Investigating the Impact of Lessons Based on Marzano's Theory of Learning on Student Attitude, Engagement, and Achievement in Grade 10 Academic Mathematics

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Abstract

Motivation is an important construct in education, both for its links to student learning and in its own right as a factor in student development. The relationship between motivation and student learning is particularly important in mathematics since numerous studies have demonstrated that motivation in mathematics is linked to student achievement, and that student achievement and student attitudes toward mathematics are reciprocally related. This study investigated the impact of an instructional intervention that specifically addressed two dimensions of motivation: engagement and student attitudes. Based on Marzano's (1998, 2007) *New Taxonomy of Educational Objectives*©, a unit of study in Grade 10 Academic Mathematics was developed that utilized targeted activities and complete lessons to positively influence student engagement and attitudes. This mixed methods study used pre–post comparisons as well as treatment-control comparisons of 70 students in 3 classes of Grade 10 mathematics to investigate the impact of the instructional intervention on student engagement, attitude, and achievement in order to determine whether such an intervention could function as an exemplar for development of similar interventions that positively impacted student learning. The results of the study showed statistically significant changes in student engagement and student attitudes, but not for student achievement. Implications of these results pointed to directions for future research in this area.

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Table of Contents

List of Tables

Table **Page**

List of Figures

CHAPTER ONE: INTRODUCTION

> Mathematical competency and its development are influenced by affective variables of the learners and characteristics of learning environments which may raise or support motivation. …The awareness of affect and motivation as impact factors on learning has led to a multi-criteria perspective of instructional goals, with a simultaneous focus both on learning goals and goals of supporting motivation. (Kuntze & Dreher, 2015, p. 296)

The purpose of this study was to investigate whether specific instructional strategies that explicitly address student affective dimensions (notably attitude and engagement) can positively impact student engagement, attitude, and achievement in mathematics. The study utilized an instructional intervention with instructional strategies and activities that explicitly target student engagement and student attitude in order to positively influence resulting student outcomes.

Background

Attitude and engagement have a major impact on mathematics achievement and related educational goals (Conner & Pope, 2013; Harlow, DeBacker, & Crowson, 2011; Li & Lerner, 2013; Ouweneel, Schaufeli, & LeBlanc, 2013). Fredricks, Blumenfeld, and Paris (2004) found that a significant percentage of mathematics students lack motivation, have low mathematics self-efficacy, and generally exhibit low levels of engagement as well as negative attitudes towards mathematics. This situation is exacerbated by reliance on a transmission style of pedagogy, which encourages students to become passive content consumers rather than active participants during their learning (Cotic & Zuljan, 2009; Moyer, Robison, & Cai, 2018). Clarkson, Bishop, and Seah (2010) have proposed the concept of Mathematical Well-Being (MWB), which integrates a number of constructs related to motivation under the superordinate construct denoted *value*. MWB consists of students' attitudes, motivation, self-efficacy, beliefs, values, and confidence in their ability to do mathematics, as well as their readiness and cognitive skills. By concatenating Bloom's cognitive taxonomy (Bloom, Engelhart, Furst, Hill, & Krathwohl, 1956), Bloom's taxonomy of the affective domain (Krathwohl, Bloom, & Masia, 1964), and an emotional taxonomy developed by Clarkson et al. (2010), Clarkson et al. produced a five-stage taxonomy of MWB (stage 1=awareness, stage 5=independently competent). However, MWB is a very broad and comprehensive construct involving both student and teacher values and is difficult to utilize as a framework for a classroom intervention. This current study instead integrated activities targeting student affect into classroom activities together with mathematical content, which has been shown to be more effective in influencing affective outcomes, particularly in mathematics (Skilling, Bobis, Martin, Anderson, & Way, 2016).

To date, globally there has been limited success in addressing the impact of attitude and motivation on student achievement (Clarkson, 2013). In Ontario, in a study that explicitly examined issues involving student motivation of over 90,000 students who wrote the EQAO Grade 9 Assessments in 2012, Pang and Rogers (2014) found large effect sizes for student attitudes and student self-confidence on achievement: Student attitude effect sizes ranged from 0.592 to 1.076; mean effect sizes for student-confidence were 0.675; effect sizes for negative attitudes toward mathematics were also large, and ranged from -0.503 to -1.004; and effect sizes related to effort and engagement with homework ranged widely from 0.392 to 1.507. Effect sizes in these ranges have major impacts on student achievement. An effect size of +1.0 represents an increase for an average student performing at the 50th percentile to the 84th percentile. Similarly, an effect size of -1.0 would move student performance from the 50th percentile to the 16th percentile. Smith and Star (2007) claim that traditional methods that focus on content and ignore student emotions, attitudes, and motivation have performed very poorly Therefore, it is imperative to identify ways to positively influence student attitudes toward and motivation to engage in mathematics.

This current study builds on research linking components of motivation (i.e., engagement and attitude) to achievement in mathematics by examining whether instructional strategies specifically targeted at both metacognition and motivation positively impacted student achievement, attitude, and engagement. Motivation is described as “an individual’s desire to act in particular ways” (Walter & Hart, 2009, p. 163); metacognition is “the knowledge about and regulation of one’s cognitive activities in learning processes” (Veenman, Van Hout-Wolters, & Afflerbach, 2006, p. 3). Both constructs have been shown to be important to student learning (Hannula, 2006; Koller, Baumert, & Schnabel, 2001; Malmivuori, 2006; Veenman et al., 2006).

Problem Statement

The purpose of this study was to assess changes in student motivation and achievement after an instructional intervention targeting engagement and attitude. Specifically, the study examined changes in student engagement, attitude, and achievement after an instructional intervention in secondary school (Grade 10) mathematics classrooms in Ontario (Canada), that explicitly addressed two levels of Marzano’s taxonomy: metacognition and self system (motivation). Using pre- and post-measures of these constructs, the study added to the literature on the links between motivation and achievement in mathematics, as well as to the literature on targeted instructional strategies. The study also recognizes that engagement and attitude are important outcomes in their own right, having been shown to be linked to intrinsic motivation and lifelong learning (Ryan & Deci, 2000). The research also provided direction for classroom instruction in mathematics that specifically impacts student engagement and attitudes. Further, this study provided an exemplar instructional intervention on the topic of quadratic relations to assist teachers in structuring similar interventions for other units in the same course or for other secondary school mathematics courses. Northey et al. (2018) identify the need for exemplars as critical: "Student engagement has been an important concern for educators for some time. However, while the benefits of student engagement have received some attention in the literature, a readily identifiable—and easy to implement—method for creating and maintaining student engagement was not clearly evident" (p. 330).

Rationale

In 1998, Marzano proposed a taxonomy of learning domains based on brain research that identified three domains, or levels of processing. Marzano's *New Taxonomy of Educational Objectives*© (MNT) (Marzano, 1998; Marzano & Kendall, 2007) identified three domains or *systems*: cognitive; metacognitive; and self, which includes aspects of student motivation. MNT differs from previous taxonomies in that it comprises three interrelated domains whereas the well-known Bloom's (1956) taxonomy addressed only the cognitive domain. Revisions to original Bloom (Anderson & Krathwohl, 2001) added metacognition, but only as a passive knowledge domain to be acted upon by the active cognitive domain.

Because MNT explicitly addresses self system constructs, such as motivation and emotions, it is appropriate to investigate whether instructional strategies based on this taxonomy can positively influence student attitude and engagement, as well as student achievement in mathematics. Although Marzano and Kendall (2008) outlined ways that MNT could be applied to learning, specifically in designing and assessing educational objectives, little empirical research was found. Indeed, no applications of MNT were found for secondary school education or secondary school mathematics education. This is surprising because MNT has the potential to address attitudes and engagement—dimensions of learning that have been identified as critical for student success and well-being (Clarkson, 2013).

Research Questions

This study was undertaken to answer the following research questions, with respect to an instructional intervention using strategies that specifically address the metacognitive and self levels of MNT (hereinafter called "the MNT intervention"):

1. What is the effect of the MNT intervention on student engagement in a Grade 10 Academic Mathematics classroom(s)?
2. What is the effect of the MNT intervention on student attitudes in a Grade 10 Academic Mathematics classroom(s)?
3. What is the effect of the MNT intervention on student achievement in a Grade 10 Academic Mathematics classroom(s)?

Theoretical Framework

The theoretical framework for development of the instructional intervention was Marzano's *New Taxonomy of Educational Objectives*© (MNT; Marzano & Kendall, 2007). MNT comprises three domains or systems: self (including motivation), metacognitive, and cognitive. These systems act on three passive knowledge domains: information, mental procedures, and psychomotor procedures, as shown in Figure 1. Marzano postulated that when confronted by a task, the self system (examining importance, efficacy, emotional response, and motivation) engages first, with decisions about whether to engage in the task. After deciding to engage in the task, the metacognitive system (specifying goals, process monitoring, monitoring clarity, monitoring accuracy) activates goal setting and monitoring. Finally, the cognitive system (retrieval, comprehension, analysis, knowledge utilization) engages in the actual cognitive work of the task. MNT has implied but not explicit feedback loops, with metacognitive goal monitoring and process monitoring, and self system monitoring comparing current task engagement to other alternatives. More details on MNT are found in Chapter 2.

The instructional intervention used in this study was developed in conjunction with the teachers and utilized the self and the metacognitive system as a foundation on which to build classroom activities and a unit plan that focus on these two systems to increase student engagement and modify student attitudes towards mathematics in Grade 10 classrooms. Details of the intervention are found in Chapter 3.

