UDC 629.735.051−52(045)

Marta M. Komnatska, post-graduate student
Anatoliy A. Tunik, D.E., Prof.

ROBUST STABILIZATION AND OPTIMIZATION OF FLIGHT CONTROL SYSTEM WITH STATE FEEDBACK AND FUZZY LOGICS

This paper deals with combination of two powerful and modern control tools as linear matrix inequality that is used for synthesis a 'crisp' controller and a fuzzy control approach for designing a soft controller. The control design consists of two stages. The first stage investigates the problem of a robust an H_2 controller design with parameters uncertainties of the handled plant in the presence of external disturbances. Stability conditions are obtained via a quadratic Lyapunov function and represented in the form of linear matrix inequalities. The second stage consists of the outer loop controller construction based on fuzzy inference system that utilizes for altitude hold mode. The parameters of the fuzzy controller are adjusted with a gradient descent method in order to improve the performance of the overall system. The case study illustrates the efficiency of the proposed approach to the flight control of small Unmanned Aerial Vehicle.

Розглянуто принцип поєднання двох потужних та сучасних засобів теорії управління як метод лінійних матричних нерівностей, який використовується для синтезу чіткого регулятора та нечіткого управління для синтезу регулятора з м'якими обчисленнями. Процедура синтезу складається з двох етапів. На першому етапі вирішено задачу синтезу робастного H_2- регулятора для безпілотного літального апарату із врахуванням зовнішніх збурень, які діють на об'єкт управління. Умови стійкості сформовано у вигляді лінійних матричних нерівностей. Другий етап присвячено задачі синтезу нечіткого регулятора для зовнішнього контуру управління в режимі стабілізації висоти, заснованого на нечіткій логіці. З метою покращення якості управління параметри нечіткого регулятора настроюються за допомогою градієнтного методу. Проведено дослідження на прикладі управління поздовжнім каналом безпілотного літального апарату.

fuzzy optimization, fuzzy robust control, model with parameters un certainties, linear matrix inequality

Introduction

During the last years, the problem of robust flight control system (FCS) has attracted a great attention from the control system society, especially in the area of Unmanned Aerial Vehicle (UAV). It is known that the control of UAV remains a challenge for the engineer. This is explained by the fact, that the parameters of UAV dynamic models are very vulnerable towards the changeable atmosphere conditions; therefore there are significant uncertainties of plant's models as well as of the exogenous disturbances spectral properties. From the other hand, the design of FCS involves manifold requirements which include a low cost design, weight and power consumption.

To satisfy the aforementioned requirements, several control methods have been proposed. Among them, it is possible to enumerate some works related to the combination of observer and linear quadratic regulator (LQR) [1; 2]. Furthermore, to preserve the required level of performance without loosing the robustness of the FCS, the H_2/H_∞ – robust optimization procedure is used. The main idea behind this technique is to seek a trade-off between the performance and the robustness of the overall closed loop system [1; 2].

Nowadays, a great attention is drawn to the Linear Matrix Inequality (LMI) approach [3; 4]. This advanced approach permits to consider the problems of optimal and robust–optimal control design in the form of LMI and formulate the stability conditions.

The LMI technique is used to design a static output feedback in [5; 6], also this method is utilized to compensate the external disturbances by the static and dynamic output feedback [7], only a few works have been devoted to the problem of H_∞ controller design with external disturbances and plant with uncertainties [7–9]. In the area of UAV robust control this approach gives a promising results, however, it is important to formulate the robust stability conditions under LMI.

In this article, the combination of two advanced approaches is used two design UAV robust flight controller. The first approach uses LMI to design the inner loop controller based on H_2 criterion, taking into account the model uncertainties and exogenous disturbances. This inner loop is designed to the stabilization of the angular motion of the aircraft. The second method is devoted to the design of outer loop controller utilizing zero order Sugeno fuzzy inference system.

The outer loop is designed to the stabilization of the UAV flight altitude and velocity. The realization of fuzzy controller requires the choice of many parameters by the designer, such as the shape and number of membership functions, the choice of the rule base to represent the control strategy and the universe of discourse, where the input/output membership function are distributed. Hence, it is important to the designer to optimize some parameters of the fuzzy controller in order to achieve the desired performance. In this work the optimization of the parameters of the input membership function as well as the position output singletons are adjusted. The performance index is formulated using the error between the desired altitude signal and the output altitude of the UAV. The method is based on the gradient descent technique, which seeks optimal parameters of the fuzzy controller using the derivative of the above performance index with respect to the membership functions parameters.

The case study and simulation results devoted to the longitudinal motion stabilization of the Aerosonde UAV. These results prove that the used method is very efficient for multivariable control from the viewpoint of its robustness and performance.

Inner loop robust controller design via Linear Matrix Inequality

This section is dedicated to the design of inner loop robust controller based on H_2 criterion taking into consideration the parametric uncertainties and external disturbances. The controller design is formulated in the context of the convex analysis via LMI, when it is necessary to find a common positive definite matrix P, which would satisfy a set of Lyapunov Inequalities [3; 4]. The LMI approach permits to obtain a state feedback controller for a set of linear models received due to the linearization of the nonlinear model for different operating conditions.

