Mass scales and stability of the proton in $[SU(6)]^3 \times Z_3$

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We prove that the proton is stable in the gauge model $[SU(6)]^3 \times Z_3$ which unifies nongravitational forces with flavors, broken spontaneously by a minimal set of Higgs fields and vacuum expectation values down to $SU(3)_C \otimes U(1)_{EM}$. We also compute the evolution of the gauge coupling constants and show how agreement with precision data can be obtained.

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Recently we proposed [1] a grand unification model of forces and flavors based on the gauge group $G=[SU(6)]^3 \times Z_3$. Our aim has been to provide some clues for the explanation of the intriguing fermion mass spectrum and mixing parameters.

The fermion content of our model includes in a single irreducible representation of G the three families of known fermions, each family being defined by the dynamics of the $SU(3)_C \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{Y_{(B-L)}}$ gauge group. This last group is the left-right symmetric (LRS) extension of the $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ standard model (SM).

Explicitly, $G \equiv SU(6)_L \otimes SU(6)_C \otimes SU(6)_R \times Z_3$ [1], where $SU(6)_C$ is a vectorlike group which includes three hadronic and three leptonic colors. $SU(6)_C$ includes as a subgroup the $SU(3)_C \otimes U(1)_{Y(B-L)}$ of the LRS model. $SU(6)_L \otimes SU(6)_R$ is the left-right symmetric flavor group which includes the $SU(2)_L \otimes SU(2)_R$ gauge group of the LRS model.

The 105 gauge fields (GF's) and the 108 Weyl fermions fields in G are explicitly depicted in Ref. [1]. Let us de-

scribe here some of them: The 105 GF's can be divided in two sets: 70 of them belonging to $SU(6)_L \otimes SU(6)_R$ and 35 associated with $SU(6)_C$. The first set includes W_L^{\pm} and Z_L^0 , the GF's of the known weak interactions, plus the GF's associated with the postulated right weak interaction, plus the GF's of the horizontal interactions, etc. All of them are $SU(3)_C$ singlets and have electrical charges 0 or \pm 1. The second set includes the eight gluon fields of $SU(3)_C$; nine leptoquark GF's, X_i , Y_i , and Z_i , i = 1, 2, 3 with electrical charges -2/3, 1/3, and -2/3, respectively, another nine leptoquark GF's charge conjugated to the previous ones, six diquark GF's, P_a^{\pm} , P^0 , and \tilde{P}^0 , a = 1, 2, with electrical charges as indicated, and three GF's associated with diagonal generators in $SU(6)_C$, where $B_{Y_{(B-L)}}$, the GF's associated with the $U(1)_{Y_{(B-L)}}$ factor in the LRS model, is one of them.

The known fermion fields are included in $\psi(108)_L = Z_3\psi(6,1,\bar{6}) \equiv \psi(6,1,\bar{6})_L + \psi(1,\bar{6},6)_L + \psi(\bar{6},6,1)_L$, with quantum numbers with respect to $(SU(3)_C, SU(2)_L, U(1)_Y)$ given by

$$\begin{split} \psi(\bar{6},6,1) &\equiv \psi_{a}^{\alpha} : 3(3,2,1/3) \oplus 6(1,2,-1) \oplus 3(1,2,1) ,\\ \psi(1,\bar{6},6) &\equiv \psi_{a}^{A} : 3(\bar{3},1,-4/3) \oplus 3(\bar{3},1,2/3) \oplus 6(1,1,2) \oplus 9(1,1,0) \oplus 3(1,1,-2) \\ \psi(6,1,\bar{6}) &\equiv \psi_{a}^{a} : 9(1,2,1) \oplus 9(1,2,-1) . \end{split}$$

As is clear, $a, b, ...; A, B, ..., \alpha, \beta, ... = 1, ..., 6$ label SU(6)_L, SU(6)_R, and SU(6)_C tensor indices, respectively.

