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Wholesale Price Discrimination with Regulatory Asymmetry

Andreas Asseyer

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Andreas Asseyer†

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Abstract

This paper studies the welfare effects of wholesale price discrimination between downstream firms operating under different regulatory systems. I model a monopolistic intermediate good market in which production cost differences between downstream firms may be due to regulatory or technological asymmetries. Price discrimination reduces regulatory distortions but may lower productive efficiency. Therefore, price discrimination increases welfare if regulation is the dominant source of cost differences. This provides a novel welfare rationale for exempting wholesale markets from the recent ban on geo-blocking in the EU.

Keywords: Price discrimination, Intermediate good markets, International price discrimination, Geo-blocking

JEL Classification: D43, L11, L42

1 Introduction

The recent EU regulation 2018/302 addresses the practice of geo-blocking, a form of geographic price discrimination whereby a customer is denied access to an offer in a webshop based on her location. The regulation bans geo-blocking on most final good markets.[1] Its provisions apply more broadly than the previously existing rules on international price discrimination following from Articles 101 and 102 of the Treaty on the Functioning of the European Union (Vesala, 2019). Interestingly, wholesale markets are exempt from the regulation in so far as the traded good is intended for "subsequent resale, transformation, processing, renting or subcontracting".[2]

This paper provides a novel welfare rationale for the exemption of wholesale markets from the ban on geo-blocking. The rationale builds on the fact that firms face different regulations in different EU member states. As regulations are often costly to firms, regulatory differences

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†Freie Universität Berlin, School of Business and Economics, Berlin (Germany), andreas.asseyer@fu-berlin.de

[1]The markets for audiovisual content and retail financial services are excluded according to Recital 9 of Regulation 2018/302.

[2]Recital 16 of Regulation 2018/302.

between states influence international production cost differences. At the same time, many regulations provide benefits to other stakeholders. The regulation of collective bargaining influences wages and employee safety which are costly for firms and beneficial for employees. Taxes, levies, and surcharges are direct costs to firms and finance public expenditures. Environmental regulations lead to abatement costs and bring health benefits to the general population.

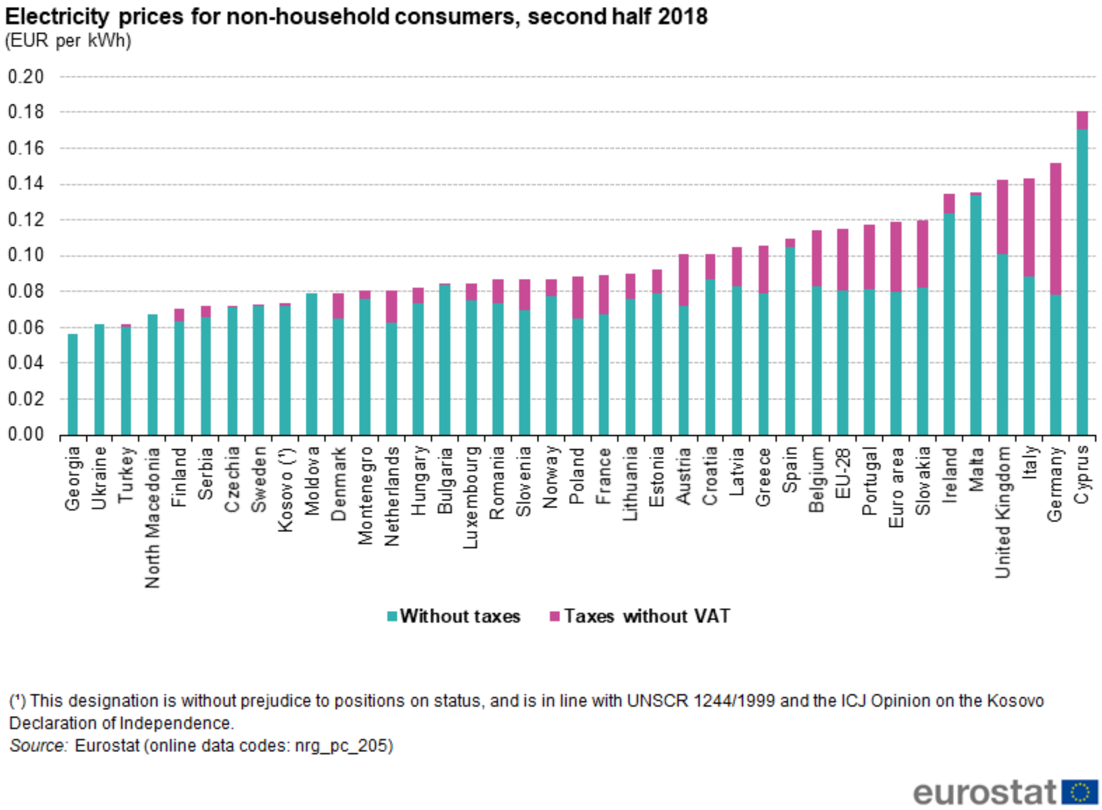
To study the welfare effects of wholesale price discrimination with regulatory differences, I model a monopolistic intermediate good market in which downstream firms' cost differences may be due to asymmetries in production technology, regulation, or both. I compare total welfare under discriminatory and uniform pricing by the upstream firm. The source of cost differences may influence the welfare effects of price discrimination as technologies alone determine the social production costs.

As the main result of this paper, I show that wholesale price discrimination often increases welfare if regulation is the dominant source of cost differences between downstream firms. This result holds even in environments where – in line with the existing literature – wholesale price discrimination reduces welfare if cost differences arise solely from technological asymmetry. Thus, the source of cost differences influences the welfare implications of wholesale price discrimination. Moreover, the exemption of wholesale markets from the ban on geo-blocking can be justified under the premise that regulation is an important source of cost variation within the EU.

The following example illustrates the effect of national regulations on the social and private production costs of firms. As Table 1 shows, electricity prices for non-household consumers vary considerably across EU member states, ranging from €0.07 per kWh in Finland to €0.18 per kWh in Cyprus. A considerable extent of this variation is due to national electricity consumption taxes. A Polish firm which requires electricity to transform some intermediate good into a final good pays €0.09 per kWh. Its German counterpart pays €0.15 per kWh. The price in Germany includes €0.07 of tax payments per kWh whereas the price in Poland includes €0.015 of taxes. In particular, German firms pay a *renewables surcharge* which finances the subsidies paid to producers of electricity from renewable energy sources.[3] All else equal, the German firm has higher private marginal costs of production than the Polish firm due to the renewable surcharge. However, the difference in social marginal costs of production is substantially lower than the

[3]In 2018, the surcharge was set at €0.0679/kWh (Bundesnetzagentur, 2017).

Table 1: Electricity prices for non-household consumers, second half 2018. Source: Eurostat



difference in private marginal costs as the renewables surcharge is a transfer to producers of low-carbon electricity.

The extant literature on the welfare effects of wholesale price discrimination assumes that the social and private costs of production coincide (Katz, 1987; DeGraba, 1990; Yoshida, 2000; Inderst and Shaffer, 2009; Herweg and Müller, 2014). This assumption is satisfied in the case of *pure technological asymmetry* where the cost differences between downstream firms are entirely due to efficiency differences in production technologies. The assumption is not satisfied if downstream firms use the same production technology but experience different costs due to exposure to different regulations. I refer to this case as *pure regulatory asymmetry*.

I add to the extant literature by analyzing the welfare effects of price discrimination if downstream firms may differ in their regulatory environment as well as their production technology. Following the literature, I sequentially analyze the cases of linear wholesale tariffs (DeGraba, 1990; Yoshida, 2000) and nonlinear wholesale tariffs. With nonlinear tariffs, I allow for complete information on production costs (Inderst and Shaffer, 2009) and private cost information of downstream firms (Herweg and Müller, 2014).

At first, I show that the source of cost differences affects the welfare implications of price discrimination if wholesale tariffs are linear. In line with the results in the literature, price discrimination reduces social welfare with pure technological asymmetry. By contrast, price discrimination increases social welfare with pure regulatory asymmetry. Independently of the source of cost differences, the upstream firm optimally offers a downstream firm with higher private costs a lower wholesale price. Under uniform pricing, all firms receive the same intermediate wholesale price. With pure technological asymmetry, price discrimination therefore shifts production from the downstream firm with lower social marginal costs to the downstream firm with higher social marginal costs. This shift tends to decrease total welfare. In the case of pure regulatory asymmetry, both downstream firms have the same production technology but one downstream firm has higher private marginal costs due to regulation. Price discrimination alleviates the regulatory burden on the high cost firm and reduces the distortion induced by the regulation. This effect tends to increase total welfare.

Second, I allow for nonlinear wholesale tariffs and show that the upstream firm's information about downstream firms' marginal costs determines whether the source of cost differences matters for the welfare effects of price discrimination. If the upstream firm knows the marginal costs, the source of cost differences does typically not matter for the welfare effects of price discrimination. In this case, the upstream firm can extract all industry profits with discriminatory two-part tariffs. With uniform pricing, Inderst and Shaffer (2009) show that the downstream firms earn positive margins. Thus, the quantities on the final good markets and total welfare are smaller with uniform pricing – independently of the source of cost differences. By contrast, if the upstream firm does not observe the downstream firms' marginal costs, a similar result to the case of linear wholesale tariffs arises. Price discrimination favors the downstream firm with the worse cost structure. If the cost disadvantage stems from technology, price discrimination induces more production by the less efficient firm and tends to reduce welfare. If the cost disadvantage is due to regulatory differences, price discrimination reduces the regulatory distortion and tends to increase welfare.

