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Photonic band structure evolution of a honeycomb lattice in the presence of an external magnetic field

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A standard plane-wave expansion technique is used to investigate the evolution of the photonic band structure of a two-dimensional honeycomb lattice composed by cylindrical shell rods with dielectric permittivities ϵ_1 and ϵ_2 , and embedded in a background with permittivity ϵ_3 . We have considered the effect of dispersive dielectric responses as well as the influence of an externally applied magnetic field aiming to obtain efficient tunable bandgaps. Present results suggest that a combination of a doped semiconductor constituent with an anisotropic geometry, which breaks symmetry and unfolds degeneracies, provides an efficient realization of photonic systems with tunable bandgaps. © 2009 American Institute of Physics. [DOI: 10.1063/1.3072668]

I. INTRODUCTION

Since the advent of nanostructured materials that exhibit photonic bandgaps, two-dimensional (2D) photonic crystals (PCs) have received a great deal of attention by researchers around the world. The reason behind such interest lies on the possibilities that these structures offer to control the properties of light. The control of spontaneous emission of a quantum dot embedded in a PC, recently evidenced,¹ is an example that illustrates the kind of novel quantum electrodynamic effects that these engineered materials provide and the promising future of such nanostructured systems in the quest for the control of light. The existence of complete or absolute photonic bandgaps (PBGs) that forbid the propagation of light for all wave vectors is crucial in the occurrence of most of the important electrodynamic effects reported in literature. A great deal of the studies devoted to the understanding of bandgaps are based on nondispersive and positive electric and magnetic responses.² Tunable bandgaps have come into play in the past decade, and other media with dispersive response functions which are strongly dependent on external parameters have been given some attention. A tunable PBG may be obtained by externally controlling the response functions, aiming to practical applications such as sensor devices. Ferrites, liquid crystals, and intrinsic semiconductors have been used to fulfill this ambition. Ferrites exhibit a permeability tensor sensitive to an external magnetic field as well as to temperature variations.³ By filling the voids of a PC with nematic liquid crystals one obtains a PBG which may be tuned by the application of an external electric field or by changing the temperature.⁴ Also, the dielectric function of intrinsic semiconductors is strongly dependent on applied magnetic fields.⁵ Along these lines, the tunability of the PBG of a square lattice composed by cylindrical air holes

embedded in a background of GaAs has been shown to be efficient for frequencies in the terahertz domain. Theoretical studies of 2D periodic lattices of dielectric cylinders have demonstrated that complete PBGs over a large range of filling factors are achieved by the hexagonal configuration.⁶ Furthermore, symmetry reduction in honeycomb lattices has been shown to increase the size of absolute PBGs. Recently, a thorough study⁷ was performed on a 2D hexagonal structure of circular rods formed by an internal region of dielectric permittivity ϵ_1 and an anisotropic^{8,9} external shell of ϵ_2 embedded periodically in a background characterized by permittivity ϵ_3 , as depicted in Fig. 1. A comparative analysis of the cases using solid Te rods and Te shells filled with another material shows that the latter exhibits wider and shifted PBGs compared to the former. Based on these studies, in this

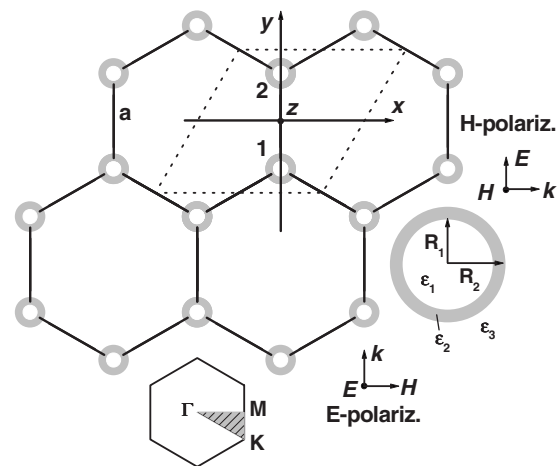


FIG. 1. Pictorial view of the 2D honeycomb PC studied in this work. The unit cell is represented by dotted lines and the basis of two shell rods is denoted by numbers 1 and 2. First-neighbor distance is denoted by a . The two radii for the shell rod, the three different dielectric constants, and the irreducible zone in the k -space are also sketched.

