

Automatic Data Processing System of Constructing an Optimal Mean/Value-at-Risk Portfolio

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Abstract — We propose a computer-based automatic system of share prices processing for constructing an optimal mean/Value-at-Risk portfolio with mixed integer linear programming algorithm based on Benati – Rizzi method. We investigate the impact of so-called Value at risk measure on the size of total capital and shares in the optimal risky portfolio is necessary for revising the classical approach of Markowitz and for adapting it to the modern requirements in the banking and financial sectors. In a classical way it is impossible to construct a portfolio when structural changes in the stock market are happened, or as the same, when a long fall in prices is replaced by a steady growth. Our work is devoted to the study of the construction of the risky portfolio using the Value at risk measure. Within this investigation, two portfolios are constructed according to the classical Markowitz algorithm and the Benati – Rizzi algorithm. The sample alpha and beta-coefficients are estimated, the riskiness and profitability of passive portfolio investments are calculated. The comparison of returns and values of such portfolios of shares included in Moscow index MICEX-10 was carried out.

Keywords — data processing system; Benati – Rizzi algorithm; Value at Risk; Markowitz method

I. INTRODUCTION

The classical Markowitz model allowing for construction and managing of the investment portfolio has been widely used for more than 50 years now [1,2]. Nevertheless, along with obvious pluses of this approach there are also some disadvantages. Firstly, calculation of a portfolio shares does not take into account possible changes in the asset parameters and depends only on its current values. Hence, any boost or fall of the asset price in future will lead to the leap of a portfolio shares when reconstructing. Secondly, it is impossible to construct an investment portfolio during the process of structural changes at the stock market when long period of price fall turns into steady growth. In this case historical returns of the prices are negative, so a few number of increasing quotes we observe at this period does not allow us to construct portfolio with positive shares. To overcome the drawback of working with a portfolio return volatilities we propose to use $VaR_\alpha(X_t)$ [3,4]:

$$VaR_\alpha(X_t) = \inf \{x \in \mathfrak{R}, P(X_t < x) \geq \alpha\}, \quad (1)$$

where X_t is a random variable describing future investment yield, α is given as threshold value and $F(x) = P(X_t < x)$ is distribution function X_t .

If $F(x)$ is a continuous strictly monotone rising function then $F(x)$ has an inverse function and (1) will turn to:

$$VaR_\alpha(X_t) = F^{-1}(\alpha).$$

Parameter $VaR_\alpha(X_t)$, or VaR in short, plays an important role in finance. Its wide implementation and use was due to the foundation of the Basel Committee on Banking Regulation and Supervisory Practices between central banks of ten leasing countries G-10 (Basel I, Basel II, Basel III, see [5] for more details).

Currently there exist a number of methods of $VaR_\alpha(X_t)$ estimation and calculation. Arbitrary they can be divided into three branches: 1) parametric (using different combinations of ARMA(p,q) and GARCH(p,q) [6]); 2) non-parametric (a historical simulation method; [7, eqs. (7), (9)]); 3) semi-parametric (a conditional autoregression model CAViAR [8]).

Despite its wide use as a risk measure, VaR is still not that popular in construction of the “return/risk” investment portfolio. For instance, in [6, eq. (6.9)] to form a VaR optimal portfolio a function dependent on losses and risk aversion is chosen. In [7] authors calculate different combined portfolio risk measures and chose the most accurate one for a particular historical data using two-step optimization. In [9,10] several optimization tasks are set up: 1) portfolio return optimization with limitation on VaR_α level; 2) quintile function minimization with restriction on portfolio return level. In [9] the optimal solution is found on the basis of mixed linear integer programming and in [10] almost optimal solution is found.

Using VaR in portfolio construction is quite complicated task due to: 1) its stochastic nature dependent on the analyzed data distribution function; 2) its incoherency in arbitrary case [3] when total portfolio price VaR is greater than the sum of each portfolio asset VaR . Moreover, the optimized during the portfolio shares function is not concave [9,10], so we do not

reach a unique stable solution. This is the reason why some researchers choose other risk measures for portfolio construction, e.g., conditional value-at-risk $CVaR_\alpha$, or $CVaR$ in short. This one is coherent for any probability distribution law:

$$CVaR_\alpha(X_t) = E\{X_t, X_t < VaR_\alpha(X_t)\}.$$

For instance, in [11] an implicit solution of a portfolio optimization task with $CVaR_\alpha$ restrictions is found within the Black-Litterman model that generalizes the classical Markowitz model. In [12] instead of portfolio volatility used in the classical Markowitz approach authors propose $CVaR_\alpha$ calculated under the condition of normal distribution and reach the implicit analytical solution of this optimization task.

