A DSGE MODEL WITH LOSS AVERSION IN CONSUMPTION AND LEISURE: AN EXPLANATION FOR BUSINESS CYCLES ASYMMETRIES

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SERIE DOCUMENTOS DE TRABAJO

No. 157

Abril de 2014

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June 12, 2014

Abstract

In this chapter, an asymmetric DSGE model is built in order to account for asymmetries in business cycles. One of the most important contributions of this work is the construction of a general utility function which nests loss aversion, risk aversion and habits formation by means of a smooth transition function. The main idea behind this asymmetric utility function is that under recession the agents over-smooth consumption and leisure choices in order to prevent a huge deviation of them from the reference level of the utility; while under boom, the agents simply smooth consumption and leisure, but trying to be as far as possible from the reference level of utility. The simulations of this model by means of Perturbations Method show that it is possible to reproduce asymmetrical business cycles where recession (on shock) are stronger than booms and booms are more long-lasting than recession. One additional and unexpected result is a downward stickiness displayed by real wages. As a consequence of this, there is a more persistent fall in employment in recession than in boom. Thus, the model reproduces not only asymmetrical business cycles but also real stickiness and hysteresis.

1 Introduction

Empirical evidence has cast doubts about the relevance of Life Cycle/Permanent Income Hypothesis (LCH/PIH) to explain the dynamics of consumption. According to this theory, the only variables determining variations in consumption are interest rate and shifts in preferences, which means that consumption should not respond to changes and expected income. In econometric terms, the previous affirmation means that in a regression of interest rate and with variables related to expectations of future income on consumption growth, the null of LCH/PIH will imply a zero vector for the expected income variables. However, empirical evidence rejects most of the times LCH/PIH. In this line, the most important and inspiring paper to uphold this thesis is Shea's (1995). He cites some papers that find rejections of the LCH/PIH: Campbell and Mankiw (1990) found a

*This is the second chapter of my Ph.D dissertation intended to explain asymmetries in the business cycle. In am very thankful to my advisor Professor Fernando Jaramillo for his guidance and to Universidad de Antioquia and Colciencias for financing my Ph.D. studies. I would like to thank comments and suggestions by Alejandro Torres, John Lopera, Andrés González, Jesus Bejarano, Frans Hamann, Jair Ojeda, colleagues attending the seminar SER at Universidad de Antioquia, colleagues attending the seminar at Universidad del Rosario. All errors and omissions are my own.

Table 1:

GC_t = Time Dummies + $\gamma GAFN_t + \sigma \log(1+r_t) + \beta(EDWAGE) + \varepsilon_t$					
Variable	(i)	(ii)	(iii)	(iv)	(v)
GAFN	0.154 (0.192) [0.802]	0.486 (0.121) [4.017]	0.226 (0.187) [1.209]	0.388 (0.093) [4.172]	0.294 (0.143) [2.056]
$\log(1+r_t)$	0.631 (2.098) [0.301]	2.702 (2.007) [1.346]	0.756 (1.675) [0.451]	4.748 (2.876) [1.651]	1.720 (1.407) [1.222]
EDWAGE	0.997 (0.703) [1.418]	0.765 (0.539) [1.419]	0.961 (0.614) [1.565]	0.867 (0.593) [1.462]	—
Positive EDWAGE	—	—	—	—	0.063 (0.785) [0.080]
Negative EDWAGE	—	—	—	—	2.242 (0.951) [2.358]
R^2:	0.010	0.067	0.014	0.121	0.011
Number of observations:	372	275	475	172	647
Split:	Zero wealth	Positive wealth	Low ratio	High ratio	Full sample

Source: Taken from Shea (1995), pp. 195.

statistically significant relationship between predictable income and increases in consumption; in Zeldes (1989), the empirical evidence rejects LCH/PIH, which is attributed to liquidity constraints; Flaving (1991) also rejects LCH/PIH stating that this is caused by myopic behavior of agents. By using data of unionized family heads (for the U.S), Shea (1995) has found three possible problems leading to the rejection of LCH/PIH: i) Panel Study of Income Dynamic (PSID) is mostly used to cover food consumption, ii) PSID is mostly interested in labor-market behavior rather than in consumption behavior, and iii) it is difficult to find variables in households information sets as good predictors of future income growth[1]. He also proposes to find correct instruments for expectations of future income to test the null hypothesis of LCH/PIH and the null of liquidity constraints. His hypothesis tests lead to rejecting the permanent income hypothesis as well as the liquidity constraint hypothesis. Moreover, not only do his findings imply that increases in consumption are related to expected income, but also to an asymmetric reaction of consumption growth: consumption reacts more strongly to expected decreases than to income increases, which is more explainable, at least qualitatively, by loss aversion. In table 1, I will show here table 4 as in Shea (1995).

Previous results by Shea (1995) for the U.S. economy were confirmed in 1995 and 1999 by Bowman, Minehart and Rabin (1995; 1999). In 1995, they formally showed the properties of more general utility functions with loss aversion for two periods. In 1999 and based on their analytical results, they tested econometrically for five OECD countries (United States, Japan, Germany and France) the existence of loss aversion in consumption by using a version of the growth consumption equation proposed and estimated by Shea (1995). Minehart and Rabin (1995; 1999) included dummy variables to capture differences in slope for negative and positive expected income variations. This means that before an expected income decrease, there is a stronger reaction

[1] Shea (1995) pag. 187.

Table 2:

Table 1
Estimates of the model

Country	Sample period	$\bar{R}_c^2$	$\bar{R}_y^2$	λ
Canada	1972 : 4–94 : 1	0.271	0.161	0.497 (5.31)
France	1971 : 2–93 : 2	0.117	0.065	0.293 (1.97)
West Germany	1961 : 2–90 : 2	0.005	0.048	0.592 (2.48)
Japan	1971 : 2–93 : 1	−0.007	−0.037	−0.269 (−0.89)
United Kingdom	1956 : 2–93 : 3	0.109	0.012	0.423 (2.63)

Table 2
Estimates of the model

Country	Obs.	Quarters			p-value
		$\Delta \ln \hat{y}_t < 0$	λ_1	λ_2	$\lambda_1 = \lambda_2$
Canada	86	21	0.270 (1.59)	1.128 (3.14)	0.067
France	78	13	0.046 (0.22)	1.045 (2.33)	0.080
West Germany	114	5	0.412 (1.91)	3.805 (2.12)	0.074
Japan	75	0	−0.269 (−0.89)	–	–
United Kingdom	118	13	0.356 (1.58)	0.651 (1.24)	0.649
Panel	471	52	0.155 (3.79)	1.136 (5.21)	0.003

Source: taken from Bowman et al (1999), pages 166 and 167.

of individuals than facing an expected income increase. The upper panel of table 2 is table 1, as reproduced from Bowman et al. (1999), shows that expected income changes are correlated to consumption growth, and the lower panel shows the estimation of the model taking into account the inclusion of dummy variables for increases and decreases in expected income. The estimated parameters (and hypothesis testing of equality between) λ_1 and λ_2[2] reveal that consumption reacts more strongly to predictable income decreases than to predictable increases in it.

Shea (1995) and Bowman et al. (1999) agree that prospect theory can provide a powerful explanation for the asymmetry response of consumption to expected variations in income. They strongly point out the importance and need for developing other research lines by using loss aversion and by taking into account dynamic models of more than two periods. Bowman et al. (1999) express that "loss aversion can usefully be incorporated into areas of economic research other than consumption and saving" (p. 168) and also noted: "Formal modeling along the lines developed in this paper may help researchers begin to systematically investigate the implications of loss aversion in a wider array of economic situations" (p. 168). On the same order, Shea (1995) states that "further research should investigate the implications of loss aversion for the dynamic behavior of consumption in more general settings and should attempt to derive additional testable implications of loss aversion beyond asymmetric rejection of the LCH/PIH" (p. 199).

Prospect theory and functional forms including loss aversion have been hardly used in DSGE research, although there are some interesting papers intended to explicitly model loss aversion and test its presence in macroeconomic time series. Rosenblatt-Wisch (2005) had introduced loss aversion in a traditional Ramsey model calibrated for the steady state. This author also estimated parameters by GMM and has tested the null hypothesis of loss aversion in the macroeconomic time series (on U.S data) (2008). Her results have been for loss aversion. Foellmi, Rosenblatt-wisch and Schenk Hoppé (2010) have found that when agents have loss aversion, their consumption

[2] These are the coefficients for expected increase and expected decrease in income repectively, in an ordinary least squares regression, as the one in table 1

path is smoother and the economy could stay in a poverty trap which can be explained by a sub-accumulation of physical capital. **However, these works use linear utility functions, exclude labor, and do not perform impulse response exercises either simulation or comparison of theoretical and sample moments.** Gaffeo et al. (2010) in a DSGE framework, employ loss aversion to study the asymmetrical responses of output and prices for the monetary policy. However, their aim was different from explaining the asymmetries of macroeconomic time series along the business cycle. The utility function in their work had additive separable labor decisions in a concave function, while the part of consumption was a convex combination of a neoclassical utility function and an exponential gain-loss function, which satisfies concavity for gains and convexity for losses, as proposed by KT (1979). The set up of their utility functions excludes smooth transition, which means that the authors must perform separated simulations for each of the regimes of the model, and need to model a two-state markov chain stochastic process for simulations. Notwithstanding, the model succeeds at reproducing the documented empirical regularities of output responses more strongly to monetary policy during recessions than during booms. But for the case of inflation responses, it does not seem to present differences during recessions and booms.

Up to now, to the best of my knowledge, prospect theory has been mostly applied in consumption-based asset pricing models and has been applied to finance. Andries (2011) has redefined preferences by using loss aversion in order to study asset pricing. Her model, which includes loss aversion, performs more efficiently than the recursive utility model as it explains the excess of returns varying with skewness of returns distribution. This model also captures an effect level on the risk-free returns assets. Han and Hsu (2004) have documented the work of researchers by using prospect theory in financial theory. Regarding the disposition effect, they cite Odean (1998), Grinblatt and Keloharju (2001), Heath, Huddart and Lang (1999), Shapita and Venezia (2001), Garvey and Murphy (2004), thus finding a tendency to sell papers experiencing gains while not selling papers experiencing losses. Loss aversion explains this behavior: if the agents sell losing papers they will realize such a loss, which is not desirable for agents; thus the utility is convex for losses, implying that they take the risk of keeping those papers until experimencing gains. Home bias is another fact explainable through prospect theory. Home bias is a tendency to hold domestic stock in a higher share proportion than the international stock share. This contradicts the results implied by the mean-variance framework (Stracca, 2002). Equity premium puzzle, discovered by Mehra and Prescott (1985), is also analyzed by means of prospect theory. In neoclassical models, risk aversion coefficient should be 30 in order to explain such a phenomenon, while the empirical evidence suggests a value of 1 for this parameter. According to Benartzi and Thaler (1995), loss aversion helps to explain equity premium by means of the attractiveness of a risky asset. This will depend on the planning horizon of the investor: the more frequently the investor evaluates his portfolio, the more likely he experiences losses, and this will imply loss aversion in such a way the investor will demand a higher return in order to hold riskier assets.

Given the state of art of RBC models and the cited application of prospects theory, the goal of this chapter is to build a DSGE model whose core is the inclusion of prospects theory utility function in order to capture the asymmetric behavior of agents along the different phases of a business cycle. The original expression of the prospects utility function is kinked in the reference point (Zero for the original proposal of T-K (1979)), which makes it non-differentiable at that point. Additionally, the prospects utility is originally defined by monetary gains and losses rather than consumption and leisure as it is commonly defined. Thus, I propose three modifications of the prospect utility function. Firstly, I defined it on an aggregator of consumption and leisure. Secondly, I redefined the reference point in such a way that for consumption there is a weighed average of the reference point for consumption in the previous period and consumption in the previous period as well. For leisure, the reference point is analogously defined. Thus, the utility function argument is a consumption and leisure aggregator and its reference point is defined as an aggregator of reference points for consumption and leisure respectively. The utility function is then defined as the consumption-leisure bundle divided by the bundle of reference points for consumption-leisure. Then, when this ratio is greater than one, the agent has gains (and is risk averse); when it is lower than one, the agent has losses (and is loss averse). Thirdly, in order to get differentiability of the utility function, I defined a smooth transition function (by using a logistic function) whose threshold is one. The importance and contribution of this work is extending knowledge of prospect theory utility function, which is a general form that nests loss aversion, risk aversion and habits formation. The simulation of the DSGE model proposed here reveals that loss aversion is a mechanism of transmission suitable to explain asymmetries in business cycles. In Section 2, I present the basic properties of prospect theory utility function. Section 3 presents the construction of prospects utility for consumption and leisure. In section 4, I discuss the reference point formulation. In Section 5, I present the a Prospect Theory-DSGE model. Section 6 has the first order and equilibrium conditions. Section 7 discuses the uniqueness of equilibrium in a model with Prospects Theory utility. Section 8 presents the calibration. Section 9 displays both deterministic and stochastic simulations results. Section 10 deals with conclusions.

2 Prospect theory utility function: Basic properties.

The properties and deduction of the "kinked" utility function were initially derived by T-K (1979) y T-K (1992) who, based on experimental data, discovered the violation of expected utility assumptions. Bowman et al. (1999) have formalized the properties of a utility function based on loss aversion, have developed a consumption-savings model, and have made estimations to test the existence of loss aversion in consumption savings for Canada, France, West Germany, Japan, United States and for the panel. The results were supportive for loss aversion. They also proposed how to model the reference point. Köbberling and Wakker (2005) formalized an index of loss aversion and derived implications for parametric forms of utility functions based on itself. Koszegi and Rabin (2006) developed a model of reference dependent preferences focused on determining the reference point, which has been a very controversial element of Prospect theory. Insofar, they found that the reference point must be rational expectations determined.