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work we investigate the photonic band structure (PBS) of Te shell rods embedded in air (see Fig. 1). Furthermore, we study the evolution of the PBS of a highly anisotropic honeycomb PC using a dispersive dielectric function characteristic of bulk GaAs in air and considering the presence of an external magnetic field parallel to the rods' axis. The dielectric response of a n -doped GaAs semiconductor used here to fill the cylindrical rods depends on the particular state of polarization of light. In the present study, we consider that the incident polarized light has its electrical component in the direction perpendicular to the rods' axis, i.e., H -polarized light. In Sec. II we deduce the wave equations governing light propagation which are numerically solved by a standard plane-wave expansion technique, in Sec. III we present the results and discussion, and finally Sec. IV has our conclusions.

II. THEORETICAL FRAMEWORK

The photonic bandgap in a periodic dielectric structure may be determined by studying the propagation of light within the framework of Maxwell's equations. Thus, the wave equation for the magnetic field in inhomogeneous dielectric materials reads^{10,11}

$$\nabla \times \left[\frac{1}{\varepsilon(\vec{r})} \nabla \times H(\vec{r}) \right] = \frac{\omega^2}{c^2} H(\vec{r}), \quad (1)$$

where c and ω are the velocity and the frequency of light, respectively. Considering the periodicity of the dielectric function $\varepsilon(\vec{r})$ one may expand the magnetic field in a sum of plane waves as follows.^{10,12}

$$H(\vec{r}) = \sum_{\vec{G}} \sum_{\lambda=1}^2 h_{\vec{G},\lambda} \hat{e}_\lambda e^{i(\vec{k}+\vec{G})\cdot\vec{r}}, \quad (2)$$

with $\vec{G}=(G_x, G_y)$ representing a reciprocal lattice vector, \vec{k} a wave vector in the first Brillouin zone, and $\hat{e}_\lambda(\lambda=1, 2)$ orthogonal unit vectors perpendicular to $\vec{k}+\vec{G}$. Substituting Eq. (2) into Eq. (1) and taking the Fourier transform of the inverse of $\varepsilon(\vec{r})$ one may obtain the following equation governing the dispersion of electromagnetic waves:¹³

$$\sum_{\vec{G}'} \left[|\vec{k} + \vec{G}'| |\vec{k} + \vec{G}'| \eta(\vec{G} - \vec{G}') \begin{pmatrix} +\hat{e}_2 \cdot \hat{e}'_2 & -\hat{e}_2 \cdot \hat{e}'_1 \\ -\hat{e}_1 \cdot \hat{e}'_2 & +\hat{e}_1 \cdot \hat{e}'_1 \end{pmatrix} - \frac{\omega^2}{c^2} \delta_{\vec{G}', \vec{G}} \right] \begin{pmatrix} h_{\vec{G}',1} \\ h_{\vec{G}',2} \end{pmatrix} = 0, \quad (3)$$

where $\eta(\vec{G})$ denotes the Fourier transform of the dielectric response function $\varepsilon^{-1}(\vec{r})$, i.e.,

$$\eta(\vec{G}) = \frac{1}{\Omega} \int_{\text{cell}} \varepsilon^{-1}(\vec{r}) e^{-i\vec{G}\cdot\vec{r}} d\vec{r}, \quad (4)$$

with the integral performed over the area Ω of a unit cell of the lattice.

Let us consider the periodic array of dielectric rods whose axes are parallel to the z -axis so that the intersection of these rods with the x - y plane forms a 2D periodic dielectric pattern according to the scheme presented in Fig. 1: a

honeycomblike structure of rods whose dielectric functions are $\varepsilon_1=n_1^2$ for the internal rod and $\varepsilon_2=n_2^2$ for the anisotropic (as in previous work⁷) external shell, embedded periodically in a background with dielectric function $\varepsilon_3=n_3^2$. The anisotropic^{8,9} external shell has the ordinary-refractive index $n_2=n_o$ and extraordinary-refractive index $n_2=n_e$. The problem defined in Eq. (1) is reduced to the problem of solving two eigenvalue equations according to the particular state of polarization of light. So, for E -polarization, i.e., with the electric field parallel to the rods, the wave equation becomes

$$\sum_{\vec{G}'} |\vec{k} + \vec{G}'| |\vec{k} + \vec{G}'| \eta(\vec{G} - \vec{G}') h_{\vec{G}',2} = \frac{\omega^2}{c^2} h_{\vec{G},2}, \quad (5)$$

whereas in the case of H -polarization, i.e., with the magnetic field parallel to the rods, the wave equation is written as

$$\sum_{\vec{G}'} (\vec{k} + \vec{G}) \cdot (\vec{k} + \vec{G}') \eta(\vec{G} - \vec{G}') h_{\vec{G}',1} = \frac{\omega^2}{c^2} h_{\vec{G},1}. \quad (6)$$