Along with univariate risk metrics multivariate are also used to estimate the shares of a constructed portfolio and get the optimization task solution. Due to the fact that VaR and $CVaR$ depend on random price increment distribution function, authors of [13-15] implement different types of copulas and investigate their impact on shares value. In [13] the GARCH-EVT-copula, ARMA-GARCH-EVT-copula, elliptic and Archimedean copulas are considered (the last two are also used in [14]). Whereas in [15] authors try different empirical copulas.

Still, despite all the apparent disadvantages of the VaR methodology in the context of an optimal risk portfolio task, in the present paper we will use it. Our decision can be explained by the fact that optimization tasks with quintile risk measures are quite complicated. This happens because even if we just simply replace portfolio volatility with VaR , the number of arithmetical operations in the Markowitz algorithm increases exponentially [9]. Furthermore, under the condition of asset returns normal distribution (elliptical in general case), VaR is coherent risk measure [3,6], and the optimization task with VaR becomes the classical Markowitz model [6, p.246]. Finally, to estimate VaR we can use non-parametrical algorithms, for instance, historical modeling method, whereas to estimate $CVaR$ we need to choose distribution function very thoroughly [16].

II. THE MODEL OF DATA PROCESSING SYSTEM

Let's choose K risky assets at the stock market. Suppose x_i is a random variable that describes portfolio return at the moment i , $1 \leq i \leq T$, where T is the moment of the portfolio construction, $F(x)$ is a distribution function of x_i . Let R_j is a random variable characterizing the relative asset return j , $1 \leq j \leq K$, λ_j is its share at the constructed portfolio, r_{ij} is observed return R_j at the moment i , $1 \leq i \leq T$ and $r_{min} = \min_{i,j} \{r_{ij}\}$ is a minimal return level for each assets of the portfolio. Let α be the quintile that fixes VaR according to (1). Finally, r_{VaR} is relative portfolio return set by its manager.

Let's construct a computer-based automatic system of share prices processing with restrictions in VaR and fixed α . The observed portfolio return in this case will be as follows:

$$x_i = \sum_{j=1}^K \lambda_j r_{ij}.$$

We will not make any suggestions about density function of relative asset return. To estimate VaR let's use the historical modeling method [7], so that we could avoid the problem of finding the best distribution function of R_j .

Let us formulate our optimal portfolio with VaR task in the following way (using the Benati-Rizzi algorithm) [9]:

$$\max_{\lambda, x, y} \sum_{i=1}^T p_i x_i; \quad (2)$$

$$x_i = \sum_{j=1}^K \lambda_j r_{ij}, 1 \leq i \leq T; \quad (3)$$

$$x_i \geq r_{min} + (r_{VaR} - r_{min}) y_i, 1 \leq i \leq T; \quad (4)$$

$$\sum_{i=1}^T p_i (1 - y_i) \leq \alpha; \quad (5)$$

$$y_i \in \{0,1\}, 1 \leq i \leq T, \sum_{j=1}^K \lambda_j = 1, \lambda_j \geq 0, \quad (6)$$

where p_i is the probability of x_i in the set of empirical observations and $1 \leq i \leq T$.

Notice that y_i in (6) are binary with only zero or unit value. This is important for the correct estimation of a portfolio risk value in (5): each time when x_i is less than r_{VaR} , we suppose y_i равным нулю. to be zero. Hence, in (4) we summarize only probabilities p_i , for which the observed return x_i is less than VaR . If the sum in (5) is greater than α , our portfolio turns into unexecutable one.

Solution of (2)-(6) is complicated in view of exponential calculation difficulties of the applied integer linear programming [9] because for the integer variables y_i , $1 \leq i \leq T$, there are 2^T possible combinations. That is the reason why in this paper we used package program IBM ILOG CPLEX Optimization Studio 12.8².

In case this package is unavailable integer solution of (2)-(6) can be accomplished with any mathematical algorithm, e.g., the Danzig-Mann method or the Benders method [17].

