Chapter 4
Luigi Cremona and Wilhelm Fiedler: the Link between Descriptive and Projective Geometry in Technical Instruction

Marta Menghini (Sapienza Università di Roma)
marta.menghini@uniroma1.it

Abstract
This paper considers Luigi Cremona's and Wilhelm Fiedler's outlook on technical instruction at school and University level, their vision about the educational role of descriptive geometry, and the relation to Monge's original conception. Like Cremona, Fiedler sees a symbiosis between descriptive and projective geometry, given by the fundamental idea of central projection. The link between projective and descriptive geometry plays a double role: an educational one - due to the graphical aspects of the two disciplines – and a conceptual one – given by the connection of theory to practice. Thus, projective and descriptive geometry contribute to the education of a scientific learned class and the link between them epitomizes – in the opinion of Cremona – the link between pure mathematics and its applications. According to Fiedler, the main scope of the teaching of descriptive geometry is the scientific construction and development of *Raumanschauung*, as he also states in a paper published in the Italian journal *Giornale di Matematiche*. The textbooks by Fiedler (1871) and Cremona (1873) were used in Italy to develop the geometry programs for the physics and mathematics section within technical secondary instruction. While the relation between projective and descriptive geometry – and thus in some way between pure and applied mathematics – had a short life at secondary school level in Italy, after the turn of the century there was a new expansion at university level, due to the role that important mathematicians had at the moment of the creation of the Faculty of Architecture.

Key Words
descriptive geometry; projective geometry; Luigi Cremona; Wilhelm Fiedler; history of technical education.

1. Introduction

In 1870s, soon after the Unification of Italy, the new country built its educational system out of the different preceding experiences (Scoth, Chapter 3). Within this system, these were crucial years for technical instruction, which was particularly influenced by French and German models. The new programs for the physics and mathematics section of the *Istituti tecnici* (technical institutes, that is, Italian technical secondary schools), set out by Luigi Cremona in 1871, represented the apex of the teaching of descriptive and projective geometry in Italy (Menghini 2006, Fiocca 1992 cited in Scoth). In 1873, Cremona published his *Projective Geometry* to fulfill the programs of the physics-mathematics section. In the same year Cremona contributed to the creation of the "Citadel of Science" in Rome: the new university site should "physically" evidence the link between mathematics and its applications, and in particular the link between projective and descriptive geometry. Furthermore, in 1874 Wilhelm Fiedler's *Descriptive Geometry* was translated into Italian, to be used, alongside Cremona's text, in the technical institutes.

The political unification of Italy in 1861 led to the various areas of Italian mathematics becoming integrated into the context of European research. The most eminent Italian mathematicians were involved in bringing Italy back to the forefront of international developments in the fields of science and economics. Of special standing in this context were the geometric studies of a synthetic nature. These achieved their greatest development in the school of geometry headed by Luigi Cremona, whose extensive correspondence (Israel, to appear) shows the many links with foreign researchers. Cremona anchored his work within the classical school of projective geometry, with particular attention given to the ramifications which came about in France with Poncelet and Chasles, and in Germany with von Staudt, Plücker, Möbius, Steiner and Clebsch.

Among the letters to Cremona we find 33 letters sent by Wilhelm Fiedler between 1862 and 1888 (Knobloch & Reich, to appear). The correspondence mostly concerns the exchange of publications, but we find also reference to the roots of the two geometers (Descartes, Desargues, Taylor, Lambert, Monge, Poncelet, Möbius, Steiner, Chasles, von Staudt, Plücker), and to authors that both appreciate (as Salmon, Culman, Clebsch, Klein, Reye, …). Very often Fiedler confirms his interest in the teaching of descriptive geometry, and in a letter written at the beginning of 1873, he praises Cremona's book and the simple way in which he introduces the topics. Furthermore, he praises the Italian technical education.

With the arrival of Cremona in Rome in 1873, an interesting project took place, based on experiences of the Polytechnical Schools in northern Italy. The project foresaw transferring a part of the structures and professors of the Faculty of Science from the old 'Sapienza' to a new site at San Pietro in Vincoli, thus setting up a sort of 'citadel of science' in which, along with other disciplines, all teaching of a mathematical nature was brought together in a newly-founded autonomous Institute of Mathematics. In the new setting, one could find, beside the School for Engineers, the School of Mathematics, the Library and the School of Drawing and Architecture. All these schools were part of the University. The position that mathematics should occupy in science is clearly reflected in this "physical" arrangement.