Considering now the extraordinary axis parallel to the z -axis, the eigenequations for E - and H -polarizations are the same as those for isotropic photonic crystals, except for the fact that the dielectric indices of the anisotropic external shell are $n_2=n_e$ and $n_2=n_o$ for E - and H -polarizations, respectively. To express the dielectric function of the system in this configuration, we introduce the step functions

$$f_{\text{rod}}(\vec{r}) = \begin{cases} 1, & 0 < r < R_1 \\ 0 & \text{otherwise} \end{cases} \quad (7)$$

and

$$f_{\text{shell}}(\vec{r}) = \begin{cases} 1, & R_1 < r < R_2 \\ 0 & \text{otherwise.} \end{cases} \quad (8)$$

In this way the dielectric response for the system in this configuration may be written as

$$\frac{1}{\varepsilon(\vec{r})} = \frac{1}{\varepsilon_3} + \left(\frac{1}{\varepsilon_1} - \frac{1}{\varepsilon_3} \right) F_{\text{rod}} + \left(\frac{1}{\varepsilon_2} - \frac{1}{\varepsilon_3} \right) F_{\text{shell}}, \quad (9)$$

with

$$F_{\text{rod(shell)}} = \sum_i \sum_{\vec{R}} f_{\text{rod(shell)}}(\vec{r} - \vec{u}_i - \vec{R}), \quad (10)$$

where the unit cell of a honeycomb array contains two rods located at $\vec{u}_1=(0, +a/2)$ and $\vec{u}_2=(0, -a/2)$. The Fourier coefficients of the honeycomb structure with shelled circular rods are given by⁷

$$\eta(\vec{G}) = \begin{cases} \frac{1}{\varepsilon_3} + \frac{2\pi R_1^2}{\Omega} \left(\frac{1}{\varepsilon_1} - \frac{1}{\varepsilon_3} \right) + \frac{2\pi}{\Omega} (R_2^2 - R_1^2) \left(\frac{1}{\varepsilon_2} - \frac{1}{\varepsilon_3} \right), & \vec{G} = 0 \\ \frac{4\pi}{\Omega G} \cos\left(\frac{aG_y}{4}\right) \left[\left(\frac{1}{\varepsilon_1} - \frac{1}{\varepsilon_3} \right) R_1 J_1(R_1 G) + \left(\frac{1}{\varepsilon_2} - \frac{1}{\varepsilon_3} \right) [R_2 J_1(R_2 G) - R_1 J_1(R_1 G)] \right], & \vec{G} \neq 0, \end{cases} \quad (11)$$

where $J_1(x)$ is the Bessel function of the first kind. In this study we shall be interested to establish tunable properties of the PBS based on the consequences of an external magnetic field applied perpendicular to the electric component of the optical field. Let us then consider the applied magnetic field pointing in the direction parallel to the rods' axes. In this case the dielectric response for E -polarization is constant and thus, independent of the external magnetic field, defined just like for an ordinary semiconductor, that is,

$$\frac{\varepsilon_{\parallel}(\omega)}{\varepsilon_0} = 1 - \frac{\omega_p^2}{\omega^2}, \quad (12)$$

where ε_0 is the background dielectric constant and ω_p is the plasma frequency, a function of the effective mass and carrier density.¹⁴ For H -polarization, the electrical field is perpendicular to the external applied magnetic field so that the electric response is modified by the applied field according to

$$\frac{\varepsilon_{\perp}(\omega)}{\varepsilon_0} = 1 - \frac{\omega_p^2}{(\omega^2 - \omega_c^2)} - \frac{\omega_p^4 \omega_c^2}{\omega^2 (\omega^2 - \omega_c^2) (\omega^2 - \omega_c^2 - \omega_p^2)}, \quad (13)$$

where $\omega_c = eB/m^*c$ is the cyclotron frequency, B is the applied magnetic field, and e is the modulus of the electron charge. From Eq. (13) it is clear that the response function of a PC with semiconductor constituents may be tuned by means of an externally applied magnetic field. In the following, we shall describe quantitatively how the PBS and PBGs are influenced by the magnetic field.