III. NUMERICAL SIMULATION RESULTS

Let us construct the portfolio P1 that consists of the Russian blue chips included in the MICEX-10 Russian Index. Let us use the Benati-Rizzi method. As the initial data we used quotation of the following companies: PAO «Aeroflot», PAO «Aviakompaniya ALROSA», PAO «Bank VTB», PAO «Gazprom», PAO «GMK Norilskij nikel», PAO «Lukoil», PAO «Magnit», PAO «MosBirzha», PAO «NK Rosneft», PAO

² Installer IBM ILOG CPLEX Optimization Studio V12.8.0 [e-resource]. URL: <http://www-01.ibm.com/support/docview.wss?uid=swg24044295>. Access date: 01.06.2019. Access is free.

«Sberbank». The data ranges from January, 3, 2017 through December, 29, 2017 for 252 observations. All the data were downloaded from Finam.ru. Access is free.

Suppose the *VaR* value is 0.95 and then find the solution of (2)-(6) with the IBM CPLEX. Table 1 reports calculated asset shares in the optimal portfolio P1.

The observed portfolio P1 return at the moment of its construction (January, 29, 2017) was 95%, that exceeds the MICEX-10 return at the same period (it constituted -23.04%).

TABLE I. STRUCTURE OF THE OPTIMAL PORTFOLIO COMPOSED BY BENATI-RIZZI METHOD

Issuer	PAO «Aehroflo»	PAO «Gazprom»	PAO «GMK Noril'skiy nikel»	PAO «Lukoil»	PAO «Magnit»	PAO «Sberbank»
Share	0.15	0.37	0.16	0.11	0.05	0.16

Using the same historical data we calculated asset shares of another portfolio P2 constructed by the Markowitz model [2]. Then we compared the achieved results for both investment portfolios. Volatility level (or acceptable risk level) for P2 was chosen to be 20%.

According to the calculations, the portfolio P2 consisted only of the common shares (share 0.26) and (share 0.74). The observed return of this portfolio at the moment of its construction was 30% per year.

Figure 1 shows prices of P1 and P2 during the first five months of 2018 (period from January, 3 to May, 28).

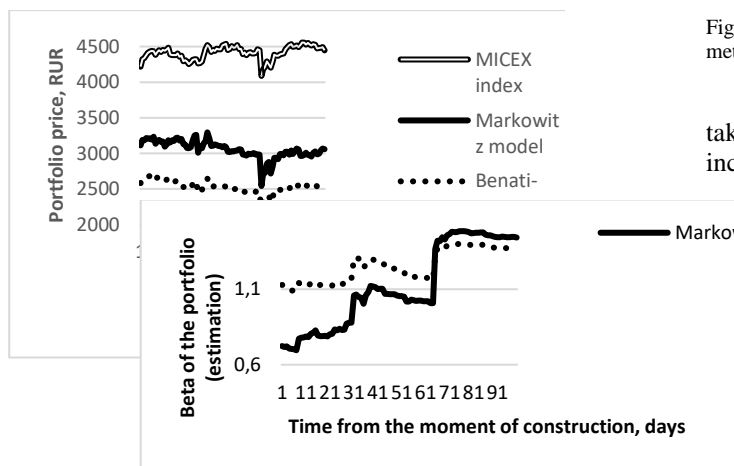


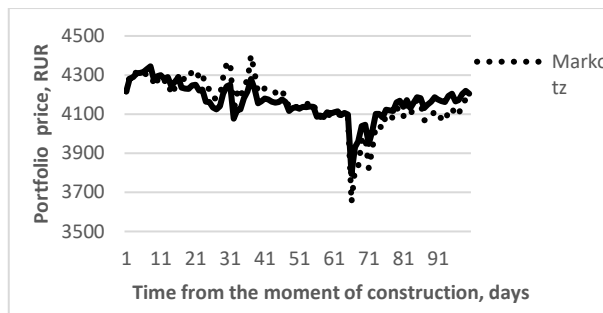
Fig. 1. The price dynamics of the portfolios and MICEX-10 index.

As it follows from fig.1, for the portfolio P1 we need less investments compared to P2. Moreover, as it is known, index investment is more resource-heavy. It demands 1.7 and 1.4 times more investments than the initial ones for P1 and P2 respectively.