The model of the controlled plant with structured uncertainties and disturbances could be represented as follows:

$$\dot{x}(t) = ((A + \Delta A_i(t))x(t) + (B + \Delta B_i(t))u(t)) + \varphi(t), \quad (1)$$

where

$x(t) \in R^n$ is a state space vector;

$u(t) \in R^m$ is a control vector;

$\varphi(t) \in R^n$ denotes the unknown disturbances with a known upper bound

$$\varphi_{up} \geq \|\varphi(t)\|, \quad (2)$$

the state space matrices $\Delta A_i \in R^{nxn}$, $\Delta B_i \in R^{nxm}$ describe all modelling uncertainties of $A \in R^{nxn}$, $B \in R^{nxm}$ and $\| \; \|_2$ denotes the H_2 – norm of external disturbances.

We suppose that all uncertainties are bounded as described in [10] as follows:

$$\begin{aligned} \Delta A_i(t) &= H_{ai} D_{ai} N_{ai}, \\ \Delta B_i(t) &= H_{bi} \Delta_{bi} N_{bi}, \end{aligned} \quad (3)$$

where

$H_{ai}, H_{bi}, N_{ai}, N_{bi}$ are known real constant matrices with appropriate dimensions;

$\Delta_{ai}(t), \Delta_{bi}(t)$ are unknown uncertainties, which satisfy the classical boundedness conditions such that $\forall t$:

$$\Delta_{ai}^T(t)\Delta_{ai}(t) \leq I, \; \Delta_{bi}^T(t)\Delta_{bi}(t) \leq I.$$

If all components of the state vector $x(t)$ could be measured, then the control law for the system (1) is given as:

$$u(t) = -Kx(t). \quad (4)$$

Thus, the closed inner loop system with uncertainties and external disturbances is obtained by substituting (4) into (3), and is given by:

$$\dot{x}(t) = [(A + \Delta A_i(t)) - (B + \Delta B_i(t))] Kx(t) + \varphi(t). \quad (5)$$

Notice that the controlled plant considered in this paper contains the uncertainties and is subjected to the external disturbances $\varphi(t)$, hence the most convenient way to attenuate them is to use H_2 – criterion, which is expressed by:

$$\int_{t_0}^{t_f} x^T(t) Q x(t) \leq \eta^2 \int_{t_0}^{t_f} \varphi^T(t)\varphi(t)dt, \quad (6)$$

where

t_f is the final time;

Q is positive definite weight matrix and η predetermine the attenuation level.

The objective now consists of evaluation the gain K in equation (4). This gain should ensure that requirements of the quadratic stability and robust H_2 – performance of the closed loop system (5) for all bounded disturbances $\varphi(t)$ (2) and for all parameters variations inside given structured uncertainties ΔA and ΔB would be satisfied.

In the next section, the LMI robust stability condition for the closed loop system (5) is formulated.

Formulation of the Linear Matrix Inequality stability condition

In order to formulate the LMI robust stability condition aforementioned control system, the following two well known lemmas are needed **[10]**:

Lemma 1. For any real matrices Z, H and N with appropriate dimensions and $Z = Z^T$, the following Lyapunov inequality

$$Z + H\,\Delta\,N + N^T\,\Delta^T\,H^T \le 0.$$

Is satisfied for all real matrices Δ satisfying $\Delta^T(t)\Delta(t) \le I \quad \forall t \in [0, \infty]$, if there exists the scalar $\sigma > 0$ such that

$$Z + \sigma H H^T + \sigma^{-1} N^T N \le 0.$$

Under the H_2 criterion we are ready to formulate the robust stability condition for the closed loop system (5) in the following theorem.

Lemma 2 (Schur's lemma). For real matrices $D, L = L^T, E = E^T > 0$, the following two conditions are equivalent

1) $L - DE^{-1}D^T > 0$;

2) $\begin{bmatrix} L & D \\ D^T & E \end{bmatrix} > 0$.

Theorem. The uncertain and disturbed system (1) is quadratically stable and satisfies H_2 criterion (6), if there exist a positive definite matrix $P = P^T > 0$, the attenuation level η and the gain matrix K such that the following condition is satisfied:

$$\begin{bmatrix} X\,A_i^T + A_i^T\,X\text{-}B_i\,M_i\text{-}M_i^T\,B_i^T & H_{ai} & H_{bi} & X^T\,N_{ai}^T & X\,Q^{1/2} & M^T\,N_{bi}^T & I \\ H_{ai}^T & -\sigma^{-1} & 0 & 0 & 0 & 0 & 0 \\ H_{bi}^T & 0 & -\sigma^{-1} & 0 & 0 & 0 & 0 \\ N_{ai}\,X & 0 & 0 & -\sigma^{-1} & 0 & 0 & 0 \\ Q^{1/2}\,X & 0 & 0 & 0 & -\mathrm{I} & 0 & 0 \\ N_{bi}\,M & 0 & 0 & 0 & 0 & -\sigma & 0 \\ I & 0 & 0 & 0 & 0 & 0 & -\eta^2\,I \end{bmatrix} \le 0, \qquad (7)$$

where

a change of variables such as $X = P^{-1},\ M = K\,P^{-1}, K = M\,P$ was used.