Related literature The literature on price discrimination in intermediate good markets starts with Katz (1987).[4] He shows that price discrimination can be detrimental for welfare if larger downstream firms have the possibility to engage in inefficient backward integration by producing an input instead of buying it.[5]

This paper belongs to the part of the literature that does not assume an option of backward integration. Downstream firms only differ with respect to their production costs.[6] All extant articles in this literature posit that social and private production costs coincide. DeGraba (1990) and Yoshida (2000) consider the case of linear wholesale tariffs and demonstrate that price discrimination reduces total welfare if demand is linear. O'Brien and Shaffer (1994), Inderst and Shaffer (2009), and Herweg and Müller (2014) analyze the case where the upstream firm sets non-linear tariffs. O'Brien and Shaffer (1994) and Inderst and Shaffer (2009) assume that the upstream firm is perfectly informed about the downstream firms' production costs. In O'Brien and Shaffer (1994), the upstream firm may secretly renegotiate the tariff with downstream firms and this renegotiation leads to lower prices with price discrimination. In Inderst and Shaffer (2009), contracts are public and a ban on price discrimination induces the upstream firm to increase the marginal wholesale price. This reduces welfare compared to price discrimination. Herweg and Müller (2014) consider the case where the downstream firms have private information about their production costs. They show that price discrimination lowers welfare if downstream markets are covered.

Regulatory asymmetry provides a welfare rationale for wholesale price discrimination which does not depend on the form of wholesale contracts or the upstream firm's information about production costs. The negative welfare results of DeGraba (1990), Yoshida (2000), and Herweg and Müller (2014) rely on the dominance of technological asymmetry. The positive welfare result of Inderst and Shaffer (2009) is robust to the introduction of regulatory asymmetry.

In the following section, I introduce the model. In Section 3, I analyze the model with linear wholesale tariffs. Section 4 presents the analysis for the case of nonlinear tariffs with either complete or private information about marginal costs. Section 5 extends the analysis to

[4] Robinson (1933) provides the first formal analysis of the welfare effects of price discrimination in final good markets. Aguirre, Cowan and Vickers (2010) and Cowan (2012) generalize her insights.

[5] Inderst and Valletti (2009) analyze the effect of price discrimination on long-run investment incentives in the model of Katz (1987). O'Brien (2014) extends Katz (1987) to allow for more general sources of bargaining power.

[6] Arya and Mittendorf (2010) study wholesale price discrimination if some downstream firms reach more final good markets. Miklós-Thal and Shaffer (2019) study price discrimination across downstream markets.

a setting with downstream competition. Section 6 provides a discussion and I conclude with Section 7.

2 The model

An upstream firm U produces an intermediate good at zero marginal cost for two downstream firms D_1 and D_2 which serve separate final good markets.[7] I refer to the final good market served by D_i as market i for $i \in \{1, 2\}$. Both downstream markets have the same thrice differentiable inverse demand function $P(q)$ with $P'(q) < 0$ for $q > 0$.[8] The revenue on market i is denoted by $R(q) \equiv qP(q)$. Each downstream firm D_i with $i \in \{1, 2\}$ can transform q_i units of the intermediate good into q_i units of the final good at a private marginal cost of x_i. The social marginal cost of this transformation is denoted by c_i and may differ from the private marginal cost. Thus, social welfare on market i for the quantity q_i and social marginal cost c_i is given by $w_i(c_i, q_i) \equiv \int_0^{q_i} (P(q) - c_i)dq$. The socially optimal quantities satisfy $q_i^o = q^o(c_i)$ for $P(q^o(c_i)) = c_i$ and $i \in \{1, 2\}$. I impose the following assumption on the inverse demand function.

Assumption 1. *The inverse demand function $P(q)$ satisfies*

a) $P'(q) + qP''(q) \leq 0$,

b) $2P'(q) + 4qP''(q) + q^2P'''(q) \leq 0$,

c) $-\frac{R'''(q)}{R''(q)} = -\frac{3P''(q)+qP'''(q)}{2P'(q)+qP''(q)} \leq -\frac{P'(q)}{P(q)}$.

By Part a) of Assumption 1, the revenue function $R(q) \equiv qP(q)$ is strictly concave and the monopoly problem on each market is well-behaved. This assumption is used throughout this paper. Part b) is equivalent to $qR''(q)$ being decreasing. Parts a) and b) jointly imply that the expression $qR'(q)$ is strictly concave. This assumption ensures that the upstream firm's choice of a linear wholesale price in Section 3 is well-behaved. Part c) of Assumption 1 is needed for the welfare analysis with private information about marginal costs in Section 4 and requires that marginal revenue $R'(q)$ is less concave in q than social welfare $w(c_i, q)$.[9]

[7] The assumption of separate markets seems a natural starting point for the analysis of international price discrimination. I consider the case of competition in Section 5.

[8] Villas-Boas (2009) shows that wholesale price discrimination tends to increase welfare with asymmetric demand. Thus, the assumption of identical markets is the strongest test of the effect of regulatory asymmetry.

[9] Given a), $R'''(q) \leq 0$ is a sufficient condition for b) and c). Assumption 1 allows for some convexity of

A microfoundation for the difference between social and private costs

I provide a microfoundation for the difference between social and private marginal costs. Many regulations are equivalent to taxing an input into the downstream firms' production process. In the example of the renewable surcharge discussed in the introduction, electricity usage is taxed. Similarly, regulations regarding the rights of labor unions affect wages and therefore tax labor.

The microfoundation follows the example of the tax on electricity consumption. Suppose each downstream firm D_i with $i \in \{1, 2\}$ requires capital and electricity to transform the intermediate good into the final good. The production function is Cobb-Douglas, i.e., to transform one unit of the intermediate good into one unit of the final good, D_i requires capital K_i and electricity E_i such that $f(K_i, E_i) = K_i^\alpha E_i^{1-\alpha} = 1$ with $\alpha \in (0, 1)$. Let R_i denote the price of capital and W_i the price of electricity. For each unit of electricity consumption, a tax of T_i has to be paid. Moreover, each unit of electricity consumption leads to a negative externality of $N_i > 0$. The externality consists of negative effects on the health of the general population, environmental damage, and the costs of climate change for future generations. D_i's cost minimization problem is given by

$$\min_{K_i, E_i} R_i K_i + (W_i + T_i) E_i \quad \text{subject to} \quad K_i^\alpha E_i^{1-\alpha} = 1.$$

The solution to this problem is $K_i^* = \left(\frac{\alpha}{1-\alpha} \frac{W_i+T_i}{R_i}\right)^{1-\alpha}$ and $E_i^* = \left(\frac{1-\alpha}{\alpha} \frac{R_i}{W_i+T_i}\right)^{\alpha}$. Thus, the private marginal cost of downstream firm D_i is

$$x_i = R_i K_i^* + (W_i + T_i) E_i^* = \left(\frac{R_i}{\alpha}\right)^\alpha \left(\frac{W_i + T_i}{1-\alpha}\right)^{1-\alpha}.$$

Under the assumption that the prices R_i and W_i equal the opportunity costs, the social marginal cost of production is given by

$$c_i = R_i K_i^* + (W_i + N_i) E_i^* = \left(\frac{R_i}{\alpha}\right)^\alpha \left(\frac{W_i + T_i}{1-\alpha}\right)^{1-\alpha} \left(\alpha + (1-\alpha)\frac{W_i + N_i}{W_i + T_i}\right)$$

marginal revenue.

We can therefore write $x_i = a_i c_i$ where

$$a_i = \left(\alpha + (1-\alpha) \frac{W_i + N_i}{W_i + T_i} \right)^{-1}. \tag{1}$$

There is a wedge between social and private marginal costs if the tax on electricity is not equal to the Pigouvian tax, i.e., $T_i \neq N_i$. There are a number of reasons why regulators in country i might not perfectly internalize the negative externality of electricity consumption. If the total negative externality N_i can be split up in a negative externality on country i denoted by N_i^i and a negative externality on the other country j given by N_i^j, the regulator might set the individually optimal tax rate $T_i = N_i^i < N_i$. The regulator in country i might set an even lower tax rate $T_i < N_i^i$ if the voters of the governing political party would be strongly affected by a higher electricity tax or have a lower valuation for an intact environment.

The wedges between private and social costs are asymmetric if the ratios $\frac{W_i+N_i}{W_i+T_i}$ differ. For two countries with $x_1 < x_2$, $N_1 \simeq N_2$, $W_1 \simeq W_2$, and $T_1 < T_2$, we have $a_1 < a_2$. The difference between social marginal costs is smaller than the difference in private marginal costs.

