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Geometric Algebra in Nonsinusoidal Power Systems: A Case of Study for Passive Compensation

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Abstract: New-generation power networks, such as microgrids, are being affected by the proliferation of nonlinear electronic systems, resulting in harmonic disturbances both in voltage and current that affect the symmetry of the system. This paper presents a method based on the application of geometric algebra (GA) to the resolution of power flow in nonsinusoidal single-phase electrical systems for the correct determination of its components to achieve passive compensation of true quadrature current. It is demonstrated that traditional techniques based on the concepts of Budeanu, Fryze or IEEE1459 fail to determine the interaction between voltage and current and therefore, are not suitable for being used as a basis for the compensation of nonactive power components. An example is included that demonstrates the superiority of GA method and is compared to previous work where GA approaches and traditional methods have also been used.

Keywords: geometric algebra; nonsinusoidal power; passive compensation; clifford algebra; circuit systems

1. Introduction

The new power grids are a major step forward for today's society, as they allow better energy management and integration with new renewable sources such as solar, wind, etc. [1]. These networks are made up of a large number of devices based on power electronics. A clear example is seen in distributed generation systems, intelligent buildings or control systems, where many receivers are installed such as cycloconverters, speed drives, household appliances, battery power converters, power inverters and more. Likewise, symmetry is a fundamental concept in art as well as science and engineering. Although, during the normal operation of the network, the system usually presents a symmetry in the waveform of voltage and current, all these elements can cause the network to supply a highly distorted current, so the symmetry is broken. In turn, this current distortion causes voltage drops in the lines that distort the voltage itself, causing problems to the neighbouring receivers. It is a situation that feeds back and causes a progressive degradation to the power quality of the supply [2–4].

There are numerous publications found in the literature specifying the problems caused by the appearance of harmonics in voltage and current, such as, for example, excessive heating, degradation of components, faults in protection and measurement equipment or inefficiencies in the transmission of energy [5–8]. All of the above can be summed up in an abnormal microgrid operation and low energy efficiency.

Therefore, it is essential to know precisely the electrical energy balances on any power grid or microgrid in order to be able to make the right decisions. Traditionally, mathematical tools used in sinusoidal conditions have been based on Steinmetz [9] theory and its decomposition into frequency components. In these circumstances, the result obtained for the apparent power is

$$\boldsymbol{S} = P + jQ \tag{1}$$

where P is the active power, Q is the reactive power and j is the imaginary unit. For the sinusoidal case, all the power theories converge because of the implicit symmetry associated to this problem, so there is no discussion about the matter. This is not the case for nonsinusoidal systems with a high harmonic content, as in modern microgrids, such as those described in [10–12].

Although there have been major contributions over the last few years [13–16], there are still some misconceptions that need to be revised [14,17]. The best-known theories, such as Budeanu's [18] or Fryze [19], have been criticised and highlighted by several authors, including Czarnecki [17,20–22], demonstrating inconsistency and errors in nonsinusoidal situations. Recently, Czarnecki's own theory has been criticised, finding weak points in the description of the nonactive components of apparent power [23]. Therefore, it is essential to find a methodology or framework that allows the unification of the concepts necessary for a correct compensation of the power factor, i.e., how to find the optimal configuration to demand active power with minimal current from source. In this sense, optimising the use of passive compensators (with energy storage) and active compensators or filters can be based on these techniques to achieve better control over the flow of electrical energy between the source and the load.

On the other hand, geometric algebra or Clifford's algebra has proven to be a powerful and flexible tool for representing the flow of energy and power in electrical systems [24,25]. Some researchers have proposed the use of Clifford's algebra as a mathematical tool to address the multicomponent nature of power in nonsinusoidal contexts [26–28]. The concept of nonactive, reactive or distorted power acquires a meaning that is more in line with its mathematical significance, allowing a better understanding of the energy balances and verification of the principle of energy conservation. It is also presented as a natural language to describe the deeper symmetry that underlies mathematical transformations such as those arising in power networks [29].

The concept of multicomponent power within the scope of geometric algebra [30] is used in this article to demonstrate its feasibility for determining the net power flow in a nonsinusoidal electrical circuit, the direction and sense of such power, as well as its use for calculating the geometric or net power factor defined as the ratio between the active power and the norm of the multivector power as defined in Section 3. This approach allows the designing of simpler and more efficient compensators than those proposed by Czarnecki [31,32]. In addition, the proposal made in this article improves other proposals based on GA such as those of Castilla [33]. The main contributions of this work are briefly presented under the following considerations:

- The use of GA to solve the problem of passive compensation of single-phase nonsinusoidal circuits.
- Determination and suppression of the current and geometric power in quadrature that make the power factor maximum.
- Evidence of the disadvantages of traditional compensation methods based on complex numbers compared to GA.
- Design of simpler and more efficient compensators.
- Comparison with other GA-based methods.