Proof. Let $V(x,t) = x(t)P\,x^T(t)$ with $\mathrm{P} = \mathrm{P}^{\mathrm{T}} > 0$ be a candidate Lyapunov function. The closed loop system (4) preserves stability and the H_2 performance (6) with attenuation level η if:

$$\dot V(x,t) + x^T(t)Q\,x(t) - \eta^2\,\varphi^T(t)\varphi(t) \le 0. \qquad (8)$$

The condition (8) leads to the following inequality:

$$x^T(t)\left\{ \left(\tilde A\text{-}\tilde B\,K\right)^T P + P\left(\tilde A\text{-}\tilde B\,K\right) + Q \right\} x(t) +$$
$$+\varphi^T(t)\,P\,x(t) + x^T(t)\,P\,\varphi(t) - \eta^2\varphi^{\mathrm{T}}(t)\,\varphi(t) \le 0$$

or it is equivalent to:

$$\begin{bmatrix} x(t) \\ \varphi(t) \end{bmatrix}^T \begin{bmatrix} (\tilde A - \tilde B K)^T P + P(\tilde A - \tilde B K) + Q & P \\ P & -\eta^2 I \end{bmatrix} \begin{bmatrix} x(t) \\ \varphi(t) \end{bmatrix} \le 0, \qquad (9)$$

where

$\tilde A = A + \Delta A_i(t)$,

$\tilde B = B + \Delta B_i(t)$.

We use the following change of variables

$$X = P^{-1},\ M = K\,P^{-1}, K = M\,P.$$

Pre-multiplying and post-multiplying right and left sides of the inequality (9) by

$X = diag\begin{bmatrix} X^T & I \end{bmatrix}$

and $X = diag\begin{bmatrix} X & I \end{bmatrix}$ respectively, we obtain:

$$\begin{bmatrix} \left\{X\,\tilde A_i^T + \tilde A_i^T\,X - \tilde B_i\,M_i - M_i^T\,\tilde B_i^T + X^T\,Q\,X\right\} & I \\ I & -\eta^2 I \end{bmatrix} \le 0.$$

In order to solve the inequality (9), which contains the unknown uncertainties, the Lemma 1 is applied.

Let:

$$F = \begin{bmatrix} Y & P \\ P & -\eta^2 I \end{bmatrix},$$

$$Y = (A_i - B_i\,K)^T\,P + P\,(A_i - B_i\,K) + Q.$$

Then the inequality (9) can be rewritten as:

$$F + \begin{bmatrix} (\Delta A_i(t) - \Delta B_i(t)K)^T P + P(\Delta A_i(t) - \Delta B_i(t)K) & 0 \\ 0 & 0 \end{bmatrix} \le 0$$

Remind that parameters uncertainties are represented as given in (3), replacing these uncertainties by their bounded quantities as described in [11] and basing on the above mentioned Lemma, the following expression is obtained:

$$F+\begin{bmatrix} H_{ai} & H_{bi} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} \Delta_{ai} & 0 & 0 \\ 0 & \Delta_{bi} & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} N_{ai}X & 0 & 0 \\ 0 & -N_{bi}M & 0 \\ 0 & 0 & 0 \end{bmatrix}+\begin{bmatrix} X^T N_{ai}^T & 0 & 0 \\ 0 & -M^T N_{bi}^T & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} \Delta_{ai}^T & 0 & 0 \\ 0 & \Delta_{bi}^T & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} H_{ai}^T & 0 & 0 \\ H_{bi}^T & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}=F+ \tag{10}$$

$$+\sigma\begin{bmatrix} H_{ai} & H_{bi} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} H_{ai}^T & 0 & 0 \\ H_{bi}^T & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}+\frac{1}{\sigma}\begin{bmatrix} X^T N_{ai}^T & 0 & 0 \\ 0 & -M^T N_{bi}^T & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} \Delta_{ai}^T & 0 & 0 \\ 0 & \Delta_{bi}^T & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} H_{ai}^T & 0 & 0 \\ H_{bi}^T & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} N_{ai}X & 0 & 0 \\ 0 & -N_{bi}M & 0 \\ 0 & 0 & 0 \end{bmatrix}\le 0.$$

Since, $\forall t:\quad \Delta_{ai}^T(t)\Delta_{ai}(t)\le I$ and $\Delta_{\mathrm{bi}}^{\mathrm{T}}(\mathrm{t})\Delta_{\mathrm{bi}}(\mathrm{t})\le \mathrm{I}$, the inequality (10) becomes:

$$F+\sigma\begin{bmatrix} H_{ai} & H_{bi} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} H_{ai}^T & 0 & 0 \\ H_{bi}^T & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}+\frac{1}{\sigma}\begin{bmatrix} X^T N_{ai}^T & 0 & 0 \\ 0 & -M^T N_{bi}^T & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} N_{ai}X & 0 & 0 \\ 0 & -N_{bi}M & 0 \\ 0 & 0 & 0 \end{bmatrix}\le 0 .$$

The above inequality results in:

$$\begin{bmatrix} XA_i^T + A_iX - B_iM_i - M_i^T B_i^T + \sigma H_{ai}H_{ai}^T + \sigma H_{bi}H_{bi}^T + \frac{1}{\sigma}X^T N_{ai}^T N_{ai}X & X Q^{1/2} & I \\ Q^{1/2}X & -I+\frac{1}{\sigma}M^T N_{bi}^T N_{bi}M & 0 \\ I & 0 & -\eta^2 I \end{bmatrix}\le 0 . \tag{11}$$

After applying Schur's lemma to the $\xi_1=\sigma H_{ai}H_{ai}^T+\sigma H_{bi}H_{bi}^T+\frac{1}{\sigma}X^T N_{ai}^T N_{ai}X$ and $\xi_2=\frac{1}{\sigma}M^T N_{bi}^T N_{bi}M$ in the inequality (11) we obtain the inequality (7).

Design of the outer loop controller based on optimized Sugeno fuzzy inference

The outer loop is devoted to the design of Sugeno fuzzy controller. The block-diagram of the overall closed loop system is depicted in fig.1, where the outer loop is represented by a Takagi-Sugeno Fuzzy Controller (TSFC).