Again, this decision reflected the close ties between aspects of a theoretical nature and aspects of a 'concrete' nature[1]—linked to the practice of drawing—in the conceptions of that time and in those of Cremona in particular, who headed the "School for Engineers" until his dead in 1903 (after Cremona's dead, another mathematician, Valentino Cerruti, headed the school). So, the 'mathematical school' of the Faculty of Science soon tasted the flavor of a relationship between applied and pure research, where the studies on descriptive geometry represented a clear example, according to the tradition inspired by Monge and by the French Polytechnics schools.

As shown in many papers of this volume, the Ecole influenced the development of technical schools in many countries in 19th century Europe (also see Schubring 1989, Barbin & Menghini, 2014). As concerns the *Politecnici* in Italy, there was a direct influence due to the French occupation (1796 – 1813); as described by Scoth (Chapter 3) the situation was maintained during the Restoration, and descriptive geometry spread widely in the Italian states, thanks to military schools and universities that had preserved the syllabuses of the French era.

The key role played by Monge is not to be considered only for its institutional influences but also for aspects of a didactic nature, concerning the teaching of geometric disciplines in the nascent technical secondary schools. The fundamental role that in such schools should be attributed to descriptive geometry and to geometric drawing fosters the learners' intelligence by giving them the habit and the feeling for precision (Monge, 1839, Vol. II, n. V).

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[1] Some of the considerations contained in this section, as well as in the last section, were developed with L. Dell'Aglio (see Dell'Aglio, Emmer & Menghini, 2001).

These aspects were taken up again by Luigi Cremona in the preface to the programs of 1871, where he stressed the educational role of descriptive geometry in secondary education, linked to the exactness of its methods. Cremona never neglected the applications of geometry and attributed considerable importance, even from a logical point of view, to drawing, such preserving Monge's ancient conception of this doctrine.

Cremona proposes a theoretical introduction in a projective realm to address an issue of 'graphic' character within his courses of Static Graphics in the Institute of higher technical education in Milan and in his courses at the University of Bologna. In his lecture course of descriptive geometry, held in Bologna from 1861 to 1867, Cremona had introduced some fundamental views of higher geometry. In a note of 1865, written with the pseudonym (anagram) Marco Uglieni, Cremona explained the solution of some graphic problems of central projection proposed in a booklet of Brook Taylor, following the methods used later by Wilhelm Fiedler in descriptive geometry.

So, with reference to teaching, the position of Cremona is characterized by the interchange of theoretical and graphical methods. This view - which encompasses the central role assigned to projective geometry and the recognition of the 'empirical' importance of a graphic discussion of its principles - is essentially classic.

2. Projective and descriptive geometry in secondary schools: Cremona and Fiedler

2.1. *Cremona's projective geometry*

A reform in 1871 concerned the teaching of geometry in the *Istituti tecnici*, the Italian secondary schools for the technical instruction (Menghini, 2006). The reform presumed the explicit introduction of the fundamental principles of projective geometry, which was deemed to be a necessary theoretical preamble to the study of descriptive geometry. The Italian aspiration was that of forming, around the nucleus of projective geometry, a learned scientific class able to compete with its counterpart in the humanities.

The main points of the programs are summarized here. Geometry includes

the theory of projections of geometric forms (projective ranges and pencils, cross ratio, complete quadrilateral) with its applications to the graphical solution of the problems of first and second degree and to the construction of the curves of the second order, seen as projections of the circle (this requires: projective ranges in a circle; self-corresponding elements of superposed forms); the theory of involution (conjugate points with respect to a circle); the duality principle in the plane; elements of stereometry and the graphic construction of the barycentres of plane figures.

Cremona's *Elementi di Geometria Projettiva*, published in 1873, was written to fulfill the part of the syllabus concerning projective geometry. The book had a great success also outside Italy, and had numerous translations. But, although the book was written for Italian secondary school, it was to be adopted mostly at university level outside Italy.

From this work, the conviction emerges clearly of the importance of the insertion of theoretical topics – concerning projective geometry – within the curriculum of coming engineers – from the secondary technical institutes to the Polytechnics – where the central role is nevertheless played by activities related to graphics. In the introduction to the book, Cremona claims that the methods of projective geometry could, one day, solve the problem of teaching geometry also in the classical instruction. He writes that he applied the methods of projective geometry to the teaching of descriptive geometry when he was in the Bologna University, and that he followed in that the methods suggested by Fiedler.

