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Five texture zeros and *CP* violation for the standard model quark mass matrices

William A. Ponce and John D. Gómez
Instituto de Física, Universidad de Antioquia, Apartado Aereo 1226, Medellín, Colombia

Richard H. Benavides
Instituto Tecnológico Metropolitano, Facultad de Ciencias, Medellín, Colombia
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The phenomenology of the five independent sets of 3×3 quark mass matrices with five texture zeros is carried through in full detail, including predictions for the CP-violation asymmetries. Our study is done without any approximation, first analytically and then numerically.

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I. INTRODUCTION

Although the gauge boson sector of the standard model (SM) with the $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ local symmetry has been very successful [1], its Yukawa sector is still poorly understood. Questions related to this sector, such as the total number of families in nature, the hierarchy of the charged fermion mass spectrum, the smallness of neutrino masses, the quark mixing angles, the neutrino oscillations, and the origin of the CP violation, remain open problems in theoretical particle physics.

The mechanisms for fermion mass generation and flavor mixing can be classified into four different categories: (i) radiative mechanisms [2] (which includes the so-called Froggat and Nielsen mechanism [3]); (ii) texture zeros in the mass matrices which may predict self-consistent and experimentally favored relations between fermion masses and flavor mixing parameters [4,5]; (iii) family symmetries—discrete [6] and continuous symmetries and global and local gauge symmetries [7], and (iv) see-saw mechanisms for electrically neutral [8] and charged particles [9], related to a natural interpretation of the smallness of some fermion masses.

In the SM after the local gauge symmetry has been spontaneously broken, the quark mass terms are given by

$$-\mathcal{L}_M = \bar{U}_{0L} M_u U_{0R} + \bar{D}_{0L} M_d D_{0R} + \text{H.c}, \quad (1.1)$$

where $\bar{U}_{0L} = (\bar{u}_0, \bar{c}_0, \bar{t}_0)_L$, $\bar{D}_{0L} = (\bar{d}_0, \bar{s}_0, \bar{b}_0)_L$, $U_{0R} = (u_0, c_0, t_0)^T_R$, and $D_{0R} = (d_0, s_0, b_0)^T_R$, (the upper T stands for transpose, and the down zero stands for weak basis states). The matrices M_u and M_d in (1.1) are in general 3×3 complex mass matrices. In the most general case, they contain 36 free parameters. In the context of the SM, such a large number of parameters can be drastically cut by making use of the polar theorem of matrix algebra, by which one can always decompose a general matrix as the product of a Hermitian times a unitary matrix. Since in the context of the SM the unitary matrix can be absorbed in a redefinition of the right-handed quark components, this immediately brings down the number of free parameters from 36 to 18 (the other 18 parameters can be hidden in the right-handed quark components in the context of the SM and some of its extensions, but not in its left-right symmetric extensions).

So, as far as the SM is concerned, we may treat without loss of generality M_u and M_d as two Hermitian quark mass matrices, with 18 real parameters in total, out of which six are phases. Since five of those phases can be absorbed in a redefinition of the quark fields [10], the total number of free parameters we may play with in M_u and M_d are 12 real parameters and one phase; this last one used to explain the CP-violation phenomena.

But in the context of the SM it is always possible to implement the so-called weak basis (WB) transformation, which leaves the two 3×3 quark mass matrices Hermitian and does not alter the physics implicit in the weak currents. Such a WB transformation is a unitary transformation acting simultaneously in the up and down quark mass matrices [11]. That is

$$M_u \to M_u^R = U M_u U^\dagger, \qquad M_d \to M_d^R = U M_d U^\dagger, \quad (1.2)$$

where U is an arbitrary unitary matrix. We say then that the two representations (M_u, M_d) and (M_u^R, M_d^R) are equivalent in the sense that they are related to the same V_{CKM} mixing matrix. This kind of transformation plays an important role in the study of the so-called flavor problem.

In the last paper of Ref. [11] it was shown that related to the mass hierarchy $m_u < m_c < m_t$ and $m_d < m_s < m_b$, it is always possible to perform a weak-basis transformation in the Hermitian quark mass matrices such that

$$(M_u^R)_{11} = (M_d^R)_{11} = (M_u^R)_{13} = (M_u^R)_{31} = 0; \quad \text{or, equivalently,} \quad (M_u^R)_{11} = (M_d^R)_{11} = (M_d^R)_{13} = (M_d^R)_{31} = 0. \quad (1.3)$$

According to this, it is always possible to have Hermitian quark mass matrices with three texture zeros that do not have any physical implication. With three texture zeros, the number of free parameters in M_u^R and M_d^R reduces from twelve to nine real plus one phase, just

enough to fit the measured values for the six quark masses, the three mixing angles, and the CP-violation phenomena. Any extra texture zero can only be a physical assumption and should imply a relationship between the quark masses and the parameters of the mixing matrix. But the maximum number of such texture zeros consistent with the absence of a zero mass eigenvalue and a nondegenerate spectrum is just six, with three in the up and three in the down quark sectors, respectively.

In what follows we are going to present analytic and numeric results for the set of SM quark mass matrices containing five texture zeros, taking special care to accommodate the latest experimental data available [12], including the CP- violation phenomena.

This paper is organized as follows. In Sec. II some features of the SM mixing matrix are presented, in Sec. III we introduce the five independent sets of 3×3 quark mass matrices with five texture zeros, and in Sec. IV we present our analytic and numerical analysis. The results are discussed in Sec. V, and the conclusions are presented in Sec. VI. Two appendixes are written at the end: the first one presents the experimental measured values used in the main text and the second one is a purely mathematical appendix, dealing with the analytic diagonalization of real 3×3 orthogonal matrices.

II. THE SM MIXING MATRIX

In the SM for the six-flavor case, the Baryon charged weak current is given by

$$J^{-}_{\mu L} = \bar{U}_{0L}\gamma_\mu D_{0L} = \bar{U}_L \gamma_\mu V_{\rm CKM} D_L, \tag{2.1}$$

where $V_{\rm CKM} = U_u U_d^\dagger$ is the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix, with U_u and U_d the unitary matrices which diagonalize the Hermitian $M_u M_u^\dagger$ and $M_d M_d^\dagger$ square mass matrices, respectively, and $\bar{U}_L = (\bar{u}, \bar{c}, \bar{t})_L$ and $D_L = (d, s, b)_L^T$ stand for the quark field mass eigenstates.

$V_{\rm CKM}$ is a 3×3 unitary matrix. Its form is not unique, but the permutation freedom between the three generations can be removed by ordering the families such that $(u_1, u_2, u_3) \rightarrow (u, c, t)$ and $(d_1, d_2, d_3) \rightarrow (d, s, b)$. The complex elements of $V_{\rm CKM}$ are thus commonly written as

$$V_{\rm CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}. \tag{2.2}$$

The unitary of the CKM mixing matrix leads to relations among the rows and columns of $V_{\rm CKM}$; in particular we have for the columns,

$$V_{ud}V_{us}^* + V_{cd}V_{cs}^* + V_{td}V_{ts}^* = 0, \tag{2.3a}$$
$$V_{us}V_{ub}^* + V_{cs}V_{cb}^* + V_{ts}V_{tb}^* = 0, \tag{2.3b}$$
$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0. \tag{2.3c}$$

Each of these three relations requires the sum of three complex quantities to vanish and so can be geometrical represented in the complex plane as a triangle. These are the unitary triangles [13], though the term "unitary triangle" is usually reserved for the relation (2.3c) only.

The three angles of the unitary triangle represented by (2.3c), which are physical quantities and can be independently measured by CP asymmetries in B decays, are defined as follows [13]:

$$\alpha \equiv \arg\left[-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right], \tag{2.4a}$$
$$\beta \equiv \arg\left[-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right], \tag{2.4b}$$
$$\gamma \equiv \arg\left[-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right]. \tag{2.4c}$$

The experimental findings at the B factories, fitted to close the triangle, are [14,15]

$$(\alpha, \beta, \gamma)^{\rm fit}_{\rm exp} = (95.9^{+2.2}_{-5.6}, 21.8 \pm 2.8, 67.2^{+4.4}_{-4.6}), \tag{2.5}$$

with an accuracy in the measurement of $\sin 2\beta$ no less than 20% [16].

The Cabbibo-Kobayashi-Maskawa $V_{\rm CKM}$ matrix, can be parametrized by three mixing angles θ_{ij}, $i < j$, $i, j = 1, 2, 3$, the angles between the ith and jth families, and only one CP violating phase [10]. Of the many possible parametrizations, the standard choice is [12,17]

$$V_{\rm CKM} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \tag{2.6}$$

but the most important fact related to this matrix is that most of its entries have been measured with high accuracy, with the following bounds [12,14],

$$V^{(\rm exp)} = \begin{pmatrix} 0.970 \le |V_{ud}| \le 0.976 & 0.222 \le |V_{us}| \le 0.226 & 0.003 \le |V_{ub}| \le 0.004 \\ 0.217 \le |V_{cd}| \le 0.237 & 0.960 \le |V_{cs}| \le 0.990 & 0.039 \le |V_{cb}| \le 0.041 \\ 0.008 \le |V_{td}| \le 0.009 & 0.038 \le |V_{ts}| \le 0.042 & 0.999 \le |V_{tb}| < 1.000 \end{pmatrix}, \tag{2.7}$$

where the experimental numbers quoted above at 95% C.L. are beyond the purely experimental bounds (which call for example for a $|V_{tb}| > 0.79$) because they are restricted to fit the unitary conditions of $V_{\rm CKM}$. The numbers quoted in (2.7) are the most convenient for our purpose, due to the fact that we are going to confront these numbers with quark mass matrices that must fit the SM constraints.