Scope and Limitations

This study involved 73 students out of a possible 81 students in three Grade 10 Academic Mathematics courses in one secondary school, which may limit the generalizability of the conclusions. In addition, teacher participants were a voluntary sample from Ontario, and thus the classes and students involved constitute a quasi random sample.

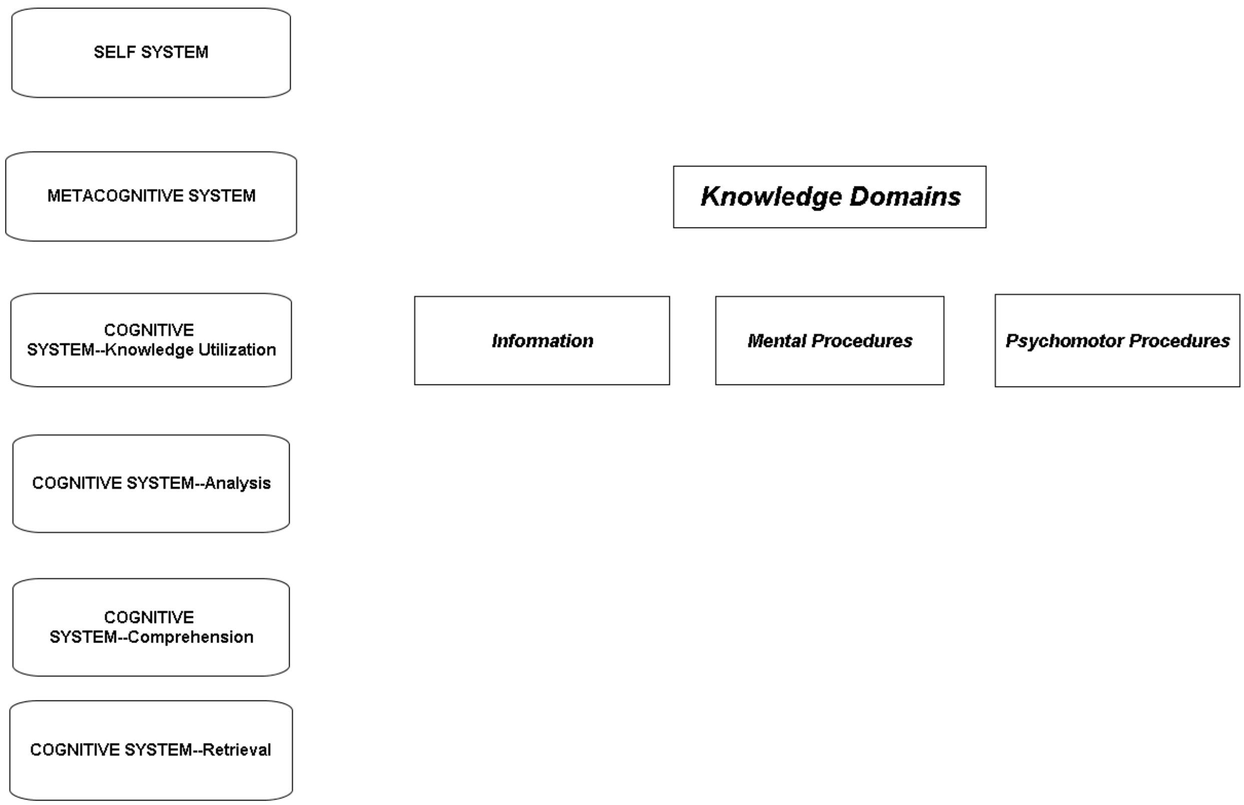

Figure 1. Marzano's New Taxonomy of Educational Objectives. Reproduced with publisher's permission from R. Marzano & J. Kendall (2007), *The New Taxonomy of Educational Objectives* (2nd ed.).

Quasi experimental studies are limited by possible confounding factors such as effects attributable to the teachers involved, student factors in the intervention classes, or interactions among these factors which cannot be controlled for. Since student participation was voluntary and required the consent of students and their parents, this may have resulted in non-response bias, further limiting the randomness of the student sample population. Another possible bias is that the data for this study were self-reported, for students and teachers alike. More details on possible impact of these biases are discussed in Chapter 3.

Importance of the Study

While MNT has been published for 20 years, to date there has been limited research on implementing the taxonomy in the classroom, and no work linking MNT to student attitude and engagement in secondary mathematics. The identification of research-affirmed combinations of strategies builds on the work of Marzano (1998), Marzano and Kendall (2007, 2008), and Marzano, Pickering, and Pollock (2001) in the implementation of MNT. It also links instructional strategies based on the taxonomy to student attitudes and engagement, which are key elements of increasing student motivation in mathematics.

An additional constraint is that bridging the theory-to-practice gap has frequently been problematic (e.g., Nuthall, 2004). This can be attributed to a number of factors, including time to learn and implement the innovation, ease of implementation, and clear and direct relationships between theory and practice (Farley-Ripple, May, Karpyn, Tilley, & McDonough, 2018). Frequently, workplace socialization and school culture mitigate against successful implementation (Allen, 2009; Lattimer, 2015). Yet, “educational research will not have any practical value if it does not affect teaching and learning in classrooms, no matter how brilliant the design or how magnificent the result" (Wang, Kretschmer, & Hartman, 2010, p. 105). By providing teachers with a complete unit instructional intervention, including classroom activities and lesson plans, and by giving teachers "on-demand" professional learning and support when requested, this study mitigates these traditional barriers to theory-practice implementation.

Ontario and Canada have placed comparatively well in international, large-scale assessments, such as the Trends in International Mathematics and Science Study (TIMSS), Programme for International Student Assessment (PISA), and Pan-Canadian Assessment Program (PCAP). However, recently there have been concerns that Canada's and Ontario's rankings, while still very good, have slipped over more recent assessments (Organisation for Economic Co-operation and Development [OECD], 2019). According to the Education Quality and Accountability Office (EQAO), while Grade 9 Academic mathematics scores have been stable in the 84% (of students at level 3 or 4) range, Grade 9 Applied mathematics scores have been consistently below 50%, and Grade 3 and Grade 6 scores have trended down for over 5 years (EQAO, 2019). The improvement of Ontario's level of performance will require increased use of research-affirmed strategies. This current study contributes to knowledge concerning effective implementation of research-affirmed instructional strategies in mathematics classrooms.

Organization of the Dissertation

Chapter 2 presents a review of the related literature, with a focus on constructs that were measured in this study, as well as Marzano's New Taxonomy, which functioned as the theoretical framework for the instructional intervention.

Chapter 3 outlines the research design and methodology, including the rationale for employing a mixed methods methodology.

Chapter 4 presents the analysis of the data collected. Quantitative data analysis techniques include correlational analysis, and both parametric and nonparametric statistical tests (Naiman, Rosenfeld, & Zirkel, 2000). Qualitative analysis employed content analysis (Krippendorff, 2013) using an a priori coding table as well as constructivist grounded theory (Charmaz, 2014) to identify additional themes.

Chapter 5 combines the data analysis from the previous chapter to draw conclusions about the impact of the intervention on engagement, attitude, and achievement in Grade 10 secondary school mathematics. This chapter also includes a discussion of implications of this research for both practice and theory, as well as suggestions for future research.

Given the importance of motivation in mathematics education, this study provides a significant contribution to the research literature, as well as a practical exemplar of how to implement theory into practice.

CHAPTER TWO: LITERATURE REVIEW

This chapter examines literature related to the three variables of interest for this study: engagement, attitude, and achievement. It then reviews the research related to *Marzano's New Taxonomy of Educational Objectives* (MNT), which formed the theoretical framework around which the instructional intervention was constructed. Finally, an alternative theoretical framework, powerful learning environments (PLE), is briefly discussed.

Many researchers treat motivation as a superordinate category that subsumes a number of related concepts such as engagement, persistence, attitude, interest, self-efficacy, and self-concept (Irvine, 2018a). Since it is considered superordinate, motivation involves a wide array of theoretical constructs—such as expectancy-value or intrinsic-extrinsic—and many related theories, including self-efficacy, goal theory, theories of intelligence, choice theory, self-determination theory, and flow, among others (Irvine, 2018a). Because motivation involves such an array of constructs and theories, this current study focused on only two dimensions of motivation: engagement and attitude.

It should be noted that while most researchers accept the superordinate position of motivation, some (e.g., Singh, Granville, & Dika, 2002) envision dimensions such as engagement and sometimes attitude as separate constructs and not within the motivation category. In addition, some researchers use motivation in a very narrow context. For example, the motivation subscale of the *Attitudes Towards Mathematics Inventory©* (Tapia & Marsh, 2005) refers only to students being motivated to take additional mathematics courses and does not address any other aspects of motivation. However, as with the majority of the literature, this current study treated motivation as the superordinate category with both engagement and attitude as subordinates.

Engagement

The first dependent variable in this study was engagement. Engagement has been described as “a positive and inspiring state of mind that is characterized by vigor, dedication, and absorption” (Ouweneel et al., 2013, p. 225). Reeve (2013) emphasizes the proactive nature of engagement, stating that “Engagement refers to a student's active involvement in a learning activity. It functions as a student-initiated pathway to highly valued educational outcomes, such as academic progress and achievement” (p. 579). Engagement is identified as “one of the most important issues facing educators today” (Conner, 2011, p. 53).