2. Background on Geometric Algebra

Geometric algebra has its origins in the work of Clifford and Grassman in the 19th century. Unfortunately, it did not have much impact until its recent impulse thanks to Hestenes and others [34–36]. Traditional concepts such as vector, spinor, complex numbers or quaternions are naturally explained as members of subspaces in GA. It can be easily extended in any number of dimensions, being this one of its main strengths. Because these are geometric objects, they all have direction, sense and magnitude. The most basic definitions of certain GA properties are presented below.

Definition 1. *A vector is considered to be a segment that has direction and meaning*

Definition 2. *The inner product of two vectors a and b corresponds to the traditional concept $a \cdot b$ and the result is a scalar.*

Definition 3. *The wedge product of two vectors, a and b, is represented by $a \wedge b$ and defines an area enclosed by the parallelogram formed by both vectors (see Figure 1). This plane has a direction and a sense, resulting in a bivector. This product complies with the anti-commutative property, i.e., $a \wedge b = -b \wedge a$.*

Definition 4. *A bivector is a novel concept that introduces geometric algebra and does not exist in vectorial calculus that engineers learn in a degree course. It is the result of the external product of 2 vectors producing a plane with direction and sense, exactly as a vector would have it. Its value is equal to the area enclosed by the parallelogram formed by the vectors (see Figure 2). Like vectors, a bivector can be written as the linear combination of a base of bivectors.*

Definition 5. *The geometric product is also another major contribution of the GA. It is defined primarily for vectors, although it can be extended to other objects. For example, given two vectors, $a = \alpha_1 e_1 + \beta_1 e_2$ and $b = \alpha_2 e_1 + \beta_2 e_2$, you can define its geometric product ab as*

$$ab = a \cdot b + a \wedge b$$

that is, the geometric product is a linear combination of the internal and external product. It can be seen how the result is made up of a scalar and a bivector, resulting in the so-called multivector.

$$A = ab = \langle A \rangle_0 + \langle A \rangle_2 = (\alpha_1 \alpha_2 + \beta_1 \beta_2) + (\alpha_1 \beta_2 - \beta_1 \alpha_2) e_1 e_2$$

$\langle A \rangle_0$ *is the scalar part and* $\langle A \rangle_2$ *is the bivector.*

Figure 1. Wedge product of 2 vectors a and b.
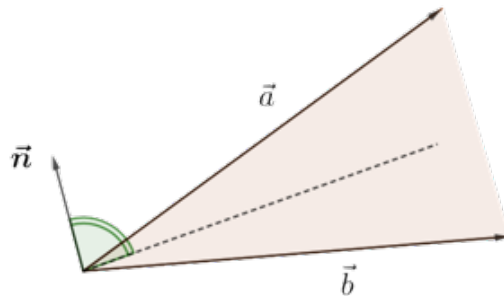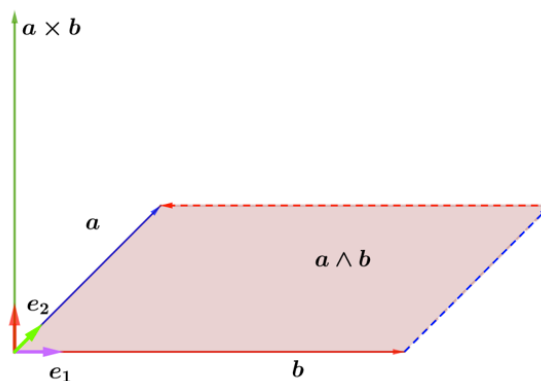

Figure 2. Representation of a bivector $a \wedge b$.

3. Power in Geometric Algebra

3.1. Vector Representation in $\mathcal{G}_\mathcal{N}$ Domain

Consider a periodic function in the time domain $x(t)$ that can represent a voltage or current waveform. A function space can be established where the following norm is defined as

$$\|x(t)\| = \sqrt{\frac{1}{T}\int_0^T x^2(t)dt} \tag{2}$$

The norm is found to be consistent with the definition of the root mean square (RMS) value. Well, this function can be represented by a linear combination of sine and cosine functions, i.e., a series of Fourier functions. Let us call these bases $\varphi_i(t)$, so that

$$x(t) = \sum_{i=1}^{n} x_i \varphi_i(t) \tag{3}$$

so that a direct transformation to $\mathcal{G}_N$ gives

$$\boldsymbol{x} = \sum_{i=1}^{n} x_i \boldsymbol{e_i} \tag{4}$$

where $\boldsymbol{e_i}$ are the new basis for the geometric space $\mathcal{G}_\mathcal{N}$. Because the new base is orthonormal, the following property is fulfilled,