III. FIVE TEXTURE ZEROS

As mentioned before, the maximum number of texture zeros in the quark mass matrices of the SM, consistent with a nondegenerate spectrum and with the absence of a zero-mass eigenvalue, are three in the up and three in the down quark sectors, respectively. With this in mind, Harald Fritzsch proposed some time ago [4] the existence of parallel three-texture-zero structures for the 3×3 quark mass matrices M_u and M_d of the SM, such that $(M_q)_{11} = (M_q)_{22} = (M_q)_{13} = (M_q)_{31} = 0$, $q = u, d$. This original Fritzsch ansatz is named today in the literature as the "parallel nearest neighbor interaction form." With six texture zeros, there are left just six real parameters plus one phase to describe the six quark masses, the three mixing angles, and the *CP*-violation phase; hence, the three mixing angles of the CKM mixing matrix can be expressed as functions of the quark masses and of the *CP*-violation phase.

As it has been shown in several places, the parallel and nonparallel six-texture-zero structure does not work properly and it has been ruled out by analytic [18] and numeric [19] studies (only a charm quark mass half its measured value can rescue this ansatz).

Later, Ramond-Robert-Ross (RRR) gave up the parallelism between the structures of M_u and M_d and found that there exist five phenomenologically allowed patterns of Hermitian quark mass matrices, each one of them with five texture zeros [5], as listed in Table I.

TABLE I. Five RRR patterns of Hermitian quark mass matrices.

Pattern	$M_u^{\#}$	$M_d^{\#}$
I	$\begin{pmatrix} 0 & a_u & 0 \\ a_u^* & c_u & b_u \\ 0 & b_u^* & d_u \end{pmatrix}$	$\begin{pmatrix} 0 & a_d & 0 \\ a_d^* & c_d & 0 \\ 0 & 0 & d_d \end{pmatrix}$
II	$\begin{pmatrix} 0 & a_u & 0 \\ a_u^* & c_u & 0 \\ 0 & 0 & d_u \end{pmatrix}$	$\begin{pmatrix} 0 & a_d & 0 \\ a_d^* & c_d & b_d \\ 0 & b_d^* & d_d \end{pmatrix}$
III	$\begin{pmatrix} 0 & a_u & 0 \\ a_u^* & 0 & b_u \\ 0 & b_u^* & d_u \end{pmatrix}$	$\begin{pmatrix} 0 & a_d & 0 \\ a_d^* & c_d & b_d \\ 0 & b_d^* & d_d \end{pmatrix}$
IV	$\begin{pmatrix} 0 & 0 & a_u \\ 0 & c_u & b_u \\ a_u^* & b_u^* & d_u \end{pmatrix}$	$\begin{pmatrix} 0 & a_d & 0 \\ a_d^* & c_d & 0 \\ 0 & 0 & d_d \end{pmatrix}$
V	$\begin{pmatrix} 0 & 0 & a_u \\ 0 & c_u & 0 \\ a_u^* & 0 & d_u \end{pmatrix}$	$\begin{pmatrix} 0 & a_d & 0 \\ a_d^* & c_d & b_d \\ 0 & b_d^* & d_d \end{pmatrix}$

The following remarks are in order:

(i) Patterns I, II, IV, and V have three different phases, so two are unphysical. Pattern III has four different phases, so three are unphysical.

(ii) Each pattern has seven real parameters, which means two physical predictions. That is, for each pattern two of the three mixing angles can be written as a function of the six quark masses and of the *CP*-violation physical phase.

(iii) The five different RRR patterns commit to at least one (some with both) of the weak basis texture arrangements in Eq. (1.3).

In what follows we are going to study in detail these independent RRR patterns, paying special attention in each case to the prediction for the Cabbibo angle and to the *CP*-violation phenomena.

IV. ANALYTIC AND NUMERIC ANALYSIS

A. Pattern I

This pattern was studied previously in Ref. [18]. The complex Hermitian quark mass matrices for this pattern can be written as

$$M_u^{wb} = \begin{pmatrix} 0 & |a_u| e^{i\alpha_u^w} & 0 \\ |a_u| e^{-i\alpha_u^w} & c_u & |b_u| e^{i\beta_u^w} \\ 0 & |b_u| e^{-i\beta_u^w} & d_u \end{pmatrix}, \tag{4.1}$$

$$M_d^{bf} = \begin{pmatrix} 0 & |a_d| e^{i\alpha_d^b} & 0 \\ |a_d| e^{-i\alpha_d^b} & c_d & 0 \\ 0 & 0 & d_d \end{pmatrix}. \tag{4.2}$$

As can be seen, for this pattern the mixing angles θ_{13} and θ_{23} between the third family and the first two families, come only from the up quark sector. The complex phases are removed by going to a prime basis using the following unitary transformations:

$$\begin{pmatrix} u_0' \\ c_0' \\ t_0' \end{pmatrix} = U_u^{wb} \begin{pmatrix} u_0 \\ c_0 \\ t_0 \end{pmatrix}, \qquad \begin{pmatrix} d_0' \\ s_0' \\ b_0' \end{pmatrix} = U_d^{bf} \begin{pmatrix} d_0 \\ s_0 \\ b_0 \end{pmatrix}, \tag{4.3}$$

where U_u^{wb} and U_d^{bf} are complex diagonal matrices as presented in Appendix B.

So, in the primed basis, the algebra reduces to diagonalize the two real symmetric mass matrices (B17) for $q = u$ and (B7) for $q = d$, which are

$$M_u^{\rm I} = \begin{pmatrix} 0 & |a_u| & 0 \\ |a_u| & c_u & |b_u| \\ 0 & |b_u| & d_u \end{pmatrix}, \qquad M_d^{\rm I} = \begin{pmatrix} 0 & |a_d| & 0 \\ |a_d| & c_d & 0 \\ 0 & 0 & d_d \end{pmatrix},$$

with diagonalization carried through in Appendix B.

From the former analysis we can evaluate the mixing matrix $V^{\rm I}_{\rm CKM} = O^{wb}_u U^{wb}_u U^{bf\dagger}_d [O^{bf}_d]^T$, where U^{wb}_u and U^{bf}_d are as defined in (4.3) and calculated in Appendix B and O^{wb}_u and O^{bf}_d are given by (B19) and (B9), respectively.

The elements of the CKM mixing matrix can be expressed in the context of this pattern as

$$(V^{\rm I}_{\rm CKM})_{lm} = (O^{wb}_u)_{l1}(O^{bf}_d)_{m1} + e^{i\phi_1}(O^{wb}_u)_{l2}(O^{bf}_d)_{m2} + e^{i\phi_2}(O^{wb}_u)_{l3}(O^{bf}_d)_{m3}, \tag{4.4}$$

where $\phi_1 = (\alpha^w_u - \alpha^b_d)$ and $\phi_2 = (\alpha^w_u + \beta^w_u)$.

The use of the matrices in Appendix B allows us to write the following analytic mixing matrix:

$$V^{\rm I}_{\rm CKM} = \begin{pmatrix} \frac{\sqrt{d_u - m_u}(\sqrt{m_c m_s m_t} + e^{i\phi_1}\sqrt{d_u m_d m_u})}{\sqrt{d_u(m_d+m_s)(m_t-m_u)(m_c+m_u)}} & \frac{\sqrt{d_u - m_u}(-\sqrt{m_c m_d m_t} + e^{i\phi_1}\sqrt{d_u m_s m_u})}{\sqrt{d_u(m_d+m_s)(m_t-m_u)(m_c+m_u)},} & -e^{i\phi_2}\frac{\sqrt{(d_u+m_c)(m_t-d_u)m_u}}{\sqrt{d_u(m_t-m_u)(m_c+m_u)}} \\ \frac{-e^{i\phi_1}\sqrt{d_u m_c m_d(d_u+m_c)} + \sqrt{(d_u+m_c)m_s m_t m_u}}{\sqrt{d_u(m_d+m_s)(m_c+m_t)(m_c+m_u)}} & -\frac{e^{i\phi_1}\sqrt{d_u m_c m_s} + \sqrt{m_d m_t m_u}}{\sqrt{\frac{d_u(m_d+m_s)(m_c+m_t)(m_c+m_u)}{d_u+m_c}}} & e^{i\phi_2}\frac{\sqrt{m_c(m_t-d_u)(d_u-m_u)}}{\sqrt{d_u(m_c+m_t)(m_c+m_u)}} \\ \frac{\sqrt{-d_u+m_t}(e^{i\phi_1}\sqrt{d_u m_d m_t} + \sqrt{m_c m_s m_u})}{\sqrt{d_u(m_d+m_s)(m_c+m_t)(m_t-m_u)}} & \frac{\sqrt{-d_u+m_t}(e^{i\phi_1}\sqrt{d_u m_s m_t} - \sqrt{m_c m_d m_u})}{\sqrt{d_u(m_d+m_s)(m_c+m_t)(m_t-m_u)}} & e^{i\phi_2}\frac{\sqrt{(d_u+m_c)m_t(d_u-m_u)}}{\sqrt{d_u(m_c+m_t)(m_t-m_u)}} \end{pmatrix},$$

where as anticipated, the last column is a function of the parameters in the up quark mass matrix only, and a common phase $e^{i\phi_2}$ in that column can be removed by a redefinition of the b quark field.