The literature usually identifies three components of engagement: behavioural, emotional or affective, and cognitive. (Ouweneel et al., 2013). Behavioural engagement includes basic behaviours such as attending class, following the rules, demonstrating effort, persistence, asking questions, paying attention, positive classroom behaviours, and making an effort (Fredricks et al., 2004). It is the most potentially observable component and has been the most studied (Conner & Pope, 2013). Emotional engagement consists of affective—usually classroom—demonstrations of emotion, such as interest, boredom, happiness, sadness, and anxiety (Fredricks et al., 2004). There are a number of components of cognitive engagement identified in the literature. These include preference for hard work, flexible problem solving, self-regulation, the use of metacognitive strategies, and coping with failure (Conner & Pope, 2013). Several scholars have restricted cognitive engagement to deep versus surface learning (Paige, Sizemore, & Neace, 2013; Smiley & Anderson, 2011).

Many of these concepts overlap with dimensions of motivation and higher-order thinking skills. Most of the definitions of engagement also suffer from identifying the behaviours associated with components of engagement, rather than the underlying constructs.

More recently, Reeve and Lee (2013) have argued for a fourth component to engagement, which they call agentic engagement. Reeve (2013) defines agentic engagement as "students' constructive contribution into the flow of the instruction they receive" (p. 579). Agentic engagement is characterized by student self-advocacy, such as asking questions, offering opinions, identifying areas of student interest, and stating preferences (Reeve & Lee, 2013). Agentic engagement can be identified through the level of response to five statements:

- During class, I ask questions;
- I tell my teacher what I like and what I don't like;
- I let my teacher know what I'm interested in;
- During class, I express my preferences and opinions; and
- I offer suggestions about how to make the class better. (Reeve & Lee, 2013, p. 580)

These questions characterize the proactive nature of agentic engagement; that is, students seek to modify the learning environment to enhance and maximize their own learning. Agentic engagement somewhat overlaps concepts from both emotional engagement and cognitive engagement; however, what is qualitatively different is the emphasis on self-advocacy. Reeve and Lee (2013) conducted a study in which principal factor analysis identified all four components of engagement, including agentic engagement, as separate constructs that are each correlated with student achievement.

There is some debate concerning whether engagement is a trait, possibly domain, variable, relatively stable over time; or a state variable, task dependent, and malleable (Vera, Le Blanc, Taris, & Salanova, 2014). Fredricks et al. (2004) point out that this may vary by engagement situation. For example, for the school domain, behavioural engagement may be high and stable; however, for some subject domains, emotional engagement may be high (or low), and cognitive engagement is likely to vary based on task.

Lilejdahl (2014) identifies engagement with Csikszentmihalyi's concept of flow. While the concept of flow is outside the scope of this study, it is informative to examine the logic behind Lilejdahl's position. He conceptualizes flow (and thus engagement) as the tension between skill and challenge. If the task requires challenge exceeding the student's skill level, the result is anxiety. If skill exceeds challenge, the result is boredom. However, if there is a balance between skill and challenge, the student is engaged and will tend to exhibit observable indicators of all three components of engagement described above. The relationship between engagement and flow is also identified by Fredricks et al. (2004) as being related to emotional engagement. Lilejdahl's characterization appears more compelling, because flow exhibits dimensions of behavioural and cognitive engagement as well as emotional engagement. However, achieving flow is an extremely high standard for identifying student engagement. Clearly, students can be engaged without reaching a state of flow.

A number of scholars have related work on engagement to other theories, usually involving constructs from motivation. These include relating engagement to self-determination theory (Fredricks et al., 2004; Reeve, 2013), social cognitive theory (Smart & Marshall, 2013), self-regulation (Reeve, 2013), and emotional response theory (Mazer, 2013).

Reeve and Lee (2013) postulate a reciprocal symbiotic relationship between motivation and engagement. They theorize that positive changes in motivation will result in positive changes in engagement. These changes in engagement will lead to more positive changes in motivation, which in turn will lead to more changes in engagement, in a reciprocal manner.

Superficially, engagement appears to be a relatively well understood construct, with general agreement on three (possibly four) major components. Fredricks et al. (2004) argue that the relationships among the components are dynamic and engagement is malleable. This is complicated by the place of engagement in the motivational literature, sometimes seen as a subcomponent of motivation and sometimes treated as a separate construct. Engagement is recognized in some studies as an important factor in student achievement, with the role of the teacher being critical in enhancing student engagement at all levels of schooling (Dotterer & Lowe, 2011). Patrick, Ryan, and Kaplan (2007) caution against attempting to link engagement to narrow conceptions of student achievement such as grades. They point out that engagement is a mediator of student beliefs, as are peer influences and teacher influences; that engagement is responsive to the classroom social environment; and that these relationships are reciprocal in that levels of student engagement also influence classroom social environments, peer behaviours, and teacher behaviours. Zyngier (2007) echoes this conception of engagement and claims an important link between engagement and student attitudes.

Measuring Engagement

Ouweneel et al.'s (2013) description of engagement cited at the outset of this section demonstrates some of the difficulties with the construct of engagement. First, it is a latent variable and is unobservable directly. The usual methods for measuring engagement are self-report surveys and observation (Mazer, 2013; Plenty & Heubeck, 2013). Both of these methods have limitations. Self-report surveys are frequently upwardly biased (Hattie & Yates, 2014) and may lack reliability. Self-report surveys may also suffer from social desirability bias (Caskie, Sutton, & Eckhardt, 2014) as well as non-response bias (Mundia, 2011) and other forms of bias that could influence the accuracy of the results. Engagement may also be assessed using interviews, which may be more frequently affected by social desirability bias due to the limited anonymity of the respondent (Desimone, Smith, & Frisvold, 2010). Engagement is a difficult to observe construct, and thus it must be infered from overt behaviours and therefore can be difficult to interpret.

Second, the three traditionally recognized components of engagement (behavioural, emotional, cognitive) are not independent. Fredricks et al. (2004) point out that there are overlapping constructs among these, and that engagement must be viewed as a dynamic interrelationship of the three components. Third, as can be seen from the above discussion on agentic engagement, there is not universal agreement on identifying the components or even how many components of engagement are valid. Finally, there is considerable overlap with other concepts from motivation (Archambault, Janosz, Morizot, & Pagani, 2009). A significant advantage of agentic engagement, as defined by Reeve (2013), is that it can be identified through student responses to the five statements outlined above. While still a self-report, the questions are more factual in nature and thus may have higher reliability.

Fredricks et al.'s (2004) literature review examined engagement measurement concepts and tools across a number of studies. They report that most measurement instruments comingled scales and questions across multiple components of engagement. Further, the actual questions related to each component varied dramatically. Behavioural measures included questions on conduct, persistence, effort, attention, participation, and helpless behaviours. Emotional engagement questions included student–teacher relations, feelings, values, work orientation and persistence, and school orientation and persistence. Cognitive questions listed psychological investment in learning, flexible problem solving, preference for hard work, independent work styles, ways of coping with perceived failure, preference for challenge, and several questions related to intrinsic motivation (Fredricks et al., 2004). Fredricks et al. point out that many of the studies used questions that overlap across two or more of the engagement components. They found that the majority of studies involved behavioural engagement since it is perceived to be the easiest component to observe.

The instrument used in this study was the *Dimensions of Student Engagement Survey*© (DSES; Reeve, 2013) which addresses engagement and disengagement across all four subscales of engagement, namely behavioural, cognitive, emotional, and agentic. The DSES has been validated as a good measure of engagement (Reeve & Lee, 2013).

Engagement in Mathematics Classes

There are limited studies of engagement that are domain specific. Often in these studies, engagement is measured as a component of, or alongside, motivation. For example, Plenty and Heubeck (2013) studied changes in overall motivation among students as they progressed through their high school years; they found that motivation in mathematics is lower than it is in school, in general, and lower than for some other subjects, and that this relationship is relatively stable over time. However, they also found that valuing of mathematics and student self-efficacy in mathematics (possibly related to emotional engagement) increased in the later years of high school compared to the early years. The study reported motivation as the metavariable, with engagement as a subvariable.

Shernoff, Csikszentmihalyi, Schneider, and Shernoff (2003) found very low levels of engagement in mathematics classes, where they reported that the classes had very high cognitive intensity but very low motivation. Students reported being more negative about mathematics and less engaged than any other subject. While many studies report correlations between engagement and achievement in mathematics (e.g., Bodovski & Farkas, 2007; Moller, Stearns, Mickelson, Bottia, & Banerjee, 2014), engagement is also recognized as an important outcome variable in its own right (Collie & Martin, 2017). Engagement has been positively linked to perceptions of mathematics (Fung, Tan, & Chen, 2018); attitudes towards mathematics (Bodovski & Farkas, 2007); student agency in mathematics (Collie & Martin, 2017); and student graduation rates as well as students pursuing higher education (Bodovski & Farkas, 2007).

Teacher Behaviours That Support Student Engagement

Classroom environment and teacher behaviours play a large role in student motivation and engagement. Conner and Pope (2013) note that student engagement declines over the course of a student's time in school, and that "by upper high school, 40% to 60% of students are disengaged" (p. 1427). Schussler (2009) identified the critical role that teachers play in maintaining and increasing student engagement, through what she calls "a synergy of care and high expectations" (p. 116). Archambault, Janosz, and Chouinard's (2012) study emphasized a similar stance in mathematics classrooms. They point out that mathematics teachers are frequently criticized for failing to engage student interest and motivation. They challenge mathematics teachers to engage in behaviours that increase student motivation, engagement, and attitudes. Archambault et al. (2012) emphasize that such teacher behaviours will require significant changes in teacher beliefs about the learning of mathematics, which may be problematic.