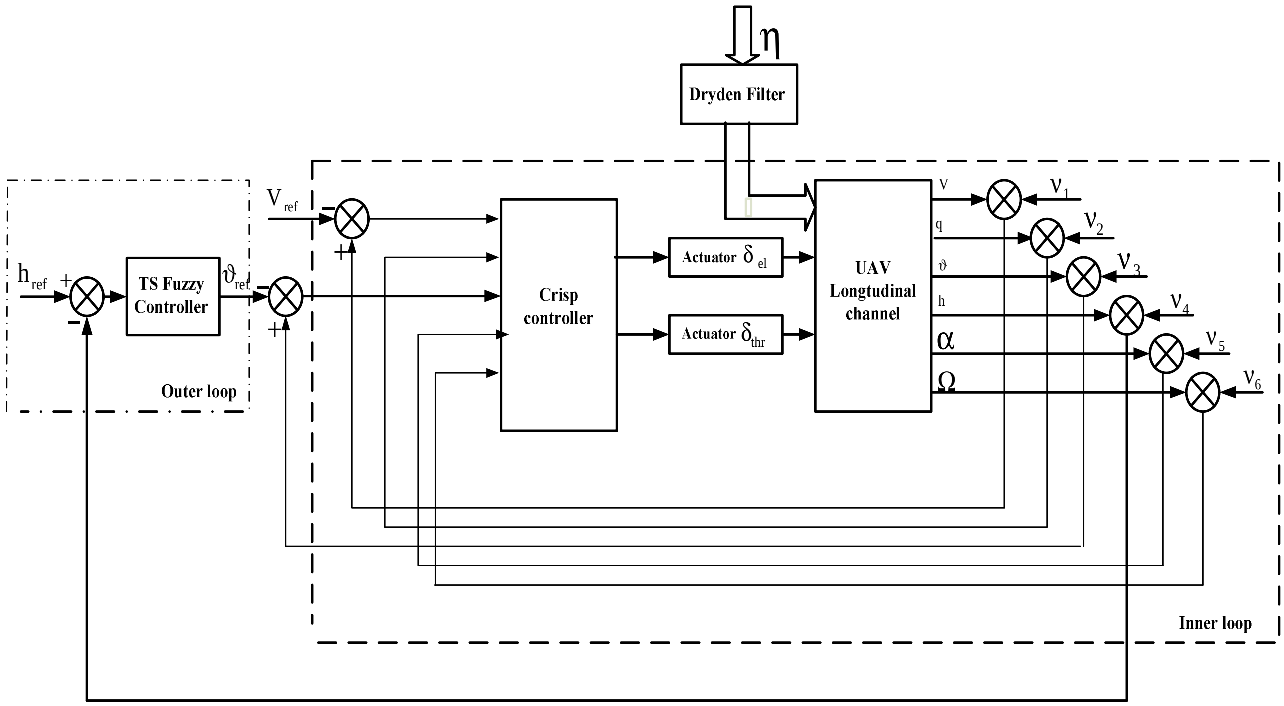

Fig. 1. Block diagram of the overall closed loop system

The input to the TSFC is the error $e(t)$ between the reference altitude signal and the actual output of the UAV, $e(t) = h_{ref}(t) - h(t)$. The output corresponds to the reference signal $\vartheta_{ref}(t)$ for the inner loop.

The TSFC considered in this paper is of type zero, where the rule base is embedded in following form:

IF e *is* X^j *THEN* u *is* b^i

where

i is number of control rules;

X^j is the linguistic values of the rule antecedent;

b^i is the output membership function centers.

We use the gaussian membership functions that are specified with the centers c^i and spreads σ^i for the premise part of control rules, the output is considered as singleton membership function. The gaussian membership function is given by:

$$\mu_i\left(e(t), c^i, \sigma^i\right) = \exp\left(-\frac{1}{2}\left(\frac{e(t)-c^i}{\sigma^i}\right)^2\right). \qquad (12)$$

Using product for the premise and implication, and center-average defuzzification, the overall output of the TSFC is computed as [12]:

$$\vartheta_{ref}\left(e(t)\middle|\theta_k\right) = \frac{\sum_{i=1}^{R} b_i \prod_{j=1}^{n} \exp\left(-\frac{1}{2}\left(\frac{e(t)-c^j}{\sigma^j}\right)^2\right)}{\sum_{i=1}^{R} \prod_{j=1}^{n} \exp\left(-\frac{1}{2}\left(\frac{e(t)-c^j}{\sigma^j}\right)^2\right)}, \qquad (13)$$

where

$i = 1, \ldots, R$;

$j = 1, \ldots, n$;

$k = n + R$.

Recall that our goal is to optimize the shape of the input and the output membership functions in order to minimize the quadratic error function given by:

$$E = \frac{1}{2}\left(h_{ref}\left(e(t)\middle|\psi_k\right) - h(t)\right)^2,$$

where

$h(t)$ is the target output of the system;

ψ is vector of parameters to be optimized, namely b_i, c^j, σ^j.

The tuning of the input and output membership function parameters of the TSFC is realized using the gradient descent method, which uses the partial derivatives of E with respect to the input and output membership functions parameters.

This idea was previously suggested and successfully realized with triangular membership functions for Mamdani fuzzy controller in [13].

In the present work the procedure of tuning the membership function parameters is applied to the TSFC with membership functions (12) and their update laws are given below.