$$\|x(t)\| = \sqrt{\sum_{i=1}^{n} x_i^2} = \|\boldsymbol{x}\| \tag{5}$$

Finally, we use the transformation proposed by Castro-Nuñez [25],

$$
\begin{aligned}
\varphi_{c1}(t) &= \sqrt{2}\cos\omega t &\longleftrightarrow&& \boldsymbol{e_1} \\[2mm]
\varphi_{s1}(t) &= \sqrt{2}\sin\omega t &\longleftrightarrow&& -\boldsymbol{e_2} \\[2mm]
\varphi_{c2}(t) &= \sqrt{2}\cos 2\omega t &\longleftrightarrow&& \boldsymbol{e_2}\boldsymbol{e_3} \\[2mm]
\varphi_{s2}(t) &= \sqrt{2}\sin 2\omega t &\longleftrightarrow&& \boldsymbol{e_1}\boldsymbol{e_3} \\[2mm]
&\quad\vdots& && \\[2mm]
\varphi_{cn}(t) &= \sqrt{2}\cos n\omega t &\longleftrightarrow&& \bigwedge_{i=2}^{n+1} \boldsymbol{e_i} \\[2mm]
\varphi_{sn}(t) &= \sqrt{2}\sin n\omega t &\longleftrightarrow&& \bigwedge_{\substack{i=1 \\ i\neq 2}}^{n+1} \boldsymbol{e_i}
\end{aligned}
\tag{6}
$$

where $\bigwedge \boldsymbol{e_i}$ represents the product of n vectors. This way, we can transform any waveform $x(t)$ to the geometric domain $\mathcal{G}_\mathcal{N}$.

3.2. Multivector Power

Several authors [14,17,30] have already shown that the traditional expression for apparent power (accepted by the IEEE1459 standard or Budeanu and Fryze's proposals) is incorrect, because it does not

comply with the principle of energy conservation and does not have a true physical correspondence with power flows. For example,

$$S^2 = P^2 + Q^2 + D^2 \quad \text{or} \quad S^2 = P_1^2 + Q_1^2 + D_I^2 + D_V^2 + S_H^2 \tag{7}$$

are expressions frequently used that violate the principle of energy conservation, so they should not be used on a regular basis, especially in nonsinusoidal scenarios as they lead to errors in the achieved results.

The addition of the concept of multivector power, geometric apparent power or net apparent power M (as labeled by Castro-Núñez), opens a door for attempting to solve the aforementioned problems. This concept is totally different form the traditional definition of the nonsinusoidal apparent power S, i.e., in general, $\|M\| \neq S$, so $\|M\|$ cannot be called apparent power. From a mathematical point of view, the expressions are simple and elegant. From a physical point of view, each term takes on a real meaning in the flow of energy between the load and the source. The geometric apparent power is defined as the geometric product between voltage and current:

$$M = ui = u \cdot i + u \wedge i \tag{8}$$

which will generally result in a scalar and a bivector for the sinusoidal case.

$$M = \underbrace{\langle M \rangle_0}_{\text{scalar}} + \underbrace{\langle M \rangle_2}_{\text{bivector}} \tag{9}$$

In fact, if we consider a sinusoidal voltage applied to a linear load, we obtain a sinusoidal current,

$$u(t) = A\cos(\omega t + \varphi) \Rightarrow u = \alpha_1 e_1 + \alpha_2 e_2 \tag{10}$$
$$i(t) = B\cos(\omega t + \delta) \Rightarrow i = \beta_1 e_1 + \beta_2 e_2 \tag{11}$$

The apparent geometric power is then

$$M = ui = (\alpha_1 e_1 + \alpha_2 e_2)(\beta_1 e_1 + \beta_2 e_2)$$
$$= \underbrace{(\alpha_1 \beta_1 + \alpha_2 \beta_2)}_{scalar} + \underbrace{(\alpha_1 \beta_2 - \alpha_2 \beta_1)e_1 e_2}_{bivector}$$

If we generalize for a nonsinusoidal voltage,

$$u(t) = \sum_{i=1}^{n} u_i(t) = D_1 \cos(\omega t) + E_1 \sin(\omega t) + $$
$$+ \sum_{h=2}^{d} D_h \cos(h\omega t) + \sum_{h=2}^{k} E_h \sin(h\omega t) \tag{12}$$

we can obtain the voltage transferred to the geometric domain $\mathcal{G}_N$.

$$u = D_1 e_1 - E_1 e_2 + \sum_{h=2}^{d} \left[D_h \bigwedge_{i=2}^{h+1} e_i \right] + \sum_{h=2}^{k} \left[E_h \bigwedge_{i=1, i \neq 2}^{h+1} e_i \right] \tag{13}$$

As Castro-Nunez [37] establishes that the geometric admittance is $Y = G_h + B_h e_1 e_2$, applying the principle of superposition yield each of the harmonic currents as $i_h = (G_h + B_h e_1 e_2)u_h$. Clearly, the total current i is the sum of all harmonic currents

$$i = \sum_{h=1}^{n} i_h \tag{14}$$

This current can be decomposed into in-phase and quadrature components with voltage.