Teacher practices must be intentional with respect to student engagement (Skilling et al., 2016). Instructional strategies play a large role in student engagement. Strategies that are active, involve students working in groups, employ problem-based learning, and ask students to explain their thinking (collectively referred to as *reform mathematics curricula*) were found by Moyer et al. (2018) to have long-lasting effects on student engagement. In a survey of Kindergarten to Grade 12, Smith and Star (2007) found that instruction that involved manipulatives, hands-on activities, real-world problems, and student groups had positive impacts on engagement across all grade levels studied.

Another important instructional strategy that encourages engagement is student choice (Irvine, 2018b). Student choice responds to students' need for autonomy, one of the three core needs identified in self-determination theory, together with needs for competence and relatedness (Deci & Ryan, 2008). By providing students with some level of choice in some activities, such as a choice of solution methods, choice of problems to be addressed, or a choice of product to demonstrate their learning, teachers support student autonomy, which has been found to foster increased engagement (Deci, Vallerand, Pelletier, & Ryan, 1991). Conversely, teacher-directed learning was found to have a significant negative effect on engagement and an increase in students' use of avoidance strategies in mathematics classes (Turner et al., 2002).

The instructional intervention designed for the classroom intervention in this study was based on the research-affirmed principles discussed above. Specific instructional strategies chosen from Table 1 recognized the need for intentionally addressing both student engagement and student attitudes through providing choice, manipulatives, hands-on activities, real-world connections, and student social needs through the use of student groups.

Attitude

The second dependent variable in this study was student attitudes. Researchers' comments concerning the importance of attitude include: "a critical construct related to learning" (Vandecandelaere, Seabrook, Velar, Frayne, & Van Damme, 2012, p. 107); "one of the most key factors that relates to achievement" (Mohd & Mahmood, 2011, p. 1857); and "most researchers have verified the link between students' attitude and their performance in mathematics" (Maat, Zakaria, Nordine, & Embe, 2010, p. 201). However, similar to studies of engagement, this position relating attitudes to achievement is not unanimous (e.g., Di Martino & Zan, 2009; Hannula, 2006).

Vandecandelaere et al. (2012) present a number of definitions of attitude, both as a general attribute of learners and also as particularly related to mathematics. Among these are: an evaluative disposition towards some object based upon cognition, affective reaction, behavioural intentions, and past behaviour that can influence cognitions, affective responses, and future intentions and behaviours.

Several studies list beliefs, emotional response, and behaviour as components of attitude (e.g., Yaratan & Kasapoglu, 2012). These attitude dimensions differ from engagement, however, in that they are a priori states, prior to engaging in the task, as compared to similar constructs in engagement, which apply during the task.

Table 1

Instructional Strategies by Domain

Self	Metacognitive	Cognitive
Explicit questions: interest, importance, efficacy	Explicit questions: goal setting, process monitoring, monitoring clarity & accuracy	Explicit questions: similarities and differences, open questions
Open questions	Open questions	Open questions
Student choice	Web and decision trees	Jigsaw
PMI	PMI	RAFT
Journals	Journals	Journals
Post it pileup	Anticipation guides	Inside/outside circle
Placemat	Placemat	Placemat
What/so what double entry	What/so what double entry	What/so what double entry
Graffiti	Graffiti	Graffiti
Four corners	Four corners	Timed retell
RAFT	Timed retell	Ticket to leave
Ticket to leave	Ticket to leave	Problem posing
Graphic organizers	Think aloud	Graphic organizers
Connect to real-life applications	Graphic organizers	Problem posing
Choice on assessments		
Crossword puzzles		

Sometimes, researchers choose to define attitude using only one dimension. For example, in his seminal work on beliefs, attitudes, and emotions, McLeod (1992) defined attitude as “a long-term positive or negative emotional disposition towards mathematics” (p. 787). While this has the advantage of brevity, and it does emphasize that attitude is a long-term disposition, the definition lacks the fullness that is obtained by inclusion of other dimensions beyond the emotional. As can be seen from the above, attitude appears to consist of several constructs chosen from among self-confidence, interest, emotional response, behavioural intentions, value, beliefs, and expected outcomes. Some authors also identify motivation as a dimension of attitude. This is problematic, as motivation is usually considered a superordinate concept and attitude a subordinate concept (Irvine, 2018a). Marzano and Kendall (2007) identify motivation as the superordinate construct with subordinate constructs consisting of importance, self-efficacy, and interest.

Attitudes Towards Mathematics

Factors influencing attitudes of students toward mathematics are complex (Mata, Monteiro, & Peixoto, 2012). Ediger (2012), in discussing quality teaching of mathematics, lists six statements about attitudes that are illuminating: attitudes are evaluative and can be presented on some continuum of favourableness; attitudes vary in intensity and direction; some attitudes are accompanied by or connected with a person’s emotions; attitudes are relatively durable; attitudes are learned and can therefore be taught; and attitudes are related to behaviour. Three of these statements directly influenced this study: attitudes are related to emotions, they are durable, and they can be taught.

Vandecandelaere et al. (2012) state that when analyzing the items on mathematics attitude in the Trends in Mathematics and Science Study (TIMSS) 2003, three dimensions of mathematics attitude were identified: self-confidence, liking mathematics, and usefulness of mathematics. These three dimensions are very similar to a model proposed by Di Martino and Zan (2009) based on qualitative analysis of essay responses of 1,496 Italian students from Grades 2 through 13 to the topic "Me and maths." Analysis produced three dimensions: perceived competence, emotions towards mathematics, and vision of mathematics.

It is also insightful to examine the origins of students' mathematical attitudes. Hannula (2002) developed a four-phase model of attitude towards mathematics. This model proposes that students evaluate a mathematics task in four sequential phases. First is an emotional response to the task; this is typically a quick response based only on emotion at the moment. The second phase is an associative evaluation, based on similarities to tasks that the student has encountered in the past. The third phase is a competency evaluation, where the student decides whether he or she feels competent to attempt the task. The fourth phase is evaluation based on the student's personal goals, both short term and longer term. Each evaluative phase may return positive or negative results, influencing the student's final decision regarding how to engage with the task. This is similar to Marzano's self system.

Lim and Chapman (2013) identify four dimensions of attitude as enjoyment, motivation, self-confidence, and value. The instrument used in this study, *Attitudes Towards Mathematics Inventory*© (ATMI; Tapia & Marsh, 2005) contains the same four subscales. The ATMI is a widely used instrument that has substantial validation in the literature (Majeed, Darmawan, & Lynch, 2013).

Various researchers have identified sources of attitude as parents, teachers, teaching methods, peer groups, self-confidence, previous experiences, motivation, and teachers' evaluations (Yaratan & Kasapoglu, 2012). Of note is the reciprocal nature of some of these, especially self-confidence, previous experiences, and motivation. In Ontario, a recent EQAO (2014) analysis emphasized this reciprocity when it identified students who had previously been unsuccessful on EQAO assessments as having negative attitudes towards mathematics, as well as lower self-confidence and lower overall motivation. These negative attitudes were reflected in lower achievement on the current EQAO assessment.

An extreme dimension of mathematics attitude is math anxiety. Math anxiety includes worry and fear, dislike, frustration, distress, tension, helplessness, and mental disorganization (Yaratan & Kasapoglu, 2012). While math anxiety is outside the scope of this current study, the impact on student achievement of this extreme attitude toward mathematics is noteworthy. In a synthesis of over 800 meta analyses of factors impacting student achievement, Hattie (2009) cites an effect size of math anxiety of -0.34 on mathematics achievement. Thus, a student who was performing at the 50th percentile in mathematics would, due to math anxiety, perform below the 38th percentile.

Teacher Behaviours That Support Positive Student Attitudes in Mathematics

As with engagement, teachers have a major impact on student attitudes. Because attitudes are formed both directly and indirectly through reactions to situations or the environment, they are, therefore, malleable (Vandecandelaere et al., 2012). Measures can be adopted to influence attitudes in a positive direction. Ediger (2012) points out that the teacher serves as a role model for students. If the teacher's attitude reflects interest, enthusiasm, and enjoyment, it is more likely that these attitudes will influence students' attitudes in similar directions (Smith & Star, 2007). Teacher attitudes have a significant effect on both student attitudes and student achievement. Hattie (2009) identified an effect size on student achievement of 1.04 for teacher attitudes with respect to influencing student achievement. Thus, teacher attitudes can change student achievement by more than 34 percentile points. In a more general study of student beliefs and goal orientation, Mesa (2012) found that students' beliefs were significantly affected by teacher behaviours. The instructional intervention in the current study was of a relatively short length (approximately 4 weeks), which McLeod (1992) would predict to have limited impact on student attitudes. However, if changes in attitude do occur, they should have a relatively longer-lived duration.

DeBellis and Golding (2006), expanding on the seminal work by McLeod (1992), identified a taxonomy of affect, consisting of four constructs: *emotions*, which are rapidly changing states of feeling, mild to very intense, usually local or embedded in context; *attitudes*, moderately stable predispositions toward ways of feeling in classes of situations, involving a balance of affect and cognition; *beliefs*, internal representations to which the believer attributes truth, validity, or applicability, usually stable and highly cognitive, sometimes highly structured; *values, ethics, and morals*, deeply held preferences, stable, highly affective as well as cognitive, may also be highly structured, sometimes characterized as "personal truth." In the current study, engagement falls in the emotions category, while attitude, characterized as "moderately stable," is considered somewhat malleable.