Our approach now is the following: first we use the free parameter d_u in order to fit the experimental value V_{cb} and then use the phase ϕ_1 to fit $|V_{us}|$; when the central quark mass values are used, we obtain for this pattern the result $d_u \approx m_t$ and $\phi_1 \approx 1.6$. The next step is to fine-tune both d_u and ϕ_1, using a random numerical analysis and quark masses m_u and m_d at 2σ values with the other four quark masses at 1σ [20] and aiming to get results as close as possible to the ones presented in $V^{(\rm exp)}$ in (2.7). Finally, for the best values obtained we calculate the CP asymmetries α, β, and γ as in (2.4).

Notice that a crude approximation for which $d_u \equiv m_t$, using further $m_t >> m_c >> m_u$, implies from the mixing matrix $V^{\rm I}_{\rm CKM}$ that

$$V_{us} \approx \sqrt{\frac{m_s}{m_s + m_d}}\left|\sqrt{\frac{m_d}{m_s}} - e^{i\phi}\sqrt{\frac{m_u}{m_c}}\right|, \tag{4.5}$$

a form advocated in several references [21]. But to set $d_u = m_t$ implies $b_u = 0$, according to Eq. (B18), and not only that but we also get back to a parallel six texture zero pattern with $\theta_{13} = \theta_{23} = 0$ (something that can be seen also from $V^{\rm I}_{\rm CKM}$, the 3×3 mixing matrix above). So, our fine-tunning approach is mandatory, d_u must be different from m_t, and any analysis done for which expression (4.5) plays a central role must produce dubious predictions. In this regard, the conclusions in Ref. [21], which are contrary to ours, may be wrong.

For pattern I, the numerical subroutines used throw the following numbers (mass parameters are in GeVs and angles in radians):

$$m_u = 0.0023, \quad m_c = 0.560, \quad m_t = 172,$$
$$m_d = 0.0029, \quad m_s = 0.06, \quad m_b = 2.89,$$
$$d_u = 171.721 < m_t, \quad \phi_1 = 1.61,$$

which imply the following 3×3 mixing matrix:

$$\begin{pmatrix} 0.97428 & 0.22532 & 0.00264 \\ 0.22517 & 0.97349 & 0.04020 \\ 0.00865 & 0.03934 & 0.99919 \end{pmatrix},$$

with numbers in quite good agreement with the experimental measured values. Finally, the three angles of the unitary triangle of the B decays CP asymmetries calculated according to (2.4) for this pattern are

$$(\alpha, \beta, \gamma)^{\rm I}_{\rm th} = (90.79, 16.51, 72.68),$$

which not only close the triangle but are such that α and γ agree with the measured value at 1σ and β at 2σ.

B. Pattern II

Notice for this pattern that the mixing angles θ_{13} and θ_{23} between the third family and the first two families come only from the down quark sector. Also, this pattern is obtained from the previous one by the following replacements: $M^{\rm I}_d \to M^{\rm II}_u$ and $M^{\rm I}_u \to M^{\rm II}_d$ and $d_u \to d_d$, where d_d is the free parameter of this pattern.

As in the previous pattern, we start by removing the complex phases from $M^{\rm II}_u$ and $M^{\rm II}_d$ and then use the results in Appendix B in order to diagonalize the orthogonal mass matrices. When done, the CKM mixing matrix for this pattern reads $V^{\rm II}_{\rm CKM} = O^{bf}_u U^{bf}_u U^{wb\dagger}_d [O^{wb}_d]^T$, where the orthogonal and unitary matrices are presented in Appendix B. The elements of the CKM mixing matrix can be expressed now as

$$(V^{\rm II}_{\rm CKM})_{lm} = (O^{bf}_u)_{l1}(O^{wb}_d)_{m1} + e^{i\phi_1}(O^{bf}_u)_{l2}(O^{wb}_d)_{m2} + e^{i\phi_2}(O^{bf}_u)_{l3}(O^{wb}_d)_{m3}, \tag{4.6}$$

where $\phi_1 = (\alpha^b_u - \alpha^w_d)$ and $\phi_2 = -(\alpha^w_d + \beta^w_d)$, which allows us to write the following analytic forms:

$$
\begin{aligned}
V_{ud}^{\mathrm{II}} &= \frac{\sqrt{m_c m_b (d_d - m_d) m_s}}{\sqrt{(m_c + m_u) d_d (m_b - m_d)(m_d + m_s)}} + \frac{e^{i\phi_1}\sqrt{m_u (d_d - m_d) m_d}}{\sqrt{(m_c + m_u)(m_b - m_d)(m_d + m_s)}}, \\
V_{us}^{\mathrm{II}} &= \frac{\sqrt{m_c m_b m_d (d_d + m_s)}}{\sqrt{(m_c + m_u) d_d (m_b + m_s)(m_d + m_s)}} - \frac{e^{i\phi_1}\sqrt{m_s m_u (d_d + m_s)}}{\sqrt{(m_c + m_u)(m_b + m_s)(m_d + m_s)}}, \\
V_{ub}^{\mathrm{II}} &= \frac{\sqrt{(-d_d + m_b) m_d m_s m_c}}{\sqrt{(m_c + m_u) d_d (m_b - m_d)(m_b + m_s)}} + \frac{e^{i\phi_1}\sqrt{m_b (-d_d + m_b) m_u}}{\sqrt{(m_c + m_u)(m_b - m_d)(m_b + m_s)}}, \\
V_{cd}^{\mathrm{II}} &= \frac{e^{i\phi_1}\sqrt{(d_d - m_d) m_d m_c}}{\sqrt{(m_c + m_u)(m_b - m_d)(m_d + m_s)}} - \frac{\sqrt{m_u m_b (d_d - m_d) m_s}}{\sqrt{(m_c + m_u) d_d (m_b - m_d)(m_d + m_s)}}, \\
V_{cs}^{\mathrm{II}} &= -\frac{e^{i\phi_1}\sqrt{m_s m_c (d_d + m_s)}}{\sqrt{(m_c + m_u)(m_b + m_s)(m_d + m_s)}} - \frac{\sqrt{m_b m_d (d_d + m_s) m_u}}{\sqrt{(m_c + m_u) d_d (m_b + m_s)(m_d + m_s)}}, \\
V_{cb}^{\mathrm{II}} &= \frac{e^{i\phi_1}\sqrt{m_c m_b (-d_d + m_b)}}{\sqrt{(m_c + m_u)(m_b - m_d)(m_b + m_s)}} - \frac{\sqrt{(-d_d + m_b) m_u m_d m_s}}{\sqrt{(m_c + m_u) d_d (m_b - m_d)(m_b + m_s)}}, \\
V_{td}^{\mathrm{II}} &= -e^{i\phi_2}\sqrt{\frac{(m_b - d_d) m_d (d_d + m_s)}{d_d (m_b - m_d)(m_d + m_s)}}, \qquad V_{ts}^{\mathrm{II}} = e^{i\phi_2}\sqrt{\frac{(m_b - d_d)(d_d - m_d) m_s}{d_d (m_b + m_s)(m_d + m_s)}}, \\
V_{tb}^{\mathrm{II}} &= e^{i\phi_2}\sqrt{\frac{m_b (d_d - m_d)(d_d + m_s)}{d_d (m_b - m_d)(m_b + m_s)}},
\end{aligned}
\tag{4.7}
$$

where again the last row is a function of the parameters in the down quark mass matrix only, and the phase $e^{i\phi_2}$ can be removed by a redefinition of the field for the quark t. The form of the former mixing entries recommends to use d_d to fit the experimental measured value V_{ts} and then ϕ_1 to fit V_{us}, which produces $d_d \approx m_b$ and $\phi_1 \approx 1.45$.

Notice that for $d_d = m_d$ and $m_c \gg m_u$, V_{us}^{II} acquires the form presented in Eq. (4.5). But again, $d_d = m_b$ drives pattern II to a six-texture-zero ansatz with $\theta_{13} = \theta_{23} = 0$, implying once more a mandatory fine-tunning for the free parameter d_d.

The random numerical analysis for pattern II throws the following numbers (mass parameters in GeVs and angles in radians as before):

$$
\begin{aligned}
&m_u = 0.0023, \qquad m_c = 0.560, \qquad m_t = 172, \\
&m_d = 0.0029, \qquad m_s = 0.060, \qquad m_b = 2.89, \\
&d_d = 2.8855 < m_b, \qquad \phi_1 = 1.6,
\end{aligned}
$$

which imply the following 3×3 mixing matrix:

$$
\begin{pmatrix}
0.97437 & 0.22493 & 0.00250 \\
0.22478 & 0.97363 & 0.03900 \\
0.00856 & 0.03813 & 0.99924
\end{pmatrix},
$$

with numbers again in good agreement with the experimental measured values. Finally, the three angles of the unitary triangle of the B decays *CP* asymmetries are calculated for this pattern to be

$$
(\alpha, \beta, \gamma)_{\mathrm{th}}^{(\mathrm{II})} = (86.78, 16.11, 77.11),
$$

which again close the triangle and are such that α and γ agree with the measured value at 1σ with a value for β at 2σ.

