Space-Time Adaptive Reduction of Unsteady Flamelets

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***Abstract*— The Wavelet Adaptive Multiresolution Representation (WAMR) code and the G-Scheme framework are used for the numerical time integration of the flamelet model. The steep gradients are efficiently captured by the WAMR algorithm with an a-priori defined accuracy and an associated large reduction of the number of degrees of freedom (DOFs). A further opportunity to reduce the complexity of the problem is represented by the G-Scheme, to achieve multi-scale adaptive model reduction along-with the time integration of the differential equations.**

I. Introduction

In this work a study of the unsteady flamelet model is proposed. Steady-state solutions are generated through the Wavelet Adaptive Multiresolution Representation (WAMR) code [1, 2]. This method was verified [2, 3] for a wide range of test cases - compressible and incompressible flows described by reacting Navier-Stokes equations in primitive variables in 1-, 2- and 3-D geometries. Using the WAMR algorithm to dynamically adapt the space resolution, the computational cost is largely reduced: steep gradients are well captured by the algorithm, with a reduced number of grid points and consequent reduction of the computational cost.

By decomposing the system dynamics into active, slow, fast and invariant subspaces, the G-Scheme algorithm allows to integrate only the DOFs belonging to the active subspace, with corresponding saving in computational work. Complete details about the G-Scheme theory can be found in the work by Valorani et al. [4].

II. Discussion and Results

We consider the unsteady flamelet proposed in [5], considered for adiabatic case, at constant pressure and unity Lewis number. The thermodynamic properties of the fluid are evaluated with the ideal gas Equation of State (EoS). The kinetic mechanism for CO, CH_2O and CH_3OH combustion [6] is considered, taking into account 12 chemical species. The flamelet equations are integrated for pressure $p = 20$ atm and scalar dissipation rate $\chi^0_{max} = 200$ s^{-1}.

The coupling between WAMR and the G-Scheme is represented in Fig. 1, where t is the time and $\vec{y}$ is the state vector, where the temperature and the species mass fractions are stored.

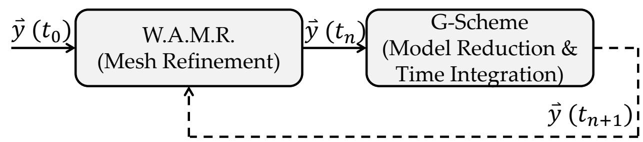

Fig. 1. Coupling between WAMR and the G-Scheme; $\vec{y}(t)$ represents the state vector.

The reference solution is built on a uniform grid using DVODE as time integrator and the steady-state is supposed to be reached when the root mean square (RMS) of the right hand side (RHS) becomes lower than a fixed prescribed minimum value.

The comparison between the reference and the adaptive solutions is represented in Fig. 2 in terms of temperature with respect to the mixture fraction z, for five values of t: the excellent level of accuracy produced by the G-Scheme and WAMR can be clearly appreciated.

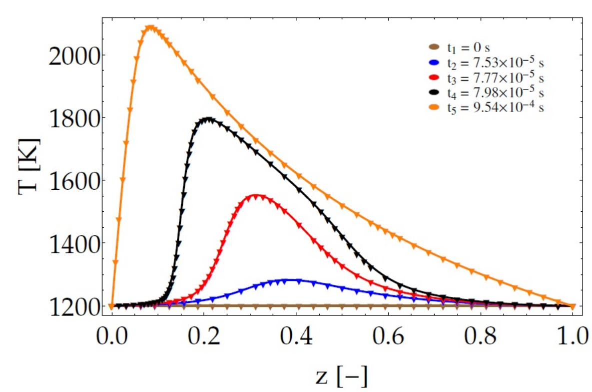

Fig. 2. Solutions comparison in terms of temperature with respect to z for five values of t - in the WAMR/G-Scheme simulation the wavelet threshold parameter is $\varepsilon = 10^{-3}$; the dashed lines represent the reference curves, while the triangles are associated with the results of the WAMR/G-Scheme simulation.