Modifications to the learning environment, including teaching strategies, can also have a powerful effect on student attitudes (Vandecandelaere et al., 2012). Domino (2009) identified three dimensions of teacher behaviours that influenced student attitudes toward mathematics: instructional strategies, ensuring student understanding, and teacher care. Instructional strategies similar to those identified for encouraging student engagement also have positive influences on student attitudes (Elçi, 2017). Other factors influencing attitude towards mathematics include perceived teacher care (Cooper & Miness, 2014); teacher's displays of enjoyment (Frenzel, Goetz, Ludke, Pekrun, & Sutton, 2009); teacher fairness (Mata et al., 2012); and classroom climate, especially social climate (Kunter et al., 2008). Ironically, teacher care may also result in social desirability bias when students who perceive their teacher as caring complete questionnaires or interviews about classroom activities (Krupa, 2017). OECD (2016a, 2016b) reported that high levels of teacher-directed activities had negative impacts on student attitudes in mathematics while a mix of direct instruction and student activities had positive impacts on attitudes. As with engagement, these teacher behaviours that support positive student attitudes towards mathematics were central to the design of the instructional intervention used in this study.

Student Attitudes and Mathematics Achievement

A number of correlational studies have identified significant correlations between attitude and mathematical achievement (e.g., Marchis, 2011; Yaratan & Kasapoglu, 2012). However, this correlation has not been universally supported by research, which in some cases found very weak correlations or no correlations at all (e.g., Di Martino & Zan, 2009; Hannula, 2002). Vandecandelaere et al. (2012) state that "attitude towards mathematics is a vital matter in mathematics education" (p. 107). This consensus is all the more surprising considering that the definitions of attitude are very diverse.

All of these dimensions with respect to attitude require teachers of mathematics to have sufficient content knowledge for teaching mathematics (Ball & Bass, 2003). Ediger

(2012) lists considerations around pedagogical content knowledge; necessary knowledge includes: what ideas about or understanding of a concept students are likely to have before instruction; typical difficulties students tend to have in learning a given concept or topic; what order to introduce concepts and skills to minimize confusion about a topic; what strategies work to help different kinds of students overcome common difficulties; how to choose and use instructional materials; what models/analogies/visualizations/activities work well to convey specific understandings; and, how to assess what students have learned about a given topic.

Achievement

Student achievement is the third dependent variable in this study. Student achievement is the demonstration of student learning, which may be subdivided into surface learning and deep learning. Higher order thinking skills (HOTS) are one of the dimensions of deep learning, together with integrative learning and critical reflection (Campbell & Cabrera, 2014). In deep learning, students make connections and integrate knowledge into internal cognitive networks. Deep learning can be contrasted with surface learning, which focuses on facts and basic procedures (Campbell & Cabrera, 2014). HOTS can be contrasted with lower order thinking skills (LOTS), which consist of basic recall of facts or procedures, or application of a known procedure in a known situation. A specific situation may require LOTS or HOTS, depending on the learner's prior knowledge (Lewis & Smith, 1993). For example, in mathematics a problem that can be solved using the sine law may be LOTS, if the learner has seen similar problems before, or the problem may require HOTS, if the problem situation is new to the learner. Lewis and Smith (1993) offer this definition of HOTS: "Higher order thinking occurs when a person takes new information and information stored in memory and interrelates and/or rearranges and extends this information to achieve a purpose or find possible answers in perplexing situations" (p. 136).

With respect to Marzano's New Taxonomy (discussed in the next section), HOTS include all the sublevels of the Metacognitive system, all sublevels in the Cognitive domain of Knowledge Utilization, and the sublevels Generalizing and Specifying of the Cognitive domain Analysis. The sublevel Specifying refers to predicting and may include formulating a hypothesis. Formulating hypotheses will also fall into the Knowledge Utilization categories of Experimenting and Investigating. LOTS would consist of the lower two levels of MNT and the sublevels of Analysis not noted above. In the Ontario mathematics curriculum, the seven mathematical processes illustrate the requirement for the inclusion of HOTS. These mathematical processes are included in the curriculum as expectations and must be assessed (Ontario Ministry of Education, 2005). The mathematical processes are: problem solving, representing, connecting, reasoning and proving, selecting tools and computational strategies, reflecting, and communicating. While the mathematical processes are not explicitly referenced in the MNT instructional intervention, these concepts are intertwined with both the cognitive and metacognitive activities in the lessons of intervention.

Teaching for HOTS was positively correlated with increased student achievement, while teaching for both LOTS and HOTS had the greatest impact (Thompson, 2011). Instructional strategies that promote HOTS include cooperative learning, graphic organizers, and think aloud (Gokhale, 1995; Thompson, 2011). Marshall and Horton (2011), in a study involving 22 middle school science and mathematics teachers, found that teacher-facilitated, inquiry-based instruction was correlated with development of HOTS in students. This present study builds on the knowledge utilization level of MNT and uses HOTS to generate metacognitive thinking as well as motivating students to take a more active role in their own learning.

Elliot, Dweck, and Yeager (2017) advocate for assessing student *competence* rather than achievement. They argue that achievement is a narrowly-construed construct typically identified by student grades; competence offers a more precise, clear, and more broadly-applicable definition. Thus, rather than discussing achievement motivation, researchers investigate competence motivation. This conceptualization is also used by researchers Scherrer and Preckel (2019) who state "The definition of competence motivation as the way in which individuals energize and direct their behavior can be directly applied to intrinsic motivation and achievement goals" (p. 212), two important concepts in motivation theory. Based on available data, the current study restricts measures of achievement to student grades; however, this gives rise to distinctions between grades and understanding, discussed later in this study.

Measuring Student Achievement

Student achievement is typically evaluated using classroom assessments such as written tests or performance tasks, final grades, or by standardized test scores (Fung, Tan, & Chen, 2018). Both classroom assessments and standardized tests may be negatively influenced by external factors such as test anxiety, student affective factors such as mood, and other student-specific factors. Thus, one-time assessments may not accurately reflect student achievement. Therefore, it is necessary to have repeated measures of student achievement over time, preferably using different assessment strategies. This stance is reiterated by the Ontario Ministry of Education (2010) in its assessment policy document, *Growing Success*: “Assessment is a process, not an event” (p. 6).

Measuring student achievement therefore must include multiple assessment items and must have measures of both LOTS and HOTS. Assessment strategies for LOTS include written tests and quizzes (evaluated with marking schemes or rubrics), assignments, and sometimes interviews (evaluated with rubrics). Assessment strategies for HOTS include rich assessment tasks, portfolios, presentations, posters or models, role play or skits, journals, and sometimes interviews. Most assessments of student demonstrations of HOTS are evaluated with rubrics. In this study, student achievement was measured with written summative assessments (evaluated with marking schemes) and a rich assessment task (evaluated with a rubric).

In recognizing the distinction between grades and understanding, Widlund, Tuominen, and Korhonen (2018) postulate that this distinction not only arises from differing goal orientation but also stems from students’ needs for academic well-being, and the necessity to evaluate student learning across more variables than simply grades, including affective dimensions. Several of the student interviewees in this study made the distinction between their understanding of concepts and their achievement measured by grades.

Theoretical Framework: Marzano’s New Taxonomy of Educational Objectives

Marzano’s *New Taxonomy of Educational Objectives*© (MNT) was utilized in this research as the theoretical framework for the instructional intervention. MNT consists of three domains or systems (self, metacognitive, cognitive) acting on three knowledge domains: information, mental procedures, and psychomotor procedures. The systems can be further subdivided by strategy (Figure 2): Self-system strategies comprise examining importance, examining self-efficacy, examining emotional response, and examining overall motivation; metacognitive system strategies are goal specification, process monitoring, and monitoring for clarity and accuracy; and cognitive system strategies encompass storage and retrieval, analysis, and knowledge utilization processes.

Unlike Bloom, MNT is not a strict hierarchy but is based on two dimensions: flow of information, and level of consciousness. In top-down fashion, the self system engages first, making decisions about whether to engage in a new task. This is followed by the metacognitive system that sets goals and strategies. Finally, the cognitive system engages at whatever levels are appropriate to resolve the task. Although Marzano specifies a hierarchy among the three systems, there is no strict hierarchy within the cognitive system.

This flow of processing is illustrated in Figure 3. Marzano also argues that his taxonomy is hierarchical based on levels of consciousness, which increase as one proceeds up the taxonomy. For example, retrieval processes may be automatic, requiring a very low level of consciousness; however, knowledge utilization requires significantly more conscious thought, as does goal setting by the metacognitive system, while self system involvement and decision making requires even more.