The analysis done in Ref. [1] shows that the most economical set of Higgs fields (HF's) and vacuum expectation values (VEV's) which breaks the symmetry from Gdown to $SU(3)_C \otimes U(1)_{EM}$ and at the same time produces what we called the *modified horizontal survival hypothesis* is formed by

$$\phi_1 = \phi(675) = \phi_{1,[a,b]}^{[A,B]} + \phi_{1,[A,B]}^{[\alpha,\beta]} + \phi_{1,[\alpha,\beta]}^{[a,b]}$$
(1)

with VEV's in the directions [a,b] = [1,6] = -[2,5] =

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-[3,4], [A,B] similar to [a,b] and $[\alpha,\beta]=[5,6],$

$$\phi_2 = \phi(1323) = \phi_{2,\{a,b\}}^{\{A,B\}} + \phi_{2,\{A,B\}}^{\{\alpha,\beta\}} + \phi_{2,\{\alpha,\beta\}}^{\{a,b\}}$$
(2)

with VEV's in the directions $\{a, b\} = \{1, 4\} = -\{2, 3\}, \{A, B\}$ similar to $\{a, b\}$ and $\{\alpha, \beta\} = \{4, 5\},$

$$\phi_3 = \phi'(675) = \phi_{3,[a,b]}^{[A,B]} + \phi_{3,[A,B]}^{[\alpha,\beta]} + \phi_{3,[\alpha,\beta]}^{[a,b]}$$
(3)

with VEV's such that $\langle \phi_{3,[a,b]}^{[A,B]} \rangle = \langle \phi_{3,[\alpha,\beta]}^{[a,b]} \rangle = 0$, and $\langle \phi_{3,[A,B]=[4,6]}^{[\alpha,\beta]=[4,6]} \rangle \equiv M_R$,

$$\phi_4 = \phi(108) = \phi_{4,\alpha}^A + \phi_{4,a}^\alpha + \phi_{4,A}^a \tag{4}$$

with VEV's such that $\langle \phi_{\alpha}^{4} \rangle = \langle \phi_{\alpha}^{\alpha} \rangle = 0$ and $\langle \phi_{\alpha}^{a} \rangle \equiv M_{Z}$, with values different from zero only in the directions $\langle \phi_{2}^{2} \rangle = \langle \phi_{4}^{2} \rangle = \langle \phi_{6}^{2} \rangle = \langle \phi_{2}^{4} \rangle = \langle \phi_{4}^{4} \rangle = \langle \phi_{6}^{4} \rangle = \langle \phi_{6}^{6} \rangle = M_{Z} \sim 10^{2} \text{ GeV}.$

In Eqs. (1)–(3), the symbols $\{.,.\}$ and [.,.] indicate symmetrization and antisymmetrization, respectively, of the indices inside the brackets. The mass hierarchy suggested in Ref. [1] is $\langle \phi_3 \rangle > \langle \phi_1 \rangle \simeq \langle \phi_2 \rangle \gg M_Z \sim 10^2$ GeV.

For the renormalization group equation (RGE) analysis which follows, we adopt the working conditions known as "the survival hypothesis" [2] and "the extended survival hypothesis" [3]. The survival hypothesis claims that [2] at each energy scale, the only fermion fields which are relevant are those belonging to chiral representations of the unbroken symmetries. The extended survival hypothesis claims that [3] at each energy scale the only scalars which are relevant are those that develop VEV's at that scale and at lower mass scales. Both hypothesis are satisfied if a particular selection of scalar fields and VEV's is made, and appropriate terms in the scalar potential and Yukawa Lagrangian are included.