Figure 3 shows the time evolution of the number of DOFs produced by WAMR, integrated by the G-Scheme and associated with the reference uniform grid. From an overall perspective, it is clear that the number of DOFs produced by WAMR is significantly lower than the one associated with the reference uniform mesh ensuring the same accuracy in space. A useful tool to evaluate the efficiency of the wavelet compression is represented by the compression degree π_w, defined as the ratio between the number of grid points of the adaptive mesh and a reference uniform grid having the same minimal spacing. In the current test this values reaches a peak at $t \simeq 7.5 \times 10^{-6}$ s ($\pi_w^{max} \simeq 26\%$), while it remains fairly constant from $t \simeq 1.5 \times 10^{-4}$ s ($\pi_w \simeq 20\%$): this excellent result shows that only a few DOFs are required to obtain solutions accurate as prescribed by the wavelet threshold parameter. In the same way, the G-Scheme efficiency can be evaluated by the index π_{gs}, representing the ratio between the number of DOFs integrated by the G-Scheme (N_A) and generated by WAMR. The peak is reached at $t \simeq 7.5 \times 10^{-6}$ ($\pi_{gs}^{max} \simeq 66\%$) and the trend remains stable from $t \simeq 1.5 \times 10^{-4}$ s ($\pi_{gs} \simeq 7.5\%$), showing that the G-Scheme is typically able to integrate a small amount of DOFs generated by WAMR, while maintaining a prescribed accuracy in time.

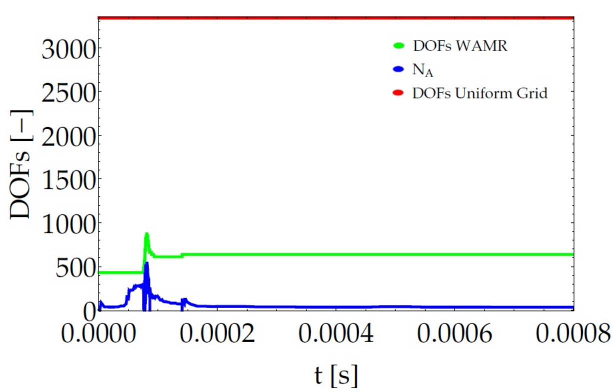

Fig. 3. Number of DOFs with respect to t.

Another important feature of the G-Scheme is the integration time step, Fig. 4: it is of the order of the fastest time scale of the active subspace, that can be several order of magnitude larger than the fastest time scale of the system.

Finally, Figure 5 shows the time evolution of the Jacobian matrix eigenvalues, in terms of orders of magnitude. The few eigenvalues associated with the active subspace and integrated are included between the red and blue continuous lines, representing the boundaries between the active and the fast/slow subspaces, respectively. The number of active DOFs remains approximatively constant from $t = 2 \times 10^{-4}$ s, $\widehat{N_A} \simeq 39.25$. The green line is associated with the Tangential Stretching Rate (TSR) [7], showing that the most energetic scale is always included in the active subspace.

Acknowledgment

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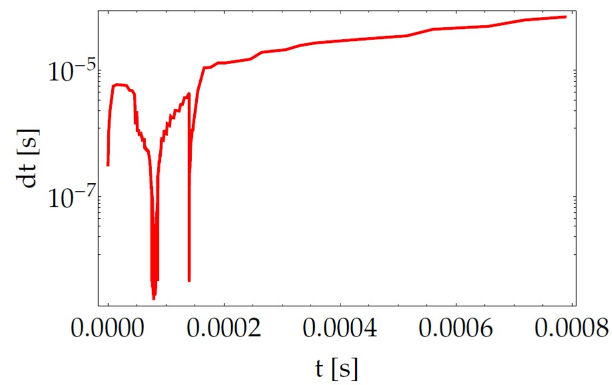

Fig. 4. Integration time step in time for the G-Scheme.

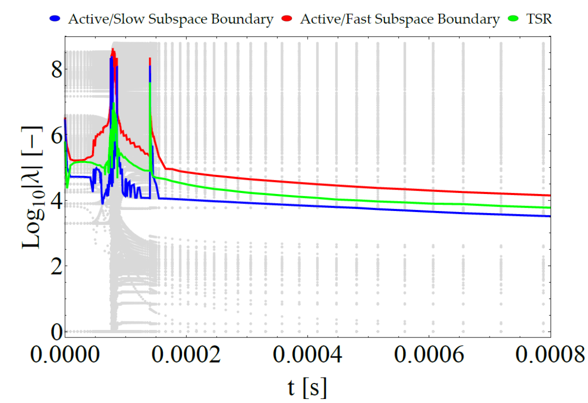

Fig. 5. Order of magnitude of the Jacobian matrix eigenvalues (gray dots) for $\varepsilon = 10^{-3}$; the active subspace is included between the red and blue lines; the green line is associated with the TSR.

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