Marzano and Kendall (2008) published *Designing and Assessing Educational Objectives* to help educators apply the taxonomy, although the work's instructional strategies are somewhat basic and need enhancement and augmentation before using them in classroom situations. MNT formed the theoretical framework for the instructional intervention used in the current study. Chapter 3 and Appendix A provide more detail on how this instructional intervention was structured

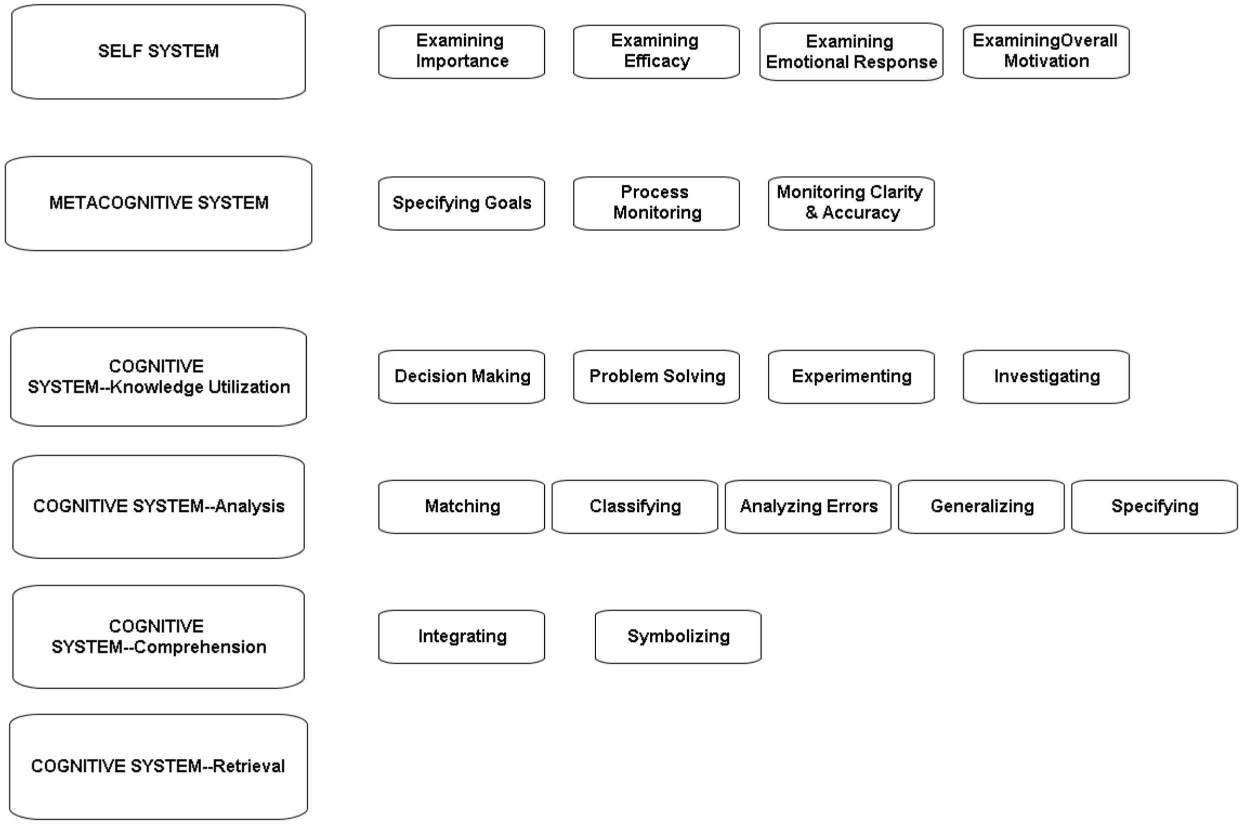

Figure 2. Marzano’s New Taxonomy showing sublevels. Reproduced with permission from R. Marzano & J. Kendall (2007), *The New Taxonomy of Educational Objectives* (2nd ed.).

Since Marzano identifies the self system as the first system to engage, followed by the metacognitive system and then the cognitive system, the discussion below reflects Marzano's sequencing in Figure 3.

Self System: Decision to Engage

Marzano's self system (see Figure 2) enumerates four subsystems: examining importance, examining efficacy, examining emotional response, and examining overall motivation, which is defined as an amalgam of importance, efficacy, and emotional response. Thus, motivation to engage in a task involves (a) perception that the task is important, (b) belief that the student possesses the ability to succeed at the task, and (c) a positive emotional response to the task (Marzano & Kendall, 2007).

This treatment of motivation has elements of expectancy-value theory (Wigfield & Eccles, 2000), self-efficacy (Bandura, 1997; Pajares, 1997), and, in the case of mathematics, MWB (Clarkson et al., 2010). Marzano suggests that repeated feedback loops to the self system occur as students engage in the task. These feedback loops involve verifying that the current task is still more important than possible alternative tasks, a re-evaluation of self-efficacy, based on task progress to date, and reassessment of emotional response to the current task. N what follows, each subsystem of the self system is examined in more detail.

Examining importance: Expectancy-value theory. Expectancy-value theory posits that students' choice of tasks, persistence, and achievement depends on two factors: students' beliefs about their probability of success and the value they place on the task (Wigfield & Eccles, 2000). Students choose a task based on degree of difficulty and the cost associated with that choice (Eccles & Wigfield, 2002).

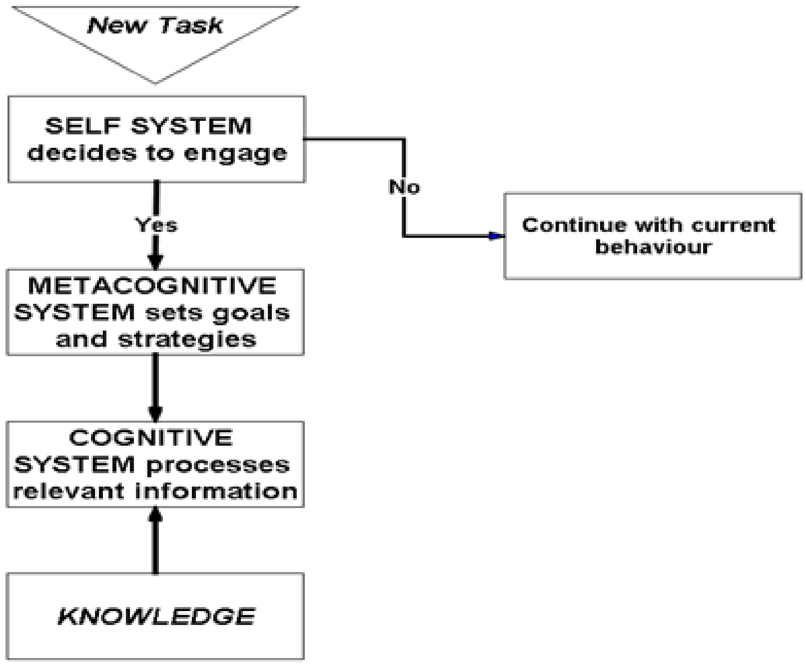

Figure 3. Flow of processing in Marzano's New Taxonomy. Reproduced with permission from R. Marzano & J. Kendall (2007), *The New Taxonomy of Educational Objectives* (2nd ed.).

We thus see the interrelationship between expectancy-value theory and self-efficacy; the students' beliefs about their own ability to accomplish a given task will influence whether they choose to engage in the task.Ball, Huang, Cotton, Rikard, and Coleman (2016) point out that while expectancy and self-efficacy are different theoretical constructs, they are often difficult to separate and hence many studies load them onto the same factors for research purposes.

The importance component of Marzano's self system is a central concept of expectancy-value theory. Marzano asks students to respond to questions such as: How important is this to you? Why do you think it might be important? Can you provide some reasons why it is important? How logical is your thinking with respect to the importance of this? The second subsystem of the self system is examining efficacy.

Examining efficacy: Self-efficacy theory. Self-efficacy (Bandura, 1997; Pajares, 1997) refers to individuals' judgment about whether they are capable of accomplishing a task. In mathematics, students' perceptions about their mathematical abilities are related to their intrinsic motivation (Middleton & Spanias, 1999). Changes in self-efficacy can result in major changes in achievement. S. Ross's (2008) study of PISA 2003 mathematics data found that a one-unit increase in self-efficacy resulted in a 32-unit increase in achievement; she also found that no other motivational variable (intrinsic motivation, goal orientation, instrumental versus relational view of instruction) had as significant an impact on student achievement. Unfortunately, self-efficacy is very resilient and difficult to change (J. Ross, 2009), and it is also domain- and task-specific (Bandura, 1997). Consequently, a student's self-efficacy will vary, sometimes dramatically, for different subjects and for different tasks within a subject; still, given the major potential impact that changes in self-efficacy can have on student achievement, it plays an important role in any study relating motivation and achievement. Self-efficacy has also been found to be positively related to effort and persistence (S. Ross, 2008).

With respect to self-efficacy, Marzano poses questions such as: How good are you at this? How well do you think you can do on this? Can you improve at this? How well can you learn this? How logical is your thinking about your ability to do this?

The third subsystem of the self system is examining emotional response. This subsystem identifies affective considerations as important in the overall decision to engage.

Examining emotional response. Regarding emotional response, Marzano asks questions such as: What are your feelings about this? What is the logic underlying these feelings? How reasonable is your thinking? These questions tend to involve affective dimensions, as well as cognitive questions concerning reasonableness. A major component of emotional response is interest, which can be construed as an emotion, as affect, or as a schema (Reeve, Lee, & Won, 2015).

If considered an emotion, "interest exists as a coordinated feeling-purposive-expressive-bodily reaction to an important life event" (Reeve et al., 2015, p. 80). Interest is activated by the opportunity for new information or greater understanding. With regard to feeling, interest involves an alert, positive feeling; in terms of purpose, it creates a motivational urge to explore and to investigate; as an expression, interest widens the eyelids, parts the lips slightly, and notably stills the head; and in terms of bodily changes, it decreases heart rate. Collectively, this coordinated pattern of reactivity facilitates attention, information processing, stimulus comprehension, and learning (Reeve et al., 2015, p. 80).