3 Linear wholesale tariffs

In this section, I suppose that the upstream firm is restricted to set linear tariffs consisting of wholesale prices w_i with $i \in \{1, 2\}$. The case of linear tariffs is considered in DeGraba (1990) and Yoshida (2000) with downstream competition. In the context of international price discrimination, independent downstream markets are a natural starting point.[10]

First, I define *pure regulatory asymmetry* and *pure technological asymmetry* as two polar cases of interest. Let $\Delta x \equiv x_2 - x_1$ and $\Delta c \equiv c_2 - c_1$ denote the differences in private and social marginal costs. Without loss of generality, let $\Delta x > 0$, i.e., D_1 is the stronger downstream firm. If $\Delta x = \Delta c$, we are in the case of *pure technological asymmetry.* Here, the difference in private marginal costs across downstream firms fully reflects the difference in social marginal costs. By contrast, if $\Delta c = 0$, we are in the case of *pure regulatory asymmetry* which represents a situation where differences in private marginal costs are not at all driven by differences in social marginal costs.

[10]The case of competition is analyzed in Section 5.

Next, I analyze the equilibria with linear tariffs under price discrimination and uniform pricing. I show that the stronger downstream firm sells a higher quantity and faces a higher wholesale price under price discrimination. Put differently, the weaker downstream firm's cost disadvantage is partially offset under price discrimination. With uniform pricing, both downstream firms face an intermediate price. The stronger downstream firm produces more and the weaker downstream firm produces less under uniform than under discriminatory pricing.

If U sets a wholesale price w_i, D_i optimally orders the quantity

$$q_i^*(w_i) \equiv \arg\max_q R(q) - (w_i + x_i)q$$

which satisfies $R'(q_i^*(w_i)) = w_i + x_i$ if the solution is interior. Under price discrimination, U's optimal wholesale price on market i is the solution to $\max_{w_i} w_i q_i^*(w_i)$. U's choice of a wholesale price w_i is equivalent to the direct choice of the quantity q_i – provided the wholesale price satisfies $w_i = R'(q_i) - x_i$. Thus, U's profit maximization problem on market i can be written as $\max_{q_i} q_i(R'(q_i) - x_i)$. This problem is well-behaved by Parts a) and b) of Assumption 1. The optimal quantities q_1^d and q_2^d solve the first-order condition

$$R'(q_i^d) - x_i + q_i^d R''(q_i^d) = 0$$

for $i \in \{1, 2\}$. As $qR'(q)$ is strictly concave by Assumption 1, $\Delta x > 0$ implies $q_1^d > q_2^d$. The optimal wholesale prices w_1^d and w_2^d satisfy

$$w_1^d = R'(q_1^d) - x_1 = -q_1^d R''(q_1^d) \geq -q_2^d R''(q_2^d) = R'(q_2^d) - x_2 = w_2^d$$

where the inequality follows from $q_1^d > q_2^d$ and part b) of Assumption 1. Thus, U optimally sets a higher wholesale price for the stronger downstream firm. However, the price difference does not offset D_1's cost advantage.

If U has to charge uniform wholesale prices, the profit maximization problem becomes $\max_w w(q_1^*(w) + q_2^*(w))$. U's choice of a uniform wholesale price is equivalent to choosing the quantities q_1 and q_2 directly for the wholesale price $w = R'(q_i) - x_i$ for $i \in \{1, 2\}$. The second interpretation gives rise to the program

$$\max_{q_1,q_2} \quad (R'(q_1)-x_1)q_1+(R'(q_2)-x_2)q_2 \quad \text{subject to} \quad R'(q_1)-x_1=R'(q_2)-x_2.$$

Using a Lagrangian approach with multiplier λ, the optimal quantities under uniform pricing q_1^u and q_2^u are given by

$$R'(q_1^u)-x_1+(q_1^u-\lambda)R''(q_1^u)=0 \quad \text{and} \quad R'(q_2^u)-x_2+(q_2^u+\lambda)R''(q_2^u)=0.$$

As $\lambda>0$, we find that $q_1^u>q_1^d$ and $q_2^d<q_2^u$. Thus, the optimal uniform wholesale tariff satisfies $w^u\in[w_2^d,w_1^d]$. It follows that price discrimination lowers the sales of the stronger downstream firm D_1 and increases the sales of the weaker downstream firm D_2.

Next, I compare total welfare under price discrimination W^d with the total welfare under uniform pricing W^u for varying sources of cost differences.

Proposition 1. *Suppose wholesale tariffs are linear and both downstream firms supply positive quantities. If the total output under price discrimination and uniform pricing is sufficiently similar, price discrimination increases welfare with pure regulatory asymmetry and decreases welfare with pure technological asymmetry. More generally, there exist* $\varepsilon>0$ *and* $\hat{a}\in(0,1)$ *such that* $|q_1^d+q_2^d-q_1^u-q_2^u|<\varepsilon\Rightarrow W^d\geq W^u\Leftrightarrow\frac{\Delta c}{\Delta x}\leq\hat{a}$.

With linear wholesale tariffs, the source of the private cost difference between downstream firms determines the welfare effect of price discrimination if the total quantity is similar under price discrimination and uniform pricing. Suppose the total quantity is the same under both regimes, i.e., $q_1^u+q_2^u=q_1^d+q_2^d=Q$. As $q_1^u>q_1^d>q_2^d>q_2^u$, a switch from uniform pricing to price discrimination reallocates output from the low price market 1 to the high price market 2. The marginal welfare effect of such a shift in output is the difference in prices net of social marginal costs $P(q_2)-P(q_1)-\Delta c$. The reallocation of one unit of the good from the low valuation marginal consumer on market 1 to the high valuation marginal consumer is beneficial. The benefit is measured by the price difference $P(q_2)-P(q_1)$. However, the reallocation leads to additional social costs of Δc. With pure regulatory asymmetry, redistributing output is cost-free and reduces the distortion induced by the regulation. Thus, price discrimination increases welfare in this case. With pure technological asymmetry, the marginal welfare effect of redistributing output from market 1 to market 2 is negative. The benefits of shifting output from the low to the high price market are smaller than the costs of redistribution. In particular, for all $q_1 \in [q_1^d, q_1^u]$ and $q_2 = Q - q_1$

$$P(q_2) - P(q_1) \leq P(q_2^u) - P(q_1^u) = \Delta x - q_2^u P'(q_2^u) + q_1^u P'(q_1^u) \leq \Delta x = \Delta c$$

where the first equality follows from the uniform pricing condition $R'(q_2^u) - R'(q_1^u) = \Delta x$ and the last inequality follows from $q_1^u > q_2^u$ and $qP'(q)$ being decreasing due to Part a) of Assumption 1. Thus, price discrimination lowers productive efficiency under pure technological asymmetry.

Proposition 1 supposes that the total quantity is similar with price discrimination and uniform pricing. For which demand functions is this a valid assumption?

Remark 1. *Suppose wholesale tariffs are linear and both downstream firms supply positive quantities. If demand is linear, the total quantity is identical under price discrimination and uniform pricing .*

The instance of linear demand is of particular interest as DeGraba (1990) and Yoshida (2000) focus on this case.

4 Nonlinear wholesale tariffs

In this section, I allow the upstream firm to offer arbitrary nonlinear tariffs. I consider two different informational setups. First, I suppose that the upstream firm perfectly observes the downstream firms' marginal costs. This case is considered in Inderst and Shaffer (2009) under the assumption of pure technological asymmetry. Second, I assume that the downstream firms have private information about their marginal costs. This case is studied in Herweg and Müller (2014) under pure technological asymmetry.

Complete cost information

At first, assume the upstream firm observes the downstream firms' costs. Under price discrimination, U cannot do better than to offer each downstream firm D_i a two-part tariff consisting of a fixed fee f_i and a per-unit price w_i. With uniform pricing, U has to offer both downstream firms the same two-part tariff (f, w).

The difference between private and social marginal costs does not influence the equilibrium analysis of Inderst and Shaffer (2009). Given a per-unit price w_i, D_i orders the quantity $q_i^*(w_i)$. With price discrimination, U optimally sets $w_i^d = 0$ and $f_i^d = \max_q R(q) - x_i q$. With these tariffs, U achieves the profit it could attain by producing the final good itself at the same marginal costs as the downstream firms. The resulting optimal quantities equal the monopoly quantities $q_i^m \equiv q_i^*(0)$ for $i \in \{1, 2\}$ on the final good markets. With uniform pricing, Proposition 3 in Inderst and Shaffer (2009) states that U optimally sets a strictly positive per-unit price w^u. Thus, the quantities on both markets are strictly smaller under uniform pricing as $q_i^*(w^u) < q_i^m$ for $w^u > 0$ and $i \in \{1, 2\}$. The reason behind this result is as follows. With uniform pricing, the optimal fixed fee f is bounded by the weaker downstream firm's profit for a given per-unit price w, i.e., $f = \max_q R(q) - (x_2 + w)q$. If U slightly increases the per-unit price w from zero to a positive value, U experiences a second-order loss in profit from the weaker downstream firm D_2 and a first-order gain in profit from the stronger downstream firm D_1.

The results of Inderst and Shaffer (2009) are robust to varying sources of cost differences if higher output increases social welfare. This is the case if there is underproduction with price discrimination and uniform pricing on both markets, i.e., $q_i^u < q_i^d = q_i^m \leq q_i^o$ for $i \in \{1, 2\}$.