Defuzzification parameters update law

Firstly, let's obtain the partial derivative of E with respect to the output membership function. By using chain rule, we obtain:

$$\frac{\partial E_t}{\partial b_i} = \left(h_{ref}\left(e(t)\middle|\psi_k\right) - h(t)\right)\frac{\partial \vartheta_{ref}\left(e(t)\middle|\psi_k\right)}{\partial b_i},$$

where $\vartheta_{ref}\left(e(t)\middle|\theta_k\right)$ is defined previously in (13).

Taking the partial derivative we get the following equation:

$$\frac{\partial E_t}{\partial b_i} = \left(h_{ref}\left(e(t)\middle|\psi_k\right) - h(t)\right)\times$$

$$\times\frac{\prod_{j=1}^{n} \exp\left(-\frac{1}{2}\left(\frac{e(t)-c^j}{\sigma^j}\right)^2\right)}{\sum_{i=1}^{R} \prod_{j=1}^{n} \exp\left(-\frac{1}{2}\left(\frac{e(t)-c^j}{\sigma^j}\right)^2\right)}. \qquad (14)$$

For convenience, we denote in (13)

$$\mu_i(e(t)) = \prod_{j=1}^{n} \exp\left(-\frac{1}{2}\left(\frac{e(t)-c^j}{\sigma^j}\right)^2\right)$$

and let

$$E_t = \left(h_{ref}\left(e(t)\middle|\psi_k\right) - h(t)\right)$$

denotes the instantaneous error.

Thus, we get the gradient descent rule to update the output membership function:

$$b_{i,k+1} = b_{i,k} - \lambda_1 E_t \frac{\mu_{i,k}(e(t))}{\sum_{i=1}^{R}\mu_{i,k}(e(t))}. \qquad (15)$$

In general, the update law can be rewritten as:

$$b_{i,k+1} = b_{i,k} - \lambda_1 \left.\frac{\partial E_t}{\partial b_i}\right|_k,$$

where

$b_{i,k+1}$ is the updated parameter;

$b_{i,k}$ is the parameter before optimization.

Input membership function centers update law

We would get the partial derivative of E with respect to the centers of the input membership functions in the same way as it is done in a previous section.

$$\frac{\partial E_t}{\partial c^j} = E_t \frac{\partial \vartheta_{ref}\left(e(t)\middle|\psi_k\right)}{\partial \mu_{i,k}(e(t))}\frac{\partial \mu_{i,k}(e(t))}{\partial c^j},$$

where

$$\frac{\partial \vartheta_{ref}\left(e(t)\,|\,\psi_k\right)}{\partial \mu_{i,k}\left(e(t)\right)} =$$

$$= \frac{\left(\sum_{i=1}^{R} \mu_{i,k}\left(e(t)\right)\right) b_{i,k} - \left(\sum_{i=1}^{R} b_{i,k}\, \mu_{i,k}\left(e(t)\right)\right)}{\sum_{i=1}^{R} \mu_{i,k}\left(e(t)\right)}$$

thus,

$$\frac{\partial \vartheta_{ref}\left(e(t)\,|\,\psi_k\right)}{\partial \mu_{i,k}\left(e(t)\right)} = \frac{b_{i,k} - \vartheta_{ref}\left(e(t)\,|\,\psi_k\right)}{\partial \mu_{i,k}\left(e(t)\right)}$$

and

$$\frac{\partial \mu_{i,k}\left(e(t)\right)}{\partial c^j} = \mu_{i,k}\left(e(t)\right)\left(\frac{e(t) - c_k^j}{\left(\sigma_k^j\right)^2}\right).$$

So, the update law for $c^j(k)$ is represented by expression (14)

$$c_{k+1}^j = c_k^j - \lambda_2 E_t \left(\frac{b_{i,k} - \vartheta_{ref}\left(e(t)\,|\,\psi_k\right)}{\sum_{i=1}^{R} \mu_{i,k}\left(e(t)\right)}\right) \mu_{i,k}\left(e(t)\right)\left(\frac{e(t) - c_k^j}{\left(\sigma_k^j\right)^2}\right). \tag{16}$$

In general, the update rule is expressed as

$$c_{k+1}^j = c_k^j - \lambda_2 \left.\frac{\partial E_t}{\partial c^j}\right|_k .$$

Input membership function spreads update law

To update the $\sigma^j(k)$ we will follow the same steps as above:

$$\sigma_{k+1}^j = \sigma_k^j - \lambda_3 \left.\frac{\partial E_t}{\partial \sigma^j}\right|_k , \tag{17}$$

using chain rule, we obtain:

$$\frac{\partial E_t}{\partial \sigma^j} = E_t \frac{\partial \vartheta_{ref}\left(e(t)\,|\,\psi_k\right)}{\partial \mu_{i,k}\left(e(t)\right)} \frac{\partial \mu_{i,k}\left(e(t)\right)}{\partial \sigma^j},$$

we have:

$$\frac{\partial \mu_{i,k}\left(e(t)\right)}{\partial \sigma^j} = \mu_{i,k}\left(e(t)\right) \frac{\left(e(t) - c_k^j\right)^2}{\left(\sigma_k^j\right)^3},$$

the update formula is described as:

$$\sigma_{k+1}^j = \sigma_{k+1}^j - \lambda_3 E_t \frac{b_{i,k} - \vartheta_{ref}\left(e(t)\,|\,\psi_k\right)}{\sum_{i=1}^{R} \mu_{i,k}\left(e(t)\right)} \mu_{i,k} \times$$

$$\times \left(e(t)\right) \frac{\left(e(t) - c_k^j\right)^2}{\left(\sigma_k^j\right)^3}.$$

In (15), (16) and (17) λ_l, $l = 1,2,3$ is a step size of the gradient descent algorithm. This completes the gradient descent method that is utilized to adjust the parameters of the fuzzy system.