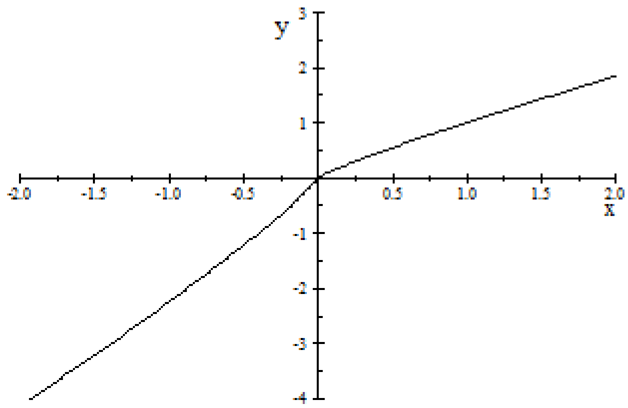
According to T-K (1979) and T-K (1992), when agents face the possibility of random losses, expected utility is an inappropriate descriptor of consumer behavior. Moreover, T-K (1979; 1992) argue that the agent is more sensitive to losses than to gains. This means that losing a quantity of x generates a disutility greater than the utility of wining x. Thus, while for gains the agent will prefer the certainty equivalent of the uncertain bundle (concavity), the certainty equivalence unpreferred for losses. This is, the agent behaves as if he were risk-loving (convexity). Thus, T-K (date) derived the basic properties of Prospect theory utility function, which are summarized as follows:

Let suppose that $u(x)$ is a concave function and $v(-x)$ is a convex function; if $x > 0$, is a gain, $-x < 0$ is a loss, then:

1. The utility of a gain is positive $u(x) > 0$;the utility (disutility) of a loss is negative $v(-x) < 0$. $u(0) = v(0) = 0$

2. The magnitude of the disutility for lossing x is greater than the magnitude of the utitility for gaining $x : |v(-x)| > u(x) \Rightarrow \frac{|v(-x)|}{u(x)} > 1$

Figure 1:



3.$v'(-x) > 0, u'(x) > 0$, and $v'(-x) > u'(x) \Rightarrow \frac{v'(-x)}{u'(x)} > 1$
4.$v''(-x) > 0, u''(x) < 0$

Based on these properties, T-K (1992) has formally proposed the following utility function:

$$U(x) = \begin{Bmatrix} u(x) = x^{\alpha}, if \ x \geq 0 \\ v(x) = -\lambda(-x)^{\beta}, if \ x < 0 \end{Bmatrix} \tag{1}$$

In the figure 1 below, I use the parameters $\alpha = 0.88, \beta = 0.88$ and $\lambda = 2.25$ as estimated by T-K (1992).

There are two additional characteristics of this utility function that are defined in this way: i) it is kinked (and consequently non-differentiable) in $x = 0$; and ii) as it is defined on gains and losses, its reference point is always the same: $x = 0$. I will deepen into the controversial problem of the reference point in section 4, and I also will deal further with the kink of the function exposed in section 3.

In accordance with Köbberling and Wakker (2005), KW (hereafter), the original formulation by TK (1992) has an implicit scaling convention: because the reference point is $x = 0$, their function implies that $u(1) = v(-1) = 1$, which also would imply $\lambda = 1$. Thus, they propose a scaling untied to the unit of payments, which would also imply that the utility function may be differentiable at the reference point. They define the *loss aversion index* as: $\lambda = \frac{v'_{\uparrow}(0)}{u'_{\downarrow}(0)}$. $v'_{\uparrow}(0)$ and $u'_{\downarrow}(0)$ are left and right derivatives respectively supposing that those derivatives do exist as positives and finites. As this loss aversion index is independent from the unit or/of? payments, it is the same for different countries and needs no adjustment (Köbberling and Wakker, 2005, p. 125). Theorem 1 of KW (2005) applies for any index of loss aversion for $\frac{v(\tau)}{u(\tau)}$ for $\tau > 0$ fixed (p. 125).

3 Prospect theory utility function for consumption and leisure

Because the original prospect theory was at first built on losses and gains of wealth, it is centered in zero. However, it is possible to re-define the utility function so that basic properties are fulfilled in order to change the reference point, the payment units, thus conserving a loss aversion utility function with desirable properties as stated by KT (1992) and KW (2005).

The purpose of this section is to build a "general" prospect utility function for consumption and leisure which can be used in a DSGE framework. Thus, we are looking for a utility function that fullfills both loss aversion properties as well as desirable properties for RBC models such as those proposed by King, Plosser, and Rebelo (2001) (see technical appendix for **"Production, growth and business cycles"**). But first, we must deal with the issue of the kink of the utility function when evaluated in the reference point. First, let us assume that the utility function satisfies the properties discussed so far. Then, we will define $U(c, l, r^c, r^l)$ as the utility function derived from consumption and leisure (c, l) and (r^c, r^l) as the reference points for them. Moreover, we will assume for the moment that utility is only derived from consumption:

$$U(c_t, r_t^c) = \left\{ \begin{array}{l} \overline{u}(c_t, r_t^c), \text{ for gains respect to } r_t^c \\ \underline{u}(c_t, r_t^c), \text{ for losses respect to } r_t^c \end{array} \right\} \tag{2}$$

$\overline{u}(c_t, r_t^c)$ and $\underline{u}(c_t, r_t^c)$ are such that:

$$\begin{aligned} \frac{\partial \overline{u}(.)}{\partial c_t} > 0, \frac{\partial^2 \overline{u}(.)}{\partial c_t^2} < 0, \text{ for gains} \\ \frac{\partial \underline{u}(.)}{\partial c_t} > 0, \frac{\partial^2 \underline{u}(.)}{\partial c_t^2} > 0, \text{ for losses} \end{aligned} \tag{3}$$

In order to build a stable DSGE model, the utility function must be consistent with a balanced growth path as demonstrated by King, Plosser and Rebelo (2001), which means that it must have a constant relative risk aversion:

$$R(c) = -c\frac{\overline{u}''(.)}{\overline{u}'(.)} = \overline{\sigma} \tag{4}$$

And for the loss-averse part of the utility function, we can also require and define analogously the constant relative loss aversion coefficient:

$$L(c) = c\frac{\underline{u}''(.)}{\underline{u}'(.)} = \underline{\sigma} \tag{5}$$

Power functions like CRRA fulfill the properties requested for risk averse behavior when agent experiences gains:

$$\overline{u}(c_t, r_t^c) = \frac{(\psi(c_t, r_t^c))^{\overline{\theta}}}{\overline{\theta}} \tag{6}$$

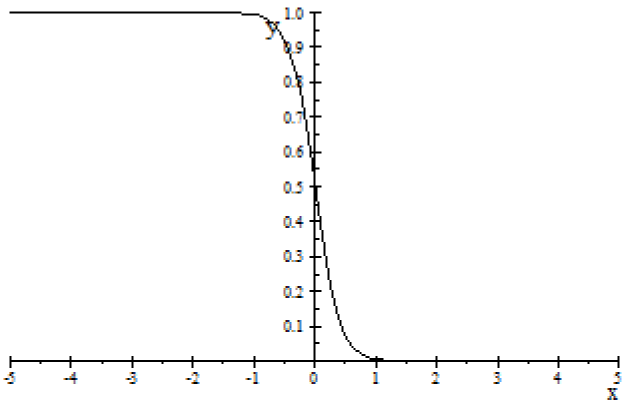
When the agent experiences loss, the utility function will be:

$$\underline{u}(c_t, r_t^c) = \frac{\lambda\, (\psi(c_t, r_t^c))^{\underline{\theta}}}{\underline{\theta}} \tag{7}$$

Being $\psi(c_t, r_t^c)$ a function in c_t and r_t^c. I will visit this point in the discussion of the reference point. Parameters values for $\underline{\theta}$, $\overline{\theta}$, and λ are such that the conditions 1-4 for loss aversion and the definition loss aversion index are fulfilled.

One of the most important features of prospect theory utility function, originally proposed by TK (1979), is its kink in the reference point, which makes the function non-differentiable at

Figure 2:



such point. To deal with the problem of non-differentiability, one of my contributions in this chapter is to include a smooth transition function between two states or regimes, for instance, boom and recession. Smooth transition functions are mostly used in nonlinear econometrics to model transitions between regimes (references here). Let $\phi_t \in [0,1]$ be such that (figure 2,$\gamma = 5$):

$$\phi_t = \frac{1}{1+\exp(\gamma(x))} \tag{8}$$

$$x \to -\infty \Rightarrow \phi_t \to 1$$
$$x \to \infty \Rightarrow \phi_t \to 0$$
$$x \to 0 \Rightarrow \phi_t \to 0.5$$
$$\gamma \to \infty \Rightarrow \phi_t \ step\ function$$
$$\gamma \to 0 \Rightarrow \phi_t \to 0.5$$

Thus, if we define

$$\phi_{ct} = \phi(c_t, r_t^c) = \frac{1}{1+\exp(\gamma(\tau(c_t, r_t^c)))} \tag{9}$$

The utility function with loss aversion and smooth transition is:

$$U_{L,S}(c_t, r_t^c) = \phi_{ct}\underline{u}(c_t, r_t^c) + (1-\phi_{ct})\overline{u}(c_t, r_t^c) = \phi_{ct}\frac{\lambda\left(\psi(c_t, r_t^c)\right)^{\underline{\theta}}}{\underline{\theta}} + (1-\phi_{ct})\frac{\left(\psi(c_t, r_t^c)\right)^{\overline{\theta}}}{\overline{\theta}} \tag{10}$$

In the limit, if γ is large enough, ϕ_{ct} becomes a step function and $U_{L,S}(c_t, r_t^c)$ will be kinked, but still differentiable at the reference point. Similarly, I will define a smooth transition loss aversion utility function only for leisure:

$$\phi_{lt} = \phi(l_t, r_t^l) = \frac{1}{1+\exp(\gamma(\tau(l_t, r_t^l)))} \tag{11}$$

$$U_{L,S}(l_t, r_t^l) = \phi_{lt}\underline{u}(l_t, r_t^l) + (1-\phi_{lt})\overline{u}(l_t, r_t^l) = \phi_{lt}\frac{\lambda\left(\psi(l_t, r_t^l)\right)^{\underline{\mu}}}{\underline{\mu}} + (1-\phi_{lt})\frac{\left(\psi(l_t, r_t^l)\right)^{\overline{\mu}}}{\overline{\mu}} \tag{12}$$

In a more general form, if we have an additive separable utility function of leisure and consumption, we will have:

$$U(c,l,r^c,r^l) = \phi_{ct}\underline{u}(c_t,r_t^c) + (1-\phi_{ct})\overline{u}(c_t,r_t^c) + \phi_{lt}\underline{u}(l_t,r_t^l) + (1-\phi_{lt})\overline{u}(l_t,r_t^l) \tag{13}$$

If there was not any additive separability, the utility function would be:

$$U(c,l,r^c,r^l) = \phi_{ct}(\underline{u}(c_t,r_t^c))^{\omega}(\underline{u}(l_t,r_t^l))^{(1-\omega)} + (1-\phi_{ct})(\overline{u}(c_t,r_t^c))^{\upsilon}(\overline{u}(l_t,r_t^l))^{(1-\upsilon)} \tag{14}$$

4 The reference point

The reference point is perhaps the most controversial element of prospect theory and reference-dependent preferences. The initial proposal by TK (1979) focused on gains and losses of wealth indicate that the reference point was zero. Bowman et al. (1999) originally proposed that the reference point should be a convex combination of both lagged reference point and past consumption. However, if I define the reference point in this same way, and given that I have power functions (CRRA and CRLA), the inmediate consequence is that in the steady state the argument of the utility function will be zero and the marginal utilities will be not defined at that point. This is a problem because the steady state will not be defined as well. To deal with this problem, Bowman et al. (1999) propose for a two-period model a utility function as follows:

$$U(c,r) = \begin{Bmatrix} wr + \frac{(b_g+c-r)^{1-\gamma}}{1-\gamma}, if\ c > r \\ wr - \frac{(b_l+c-r)^{1-\lambda}}{1-\lambda}, if\ c \leq r \end{Bmatrix} \tag{15}$$

$$r_2 = (1-\alpha)r_1 + \alpha c_1 \tag{16}$$

Thus, for a multiperiod model, if we write $r_t = (1-\alpha)r_{t-1} + \alpha c_{t-1}$ and replace into the utility function:

$$U(c_t,r_t) = \begin{Bmatrix} wr_t + \frac{(b_g+c_t-(1-\alpha)r_{t-1}-\alpha c_{t-1})^{1-\gamma}}{1-\gamma}, if\ c > r \\ wr_t - \frac{(b_l+c_t-(1-\alpha)r_{t-1}-\alpha c_{t-1})^{1-\lambda}}{1-\lambda}, if\ c \leq r \end{Bmatrix} \tag{17}$$

Which is a prospect theory utility function; it also generalizes a utility function with habits formation and wr_t is the utility derived from consuming the reference point. In this model of multiple periods by imposing the steady state condition, it results: $r = c$ and

$$U(c,r) = \begin{Bmatrix} wc + \frac{(b_g)^{1-\gamma}}{1-\gamma}, if\ c > r \\ wc - \frac{(b_l)^{1-\lambda}}{1-\lambda}, if\ c \leq r \end{Bmatrix} \tag{18}$$

if we impose that (which is not a condition required for Bowman's model) $wc + \frac{(b_g)^{1-\gamma}}{1-\gamma} = wc - \frac{(b_l)^{1-\lambda}}{1-\lambda} \Rightarrow \frac{(b_g)^{1-\gamma}}{1-\gamma} = -\frac{(b_l)^{1-\lambda}}{1-\lambda} \Rightarrow b_g = \left[\left[-\frac{(b_l)^{1-\lambda}}{1-\lambda}\right](1-\gamma)\right]^{\frac{1}{1-\gamma}}, 0 < b_l$

4.1 The alternatives of modeling the utility

4.1.1 In the way of Bowman et al.

In a similar way, by defining

$$\widetilde{c_t} = c_t - \overline{c}_t \tag{19}$$

Where

$$\overline{c}_t = \vartheta \overline{c}_{t-1} + (1-\vartheta)(c_{t-1} - c^*), \text{ is the reference point} \tag{20}$$

and c^* *is not necessarily the consumption in the steady state.*

$$U(c_t, \overline{c}_t) = \begin{Bmatrix} \frac{(c-\overline{c})^{\overline{\theta}}}{\overline{\theta}}, if\ c_t > \overline{c}_t \\ \frac{\lambda(c-\overline{c})^{\underline{\theta}}}{\underline{\theta}}, if\ c_t \leq \overline{c}_t \end{Bmatrix} \tag{21}$$