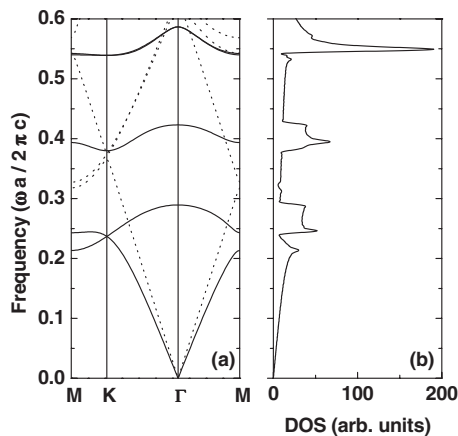


FIG. 2. (a) Photonic band structures of the 2D honeycomb lattice of Te shell rods in air ($R_1=R_2/2$; $R_2=0.1a$; $\varepsilon_1=\varepsilon_3=1.00$) for E -polarization (solid lines, $\varepsilon_2=38.44$) and H -polarization (dotted lines, $\varepsilon_2=23.04$) modes. In (b) the total (for both polarizations) density of states is plotted, illustrating the absence of complete bandgaps.

III. RESULTS AND DISCUSSION

We have solved Eqs. (5) and (6) numerically (we have used a total of 253 plane waves), for E - and H -polarizations, in order to obtain the PBS of a honeycomb lattice formed by Te shell rods in air, for different values of the area occupied by the dielectric material, as shown in Figs. 2 and 3. We find that, in the E -polarized configuration, light propagation is forbidden, for the small values of radius chosen in Fig. 2, in the frequency range between (0.28–0.38) ($2\pi c/a$) and (0.42–0.53) ($2\pi c/a$). Using the same parameter values, the corresponding H -polarized configuration does not exhibit any bandgap and the PBS resembles that of light propagating in vacuum. This could be attributed to the joint effects of a higher value of the dielectric constant assumed by the E -polarized configuration and a much more intense coupling between the electric field and the dielectric medium in this configuration. Compared with Fig. 3 where the radius of the dielectric region is increased with respect to the radius used in Fig. 2, we find that the dispersive modes corresponding to the H -polarization lose its vacuum resemblance so that bandgaps are now exhibited by both E -polarized and H -polarized configurations. The overlap of the PBG corresponding to E -polarized and H -polarized light defines absolute PBGs, as illustrated in Fig. 4, where the photonic gap map is presented as a function of the relative radius R_2/a . This figure shows that for small relative radii there are bandgaps only for E -polarization, whereas the bandgaps corresponding to H -polarization begin to appear for larger relative radii. In Fig. 5 one may note that in the case of equal areas occupied by the dielectric, the configuration with the electric field parallel to the rods produces wider bandgaps for smaller dielec-

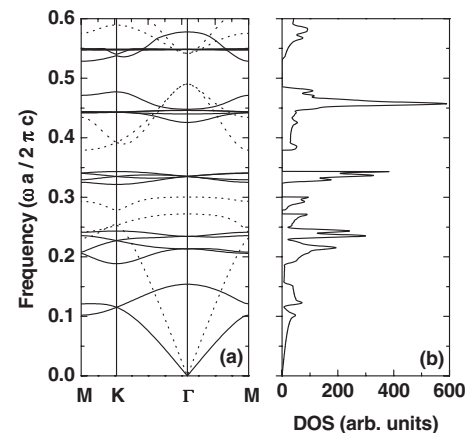


FIG. 3. (a) PBS of the 2D honeycomb lattice of Te shell rods in air ($R_1=R_2/2$; $R_2=0.3a$; $\varepsilon_1=\varepsilon_3=1.00$) for both E -polarization (solid lines, $\varepsilon_2=38.44$) and H -polarization (dotted lines, $\varepsilon_2=23.04$) modes. In (b) the corresponding total density of states is shown, in arbitrary units.