Proposition 2. *Suppose wholesale tariffs may be nonlinear, marginal costs are publicly known, both downstream firms supply positive quantities, and there is underproduction with price discrimination and uniform pricing on both markets. Price discrimination increases welfare. In particular, price discrimination increases welfare with pure regulatory and pure technological asymmetry.*

With public information on costs, the source of cost differences is not relevant for the welfare effect of price discrimination. As Inderst and Shaffer (2009) show, price discrimination leads to higher quantities on both downstream markets. If price discrimination leads to underproduction from a social perspective, i.e., $q_i^m \leq q_i^o$, higher quantities lead to higher total welfare independently of the source of cost differences.

As the next section demonstrates, the source of cost difference is only irrelevant with nonlinear tariffs if marginal costs are public information. With private information, the difference between private and social marginal costs changes the welfare effect of price discrimination.

Private cost information

Suppose now that the upstream firm cannot observe marginal costs. Each downstream firm D_i with $i \in \{1,2\}$ is privately informed about its private marginal cost $x_i \in [\underline{x}_i, \overline{x}_i]$ drawn from the distribution $G_i(x_i)$. Let the densities $g_i(x_i) = G'(x_i)$ be strictly positive on $(\underline{x}_i, \overline{x}_i)$. For a private marginal cost x_i, the social marginal cost of D_i is given by $c_i = s_i(x_i)$ where $s_i(x)$ is a strictly increasing function with the image $[\underline{c}, \overline{c}] \subseteq \mathbb{R}_+$. Thus, the inverse function $\sigma_i(c_i) \equiv s_i^{-1}(c_i)$ exists and gives for each value of social marginal cost the associated private marginal cost. It follows that the social marginal cost c_i of D_i is distributed according to the distribution function $F_i(c_i) \equiv G_i(\sigma_i(c_i))$.[11]

Next, I define *pure technological asymmetry* and *pure regulatory asymmetry* in this context. For $\sigma_1(c) = \sigma_2(c)$, the cost difference between D_1 and D_2 is entirely due to technology. This case of *pure technological asymmetry* between downstream firms is studied in Herweg and Müller (2014). With *pure regulatory asymmetry*, the downstream firms use identical production technologies which implies $F_1(c) = F_2(c)$ for all $c \in [\underline{c}, \overline{c}]$. In this case, the cost difference between D_1 and D_2 can entirely be attributed to regulatory differences. If the downstream firms in the microfoundation of Section 2 have private information about their opportunity cost of capital, then we have $\sigma_i(c) = a_i c$ where a_i is defined in equation (1).

I make the following assumption regarding G_1 and G_2.

Assumption 2. *The distribution functions $\{G_i(x)\}_{i=1,2}$ satisfy the following conditions:*

1. $\Gamma_i(x) \equiv x + \frac{G_i(x)}{g_i(x)}$ *is strictly increasing,*

2. $\overline{\Gamma}(x) \equiv x + \frac{G_1(x)+G_2(x)}{g_1(x)+g_2(x)}$ *is strictly increasing,*

3. $\gamma(x) \equiv \frac{g_2(x)}{g_1(x)}$ *is weakly increasing.*

Parts 1 and 2 of the assumption ensure that the optimal tariffs under price discrimination (Part 1) and uniform pricing (Part 2) do not lead to bunching of different cost types. These conditions are equivalent to the standard *regularity* assumption in the literature on non-linear pricing. The third part of the assumption states that the cost distribution of D_2 is less favorable than the cost distribution of D_1 in the sense of the monotone likelihood ratio property. Thus, D_2 is assumed to be the weaker firm.

[11] The relationship between social and marginal cost need not be deterministic. As only the conditional expected social marginal cost $\mathbf{E}[c_i|x_i]$ matters for welfare, one may define $s_i(x_i) \equiv \mathbf{E}[c_i|x_i]$ and is back to the original model.

The timing of the interaction between U, D_1, and D_2 unfolds as follows. At the beginning of the game, marginal costs x_1 and x_2 are drawn. Next, U offers a tariff $T_1(q_1)$ to D_1 and a tariff $T_2(q_2)$ to D_2. These tariffs imply that D_i can order a quantity q_i of the intermediate good at a total payment of $T_i(q_i)$. D_1 and D_2 can accept or reject their offer. If D_i rejects, it receives an outside option of value zero. Finally, production takes place and payoffs realize. I impose no restrictions – such as linearity – on the tariffs $T_1(q_1)$ and $T_2(q_2)$.

Optimal tariffs

I start with the analysis of U's optimal wholesale tariffs under price discrimination. An optimal tariff solves U's optimization problem

$$\mathcal{P}_i^d: \quad \max_{T_i(\cdot)} \int T_i(q_i(x_i))dG_i(x_i)$$

$$\text{s.t.} \quad q_i(x_i) \in \arg\max_q R(q) - x_i q - T_i(q) \quad \forall x_i \in [\underline{x}_i, \overline{x}_i], \tag{IC_i}$$

$$\max_q R(q) - x_i q - T_i(q) \geq 0 \quad \forall x_i \in [\underline{x}_i, \overline{x}_i]. \tag{PC_i}$$

The constraint (IC_i) captures that D_i orders its preferred quantity for any value of private marginal cost. The constraint (PC_i) ensures that D_i accepts the offered tariff. The profit maximization problem can be solved using standard techniques as provided in the clear exposition by Martimort and Stole (2009). The solution to this problem is presented in the following Lemma.

Lemma 1. *With price discrimination, U offers the tariff $T_i^d(q_i)$ to D_i, D_i accepts the offer, and orders a quantity $q_i^d(x_i)$. $T_i^d(q_i)$ and $q_i^d(x_i)$ are given by*

$$T_i^d(q) = R(q) - \int_0^q \left(q_i^d\right)^{-1}(y)dy, \tag{2}$$

$$q_i^d(x_i) = \arg\max_q R(q) - \Gamma_i(x_i)q, \tag{3}$$

where $\left(q_i^d\right)^{-1}(\cdot)$ is the inverse of $q_i^d(c_i)$ with $\left(q_i^d\right)^{-1}(0) = \tilde{x}_i \equiv \inf\left\{x \in [\underline{x}_i, \overline{x}_i] : q_i^d(x) = 0\right\}$.

Under the optimal tariff, the *virtual* industry profit on market i is maximized. As equation (3) shows, the virtual industry profit is the difference between revenue and the virtual private marginal cost $\Gamma_i(x_i)$ which consists of the private marginal cost and the margin of D_i. Under the optimal tariff $T_i^d(\cdot)$, D_i chooses the quantity that maximizes the virtual industry profit.

The following remark addresses the question which downstream firm is favored under price discrimination.

Remark 2. *Under price discrimination, U favors the weaker downstream firm D_2.*

The monotone likelihood ratio property implies that the inverse hazard rates are ranked as $G_1(c)/g_1(c) \geq G_2(c)/g_2(c)$. Thus, the remark follows from the classical trade-off between efficiency and rent extraction. D_1 has a more favorable distribution of private marginal cost. Thus, the upstream firm has a stronger incentive to reduce the output of D_1 for high cost realizations to lower the firm's information rent for low cost realizations.

Next, I analyze optimal uniform pricing. If price discrimination is not permitted, U has to offer the same tariff $T(q)$ to both downstream firms. As under price discrimination, U needs to take into account the downstream firms' optimal quantity choices and their participation incentives. Formally, U's optimization problem is

$$\mathcal{P}^u: \quad \max_{T(\cdot)} \sum_{i=1}^{2} \int T(q_i(x_i))dG_i(x_i)$$

$$\text{s.t.} \quad q_i(x_i) \in \arg\max_q R(q) - x_i q - T(q), \quad \forall x_i \in [\underline{x}_i, \overline{x}_i], \forall i \in \{1,2\} \qquad (IC_i^u)$$

$$\max_q R(q) - x_i q - T(q) \geq 0 \quad \forall x_i \in [\underline{x}_i, \overline{x}_i], \forall i \in \{1,2\} \qquad (PC_i^u)$$

The problem $\mathcal{P}^u$ is technically equivalent to the problems $\mathcal{P}_1^d$ and $\mathcal{P}_2^d$ with the additional non-discrimination constraint $T_1(\cdot) = T_2(\cdot)$. This constraint connects the two otherwise independent problems. The optimal tariff under uniform pricing is given in the following lemma.

Lemma 2. *With uniform pricing, U offers the tariff $T^u(q)$ to D_1 and D_2, both accept the tariff and order each the quantity $q^u(x)$. $T^u(q)$ and $q^u(x)$ are given by*

$$T^u(q) = R(q) - \int_0^q (q^u)^{-1}(y))dy, \tag{4}$$

$$q^u(x) = \arg\max_q R(q) - \overline{\Gamma}(x)q, \tag{5}$$

where $(q^u)^{-1}(\cdot)$ is the inverse of $q^u(x)$ with $(q^u)^{-1}(0) = \tilde{x}^u \equiv \inf\{x \in [\underline{x}, \overline{x}] : q^u(x) = 0\}$ and $q^u(x) \in [q_1^d(x), q_2^d(x)]$.

