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Quark masses without Yukawa hierarchies

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Abstract. – A model based on the local gauge group $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$ without particles with exotic electric charges is shown to be able to provide the quark mass spectrum and their mixing, by means of universal see-saw mechanisms, avoiding a hierarchy in the Yukawa coupling constants.

The Standard Model (SM), with all its successes, is in the unaesthetic position of having no explanation of fermion masses and mixing angles, both in the quark and lepton sectors. Besides, recent experimental results on neutrino oscillations [1], which imply physics beyond the SM, call for extensions of the model. In this regard, models based on the local gauge group $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$ (named in the literature 3-3-1 models) have been advocated recently, due to the fact that several versions of the model can be constructed so that anomaly cancellation is achieved [2, 3] under the condition that the number of families N_f equals the number of colors $N_c = 3$. Among those models we have chosen to work with a particular one that avoids the inclusion of fermion fields with exotic electric charges.

As has been recently pointed out [4], the appearance of see-saw mechanisms could be in itself a guiding principle to distinguish between fundamental scales and those which are not; if this is so, then the explanation of the five orders of magnitude spanned by the quark mass spectrum would require a new mass scale unconnected with the electroweak symmetry breaking mass scale, which may come from new physics, *e.g.*: supersymmetry, left-right symmetric models, or something else. See-saw origin for all fermion masses has been analyzed in the past in the context of several models [5].

In the framework provided by a 3-3-1 model, and by using a convenient set of Higgs fields, we show that one can avoid hierarchies in the Yukawa couplings. The presence of a new scale $V \gg v$ related to the breaking of $SU(3)_L \otimes U(1)_X$, triggers see-saw mechanisms that provide a sensible mass spectrum for quarks. At the same time, these mechanisms provide relations between the mass eingenstates and the weak interaction eingenstates, and thus a Cabbibo-Kobayashi-Maskawa (CKM) mixing matrix emerges.

The model based on the local gauge group $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$ has 17 gauge Bosons: one gauge field B^μ associated with $U(1)_X$, the 8 gluon fields G^μ associated with $SU(3)_c$

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which remain massless after breaking the symmetry, and another 8 gauge fields associated with $SU(3)_L$ and that we write for convenience as [6]

$$\frac{1}{2}\lambda_\alpha A^\mu_\alpha = \frac{1}{\sqrt{2}}\begin{pmatrix} D_1^\mu & W^{+\mu} & K^{+\mu} \\ W^{-\mu} & D_2^\mu & K^{0\mu} \\ K^{-\mu} & \bar{K}^{0\mu} & D_3^\mu \end{pmatrix},$$

where $D_1^\mu = A_3^\mu/\sqrt{2} + A_8^\mu/\sqrt{6}$, $D_2^\mu = -A_3^\mu/\sqrt{2} + A_8^\mu/\sqrt{6}$, and $D_3^\mu = -2A_8^\mu/\sqrt{6}$. λ_i, $i = 1, 2, ..., 8$, are the eight Gell-Mann matrices normalized as $\mathrm{Tr}(\lambda_i\lambda_j) = 2\delta_{ij}$.

The charge operator associated with the unbroken gauge symmetry $U(1)_Q$ is given by $Q = \lambda_{3L}/2 + \lambda_{8L}/(2\sqrt{3}) + XI_3$, where $I_3 = \mathrm{Diag}(1, 1, 1)$ (the diagonal 3×3 unit matrix), and the X values, related to the $U(1)_X$ hypercharge, are fixed by anomaly cancellation. The sine square of the electroweak mixing angle is given by $S_W^2 = 3g_1^2/(3g_3^2 + 4g_1^2)$, where g_1 and g_3 are the coupling constants of $U(1)_X$ and $SU(3)_L$, respectively, and the photon field is given by

$$A_0^\mu = S_W A_3^\mu + C_W\left[\frac{T_W}{\sqrt{3}}A_8^\mu + \sqrt{(1 - T_W^2/3)}B^\mu\right], \quad (1)$$

where C_W and T_W are the cosine and tangent of the electroweak mixing angle, respectively.