$$i = i_{||} + i_{\perp} = i_g + i_b \tag{15}$$

where

$$i_g = G_1 D_1 e_1 - G_1 E_1 e_2 + \sum_{h=2}^{d} \left[G_h D_h \bigwedge_{i=2}^{h+1} e_i \right] +$$
$$+ \sum_{h=2}^{k} \left[G_h E_h \bigwedge_{i=1, i \neq 2}^{h+1} e_i \right] \tag{16}$$

$$i_b = -B_1 E_1 e_1 - B_1 D_1 e_2 + \sum_{h=2}^{d} \left[B_h D_h \bigwedge_{i=1, i \neq 2}^{h+1} e_i \right] -$$
$$- \sum_{h=2}^{k} \left[B_h E_h \bigwedge_{i=2}^{h+1} e_i \right] \tag{17}$$

Finally, the apparent multivector geometric power M can be obtained as the product between u and i,

$$M = ui = M_g + M_b = \underbrace{\overbrace{\langle M_g \rangle_0}^{P} + \overbrace{\sum_{i=1}^{n+1} \langle M_g \rangle_i}^{CN_d} +}_{M_g}$$
$$+ \underbrace{CN_{r(ps)} + CN_{r(hi)}}_{M_b = CN_r} \tag{18}$$

where

   M_g is the in phase geometric apparent power

   M_b is the cuadrature geometric apparent power

   P is the active power

   CN_d is the degraded power

   CN_r is the geometric reactive power

$CN_{r(ps)}$ is the geometric reactive power due to voltage and current phase shift

$CN_{r(hi)}$ is the geometric reactive power due to voltage and current cross products

Based on the above definitions, the net or geometric power factor can be defined as

$$pf = \frac{P}{\|M\|} = \frac{\langle M \rangle_0}{\sqrt{\langle M^\dagger M \rangle_0}} \tag{19}$$

4. Power Factor Compensation Using Multivector Apparent Power

Once the effectiveness of the geometric power has been induced to represent the mathematical and physical energy flows, it is time to analyse how it is possible to propose compensation schemes that increase the power factor of the facilities in a microgrid.

To improve the power factor, it is necessary to eliminate any current that is not in phase with the voltage. This strategy implies that the load-compensating combination is seen as a pure resistance by the source. An example of a compensator can be seen in Figure 3, which shows the load admittance Y_{load} as well as the compensator admittance Y_{cp}. The values of these admittances in the geometric domain are

$$Y_{load} = \frac{1}{Z_{load}} = \frac{1}{R + \left(\frac{1}{\omega C} - \omega L\right) e_{12}} = \frac{R}{R^2 + \left(\frac{1}{\omega C} - \omega L\right)^2} +$$

$$+ \frac{\omega L - \frac{1}{\omega C}}{R^2 + \left(\frac{1}{\omega C} - \omega L\right)^2} e_{12} = G_l + B_l e_{12} \tag{20}$$

$$Y_{cp} = \left(\frac{1}{\omega C} - \omega L\right) e_{12} = B_{cp} e_{12}$$
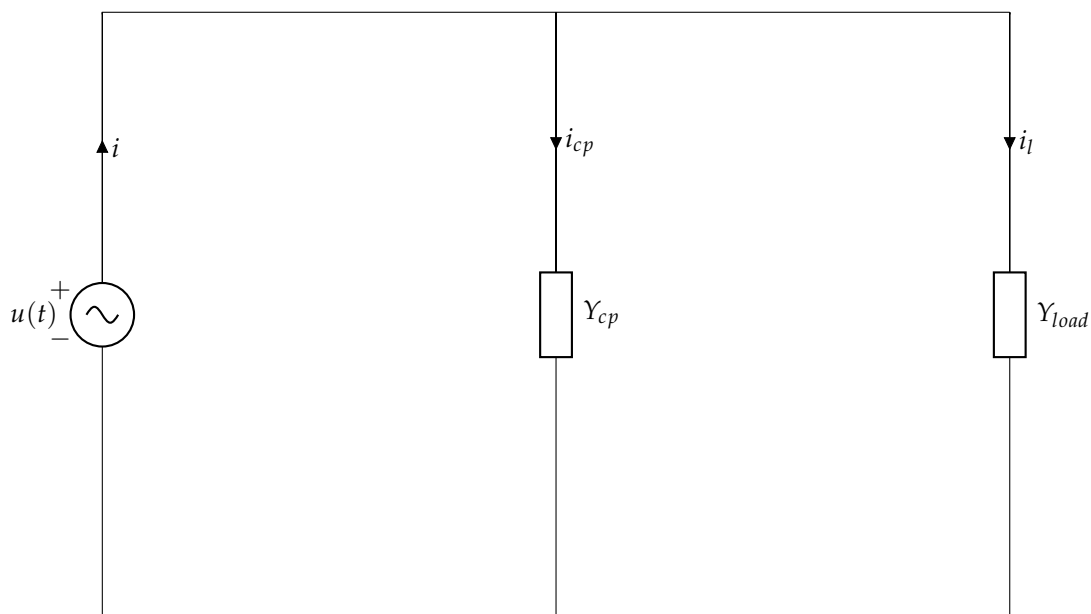

Figure 3. Circuit compensation proposal.