C. Pattern III

Proceeding as in the two previous patterns and using the notation introduced in Appendix B, we have now $V_{\mathrm{CKM}}^{\mathrm{III}} = O_u^{nn} U_u^{nn} U_d^{wb\dagger} [O_d^{wb}]^T$, where the elements of the CKM mixing matrix can be expressed as

$$
\begin{aligned}
(V_{\mathrm{CKM}}^{\mathrm{III}})_{lm} = {} & (O_u^{nn})_{l1} (O_d^{wb})_{m1} + e^{i\phi_1} (O_u^{nn})_{l2} (O_d^{wb})_{m2} \\
& + e^{i\phi_2} (O_u^{nn})_{l3} (O_d^{wb})_{m3},
\end{aligned}
\tag{4.8}
$$

where $\phi_1 = (\alpha_u^n - \alpha_d^w)$ and $\phi_2 = (\alpha_u^n - \alpha_d^w + \beta_u^n - \beta_d^w)$, which allows us to write the following analytic forms:

$$V_{ud}^{\mathrm{III}} = \sqrt{\frac{(m_t - m_c)}{(m_t - m_u)(m_c + m_u)(m_b - m_d)(m_d - m_s)}}\left[\sqrt{\frac{m_t m_c m_b m_s (d_d - m_d)}{d_d(m_t - m_c + m_u)}} + e^{i\phi_1}\sqrt{m_u m_d (d_d - m_d)} + e^{i\phi_2}\sqrt{\frac{m_u m_d (m_t + m_u)(m_c - m_u)(m_b - d_d)(d_d + m_s)}{d_d(m_t - m_c + m_u)(m_t - m_c)}}\right],$$

$$V_{us}^{\mathrm{III}} = \sqrt{\frac{(m_t - m_c)}{(m_t - m_u)(m_c + m_u)(m_b + m_s)(m_d + m_s)}}\left[\sqrt{\frac{m_t m_c m_b m_d (d_d + m_s)}{d_d(m_t - m_c + m_u)}} - e^{i\phi_1}\sqrt{m_u m_s (d_d + m_s)} - e^{i\phi_2}\sqrt{\frac{m_u m_s (m_t + m_u)(m_c - m_u)(m_b - d_d)(d_d + m_d)}{d_d(m_t - m_c + m_u)(m_t - m_c)}}\right],$$

$$V_{ub}^{\mathrm{III}} = \sqrt{\frac{(m_t - m_c)}{(m_t - m_u)(m_c + m_u)(m_b + m_s)(m_b + m_d)}}\left[\sqrt{\frac{m_t m_c m_s m_d (m_b - d_d)}{d_d(m_t - m_c + m_u)}} + e^{i\phi_1}\sqrt{m_u m_b (m_b - d_d)} - e^{i\phi_2}\sqrt{\frac{m_u m_b (m_t + m_u)(m_c - m_u)(d_d + m_s)(d_d - m_d)}{d_d(m_t - m_c + m_u)(m_t - m_c)}}\right],$$

$$V_{cd}^{\mathrm{III}} = \sqrt{\frac{(m_t + m_u)}{(m_t + m_c)(m_c + m_u)(m_b - m_d)(m_d + m_s)}}\left[\sqrt{\frac{m_t m_u m_b m_s (d_d - m_d)}{d_d(m_t - m_c + m_u)}} - e^{i\phi_1}\sqrt{m_c m_d (d_d - m_d)} - e^{i\phi_2}\sqrt{\frac{m_c m_d (m_t - m_c)(m_c - m_u)(m_b - d_d)(d_d + m_s)}{d_d(m_t - m_c + m_u)(m_t + m_u)}}\right],$$

$$V_{cs}^{\mathrm{III}} = \sqrt{\frac{(m_t + m_u)}{(m_t + m_c)(m_c + m_u)(m_b + m_s)(m_d + m_s)}}\left[\sqrt{\frac{m_t m_u m_b m_d (d_d + m_s)}{d_d(m_t - m_c + m_u)}} + e^{i\phi_1}\sqrt{m_c m_s (d_d + m_s)} + e^{i\phi_2}\sqrt{\frac{m_c m_s (m_t - m_c)(m_c - m_u)(m_b - d_d)(d_d - m_d)}{d_d(m_t - m_c + m_u)(m_t + m_u)}}\right],$$

$$V_{cb}^{\mathrm{III}} = \sqrt{\frac{(m_t + m_u)}{(m_t + m_c)(m_c + m_u)(m_b + m_s)(m_b - m_d)}}\left[\sqrt{\frac{m_t m_u m_s m_d (m_b - d_d)}{d_d(m_t - m_c + m_u)}} - e^{i\phi_1}\sqrt{m_c m_b (m_b - d_d)} + e^{i\phi_2}\sqrt{\frac{m_c m_b (m_t - m_c)(m_c - m_u)(d_d - m_d)(d_d + m_s)}{d_d(m_t - m_c + m_u)(m_t + m_u)}}\right],$$

$$V_{td}^{\mathrm{III}} = \sqrt{\frac{(m_c - m_u)}{(m_t + m_c)(m_t - m_u)(m_s + m_d)(m_b - m_d)}}\left[\sqrt{\frac{m_c m_u m_b m_s (d_d - m_d)}{d_d(m_t - m_c + m_u)}} + e^{i\phi_1}\sqrt{m_t m_d (d_d - m_d)} - e^{i\phi_2}\sqrt{\frac{m_t m_d (m_t - m_c)(m_t + m_u)(m_b - d_d)(d_d + m_s)}{d_d(m_t - m_c + m_u)(m_c - m_u)}}\right],$$

$$V_{ts}^{\mathrm{III}} = \sqrt{\frac{(m_c - m_u)}{(m_t + m_c)(m_t - m_u)(m_b + m_s)(m_s + m_d)}}\left[\sqrt{\frac{m_c m_u m_b m_d (d_d + m_s)}{d_d(m_t - m_c + m_u)}} - e^{i\phi_1}\sqrt{m_t m_s (d_d + m_s)} + e^{i\phi_2}\sqrt{\frac{m_t m_s (m_t - m_c)(m_t + m_u)(m_b - d_d)(d_d - m_d)}{d_d(m_t - m_c + m_u)(m_c - m_u)}}\right],$$

$$V_{tb}^{\mathrm{III}} = \sqrt{\frac{(m_c - m_u)}{(m_t + m_c)(m_t - m_u)(m_b + m_s)(m_b - m_d)}}\left[\sqrt{\frac{m_c m_u m_d m_s (m_b - d_d)}{d_d(m_t - m_c + m_u)}} + e^{i\phi_1}\sqrt{m_t m_b (m_b - d_d)}\right.$$
$$\left. + e^{i\phi_2}\sqrt{\frac{m_t m_b (m_t - m_c)(m_t + m_u)(d_d + m_s)(d_d - m_d)}{d_d(m_t - m_c + m_u)(m_c - m_u)}}\right]. \tag{4.9}$$

Notice that for $d_d = m_d$ and $m_t \gg m_c \gg m_u$, V_{us}^{III} acquires the form presented in Eq. (4.5). But again, $d_d = m_b$ drives pattern III to a six-texture-zero ansatz.

Written in the previous form, all the entries for $V_{\mathrm{CKM}}^{\mathrm{III}}$ include two different phases, ϕ_1 and ϕ_2, none of them absorbed immediately by a redefinition of a single quark field; but since it is a well-known fact that the SM mixing matrix can be parametrized with only one single phase, our analysis makes sense only for the following three different cases, which must be studied separately:

(i) Case 1: $\phi_1 \neq 0$, $\phi_2 = 0$.
(ii) Case 2: $\phi_1 = 0$, $\phi_2 \neq 0$.
(iii) Case 3: $\phi_1 = \phi_2 \neq 0$.

The numerical analysis for pattern III and for $\phi_1 = \phi_2$ throws the following numbers (mass parameters in GeVs and angles in radians as before):

$$m_u = 0.0023, \quad m_c = 0.560, \quad m_t = 172,$$
$$m_d = 0.0029, \quad m_s = 0.06, \quad m_b = 2.89,$$
$$d_d = 2.889, \quad \phi_1 = \phi_2 = 1.6,$$

which imply the following 3×3 mixing matrix:

$$\begin{pmatrix} 0.97437 & 0.22493 & 0.00247 \\ 0.22408 & 0.97365 & 0.03837 \\ 0.00817 & 0.03757 & 0.99926 \end{pmatrix},$$

with numbers again in good agreement with the experimental measured values. Finally, the three angles of the unitary triangle of the B decays *CP* asymmetries calculated for this pattern are

$$(\alpha, \beta, \gamma)_{\mathrm{th}}^{(\mathrm{III})} = (92.73, 16.22, 71.06),$$

which again close the triangle and are such that α and γ agree with the measured value at 1σ and β at 2σ.