The two weak flavor diagonal neutral currents in the model are coupled to the gauge Bosons:

$$Z_0^\mu = C_W A_3^\mu - S_W\left[\frac{T_W}{\sqrt{3}}A_8^\mu + \sqrt{(1 - T_W^2/3)}B^\mu\right];\ Z_0'^\mu = -\sqrt{(1 - T_W^2/3)}A_8^\mu + \frac{T_W}{\sqrt{3}}B^\mu, \quad (2)$$

where Z_0^μ coincides with the neutral gauge boson of the SM [3]. There is also an electrically neutral current associated with the flavor non-diagonal gauge boson $K^{0\mu}$ which is charged in the sense that it has a kind of weak V isospin charge.

The quark content of the model is [3, 6] $Q_L^i = (u^i, d^i, D^i)_L \sim (3, 3, 0)$, $i = 1, 2$ for two families, where D_L^i are two extra quarks of electric charge $-1/3$ (numbers inside the parenthesis stand for the $[SU(3)_c, SU(3)_L, U(1)_X]$ quantum numbers); $Q_L^3 = (d^3, u^3, U)_L \sim (3, 3^*, 1/3)$, where U_L is an extra quark of electric charge $2/3$. The right-handed quarks are $u_L^{ac} \sim (3^*, 1, -2/3)$, $d_L^{ac} \sim (3^*, 1, 1/3)$ with $a = 1, 2, 3$, a family index, $D_L^{ic} \sim (3^*, 1, 1/3)$, $i = 1, 2$, and $U_L^c \sim (3^*, 1, -2/3)$. The lepton content of the model is $L_{aL} = (e_a^-, \nu_a^0, N_a^0)_L \sim (1, 3^*, -1/3)$, for $a = 1, 2, 3 = e, \mu, \tau$, respectively (three $SU(3)_L$ anti-triplets), and the three singlets $e_{aL}^+ \sim (1, 1, 1)$, with ν_a^0 the neutrino field associated with the lepton e_a and N_a^0 playing the role of the corresponding right-handed neutrinos. There are not exotic charged leptons, and universality for the known leptons in the three families is present at tree level in the weak basis. With the former quantum numbers the model is free of all the gauge anomalies [6].

Instead of using the set of Higgs fields introduced in the original papers [3], we use the following set of four scalar triplets, with their Vacuum Expectation Values (VEV) as stated:

$$\begin{aligned}
\langle\phi_1^T\rangle &= \langle(\phi_1^+, \phi_1^0, \phi_1'^0)\rangle = \langle(0, 0, v_1)\rangle \sim (1, 3, 1/3);\\
\langle\phi_2^T\rangle &= \langle(\phi_2^+, \phi_2^0, \phi_2'^0)\rangle = \langle(0, v_2, 0)\rangle \sim (1, 3, 1/3);\\
\langle\phi_3^T\rangle &= \langle(\phi_3^0, \phi_3^-, \phi_3'^-)\rangle = \langle(v_3, 0, 0)\rangle \sim (1, 3, -2/3);\\
\langle\phi_4^T\rangle &= \langle(\phi_4^+, \phi_4^0, \phi_4'^0)\rangle = \langle(0, 0, V)\rangle \sim (1, 3, 1/3),
\end{aligned}$$

with the hierarchy $v_1 \sim v_2 \sim v_3 \sim v \sim 10^2\,\mathrm{GeV} \ll V \sim \mathrm{TeV}$. The analysis shows that this set of VEV breaks the $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$ symmetry in two steps following the scheme

$$SU(3)_c \otimes SU(3)_L \otimes U(1)_X \xrightarrow{V} SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \xrightarrow{v} SU(3)_c \otimes U(1)_Q,$$

where the first scale comes from $V + v_1 \approx V$ and the second one from $v_2 + v_3 \approx v$. The breaking allows for the matching conditions $g_2 = g_3$ and $1/g_Y^2 = 1/g_1^2 + 1/(3g_2^2)$, where g_2 and g_Y are the gauge coupling constants of the $SU(2)_L$ and $U(1)_Y$ gauge groups in the SM.