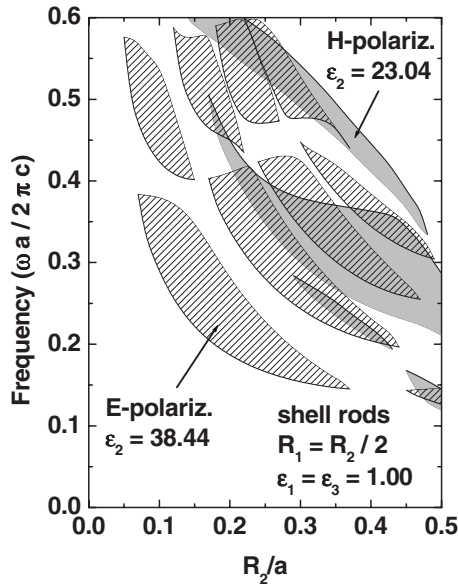


FIG. 4. Photonic gap map for the 2D honeycomb lattice of Te shell rods in air ($R_1=R_2/2$, $\epsilon_1=\epsilon_3=1.00$) for E -polarization (striped regions, $\epsilon_2=38.44$) and H -polarization (gray regions, $\epsilon_2=23.04$) modes. Absolute bandgaps are represented by the intersection between the striped and gray regions.

tric regions. As the dielectric area increases, these bandgaps split up and shrink, up to the point where most of them disappear. On the other hand, in the case of a configuration where the electric field is perpendicular to the rods, there are no gaps at all for small dielectric regions. As the dielectric region is increased the bandgaps begin to appear. This analysis shows that one must have high index contrast to produce absolute bandgaps.

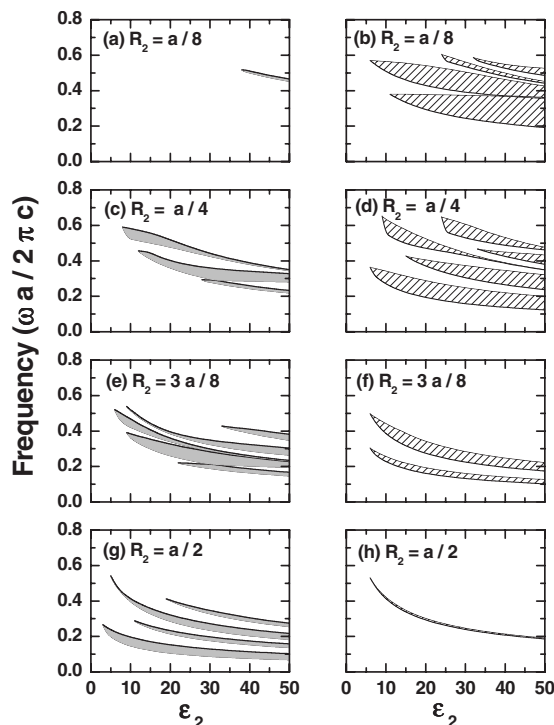


FIG. 5. Photonic gap map vs dielectric constant in the rod material (ϵ_2) for the 2D honeycomb lattice of solid rods in air ($R_1=0$, $\epsilon_3=1.00$) for both H -polarization (left panel) and E -polarization (right panel) modes.

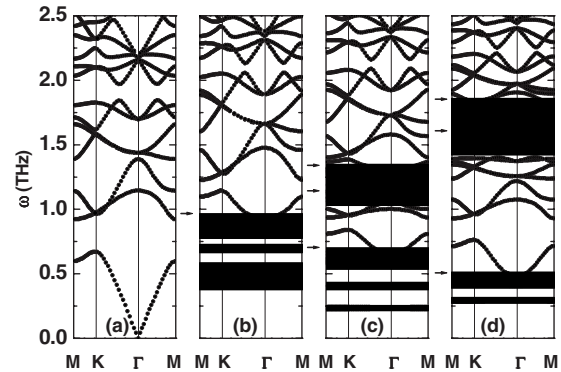


FIG. 6. PBS for H -polarization in a honeycomb structure composed by circular rods of GaAs (a) and n -doped (Ref. 5) GaAs [(b)–(d)] embedded in air and for different values of an applied magnetic field parallel to the rods: $B=0$ [(a) and (b)], 0.25 T (c), and 0.5 T (d). Here $R=a/2$ (a is the first-neighbor distance and R is the radius of the rods), with $a=0.06$ cm, $\epsilon_0=12.9$ for the static dielectric constant of the GaAs, and $m^*=0.0665m_0$ (m_0 is the free-electron mass). In (a) a nondispersive GaAs dielectric constant is considered, whereas a dispersive one is considered for n -doped GaAs in (b)–(d) with $\omega_p=0.942$ THz. The zones of flat bands are visualized by black regions.