In his book, Cremona maintains reference to the affine formulation of the theorems as much as possible, considering projections of figures from a plane onto a parallel plane, thus using parallelism and points at infinity. For example, he introduces homothetic (similar) triangles as a significant case of homological (perspective) triangles, or parallelograms as a particular case of quadrilaterals; he used length and sign of a segment, together with similitude, to prove the invariance of cross ratio, in accordance with Moebius' barycentric calculus, rather than basing it on the complete quadrilateral, as von Staudt did.

After establishing the fundamental concepts of space, as surface, line, point, straight line and plane, Cremona introduces the important concept of collineation. Like Fiedler, Cremona starts from central projection, but, differently from Fiedler, he considers only the graphic (projective) properties.

Fig. 1. Fig. 3.

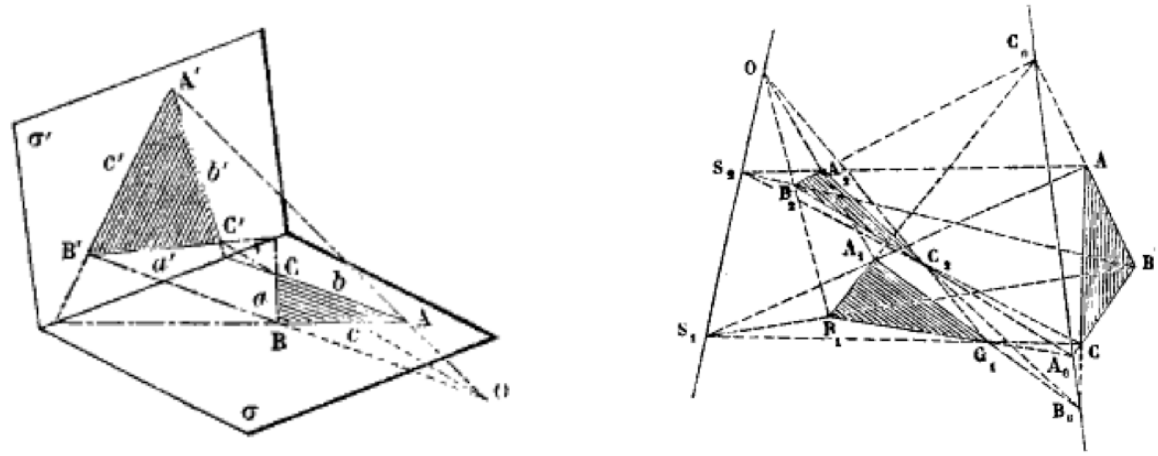


Fig. 1: From the table of figures in Cremona (1873)

So he considers a centre of projection O, a figure made of points and lines *ABCabc*.., the rays that connect the centre with the points or the lines of the given figure (which form straight lines and planes), and the intersection of these rays with a plane called the picture plane (Fig. 1, left).

Then he gives some properties: for instance, if the figure *ABCabc*.. lies on a plane, we can have a one to one correspondence between the picture plane and the plane of the figure by introducing the points at infinity. The image of a point at infinity is a vanishing point. Cremona can such introduce the Desargues theorem and homology in space. The Desargues theorem on the plane is obtained applying twice the construction of Fig. 1 (left): first from σ to σ', and then from σ' to another triangle on σ (Fig. 1, right).

The focal properties of conic sections, mentioned in the programs of 1871, were to have been covered in the second volume, but this volume was never published because the syllabuses were reduced in 1876.

The section of the same programs of 1871 for the physics-mathematics class concerning the teaching of descriptive geometry states that the teacher will start from central projection and from the projective properties of figures and will handle the theory of collinearities, of affinities, of similarities, with attention to homology, up to the construction of intersections of surfaces of the second degree. The teachers of mathematics and descriptive geometry shall cooperate, as both are concerned with the projection of geometric figures (Ministero etc., 1871, p. 52-63).

In 1874, the translation of the book on *descriptive geometry* by Wilhelm Fiedler (Fiedler 1874) appeared[2]. Although it was written for the *Technische Hochschulen*, which were university level schools in Germany, the Italian edition was explicitly translated and adapted for use at the secondary school level, in the Italian Technical Institutes.