The additional constraint $T_1(\cdot) = T_2(\cdot)$ can be expressed in terms of virtual private marginal costs. In particular, the expression of virtual costs in equation (5) satisfies

$$\overline{\Gamma}(x) = \frac{g_1(x)}{g_1(x) + g_2(x)}\Gamma_1(x) + \frac{g_2(x)}{g_1(x) + g_2(x)}\Gamma_2(x). \tag{6}$$

Thus, the virtual private marginal costs under uniform pricing are a weighted average of the virtual private marginal costs with price discrimination. As $\overline{\Gamma}(x) \in [\Gamma_2(x), \Gamma_1(x)]$, the quantity schedule and the optimal tariff under uniform pricing lie between the quantity schedules and the optimal tariffs under price discrimination.

Welfare analysis

In the welfare analysis, I compare the expected welfare under price discrimination and uniform pricing. First, I derive sufficient conditions for price discrimination to increase or decrease social welfare. I then use this result in the cases of pure regulatory and pure technological asymmetry as well as intermediate cases.

It is helpful for the welfare analysis to state the quantity schedules as functions of social marginal cost instead of private marginal costs. Define the inverse function of marginal revenue

$$\rho(z) = \begin{cases} R'^{-1}(z), & z \leq P(0) \\ 0, & z > P(0) \end{cases}$$

which exists due to the strict concavity of $R(q)$. Using equations (3) and (5), define $\hat{q}_i^d(c) \equiv q_i^d(\sigma_i(c)) = \rho(\Gamma_i(\sigma_i(c)))$ and $\hat{q}_i^u(c) \equiv q^u(\sigma_i(c)) = \rho(\overline{\Gamma}(\sigma_i(c)))$ for $i, j \in \{1, 2\}, i \neq j$ as the mappings from social marginal costs to quantity. Given a quantity schedule $\hat{q}_i(c_i)$, the (expected) welfare on market i is $W_i(\hat{q}_i(\cdot)) \equiv \int w_i(c_i, \hat{q}_i(c_i))dF_i(c_i)$. Total (expected) welfare is given by $W(\hat{q}_1(\cdot), \hat{q}_2(\cdot)) \equiv W_1(\hat{q}_1(\cdot)) + W_2(\hat{q}_2(\cdot))$.

In the subsequent analysis, I suppose that there is underproduction with price discrimination and uniform pricing. Formally, this requires that the equilibrium quantities $\hat{q}_i^j(c)$ are lower than the socially optimal quantity $q^o(c)$ for all $i \in \{1, 2\}$, $j \in \{d, u\}$, and $c \in [\underline{c}, \overline{c}]$. The following important intermediate result provides sufficient conditions for price discrimination to increase or decrease total welfare.

Lemma 3. *Suppose wholesale tariffs may be nonlinear, marginal costs are private information* *of downstream firms, both downstream firms always supply positive quantities, and there is underproduction with price discrimination and uniform pricing on both markets.*

1. *Price discrimination increases welfare if for all* $c \in [\underline{c}, \overline{c}]$

$$\Gamma_1(\sigma_1(c)) \leq \Gamma_2(\sigma_2(c)) \quad \textit{and} \tag{7}$$

$$f_1(c)\Gamma_1(\sigma_1(c)) + f_2(c)\Gamma_2(\sigma_2(c)) \leq f_1(c)\overline{\Gamma}(\sigma_1(c)) + f_2(c)\overline{\Gamma}(\sigma_2(c)). \tag{8}$$

2. *Price discrimination decreases welfare if for all* $c \in [\underline{c}, \overline{c}]$

$$\Gamma_1(\sigma_1(c)) \geq \overline{\Gamma}(\sigma_2(c)), \quad \Gamma_2(\sigma_2(c)) \leq \overline{\Gamma}(\sigma_1(c)), \quad \textit{and} \tag{9}$$

$$f_1(c)\Gamma_1(\sigma_1(c)) + f_2(c)\Gamma_2(\sigma_2(c)) \geq f_1(c)\overline{\Gamma}(\sigma_1(c)) + f_2(c)\overline{\Gamma}(\sigma_2(c)). \tag{10}$$

First, I explain the sufficient conditions for price discrimination to be welfare-increasing. Condition (7) requires that – for the same social marginal cost c – the weaker downstream firm receives a lower quantity than the stronger downstream firm under price discrimination. If this is the case, the better contractual terms for the weaker downstream firm under price discrimination offset its disadvantage only partially. Moreover, condition (7) implies that the price on market 1 is lower than the price on market 2 for the same social marginal cost of serving these markets. Under uniform pricing, the disadvantage for the weaker downstream firm is larger and the gap between prices on the final good markets is wider. Price discrimination reallocates the good from the marginal consumer on market 1 to the marginal consumer on market 2. As the marginal consumer on market 1 has a lower valuation for the good, price discrimination tends to increase total welfare.

However, the welfare effect of price discrimination depends on the total quantity produced under price discrimination and uniform pricing. If the total quantity is larger under price discrimination and there is underproduction, total welfare is higher than under uniform pricing. Even if total quantity is slightly lower under price discrimination, total welfare is still higher as the allocation of goods across the final good markets is more efficient. Condition (8) requires that the weighted sum of virtual marginal costs of both downstream firms is smaller under price discrimination than under uniform pricing. Together with part c) of Assumption 1, this implies that the total quantity under price discrimination is sufficiently large – albeit potentially smaller than the total quantity under uniform pricing – to ensure that price discrimination leads to higher total welfare than uniform pricing.

Next, I discuss the sufficient conditions for price discrimination to be welfare-decreasing. Conditions (9) and (10) mirror conditions (7) and (8). Under condition (9), the weaker downstream firm receives a larger quantity under uniform pricing than the stronger downstream firm under price discrimination. Moreover, the weaker firm receives a higher quantity under price discrimination than the stronger firm under uniform pricing. Both statements hold for identical values of social marginal cost. The first inequality in (9) further implies that the weaker downstream firm receives a higher quantity than the stronger firm with price discrimination and identical social marginal costs. Thus, the price on market 2 is smaller than the price on market 1 for the same social marginal cost under price discrimination. A shift to uniform pricing reduces the price difference without completely offsetting it. Thus, a ban on price discrimination reallocates the good from a low valuation marginal consumer on market 1 to a high valuation marginal consumer on market 2. This reallocation is welfare-improving. Under condition (10), the weighted sum of virtual marginal costs – for the same value of social marginal costs – is lower under uniform pricing. Together with Assumption 1, this implies that the total quantity under uniform pricing is sufficiently large to ensure that – given underproduction – total welfare is higher under uniform pricing.

Next, I use Lemma 3 to derive welfare results for pure regulatory and pure technological asymmetry as well as intermediate cases.

Proposition 3. *Suppose wholesale tariffs may be nonlinear, marginal costs are private information of downstream firms, both downstream firms always supply positive quantities, and there is underproduction with price discrimination and uniform pricing on both markets.*

1. *Price discrimination increases welfare with pure regulatory asymmetry if* $\sigma_1'(c) \leq \sigma_2'(c)$.
2. *Price discrimination decreases welfare with pure technological asymmetry.*
3. *The more the cost advantage of the stronger downstream firm* D_1 *is based on technology, the smaller the welfare gain from price discrimination.*

The proposition shows that the source of cost differences between downstream firms may play a crucial role with nonlinear tariffs if the upstream firm does not know marginal costs. If downstream firms differ only with respect to their production technology, price discrimination is detrimental to welfare. However, if downstream firms use the same production technology but face different regulatory regimes, price discrimination may increase welfare. In particular, price discrimination increases welfare if the sensitivity of the private marginal cost to changes in the social marginal cost is weakly larger for the weaker downstream firm. This condition is for instance satisfied in the microfoundation of Section 2 if the downstream firms have private information about their cost of capital and the opportunity cost of capital is identical. In this case, $\sigma_1'(c) = a_1 < \sigma_2'(c) = a_2$. A higher cost of capital induces a downstream firm to substitute capital for electricity consumption which is less expensive for the downstream firm with a lower tax on electricity.

Price discrimination becomes less beneficial for welfare, the larger the extent to which the cost advantage of the stronger downstream firm D_1 is due to technology. Thus, the intermediate cases between pure technological and pure regulatory asymmetry also lead to intermediate outcomes with respect to welfare. Formally, the welfare effect of price discrimination can be expressed as

$$\begin{aligned}\Delta W &= \sum_{i=1}^{2} \Big(w(q_i^d(x_i), s_i(x_i)) - w(q_i^u(x_i), s_i(x_i)) \Big)\, dG_i(x_i) \\ &= \sum_{i=1}^{2} \left(\int_{q_i^u(x_i)}^{q_i^d(x_i)} P(q)dq - s_i(x_i)(q_i^d(x_i) - q_i^u(x_i)) \right) dG_i(x_i).\end{aligned}$$

Consider the addition of a positive function $\varepsilon(x_1)$ to $s_1(x_1)$ holding the distribution G_1 fixed. This reduces the part of the cost advantage of D_1 which is based on technology. As D_1 produces less under price discrimination than under uniform pricing, adding $\varepsilon(x_1)$ to $s_1(x_1)$ makes price discrimination relatively more attractive from a welfare perspective. If the positive function $\varepsilon(x_2)$ is added to $s_2(x_2)$, then a smaller share of D_2's cost disadvantage can be explained by a technology difference. As D_2 produces more under price discrimination than under uniform pricing, the welfare effect of price discrimination increases.