Case study

To illustrate the efficiency of the proposed approach a longitudinal channel of the UAV (Aerosonde UAV) is used as a case study. The state space vector of the longitudinal channel is

$$X = [u, w, q, \vartheta, h, \Omega],$$

where

u is horizontal velocity component;

w − vertical velocity component;

q is pitch rate;

ϑ is pitch angle;

Ω is the engine spin (r.p.m).

The control vector consists of the elevator deflection and throttle. The nonlinear model of the Aerosonde model is linearized for three operating conditions: the nominal model at true airspeed of 26 m/s and two parametrically perturbed models at 23 m/s and 30 m/s.

The linear state space models are represented by matrices A, B, C, D:

$$A_n = \begin{bmatrix} -0.2489 & 0.4990 & -1.0564 & -9.8131 & 0 & 0.0120 \\ -0.5634 & -4.6466 & 25.3584 & -0.4058 & 0.0009 & 0 \\ 0.4079 & -4.9891 & -5.3614 & 0 & 0 & -0.0093 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0.0413 & -0.3331 & 0 & 25.9997 & 0 & 0 \\ 35.9563 & 1.4867 & 0 & 0 & -0.0417 & -3.2272 \end{bmatrix};$$

$$B_n = \begin{bmatrix} 0.3667 & 0 \\ -2.7579 & 0 \\ -38.0640 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 816.6240 \end{bmatrix};$$

$$A_{p1} = \begin{bmatrix} -0.2197 & 0.6002 & -1.4881 & -9.7969 & -0.0001 & 0.0108 \\ -0.5820 & -4.1207 & 22.4024 & -0.6460 & 0.0009 & 0 \\ 0.4823 & -4.5287 & -4.7515 & 0 & 0 & -0.0084 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0.0658 & -0.9978 & 0 & 22.9997 & 0 & 0 \\ 32.1031 & 2.1170 & 0 & 0 & -0.0294 & -2.7813 \end{bmatrix};$$

$$B_{p1} = \begin{bmatrix} 0.3246 & 0 \\ -2.1521 & 0 \\ -29.8233 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 448.6133 \end{bmatrix};$$

$$A_{p2} = \begin{bmatrix} -0.2933 & 0.3877 & -0.5578 & -9.7843 & 0 & 0.0138 \\ -0.5509 & -5.3691 & 29.2779 & -0.1849 & 0.0009 & 0 \\ 0.3382 & -5.6317 & -6.1948 & 0 & 0 & -0.1017 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0.0189 & -0.9998 & 0 & 29.9997 & 0 & 0 \\ 41.5394 & 0.7850 & 0 & 0 & -0.6355 & -3.8541 \end{bmatrix};$$

$$B_{p2} = \begin{bmatrix} -0.3 & 0 \\ -3.7 & 0 \\ -50.2 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 2663.9 \end{bmatrix};$$

$C = [I_{6\times6}],\ D = [0_{6\times2}]$.

In order to simulate the atmospheric turbulence a Dryden filter is used. Its state space description is given as follows [14]:

$$A_{dr} = \begin{bmatrix} -1/\lambda_u & 0 & 0 \\ 0 & -1/\lambda_w & 0 \\ 0 & -K_q/\lambda_q^2 & -1/\lambda_q \end{bmatrix};$$

$$B_{dr} = \begin{bmatrix} K_u/\lambda_u & 0 \\ 0 & K_w/\lambda_w \\ 0 & 0 \end{bmatrix};$$

$$C_{dr} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & K_q/\lambda_q & 1 \end{bmatrix},$$

where the subscript w corresponds to the true airspeed vertical component and u for the longitudinal one. In our case the Aerosonde flies at an altitude of 200 m in moderate turbulence. The parameters appearing in the state space of Dryden filter are given as follows [14; 15]:

$\mathrm{K_u} = \sigma_u \sqrt{(2\mathrm{L_u}/\pi \mathrm{V})}$,

$\lambda_u = L_u/V$,

$K_w = 1.42$,

$\lambda_w = 6.67$,

$K_q = 1/V$,

$\lambda_q = 4b/\pi V$,

where b is the Aerosonde's wing span: $b = 2.9\,m$.

The same parameters are defined for different models corresponding to different true airspeeds.

The inner loop is designed using LMI approach for uncertain model with external disturbances. The measured variables for the inner loop are $X = [u, w, q, \vartheta, \Omega]$. To apply LMI for inner loop design it is necessary to rewrite the matrices of uncertainties as $\Delta A_i(t) = H_{ai}\Delta_{ai}N_{ai}$

with: $$H_{a1} = H_{a2} = H_{a3} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$N_{a1} = N_{a2} = \begin{bmatrix} 0 & 0.1012 & -0.4317 & 0 & 0 \\ 0 & 0.5259 & -2.9560 & 0 & 0 \\ 0 & 0.4604 & 0.6099 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix};$$

$$N_{a3} = \begin{bmatrix} 0 & -0.1058 & 0.5007 & 0 & 0 \\ 0 & -0.8883 & 4.8867 & 0 & 0 \\ 0 & -0.8018 & -1.0167 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

The attenuation level is estimated as $\eta =$ 0,8010. The obtained gain matrix is defined as follows:

$$K = \begin{bmatrix} -0,0556 & -0,0593 & 0,7976 & 5,4622 & -0,0004 \\ -0,9842 & -0,1059 & -0,0204 & 1,6761 & -0,0025 \end{bmatrix}.$$

The indices of performance and robustness of the inner loop control for the nominal and perturbed models are given in tab. 1.