Let us normalize the value of the initial capital so that it was of the same value for P1 and P2. This step allows us to compare

portfolio price dynamic in case of a sharp increase/decrease of its underlying asset price (fig.2). It is also possible to detect a portfolio with the lowest investment losses.

According to fig.2, P1 is less sensible to weak fluctuations (up to 4%) at falling market (01-65th trading days) and has lower price compared to P2. At the same time, at the moment of sharp fall (66th trading day) or increase (67-99th trading days) of the market, the price is higher for P1 than for P2. This is due to the



fact that during the gradual fall at the market *VaR* value changes slower than volatility value at the Markowitz model.

Fig. 2. The normalized price dynamics of the portfolios built by different methods.

Notice that, unlike the Markowitz approach, *VaR* doesn't take into account negative historical data, this allows us to increase shares of the risky assets included into our portfolio.

To estimate riskiness of the passive portfolio management during the first five months of 2018 we calculated sample beta-coefficient:

$$\bar{\beta} = \frac{\text{cov}(R_{\pi}, R)}{\sigma(R)},$$

where R_{π} is relative portfolio return (P1 and P2); R is relative MICEX-10 return and $\sigma(R)$ is an index sample standard deviation.

Figure 3 illustrates $\bar{\beta}$ dynamic for P1 and P2.

Fig. 3. Dynamics of statistical estimates of beta coefficients for the first five months of 2018.

It is well known that $\bar{\beta} = 1$ indicates market risk level [1], i.e. that risk of the constructed investment portfolio is the same as the market index portfolio. Coefficient value $\bar{\beta} > 1$ means that portfolio investment risk is higher than average market risk, whereas $\bar{\beta} < 1$ indicates that it is lower. Considering fig. 3 data, we can conclude that from the moment of construction and up to the 65th trading day P1 is more risky than P2 and MICEX-10 index portfolio. Nevertheless, from 66th to 99th trading days (sharp fall and further market correction) P2 showed higher risk level compared to P1. In general, P1 revealed to be more risky investment portfolio than P2 and MICEX index portfolios.

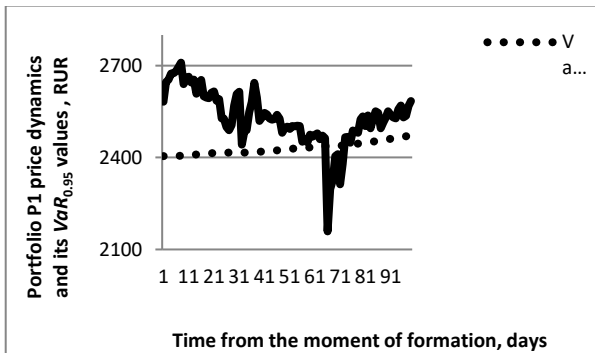


Fig. 4. Portfolio P1 price dynamics and its $VaR_{0.95}$ values for the first five months of 2018.

Figure 4 indicates dynamics of P1 price and $VaR_{0.95}$, (calculated with the historical modeling method) during the first five months of 2018. Numerical calculations allowed to detect seven $VaR_{0.95}$, level strikes that constitutes 7.1% of total number of trading days during the considered period (18.1% if we turn to annual basis with 252 trading days). This risk level is consistent with volatility $\sigma = 20\%$ in the Markowitz model. Therefore, P1 provides higher income level compared to P2 under the same risk profile.

Observed sample alpha-coefficients showed that their values were changing during the first five months of 2018 within the range from -10^{-3} to 2×10^{-3} . This, according to [1], corresponds to the market index portfolio return level.

Realized return of the MICEX index within the period from January, 3 to May, 28, 2018 was 13.88%, for P2 -4.10% and -1.21% for P1.

IV. CONCLUSION

We proposed a computer-based automatic system (2)-(6) of share prices processing for constructing an optimal mean/Value-at-Risk portfolio with mixed integer programming algorithm. This system allows the constructing of investment portfolio taking into account marginal risk level. Usage of this approach demands lower initial investments,

weakens influence of critical market drop on portfolio price and increases realized investment return compared to the classical Markowitz model. As well, the Benati-Rizzi method is suitable for construction of wide range of investment portfolios managed by unskilled investors with different risk aversion.

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