5 Downstream competition

In this section, I allow for competition on the final good market. The assumption of separate markets is a natural starting point to analyze international price discrimination. Nevertheless, even small downstream firms may reach consumers in other states via online trading platforms. Therefore, downstream competition may be relevant.

I focus on the case of linear wholesale tariffs and linear demand. In particular, suppose there is a representative consumer with the quadratic utility function

$$U(q_1, q_2) = q_1 + q_2 - \frac{1}{2}q_1^2 - \frac{1}{2}q_2^2 - \beta q_1 q_2.$$

For the final good prices p_1 and p_2, the first-order conditions of the consumer's utility maximization problem $\max_{q_1,q_2} U(q_1, q_2) - p_1 q_1 - p_2 q_2$ imply the linear demand system

$$P_i(q_i, q_j) = 1 - q_i - \beta q_j \quad \text{for} \quad i, j \in \{1, 2\}, i \neq j.$$

The goods are imperfect substitutes if $\beta \in [0, 1)$. Given the wholesale prices w_1 and w_2, the downstream firms compete in quantities.

I analyze the equilibrium of this game by backwards solution. After U has chosen the wholesale prices, the downstream firms play the unique Cournot equilibrium $(q_1^*(w_1, w_2), q_2^*(w_2, w_1))$ with

$$q_i^*(w_i, w_j) = \frac{2(1 - x_i - w_i) - \beta(1 - x_j - w_j)}{4 - \beta^2} \quad \text{for} \quad i, j \in \{1, 2\}, i \neq j.$$

Under price discrimination, the upstream firm's optimal wholesale prices (w_1^d, w_2^d) are the solution to the profit maximization problem $\max_{w_1,w_2} w_1 q_1^*(w_1, w_2) + w_2 q_2^*(w_2, w_1)$. It is easy to check that the optimal wholesale prices are given by $w_i^d = \frac{1-x_i}{2}$ for $i \in \{1, 2\}$. Thus, the stronger downstream firm pays a higher wholesale price. The equilibrium quantities (q_1^d, q_2^d) can be computed as $q_i^d = q_i^*(w_i^d, w_j^d) = \frac{2(1-x_i)-\beta(1-x_j)}{2(4-\beta^2)}$ for $i, j \in \{1, 2\}$ and $i \neq j$.

Under uniform pricing, U's profit maximization problem becomes $\max_w w \sum_{i=1}^2 q_i^*(w, w)$. The optimal uniform wholesale price $w^u = \frac{2-x_1-x_2}{4}$ solves the profit maximization problem. The equilibrium quantities are given by (q_1^u, q_2^u) with $q_i^u = \frac{2(2-3x_i+x_j)-\beta(2-3x_j+x_i)}{4(4-\beta^2)}$ for $i, j \in \{1, 2\}$

and $i \neq j$. Observe that w^u lies between the discriminatory prices w_1^d and w_2^d. Moreover, the stronger downstream firm produces more and the weaker downstream firm produces less under uniform pricing. Finally, note that the total quantities are identical under price discrimination and uniform pricing, i.e., $q_1^d + q_2^d = q_1^u + q_2^u$.

The equilibrium analysis implies the following result with respect to welfare.

Proposition 4. *Suppose the downstream firms compete, wholesale tariffs are linear, and both downstream firms supply positive quantities. If demand is linear and goods are imperfect substitutes, price discrimination increases welfare with pure regulatory asymmetry and decreases welfare with pure technological asymmetry. More generally, there exists* $\hat{a} \in (0, 1)$ *such that* $W^d \geq W^u \Leftrightarrow \frac{\Delta c}{\Delta x} \leq \hat{a}$.

The underlying reason for this result is the following. As in the case of linear demand with separate markets, the total quantity is identical under price discrimination and uniform pricing. Thus, a switch from uniform pricing to price discrimination shifts production from the stronger to the weaker downstream firm without affecting total output. The marginal welfare effect of this reallocation is $P_2(q_2, q_1) - P_1(q_1, q_2) - \Delta c$. $P_2(q_2, q_1) - P_1(q_1, q_2)$ measures the benefit from redistributing production from the low to the high price firm. This benefit is strictly positive if the goods are imperfect substitutes, i.e., if $\beta < 1$. Δc is the additional production cost that results from the shift of production and can therefore be seen as the cost of reallocation. With pure regulatory asymmetry, the reallocation is cost-free. Price discrimination lowers the regulatory distortion and increases welfare. With pure technological asymmetry, the marginal cost difference $\Delta x = \Delta x$ exceeds the price difference $P_2(q_2, q_1) - P_1(q_1, q_2)$ for all $q_1 \in [q_1^d, q_1^u]$ and $q_2 = Q - q_1$.[12] Thus, price discrimination reduces welfare with pure technological asymmetry.

6 Discussion

The model and analysis of this paper apply to any setting of wholesale price discrimination by a monopolistic upstream firm in which the social and private costs of dowstream firms differ. Section 2 provides a microfoundation for the difference between social and marginal costs which is based on suboptimal regulation. If the regulation of a negative production externality in one country does not take into account the externality on other countries, corrective tax rates are

[12]See the proof of Proposition 4 for details.

too low. Private costs therefore do no internalize the social costs completely. In the following, I discuss two alternative microfoundations.

Differences between social and marginal costs may also be driven by varying preferences of the populations in different states. The median attitude towards labor unions may influence regulations of collective bargaining and the resulting bargaining power of labor unions. If two firms are located in countries with starkly different attitudes towards collective bargaining, the firms might have very different labor costs even with identical production technologies. Within the EU, labor union coverage is indeed very diverse. In France, union coverage is almost universal. By contrast, only slightly more than a quarter of employees enjoy the right to bargain in the UK (OECD, 2019, pp.44). For a formalization, reconsider the example of Section 2 with the interpretation that input E_i represents labor, W_i is the opportunity cost of labor, and T_i is the mark-up above W_i through collective bargaining. The externality term N_i can be set to zero. If two countries have similar opportunity costs of labor $W_1 \simeq W_2$ but different mark-ups $T_1 > T_2$, the difference between social marginal costs across the two countries is smaller than the difference betwen private marginal costs.

The model can also be applied to settings where social and private costs differ for other reasons than regulation. I describe an example where an asymmetric market structure induces a wedge between social and private costs. Consider the following extension of the model of Section 2. Both downstream firms require one unit of a second intermediate good to transform the good sold by the upstream firm U into a final good. Downstream firm D_i with $i \in \{1, 2\}$ can buy this second intermediate good from one of two suppliers S_i^a or S_i^b. The suppliers have constant marginal costs of c_i^a and c_i^b. They simultaneously set the wholesale prices ω_i^a and ω_i^b before U offers a wholesale tariff to D_i. Suppose $c_1^a = c_1^b = c_2^a = c < c_2^b = x$. If x is not too far from c, Bertrand competition between the suppliers leads to the equilibrium prices $\omega_1^a = \omega_1^b = c$ and $\omega_2^a = \omega_2^b = x$. Thus, the private marginal costs of transforming one unit of the intermediate good sold by U into the final good are $x_1 = c$ for D_1 and $x_2 = x$ for D_2. The social marginal costs are $c_1 = c$ for D_1 and $c_2 = c$ for D_2. As in the case of pure regulatory asymmetry, private costs differ across downstream firms whereas the social costs are the same.

7 Conclusion

This paper studies the welfare effects of wholesale price discrimination between downstream firms operating under different regulatory systems. I analyze a model of a monopolistic intermediate good market where production cost differences between downstream firms may be due to differences in production technology, in the regulatory environment, or both. Like production technologies, regulations influence the production costs of firms. In addition, many regulations provide benefits to other members of the economy. Thus, the difference in private production costs of downstream firms operating under different regulations may exceed the difference in social production costs.

I show that the source of cost differences matters for the welfare effects of price discrimination. Price discrimination reduces regulatory distortions but may lower productive efficiency. Therefore, price discrimination increases welfare if regulation is the dominant source of cost differences and decreases welfare if cost differences are mainly due to technological asymmetry. Thus, this paper provides a novel welfare rationale for the exemption of wholesale markets from the recent ban on geo-blocking in the EU under the premise that regulatory asymmetries are an important source of cost differences.

The main result suggests a positive relation between the degree of regulatory harmonization of states and the benefits of regulating geographic wholesale price discrimination. If regulatory differences are removed, technology becomes the dominant source of cost differences. A ban on geo-blocking in intermediate good markets might therefore become more attractive after a process of additional regulatory harmonization within the EU.