When the symmetry is broken in two steps, $G \xrightarrow{M} SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \xrightarrow{M_Z} SU(3)_C \otimes U(1)_{EM}$, where $M = \langle \phi_1 \rangle + \langle \phi_2 \rangle + \langle \phi_3 \rangle$ and $M_Z = \langle \phi_4 \rangle$, the one loop running coupling constants of the standard model satisfy

$$\alpha_i^{-1}(M_Z) = \alpha_i^{-1}(M) - b_i^0 \ln(M/M_Z), \tag{5}$$

where $\alpha_i = g_i^2/4\pi$, i = 1, 2, 3 refers to $U(1)_Y$, $SU(2)_L$, and $SU(3)_C$, respectively, and

$$b_i^{\kappa} = \{\frac{11}{3}C_i^{\kappa}(\text{vectors}) - \frac{2}{3}C_i^{\kappa}(\text{Weyl fermions}) \\ -\frac{1}{6}C_i^{\kappa}(\text{scalars})\}/4\pi$$
(6)

with $C_i^{\kappa}(\cdots)$ the index of the representation to which the (\cdots) particles are assigned. For a complex field the value of $C_i^{\kappa}(\text{scalars})$ should be doubled. With the normalization of the generators of G such that $\alpha_1(M) = \alpha_2(M) = \alpha_3(M)$, the relationship

$$\alpha_{\rm EM} = \frac{1}{3} \alpha_2 \sin^2 \theta_W = \frac{3}{14} \alpha_1 \cos^2 \theta_W, \tag{7}$$

where θ_W is the weak mixing angle, is valid at all energy scales. This last equation implies also that

$$3\alpha_{\rm EM}^{-1} = 14\alpha_1^{-1} + 9\alpha_2^{-1}.$$
 (8)

Equations (5)-(8) give straightforwardly

$$\frac{3}{23}\alpha_{\rm EM}^{-1}(M_Z) = \alpha_3^{-1}(M_Z) + (b_3^0 - \frac{14}{23}b_1^0 - \frac{9}{23}b_2^0)\ln(M/M_Z)$$
(9)

and

$$\sin^2 \theta_W(M_Z) = 3\alpha_{\rm EM}(M_Z) [\alpha_3^{-1}(M_Z) + (b_3^0 - b_2^0) \ln(M/M_Z)],$$
(10)

where $b_3^0 = (11-4)/2\pi$, $b_2^0 = [\frac{22}{9} - \frac{4}{3}(3-n_2^0) - \frac{N_H}{18}]/2\pi$, $b_1^0 = -[\frac{4}{3}(3-n_1^0) + \frac{N_H}{28}]/2\pi$, $N_H = 9$ is the number of low-energy Higgs fields doublets in $\langle \phi_4 \rangle$, and $n_2^0 = 2$, $n_1^0 = 27/14$ are related to the number of fermion fields which decouple from $\psi(108)_L$ according to the survival hypothesis and the Appelquist-Carrazone theorem [4] $[n_1^0 = n_2^0 = 0$ when all the fermion fields in $\psi(108)_L$ contribute to $b_{1,2}^0$].

Substituting in the last two equations the experimental values [5] $\sin^2\theta_W(M_Z) = 0.233$, $\alpha_{\rm EM}^{-1}(M_Z) = 127.9$, and $\alpha_3(M_Z) = 0.122$ we get from Eq. (9) $\ln(M/M_Z) = 6.3$, while from Eq. (10) $\ln(M/M_Z) = 1.1$ which are widely incompatible solutions. Therefore the model with only two mass scales is excluded.

When the symmetry is broken in three steps: $G \xrightarrow{M} G_L \otimes G_C \otimes G_R \otimes \cdots \xrightarrow{M_I} SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \xrightarrow{M_Z} SU(3)_C \otimes U(1)_{EM}$ where $M \gg M_I \gg M_Z = \langle \phi_4 \rangle$, the one loop running coupling constants of the SM satisfy

$$\alpha_i^{-1}(M_Z) = \alpha_i^{-1}(M) - b_i^0 \ln(M_I/M_Z) - b_i^1 \ln(M/M_I).$$
(11)

It is easy then to show that

$$\frac{3}{23}\alpha_{\rm EM}^{-1}(M_Z) = \alpha_3^{-1}(M_Z) + (b_3^0 - \frac{14}{23}b_1^0 - \frac{9}{23}b_2^0)\ln\left(\frac{M_I}{M_Z}\right) \\ + (b_C^1 - \frac{14}{23}b_Y^1 - \frac{9}{23}b_L^1)\ln\left(\frac{M}{M_I}\right)$$
(12)