If we apply the voltage given by (13), the current flowing through the compensator will be

$$i_{cp} = -B_{cp1} E_1 e_1 - B_{cp1} D_1 e_2 + \sum_{h=2}^{d} \left[B_{cph} D_h \bigwedge_{i=1, i \neq 2}^{h+1} e_i \right] -$$

$$- \sum_{h=2}^{k} \left[B_{cph} E_h \bigwedge_{i=2}^{h+1} e_i \right] \tag{21}$$

Therefore, it is pretty obvious that $Bcp = -B_{load}$ to fully compensate the reactive term. In this case, the total current i is reduced to i_g as $i_b + i_{cp}$ is equal to 0 after applying Kirchhoff laws.

5. Application to Real Circuits

To demonstrate the robustness of geometric algebra in the resolution of nonsinusoidal electrical circuits and to verify that it is a useful and valid method, the circuit shown in Figure 4, already exposed in [33], will be solved. This theoretical circuit represents a hypothetical electrical circuit in a modern

microgrid building, where the application of a nonsinusoidal voltage to a linear load results in the circulation of a nonsinusoidal current. The power involved has several components, although it will be shown how the traditional approach to power factor improvement is not very successful. Note that the proposal made in this paper also improves the one made by [33], going from a compensated power factor of 0.63 to a higher one of 0.83.

Let the nonsinusoidal voltage, $u(t)$, be

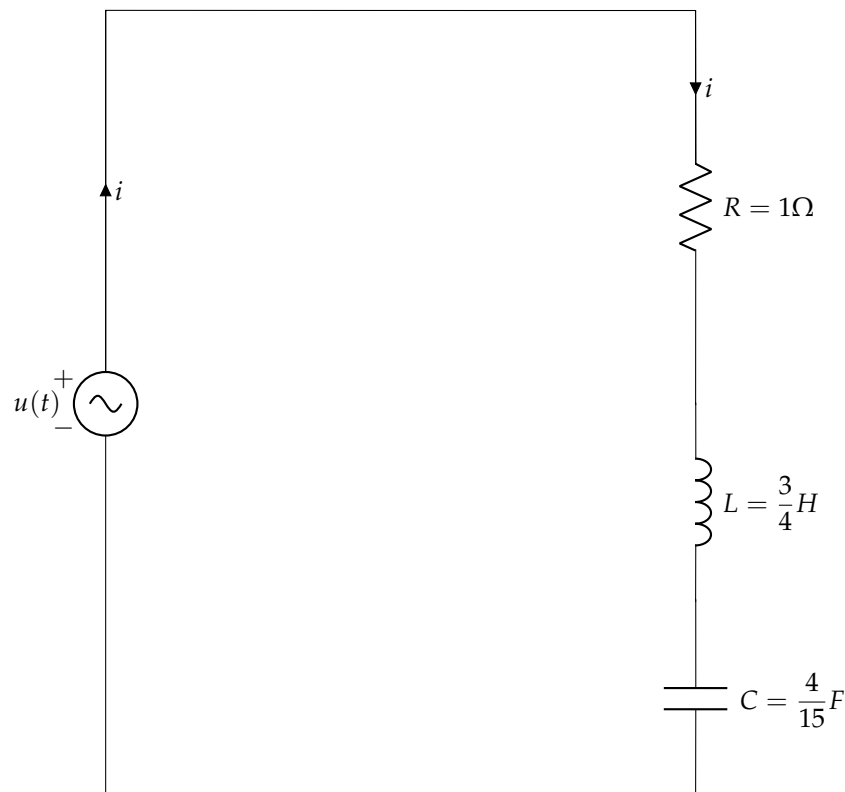$$u(t) = 200\sqrt{2}(\sin \omega t + \sin(3\omega t)) \tag{22}$$

Figure 4. Building equivalent circuit.

The geometric impedance z will have two different values, one for each voltage harmonic. According to the authors of (20), the geometric impedance is defined as

$$z_h = R + \left(\frac{1}{h\omega C} - h\omega L \right) e_{12} \tag{23}$$

where z_h is the value of the impedance for the harmonic of order h. Applying the above expression for each harmonic present, we obtain $z_1 = 1 + 3e_{12}$ y $z_3 = 1 - e_{12}$.