A second way of viewing interest is as affect or mood. The two dimensions of affect are pleasure/displeasure and activation/deactivation. The goal of instruction is to place the student's affect/mood in the pleasure-activated quadrant, increasing motivation and stimulating engagement. The third way of viewing interest is as an emotion schema, which is "an acquired, process-oriented, highly individualized, and developmentally rich construct in which an emotion is highly intertwined with appraisals, attributions, knowledge, interpretations, and higher-order cognitions such as the self-concept" (Reeve et al., 2015, p. 82). This conceptualization of interest is closely related to identification of value that enables a shift from situational interest to individual interest. Interest is a predictor of engagement and has been shown to replenish motivational and cognitive resources when an interested student is engaged in an activity.

Interest is positively and reciprocally correlated with self-efficacy (Bong, Lee, & Woo, 2015), self-concept (Durik, Hulleman, & Harackiewicz, 2015), self-regulation (Sansone, Thoman, & Fraughton, 2015), and is also related to valuing of content (Kim, Jiang, & Song, 2015). The value that a student places on particular content is related to the level of interest that the student has for that content. Kim et al. (2015) also demonstrated that interest and value have an impact on engagement and achievement, with self-efficacy acting as a moderator variable. For specific content, it has also been shown that value impacts interest. The greater the value a student places on particular content, the higher the likelihood that the student will demonstrate interest in that content (Ainley & Ainley, 2015).

The four-phase model of interest development (Hidi & Reninger, 2006) presents a taxonomy of interest development. This model postulates that initial interest is triggered by a situation or topic (Triggered Situational Interest), which may be fleeting, and may be positive or negative. If interest in the situation becomes more sustained (Maintained Situational Interest), this phase is characterized by positive student focus and persistence with the material. If the student develops Emerging Individual Interest, they are likely to independently re-engage with the material or classes and ask curiosity questions, building stored knowledge and stored value about the material. Finally, at the Well-Developed Individual Interest stage, the student willingly re-engages with the content, self-regulating to reframe questions and seek answers. This level is characterized by positive feelings towards the material, perseverance through frustration and challenges, and actively seeking feedback on his or her learning. The four-phase model has abundant research evidence supporting it. This present research study focused on the first two levels of the four-phase model, triggered situational interest and maintained situational interest, with the hope that some students will become sufficiently engaged in the material to proceed to the higher two stages of the model.

The last subsystem, examining overall motivation, is an amalgamation of the other subsystems of the self system.

Examining overall motivation. Marzano’s concept of overall motivation is a synthesis of importance (expectancy-value), self-efficacy, and emotional response. In this, Marzano is consistent with Hannula’s (2006) model of attitude as well as Di Martino and Zan’s (2009) three dimensions of attitude. Marzano’s treatment recognizes that students may be motivated across all three of these dimensions, or some subset of them.

Therefore, the strength of a student's motivation will vary depending on the number of dimensions (importance, self-efficacy, emotional response) that are engaged at a specific point in time. Thus, the level of motivation can and will fluctuate across tasks as well as within tasks. A student may approach a task with high motivation but become disinterested as the task progresses. Alternatively, a student may approach a task with low initial motivation, but become more motivated while engaging in the task due to increased self-efficacy and confidence that they can successfully accomplish that task.

Questions posed by Marzano in relation to overall motivation include: How interested are you in this? How motivated are you to learn this? How would you explain your level of interest in this? How reasonable is your thinking about your motivation for this?

Instructional strategies that support the self system and motivation include: choice, open questions, connections to real life, RAFT (role, audience, format, topic), journals, placemat, PMI (plus, minus, interesting), and explicit questioning about aspects of motivation. Additional instructional strategies can be found in Table 1.

Motivation and Achievement in Mathematics

A significant body of evidence shows that motivation in mathematics (or one or more of the dimensions of motivation) has a major role in mathematics achievement (Hannula, 2006; Koller et al., 2001; Malmivuori, 2006). In addition, there is a demonstrated reciprocal symbiotic relationship between motivation and achievement (Koller et al., 2001; Middleton & Spanias, 1999). Further, a *Toronto Star* article reporting on the practice of streaming students in secondary schools stated, "The problem is that student achievement often has more to do with motivation than innate intelligence"

(Maharaj, 2014, para. 1). Thus, low achievement leads to low motivation and low motivation leads to low achievement in a debilitating spiral.

These findings emphasize the important role that teachers play in addressing students' motivation, and breaking the prioritization that Middleton (1995) identified as content goals taking precedence over student motivational considerations in mathematics instruction:

> If mathematics is intrinsically motivating to some students but not to others, it seems reasonable to assume, then, that individual differences among students, and the ways in which mathematics education complements these differences, determine to a large extent the degree to which mathematics is perceived as motivating. (p. 255)

Middeleton and Spanias (1999) found that motivation in mathematics develops early, is highly stable, but is greatly influenced by teacher actions and attitudes. Therefore, motivation in mathematics can be affected through careful instructional designs. For example, Cotic and Zuljan (2009) found that mathematics activities involving problem solving and problem posing had a significant positive impact on cognition and affect, and thus on motivation. Since motivation has been linked to mathematics achievement, teacher actions and attitudes are an important consideration in enhancing student achievement. This current study focuses on two dimensions of motivation, engagement and attitude. While self-efficacy has been shown to have a large impact on achievement, self-efficacy is very resilient and difficult to change, especially in the short term. Since this current study was limited to approximately four weeks duration, the probability of modifying self-efficacy is low, while engagement is malleable in the short term, and attitude can be modified in the intermediate term. Thus, engagement and attitude are the constructs of motivation that served as dependent variables in this study, as well as student achievement.

The primacy of the self system in MNT indicates the importance of student motivation in deciding to engage in a mathematics task. Once the decision to engage has been made, the metacognitive system is activated to set goals and plan strategy to address the requirements of the task.

Metacognitive System: Planning and Goal Setting

The second system in MNT is metacognition, defined by Marzano as a separate system, based on four subsystems: goal specification, process monitoring, monitoring clarity, and monitoring accuracy. As noted previously, Marzano treats the beliefs and self attributes that are included in metacognition by RBT as a separate self system, which Marzano places at the highest level of his taxonomy. There is support for this positioning of metacognition in McCombs and Marzano's (1990) earlier work.

Metacognition has been defined as "the knowledge about and regulation of one's cognitive activities in learning processes" (Veenman et al., 2006, p. 3). Flavell (1979) separates metacognition into two substrata: Knowledge about cognition; and self-regulation, which encompasses control monitoring and regulation of cognitive processes. This dichotomous treatment can be seen in how metacognition is treated in two taxonomies of education—revised Bloom (RBT) (Anderson & Krathwohl, 2001) and MNTR. RBT's focus is on the first substrate, and metacognition is identified as a knowledge domain. The authors indicate that there was significant debate about this placement (Anderson & Krathwohl, 2001, p. 44). It was felt that placement of metacognition as a process would be redundant, since elements of metacognition infuse all the other cognitive processes.

The stance in RBT is consistent with researchers who treat metacognition as declarative knowledge (Veenman et al., 2006). However, Veenman et al. (2006) point out that metacognition subsumes a number of distinctly different constructs, of which declarative knowledge is only one.

MNT identifies metacognition as a separate active system, based on Flavell’s second substrate of self-regulation. This is consistent with Jans and Leclercq (1997), who identified metacognition as active judgments that occur before, during, and after learning. Similarly, Nunes, Nunes, and Davis (2003) commented that a metacognitive approach to instruction can help students learn to take control of their own learning by defining learning goals and monitoring their progress towards them. Thus, metacognitive strategies form part of this current study, both as autonomy support for students and to promote student self-regulation.

The literature is inconclusive regarding whether metacognition is domain specific or general. A literature review by Veenman et al. (2006) found studies that support both positions. Veenman et al. postulate that these conflicting positions may be due to the grain size of the various studies. For example, metacognition with respect to reading strategies (a fine grain size) may have attributes in common with metacognition involved in problem solving (a coarser grain size), since one of the activities in problem solving is reading the problem with comprehension (Veenman et al., 2006). The former position would support the treatment of metacognition in RBT, while the latter position supports Marzano’s treatment.

The different treatments of metacognition have a number of consequences. RBT’s placement of metacognition in the knowledge domains renders it a passive object to be acted upon (Irvine, 2017a). Marzano identifies metacognition as an important, active domain, placed second highest after the self-system in MNT. Activation of the metacognitive system is identified as critical in the chain of processes, falling between motivation to take on a task (self system) and activation of the cognitive processes required for that task. A survey of the vignettes in revised Bloom provides little evidence that their placement of metacognition is appropriate, since there are very few instances where a learning objective is shown to address metacognitive knowledge (Anderson & Krathwohl, 2001, chapters 7-13). Marzano, on the other hand, is able to argue that the treatment of metacognition in MNT gives appropriate recognition to aspects such as goal setting, which is identified in the literature as an important aspect of learning. In a synthesis of over 800 meta-analyses, Hattie (2009) identifies an effect size of 0.56 on student achievement from explicitly teaching goal setting, and also cites an effect size of 0.69 from explicitly teaching metacognitive strategies. This contrasts with Hattie's identification of an effect size of 0.40 for average instructional strategies. There is other support for Marzano's identification of metacognition as an active strategy rather than a passive object. For example, Meijer, Veenman, and Hout-Wolters (2006), in developing their metacognitive taxonomy, conducted a literature review that cites numerous researchers who consider metacognition as active and ongoing throughout a cognitive activity.