 \mathbf{and}

$$\sin^2 \theta_W(M_Z)$$

$$= 3\alpha_{\rm EM}(M_Z) \left[\alpha_3^{-1}(M_Z) + (b_3^0 - b_2^0) \ln\left(\frac{M_I}{M_Z}\right) + (b_C^1 - b_L^1) \ln\left(\frac{M}{M_I}\right) \right], \quad (13)$$

where $b_i^0, i = 1, 2, 3$ are the same as above, but $b_i^1, i = C, Y, L$ depend upon the structure of the subgroup $G_L \otimes G_C \otimes G_R \otimes \cdots \equiv G_I$.

Equation (13) indicates that G_I cannot be $\mathrm{SU}(3)_C \otimes \mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R \otimes \mathrm{U}(1)_{Y(B-L)}$ because if that were the case $b_C^1 = b_3^0$ and $b_L^1 = b_2^0$, and since the first entry of the right-hand side of Eq. (13) has an experimental value of 0.192, $M_I \simeq 3M_Z$ is required to be satisfied and, therefore, there is a very small value for the masses of the gauge bosons associated with $\mathrm{SU}(2)_R$. With the minimal set of Higgs fields G_I contains therefore flavor-changing

neutral currents and thus M_I has to be greater than 100 TeV. The first two terms of the right-hand side of Eq. (13) have then a lower bound of 0.36, and the experimental value for $\sin^2\theta_W(M_Z)$ requires that $(b_C^1 - b_L^1) < 0$ which is not satisfied by the minimal set of Higgs fields and VEV's [1].

We are therefore led to consider introducing a minimum change in the set of HF's and/or VEV's such that the new set properly breaks the symmetry, guarantees the survival hypothesis, produces appropriate values for $\sin^2\theta_W(M_Z)$, and satisfies the mass hierarchy $M \gg M_I \gg M_Z \sim 10^2$ GeV. An analysis of Table I shows that a symmetry-breaking pattern in three steps with $G_I = SU(6)_L \otimes SU(4)_C \otimes U(1)_Y \otimes \cdots$ produces consistent results provided we introduce the following two changes. First, add a new set of Higgs fields

$$\phi_2' = \phi'(1323) = \phi_{2,\{a,b\}}'^{\{A,B\}} + \phi_{2,\{A,B\}}'^{\{a,\beta\}} + \phi_{2,\{\alpha,\beta\}}'^{\{a,b\}}$$
(14)

with VEV's in the directions $\{a, b\} = \{3, 6\} = -\{4, 5\},$ $\{A, B\}$ similar to $\{a, b\}$ and $\{\alpha, \beta\} = \{5, 5\},$ and second, orient the VEV's such that $\langle \phi_{2, \{\alpha, \beta\}}^{\prime, \{a, b\}} \rangle = \langle \phi_{2, \{a, b\}}^{\prime, \{A, B\}} \rangle = 0$. For this particular choice of VEV's we have that $b_C^1 = (\frac{88}{3} - \frac{2 \times 12}{3} - \frac{148}{3})/4\pi,$ $b_L^1 = (\frac{132}{3} - \frac{2 \times 12}{3} - \frac{107}{3})/4\pi$ and $b_Y^1 = -(\frac{2 \times 12}{3} + \frac{9}{14})/4\pi,$ where the extended survival hypothesis [3] was taken into account for the contribution of the HF's.

As can be seen, the Higgs fields play a fundamental role in Eqs. (12) and (13). Notice also that the symmetrybreaking pattern is achieved with $M = \langle \phi_3 \rangle$, $M_I = \langle \phi_1 \rangle + \langle \phi_2 \rangle + \langle \phi'_2 \rangle$, $M_Z = \langle \phi_4 \rangle$, and that ϕ_3 plays no role in the evolution of the gauge coupling constants.