H_2 and H_∞ of the closed loop system

Plant		H_2 Deterministic case	H_2 Stochastic case	H_∞-norm
Vn=26 m/s	Nominal	0,6014	0,3543	0,5593
Vp_1=23 m/s	Perturbed 1	0,5214	0,3698	0,6711
Vp_1=30 m/s	Perturbed 2	0,7099	0,3453	0,5591

As stated before, the outer loop controller is designed using TSFC for altitude hold mode at the reference signal. The error between the reference signal and actual position of the UAV is removed through the fuzzy controller by adjusting the parameters using the gradient descent algorithm. TSFC comprises one input and one output.

Three input Gaussian shaped membership functions are used to represent the "crisp" values on the universe of discourse and singletons are used for output. Since there are a total two fuzzy variables (one input and one output) and each fuzzy variable have three membership functions. Thus, the total number of fuzzy parameters to be tuning is 9.

The simulation results of the closed loop system, before and after the optimization, are given in the fig. 2.

The maximum deflections of the angle of attack and pitch angle are enclosed within acceptable intervals: $-3 < \alpha < 3\,\mathrm{deg}$ and $-4 < \theta < 8\,\mathrm{deg}$, respectively. The altitude h and velocity V are also held at their reference signals $\mathrm{h\_ref} = 50\,\mathrm{m}$ and $\mathrm{V\_ref} = 5\,\mathrm{m/s}$ respectively with acceptable deflections.

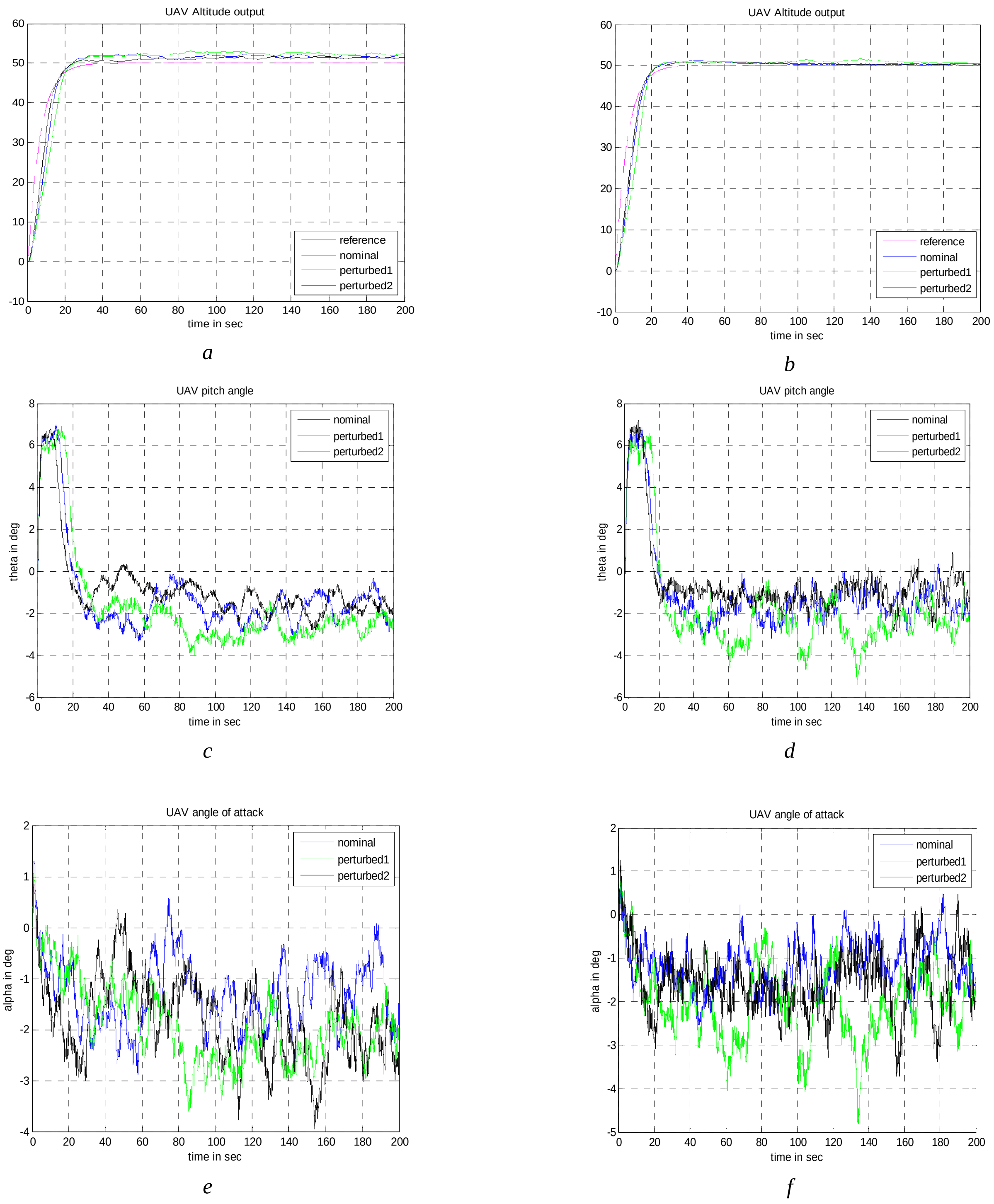

Fig. 2. Simulation results for longitudinal channel of Aerosonde:
non-optimized fuzzy controller:
a − altitude of UAV nominal and perturbed models;
c − pitch angle of UAV nominal and perturbed models;
e − angle of attack of UAV nominal and perturbed models;
optimized fuzzy controller:
b − altitude of UAV nominal and perturbed models;
d − pitch angle of UAV nominal and perturbed models;
f − angle of attack of UAV nominal and perturbed models