The existence of a kinked point in the utility function means that at the reference point, the risk aversion as part of the utility function intersects the loss aversion part of it; thus, they are equal at that point:

$$\frac{\lambda\left(c-\overline{c}\right)^{\underline{\theta}}}{\underline{\theta}} = \frac{\left(c-\overline{c}\right)^{\overline{\theta}}}{\overline{\theta}} \tag{22}$$

Being $c - \overline{c} = \widetilde{c}$, at some moment $\widetilde{c}$ will reach a level where the agent neither loses nor wins. The previous equation can be solved analytically to get:

$$c^* = \left(\frac{\lambda\overline{\theta}}{\underline{\theta}}\right)^{\frac{1}{\overline{\theta}-\underline{\theta}}} \tag{23}$$

Thus the utility function becomes

$$U(c_t, \overline{c}_t) = \begin{Bmatrix} \frac{\left(c-(\vartheta\overline{c}_{t-1}+(1-\vartheta)(c_{t-1}-\left(\frac{\lambda\overline{\theta}}{\underline{\theta}}\right)^{\frac{1}{\overline{\theta}-\underline{\theta}}}))\right)^{\overline{\theta}}}{\overline{\theta}}, if\ c_t > \overline{c}_t \\ \frac{\lambda\left(c-(\vartheta\overline{c}_{t-1}+(1-\vartheta)(c_{t-1}-\left(\frac{\lambda\overline{\theta}}{\underline{\theta}}\right)^{\frac{1}{\overline{\theta}-\underline{\theta}}}))\right)^{\underline{\theta}}}{\underline{\theta}}, if\ c_t \leq \overline{c}_t \end{Bmatrix} \tag{24}$$

For the case of consumption, these definitions imply that, in the steady state $\widetilde{c_t} = c_t - \overline{c}_t \to c^*$, this is a desirable property. However, the most important thing is the dynamics of the steady state. By means of backward induction, I found:

$$\widetilde{c_t} = c_t - \overline{c}_t = c_t - \vartheta\overline{c}_{t-1} - (1-\vartheta)(c_{t-1} - c^*) \tag{25}$$

$$\widetilde{c_t} = c_t - \vartheta^m \overline{c}_{t-m} - (1-\vartheta)\sum_{j=1}^{m} \vartheta^{j-1}(c_{t-j} - c^*)$$

if $m \to \infty$, $\widetilde{c_t}$ an infinite moving average procces centered in c^* :

$$\widetilde{c_t} = c^* + c_t - (1-\vartheta)\sum_{j=1}^{\infty} \vartheta^{j-1} c_{t-j} \tag{26}$$

Thereby, I can express this equation as:

$$\widetilde{c_t} = c_t - \left[(1-\vartheta)\sum_{j=1}^{\infty}\vartheta^{j-1}c_{t-j} - c^*\right] \tag{27}$$

Then, I can define the reference point as an infinite moving average of the past deviations of consumptions with respect to its steady state (a moving average of consumption gaps with respecto to c^*). This definition is rather general because the reference point is a dynamic one and also includes habits formation.

By defining $\tau(c_t, r_t^c) = c - (\vartheta\overline{c}_{t-1} + (1-\vartheta)(c_{t-1} - \left(\frac{\lambda\overline{\theta}}{\underline{\theta}}\right)^{\frac{1}{\overline{\theta}-\underline{\theta}}}))$ and $\psi(c_t, r_t^c) = c - (\vartheta\overline{c}_{t-1} + (1-\vartheta)(c_{t-1} - \left(\frac{\lambda\overline{\theta}}{\underline{\theta}}\right)^{\frac{1}{\overline{\theta}-\underline{\theta}}}))$, the smooth transition loss aversion (STLA) utility funciton will be as follows:

$$\begin{aligned} U_{L,S}(c_t, r_t^c) = \phi_{ct}&\frac{\lambda\left(c - (\vartheta\overline{c}_{t-1} + (1-\vartheta)(c_{t-1} - \left(\frac{\lambda\overline{\theta}}{\underline{\theta}}\right)^{\frac{1}{\overline{\theta}-\underline{\theta}}}))\right)^{\underline{\theta}}}{\underline{\theta}} \\ &+ (1-\phi_{ct})\frac{\left(c - (\vartheta\overline{c}_{t-1} + (1-\vartheta)(c_{t-1} - \left(\frac{\lambda\overline{\theta}}{\underline{\theta}}\right)^{\frac{1}{\overline{\theta}-\underline{\theta}}}))\right)^{\overline{\theta}}}{\overline{\theta}} \end{aligned} \tag{28}$$

$$\phi_{ct} = \phi(c_t, r_t^l) = \frac{1}{1 + \exp(\gamma(c - (\vartheta\overline{c}_{t-1} + (1-\vartheta)(c_{t-1} - \left(\frac{\lambda\overline{\theta}}{\underline{\theta}}\right)^{\frac{1}{\overline{\theta}-\underline{\theta}}})))} \tag{29}$$

4.1.2 In the way of habits formation as defined by Carrol (2000)

It is also possible to define the STLA utility function in a more intuitive way: if we define

$$\overline{c}_t = (1-\vartheta)\overline{c}_{t-1} + \vartheta(c_{t-1}) \Rightarrow \tag{30}$$

$$\overline{c}_t - \overline{c}_{t-1} = \vartheta\left(c_{t-1} - \overline{c}_{t-1}\right) \tag{31}$$

This equation is equivalent to equation (2) in Carrol (2000). In the steady state,

$$\overline{c}_t = \overline{c}_{t-1} = \overline{c}_\infty, \Rightarrow c_\infty = \overline{c}_\infty. \tag{32}$$

This is, the reference point in the steady state equals per capita consumption (things will change in the presence of population growth, see Carrol (2000)). However, out of the steady state, $c_t \neq \overline{c}_t$.

$$z_y = \frac{c_t}{\overline{c}_t} = \frac{c_t}{(1-\vartheta)\overline{c}_{t-1} + \vartheta(c_{t-1})} \tag{33}$$

$$U(c_t, \overline{c}_t) = \begin{cases} (z_t)^{\beta} & if\ z_t > 1,\ c > c_\infty,\ 0 < \beta < 1,\ concavity \\ \lambda\,(z_t)^{\alpha} & z_t < 1,\ c < c_\infty,\ \alpha > 1, convexity \end{cases}, \tag{34}$$

Note that $\frac{c_t}{\bar{c}_t} = z_t \in (0, +\infty)$. Defined in this way, the utility function fulfills the loss aversion index proposed by Köbberling and Wakker (2005): $\frac{v'_{\uparrow}(1)}{u'_{\downarrow}(1)} = \frac{\alpha\lambda}{\beta}$. Thus, the STLA utility function becomes:

$$\phi_{ct} = \frac{1}{1 + \exp\left(\gamma(z-1)\right)} \tag{35}$$

$$U(z_t) = \left(\frac{1}{1 + \exp\left(\gamma(z-1)\right)}\right) \lambda \left(z\right)^{\alpha} + \left(\frac{\exp\left(\gamma(z-1)\right)}{1 + \exp\left(\gamma(z-1)\right)}\right) \left(z\right)^{\beta} \tag{36}$$

In this chaptrer of the disertation, I will use this utility function as it turns out to be more intuitive, easier to deal with, and permits to generalize habits formation, which is already a conventional form to induce persistence in consumption and reference-dependence modeling.

4.1.3 Defining percentage deviations from the reference point

It is also possible to define losses and gains in percentage deviations with respect to the reference level: $x = \frac{c - \bar{c}}{\bar{c}}$, $x \in [-1, \infty)$, and the utility function can be writen as:

$$U(c) = \begin{cases} (1+x)^{\beta} & if\ x > 0,\ 0 < \beta < 1,\ concavity \\ (1+x)^{\alpha} & if\ x < 0,\ \alpha > 1, convexity \end{cases} \tag{37}$$

Using the definition by Booij and van de Kuilen (2006), and Köbberling and Wakker (2005), the loss aversion coefficient will take place

$$loss\ aversion\ coefficient = \frac{U'_{\uparrow}(0)}{U'_{\downarrow}(0)} = \frac{\alpha\left(1+x\right)^{\alpha-1}}{\beta\left(1+x\right)^{\beta-1}} = \frac{\alpha}{\beta}, \tag{38}$$

where is the left derivative and $U'_{\downarrow}(c)$ the right derivative. Thus, the STLA will become:

$$U(x) = \phi_{ct}\left(1+x\right)^{\alpha} + \left(1 - \phi_{ct}\right)\left(1+x\right)^{\beta} \tag{39}$$

$$\phi_{ct} = \left(\frac{1}{1 + \exp\left(\gamma x\right)}\right) \tag{40}$$

5 Uniqueness of the equilibrium

Our prospects utility function is re-defined around the reference point z_{ct} and is given by:

$$U(c_t, \bar{c}_t) = \begin{cases} (z_{ct})^{\overline{\theta}} & if\ z_t > 1,\ c > c_{\infty},\ 0 < \overline{\theta} < 1,\ concavity \\ \lambda\left(z_{ct}\right)^{\underline{\theta}} & z_t < 1,\ c < c_{\infty},\ \underline{\theta} > 1, convexity \end{cases}, \tag{41}$$

Proposition: In a two period economy, the prospects utility function has only one optimum and therefore a unique equilibrium.

Proof: Note that $u(z_{ct}) = (z_{ct})^{\overline{\theta}}$ would be also defined for any value of z_t greater than zero and not only for values greater than 1. similarly, $v(z_{ct}) = (z_{ct})^{\underline{\theta}}$ would be also defined for values greater than 1. This function is kinked but its respective approximated function with smooth transition is not kinked.

Lets suppose the function $y = x$ which is the 45 degrees line with slope equal to 1.

1. By construction, $u(z_{ct}) = (z_{ct})^{\overline{\theta}}$ is concave in S such that $S = [0, \infty)$ and particularly for $S' \subset S$. y $S' = [1, \infty)$.Thus, $\forall\ x \in [1, \infty)$, $u'(x) = \overline{\theta}\,(x)^{\overline{\theta}-1} < 1$, and $\nexists x_o \in [1, \infty) : u'(x_o) = \overline{\theta}\,(x_o)^{\overline{\theta}-1} = 1$.

2. Also by construction, $v(z_{ct}) = (z_{ct})^{\underline{\theta}}$ is convex in $S'' = [0, 1]$, and is also convex in $S = [0, \infty)$. Thus,

a. $\exists x_1 \in [0, 1] : v'(x_1) = \overline{\theta}\,(z_{ct})^{\underline{\theta}-1} = 1$

b. $\forall x \in (x_1, 1], v'(x) = \overline{\theta}\,(x)^{\underline{\theta}-1} > 1$

c. $\forall x \in [0, x_1), v'(x) = \overline{\theta}\,(x)^{\underline{\theta}-1} < 1$

From (a) and (b) it is deduced that $\nexists x_2 \in [x_1, 1] : v'(x_2) = \overline{\theta}\,(x_2)^{\underline{\theta}-1} = u'(x_3) = \overline{\theta}\,(x_3)^{\overline{\theta}-1}, x_3 \in [1, \infty)$. Thus, there does not exist a straight line touching more than one point of the function $U(c_t, \overline{c}_t)$ in the interval $[x_1, \infty)$.

What about the interval $[0, x_1)$ where $v'(x) = \overline{\theta}\,(x)^{\underline{\theta}-1} < 1$?

Given that $u'(x) = \overline{\theta}\,(x)^{\overline{\theta}-1} < 1, \forall\ x \in [1, \infty)$ and $v'(x) = \overline{\theta}\,(x)^{\underline{\theta}-1} < 1, \forall x \in [0, x_1)$, hence $\exists x_4 \in [0, x_1)$,$\exists\ x_5 \in [1, \infty)$ shuch that $v'(x_4) = \overline{\theta}\,(x_4)^{\underline{\theta}-1} = u'(x_5) = \overline{\theta}\,(x_5)^{\overline{\theta}-1} < 1$.

But given $x_4 < x_5$ and $v(x_4) < u(x_5)$ there does not exist a straight line simultaneously touching the utility function in $v(x_4)$ and $u(x_5)$.

3. Suppose now, that in this two period economy the agent has an income M such that $M = P_1C_1 + P_2C_2$. The agent seeks maximizing her inter-temporal utility $U(c_1, \overline{c}_1) + \beta U(c_2, \overline{c}_2)$, subjec to $M = P_1C_1 + P_2C_2$.

Thus, $U'(c_1, \overline{c}_1) = \lambda P_1$ and $U'(c_2, \overline{c}_2)\beta = \lambda P_2$. Then, if the price for consumption in the first period is one and the price for consumption in period 2 is $1/1 + r$, first order conditions will be $U'(c_1, \overline{c}_1) = \lambda$ and $U'(c_2, \overline{c}_2)\beta = \lambda\frac{1}{1+r}$. Thus, $U'(c_2, \overline{c}_2)\beta = U'(c_1, \overline{c}_1)\frac{1}{1+r}$. Note that it is possible to have four cases:

3.1. $u'(c_2, \overline{c}_2)\beta = u'(c_1, \overline{c}_1)\frac{1}{1+r}$

3.2. $v'(c_2, \overline{c}_2)\beta = v'(c_1, \overline{c}_1)\frac{1}{1+r}$

3.3. $u'(c_2, \overline{c}_2)\beta = v'(c_1, \overline{c}_1)\frac{1}{1+r}$

3.4. $v'(c_2, \overline{c}_2)\beta = u'(c_1, \overline{c}_1)\frac{1}{1+r}$

Thus, $U(c_t, \overline{c}_t)$ has one and only one constrained maximum and therefore, only one equilibrium (QED).

6 A DSGE model with loss aversion

In this thesis, one of the departure points of the mainstream literature on RBC is the use of prospect theory to micro-found decisions of agents in an uncertain environment. The purpose of this section is to build a model of a representative agent in a closed economy; this agent owns the firms, chooses consumption and leisure, and accumulates physical capital; there is a neoclassical production function and a stochastic technology shock introduces uncertainty into the model.