Related models to this, with the same fermion content but different scalar sector (ϕ_1 is absent) are analyzed in the papers in refs. [3]. Other 3-3-1 models without exotic electric charges, but with different fermion contents, can be found in refs. [7]

The Higgs scalars introduced above are used to write the Yukawa terms for the quarks. In the case of the Up quark sector, the most general invariant Yukawa Lagrangian is given by

$$\mathcal{L}_Y^u = \sum_{\alpha=1,2,4} Q_L^3 \phi_\alpha C \left(h_\alpha^U U_L^c + \sum_{a=1}^{3} h_{a\alpha}^u u_L^{ac} \right) + \sum_{i=1}^{2} Q_L^i \phi_3^* C \left(\sum_{a=1}^{3} h_{ia}^u u_L^{ac} + h_i'^U U_L^c \right) + \text{H.c.}, \quad (3)$$

where the $h^{u,U\prime}$s are couplings that we assume of order one. C is the charge conjugation operator. In order to restrict the number of Yukawa couplings, and produce a realistic fermion mass spectrum, we introduce the following anomaly-free [8] discrete Z_2 symmetry:

$$Z_2(Q_L^a, \phi_2, \phi_3, \phi_4, u_L^{ic}, d_L^{ac}) = 1 \ , \quad Z_2(\phi_1, u_L^{3c}, U_L^c, D_L^{ic}, L_{aL}, e_{aL}^+) = 0, \quad (4)$$

where $a = 1, 2, 3$ ($= e$, μ, τ for the leptons) and $i = 1, 2$ are family indices.

Then in the basis (u^1, u^2, u^3, U) we get, from eqs. (3)-(4), the following tree level Up quark mass matrix:

$$M_u = \begin{pmatrix} 0 & 0 & 0 & h_{11}^u v_1 \\ 0 & 0 & 0 & h_{21}^u v_1 \\ h_{13}^u v_3 & h_{23}^u v_3 & h_{32}^u v_2 & h_{34}^u V \\ h_1'^U v_3 & h_2'^U v_3 & h_2^U v_2 & h_4^U V \end{pmatrix}, \quad (5)$$

which is a see-saw–type mass matrix, with one eigenvalue equal to zero.

On the other hand, the Yukawa terms for the Down quark sector, using the four Higgs scalars introduced, are

$$\mathcal{L}_Y^d = \sum_{\alpha=1,2,4} \sum_i Q_L^i \phi_\alpha^* C \left(\sum_a h_{ia\alpha}^d d_L^{ac} + \sum_j h_{ij\alpha}^D D_L^{jc} \right) + Q_L^3 \phi_3 C \left(\sum_i h_i^D D_L^{ic} + \sum_a h_a^d d_L^{ac} \right) + \text{H.c.} \quad (6)$$

In the basis $(d^1, d^2, d^3, D^2, D^3)$ and using the discrete symmetry Z_2, the former expression produces the following tree level Down quark mass matrix:

$$M_d = \begin{pmatrix} 0 & 0 & 0 & h_{11}^d v_1 & h_{21}^d v_1 \\ 0 & 0 & 0 & h_{12}^d v_1 & h_{22}^d v_1 \\ 0 & 0 & 0 & h_{13}^d v_1 & h_{23}^d v_1 \\ h_{11}^D v_2 & h_{21}^D v_2 & h_1^D v_3 & h_{114}^D V & h_{214}^D V \\ h_{12}^D v_2 & h_{22}^D v_2 & h_2^D v_3 & h_{124}^D V & h_{224}^D V \end{pmatrix}, \quad (7)$$

where we have used $h_{ia\alpha}^{D(d)} v_\alpha = h_{ia}^{D(d)} v_\alpha$. The mass matrix M_d is again a see-saw type, with at least one eigenvalue equal to zero.