Conclusion

In this paper the robust combined autopilot consisting of inner and outer loops for plant with parameter uncertainties and external disturbances is considered. The inner loop is the system of angular motion and velocity stabilization, while the outer loop is the flight altitude control. The stability conditions have been obtained via a quadratic Lyapunov function. The outer loop is represented with fuzzy control approach. It is shown that the membership functions of the fuzzy controller can be optimized via the gradient descent method that results in adjusting (tuning) the fuzzy controller in an automatic mode. The simulation results demonstrate that all flight requirements are held.

References

1. *Tunik A.A.* Robust Stabilization and Nominal Performance of the Flight Control System for Small UAV / A.A. Tunik, T.A. Galaguz // Applied and computation mathematics.– 2004. – Vol. 3, № 1.– P. 34–45.

2. *Оптимальне* управління боковим каналом БПЛА при неповних вимірах вектора стану / А.А. Тунік, Т.А. Галагаз, М.М. Комнацька, Т.А. Білий // Проблеми інформатизації та управління. – 2005. – № 12.– С. 143–146.

3. *Tanaka K.* Fuzzy Control System Design and Analysis. A Linear Matrix Inequality Approach / K. Tanaka, H.O. Wang. – New York.: John Wiley& Sons, Inc,2001. – 305 p.

4. *Boyd S.* Linear Matrix Inequalities in System and Control Theory / S. Boyd, L. El Ghaoui, E. Feron, V. Balakrishnan. – Philadelphia, PA SIAM, 1994. – 416 p.

5. *Grman L.* Robust Output Feedback Quadratic Controller Design / L. Grman, V. Veselý // Journal of Electrical Engineering. – 2005. – Vol. 56.–P. 128–134.

6. *Ларин В.Б.* О компенсации внешних возмущений динамической обратной связью по выходной переменной / В.Б. Ларин, А.А. Туник // Прикладная механіка.– 2006.– Т. 42, №5.– С. 132 –144.

7. *Feng J.* Quadratic Stabilization for Uncertain Stochastic Systems / J. Feng, W. Zhang // Journal of Control Theory and Applications. – 2005. – № 3. – P. 252–258.

8. *Output* Feedback LMI Tracking Control Conditions with H_∞ criterion for Uncertain and Disturbed T-S models / B. Mansouri, N. Manamanni, K. Guelton, et al // Information Sciences. – (179) 2009. – P. 446−457.

9. *An LMI* Approach To Robust H_∞ Control For Uncertain Continuous- Time Systems / S.-W. Kau, Y.-S. Liu, C.-H. Lee, et al // Asian Journal of Control.– June 2005. – Vol. 7, No. 2. – P. 182–186.

10. *Xie L.* H_∞ Control and Quadratic Stabilization of Systems with Parameter Uncertainty Via Output Feedback / L. Xie // IEEE Trans. Automat. Contr. – Vol. 37, No. 8. – P. 1253–1256.

11. *Zhou K.* Robust Stabilization of Linear Systems with Norm – Bounded Timevarying Uncertainty / K. Zhou, P. Khargonekar // Systems and Control Letters. – 1998. – № 10 (1). – P. 17–20.

12. *Passino, K.M.*, Fuzzy Control / K.M Passino, S. Yurkovich.–Addison–Wesley.– Menlo Park, Reading, Harlow, Berkley, Sidney, Bonn, Amsterdam, 1998. – 502 p.

13. *Simon D.* Sum Normal Optimization of Fuzzy Membership Functions / D. Simon // International Journal of Uncertainty, Fuzziness and Knowledge – Based Systems. – 2002. – Vol.10. − P.363−384.

14. *McLean D.* Automatic Flight Control Systems / D. McLean. Prentice Hall Inc., Englewood Cliffs. – 1990.– 593 p.

15. *Ruaw M.A.* Simulink toolbox for Flight Dynamic and Control / M.A. Ruaw. TUDelft, Delft university of Technology. – 2005. – 253 p.

16. *Layne J. R.* Fuzzy Model Reference Learning Control / J. R. Layne, K.M. Passino // Journal of Intelligent and Fuzzy Systems.–1996. –Vol. 4, No. 1, P. 33–47.

17. *Tunik A.A.* Combination of fuzzy and crisp control in airborne robust multivariable systems / A.A. Tunik, M.A. Touat, M.M. Komnatska // Proc. of the Third World Congress in the XXI Century "Safety in Aviation and Space technology". – Kyiv, Ukraine. – 2008. – Vol. 2, September 22−24. – P. 33.38–33.47.

18. *Habbi A.* An Improved Self- Tuning Mechanism of Fuzzy Control by Gradient Descent Method / A. Habbi, M. Zelmat // 17-th European Simulation Multiconference 9−12 June, 2003, Foundations For Successful Modelling and Simulation ESM 2003. − P. 43−48.

The editor received the article on 21 October 2009.