Appendix

A Omitted proofs

Proof of Proposition 1

With linear tariffs, the welfare effect of price discrimination is given by

$$\begin{aligned}\Delta W &= \int_0^{q_1^d}(P(q)-c_1)dq + \int_0^{q_2^d}(P(q)-c_2)dq - \int_0^{q_1^u}(P(q)-c_1)dq - \int_0^{q_2^u}(P(q)-c_2)dq \\ &= \int_{q_2^u}^{q_2^d}(P(q)-c_2)dq - \int_{q_1^d}^{q_1^u}(P(q)-c_1)dq \\ &= \int_{q_2^u}^{q_2^d}(P(q)-a\Delta x)dq - \int_{q_1^d}^{q_1^u}P(q)dq - c_1\delta \end{aligned} \tag{11}$$

where the last equality uses $q_1^d + q_2^d - q_1^u - q_2^u \equiv \delta$ and $a \equiv \frac{\Delta c}{\Delta x}$. The proposition is implied by the following three observations. First, expression (11) is strictly decreasing in a. Second, expression (11) is strictly positive for $a = 0$ and $\delta \to 0$ as $P(q)$ is strictly decreasing, $q_2^u < q_1^d$, and $q_2^d - q_2^u \simeq q_1^u - q_1^d$ for $\delta \to 0$. Third, expression (11) is strictly negative for $a = 1$ and $\delta \to 0$:

$$\begin{aligned}\int_{q_2^u}^{q_2^d}(P(q)-\Delta x)dq - \int_{q_1^d}^{q_1^u}P(q)dq - c_1\delta &< \int_{q_2^u}^{q_2^d}(P(q_2^u)-\Delta x)dq - \int_{q_1^d}^{q_1^u}P(q_1^u)dq - c_1\delta \\ &= (q_2^d - q_2^u)(P(q_2^u) - P(q_1^u) - \Delta x) - (c_1 - P(q_1^u))\delta \\ &= (q_2^d - q_2^u)(q_1^u P'(q_1^u) - q_2^u P'(q_2^u)) - (c_1 - P(q_1^u))\delta \end{aligned}$$

where the step from the second to the third line follows from $R'(q_1^u) - R'(q_2^u) = \Delta x$. As $\delta \to 0$, the expression in the third line becomes negative as $q_1^u > q_2^u$ and $qP'(q)$ is decreasing by Part a) of Assumption 1. □

Proof of Remark 1

Suppose demand is linear, i.e., $P(q) = 1 - q$. Given a wholesale price w_i, D_i orders the quantity $q_i^*(w_i) \equiv \arg\max_q q(1 - q - w_i - x_i) = \frac{1-w_i-x_i}{2}$. U's problem under price discrimination is $\max_{w_1,w_2}\sum_{i=1}^2 w_i q_i^*(w_i) = \max_{w_1,w_2}\sum_{i=1}^2 \frac{w_i(1-x_i-w_i)}{2}$. It is easy to verify that U optimally sets $w_i^d = \frac{1-x_i}{2}$ with $i \in \{1,2\}$. This induces the quantities $q_i^d = \frac{1-x_i}{4}$ with $i \in \{1,2\}$. With uniform pricing, U's problem is $\max_w \sum_{i=1}^2 w q_i^*(w_i) = \max_w \sum_{i=1}^2 \frac{w(1-x_i-w)}{2}$. It can be quickly verified that U optimally sets $w^u = \frac{2-x_i-x_j}{4}$ which induces the quantities $q_i^u = \frac{2-3x_i+x_j}{8}$ with $i,j \in \{1,2\}$, $i \neq j$. Finally, note that $q_1^u + q_2^u = \frac{4-2x_1-2x_2}{8} = q_1^d + q_2^d$. □

Proof of Proposition 2

The proposition follows from the discussion in the main text. □

Proof of Lemma 1

Define the variable $\Pi_i(x_i) = \max_q R(q) - x_i q - T_i(q)$. By standard arguments, the incentive compatibility constraint IC_i is equivalent to $\Pi_i'(x_i) = -q_i(x_i)$ and $q_i(x_i)$ being non-increasing. Using integration by parts, the problem $\mathcal{P}_i^d$ can be restated as

$$\max_{q_i(\cdot),\Pi_i(\overline{x}_i)} \int_{\underline{x}_i}^{\overline{x}_i} \left[R(q_i(x_i)) - \left(x_i + \frac{G_i(x_i)}{g_i(x_i)}\right) q_i(x_i)\right] dG_i(x_i) - \Pi_i(\overline{x}_i)$$

subject to $\Pi_i(\overline{x}_i) \geq 0$ and $q_i(x_i)$ non-decreasing in x_i. Under Assumption 2, the solution to this problem is given by $\Pi_i(\overline{x}_i) = 0$ and $q_i^d(x_i) = \arg\max_q R(q) - \Gamma_i(x)q$. $T_i^d(\cdot)$ can be computed using the condition $R'(q_i^d(x)) - x = (T_1^d)'(q_i^d(x))$ which implies $R'(q) - (q_i^d)^{-1}(q) = (T_i^d)'(q)$ where $(q_i^d)^{-1}(y)$ is the inverse function of $q_i^d(x)$, precisely defined in the Lemma. Using this and $T_i^d(0) = 0$ gives the result. □

Proof of Remark 2

As $\gamma(x)$ is weakly increasing by Assumption 2, it follows for $x > x'$

$$\frac{g_2(x)}{g_1(x)} \geq \frac{g_2(x')}{g_1(x')} \Rightarrow \int_{\underline{c}}^{x} g_2(x)g_1(x')dx' \geq \int_{\underline{c}}^{x} g_2(x')g_1(x)dx' \Leftrightarrow \frac{G_1(x)}{g_1(x)} \geq \frac{G_2(x)}{g_2(x)}$$

Thus, equation (3) implies $q_1^d(x) \leq q_2^d(x)$ for all x and $T_1^d(q) \geq T_2^d(q)$ for all q. □

Proof of Lemma 2

Define the variable $\Pi(x_i) = \max_q R(q) - x_i q - T(q)$. Note that (IC_1^u) and (IC_2^u) imply $q_i(x_i) = q(x_i)$ for $i \in \{1,2\}$. By standard arguments, the incentive compatibility constraints (IC_i^u) with $i \in \{1,2\}$ are equivalent to $\Pi'(x_i) = -q(x_i)$ and $q(x_i)$ being non-increasing. Define $\underline{x} \equiv \min_i\{\underline{x}_i\}$

and $\overline{x} \equiv \max\{\overline{x}_i\}$. Using integration by parts, the problem $\mathcal{P}^u$ can then be restated as

$$\max_{q(\cdot),\Pi(\overline{x})} \int_{\underline{x}}^{\overline{x}} \left[R(q(x)) - \left(x + \frac{G_1(x)+G_2(x)}{g_1(x)+g_2(x)} \right) q(x) \right] (g_1(x)+g_2(x))dx - 2\Pi(\overline{x})$$

subject to $\Pi(\overline{x}_i) \geq 0$ and $q(x)$ non-decreasing in x. Under Assumption 2, the solution to this problem is given by $\Pi(\overline{x}) = 0$ and $q^u(x) = \arg\max_q R(q) - \overline{\Gamma}(x)q$. $T^u(\cdot)$ can be computed using the condition $R'(q^u(x)) - x = (T^u)'(q^u(x))$ which implies $R'(q) - (q^u)^{-1}(q) = (T^u)'(q)$ where $(q^u)^{-1}(y)$ is the inverse function of $q^u(x)$, precisely defined in the Lemma. Using this and $T^u(0) = 0$ gives the result. Finally, $\overline{\Gamma}(x) \in [\Gamma_2(x), \Gamma_1(x)]$ implies $q^u(x) \in [q_1^d(x), q_2^d(x)]$. □

Proof of Lemma 3

To prove the proposition, I use the following property of concave functions. Consider the function $k : \mathbb{R} \to \mathbb{R}$ and the vector $\mathbf{x} = (x_1', x_2', x_1'', x_2'') \in \mathbb{R}^4$. Without loss of generality, let $x_1' \leq x_2'$ and $x_1'' \leq x_2''$.