D. Pattern IV

Proceeding in a similar way, we have for this pattern that the CKM mixing matrix reads now $V_{\mathrm{CKM}}^{\mathrm{IV}} = O_u^{ss} U_u^{ss} U_d^{bf\dagger} [O_d^{bf}]^T$, and the elements of the CKM mixing matrix can be expressed as

$$(V_{\mathrm{CKM}}^{\mathrm{IV}})_{lm} = (O_u^{ss})_{l1}(O_d^{bf})_{m1} + e^{i\phi_1}(O_u^{ss})_{l2}(O_d^{bf})_{m2} + e^{i\phi_2}(O_u^{ss})_{l3}(O_d^{bf})_{m3}, \tag{4.10}$$

where $\phi_1 = (\alpha_u^d - \alpha_d^w)$ and $\phi_2 = (\alpha_u^d + \beta_u^w - \alpha_d^w)$, which allows us to write the following analytic forms:

$$V_{ud}^{\mathrm{IV}} = \frac{\sqrt{m_t m_c m_s}(m_t - d_u + m_c) + e^{i\phi_1}\sqrt{m_d m_u (d_u - m_c + m_u)(m_t + m_c - d_u)(m_t - m_u - d_u)}}{\sqrt{(m_d + m_s)(-d_u + m_c + m_t)(m_c + m_u)(-d_u + m_c + m_t - m_u)(m_t + m_u)}},$$
$$V_{us}^{\mathrm{IV}} = \frac{-\sqrt{m_t m_c m_d}(m_t - d_u + m_c) + e^{i\phi_1}\sqrt{m_s m_u (d_u - m_c + m_u)(m_t + m_c - d_u)(m_t - m_u - d_u)}}{\sqrt{(m_d + m_s)(-d_u + m_c + m_t)(m_c + m_u)(-d_u + m_c + m_t - m_u)(m_t + m_u)}},$$
$$V_{ub}^{\mathrm{IV}} = e^{i\phi_2}\frac{\sqrt{m_u(m_t - d_u + m_c)}}{\sqrt{(m_c + m_u)(m_t + m_u)}},$$
$$V_{cd}^{\mathrm{IV}} = \frac{-e^{i\phi_1} m_c \sqrt{m_d (m_t - d_u + m_c)(d_u - m_c + m_u)} + \sqrt{m_s m_c m_u m_t (m_t - d_u - m_u)}}{\sqrt{m_c (m_d + m_s)(m_c + m_u)(m_t - m_c)(m_t + m_c - m_u - d_u)}},$$
$$V_{cs}^{\mathrm{IV}} = -\frac{e^{i\phi_1} m_c \sqrt{m_s (m_t - d_u + m_c)(d_u - m_c + m_u)} + \sqrt{m_d m_c m_u m_t (m_t - d_u - m_u)}}{\sqrt{m_c (m_d + m_s)(m_c + m_u)(m_t - m_c)(m_t + m_c - m_u - d_u)}},$$

$$
\begin{aligned}
V_{cb}^{\mathrm{IV}} &= e^{i\phi_2}\frac{\sqrt{m_c(m_t-d_u+m_u)}}{\sqrt{(m_t-m_c)(m_c+m_u)}},\\
V_{td}^{\mathrm{IV}} &= \frac{e^{i\phi_1}\sqrt{m_d m_t(m_t-d_u+m_c)(d_u-m_c+m_u)(m_t-d_u-m_u)}+\sqrt{m_s m_c m_u}(d_u-m_c+m_u)}{\sqrt{(m_d+m_s)(d_u-m_c+m_u)(m_t+m_c-m_u-d_u)(m_t-m_c)(m_t+m_u)}},\\
V_{ts}^{\mathrm{IV}} &= \frac{e^{i\phi_1}\sqrt{m_s m_t(m_t-d_u+m_c)(d_u-m_c+m_u)(m_t-d_u-m_u)}+\sqrt{m_d m_c m_u}(-d_u+m_c-m_u)}{\sqrt{(m_d+m_s)(d_u-m_c+m_u)(m_t+m_c-m_u-d_u)(m_t-m_c)(m_t+m_u)}},\\
V_{tb}^{\mathrm{IV}} &= e^{i\phi_2}\frac{\sqrt{m_t(d_u-m_c+m_u)}}{\sqrt{(m_t-m_c)(m_t+m_u)}},
\end{aligned}
\tag{4.11}
$$

where again the common phase $e^{i\phi_2}$ in the last column can be removed by a redefinition of the b quark field.

The numerical analysis for pattern IV is (mass parameters in GeVs and angles in radians as before)

$$
\begin{aligned}
&m_u = 0.00127, \qquad m_c = 0.619, \qquad m_t = 171.7,\\
&m_d = 0.0028, \qquad m_s = 0.055, \qquad m_b = 2.89,\\
&d_u = 171.39, \qquad \phi_1 = 1.9,
\end{aligned}
$$

which imply the following 3×3 mixing matrix:

$$
\begin{pmatrix}
0.97331 & 0.22549 & 0.00333\\
0.22934 & 0.97242 & 0.04144\\
0.00889 & 0.04163 & 0.99909
\end{pmatrix},
$$

with numbers again in good agreement with the experimental measured values.

Finally, the three angles of the unitary triangle of the B decays CP asymmetries calculated for this pattern are

$$
(\alpha,\beta,\gamma)_{\mathrm{th}}^{(\mathrm{IV})} = (95.23, 19.36, 65.42),
$$

which not only close the triangle, but the three agree with the measured value at 1σ.

E. Pattern V

Proceeding in a similar way, we have for this pattern that the CKM mixing matrix reads now $V_{\mathrm{CKM}}^{\mathrm{V}} = O_u^{dd}U_u^{dd}U_d^{wb\dagger}[O_d^{wb}]^T$, and the elements of the CKM mixing matrix can be expressed as

$$
\begin{aligned}
(V_{\mathrm{CKM}}^{\mathrm{V}})_{lm} = {} & (O_u^{dd})_{l1}(O_d^{wb})_{m1} + e^{i\phi_1}(O_u^{dd})_{l2}(O_d^{wb})_{m2}\\
& + e^{i\phi_2}(O_u^{dd})_{l3}(O_d^{wb})_{m3},
\end{aligned}
\tag{4.12}
$$

where $\phi_1 = -\alpha_d^w$ and $\phi_2 = (\alpha_u^d - \alpha_d^w - \beta_d^w)$, which allows us to write the following analytic forms:

$$
\begin{aligned}
V_{ud}^{\mathrm{V}} &= \frac{\sqrt{m_b m_s m_t(d_d-m_d)}-e^{i\phi_2}\sqrt{(m_b-d_d)(d_d+m_s)m_d m_u}}{\sqrt{d_d(m_b-m_d)(m_d+m_s)(m_t+m_u)}},\\
V_{us}^{\mathrm{V}} &= \frac{\sqrt{m_b m_d m_t(d_d+m_s)}+e^{i\phi_2}\sqrt{(m_b-d_d)(d_d-m_d)m_s m_u}}{\sqrt{(m_t+m_u)d_d(m_b+m_s)(m_s+m_d)}},\\
V_{ub}^{\mathrm{V}} &= \frac{\sqrt{m_d m_s m_t(m_b-d_d)}+e^{i\phi_2}\sqrt{m_b m_u(m_s+d_d)(d_d-m_d)}}{\sqrt{(m_t+m_u)d_d(m_b-m_d)(m_b+m_s)}},\\
V_{cd}^{\mathrm{V}} &= e^{i\phi_1}\frac{\sqrt{m_d(d_d-m_d)}}{\sqrt{(m_b-m_d)(m_d+m_s)}}, \qquad V_{cs}^{\mathrm{V}} = -e^{i\phi_1}\frac{\sqrt{m_s(d_d+m_s)}}{\sqrt{(m_b+m_s)(m_d+m_s)}},\\
V_{cb}^{\mathrm{V}} &= e^{i\phi_1}\frac{\sqrt{m_b(m_b-d_d)}}{\sqrt{(m_b-m_d)(m_b+m_s)}}, \qquad V_{td}^{\mathrm{V}} = \frac{-\sqrt{m_u m_b m_s(d_d-m_d)}-e^{i\phi_2}\sqrt{m_t m_d(m_b-d_d)(d_d+m_s)}}{\sqrt{(m_t+m_u)d_d(m_b-m_d)(m_s+m_d)}},\\
V_{ts}^{\mathrm{V}} &= \frac{-\sqrt{m_u m_b m_d(d_d+m_s)}+e^{i\phi_2}\sqrt{m_t m_s(m_b-d_d)(d_d-m_d)}}{\sqrt{(m_t+m_u)d_d(m_b+m_s)(m_s+m_d)}},\\
V_{tb}^{\mathrm{V}} &= \frac{-\sqrt{m_u m_d m_s(m_b-d_d)}+e^{i\phi_2}\sqrt{m_t m_b(d_d+m_s)(d_d-m_d)}}{\sqrt{(m_t+m_u)d_d(m_b+m_s)(m_b-m_d)}},
\end{aligned}
\tag{4.13}
$$

where the phase ϕ_1 can be removed by a redefinition of the charm quark field.

The numerical analysis throws now the following numbers (mass parameters in GeVs and angles in radians as before):

$$m_u = 0.00127, \qquad m_c = 0.619, \qquad m_t = 171.7,$$
$$m_d = 0.0029, \qquad m_s = 0.055, \qquad m_b = 2.89,$$
$$d_d = 2.885, \qquad \phi_2 = 1.9,$$

which imply the following 3×3 mixing matrix:

$$\begin{pmatrix} 0.97463 & 0.22377 & 0.00266 \\ 0.22361 & 0.97381 & 0.04122 \\ 0.00891 & 0.04034 & 0.99915 \end{pmatrix},$$

with numbers again in quite good agreement with the experimental measured values. Finally, the three angles of the unitary triangle of the B decays *CP* asymmetries calculated for this pattern are

$$(\alpha, \beta, \gamma)^{(\mathrm{V})}_{\mathrm{th}} = (88.83, 16.36, 74.81),$$

which close the triangle and are such that α and γ agree with the measured value at 1σ, with a value for β at 2σ.

F. Related patterns

The analysis shows that a weak basis transformation using the unitary matrix

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} = U^{\dagger}$$

shows that $UM^{ss}U = M^{wb}$ and also that $UM^{bf}U = M^{dd}$. So, a pattern like

$$M_u^{\mathrm{VI}} = \begin{pmatrix} 0 & 0 & a_u \\ 0 & b_u & 0 \\ a_u^* & 0 & c_u \end{pmatrix}; \qquad M_d^{\mathrm{VI}} = \begin{pmatrix} 0 & 0 & a_d \\ 0 & d_d & b_d \\ a_d^* & b_d^* & c_d \end{pmatrix}$$

should be physically equivalent to pattern II.