Before entering into a more detailed analysis of M_u and M_d, let us insist on the resulting see-saw character of these matrices. In both cases there is a zero eigenvalue that we immediately identify with the u and d quarks of the first family, respectively. Then, they are massless at tree level in the model considered here. In the U sector, the c quark acquires a see-saw

mass, while in the D sector, both s quark and b quark have see-saw masses (nevertheless, with a particular election of parameters, one can end up with a massless s quark too). The U sector structure is in some sense singular because the top mass is of the order of the electroweak scale; in fact it gets already a tree level mass of this order.

A further numerical check of the matrices is definitive in the sense that the model provides a see-saw mass hierarchy defined by the relationship between the symmetry-breaking scales v/V. In what follows, and without loss of generality, we are going to impose the condition $v_1 = v_2 = v_3 \equiv v \ll V$, with the value for v fixed by the mass of the charged weak gauge boson $M^2_{W^\pm} = g_3^2(v_2^2 + v_3^2)/2 = g_2^2 v^2$ which implies $v = 246/2 = 123\,\mathrm{GeV}$ (v_1 is associated with an $SU(2)_L$ singlet and does not contribute to the $W^\pm$ mass).

Starting with the U matrix, the analysis shows that $M_u^\dagger M_u$ has one zero eigenvalue, related to the eigenvector $[(h^u_{32}h'^U_2 - h^u_{23}h^U_2), (h^u_{13}h^U_2 - h^u_{32}h'^U_1), (h^u_{23}h'^U_1 - h^u_{13}h'^U_2), 0]$, that we may identify with the up quark u in the first family, which remains massless at tree level.

In order to simplify the otherwise cumbersome calculations and to avoid the proliferation of unnecessary parameters at this stage of the analysis, we propose to start with the following simple matrix:

$$M'_u = hv \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & h^u_{32}/h & \delta^{-1} \\ 1 & 1 & 1 & \delta^{-1} \end{pmatrix}, \tag{8}$$

where $\delta = v/V$ is the expansion parameter and h is a parameter that can take any value of order 1. The results below show that this matrix has the necessary ingredients to produce a consistent mass spectrum.

Neglecting terms of order δ^5 and higher, the four eigenvalues of $M'^\dagger_u M'_u$ are: one zero eigenvalue related to the eigenstate $(u^1 - u^2)/\sqrt{2}$ (notice the maximal mixing present); a see-saw eigenvalue $4h^2V^2\delta^4 = 4h^2v^2\delta^2 \approx m_c^2$ associated to the charm quark, and the other two

$$\frac{h^2V^2\delta^2}{2}[e_-^2 + \delta^2 e_+^2(4 - e_-^2)/4] \approx \frac{v^2}{2}(h - h^u_{32})^2 \approx m_t^2,$$

$$h^2V^2[2 + \delta^2(6 + e_+/2) + \delta^4(4e_+^2 - e_+^2e_-^2 - 32)/8] \approx m_U^2,$$

where $e_\pm = (1 \pm h^u_{32}/h)$. The eigenvectors are given by the rows of the following 4×4 unitary matrix:

$$U^U_L = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{C_{\eta_1}}{\sqrt{2}} & \frac{C_{\eta_1}}{\sqrt{2}} & 0 & -\frac{S_{\eta_1}}{\Delta} \\ 0 & 0 & \Delta^{-1} & -\frac{\delta\, e_+}{2\Delta} \\ \frac{S_{\eta_1}}{\sqrt{2}} & \frac{S_{\eta_1}}{\sqrt{2}} & \frac{C_{\eta_1}\delta\, e_+}{2\Delta} & \frac{C_{\eta_1}}{\Delta} \end{pmatrix}, \tag{9}$$

where C_{η_1} and $S_{\eta_1} \approx \sqrt{2}\delta(1 - 3\delta^2)$ are the cosine and sine of a mixing angle η_1, and $\Delta = \sqrt{(1 + \delta^2 e_+/4)}$.