Lemma 4. *Suppose k is decreasing and concave on $[\min\{\mathbf{x}\}, \max\{\mathbf{x}\}]$. If $\mathbf{x}$ satisfies $x_1' \geq x_1''$, and $\alpha x_1' + (1-\alpha)x_2' \leq \alpha x_1'' + (1-\alpha)x_2''$ for some $\alpha \in [0,1]$, then $\alpha k(x_1') + (1-\alpha)k(x_2') \geq \alpha k(x_1'') + (1-\alpha)k(x_2'')$.*

Proof. Define $\hat{x}_2' \in \mathbb{R}$ by $\alpha x_1' + (1-\alpha)\hat{x}_2' = \alpha x_1'' + (1-\alpha)x_2''$. Note that $x_2' \leq \hat{x}_2' \leq x_2''$. Together with $x_1' \geq x_1''$, this implies that there exist $\beta_1 \in [0,1]$ and $\beta_2 \in [0,1]$ such that $x_1' = \beta_1 x_1'' + (1-\beta_1)x_2''$ and $\hat{x}_2' = \beta_2 x_1'' + (1-\beta_2)x_2''$. The definition of $\hat{x}_2'$ implies $\alpha\beta_1 + (1-\alpha)\beta_2 = \alpha$. Note that

$$\begin{aligned} \alpha k(x_1') + (1-\alpha)k(x_2') &\geq \alpha k(x_1') + (1-\alpha)k(\hat{x}_2') \\ &\geq \alpha(\beta_1 k(x_1'') + (1-\beta_1)k(x_2'')) + (1-\alpha)(\beta_2 k(x_1'') + (1-\beta_2)k(x_2'')) \\ &= \alpha k(x_1'') + (1-\alpha)k(x_2'') \end{aligned}$$

where the first step follows from k being decreasing on $[\min\{\mathbf{x}\}, \max\{\mathbf{x}\}]$, the second step is implied by concavity of k, and the last step follows from $\alpha\beta_1 + (1-\alpha)\beta_2 = \alpha$. □

The welfare effect of price discrimination is

$$\Delta W = \int \Big\{ f_1(c)w(\hat{q}_1^d(c),c) + f_2(c)w(\hat{q}_2^d(c),c) - f_1(c)w(\hat{q}_1^u(c),c) - f_2(c)w(\hat{q}_2^u(c),c) \Big\}\, dc.$$

For each $c \in [\underline{c}, \overline{c}]$, it holds that

$$f_1(c)w(\hat{q}_1^d(c),c) + f_2(c)w(\hat{q}_2^d(c),c) - f_1(c)w(\hat{q}_1^u(c),c) - f_2(c)w(\hat{q}_2^u(c),c) =$$
$$f_1(c)w(\rho(\Gamma_1(\sigma_1(c)),c) + f_2(c)w(\rho(\Gamma_2(\sigma_2(c)),c) - f_1(c)w(\rho(\overline{\Gamma}(\sigma_1(c)),c) - f_2(c)w(\rho(\overline{\Gamma}(\sigma_2(c)),c).$$

I now want to apply Lemma 4 to the function $w(\rho(\cdot),c)$ to prove the proposition. Note first that $w(\rho(z),c)$ is decreasing in z for $z \in Z(c) \equiv Conv\{\Gamma_1(\sigma_1(c)), \Gamma_2(\sigma_2(c)), \overline{\Gamma}(\sigma_1(c)), \overline{\Gamma}(\sigma_2(c))\}$. This follows from $\rho(z)$ being decreasing in z and underproduction under price discrimination and uniform pricing. Next, I show that under part c) of Assumption 1, $w(\rho(z),c)$ is concave in z for $\rho(z) > 0$. For $z \in Z(c)$ and $\rho(z) > 0$, the second derivative satisfies

$$\frac{\partial^2}{\partial z^2} w(\rho(z),c) = (P(\rho(z)) - c)\rho''(z) + P'(\rho(z))\rho'(z) = \frac{P(\rho(z)) - c}{R''(\rho(z))^2}\left(\frac{P'(\rho(z))}{P(\rho(z)) - c} - \frac{R'''(\rho(z))}{R''(\rho(z))}\right)$$

which is negative under Part c) of Assumption 1 as $P(\rho(z)) - c > 0$ due to underproduction.

The result now follows from Lemma 4 as condition (7) implies $\overline{\Gamma}(\sigma_1(c)) \le \Gamma_1(\sigma_1(c)) \le \Gamma_2(\sigma_2(c)) \le \overline{\Gamma}(\sigma_2(c))$ and condition (9) implies $\overline{\Gamma}(\sigma_i(c)) \in [\Gamma_2(\sigma_2(c)), \Gamma_1(\sigma_1(c))]$ for $i \in \{1,2\}$. □

Proof of Proposition 3

I start with the following helpful lemma.

Lemma 5. *If $\sigma_2'(c) \ge \sigma_1'(c)$ for all $c \in [\underline{c}, \overline{c}]$, then inequality* (7) *implies inequality* (8).

Proof. Using $f_i(c) = g_i(\sigma_i(c))\sigma_i'(c)$ for $i \in \{1,2\}$ and factoring out $\sigma_1'(c)$ and $\sigma_2'(c)$, (8) can be written as

$$\sigma_1'(c)\frac{G_1(x)g_2(x) - G_2(x)g_1(x)}{g_1(x) + g_2(x)}\Big|_{x=\sigma_1(c)} \le \sigma_2'(c)\frac{G_1(x)g_2(x) - G_2(x)g_1(x)}{g_1(x) + g_2(x)}\Big|_{x=\sigma_2(c)}.$$

Note that (7) implies $\sigma_2(c) \geq \sigma_1(c)$. For $\sigma_2'(c) \geq \sigma_1'(c)$, the inequality above is satisfied as

$$\frac{\partial}{\partial x}\left(\frac{G_1(x)g_2(x) - G_2(x)g_1(x)}{g_1(x)+g_2(x)}\right) = \frac{(G_1(x)+G_2(x))(g_1(x)g_2'(x) - g_1'(x)g_2(x))}{((g_1(x)+g_2(x))^2} \geq 0$$

due to $g_2(x)/g_1(x)$ being increasing by Assumption 2. □

I can now prove the proposition as follows. For the case of pure regulatory asymmetry, it is by Lemma 5 sufficient to check that $\sigma_1'(c) \leq \sigma_2'(c)$ implies inequality (7). As $F_1(c) = F_2(c) = F(c) = G_i(\sigma_i(c))$, it holds that $\Gamma_i(\sigma_i(c)) = \sigma_i(c) + \sigma_i'(c)\frac{F(c)}{f(c)}$. Equation (7) is therefore satisfied if $\sigma_1(c) \leq \sigma_2(c)$ and $\sigma_1'(c) \leq \sigma_2'(c)$ where the latter implies the first as $\sigma_1(\underline{c}) \leq \sigma_2(\underline{c})$.

For the case of pure technological asymmetry, $\sigma_1(c) = \sigma_2(c) = \sigma(c)$ implies that condition (9) is satisfied due to equation (6) and $\Gamma_1(x) \geq \Gamma_2(x)$. Next, note that the right-hand side of inequality (10) simplifies to $(f_1(c)+f_2(c))\overline{\Gamma}(\sigma(c))$. Furthermore, due to $f_i(c) = g_i(\sigma_i(c)\sigma_i'(c)$, the left-hand side of (10) satisfies

$$\begin{aligned} f_1(c)\Gamma_1(\sigma(c)) + f_2(c)\Gamma_2(\sigma(c)) &= (f_1(c)+f_2(c))\sigma(c) + \sigma'(c)(G_1(\sigma(c)) + G_2(\sigma(c))) \\ &= (f_1(c)+f_2(c))\overline{\Gamma}(\sigma(c)). \end{aligned}$$

Thus, condition (10) is satisfied with equality. Point 3. follows from the discussion in the main text. □

Proof of Proposition 4

Denote the total quantity under price discrimination and uniform pricing by Q. Define

$$\begin{aligned} W_Q(q_1) &\equiv q_1 + (Q-q_1) - \frac{1}{2}q_1^2 - \frac{1}{2}(Q-q_1)^2 - \beta q_1(Q-q_1) - c_1q_1 - c_2(Q-q_1) \\ &= Q(1-c_2) - \frac{1}{2}q_1^2 - \frac{1}{2}(Q-q_1)^2 - \beta q_1(Q-q_1) + a\Delta x q_1 \end{aligned}$$

where $a = \frac{\Delta c}{\Delta x}$ and note that $W^d = W_Q(q_1^d)$ and $W^u = W_Q(q_1^u)$. The marginal welfare effect of redistributing from market 1 to market 2 is given by $W_Q'(q_1) = (1-\beta)(Q-2q_1) + a\Delta x$.

First, I prove $W^d > W^u$ for $a = 0$. As $W_Q'(q_1)$ is decreasing in q_1, $W_Q'(q_1) < 0$ for $q_1 \in [q_1^d, q_1^u]$ is equivalent to $W_Q'(q_1^d) < 0$. Moreover, $W_Q'(q_1^d) = (1-\beta)(q_2^d - q_1^d) < 0$ as the stronger downstream firm produces more under price discrimination and $\beta < 1$.

Second, I prove that $W^u > W^d$ for $a = 1$. For $a = 1$, $W'_Q(q_1^u) > 0$ implies $W'_Q(q_1) > 0$ for $q_1 \in [q_1^d, q_1^u]$ as $W'_Q(q_1)$ is decreasing in q_1. Moreover,

$$W'_Q(q_1^u) = (1-\beta)(Q - 2q_1^u) + \Delta x = (1-\beta)(q_2^u - q_1^u) + \Delta x = -\frac{1-\beta}{2-\beta}\Delta x + \Delta x = \frac{\Delta x}{2-\beta} > 0.$$

Third, there exists a cutoff value $\hat{a} \in (0,1)$ such that $W^d \geq W^u \Leftrightarrow \frac{\Delta c}{\Delta x} \leq \hat{a}$. This result follows from $W^d - W^u$ being strictly decreasing in a as

$$W^d - W^u = -\int_{q_1^d}^{q_1^u} W'_Q(q)dq = -\int_{q_1^d}^{q_1^u} [(1-\beta)(Q-2q) + a\Delta x]dq$$

and the previous observations that $W^d > W^u$ for $a = 0$ and $W^d < W^u$ for $a = 1$. □

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