Substituting now in Eqs. (12) and (13) the experimental values for $\sin^2 \theta_W(M_Z)$, $\alpha_{\rm EM}(M_Z)$, $\alpha_3(M_Z)$ we obtain the equations

$$egin{aligned} 1.10 &= 1.02 \ln \left(rac{M_I}{M_Z}
ight) - 2.26 \ln \left(rac{M}{M_I}
ight), \ 7.83 &= 1.25 \ln \left(rac{M_I}{M_Z}
ight) - 1.82 \ln \left(rac{M}{M_I}
ight), \end{aligned}$$

which for $M_Z = 91$ GeV have the solutions $M_I \sim 10^9$ GeV and $M \sim 10^{12}$ GeV. These results are in good agreement with those obtained from the analysis of the generational seesaw mechanism in this model [6].

STABILITY OF THE PROTON

A. Baryon number for the particles

The elementary particles in the model are the ones associated with the 105 GF's, the 108 Weyl fields in $\psi(108)_L$ and the 4104 HF's in ϕ_i , i = 1-4 and ϕ'_2 . Now, all the elementary particles in our model have a well-defined Baryon Number *B*. Let us note the following.

(1) The GF. The 70 GF's associated with $SU(6)_L \otimes SU(6)_R$ have B = 0. For $SU(6)_C$ we have that the 9 leptoquarks have B equal to 1/3, and the other 17 GF's have B = 0 (including the 8 gluon fields).

(2) The Weyl fermion fields. The quark fields in $\psi(\bar{6}, 6, 1)_L$ have B = 1/3, the quark fields in $\psi(1, \bar{6}, 6)_L$ have B = -1/3 and all the other fields in $\psi(108)_L$ have B = 0.

(3) The HF. B for the 4104 HF's of the model is given in Table II.

B. Baryon number as a symmetry of the model

In the subspace of the fundamental representation of $SU(6)_C B$ can be associated with the 6×6 diagonal matrix B = Diag(1/3,1/3,1/3,0,0,0). This matrix does not correspond to a generator of $SU(6)_C$ neither of G. Now, the full Lagrangian $\mathcal{L} = \mathcal{L}_{gauge} + \mathcal{L}_{Higgs} + \mathcal{L}_{Yukawa}$ has a $U(1)_{\chi}$ global symmetry, where χ is a constant whose magnitude depends solely on the label of the $SU(6)_C$ representation. For example $\chi = 1$ for $\psi(\bar{6}, 6, 1), \chi = 0$ for $G(1,35,1), \chi = -2$ for $\phi(1,1\bar{5},15)$, etc. Conveniently normalized, the $U(1)_{\chi}$ generator may be written in the fundamental representation of $SU(6)_C$ as $\chi = Diag(1,1,1,1,1,1)/\sqrt{12}$, which is not an element of the Lie algebra of G either.

On the other hand, in the Lie algebra of G there is a generator, an element of the $SU(6)_C$ subalgebra, of the form

$\langle \phi angle$	${ m SU}(4)_C$	$\mathrm{C}^1_C(\mathrm{scalars})$	${ m SU(6)}_L$	$\mathrm{C}^1_L(\mathrm{scalars})$
$\langle \phi^{[A,B]}_{1,[a,b]} angle$		0	fourteen 15 's	56
$\langle \phi^{[m{lpha},m{eta}]}_{1,[m{A},B]} angle$	fourteen 4's	14		0
$\langle \phi^{[a,b]}_{1,[lpha,eta]} angle$	fifteen 4's	15	four 15 's	16
$\langle \phi^{\{A,B\}}_{2,\{a,b\}} angle$		0	fourteen 21 's	112
$\langle \phi_{2,\{A,B\}}^{\{lpha,eta\}=\{4,5\}} angle$	fourteen 4's	14		0
$\langle \phi^{\{a,b\}}_{2,\{lpha,eta\}\{4,5\}} angle$	twenty-one 4's	21	four 21 's	32
$\langle \phi^{\prime \{A,B\}}_{2,\{a,b\}} angle$		0	fourteen 21's	112
$\langle \phi_{2,\{A,B\}}^{\prime\{lpha,eta\}=\{5,5\}} angle$	fourteen 10's	84		0
$\langle \phi^{\prime \{a,b\}}_{2,\{lpha,eta\}=\{5,5\}} angle$	twenty-one 10's	126	ten 21 's	80
$\langle \phi^a_{4,A} angle$		0	three 6 's	3

TABLE I. The index of the scalars for $SU(4)_C$ and $SU(6)_L$.