Thus, considering the utility function specification of section 4.1.2, defining

$$z_{ct} = \frac{c_t}{\overline{c}_t} = \frac{c_t}{(1-\vartheta)\overline{c}_{t-1} + \vartheta(c_{t-1})} \tag{42}$$

In the steady state:

$$\bar{c}_t = \bar{c}_{t-1} = \bar{c}_\infty, \Rightarrow c_\infty = \bar{c}_\infty \Rightarrow z_{c\infty} = 1 \tag{43}$$

When the economy is not in the steady state $c_t \neq \bar{c}_t$

$$z_{ct} = \frac{c_t}{\bar{c}_t} = \frac{c_t}{(1-\vartheta)\bar{c}_{t-1} + \vartheta(c_{t-1})} \neq 1 \tag{44}$$

$$U(c_t, \bar{c}_t) = \begin{cases} (z_{ct})^{\overline{\theta}} & if\ z_t > 1,\ c > c_\infty,\ 0 < \overline{\theta} < 1,\ concavity \\ \lambda\,(z_{ct})^{\underline{\theta}} & z_t < 1,\ c < c_\infty,\ \underline{\theta} > 1, convexity \end{cases}, \tag{45}$$

Note that $\frac{c_t}{\bar{c}_t} = z_{ct} \in (0, +\infty)$ and, when defined in this way, the utility function fulfills the loss aversion index proposed by Köbberling and Wakker (2005): $\frac{v'_\uparrow(1)}{u'_\downarrow(1)} = \frac{\underline{\theta}\lambda}{\overline{\theta}}$. Thus, the STLA utility function becomes:

$$U(z_{ct}) = \phi_{ct}\lambda\,(z_{ct})^{\underline{\theta}} + (1-\phi_{ct})\,(z_{ct})^{\overline{\theta}} \tag{46}$$

$$\phi_{ct} = \frac{1}{1+\exp\left(\gamma(z_{ct}-1)\right)} \tag{47}$$

$$U(z_{ct}) = \left(\frac{1}{1+\exp\left(\gamma(z_{ct}-1)\right)}\right)\lambda\,(z_{ct})^{\underline{\theta}} + \left(\frac{\exp\left(\gamma(z_{ct}-1)\right)}{1+\exp\left(\gamma(z_{ct}-1)\right)}\right)(z_{ct})^{\overline{\theta}} \tag{48}$$

For leisure, it is also possible to define a loss averse utility function and also a reference point:

$$z_{lt} = \frac{l_t}{\bar{l}_t} = \frac{l_t}{(1-\chi)\bar{l}_{t-1} + \chi(l_{t-1})} \tag{49}$$

In the steady state:

$$\bar{l}_t = \bar{l}_{t-1} = \bar{l}_\infty, \Rightarrow l_\infty = \bar{l}_\infty \Rightarrow z_{l\infty} = 1 \tag{50}$$

When the economy is not in the steady state $l_t \neq \bar{l}_t$

$$z_{lt} = \frac{l_t}{\bar{l}_t} = \frac{l_t}{(1-\chi)\bar{l}_{t-1} + \chi(l_{t-1})} \neq 1 \tag{51}$$

$$U(l_t, \bar{l}_t) = \begin{cases} (z_{lt})^{\overline{\mu}} & if\ z_{lt} > 1,\ l > l_\infty,\ 0 < \overline{\mu} < 1,\ concavity \\ \lambda\,(z_{lt})^{\underline{\mu}} & z_t < 1,\ l < l_\infty,\ \underline{\mu} > 1, convexity \end{cases}, \tag{52}$$

Note that $\frac{l_t}{\bar{l}_t} = z_{lt} \in (0, +\infty)$ and, when defined in this way, the utility function fulfills the loss aversion index proposed by Köbberling and Wakker (2005): $\frac{v'_\uparrow(1)}{u'_\downarrow(1)} = \frac{\underline{\mu}\lambda}{\overline{\mu}}$. Thus, if the only argument of the utility were leisure, the STLA utility function would be:

$$U(z_{lt}) = \phi_{lt}\lambda\,(z_{lt})^{\underline{\mu}} + (1-\phi_{lt})\,(z_{lt})^{\overline{\mu}} \tag{53}$$

$$\phi_{lt} = \frac{1}{1+\exp\left(\gamma(z_{lt}-1)\right)} \tag{54}$$

$$U(z_{lt}) = \left(\frac{1}{1+\exp\left(\gamma(z_{lt}-1)\right)}\right)\lambda\,(z_{lt})^{\underline{\mu}} + \left(\frac{\exp\left(\gamma(z_{lt}-1)\right)}{1+\exp\left(\gamma(z_{lt}-1)\right)}\right)(z_{lt})^{\overline{\mu}} \tag{55}$$

Another contribution in this thesis is the inclusion of leisure into the utility function, given that consumption is not the only good delivering utility to individuals. In this respect, I invoke Veblen from his seminal work "The theory of the leisure class". He argues that manual work or industrious run puts the individual as one belonging to a lower social and economic class. Thus, leisure demand is not due only to the fact thatoffers utility itself, but also due to an intention of emulation by those who truly want to look for boasts in a higher social and economic class. In this sense, it is supposed that leisure and consumption are not additive separable in the instantaneous utility function. Thus, the prospect theory utility function for consumption and leisure would be:

$$U(c,l,r^c,r^l) = \phi_{clt}(\lambda\,(z_{ct})^{\underline{\theta}}))^{\omega}(\lambda\,(z_{lt})^{\underline{\mu}})^{(1-\omega)} + (1-\phi_{clt})((z_{ct})^{\overline{\theta}}))^{\upsilon}((z_{lt})^{\overline{\mu}})^{(1-\upsilon)} \tag{56}$$

Note also that for the non-additive separable function, if $\theta_i = \mu_i$, and the aggregation parameters of consumption and labor do not change between states (i.e. $\omega = \upsilon$)

$$U(c,l,r^c,r^l) = \phi_{clt}\lambda\left(z_{ct}^{\omega}z_{lt}^{(1-\omega)}\right)^{\underline{\theta}} + (1-\phi_{clt})\left(z_{ct}^{\omega}z_{lt}^{(1-\omega)}\right)^{\overline{\theta}} \tag{57}$$

thus, the reference point for the agent is $agr_t = z_{ct}^{\omega}z_{lt}^{(1-\omega)}$, which I will call the *"aggregator"*; thus, the transition function in this case will be:

$$\phi_{clt} = \frac{1}{1+\exp\left(\gamma(z_{ct}^{\omega}z_{lt}^{(1-\omega)}-1)\right)} = \frac{1}{1+\exp\left(\gamma(agr_t-1)\right)} \tag{58}$$

Regarding the values of $agr_t = z_{ct}^{\omega}z_{lt}^{(1-\omega)}$, it is obvious that when z_{ct} and $z_{lt} > 1$, $agr_t > 1$, and when z_{ct} and $z_{lt} < 1$, $agr_t < 1$. However, it is not so evident what happens when $z_{ct} > 1$ and $z_{lt} < 1$ or vice versa. In a situation where $z_{ct} > 1$ and $z_{lt} < 1$, we take logs of agr_t and ask ourselves under what cases it would be greater than one when we have $\frac{lnz_{ct}}{-lnz_{lt}} > \frac{(1-\omega)}{\omega} = \varrho$. Thus, the size of agr_t will depend directly on the size of $\frac{lnz_{ct}}{-lnz_{ct}}$ and the size of ϱ will be the size of . The same line of reasoning applies for the situation where $z_{ct} < 1$ and $z_{lt} > 1$ and the relevant expression is $\frac{lnz_{lt}}{-lnz_{ct}} > \frac{\omega}{1-\omega} = \varpi$. In the calibration proposed for the simulation exercises, $\omega = 0.5$, which is a capricious choice in order to free the results from bias. However, empirical work is yet needed in order to know the model parameters suggested by the data.

7 The consumer problem, first order conditions and equilibrium

The production function, physical capital accumulation, and technological shocks are the same as those of a canonical RBC model:

$$Y_t = A_t K_t^{\alpha} n_t^{(1-\alpha)} \tag{59}$$

$$K_{t+1} = (1-\delta)K_t + Y_t - C_t \tag{60}$$

$$\ln A_t = \rho \ln A_{t-1} + \varepsilon_t \tag{61}$$

$$\varepsilon_t \backsim N(0,\sigma) \tag{62}$$

$$l_t = 1 - n_t \tag{63}$$

The Lagrangian function for this central planer problem will be:

$$\pounds_t = E_t \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{c} \left[U(c,l,r^c,r^l) = \phi_{clt}\lambda \left(z_{ct}^{\omega} z_{lt}^{(1-\omega)} \right)^{\underline{\theta}} + (1-\phi_{clt}) \left(z_{ct}^{\upsilon} z_{lt}^{(1-\upsilon)} \right)^{\overline{\theta}} \right] \\ +\mu_t \left[-K_{t+1} + (1-\delta)K_t + Y_t - C_t \right] \end{array} \right\} \tag{64}$$

Where μ_t is the Lagrange multiplier, and I define:

$$\overline{u}_t(.) = \left(z_{ct}^{\upsilon} z_{lt}^{(1-\upsilon)} \right)^{\overline{\theta}} \tag{65}$$

$$\underline{u}_t(.) = \lambda \left(z_{ct}^{\omega} z_{lt}^{(1-\omega)} \right)^{\underline{\theta}} \tag{66}$$

It is possible to write in compact form:

$$\pounds_t = E_t \sum_{t=0}^{\infty} \beta^t \left\{ \left[\phi_{clt}\underline{u}_t(.) + (1-\phi_{clt})\overline{u}_t(.) \right] + \mu_t \left[-K_{t+1} + (1-\delta)K_t + Y_t - C_t \right] \right\} \tag{67}$$

$$\pounds_t = E_t \sum_{t=0}^{\infty} \beta^t \left\{ \left[\overline{u}_t(.) + \phi_{clt} \left[\underline{u}_t(.) - \overline{u}_t(.) \right] \right] + \mu_t \left[-K_{t+1} + (1-\delta)K_t + Y_t - C_t \right] \right\} \tag{68}$$

control variables are c_t, K_{t+1}, n_t. and the first order conditions are:

$$\begin{aligned} \frac{\partial \pounds_t}{\partial c_t} = 0 = & \left\{ \frac{\partial \overline{u}_t(.)}{\partial z_{ct}} + \phi_{clt} \left[\frac{\partial \underline{u}_t(.)}{\partial z_{ct}} - \frac{\partial \overline{u}_t(.)}{\partial z_{ct}} \right] + \frac{\partial \phi_{clt}}{\partial z_{ct}} \left[\underline{u}_t(.) - \overline{u}_t(.) \right] \right\} \frac{\partial z_{ct}}{\partial c_t} \\ & - \mu_t + \beta E_t \left\{ \begin{array}{c} \frac{\partial \overline{u}_{t+1}(.)}{\partial z_{ct+1}} + \phi_{clt+1} \left[\frac{\partial \underline{u}_{t+1}(.)}{\partial z_{ct+1}} - \frac{\partial \overline{u}_{t+1}(.)}{\partial z_{ct+1}} \right] \\ + \frac{\partial \phi_{clt+1}}{\partial z_{ct+1}} \left[\underline{u}_{t+1}(.) - \overline{u}_{t+1}(.) \right] \end{array} \right\} \frac{\partial z_{ct+1}}{\partial c_t} \end{aligned} \tag{69}$$

$$\begin{aligned} \frac{\partial \pounds_t}{\partial l_t} = 0 = & \left\{ \frac{\partial \overline{u}_t(.)}{\partial z_{lt}} + \phi_{clt} \left[\frac{\partial \underline{u}_t(.)}{\partial z_{lt}} - \frac{\partial \overline{u}_t(.)}{\partial z_{lt}} \right] + \frac{\partial \phi_{clt}}{\partial z_{lt}} \left[\underline{u}_t(.) - \overline{u}_t(.) \right] \right\} \frac{\partial z_{lt}}{\partial l_t} \\ & - \mu_t \frac{\partial Y_t}{\partial n_t} + \beta E_t \left\{ \begin{array}{c} \frac{\partial \overline{u}_{t+1}(.)}{\partial z_{lt+1}} + \phi_{clt+1} \left[\frac{\partial \underline{u}_{t+1}(.)}{\partial z_{lt+1}} - \frac{\partial \overline{u}_{t+1}(.)}{\partial z_{lt+1}} \right] \\ + \frac{\partial \phi_{clt+1}}{\partial z_{lt+1}} \left[\underline{u}_{t+1}(.) - \overline{u}_{t+1}(.) \right] \end{array} \right\} \frac{\partial z_{lt+1}}{\partial l_t} \end{aligned} \tag{70}$$

$$\frac{\partial \pounds_t}{\partial K_{t+1}} = 0 = -\mu_t + \beta E_t \left\{ \mu_{t+1} \left[(1-\delta) + \frac{\partial Y_{t+1}}{\partial K_{t+1}} \right] \right\} \tag{71}$$

$$\frac{\partial \pounds_t}{\partial \mu_t} = 0 = -K_{t+1} + (1-\delta)K_t + Y_t - C_t \tag{72}$$

We define the regime switching marginal utility of consumption and leisure respectively as:

$$\varphi_{ct} = \left\{ \frac{\partial \overline{u}_t(.)}{\partial z_{ct}} + \phi_{clt} \left[\frac{\partial \underline{u}_t(.)}{\partial z_{ct}} - \frac{\partial \overline{u}_t(.)}{\partial z_{ct}} \right] + \frac{\partial \phi_{clt}}{\partial z_{ct}} \left[\underline{u}_t(.) - \overline{u}_t(.) \right] \right\} \frac{\partial z_{ct}}{\partial c_t} \tag{73}$$

$$\varphi_{lt} = \left\{ \frac{\partial \overline{u}_t(.)}{\partial z_{lt}} + \phi_{clt} \left[\frac{\partial \underline{u}_t(.)}{\partial z_{lt}} - \frac{\partial \overline{u}_t(.)}{\partial z_{lt}} \right] + \frac{\partial \phi_{clt}}{\partial z_{lt}} \left[\underline{u}_t(.) - \overline{u}_t(.) \right] \right\} \frac{\partial z_{lt}}{\partial l_t} \tag{74}$$