Let us now turn to Fig. 6, where four panels are presented, for comparison purposes, depicting the PBS of the honeycomb structure made of circular rods, for H -polarization in the following four situations: (a) GaAs circular rods embedded in air and (b)–(d) n -doped GaAs circular rods embedded in air with the dispersive dielectric function defined in Eq. (13). In the case of n -doped GaAs, $\omega_p=0.942$ THz and, in Fig. 6(b), right below this value we find the appearance of flat dispersionless bands, denoted by black regions for which the dielectric function assumes negative values, in accordance with the result reported in Refs. 15 and 16. The top edge of the top dispersionless band appears close to frequencies that produce a null dielectric response.⁵ Similar results have been reported in periodic structures containing single or double dispersive materials.^{17,18} These results suggest that such dispersionless bands may be attributed to the surface-plasmon-polariton (SPP) collective couplings among photon and surface-plasmon modes which arise due to the presence of the dispersive material in the structure. In the next panels we show the evolution of the PBS presented in (b) in the presence of a magnetic field parallel to the rods, in (c) $B=0.25$ T and in (d) $B=0.5$ T. The photonic gap map for varying applied magnetic fields is shown in Fig. 7. We find that the applied magnetic field modifies the PBS structure due to the variation in the dielectric function with the cyclotron frequency. Furthermore, some of the flat bands move upward and, by increasing the field, additional bands also move upward. As one might suspect, by studying the zeros of the dielectric function defined in Eq. (13), one finds that the regions where they appear are characterized by a negative response with the top edge located near a zero of the dielectric response. In the panels we have indicated with arrows the frequencies for which the dielectric function is null or assumes asymptotic values.⁵ Furthermore, inspection of Eq. (13) shows that, in the presence of very high fields, i.e., for high values of the cyclotron frequency, the dielectric response becomes constant and the externally applied magnetic field does not influence the PBS at all.

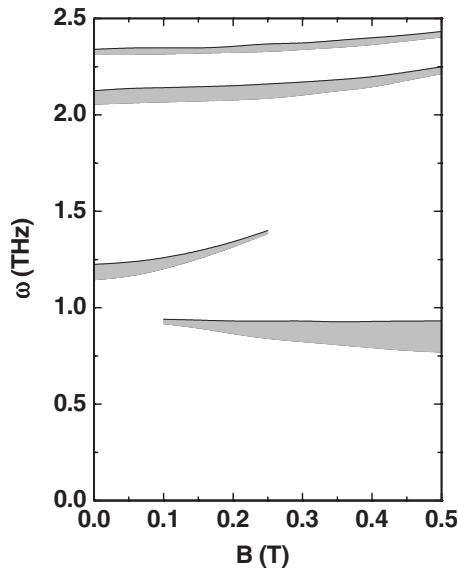


FIG. 7. Photonic gap map, for H -polarization, as a function of the external magnetic field in a honeycomb structure with parameters as in Fig. 6. The applied magnetic field is parallel to the rods.

It is worthwhile to mention that the present model calculation does not include absorption effects, although losses occur in real PCs and might quantitatively modify the PBS results presented here. Further work in that direction is certainly needed, and here we just note that effects of losses may be qualitatively taken into account by considering an imaginary part in the dielectric permittivity through a phenomenological lifetime related to the imaginary part of the wave frequency.

IV. CONCLUSIONS

In conclusion, we have studied the evolution of the PBS of a highly anisotropic hexagonal lattice, composed by shell rods of n -doped GaAs embedded in air in the presence of an external magnetic field. We have chosen a hexagonal geometry known to produce wide bandgaps combined with a n -doped semiconductor to obtain an enhanced response. We find that the dispersive character of the dielectric response introduces dispersionless bands below the region where the dielectric function changes sign from positive to negative. The effect of the externally applied magnetic field moves the

dispersionless flat bands to higher and lower frequency ranges, according to the zeros of the dielectric function. Consequently, one may tune the frequency band for which SPP modes appear, providing significant flexibility for tailoring such modes. Present theoretical results indicate that the use of PCs based on a combination of a doped semiconductor constituent, highly sensitive to the action of an external magnetic field, with an anisotropic geometry, which breaks symmetry and unfolds degeneracies, might be an efficient strategy in the realization of photonic systems with tunable bandgaps as well as tunable SPP modes.

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