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TABLE II. Baryon number of the 4104 Higgs fields.

φ	lpha,eta	a, b	A, B	В
$\phi^{[A,B]}_{1(3),[a,b]}$		a,b=1,,6	A, B = 1,, 6	0
$\phi_{1(3),[A,B]}^{[\alpha,\beta]}$	lpha,eta=1,2,3		A, B = 1,, 6	-1/3
	lpha,eta=4,5,6		A,B=1,,6	0
	lpha=1,2,3;eta=4,5,6		A,B=1,,6	1/3
$\phi_{1(3),[lpha,eta]}^{[a,b]}$	lpha,eta=1,2,3	a,b=1,,6		1/3
	lpha,eta=4,5,6	a,b=1,,6		0
	lpha=1,2,3;eta=4,5,6	a,b=1,,6		-1/3
$\phi_{2,\{a,b\}}^{(\prime)\{A,B\}}$		a, b = 1,, 6	A, B = 1,, 6	0
$\phi_{2,\{A,B\}}^{(\prime)\{\alpha,\beta\}}$	lpha,eta=1,2,3		A, B = 1,, 6	2/3
	lpha,eta=4,5,6		A,B=1,,6	0
	lpha=1,2,3;eta=4,5,6		A,B=1,,6	1/3
$\phi_{2,\{\alpha,\beta\}}^{(\prime)\{a,b\}}$	lpha,eta=1,2,3	a, b = 1,, 6		-2/3
	lpha,eta=4,5,6	a,b=1,,6		0
	lpha=1,2,3;eta=4,5,6	a,b=1,,6		-1/3
$\phi^a_{4,A}$		a = 1,, 6	A = 1,6	0
$\phi^{lpha}_{4,a}$	lpha=1,2,3	a = 1,, 6		1/3
	lpha=4,5,6	a = 1,, 6		0
$\phi^A_{4,lpha}$	lpha=1,2,3		A = 1,, 6	-1/3
	lpha=4,5,6		A = 1,, 6	0

$$B' = \text{Diag}(1, 1, 1, -1, -1, -1) / \sqrt{12} , \qquad (15)$$

which distinguishes between quarks and leptons in our model. Therefore B can be written as $B = [\chi + B']/\sqrt{3}$.

Since the elementary particles of this model have a well-defined B, it is obvious that in the unbroken theory the exchange of particles cannot break B. This statement is also true after breaking the symmetry due to the following two facts: The baryon number is not gauged (there is no gauge boson associated to B); $\phi_i, i=1-4$, and ϕ'_2 with the VEV's as stated do not break B spontaneously. That is, $B\langle\phi_i\rangle = B\langle\phi'_2\rangle = 0$, i=1,2,3,4. Therefore, B is conserved in our model [7]. Since B is conserved, the proton is perturbatively stable.

Now, the single Goldstone boson associated with the broken orthogonal combination is absorbed by the massive gauge field associated with B'. Therefore, there are

no physical Goldstone bosons, and there is no extra long range force. This mechanism in which a global symmetry emerges from the simultaneous breaking of a gauge and global symmetry is due to 't Hooft [8] and was implemented in the context of grand unified models in Ref. [9].

Finally we would like to mention that, contrary to baryon number, lepton number is violated in this model due to the fact that the GF's associated with the $U(1)_{Y_{(B-L)}}$ generator is gauged. Therefore, neither L, (B-L) or (B+L) are conserved quantities.

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