B]}^{[\alpha, \beta]}$ | $\alpha, \beta=1,2,3$ |  | $A, B=1, \ldots, 6$ | $-1 / 3$ |
|  | $\alpha, \beta=4,5,6$ |  | $A, B=1, \ldots, 6$ | 0 |
|  | $\alpha=1,2,3 ; \beta=4,5,6$ |  | $A, B=1, \ldots, 6$ | 1/3 |
| $\overline{\phi_{1(3),[\alpha, \beta]}^{[a, b]}}$ | $\alpha, \beta=1,2,3$ | $a, b=1, \ldots, 6$ |  | 1/3 |
|  | $\alpha, \beta=4,5,6$ | $a, b=1, \ldots, 6$ |  | 0 |
|  | $\alpha=1,2,3 ; \beta=4,5,6$ | $a, b=1, \ldots, 6$ |  | -1/3 |
| $\phi_{2,\{a, b\}}^{(1)\{A, B\}}$ |  | $a, b=1, \ldots, 6$ | $A, B=1, \ldots, 6$ | 0 |
| $\phi_{2,\{A, B\}}^{(1)\{\alpha, \beta\}}$ | $\alpha, \beta=1,2,3$ |  | $A, B=1, \ldots, 6$ | 2/3 |
|  | $\alpha, \beta=4,5,6$ |  | $A, B=1, \ldots, 6$ | 0 |
|  | $\alpha=1,2,3 ; \beta=4,5,6$ |  | $A, B=1, \ldots, 6$ | 1/3 |
| $\phi_{2,\{\alpha, \beta\}}^{(\prime)\{a, b\}}$ | $\alpha, \beta=1,2,3$ | $a, b=1, \ldots, 6$ |  | $-2 / 3$ |
|  | $\alpha, \beta=4,5,6$ | $a, b=1, \ldots, 6$ |  | 0 |
|  | $\alpha=1,2,3 ; \beta=4,5,6$ | $a, b=1, \ldots, 6$ |  | -1/3 |
| $\phi_{4, A}^{a}$ |  | $a=1, \ldots, 6$ | $A=1, \ldots 6$ | 0 |
| $\phi_{4, a}^{\alpha}$ | $\alpha=1,2,3$ | $a=1, \ldots, 6$ |  | 1/3 |
|  | $\alpha=4,5,6$ | $a=1, \ldots, 6$ |  | 0 |
| $\phi_{4, \alpha}^{\text {A }}$ | $\alpha=1,2,3$ |  | $A=1, \ldots, 6$ | $-1 / 3$ |
|  | $\alpha=4,5,6$ |  | $A=1, \ldots, 6$ | 0 |

$$
\begin{equation*}
B^{\prime}=\operatorname{Diag}(1,1,1,-1,-1,-1) / \sqrt{12} \tag{15}
\end{equation*}
$$

which distinguishes between quarks and leptons in our model. Therefore $B$ can be written as $B=\left[\chi+B^{\prime}\right] / \sqrt{3}$.

Since the elementary particles of this model have a well-defined $B$, it is obvious that in the unbroken theory the exchange of particles cannot break $B$. This statement is also true after breaking the symmetry due to the following two facts: The baryon number is not gauged (there is no gauge boson associated to $B$ ); $\phi_{i}, i=1-4$, and $\phi_{2}^{\prime}$ with the VEV's as stated do not break $B$ spontaneously. That is, $B\left\langle\phi_{i}\right\rangle=B\left\langle\phi_{2}^{\prime}\right\rangle=0, i=1,2,3,4$. Therefore, $B$ is conserved in our model [7]. Since $B$ is conserved, the proton is perturbatively stable.

Now, the single Goldstone boson associated with the broken orthogonal combination is absorbed by the massive gauge field associated with $B^{\prime}$. Therefore, there are
no physical Goldstone bosons, and there is no extra long range force. This mechanism in which a global symmetry emerges from the simultaneous breaking of a gauge and global symmetry is due to 't Hooft [8] and was implemented in the context of grand unified models in Ref. [9].
Finally we would like to mention that, contrary to baryon number, lepton number is violated in this model due to the fact that the GF's associated with the $\mathrm{U}(1)_{Y_{(B-L)}}$ generator is gauged. Therefore, neither $L,(B-L)$ or $(B+L)$ are conserved quantities.

This work was partially supported by CONACyT in México and COLCIENCIAS in Colombia. Two of us (W.A.P. and A.Z.) thank Gordon Kane for a useful conversation.
[1] A.H. Galeana, R. Martinez, W.A. Ponce, and A. Zepeda, Phys. Rev. D 44, 2166 (1991); W.A. Ponce and A. Zepeda, ibid. 48, 240 (1993).
[2] H. Georgi, Nucl. Phys. B156, 126 (1979); R. Barbieri and D.V. Nanopoulos, Phys. Lett. 91B, 369 (1980).
[3] F. Del Aguila, and L. Ibañez, Nucl. Phys. B177, 60 (1981); H. Georgi and S. Dimopoulos, Phys. Lett. 140B, 67 (1984).
[4] T. Appelquist and J. Carazzone, Phys. Rev. D 11, 2856 (1975).
[5] U. Amaldi et al., Phys. Lett. B 281, 374 (1992).
[6] W.A. Ponce, A. Zepeda, and R. Gaitán, Phys. Rev. D (to be published).
[7] J.B. Florez, Ph.D. thesis, CINVESTAV, 1993 (unpublished).
[8] G. 't Hooft, Nucl. Phys. B35, 167 (1971).
[9] P. Langacker, G. Segre, and A. Weldon, Phys. Lett. 73B, 87 (1978); Phys. Rev. D 18, 552 (1978); M. Gell-Mann, P. Ramond, and R. Slansky, Rev. Mod. Phys. 50, 721 (1978).