So, in the Up quark sector the heavy quark gets a large mass of order V, the top quark gets a mass at the electroweak scale (times a difference of Yukawas that in the general case of matrix (5) is $(h^U_2 - h^u_{32})$), the charm quark gets a see-saw mass, and the first family up quark u remains massless at tree level. From the former expressions we get $|h^U_2 - h^u_{32}| \sim 2$ and $m_c \approx 2hv^2/V$, which in turn implies $V \approx hm_t^2/m_c \approx 19.4h\,\mathrm{TeV}$, fixing in this way an upper limit for the 3-3-1 mass scale V (experimental values are taken from ref. [9]).

We go now to the D quark mass matrix. This matrix is full of physical possibilities, depending upon the particular values assigned to the Yukawa couplings. For example, if all of

them are different from each other, then the matrix $M_d^\dagger M_d$ has rank one with a zero eigenvalue related to the eingenvector $[(h^D_{22}h^D_1 - h^D_{21}h^D_2),(h^D_{11}h^D_2 - h^D_{12}h^D_1),(h^D_{21}h^D_{12} - h^D_{11}h^D_{22}),0,0]$, that we may identify with the down quark d in the first family (which in any case remains massless at tree level); for this case the general analysis shows that we have two see-saw eigenvalues associated with the bottom b and strange s quarks.

In the particular case when all the Yukawas are equal to one but $h^D_{114} = h^D_{224} = H^D \neq 1$, the null space of $M_d^\dagger M_d$ has rank two, with the eigenvectors associated with the zero eigenvalues given by $[-2,1,1,0,0]/\sqrt{6}$ and $[0,-1,1,0,0]/\sqrt{2}$, which in turn implies only one see-saw eigenvalue associated with the bottom quark b, with a value for its mass approximately equal to $6v\delta/(1+H^D)$, (with masses for the two heavy states of the order of $V(1 \pm H^D)$).

For the first case mentioned, the chiral symmetry remaining at tree level is $SU(2)_f$ (quarks u and d massless), and for the second case the chiral symmetry is $SU(3)_f$ (quarks u, d and s are massless). In both cases the chiral symmetry is broken by the radiative corrections.

In any way, a realistic analysis of the down sector requires to have in mind the mixing matrix (9) of the up quark sector and the fact that the CKM mixing matrix is almost unitary and diagonal. Aiming at this and in order to avoid again a proliferation of parameters, let us analyze the particular case given by the following left-right symmetric (Hermitian) down quark mass matrix:

$$M'_d = h'v \begin{pmatrix} 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & f & g \\ 1 & 1 & f & H^D\delta^{-1} & \delta^{-1} \\ 1 & 1 & g & \delta^{-1} & H^D\delta^{-1} \end{pmatrix}, \tag{10}$$

where f and g are parameters of order one. This is the most general Hermitian mass matrix with only one eigenvalue equal to zero, related with the state $(d^1 - d^2)/\sqrt{2}$, as required in order to end up with an almost diagonal CKM mixing matrix.

The two see-saw exact eigenvalues of M'_d are

$$-h'\,v\,\frac{\delta}{4}\left\{\left[\frac{(f-g)^2}{H^D-1} + \frac{8+(f+g)^2}{1+H^D}\right] \pm \sqrt{\left[\frac{(f-g)^2}{H^D-1} + \frac{8+(f+g)^2}{1+H^D}\right]^2 - \frac{8(f-g)^2}{1-(H^D)^2}}\right\}. \tag{11}$$

Moreover, notice that for the particular case $g = -f$, the five eigenvalues of the Hermitian matrix above get the following simple exact analytical expressions:

$$\frac{h'\,\delta^{-1}\,v}{2}\left[0, H^D_+(1 \pm \sqrt{1+16\delta^2/(H^D_+)^2}),\ H^D_-(1 \pm \sqrt{1+8f^2\delta^2/(H^D_-)^2})\right], \tag{12}$$

where $H^D_\pm = H^D \pm 1$. The see-saw values are thus $-4\,\delta h'v/H^D_+$ and $-2\,\delta\,f^2\,h'\,v/H^D_-$; they imply $f^2h'/h \approx m_bH^D_-/m_c \approx 3H^D_-$ and $2h'/h \approx H^D_+m_s/m_c$, that can be seen as either a mild hierarchy between h and h', or implying a detailed tuning of some of the parameters of order one (inconvenience that could be avoided by working in a frame where $SU(3)_f$ becomes the original chiral symmetry).