We define the switching marginal disutily of consumption and leisure of time t in the period $t+1$, caused by the efect of habits on the reference point.

$$\xi_{ct+1} = \left\{ \frac{\partial \overline{u}_{t+1}(.)}{\partial z_{ct+1}} + \phi_{clt+1} \left[\frac{\partial \underline{u}_{t+1}(.)}{\partial z_{ct+1}} - \frac{\partial \overline{u}_{t+1}(.)}{\partial z_{ct+1}} \right] + \frac{\partial \phi_{clt+1}}{\partial z_{ct+1}} \left[\underline{u}_{t+1}(.) - \overline{u}_{t+1}(.) \right] \right\} \frac{\partial z_{ct+1}}{\partial c_t} \tag{75}$$

$$\xi_{lt+1} = \left\{ \frac{\partial \overline{u}_{t+1}(.)}{\partial z_{lt+1}} + \phi_{clt+1} \left[\frac{\partial \underline{u}_{t+1}(.)}{\partial z_{lt+1}} - \frac{\partial \overline{u}_{t+1}(.)}{\partial z_{lt+1}} \right] + \frac{\partial \phi_{clt+1}}{\partial z_{lt+1}} \left[\underline{u}_{t+1}(.) - \overline{u}_{t+1}(.) \right] \right\} \frac{\partial z_{lt+1}}{\partial l_t} \tag{76}$$

thus the first order conditions for consumption and leisure can be written in a compact form as:

$$\varphi_{ct} - \mu_t + \beta E_t \{\xi_{ct+1}\} = 0 \tag{77}$$

$$\varphi_{lt} - \mu_t \frac{\partial Y_t}{\partial n_t} + \beta E_t \{\xi_{lt+1}\} = 0 \tag{78}$$

Thus the dynamic equilibrium equations for consumption and leisure respectively become:

$$\varphi_{ct} = \beta E_t \left\{ [\varphi_{ct+1} + \beta E_t \{\xi_{ct+2}\}] \left[(1-\delta) + \frac{\partial Y_{t+1}}{\partial K_{t+1}} \right] - E_t \{\xi_{ct+1}\} \right\} \tag{79}$$

$$\varphi_{lt} = \{\varphi_{ct} + \beta E_t \{\xi_{ct+1}\}\} \frac{\partial Y_t}{\partial n_t} - \beta E_t \{\xi_{lt+1}\} \tag{80}$$

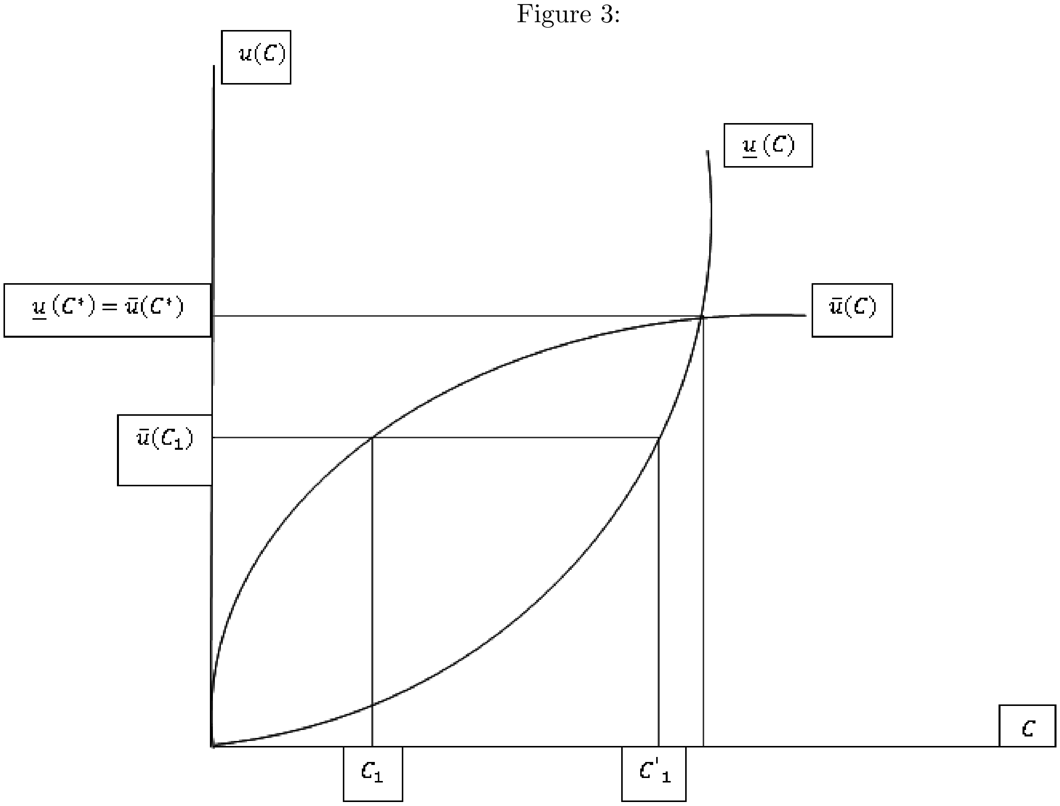
These equations jointly with transition equation for physical capital and the stochastic process for technology shocks conform the dynamical equilibrium of this ecnomiy populated by the representative prospect theory agent.

7.1 On why loss aversion could be a good explanation for business cycles asymmetries

Let us suppose that the economy is in a situation such that the steady state coincides with the reference point. Thus, the representative agent has a utility equal to "reference utility": $C_t = C^*, K_t = K^*, Y_t = Y^*, \underline{u}(C^*) = \overline{u}(C^*)$. Now, let us suppose that the economy is negatively shocked. If the agent were risk averse, he would choose C_1 and would enjoy $\overline{u}(C_1)$. But if he were loss averse he would choose a consumption level such that the departure from the reference level (C^*) would not be too large, minimizing therefore their loss of welfare. Note that by choosing C_1, the loss averse agent would have $\underline{u}(C_1) < \overline{u}(C_1)$. Thus, a loss averse agent needs to choose a consumption level such that he can, at least, enjoy a utility equivalent to $\underline{u}(C_1)$; this is, she has to choose $C_1^{'}$ such that $\underline{u}(C_1^{'}) = \overline{u}(C_1)$, which means that $C_1^{'} = \underline{u}^{-1}(\overline{u}(C_1)) > C_1$. Thus, in face of a negative shock, the loss averse agent would choose a consumption level lower than the one before the shock, but greater than the one the agent would choose if he were risk averse (figure 3).

What are the consequences on saving and investment? Let us suppose now that $Y_1 = A_1 K_1^{\alpha} n_1^{(1-\alpha)}$, the outcome in steady state, and $Y_1 = A_1 K_0^{\alpha} n_0^{(1-\alpha)}$, the outcome on shock, being $A_1 < A_0$. Thus,

Figure 3:



the capital accumulation will be $K_1^{LA} = (1-\delta)K_0 + Y_1 - C_1^{'}$ and $K_1^{RA} = (1-\delta)K_1 + Y_1 - C_1$ for the loss-averse and for the risk-averse agents respectively, and $K_0 = (1-\delta)K_0 + Y_0 - C_0$ would correspond to the capital accumulation in the steady state. As the economy was negatively shocked, savings and investment will fall below their steady state levels; thus, $I_1^{LA} < I_0$, $I_1^{RA} < I_0$. After substracting $I_1^{LA} - I_0$ from $I_1^{RA} - I_0$, we will have $I_1^{RA} - I_1^{LA} = -C_1 + C_1^{'} > 0$, which is equivalent to stating that $I_1^{LA} < I_1^{RA}$. Thus, loss aversion amplifies the effect of a negative shock to the economy on capital accumulation and increases the variability of investment during recessions. Moreover, as the technology suffers a negative shock, marginal product of capital decreases. This means that savers will require a premium on the return of savings; otherwise, they will consume more while saving less. We need to keep in mind that the marginal product equals the interest rate in equilibrium; thus, $U'(c_2, \overline{c}_2)\beta = U'(c_1, \overline{c}_1)\frac{1}{1+MPK_2}$ and $MPK_2 = \alpha A_2 K^{\alpha-1} n^{(1-\alpha)}$. Thus, when marginal product of capital falls marginal, utility and consumption fall as well because consumers save less and consume more. This reaction in consumption is more severe when the agent is loss-averse than when the agent is risk-averse.

This result is, however, a very particular case of a more general one where $\underline{u}$ could be not only a convex but also a concave function with less curvature than that of $\overline{u}$.

8 Steady state and calibration

The steady state for this economy is a situation such that: $\overline{c}_t \to c^*, \overline{l}_t \to l^*, k_t \to k^*$ and thus $\overline{u} = \underline{u} = 1, \phi_t \to \phi^* = 0.5$:

$$\varphi_c^* = \left\{\overline{\theta} + 0.5\left[\underline{\theta} - \overline{\theta}\right]\right\}\omega(c^*)^{-1} \tag{81}$$

$$\varphi_l^* = \left\{\overline{\theta} + 0.5\left[\underline{\theta} - \overline{\theta}\right]\right\}(1-\omega)(l^*)^{-1} \tag{82}$$

$$\xi_{ct+1} = -\left\{\overline{\theta} + 0.5\left[\underline{\theta} - \overline{\theta}\right]\right\}\vartheta\omega(c^*)^{-1} \tag{83}$$

$$\xi_{lt+1} = -\left\{\overline{\theta} + 0.5\left[\underline{\theta} - \overline{\theta}\right]\right\}\chi(1-\omega)(l^*)^{-1} \tag{84}$$

Thus, from the Euler equation for consumption we have:

$$\left\{0.5\left[\underline{\theta} + \overline{\theta}\right]\right\}\omega(c^*)^{-1} = \beta E_t \left\{ \begin{array}{c} \left[\begin{array}{c} \left\{0.5\left[\underline{\theta} + \overline{\theta}\right]\right\}\omega(c^*)^{-1} \\ +\beta E_t\left\{-\left\{0.5\left[\underline{\theta} + \overline{\theta}\right]\right\}\vartheta\omega(c^*)^{-1}\right\} \end{array} \right] \left[(1-\delta) + \frac{\partial Y_{t+1}}{\partial K_{t+1}}\right] \\ -E_t\left\{-\left\{0.5\left[\underline{\theta} + \overline{\theta}\right]\right\}\vartheta\omega(c^*)^{-1}\right\} \end{array} \right\} \tag{85}$$

$$1 = \beta\left\{\left[(1-\delta) + \frac{\partial Y}{\partial K}\right]\right\} \tag{86}$$

This is the well-known equation for the stochastic (symmetric) growth model without population or technological long-run growth.

For the Euler equation for leisure, we have:

$$\left\{0.5\left[\underline{\theta}+\overline{\theta}\right]\right\}(1-\omega)(l^*)^{-1} = \left\{\left\{0.5\left[\underline{\theta}+\overline{\theta}\right]\right\}\omega(c^*)^{-1} + \beta E_t\left\{-\left\{0.5\left[\underline{\theta}+\overline{\theta}\right]\right\}\vartheta\omega(c^*)^{-1}\right\}\right\}\frac{\partial Y_t}{\partial n_t} - \beta E_t\left\{-\left\{0.5\left[\underline{\theta}+\overline{\theta}\right]\right\}\chi(1-\omega)(l^*)^{-1}\right\} \tag{87}$$

$$\{1-\beta\chi\}(1-\omega)(l^*)^{-1} = \{1-\beta\vartheta\}\omega(c^*)^{-1}\frac{\partial Y_t}{\partial n_t} \tag{88}$$

thus for calibration purposes, the key equations equations will be:

$$1 = \beta\left\{\left[(1-\delta) + \alpha K^{\alpha-1}n^{1-\alpha}\right]\right\} \tag{89}$$

$$\frac{c^*}{l^*} = \frac{\{1-\beta\vartheta\}\omega}{\{1-\beta\chi\}(1-\omega)}(1-\alpha)K^{\alpha}n^{-\alpha} \tag{90}$$

$$\delta k^* = y^* - c^* \tag{91}$$

Note that since there is neither population nor technological growth in the long run, the concavity-convexity of the utility function does not play any role in the steady state determination (this seems to be very useful because it helps to compare steady state results with those of other models). Also, it has $\lambda = 1$ by construction, disappearing either in the long run equations, and in the transitional dynamics.

9 Simulations and results

9.1 Deterministic simulation

In order to test the consistency of the model construction, deterministic simulations have been run initially. To this end, technology is shocked one time (negative and positive). However, instead of solving it by any approximation algorithm, I have used the extended path method implemented in Dynare by imposing that $a = 1.06$ (positive shock) and $a = 0.9433$, which is equivalent to having $e = 0.058268908$ and $e = -0.0566$ respectively.[3] Figures 4 and 5 show path time of key macro variables: logaritms of consumption, income, capital, investment, labor and technology $(lc_t, ly_t, lk_t, li_t, ln_t, la_t)$, and marginal products of labor and capital $(pml_t,\ pmk_t)$.

9.2 Stochastic simulation: Impulse response

Impulse response is one of the most used analysis tools in macro-econometrics. However, it must be used carefully. Because the DSGE model studied in this thesis is non-linear and asymmetric, impulse response analysis should not be performed as usual assuming that the DGP is linear-multivariate. Moreover, it would be a mistake to simply shock technology once and then follow the adjustment of the whole system. Therefore, in order to gauge asymmetric effects of shocks in this

[3] It would be also possible to impose a symmetric e (this is, the same size of the shock in absolute value) but there would not be a great difference in the results.