A pattern like

$$M_u^{\mathrm{VII}} = \begin{pmatrix} 0 & a_u & 0 \\ a_u^* & c_u & b_u \\ 0 & b_u^* & d_u \end{pmatrix}; \qquad M_d^{\mathrm{VII}} = \begin{pmatrix} 0 & 0 & a_d \\ 0 & c_d & 0 \\ a_d^* & 0 & d_d \end{pmatrix}$$

should be physically equivalent to pattern IV.

And a pattern like

$$M_u^{\mathrm{VIII}} = \begin{pmatrix} 0 & a_u & 0 \\ a_u^* & b_u & 0 \\ 0 & 0 & c_u \end{pmatrix}; \qquad M_d^{\mathrm{VIII}} = \begin{pmatrix} 0 & 0 & a_d \\ 0 & c_d & b_d \\ a_d^* & b_d^* & d_d \end{pmatrix}$$

should be physically equivalent to pattern V.

V. DISCUSSION OF THE RESULTS

All the five-texture-zero patterns studied here produce numerical results for the CKM mixing matrix and *CP* violating parameters (α, β, γ) in agreement with the measured values up to 2σ. Pattern IV reproduces them up to 1σ. Since the results are presented for the best-fit values, all the patterns are predictive, as far as the up-to-date measured numbers are concerned.

But, how sensitive is our numerical analysis to the variations of the quark mass values? Since the answer to this question is pattern dependent, let us do it for pattern IV, the most predictive one.

Since $m_t \gg m_c \gg m_u$ and the experimental uncertainty in the top quark mass Δm_t is such that $\Delta m_t > m_c > m_u$, then for the up quark sector it is enough to consider variations to the top quark mass in the interval $168.7 \leq m_t\ (\mathrm{GeV}) \leq 174.7$.

Our results show that the numerical predicted values remain stable for the entire interval, as far as the difference $(m_t - d_u)$ stays constant, in a range from 0.28 to 0.33 [notice that all the CKM entries for pattern IV but $V_{tb}^{\mathrm{IV}} \sim 1$ in Eq. (4.11) depend upon the difference $(m_t - d_u)$].

Now, for the quark masses in the down sector, the CKM mixing matrix for pattern IV does not depend upon the b quark mass (due to the block form of the down quark mass matrix in this particular pattern). For the other down quark masses we have that the numerical results are stable as far as the strange quark mass is in the range $0.051 \leq m_s\ (\mathrm{GeV}) \leq 0.060$ and for the down quark mass $0.025 \leq m_d\ (\mathrm{GeV}) \leq 0.030$.

Can we relate our different parametrization of the CKM mixing matrix with other different parametrizations, especially to the advocated standard parametrization [17] in matrix (2.6)? The answer to this question is an academic exercise without much physical sense, because the adoption of a parametrization of the flavor mixing is an arbitrary fact and not a physical issue. In particular, our phase ϕ_1 in pattern IV and the phase δ of the standard parametrization are not physical quantities. The real physical parameters are in this case the *CP*-violating mixing angles (α, β, γ).

Also, the rotation parameters θ_{12}, θ_{23}, and θ_{13} of the standard parametrization are not physical entities either. For example, in the standard parametrization, $\theta_{12} = \tan^{-1}(V_{us}/V_{ud})$, which is a real number. In any of our parametrizations, (V_{us}/V_{ud}) is a complex number, without direct relation to θ_{12}.

Since generally one expects that the flavor-mixing parameters do depend on the elements of the quark mass matrices, we feel that our CKM mixing entries are more realistic that any *ad hoc* parametrization.

Can we do the mathematical exercise to relate the phase ϕ_1 in pattern IV to the phase δ of the standard parametrization (or to any other phase of an arbitrary parametrization)? The answer is yes'' and the simplest

way to do it is to use the Jarlskog [22] invariant, which for the sake of convenience we use as

$$J = \pm \mathrm{Im}.(V_{us} V_{cb} V_{ub}^{*} V_{cs}^{*}), \tag{5.1}$$

which for the standard parametrization takes the form $J = c_{12} c_{13}^{2} c_{23} s_{12} s_{13} s_{23} \sin\delta$, with a numerical evaluation producing [12] $(2.80 \leq J \leq 3.16) \times 10^{-5}$.

For pattern IV, an analytic expression for J can be obtained, but it is prohibitively long to quote here. A numerical evaluation is more viable, and for the best-fit values for the quark masses, we get $J = 2.87 \times 10^{-5}$, which falls in the allowed range of the standard parametrization. Of course, this value is sensitive to the variation of the quark mass values. In particular, we see that a strange quark mass (keeping the other masses in the best-fit values) in the interval $0.048 \leq m_s\ (\mathrm{GeV}) \leq 0.056$ is compatible with the range of the Jarlskog invariant of the standard parametrization.

VI. CONCLUSIONS

In this paper we have presented our exhaustive study of all the possible five-texture-zero quark mass matrices and their connection with the CP-violation phenomena for three SM families. Analytic and numeric studies were performed in full detail.

Let us provide the following concluding remarks:

(1) In the context of the SM or its extensions without right-handed currents, the quark mass matrices M_u and M_d can be taken to be Hermitian without loss of generality. Non-Hermitian quark mass matrices are relevant only when physics beyond the SM is being considered.

(2) By counting free parameters we have that three texture zeros or less in the Hermitian quark mass matrices of the SM do not imply physical predictions for the elements of the flavor-mixing matrix, because these texture zeros can always be obtained in a trivial way by using weak basis transformations.

(3) Each one of the five RRR patterns studied here includes two physical relationships between the quark masses and the mixing angles, as can be seen from $V_{\mathrm{CKM}}^{\#}; \# = \mathrm{I}, \mathrm{II}, \ldots, \mathrm{V}$. These predictions are case dependent but far from being trivial.

(4) Four and six texture zeros imply one and three physical relationships, respectively, and allow us to write one of the three mixing angles as functions of the six quark masses and the CP-violation phase. But six texture zeros are already ruled out by analytic and numerical considerations [18,19].

(5) More than six texture zeros are not possible because they imply either Det $M_q = 0$ or a degenerate quark mass spectrum, both of which are incompatible with the real world.

(6) For the five patterns studied here, number IV (or equivalent number VII) reproduce all the experimental measured values, including the CP-violation asymmetries, at 1σ. For the other patterns, the β angle appears at 2σ. So, five texture zeros are far from being ruled out.

(7) In our analysis for pattern IV, the several constraints for the strange quark mass imply $0.051 \leq m_s\ (\mathrm{GeV}) \leq 0.056$.

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APPENDIX A: MASS VALUES

For the experimental mass values used for carrying out the numerical calculations in the main text, we have adopted the following ranges of quark masses [20] at the M_Z energy scale (where the V_{CKM} matrix elements are measured).

Up sector	Down sector
$m_t = 171.7 \pm 3.0$ GeV	$m_b = 2.89 \pm 0.009$ GeV
$m_c = 0.619 \pm 0.084$ GeV	$m_s = 55^{+16}_{-15}$ MeV
$m_u = 1.27^{+0.5}_{-0.42}$ MeV	$m_d = 2.90^{+1.24}_{-1.19}$ MeV

The light quark masses m_u, m_d and m_s can be further constrained using the mass ratios [23]

$$m_u/m_d = 0.553 \pm 0.043 \qquad m_s/m_d = 18.9 \pm 0.8. \tag{A1}$$

Notice also that due to the experimental errors,

$$m_t \pm m_c \pm m_u \approx m_t.$$

APPENDIX B: ORTHOGONAL MATRICES

In this appendix we derive the analytic orthogonal matrices that diagonalize the several 3×3 real quark mass matrices coming from the five different RRR patterns. As can be seen, the ten quark mass matrices in Table I can be grouped in only five different forms.

For the analysis we write the parameters of the several real mass matrices as functions of the quark mass eigenvalues m_1, m_2, and m_3, with the hierarchy $|m_1| < |m_2| < |m_3|$, by making use of the following invariant forms: $\mathrm{tr}[M_q]$, $\mathrm{tr}[(M_q)^2]$, and $\det[M_q]$.

1. Bidiagonal form

For this form the Hermitian quark mass matrix is

$$M_q^{dd} = \begin{pmatrix} 0 & 0 & |a_q| e^{i\alpha_q^d} \\ 0 & c_q & 0 \\ |a_q| e^{-i\alpha_q^d} & 0 & d_q \end{pmatrix}, \tag{B1}$$

where the nonzero elements are only in the two diagonals. From the table, only matrix M_u^{V} belongs to this form.