The eigenvectors are now given by the rows of the following 5×5 unitary matrix:

$$U^D_L = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 & 0 & 0 & 0 \\ C_{\eta_2} & C_{\eta_2} & 0 & -S_{\eta_2} & S_{\eta_2} \\ 0 & 0 & \sqrt{2}\,C_{\eta_3} & -S_{\eta_3} & -S_{\eta_3} \\ S_{\eta_2} & S_{\eta_2} & 0 & C_{\eta_2} & -C_{\eta_2} \\ 0 & 0 & \sqrt{2}\,S_{\eta_3} & C_{\eta_3} & C_{\eta_3} \end{pmatrix}, \tag{13}$$

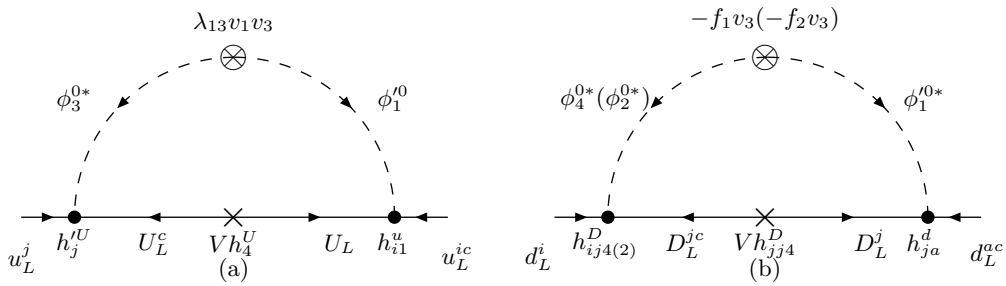

Fig. 1 – The five one-loop diagrams that produce the radiative masses for the quarks u and d.

where: C_{η_2} and $S_{\eta_2} \approx 2\,\delta[1-\delta^2/(H_+^D)^2]/H_+^D$; C_{η_3} and $S_{\eta_3} \approx \sqrt{2}\,\delta\, f[1-3\,f^2\delta^2/(H_-^D)^2]/H_-^D$ are the cosines and sines of other two mixing angles η_2 and η_3. Notice that up to this point, the CKM matrix $U_{CKM}^{(0)} = U_L^{u\,\dagger}\,U_L^d$ deviates from the identity just by terms of the order δ^2 and higher; where U_L^u is the 3×3 upper sector of U_L^U of eq. (9) and the same for U_L^d.

The consistency of the model requires that one can identify mechanisms able to produce masses for the first family, and to generate the CKM mixing angles. A detailed study of the Lagrangian for the Up quark sector (3), the discrete Z_2 symmetry (4) and the scalar potential, allows us to draw the radiative diagram in fig. 1a, which is the only diagram available to produce a finite one-loop radiative correction in the quark subspace (u^1, u^2) of the Up quark sector. The mixing of the Higgs Bosons comes from a term in the scalar potential of the form $\lambda_{13}(\phi_1^*\phi_1)(\phi_3^*\phi_3)$, which turns on the radiative correction.