Table 3:

Preliminary calibration			
parameter	value	variable	value
risk averse		c^*	1.69
$\overline{\theta}$	0.7	y^*	2.41
ω	0.5	i^*	0.725
loss averse		n^*	0.46
$\underline{\theta}$	1.5	l^*	0.54
ω	0.5	k^*	29.03
λ	1	c/y^*	0.7
production		n/y^*	0.19
α	0.4	k/y^*	12.04
δ	0.025	i/y^*	0.3
A	1	$Mgpk$	0.033
ρ	0.9	$Mgpn$	3.144
σ_ε	0.0018		
smooth transition			
γ	100		
reference points			
ϑ	0.5		
χ	0.5		
discount factor			
β	0.9917		

Figure 4:

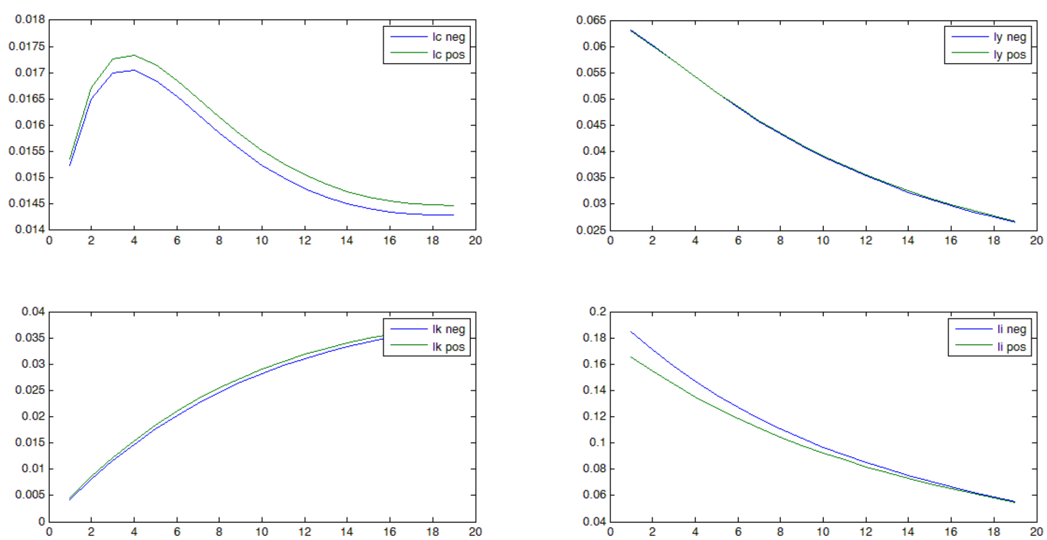
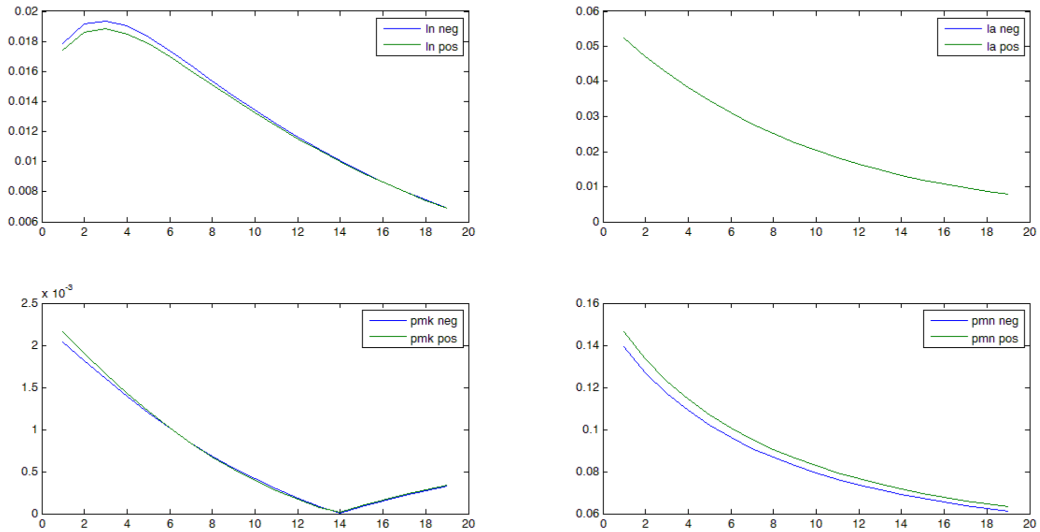


Figure 5:



hypothetical economics General Impulse Response Function (Koop et al., 1996) (GIRF hereafter) are to be adopted.[4]

Because asymmetric DGP of this DSGE model, multivariate data therein simulated lack the following properties: Symmetry property, linearity property and history independence property. Thus, linear impulse response functions (VAR-based) are inappropriate tools for analyzing the dynamics of such a DSGE model. The GIRF as defined by Koop et al. (1996) is conditioned on shocks and/or history:

$$GI_Y(n, v_t, \omega_{t-1}) = E[Y_{t+n}|v_t, \omega_{t-1}] - E[Y_{t+n}|\omega_{t-1}], \quad for \quad n = 0, 1...$$

Being Y_t a vector of variables, v_t a current shock, ω_{t-1} the history, and n the forecasting horizon. Koop et al. (1996) also describes a simple algorithm to compute these conditional expectatios through Monte Carlo integration. According to this method, GIRF could resemble a distribution of impulse-responses for each period in the forecast horizon. Impulse responses computed in this fashion are calculated and reported by Dynare. By default, Dynares drops the first 100 observations and then reports GIRF for a horizon of 40 periods ahead. Figures 9.2.1 and 9.2.1 show impulse responses (50 draws) for one standard deviation shock (positive and negative) on the perturbation term of the technology process.

9.2.1 Conditioning on a particular shock

The first simulation exercise consisted in giving a once standard deviation shock (positive and negative) to the technology process in the asymmetric model. The simulation was performed for

[4] Local Projections Impulse Response (Jordá, 2005) could also be used, but this technique is susceptible of symmetry, thus it would not be possible to detect asymmetry in data of this hipothetical model.

fifty replications; the response of macroeconomic variables in this hypothetical economy to negative shocks (in average) are asymmetric with respect to positive shocks. Thus, the GIRF computed was $GI_Y(n, v_t, \Omega_{t-1}) = E[Y_{t+n}|v_t, \Omega_{t-1}] - E[Y_{t+n}|\Omega_{t-1}]$, being Ω_{t-1} an information set of the previous history, and v_t a particular negative and positive shock. Figures 9.2.1 and 9.2.1 show these impulse response functions.

For consumption, capital, income, investment, and technology, the graphs show log-deviations while labor and marginal products of capital and labor are in levels. As it can be seen, the reaction of consumption to a negative shock is stronger than the reaction to a positive shock. However, the fall in consumption during recession is less deep and less long-lasting than the increase during boom. This can be explained by the loss-averse nature of the agents in this model. When the agent suffers a fall in income which deviates him from the reference point, he minimizes the loss induced by such deviation. Thus, his fall in consumption will be as small as possible. To this end, the agent reduces savings which brings about reduction in investment and consequently in physical capital. For the case of income, the reaction to a negative shock seems to be greater than the reaction to the positive shock, although the difference between them is almost imperceptible. For capital, the responses to perturbations on shock are very similar. Nevertheless, for positive shock, capital increase during the boom seems to be deeper and more long-lasting than the decrease during the recession. For investment, the fall in recession is very severe compared with the increase in the boom. Investment boom is less deep than during recession, but lasts longer.

For labor, interesting results were also found: the negative shock generates a stronger reaction than the positive shock and is matched by a significative smaller fall in wage (compared with the increase of wage induced by the positive shock). This means that although this model does not have either involuntary unemployment or (explicitly modeled) rigidities, a greater negative reaction of labor during a recession is accompanied by a smaller reaction in real wage; certainly, the opposite does occur after a positive technological shock. Because the utility function also includes leisure, the mechanics is the same as for consumption: a fall in income and consumption will induce an increase in leisure (as big as possible) in order to dampen the utility loss. For the physical capital, its marginal product does also seem to show some rigidity during recessions compared to booms.

9.2.2 Conditioning on a particular history

Because asymmetric models are history-dependent, it is necessary to ask what the time path of the economy in boom would be, or ask what the time path of the economy in recession would be, either positively or negatively shocked.. The results of simulating a positive shock as the economy undergoes a boom or simulating a negative shock as the economy undergoes a recession are trivial: a recession deepening and boom sharpening take place. However, since business cycles are asymmetric, it would be necessary to perform the simulation in order to know the quantitative effects. Nonetheless, it would be more interesting to know the quantitative effects of a negative shock during boom and a positive shock during recession. To perform the exercise above proposed, it must be supposed that the economy is initially shocked (positively or negatively) in period one, and in period four it receives a shock in the opposite direction to the one received in period one. Thus, the exercise consisted in computing $GI_Y(n, v_t, \tilde{\Omega}_{t-1}) = E[Y_{t+n}|v_t, \tilde{\Omega}_{t-1}] - E[Y_{t+n}|\tilde{\Omega}_{t-1}]$, being $\tilde{\Omega}_{t-1}$ the state of the economy (being in boom or in recession) and v_t a positive or negative shock.

There is another important detail to take into account: this exercise is time-dependent. This implies that the new position of the economy after the second shock would depend on how far it is

Figure 6:

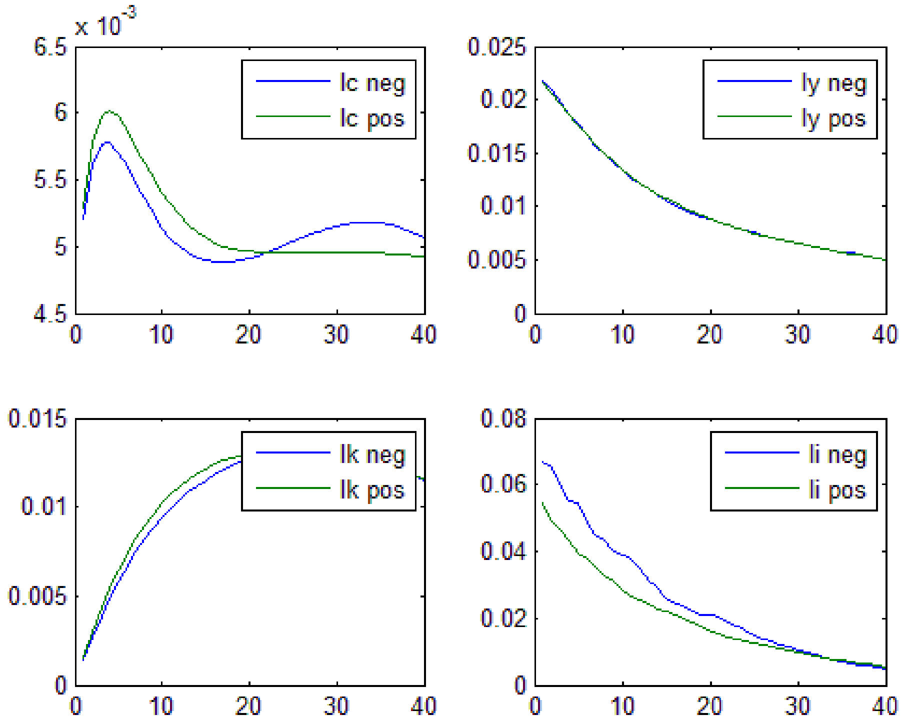


Figure 7:

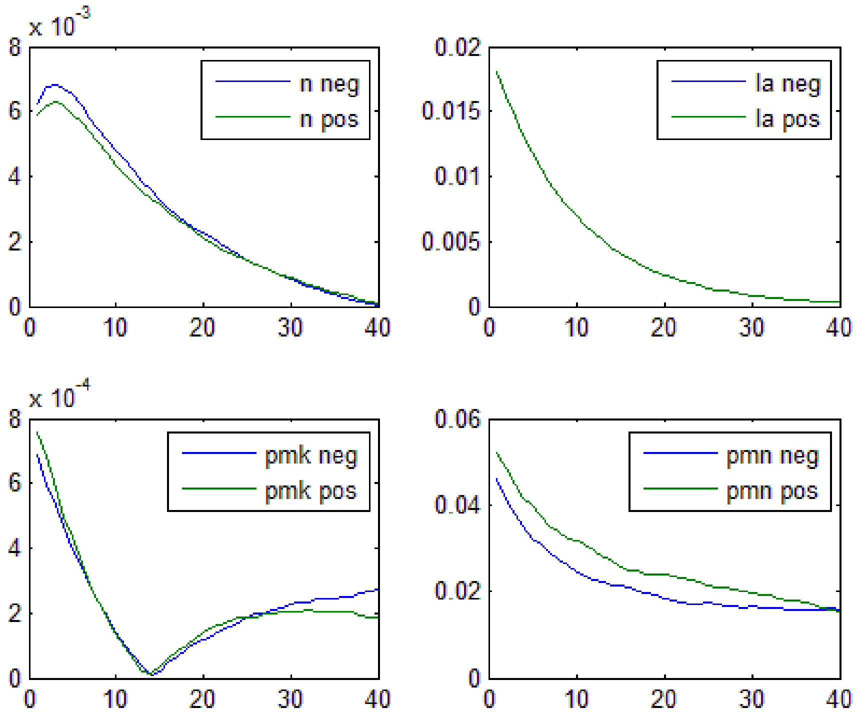
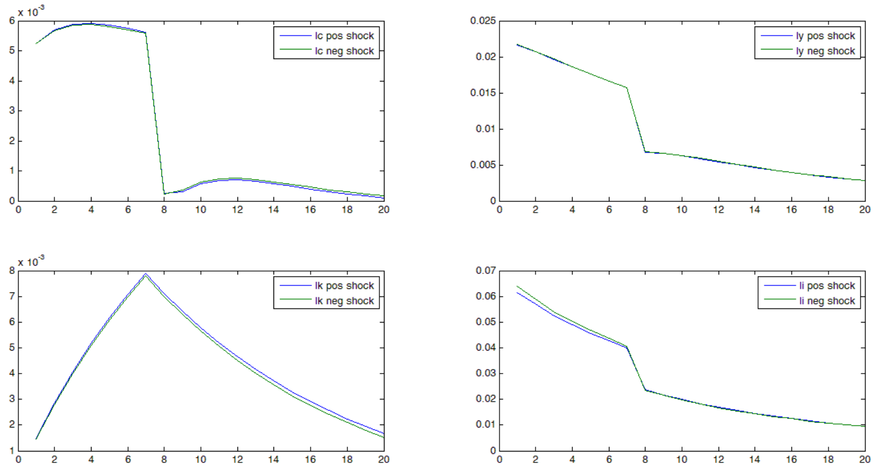

x 10^-3
n neg
n pos
0.02
0.015
0.01
0.005
la neg
la pos
x 10^-4
pmk neg
pmk pos
0.06
0.04
0.02
pmn neg
pmn pos

Figure 8:



from the steady state. That is to say, the longer the horizon of GIRF, the closer the economy will be to the steady state and, therefore, depending on the size of the shock (and on the asymmetric structure of the economy), the economy could jump (suddenly perhaps) from a boom into a recession and vice versa. In order to standardize the problem of timing, the exercise was performed as follows: the second (positive or negative) shock was introduced in a time $t0$ in such a way that the technology gap were a half of its initial value on shock. In this section, all variables are measured in logarithms. Then, a gap of variables can be interpreted as log-deviations from the steady state.