Taking into account the proposed transformation into (6), the voltage becomes

$$u = \underbrace{-200e_2}_{\langle u \rangle_1} + \underbrace{200e_{134}}_{\langle u \rangle_3} \tag{24}$$

Applying the generalized Ohm law

$$
\begin{aligned}
i = z^{-1}u &= z_1^{-1}\langle u\rangle_1 + z_3^{-1}\langle u\rangle_3 \\
&= (1+3e_{12})^{-1}(-200e_2) + (1-e_{12})^{-1}(200e_{134}) \\
&= \underbrace{-20e_2 + 100e_{134}}_{i_g} + \underbrace{60e_1 - 100e_{234}}_{i_b}
\end{aligned}
\tag{25}
$$

The total current value is $\|i\| = 154.91$A. The current obtained has two clearly differentiated components, i_g which is the component in phase with the voltage and i_b which is the component in quadrature. As a matter of fact

$$
i_g \cdot i_b = 0 \tag{26}
$$

which proves that both are orthogonal. The power balance can be obtained by using Equations (8) and (9):

$$
M = ui = 10^3 \left(\underbrace{24 + 16e_{1234}}_{M_g} + \underbrace{32e_{12} + 32e_{34}}_{M_b} \right) \tag{27}
$$

The analysis of the multivector apparent power M results in Table 1. It shows the active power P, degradation power CN_d and reactive power CN_r. This power clearly differs from that obtained by Budeanu or that obtained by the authors of [33]. A comparison of these theories is also shown in Table 2.

Table 1. Power multivector decomposition.

Description	Value
P	24,000 W
$\|CN_d\|$	16,000 VA
$\|CN_r\|$	45,254 VA

Table 2. Power before compensation.

Description	Budeanu	Castilla	Castro-Núñez
Active Power	24,000	24,000	24,000
Reactive Power	8000	8000	45,254
Distortion/Degraded Power	35,780	35,780	16,000
Apparent/Geometric Power	43,820	43,820	53,666
Power factor	0.55	0.55	0.44

It is interesting to note how the multivector power presented here has a very similar correspondence with the power in the time domain. Indeed, if we take into account that the voltage and current are (see Figure 5)

$$
\begin{aligned}
u(t) &= \sqrt{2}\,[200\sin(\omega t) + 200\sin(3\omega t)] \\
i(t) &= \sqrt{2}[60\cos(\omega t) + 20\sin(\omega t) + 100\sin(3\omega t) - \\
&\quad - 100\cos(3\omega t)]
\end{aligned}
\tag{28}
$$

we can make the product and get the time domain power $p(t)$

$$p(t) = u(t) \cdot i(t) = 2\left[200 \sin \omega t + 200 \sin 3\omega t\right] \cdot \left[60 \cos \omega t +\right.$$
$$+ 20 \sin \omega t + 100 \sin 3\omega t - 100 \cos 3\omega t\right] =$$
$$= 2[4000 \sin^2 \omega t + 20000 \sin^2 3\omega t + 20000 \sin \omega t \sin 3\omega t +$$
$$+ 4000 \sin 3\omega t \sin \omega t + 12000 \sin \omega t \cos \omega t -$$
$$- 20000 \sin \omega t \cos 3\omega t + 12000 \sin 3\omega t \cos \omega t$$
$$- 20000 \sin 3\omega t \cos 3\omega t] \tag{29}$$

so we can rearrange as

$$\begin{aligned}
P &= 4000 \sin^2 \omega t + 20000 \sin^2 3\omega \\
CN_d &= 20000 \underbrace{\sin \omega t \sin 3\omega t}_{e_{1234}} + 4000 \underbrace{\sin 3\omega t \sin \omega t}_{-e_{1234}} \\
CN_{r(ps)} &= 12000 \underbrace{\sin \omega t \cos \omega t}_{e_{12}} - 20000 \underbrace{\sin 3\omega t \cos 3\omega t}_{-e_{12}} \\
CN_{r(hi)} &= -20000 \underbrace{\sin \omega t \cos 3\omega t}_{-e_{34}} + 12000 \underbrace{\sin 3\omega t \cos \omega t}_{e_{34}}
\end{aligned} \tag{30}$$

$$\begin{aligned}
P &\rightarrow 24000 \text{ W} \\
CN_d &\rightarrow 16000 \text{ VA} \\
CN_{r(ps)} &\rightarrow 32000 \text{ VA} \\
CN_{r(hi)} &\rightarrow 32000 \text{ VA}
\end{aligned} \tag{31}$$