In order to have a contribution different from zero we must avoid maximal mixing in the first two weak interaction states, otherwise a submatrix of the democratic type arises. This is done by taking $h_{11}^u = 1-k$ and $h_1'^U = 1+k$ in matrix (8) insted of 1, where k must be a very small parameter inorder to guarantee the see-saw character of the Up sector quark mass matrix. Evaluating the contribution coming from the diagram in fig. 1a we get

$$\Delta_{ji} = N_{ji}[M^2m_1^2\ln(M^2/m_1^2) - M^2m_3^2\ln(M^2/m_3^2) + m_3^2m_1^2\ln(m_1^2/m_3^2)], \tag{14}$$

where $N_{ji} = h_j'^U h_{i1}^u \lambda_{13}v_1v_3M/[16\pi^2(m_3^2-m_1^2)(M^2-m_1^2)(M^2-m_3^2)]$, $M = h_4^U V$ is the mass of the heavy Up quark, and m_1 and m_3 are the masses of $\phi_1'^0$ and ϕ_3^0, respectively. To estimate the contribution given by this diagram we assume the validity of the "extended survival hypothesis" [10] which in our case means $m_1 \approx m_3 \approx v$, implying

$$m_u \approx \lambda_{13}v\delta\ln(V/v)/8\pi^2 \approx 0.85\lambda_{13}\,\mathrm{MeV}, \tag{15}$$

which for $\lambda_{13}\sim 2$ produces $m_u \approx 1.7\,\mathrm{MeV}$, which is of the correct order of magnitude [9] (result independent of the value of k in first approximation). Due to the fact that the parameter $k \neq 0$, the state related to the u quark looses its maximal mixing, becoming now $\{-(h-h_{32}^u)u^1+[h-h_{32}^u(1-k)]u^2+ku^3\}/N$, with N being the normalization factor. The value of k is estimated with the value of the Cabbibo angle to be $k\approx 0.1$.

For the Down quark sector there are four one-loop diagrams, two for D^1 and other two for D^2 as depicted in fig. 1b. The mixing in the Higgs sector comes from terms in the scalar potential of the form $f_1\phi_1\phi_3\phi_4 + f_2\phi_1\phi_2\phi_3 + \mathrm{H.c.}$. When the algebra is done we get

$$m_d \approx 2(f_1+f_2)\delta\ln(V/v)/8\pi^2, \tag{16}$$

which for $f_1 = f_2 \approx v$ implies $m_d \approx 2m_u$, without introducing a new mass scale in the model.

The discrete Z_2 symmetry introduced eliminates possible tree level lepton mass terms of the form $L_{aL}\phi_3 Ce_{bL}$ and $L_{aL}L_{bL}\phi_3$. Then in order to generate masses for the leptons we must use either leptoquark Higgs Fields if we intend to use the radiative mechanism, or exotic leptons if we want to use see-saw mechanisms. For the neutrinos, for example, this analysis has been done in ref. [11], where new $SU(3)_L$ Higgs scalar multiplets are introduced.

In a model like this with four scalar triplets, we should worry about possible flavor-changing neutral current (FCNC) effects. First, we notice that due to our Z_2 symmetry, they do not occur at tree level because each flavor couples only to a single multiplet. They can enter as a consequence of the violation of unitarity of the CKM matrix U^0_{CKM} which is a 3×3 submatrix of a rectangular 4×5 matrix. The violation of unitarity in our analysis is proportional to δ^2, implying FCNC proportional to δ^4. Then, a value of $\delta \approx 10^{-2}$ is perfectly safe as far as violation of unitarity of the CKM matrix and possible FCNC effects are concerned. Experimental constraints on the posible violation of unitarity of the CKM matrix are discussed in Section 11 of ref. [9].

In several 3-3-1 models with three scalar triplets [2, 3] a discrete symmetry can suppress mass terms for the neutral Higgs bosons and to produce axion states [12]. The preliminary analysis shows that the Z_2 symmetry introduced in our model with four scalar triplets, provides only with the eight Goldstone bosons needed, and nothing else.

In conclusion, we have presented a model with only two energy scales, that has the power of avoiding hierarchies among Yukawa couplings. Throughout the analysis, all the Yukawas are of order one, as also is the case for the dimensionless Higgs coupling λ_{13}. The new ingredients of the model are: the mass scale V used to define the expansion parameter δ, a new set of Higgs scalars and VEV and the discrete anomaly-free symmetry Z_2. All this triggers generalized see-saw mechanism in the Up and Down quark sectors.

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