Since $\det[M_q^{dd}] = -c_q|a_q|^2 < 0$ for $c_q > 0$, we use for the diagonal quark mass matrix $\mathrm{Dg}(m_1, m_2, -m_3)$. Now using $U_q^{dd} = \mathrm{Dg.}(1, 1, e^{i\alpha_q^d})$, we have

$$U_q^{dd} M_q^{dd} [U_q^{dd}]^{-1} = \begin{pmatrix} 0 & 0 & |a_q| \\ 0 & c_q & 0 \\ |a_q| & 0 & d_q \end{pmatrix} = M_{q(R)}^{dd}. \quad \text{(B2)}$$

The three matrix invariants provide us with the following set of equations:

$$c_q + d_q = m_1 - m_2 + m_3, \qquad |a_q|^2 c_q = m_1 m_2 m_3,$$
$$|a_q|^2 - c_q d_q = m_1 m_2 + m_2 m_3 - m_1 m_3, \quad \text{(B3)}$$

which allow us to write

$$d_q = m_1 - m_3, \qquad c_q = m_2, \qquad |a_q| = \sqrt{m_1 m_3} \quad \text{(B4)}$$

with the exact diagonalizing transformation O_q^{dd1} given by

$$O_q^{dd} = \begin{pmatrix} \frac{\sqrt{m_3}}{\sqrt{m_3+m_1}} & 0 & \pm\frac{\sqrt{m_1}}{\sqrt{m_1+m_3}} \\ 0 & 1 & 0 \\ \mp\frac{\sqrt{m_1}}{\sqrt{m_1+m_3}} & 0 & \frac{\sqrt{m_3}}{\sqrt{m_3+m_1}} \end{pmatrix}, \quad \text{(B5)}$$

which implies a mixing between the first and third quark families proportional to

$$\sin\theta_{13} = \sqrt{\frac{m_1}{m_1 + m_3}}.$$

2. Block Form

For this form, the Hermitian quark mass matrix is

$$M_q^{bf} = \begin{pmatrix} 0 & |a_q| e^{i\alpha_q^b} & 0 \\ |a_q| e^{-i\alpha_q^b} & c_q & 0 \\ 0 & 0 & d_q \end{pmatrix}. \quad \text{(B6)}$$

From the Table, matrices M_d^{I}, M_u^{II}, and M_d^{IV} belong to this form.

Using $U_q^{bf} = \mathrm{Dg.}(1, e^{i\alpha_q^b}, 1)$ we have

$$U_q^{bf} M_q^{bf} [U_q^{bf}]^{-1} = \begin{pmatrix} 0 & |a_q| & 0 \\ |a_q| & c_q & 0 \\ 0 & 0 & d_q \end{pmatrix} = M_{q(R)}^{bf}. \quad \text{(B7)}$$

We use for the diagonal quark mass matrix $\mathrm{Dg}(m_1, -m_2, m_3)$. The 3×3 matrix invariants combined with the hierarchy $|m_1| < |m_2| < |m_3|$ allow us to write

$$d_q = m_3, \quad c_q = m_1 - m_2, \quad |a_q| = \sqrt{m_1 m_2}, \quad \text{(B8)}$$

with the solution associated with the exact diagonalizing transformation $O_q^{(bf)}$ given now by

$$O_q^{bf} = \begin{pmatrix} \sqrt{\frac{m_2}{m_1+m_2}} & \pm\sqrt{\frac{m_1}{m_1+m_2}} & 0 \\ \mp\sqrt{\frac{m_1}{m_1+m_2}} & \sqrt{\frac{m_2}{m_1+m_2}} & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad \text{(B9)}$$

3. Nearest-neighbor form

For this form, the Hermitian quark mass matrix is

$$M_q^{nn} = \begin{pmatrix} 0 & |a_q| e^{i\alpha_q^n} & 0 \\ |a_q| e^{-i\alpha_q^n} & 0 & |b_q| e^{i\beta_q^n} \\ 0 & |b_q| e^{-i\beta_q^n} & d_q \end{pmatrix}. \quad \text{(B10)}$$

From the table, only M_u^{III} belong to this form.

Using $U_q^{nn} = \mathrm{Dg.}(1, e^{i\alpha_q^n}, e^{i(\alpha_q^n + \beta_q^n)})$ we have

$$U_q^{nn} M_q^{nn} [U_q^{nn}]^{-1} = \begin{pmatrix} 0 & |a_q| & 0 \\ |a_q| & 0 & |b_q| \\ 0 & |b_q| & d_q \end{pmatrix} = M_{q(R)}^{nn}. \quad \text{(B11)}$$

Using the definitions

$$M_{q(nn)}^{\mathrm{diag}} = [O_q^{nn}]^T M_{q(R)}^{nn} O_q^{nn} = \mathrm{Diag}(m_1, -m_2, m_3), \quad \text{(B12)}$$

where the subindices 1, 2, 3 in the diagonal forms refer, respectively, to the masses for the quarks u, c, and t for the up sector, as well as d, s, and b for the down sector.

The three matrix invariants allow us now to write

$$d_q = m_1 - m_2 + m_3, \qquad |a_q|^2 = \frac{m_1 m_2 m_3}{m_1 - m_2 + m_3},$$
$$|b_q|^2 = \frac{(m_3 - m_2)(m_3 + m_1)(m_2 - m_1)}{m_1 - m_2 + m_3}, \quad \text{(B13)}$$

and the exact diagonalizing transformation O_q^{nn} for this particular form is expressed as

$$O_q^{nn} = \begin{pmatrix} \pm\sqrt{\frac{m_2 m_3 (m_3 - m_2)}{(m_3 - m_1)(m_2 + m_1)(m_1 - m_2 + m_3)}} & \pm\sqrt{\frac{m_1 (m_3 - m_2)}{(m_3 - m_1)(m_1 + m_2)}} & \mp\sqrt{\frac{m_1 (m_2 - m_1)(m_1 + m_3)}{(m_3 - m_1)(m_1 + m_2)(m_1 - m_2 + m_3)}} \\ \pm\sqrt{\frac{m_1 m_3 (m_1 + m_3)}{(m_2 + m_1)(m_3 + m_2)(m_1 - m_2 + m_3)}} & \mp\sqrt{\frac{m_2 (m_1 + m_3)}{(m_2 + m_3)(m_1 + m_2)}} & \pm\sqrt{\frac{m_2 (m_3 - m_2)(m_2 - m_1)}{(m_2 + m_1)(m_3 + m_2)(m_1 - m_2 + m_3)}} \\ \pm\sqrt{\frac{m_1 m_2 (m_2 - m_1)}{(m_2 + m_3)(m_3 - m_1)(m_1 - m_2 + m_3)}} & \pm\sqrt{\frac{m_3 (m_2 - m_1)}{(m_2 + m_3)(m_3 - m_1)}} & \pm\sqrt{\frac{m_3 (m_3 - m_2)(m_1 + m_3)}{(m_2 + m_3)(m_3 - m_1)(m_1 - m_2 + m_3)}} \end{pmatrix}, \quad \text{(B14)}$$

where one has the freedom to choose two equivalent possibilities of phases (the up or down signs).

For the up quark sector, due to the fact that $m_t \gg m_c \gg m_u$, the former matrix (B14) can be expanded as

$$O_u^{nn} \approx \begin{pmatrix} \pm(1 - m_{uc}/2) & \pm\sqrt{m_{uc}}(1 - m_{ct}/2 - m_{uc}/2) & \mp\sqrt{m_{ut}}(1 - m_{uc} + m_{ct}/2) \\ \pm\sqrt{m_{uc}}(1 - m_{uc}/2) & \mp(1 - m_{ct}/2 - m_{uc}/2) & \pm\sqrt{m_{ct}}(1 - m_{uc} - m_{ct}/2) \\ \pm m_{ct}\sqrt{m_{ut}} & \pm\sqrt{m_{ct}}(1 - m_{uc}/2 - m_{ct}/2) & \pm(1 - m_{ct}/2) \end{pmatrix}, \tag{B15}$$

where $m_{ij} \equiv m_i/m_j$, $i < j$; $i, j = 1, 2, 3 = u, c, t$, respectively, and we have taken $m_{ut} = 0$.

4. Weak basis form

For this form, the Hermitian quark mass matrix is

$$M_q^{wb} = \begin{pmatrix} 0 & |a_q|e^{i\alpha_q^w} & 0 \\ |a_q|e^{-i\alpha_q^w} & c_q & |b_q|e^{i\beta_q^w} \\ 0 & |b_q|e^{-i\beta_q^w} & d_q \end{pmatrix}. \tag{B16}$$

From the Table, matrices M_u^{I}, M_d^{II}, M_d^{III}, and M_d^{V} belong to this form.

Using $U_q^{wb} = \mathrm{Dg.}(1, e^{i\alpha_q^w}, e^{i(\alpha_q^w + \beta_q^w)})$ we have

$$U_q^{wb} M_q^{wb} [U_q^{wb}]^{-1} = \begin{pmatrix} 0 & |a_q| & 0 \\ |a_q| & c_q & |b_q| \\ 0 & |b_q| & d_q \end{pmatrix} = M_{q(R)}^{wb}. \tag{B17}$$

The matrix invariants allow us now to write

$$c_q = m_1 - m_2 + m_3 - d_q, \qquad a_q^2 = \frac{m_1 m_2 m_3}{d_q}, \qquad b_q^2 = \frac{(d_q - m_1)(d_q + m_2)(m_3 - d_q)}{d_q}, \tag{B18}$$

where we use for the diagonal quark mass matrix $\mathrm{Dg}(m_1, -m_2, m_3)$. Notice here that for $d_q = m_3$, $b_q = 0$ and thus M_q^{wb} acquires the block form with an extra texture zero.