It was certainly appropriate to have Fiedler's book alongside Cremona's Projective Geometry in the parallel course of descriptive geometry at the Technical Institutes. According to Fiedler, the main scope of the teaching of descriptive geometry is the scientific construction and development of "Raumanschauung" (space-intuition). Fiedler reinforced this point of view in a paper translated and published in the Giornale di Matematiche. Fiedler sees a complete symbiosis between descriptive and projective geometry and holds that starting from central projection, which corresponds to the process of viewing, we can develop the fundamental part of projective geometry in a natural and complete way (Fiedler, 1878, p.248). He feels supported by the Swiss pedagogue and educator Johann Heinrich Pestalozzi, who argues that teaching must start with intuition. Fiedler sees these strategies as the best method for the reform of geometry teaching at all levels (to this respect, he shares Monge's opinion about the role of descriptive geometry in rethinking secondary education). Moreover, a great importance is given to the parallel development of plane and space geometry, to the duality principle, to movement and geometrical transformations.

According to Fiedler, projective geometry allows starting from few fundamental relations to construct all geometry. History shows that these fundamental relations are substantially linked to the methods of descriptive geometric and to the fact that Poncelet put at the basis of his work the methods of perspective, already developed by Desargues. Fiedler also refers to the 'elementary forms' of Jacob Steiner and to the search for the projective properties of figures by means of projections. He also refers to Möbius looking at congruences, similarities and affinities as particular collineations. Collineations can be completely understood starting from central projection. Thus, the whole geometry must become descriptive (Fiedler, 1878, pp. 243- 248). The representation methods are a premise to 'movement', which can be organically introduced in geometry from the beginning: Geometry is the science of comparison of the figures that are created one from another by means of representation.

In a letter to the editor of the Journal (the *Giornale di Matematiche*), which is annexed to the paper, Fiedler explains better his view on geometry teaching, proposing a chronology of the arguments to be developed:

- START. Intuitive geometric teaching. Drawings from models made by bars, building of nets. Geometric drawing.
- BEGINNING OF THEORY: Deductions and definitions with many compositions of forms. Exercises on definitions and their use. Central projection for simple forms in plane and in space. Duality principle.
- GEOMETRY AS THE SCIENCE OF COMPARISON. Comparison of the figures obtained the one from the other through representation. Visual parallel rays: congruence, reflection (in a line/plane), affinity. Trigonometry and Cartesian coordinates together with descriptive geometry (Monge). Visual concurring rays: similarity, reflection in a point, collinearity, involution.

[2] The original title "Die darstellende Geometrie in organischer Verbindung mit der Geometrie der Lage" (1871) was simply translated as "Trattato di Geometria descrittiva". In the preface of his *Elementi di Geometria Projettiva,* Cremona considers 'Geometrie der Lage' (geometria di posizione/ geometry of position), equivalent to projective geometry (geometria projettiva).

- General idea of projectivity and extension through non real elements. Projective coordinates and algebraic and geometric treatment of the different forms, that is, synthetic and analytic teaching o geometry of position.

In his textbook translated in 1874 Fiedler explains that his aim is to deduce from the methods of representation all the elements that are necessary to the study of the properties of figures. In the first part he indicates those methods by which one passes from a given figure in space to another figure in the plane or in space. He looks for the properties common to the two figures and for those properties that do not change in the given representation. The simplest and most natural method of representation is central projection. So he starts from this (see Volkert, this volume, ch. 11).

In the third section of Fiedler's book we find homology in three dimensions, which allows representing any figure given in space, and only in the final section we find Monge's method and axonometry. All the representation methods are deduced from central projection. In fact, with rays starting from a finite center we obtain similarity, central symmetry, central collineations, involution; with rays having the same direction, we obtain congruence, axial and plane symmetry, affinity. Finally we obtain Monge's descriptive geometry.

Particularly in Italy, Fiedler's merit is recognized to have realized the "fusion" between projective and descriptive geometry, framing the method of central projection in descriptive geometry and realizing a complete 1-1 correspondence between plane and space (Comesatti, 1937-38; Loria, 1921). Let us see how Fiedler establishes a 1-1 correspondence between plane and space at the beginning of his work. To this purpose, metric relations are of course necessary. He considers the orthogonal projection of the centre C onto the plane of the drawing, C_1. If we consider a point P on the plane of the drawing, we call d the distance CC_1, r the distance PC_1, and l the distance PC; l is the hypotenuse of a rectangular triangle. We have the relations $r\tan\beta$ b $= d$, $l\sin\beta = d$, where β is the angle CPC_1. We call the circles with centre C_1 circles of inclination. If $\beta = 45°$, then $r = d$; if $\beta > 45°$ $r < d$. a.s.o. If $\beta = 90°$ $r = 0$, if $\beta = 0°$, $r = \infty$; that is the infinity line of the picture plane has inclination equal zero (Fig. 1).