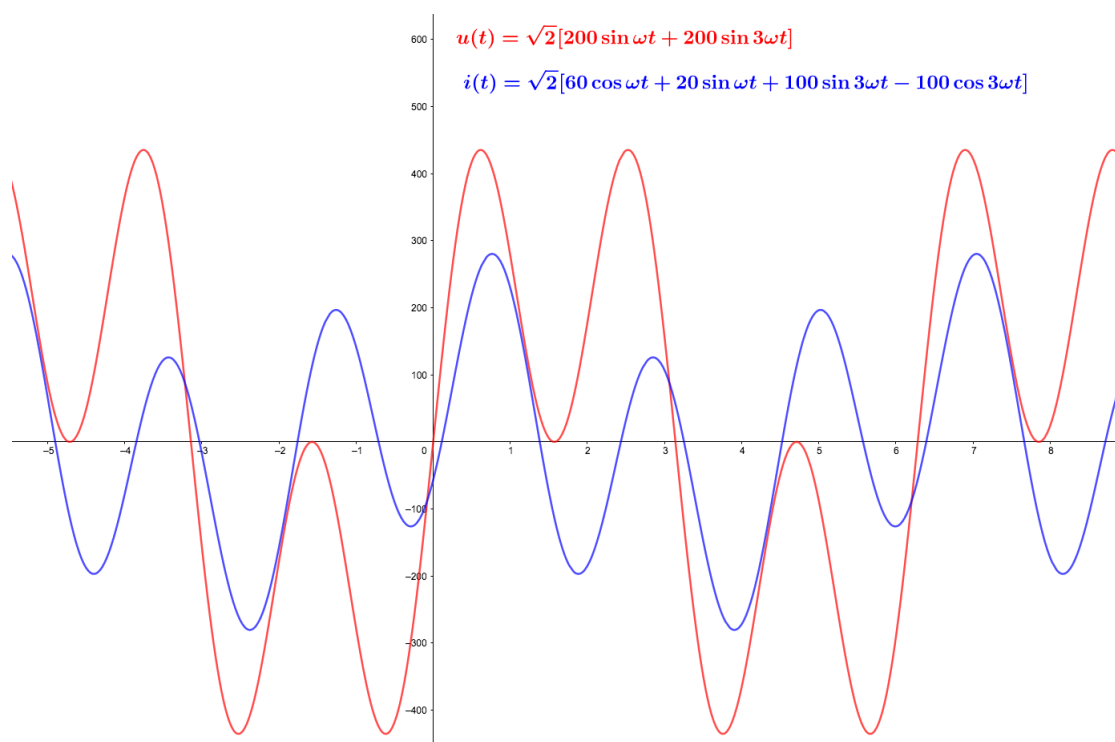achieving the same results as in (27).

Figure 5. Nonsinusoidal voltage and current waveforms.

All the power theories agree on finding the active power, P, but this is not the case for the rest of the other concepts. Both Budeanu and Castilla [33] obtain a lower reactive and apparent power, as well as a higher distortion (degraded) power. The power factor obtained by Budeanu and

Castilla is also higher, giving the impression that the system is not really as degraded as it really is. As Castro-Núñez demonstrates, the other theories fail to consider the interaction between harmonics of different frequencies, and are therefore unable to fully capture the physical sense of energy flows.

This is clearly evident when trying to design a compensator that improves the power factor as much as possible. According to Castilla, this compensator is achieved by installing a capacitor in parallel with the load of a value of $C = 0.12$F, resulting in a new power factor of 0.63. Well, if we apply our theory, we can achieve a much better power factor by simply addressing the need for reactive current i_b, according to Equations (15) and (17). See Figure 6 for the placement of the compensator in parallel with the load.
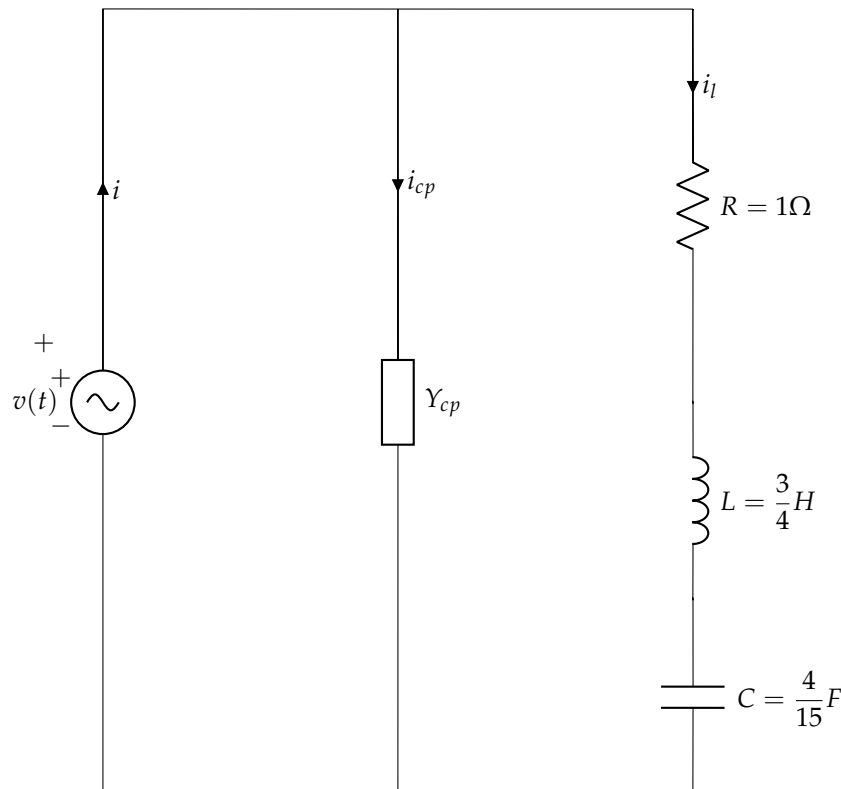

Figure 6. Building equivalent circuit with compensator.