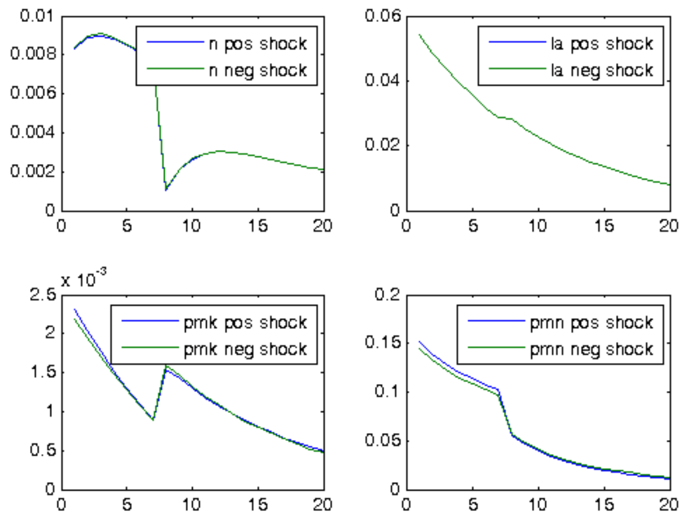
9.2.3 A second shock in the opposite direction of the first shock

Figures 8 and 9 show the adjustment path of the economy after receiving a positive shock during a recession and a negative shock during a boom. In this exercise, it seems clear that a shock in the opposite direction pushes the economy to the other phase of the cycle, this is, making it fall from a boom into a recession or makes it jump from a recession into a boom. In the case of capital, it reverses, however, the accumulation slowly (de-accumulation) process induced by a positive (negative shock).

In figures 10 and 11 (absolute values) the path of the economy is shown from the period it receives the second negative (positive) during a boom (recession).

When the economy is perturbed by a negative shock while in a boom, the asymmetrical nature of this model can be seen again. The reaction of consumption on shock when the economy is negatively shocked is greater than the reaction when the shock is positive, but this is only for the first period. However, in general terms, the recession in consumption induced by the negative shock during the boom is less deep and less long-lasting than the boom induced by the positive shock during a recession. For income, the previous result holds even since the period when the economy receives the second shock. In the case of capital, the negative shock during a boom induces a more severe capital deaccumulation than the accumulation induced by the positive shock during the recession. The dynamics of investment is consistent with what happens in capital: the reaction

Figure 9:



of investment to the second negative shock is stronger and more long-lasting than the reaction to the second positive shock. Why do these facts result like that? When the economy is in a boom, the risk aversion households makes them to desire being as far as possible from the reference point. But when the economy is negatively shocked and income falls and the household needs to adjust its consumption level, it wants to stay as close as possible to the reference point because of its loss aversion.

Figure 11 shows what happens to labor, wages, and interest rate. The reaction of labor is very similar for both shocks, although there is an important difference between the wage reactions. When the economy is shocked by a negative perturbation during a boom, the marginal product of labor shows a reaction weaker than the one shown when the economy receives a positive shock during a recession. Once again, this model seems to exhibit some rigidity in wage: in recession, the fall in wage is smaller than the increase in boom. For the case of physical capital, when the economy is negatively shocked during the boom, the interest rate fall is smaller (during five periods) than the increase when positively shocked during a recession, which is consistent with the greater fall in capital when the negative shocks occur during boom.

9.2.4 A second shock in the same direction of the first shock

We would also feel eager to ask about the effect of a positive shock during a boom or about the effect of a negative shock during recession. To answer these questions, we have performed an exercise as the previous one, but instead of giving a negative shock after a positive one, we give both: a first and a second positive shocks. First negative shock and second negative shock are also simulated.

Figures 12 and 13 show the trajectories of the economy when it is shocked by a second positive (negative) technological perturbation. The reactions observed after the first positive (negative) shock exacerbate with the second positive (negative) shock, although the asymmetrical nature of

Figure 10:

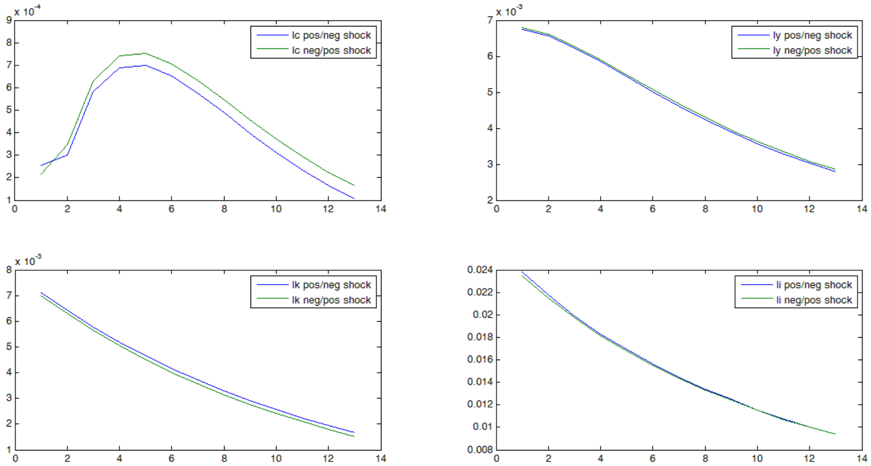


Figure 11:

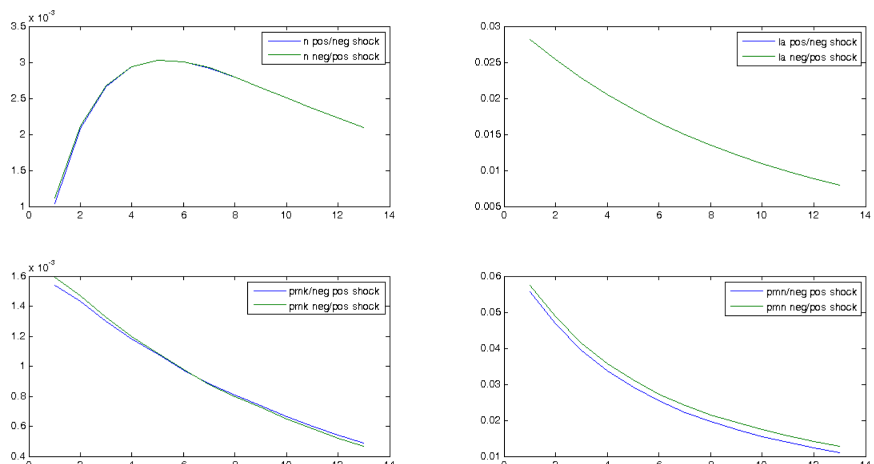
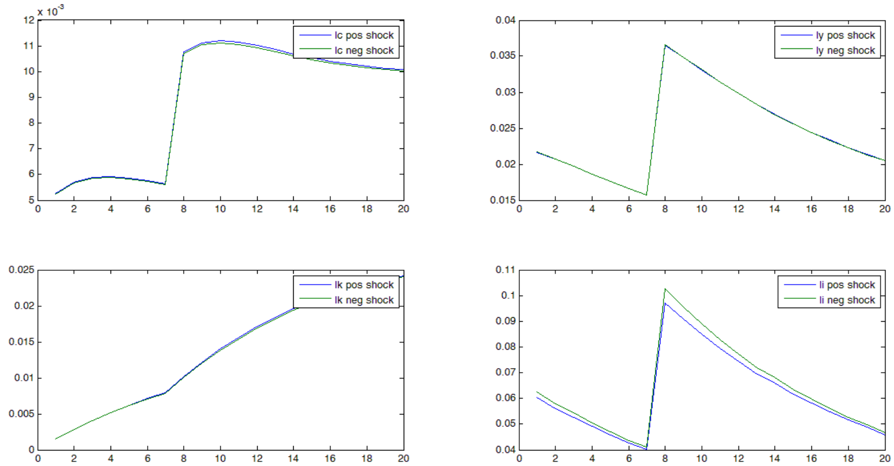


Figure 12:



the model is revealed once again. For consumption, positive shock induces a higher reaction than the negative shock. While for income, the asymmetry is almost imperceptible. For capital, the positive shock accelerates the accumulation process while the negative shock accelerates its deaccumulation. Asymmetry in investment reaction is also evident, The effect of the negative shock is larger than the one of the positive shock. This is also consequence of the loss aversion of households. Labor paths are very similar, but the wage paths show the same asymmetry as in the first shock. Again, some rigidity is shown by wage after a negative shock.

Figures 14 and 15 shows more clearly that consumption reacts strongly and deeper to positive shocks. They also display that the boom induced in consumption by the positive shock lasts longer than the recession. The reactions in income are, again, almost the same while the reaction in capital to the positive shock is stronger than its reaction to the negative shock. The fall in investment caused by the negative shock is higher than the increase induced by the positive shock; thus, there is a severe and apparently long-lasting fall of investment. Labor has a stronger reaction to the negative shock than to the positive, which explains why income reactions are not very different as noted above. The smaller fall in wage is also evident here in recession. Furthermore, while in boom there is a higher increase in wage, and the same for marginal product of capital.

9.2.5 Shocks in the same direction during different phases of cycle

At this point, it is necessary to compare shocks in the same direction, but in a different phase of the cycle. This means comparing the reaction of the economy receiving a negative shock during a boom with the reaction of the economy receiving a negative shock during recession, and the same comparison is pursued for the case of positive shocks.

Figures 16 and 17 show the time path for the experiment of perturbing the economy with a positive shock, both during boom and during recession. It is obvious that positive shocks exacerbate booms in consumption, income, investment, labor, real wage, and real interest rate, inducing an increased process of capital accumulation. The effect of the positive shock during recession is more

Figure 13:

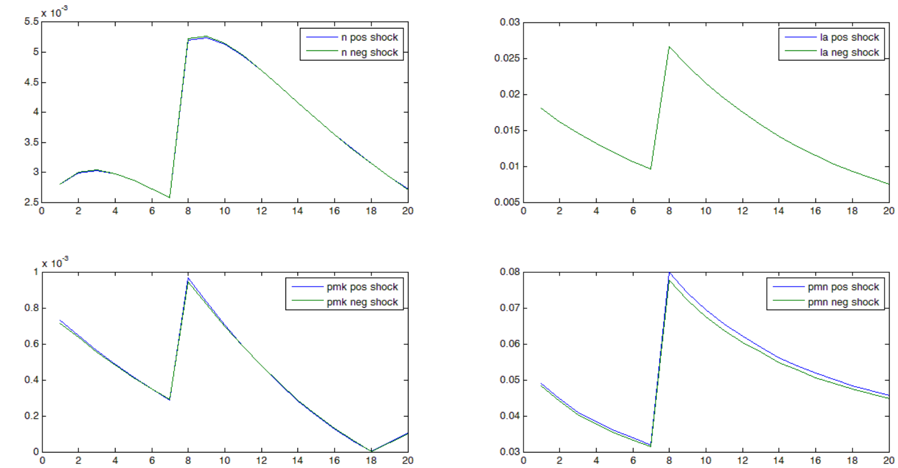


Figure 14:

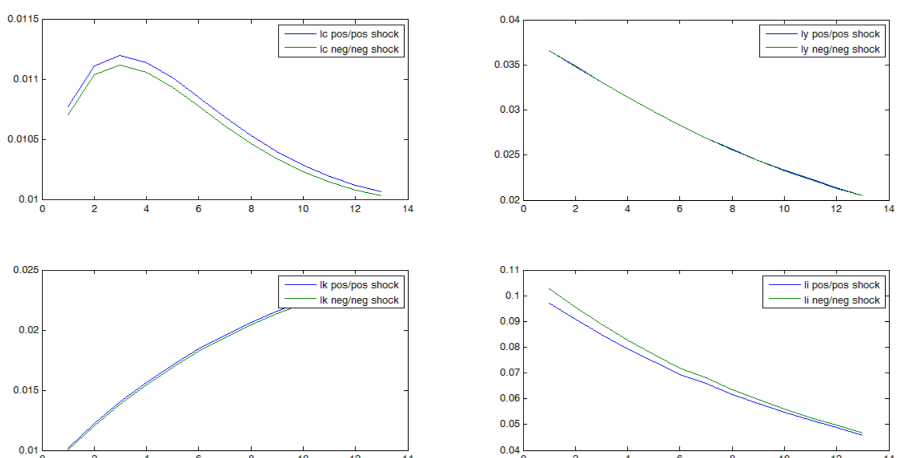
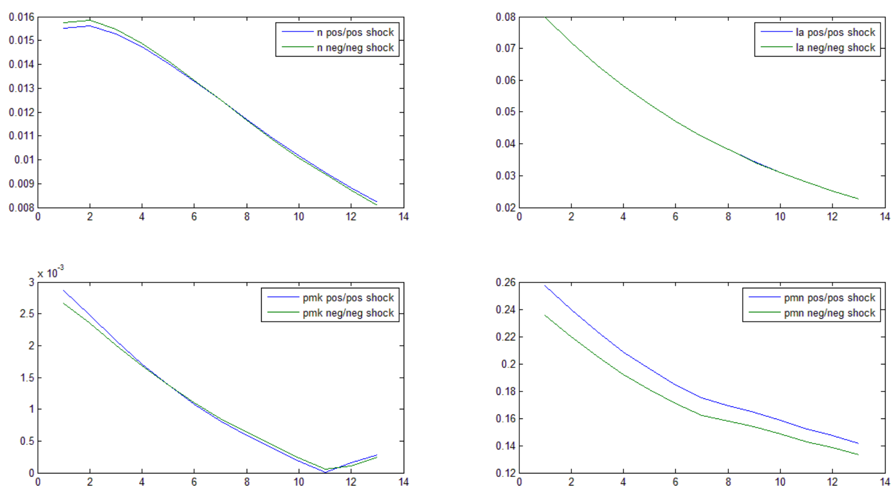


Figure 15:



interesting because it induces a rapid recovering of the whole system: for consumption, income, investment, labor, real wage, and, real interest rate, the trajectories go above the steady state (not too far from the steady state, though). For the case of physical capital, the positive shock during recession reverses the deaccumulation process induced initially by the negative shock.