Fig. 1.

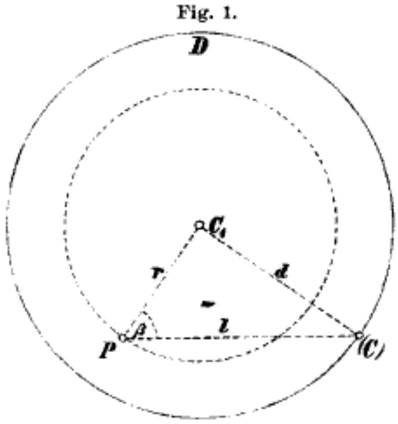
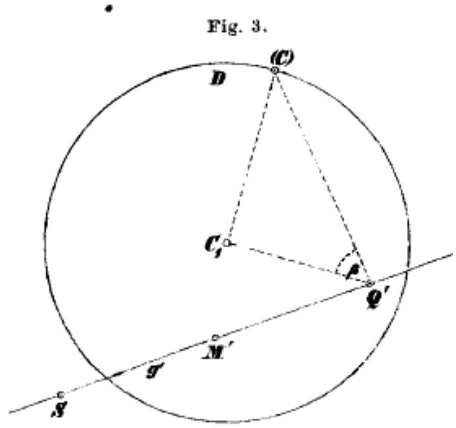


Figure 1. From page 6 of Fiedler (1874).

If a line *g* does not pass through the centre *C*, its image on the picture plane is given by the line passing through the point *S* (intersection of *g* with the picture plane) and its *vanishing point Q′* (that is, the intersection with the picture plane of the line through *C* parallel to *g*) (Fig. 2, left). The image of a point *M* cannot be given only by the intersection *M′* of the ray *CM* with the picture plane, because any other point of this line has the same image. So, a point has to be characterized by two elements: M′ and the image *g′* of a line *g* passing through it (Fig. 2, right is in three dimensions and the projection plane is represented).



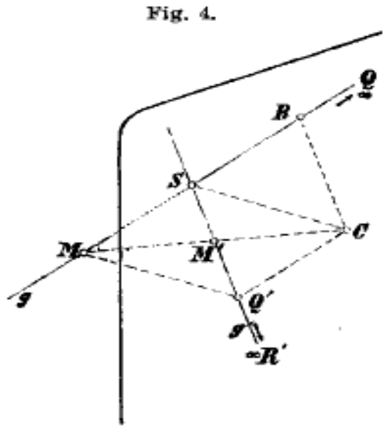


Figure 2. From pages 8 and 9 of Fiedler (1874).

The syllabuses of 1871 undoubtedly covered a great deal of ground. Indeed, the original aim was to prepare students in the physics-mathematics classes for direct entry to the School for Engineers without having to attend the two-year preparatory course. This was in the end not permitted, and in 1876 a new reform took place, based on proposals coming from teacher councils. The aim of mathematics teaching is still that of enhancing the faculties of the mind while acquiring notions which are fundamental for further studies at the University. The syllabuses were reduced. The teaching of projective and descriptive geometry was unified and appeared only in the fourth year. After two years of plane geometry, and one year of solid geometry and trigonometry, the study of projective geometry was whittled down to the study of the projective ranges and pencils, and of the harmonic properties and projective relationships in a circle. Descriptive geometry was restricted to orthogonal and central projections, which were taught together with equalities, similarities, affinities and perspective collinearities. In the following years the teaching of these subjects was further reduced and only some elements of technical drawing remained. Some of the motivations for this reduction are described in Scoth (this volume).

As a consequence, also the idea of deriving descriptive geometry from projective geometry gets loss in the following decades, and from about 1880 textbooks for the Technical Institutes only treat orthogonal projections, while we find the texts by Cremona and Fiedler used in the universities.

Theory and Applications in Italy: a slow separation

The “classic” point of view of Cremona was shared by the majority of the geometers who taught in Italian Universities in the successive years. For instance, Federigo Enriques writes

“The fact of considering under a single aspect (i.e. as falling within the method of the projections) all the methods of representation, is due to the influence of the projective geometry over the descriptive …; moreover, projective geometry returned to descriptive geometry – in a more general and fruitful form - those principles which originated from the latter (Enriques 1894-95, p. 5).