The exact diagonalizing transformation O_q^{wb} can be expressed now as [24]

$$O_q^{wb} = \begin{pmatrix} \pm\sqrt{\frac{m_2 m_3 (d_q - m_1)}{(m_3 - m_1)(m_2 + m_1) d_q}} & \pm\sqrt{\frac{m_1 m_3 (d_q + m_2)}{(m_2 + m_1)(m_3 + m_2) d_q}} & \pm\sqrt{\frac{m_1 m_2 (m_3 - d_q)}{(m_2 + m_3)(m_3 - m_1) d_q}} \\ \pm\sqrt{\frac{m_1 (d_q - m_1)}{(m_3 - m_1)(m_1 + m_2)}} & \mp\sqrt{\frac{m_2 (d_q + m_2)}{(m_2 + m_3)(m_1 + m_2)}} & \pm\sqrt{\frac{m_3 (m_3 - d_q)}{(m_2 + m_3)(m_3 - m_1)}} \\ \mp\sqrt{\frac{m_1 (m_3 - d_q)(m_2 + d_q)}{(m_3 - m_1)(m_1 + m_2) d_q}} & \pm\sqrt{\frac{m_2 (m_3 - d_q)(d_q - m_1)}{(m_2 + m_1)(m_3 + m_2) d_q}} & \pm\sqrt{\frac{m_3 (d_q - m_1)(d_q + m_2)}{(m_2 + m_3)(m_3 - m_1) d_q}} \end{pmatrix}^T. \tag{B19}$$

5. See-saw form

For this form, the Hermitian quark mass matrix is

$$M_q^{ss} = \begin{pmatrix} 0 & 0 & |a_q|e^{i\alpha_q^s} \\ 0 & c_q & |b_q|e^{i\beta_q^s} \\ |a_q|e^{-i\alpha_q^s} & |b_q|e^{-i\beta_q^s} & d_q \end{pmatrix}. \tag{B20}$$

From the table, only matrix M_{tt}^{V} belongs to this form. Now, for the diagonal quark mass matrix we use $\mathrm{Dg}(-m_1, m_2, m_3)$. Using now $U_q^{ss} = \mathrm{Dg.}(1, e^{i(\alpha_q^s - \beta_q^s)}, e^{i\alpha_q^s})$ we have

$$U_q^{ss} M_q^{ss} [U_q^{ss}]^{-1} = \begin{pmatrix} 0 & 0 & |a_q| \\ 0 & c_q & |b_q| \\ |a_q| & |b_q| & d_q \end{pmatrix} = M_{q(R)}^{ss}. \tag{B21}$$

The 3×3 matrix invariants allow us to write

$$|b_q|^2 = \frac{(m_3 + m_2 - d_q)(d_q - m_2 + m_1)(-d_q + m_3 - m_1)}{-d_q - m_1 + m_2 + m_3}, \qquad c_q = -d_q - m_1 + m_2 + m_3,$$
$$|a_q|^2 = \frac{m_1 m_2 m_3}{-d_q - m_1 + m_2 + m_3}, \tag{B22}$$

where $d_q = m_3 - m_1$, $b_q = 0$, and M_q^{ss} becomes the two-diagonal form with an extra texture zero.

The exact diagonalizing transformation O_q^{ss} is given now by

$$O_q^{ss} = \begin{pmatrix} -\sqrt{\frac{m_2 m_3(-d_q+m_2+m_3)}{(m_1+m_2)(m_1+m_3)(-d_q-m_1+m_2+m_3)}} & -\sqrt{\frac{m_1(d_q-m_2+m_1)(-d_q+m_3+m_1)}{(m_1+m_2)(m_1+m_3)(-d_q-m_1+m_2+m_3)}} & \sqrt{\frac{m_1(-d_q+m_2+m_3)}{(m_1+m_2)(m_1+m_3)}} \\ \sqrt{\frac{m_1 m_3(-d_q+m_3-m_1)}{(m_1+m_2)(-d_q-m_1+m_2+m_3)(-m_2+m_3)}} & \sqrt{\frac{m_2(d_q+m_1-m_2)(-d_q+m_2+m_3)}{(m_1+m_2)(-d_q-m_1+m_2+m_3)(-m_2+m_3)}} & \sqrt{\frac{m_2(-d_q-m_1+m_3)}{(m_1+m_2)(-m_2+m_3)}} \\ \sqrt{\frac{m_1 m_2(d_q-m_2+m_1)}{(-m_2+m_3)(-d_q-m_1+m_2+m_3)(m_1+m_3)}} & \sqrt{\frac{m_3(-d_q+m_2+m_3)(-d_q-m_1+m_3)}{(-d_q-m_1+m_2+m_3)(-m_2+m_3)(m_1+m_3)}} & \sqrt{\frac{(d_q-m_2+m_1)m_3}{(-m_2+m_3)(m_1+m_3)}} \end{pmatrix}. \tag{B23}$$

[1] For an excellent compendium of the SM, see J. F. Donoghue, E. Golowich, and B. R. Holstein, *Dynamics of the Standard Model* (Cambridge University Press, Cambridge, England, 1992).

[2] S. Weinberg, Phys. Rev. D **5**, 1962 (1972); A. Zee, Phys. Lett. **93B**, 389 (1980).

[3] C. D. Froggatt and H. B. Nielsen, Nucl. Phys. **B147**, 277 (1979).

[4] H. Fritzsch, Phys. Lett. **70B**, 436 (1977); **73B**, 317 (1978); Nucl. Phys. **B155**, 189 (1979). For a review with extensive references, see H. Fritzsch and Z. Z. Xing, Prog. Part. Nucl. Phys. **45**, 1 (2000).

[5] P. Ramond, R. G. Roberts, and G. G. Ross, Nucl. Phys. **B406**, 19 (1993); L. Ibañez and G. G. Ross, Phys. Lett. B **332**, 100 (1994).

[6] F. Wilczek and A. Zee, Phys. Lett. **70B**, 418 (1977); S. Pakvasa and H. Sugawara, Phys. Lett. **73B**, 61 (1978); Y. Yamanaka, H. Sugawara, and S. Pakvasa, Phys. Rev. D **25**, 1895 (1982); K. S. Babu and X. G. He, Phys. Rev. D **36**, 3484 (1987).

[7] F. Wilczek and A. Zee, Phys. Rev. Lett. **42**, 421 (1979); A. Davidson, M. Koca, and K. C. Wali, Phys. Rev. Lett. **43**, 92 (1979).

[8] P. Minkowski, Phys. Lett. **67B**, 421 (1977); R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. **44**, 912 (1980); S. Glashow, in *Quarks and Leptons*, Cargese 1979, edited by M. Levy *et al.* (Plenum, New York, 1980); M. Gell-Mann, P. Ramond, and R. Slansky, in *Proceedings of the Supergravity Stony Brook Workshop, New York, 1979*, edited by P. Van Niewenhuizen and D. Freedman (Amsterdam, North Holland, 1979); T. Yanagida, Prog. Theor. Phys. **64**, 1870 (1980).

[9] A. Davidson and K. C. Wali, Phys. Rev. Lett. **59**, 393 (1987); S. Rajpoot, Phys. Rev. D **36**, 1479 (1987); D. Chang and R. N. Mohapatra, Phys. Rev. Lett. **58**, 1600 (1987); B. S. Balakrishna, Phys. Rev. Lett. **60**, 1602 (1988); K. S. Babu and R. N. Mohapatra, Phys. Rev. Lett. **62**, 1079 (1989); Phys. Rev. D **41**, 1286 (1990).

[10] M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49**, 652 (1973); L. Maiani, Phys. Lett. **62B**, 183 (1976).

[11] G. C. Branco, L. Lavoura, and F. Mota, Phys. Rev. D **39**, 3443 (1989); H. Fritzsch and Z. Z. Xing, Nucl. Phys. **556B**, 49 (1999); G. C. Branco, D. Emmanuel-Costa, and R. G. Felipe, Phys. Lett. B **477**, 147 (2000).

[12] J. Beringer *et al.* (Particle Data Group), Phys. Rev. D **86**, 010001 (2012).

[13] C. O. Dib, I. Dunietz, F. J. Gilman, and Y. Nir, Phys. Rev. D **41**, 1522 (1990).

[14] J. Charles, A. Höcker, H. Lacker, S. Laplace, F. R. Diberder, J. Malclés, J. Ocariz, M. Pivk, and L. Roos (CKMfitter Group), Eur. Phys. J. C **41**, 1 (2005). Updated results and plots available at http://ckmfitter.in2p3.fr/.

[15] *BABAR* Collaboration, The BABAR Physics Book: Physics at an Asymmetric B Factory, edited by P. F. Harrison and H. R. Quinn, Report No. SLAC-R-0504.

[16] Y. Nir, Nucl. Phys. B, Proc. Suppl. **117**, 111 (2003); E. Lunghi and A. Soni, Phys. Rev. Lett. **104**, 251802 (2010).

[17] L. L. Chau and W. Y. Keung, Phys. Rev. Lett. **53**, 1802 (1984); F. J. Gilman *et al.*, Phys. Lett. B **592**, 793 (2004).
[18] W. A. Ponce and R. H. Benavides, Eur. Phys. J. C **71**, 164 (2011).
[19] N. Mahajan, R. Verma, and M. Gupta, Int. J. Mod. Phys. A **25**, 2037 (2010).
[20] For updated values of running quark and lepton masses, see Z.-z. Xing, H. Zhang, and S. Zhou, Phys. Rev. D **77**, 113016 (2008).
[21] H. D. Kim, S. Raby, and L. Schradin, Phys. Rev. D **69**, 092002 (2004).
[22] C. Jarlskog, Phys. Rev. Lett. **55**, 1039 (1985).
[23] H. Leutwyler, Phys. Lett. B **378**, 313 (1996).
[24] H. Fritzsch and Z. Z. Xing, Phys. Lett. B **353**, 114 (1995); **555**, 63 (2003); Z. Z. Xing and H. Zhang, J. Phys. G **30**, 129 (2004); R. Verma, G. Ahuja, N. Mahajan, M. Randhawa, and M. Gupta, J. Phys. G **37**, 075020 (2010).