We use Equation (21) to find the proper admittance of the compensator.

$$i_{cp} = -i_b \tag{32}$$

so

$$-200B_{cp1}e_1 - 200B_{cp3}e_{234} = -60e_1 + 100e_{234} \tag{33}$$

Solving the above equation yields $B_{cp1} = 0.3$ and $B_{cp3} = -0.5$. Obviously, it is not possible to effectively compensate by means of a single element, as proposed in [33]. A parallel LC compensator (Other circuits may be compensated with a serial model) is therefore proposed to be installed, which will result in

$$-200 \left(\frac{1 - \omega^2 L_{cp}C_{cp}}{\omega L_{cp}} \right) e_1 = -60e_1 \tag{34}$$

$$-200 \left(\frac{1 - 9\omega^2 L_{cp}C_{cp}}{3\omega L_{cp}} \right) e_{234} = 100e_{234} \tag{35}$$

Solving the previous system yields

$$L_{cp} = \frac{40}{21} = 1.90 \qquad C_{cp} = \frac{9}{40} = 0.22$$

For these compensator values, the new total current value becomes

$$i_{scp} = -20e_2 + 100e_{134} \tag{36}$$

where $\|i_{scp}\| = 101.98$A which is significantly lower than the initial 154.91A. The time domain representation of compensated current is

$$i_{scp}(t) = \sqrt{2}[20\sin\omega t + 100\sin 3\omega t] \tag{37}$$

Comparing (37) with (28) give us an idea about the current reduction thanks to the compensator. If we calculate the reactive power as in (29) and (30), the new result is

$$\begin{aligned}
p(t) = u(t) \cdot i(t) &= 2\,[200\sin\omega t + 200\sin 3\omega t]\cdot \\
&\quad [+20\sin\omega t + 100\sin 3\omega t] = 2[4000\sin^2\omega t + \\
&\quad + 20000\sin^2 3\omega t + 20000\sin\omega t\sin 3\omega t + \\
&\quad + 4000\sin 3\omega t\sin\omega t]
\end{aligned} \tag{38}$$

which can be arranged as

$$\begin{aligned}
P &= 4000\sin^2\omega t + 20000\sin^2 3\omega \\
CN_d &= 20000\underbrace{\sin\omega t\sin 3\omega t}_{e_{1234}} + 4000\underbrace{\sin 3\omega t\sin\omega t}_{-e_{1234}}
\end{aligned} \tag{39}$$

Equations (38) and (39) clearly state that all of the reactive power has been corrected through the new compensator. Furthermore, the principle of conservation of energy has been fulfilled as demonstrated in both in time and geometric domain.

Table 3 shows a summary of the compensation status. It can be seen that all the reactive current coming from the source has been suppressed, resulting in a significant reduction in geometric apparent power (from 53,666 VA to 28,844 VA). Naturally, the power factor increases considerably to 0.83, far exceeding the compensation obtained by the Budeanu or Castilla methods, in which only the placement of a capacitor in parallel with the load is considered.

Table 3. Power after LC compensation.

Description	Budeanu	Castilla	Castro-Núñez
Active Power	24,000	24,000	24,000
Reactive Power	11,000	11,000	0
Distortion/Degraded Power	27,530	27,530	16,000
Apparent/Geometric Power	38,200	38,200	28,844
Power factor	0.63	0.63	0.83

6. Conclusions

This work deepens the new advances in nonsinusoidal power theory thanks to geometric algebra. Due to the large deployment of electronic loads in today's microgrid, it is increasingly common to find a more distorted supply and with high harmonic content. This situation generates noise and harmonic pollution, degrading the power supply of the existing electrical receivers on the microgrid. In this work, a detailed study of new mathematical techniques applied to the analysis of nonsinusoidal cases is carried out, and a compensation method based on the use of geometric algebra is proposed.

Thanks to this technique, it is possible to reduce the geometric reactive power component, something that other traditional methods such as Budeanu or Fryze cannot do. It is also demonstrated that the technique proposed by Castro-Nuñez is far superior to the one proposed by Castilla, as it is able to better identify the power flows due to crossed voltage and current products (CN_d and CN_r), which allowed identifying those components of the current not in phase with the voltage, and thus suppressing them with the appropriate compensator. The main contribution of this work is in the application of geometric algebra to the resolution of power flows in nonsinusoidal electrical systems so that their direction and sense can be correctly determined when considering compensation models. This approach opens up new perspectives in the field of nonsinusoidal systems optimisation, as well as a proper and adequate definition of indices associated with power quality.

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