Starting from the end of the 19th century the principal Italian books on descriptive geometry at university level contained a list of the methods of representation of the space onto a plane. As most books of his period, Enriques starts from central projection, beginning with graphic issues and then passing to metric ones. Only in the second chapter he “moves” the centre of projection to infinity and treats orthogonal (Monge’s) projections, again starting from graphic issues and the passing to metric ones. In the third chapter he treats axonometry. This order allows, in Enriques’s point of view, to underline the passage from pure to applied mathematics. This order of exposition changed in the following decades.

In fact, it is possible to make a clear distinction between 19th century mathematical thought, which is characterized by a general symbiosis between theory and application, and that of the 20th century, which shows a clear division between the two aspects, especially on formalist bases. In geometry, this position was to lead to the complete separation of the teaching of projective geometry from that of descriptive geometry and, at university level, to a complete exclusion of descriptive geometry from faculties other than for engineering and architecture. In the universities this separation was anyway slower than in the Technical Institutes (Scoth, chapter 3).

Of a particular interest is the relationship between mathematicians and architects in the first decades of 1900s.

The introduction of mathematical studies in the curriculum of every architect represented one of the main novelties of the early institution, in 1921, of the Superior School for Architecture’. In 1935, the foundation of the Faculty of Architecture in Rome saw the substantial participation of mathematicians in its establishment. Right from its opening, courses of descriptive geometry, mathematical analysis and applications of descriptive geometry were taught to architecture students by some of the leading exponents of the Roman mathematics community of the period, as Enrico Bompiani, Francesco Severi, Ugo Amaldi (dell’Aglio, Emmer & Menghini, 2001).

Outstanding is, for example, the assignment to Severi (to 'His Excellency' Severi, Rector of the University from 1923 to 1925) of a teaching as 'Applications of Descriptive Geometry’ which seems rather marginal, at least from today’s point of view, in particular if compared to the one held in parallel by Bompiani.

How do we explain such an intense participation of Italian mathematicians to the birth of the Faculty of Architecture? Although the answer to this question invests also aspects of a global nature, related to the political weight and image assigned to the Faculty of Architecture during Fascism – we have mainly to consider a conceptual point of view, again linked to the general issue of the evolution of the teaching of geometry in the faculties of a technical nature.

In the Faculty of Science we can indeed notice a gradual weakening of the presence of descriptive geometry as an autonomous course; this corresponds precisely to a separation between the theoretical aspects of the subject and its applications. In 1910 the Italian law provided for two courses: analytic geometry and projective and descriptive geometry with drawing. For a certain period in Rome, and also in other cities, Cremona’s idea was still followed, having the teaching of analytic and projective geometry and the teaching of descriptive geometry with drawing. So, for a certain time, more weight was still given to descriptive geometry, but the road was marked (in fact the teaching of descriptive geometry disappeared from the course in mathematics only in 1960, with the separation of the first two years common to mathematics and engineering).

The climax of the separation is anyway in 1935: the birth of the Faculty of Architecture creates a sudden expansion and autonomy of descriptive geometry and drawing, in contrast with what takes place in the Faculty of Sciences.

In the Faculty of Architecture the teaching of descriptive geometry and drawing can still represent a challenge for Bompiani and Severi. It appears as a return to the origins in that it re-proposes the classical problem of the relationship between concrete and theoretical aspects in geometry, and particularly in projective geometry. It is not surprising that these authors tend to re-propose some aspects of the discussion on geometry teaching present at the beginning of the 20th century.

In the prefaces of the texts of that period, as those by Amaldi or Bompiani, authors always refer to the derivation of descriptive geometry from painting and perspectivity (what is not done by Cremona), nevertheless, the order of the presentation is changed: even if central projection is still treated, the treatises start by Monge’s orthogonal projections, surely most important for the coming architects.

But the link to projective geometry is always present: epitomic is the reference of Bompiani to the classic harmony between descriptive and projective geometry (Bompiani 1942). In the Introduction to his Lessons of Descriptive Geometry he again underlines the didactical role of descriptive geometry. And, quoting Monge, he claims:

«(...) it seems that we can still - and even more - repeat with Monge that Descriptive Geometry is a "means to search for the truth". More modestly we can assert with certainty that it is a necessary link in the chain that leads to secure understanding of higher geometrical truths» (pp. 7-8).

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