Figures 18 and 19 show the time path for the experiment of perturbing the economy with a positive shock, both during boom and during recession. The negative shock during recession, as in the case of positive shock during boom, exacerbates the bad situation of the economy and deepens the deaccumulation process of capital. For the case of the negative shock during boom, the effect is somewhat catastrophic: the fall of the economy is mostly severe for labor, investment, and capital because of the loss-averse behavior of agents, which helps the agents mantain their consumption as close as possible to the steady state.

10 Conclusions

In this chapter it was possible to build a DSGE model including loss aversion and risk aversion in a more general functional form known as prospects theory utility function following TK (1979) and KT (1992). I call my model Prospects Theory-DSGE Model (PT-DSGE). The main contribution of my work is extending the original (and rather simple) prospect theory utility function, developed by TK(1979, 1992), into a general form that nests loss aversion, risk aversion, and habits formation. In order to achieve this, I proposed three modifications of the prospect utility function. First, I have defined it on an aggregator of consumption and leisure. Second, I have redefined the reference point. For consumption, it is a weighted average of its reference point in the previous period and consumption in the previous period as well. Consequently, I used the same definition for the leisure reference point. Thus, the utility function argument is an aggregator of both consumption and leisure, and its reference point is defined as an aggregator of reference points for consumption and

Figure 16:

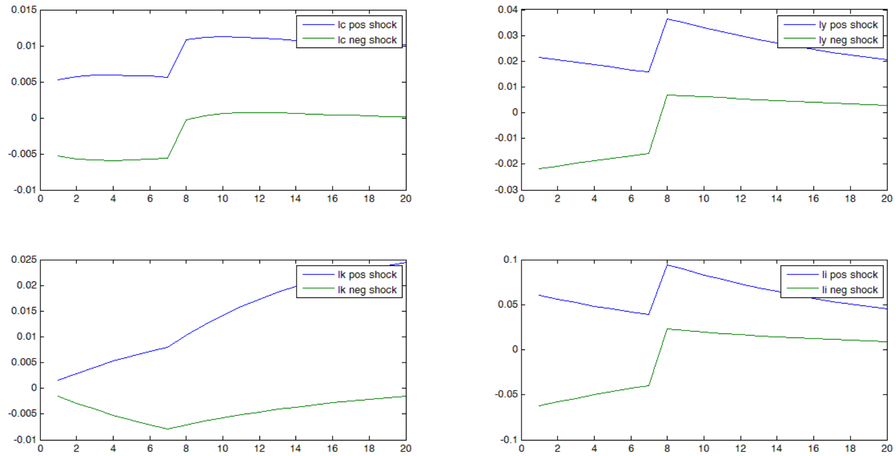


Figure 17:

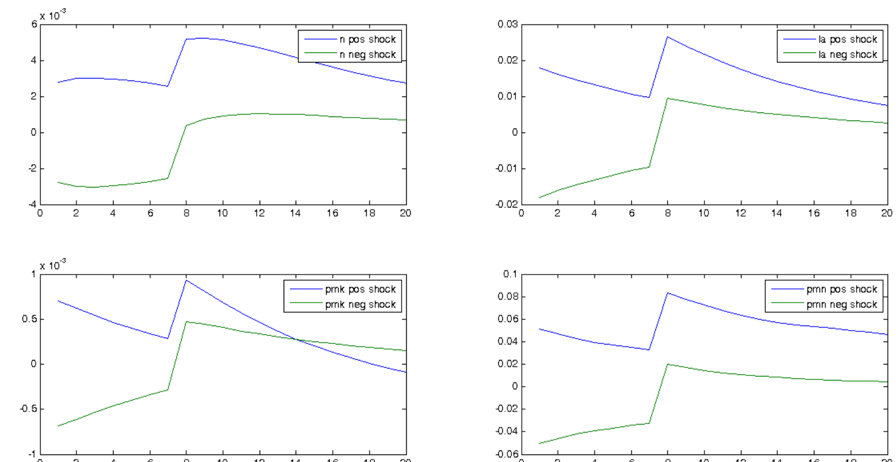


Figure 18:

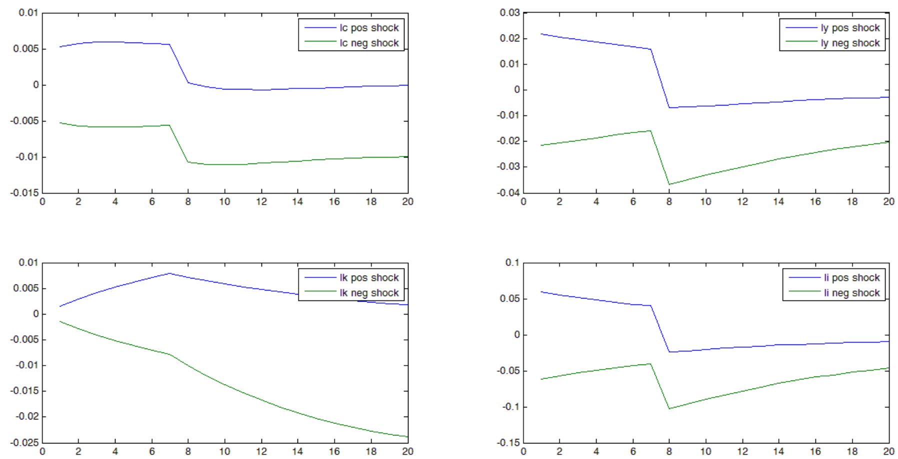
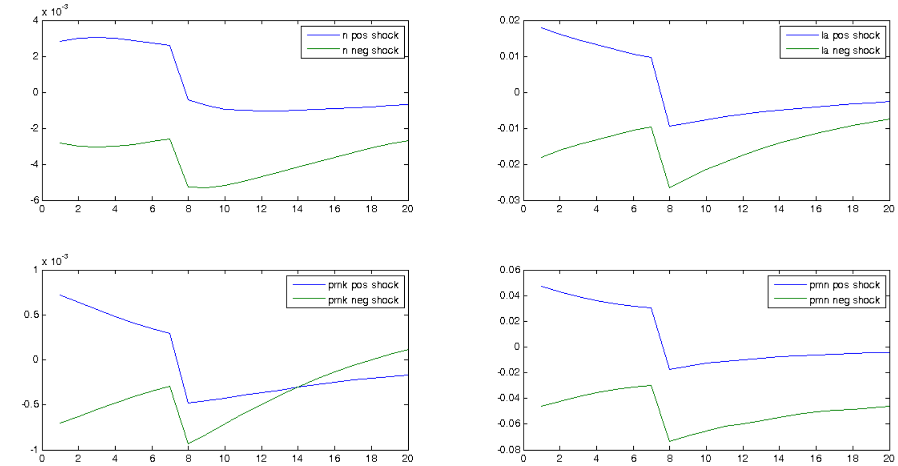


Figure 19:



leisure respectively. The utility function is defined as the consumption-leisure bundle divided by the bundle of reference points for consumption-leisure. Then, when this ratio is greater than one, the agent has gains (and is risk-averse); and, when it is lower than one, he has losses (and is loss-averse). Third, in order to get differentiability of the utility function (in the kinked point), I defined a smooth transition function (by using a logistic function) whose threshold is 1. Given that my PT-DSGE model has a utility function which is convex below the reference point and concave above, it was necessary to establish a result about optimum uniqueness of the (restricted) optimum for such a utility function.

The aim of building this PT-DSGE model was to track the link between the dynamics of the business cycle asymmetries and the asymmetric behavior of agents along the phases of the cycle. In order to evaluate the effectiveness of the model to generate asymmetrical business cycles, it is necessary to simulate the model deterministically and stochastically using extended path (or exact solution) and perturbation method (third-order approximation) respectively by means of Dynare. In the deterministic simulation, two exercises were performed: i) positive shock to technology and ii) negative shock to technology. In the stochastic simulation, General Impulse Response Functions were calculated for positive and negative shocks as well, so that the shocks were equivalent to those in the deterministic case. Results for stochastic simulations were qualitatively the same and quantitatively similar to the ones in the deterministic procedure. On shock, the reaction of consumption to a positive shock is stronger than the reaction to a negative shock, but the fall in consumption in recession is less deep and less long-lasting than the increase in the boom phase. For physical capital, the responses to perturbations on shock are similar. However, for positive shock, capital increases during the boom seem to be deeper and more long-lasting than the decrease during the recession. For investment, the fall in recession is greater as compared with the increase in the boom phase. A boom of investment is less deep than during recession, but it lasts longer. For labor, interesting results were also found: the negative shock generates a stronger reaction than the positive shock and is matched by a significatively smaller fall in wage (compared with the increase of wage induced by the positive shock), which means that although this model has neither involuntary unemployment nor (explicitly modeled) rigidities, a greater negative reaction of labor during a recession is accompanied by a smaller reaction in real wage. Meanwhile, the opposite does occur after a positive technological shock. This seems to be consistent with the stylized facts of business cycles. For the physical capital, its marginal product also seems to show some rigidity during recessions as compared to booms. For the case of income, the results have been almost trivial. The reaction to a negative shock seems to be greater than the reaction to the positive shock, although the difference between them is almost imperceptible. This can be explained by the fact that the production function is symmetrical, the shocks are also symmetrical, and the movement in capital is compensated by a move in labor in the opposite way.

Since the model proposed here is state-dependent, it is necessary to simulate shocks (positive and negative) while the economy is in boom or in recession. Then, the first exercise was based on the supposal that the economy receives a second shock in the same direction of the first shock. That is, during a boom phase induced by a positive shock, the economy is shocked one more time by a positive shock; during a recession, a new negative shock is received by the economy. Indeed, consumption reacts strongly and deeper to positive shocks, and the boom induced in consumption by the positive shock lasts longer than the recession. The reactions in income are again almost the same, given that the reaction in capital to the positive shock is stronger than its reaction to the negative shock. The fall in investment caused by the negative shock is higher than the increase induced by the positive shock. Thus, there is a severe and apparently long-lasting investment fall.

Labor has a stronger reaction to the negative shock than to the positive, which explains why income reactions are not very different as noted above. The smaller fall in wage reveals wage stickiness as an endogenous result in this model. In recession, it is also evident here that, while in boom, there is a higher increase in wage, and the same occurs for marginal capital product.

It was also necessary to simulate positive (negative) shocks while the economy is in recession (expansion). This means a simulation of shocks in the opposite direction of the first shock. In this exercise, it has been clear that a shock in the opposite direction pushes the economy to the other phase of the cycle, thus making it fall from a boom into a recession or jump from a recession into a boom. In the case of capital, it reverses (however) slowly the accumulation (deaccumulation) process induced by a positive (negative shock).

We have seen again the asymmetrical nature of this model when the economy is perturbed by a negative shock while being in a boom. The reaction of consumption on shock when the economy is negatively shocked is greater than the reaction when the shock is positive, but this is only for the first period. Despite that, in general terms, the recession in consumption induced by the negative shock during the boom is less deep and less long-lasting than the boom induced by the positive shock during a recession. For income, the previous result holds even since the period when the economy receives the second shock. In the case of capital, the negative shock during a boom induces a more severe capital deaccumulation than the accumulation induced by the positive shock during the recession. The dynamics of investment is consistent with what happens in capital: the reaction of investment to the second negative shock is stronger and more long-lasting than the reaction to the second positive shock. The explantion for this is the fact that as the economy goes through a boom risk aversion of households, they are led to choose a consumption level as far as possible from the reference point. However, when the economy is in recession, the income fall raises the need of households to adjust their consumption lever in such a way that the consumption level is as close as possible to the reference point, which can be explained by loss aversion. For wages and interest rate, the reaction of labor is similar for both shocks, although there is an important difference between the reactions of wage. When the economy is shocked by a negative perturbation during a boom the marginal product of labor shows a reaction weaker than the one showed when the economy receives a positive shock during a recession. Once again this model seems to exhibit rigidity in wage: in recession, the fall in wage is smaller than the increase in boom. For the case of physical capital, when the economy is negatively shocked during the boom, interest rate fall is smaller (during five periods) than the increase when positively shocked during a recession, which is consistent with the greater fall in capital when the negative shocks occurs during a boom.

In general, the model built in this thesis is able to generate asymmetrical business cycles, which proves that asymmetrical behavior of consumers modeled by prospects utility function is a suitable transmission mechanism. In the model, expansions are deeper and more long-lasting for consumption and capital than contractions, while the over smoothing of consumption, facing a negative shock, causes (on shock) a more severe, deeper, and more long-lasting reaction of investment than facing a positive shock. The fall in employment in a recession is severe, deeper, and more long-lasting than in expansion, and is accompanied by a fall in wages less intense than the increase shown by them during the expansion. Thus, this model also reproduces real rigidities in wages and some hysteresis in unemployment. According to Bowman et al. (1999) and with Shea (1995), Loss Aversion implies that the reaction of consumption facing a reduction in expected income is stronger than the reaction facing increase in expected income. This is not inconsistent with the smoother reaction in consumption, because their results are based on a growth rate regression of consumption on interest rate and instruments for expected income, whereas the results in this thesis are derived from GIRF. Moreover, as it can be seen that as expected income decreases, consumption reacts stronger from period 4 on, while as expected income increases, consumption reacts less intensely.

Asymmetries in RBC models could be more adequately captured by General Impulse Response Functions than by higher-order moments. However, a more rigorous test for the properties of the asymmetric model proposed here would entail the application of nonlinear econometric tools. Even though nonlinear econometrics could be a useful tool for this purpose, several issues remain open for the research agenda: i) Structural parameters of the model need to be estimated; ii) the Loss Aversion DSGE model I propose can be used to study issues in policy making, asset pricing, risk premium puzzle, international asymmetric business cycles, risk sharing, home bias, among other areas and disciplines.

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