Veenman et al. (2006) point to the importance of teaching metacognitive strategies to enhance student learning, and they identify three research-affirmed principles for successful metacognition instruction: embedding metacognitive instruction in the content matter to ensure connectivity, informing learners about the usefulness of metacognitive activities to make them exert the initial extra effort, and prolonged training to guarantee the smooth and maintained application of metacognitive activity. They refer to these principles as the WWW&H rule: what to do, when, why, and how (Veenman et al., 2006, p. 9).

Marzano and Kendall (2008) apply a rather simplistic version of these principles in their text concerning design and assessment of educational objectives, in which they limit metacognition to goal setting, process monitoring, and monitoring clarity and accuracy. Their text ignores other metacognitive strategies such as anticipation guides, think aloud, timed retell, plus/minus/interesting (PMI), and ticket to leave. A number of instructional strategies can be tailored to address any of the three systems specified in MNT. For a more detailed list, please see Table 1.

This current study is consistent with Veenman et al.'s three principles in that the metacognitive instruction is embedded in the mathematics unit involved in the study; students are made aware of the metacognitive strategies being used; and metacognitive strategies are embedded throughout the instructional intervention to help foster maintained application of the strategies.

Marzano's dimensions of metacognition (goal specification, process monitoring, monitoring clarity, and monitoring accuracy) omit some important aspects; namely, planning and evaluating. Meijer et al. (2006) identify these aspects as components of the highest level of metacognition. Because metacognition plays an important role in MNT as well as in his theory of behaviour, this study implemented metacognitive instructional strategies throughout the intervention. Once the metacognitive system has set goals and formulated a plan of action, the cognitive system engages to analyze and perform the required task.

Cognitive System: Performing the Task

The third system of MNT is the cognitive system, with four sublevels: retrieval, comprehension, analysis, and knowledge utilization. Cognition is “the mental action or process of acquiring knowledge and understanding through thought, experience, and the senses” (“Cognition,” 2017, para. 1). Cognition has been identified as an important component of student learning. Since cognition is involved in all learning, the cognitive system was present in all control and treatment lessons of the MNT intervention. The MNT intervention involved modifying or adding to base lessons to explicitly focus on metacognitive and self-system attributes, in addition to the cognitive activities already included in the lessons.

Prior knowledge has been identified as the key cognitive factor in learning mathematics (Milic et al., 2016). Cognitive competence has been shown to be significantly related to mathematics achievement as well as students’ self-rating of mathematical ability (Milic et al., 2016). Of particular note is the notion that “cognition is always for action” (Nathan et al., 2016, p. 1692) since the instructional intervention in this study took an active stance with respect to student learning, which may be different than the more passive mathematics lessons that students had experienced up to this point in their academic careers.

MNT identifies four levels within the cognitive system (lowest to highest): retrieval, comprehension, analysis, and knowledge utilization. Marzano states that they are ordered based on the level of processing required. This position is supported by Nokes and Belenky (2011) who claim that knowledge utilization that supports *far transfer* requires a significantly higher level of processing than other cognitive tasks. The two lower levels (retrieval, comprehension) share similarities with the corresponding levels of RBT. Below is a discussion of the four levels of the cognitive system, beginning with the lowest level, retrieval.

Cognitive system: Retrieval. Retrieval, the lowest level, involves the activation and transfer of knowledge from permanent memory to working memory, usually done without conscious thought. This retrieval may take the form of recognition or recall. Recognition is a simple matching of a prompt or stimulus with information in permanent memory. Recall involves recognition and production of related information. Marzano and Kendall (2007) give the example of selecting a synonym for a word (recognition) contrasted with producing the definition of a word (recall).

Cognitive system: Comprehension. The next level of MNT is comprehension, which consists of two subsystems: integrating and symbolizing. Integrating involves taking knowledge in a microsystem form and producing a macrosystem form for that knowledge. This may involve deleting extraneous information, replacing specific propositions with more generalized ones, or constructing a single proposition to replace a set of less general propositions. Symbolizing involves creating symbolic representations of knowledge, in both linguistic form and imagery. The linguistic form is semantic, while the imagery form involves mental pictures or physical sensations to support cognition. Thus, teachers may frequently employ graphic organizers, which combine both the semantic and imagery forms for a specific knowledge set.

Cognitive system: Analysis. The third level of the cognitive system in MNT is analysis, which has several sublevels: matching, classifying, analyzing errors, generalizing, and specifying (predicting). Matching involves identification of similarities and differences. Matching has been identified by Atkinson, Derry, Renkl, and Wortham

(2000) as a critical component of learning from worked examples. Matching is also important in *near transfer* (Nokes & Belenky, 2011) and in learning through comparison (Rittle-Johnson & Star, 2011). Classifying requires organizing knowledge into meaningful categories. Thus, classifying involves identifying defining characteristics, identifying superordinate and subordinate categories and justifying these categories. Classifying is used in concept comparison throughout formal education (Rittle-Johnson & Star, 2011). Analyzing errors involves the accuracy, reasonableness, and logic of knowledge. Generalizing is the process of constructing new generalizations or inferences from knowledge that is already known. Rittle-Johnson and Star (2011) point out that generalizing typically involves examination of a range of specific cases in order to identify commonalities and critical features. Finally, specifying (predicting) extends a known generalization to other similar situations, and draws conclusions about these new situations.

Cognitive system: Knowledge utilization. The highest and most complex level of the cognitive system in MNT is knowledge utilization, which has four sublevels: Decision making, problem solving, experimenting, and investigating. The knowledge utilization level is unique to MNT, and no similar level exists in RBT, although Bloom’s synthesis category has elements of some of the subcategories of knowledge utilization, without specifically addressing knowledge utilization. Decision making requires selecting among two or more alternatives. This involves thoughtful generation of alternatives and selecting among them based on sound criteria. Problem solving is a cognitive process directed at achieving a goal when no solution method is obvious to the problem solver. Problem solving has also been described as a situation having an initial undesired situation, a desired end situation, and an obstacle preventing the movement from the initial situation to the end situation (Irvine, 2015).

Thus, problem solving requires identification of obstacles, generating alternative ways to accomplish the goal, evaluating the alternatives, and selecting and executing the optimal alternative. Experimenting requires the generation and testing of hypotheses to understand or explain a phenomenon, typically from primary data collection. Alternatively, investigating relates to generating and testing hypotheses based on secondary or historical data. Instructional strategies that specifically address the cognitive system include concept attainment, problem posing, timed retell, jigsaw, open questions, explicit questioning, what/so what double entry, decision trees, and flowcharts (Table 1). The sublevels of knowledge utilization may also serve as significant motivational factors since they have a more active stance for students and involve activities such as investigation and problem solving. Since all learning involves cognition, activities in the instructional intervention utilized in this study employ cognitive strategies as vehicles to stimulate student engagement and interest. These cognitive strategies are not assessed other than through the achievement measures described below.

Powerful Learning Environments

A possible alternative theoretical framework that was considered was powerful learning environments. Vandecandelaere et al. (2012) provide a framework of "powerful learning environments" and identify teaching strategies across four dimensions. The first dimension is *motivate to exert learning*. Examples of these strategies include arousing interest by connecting to the real world, fostering a desire for intrinsic motivation and deep learning, and providing a variety of learning opportunities. The second dimension is *activate towards self-regulated learning*, which includes strategies such as cooperative learning, connecting to prior knowledge, communicating, and offering challenging yet achievable tasks for all learners (similar to Vygotsky's zone of proximal development). The third dimension is *give feedback and coach*; feedback should be given before, during, and after the task, and should focus on next steps. The final dimension is *structure and steer*, which emphasizes planning and sequencing, with the constant goal of deep learning and transfer. The current study addresses all four of these dimensions, as discussed in Chapter 5. However, MNT was selected as the theoretical framework based on its explicit linking of three major constructs, motivation (self system), metacognition, and cognition.

Summary

Engagement, the first variable of interest in this study, is closely related to the *emotional response* subsystem of the self system in MNT. Engagement is context and task specific and therefore malleable in the short term. In the research literature, engagement is frequently identified as a mediator variable with respect to achievement (e.g., Koller et al., 2001).

Student attitudes, the second variable of interst, are related to the *examining importance* and *examining efficacy* subsystems of MNT. This echoes expectancy-value theory (Wigfield & Eccles, 2000), as well as self-efficacy (Bandura, 1997; Pajares, 1997). Attitude is an a priori set of dispositions which are malleable in the intermediate term (McLeod, 1992) but require repeated positive experiences in order to be changed. Attitude is often cited as a moderator variable with respect to achievement (e.g., Middleton, 1995).

Achievement, the third variable of interest, is influenced by the teaching of metacognitive strategies. However, measuring achievement is problematic, with student grades, the most common meaure, being a very narrow view of student achievement. Through structuring an instructional intervention in Grade 10 mathematics using MNT as a theoretical framework, this study investigated whether engagement and attitude can be improved in mathematics classrooms, and thereby improve student achievement.

CHAPTER THREE: RESEARCH DESIGN AND METHODOLOGY

This research investigated the impact of a classroom intervention designed using Marzano's *New Taxonomy of Educational Objectives*© (MNT) on student engagement, attitude, and achievement. This current chapter provides details on methodology, research design, participant selection, instrumentation, and data collection and analysis. The researcher designed the classroom intervention using the instructional strategies identified in Table 1, and selected and administered appropriate instruments for measuring student engagement and attitudes. However, during the administration of the classroom intervention the researcher functioned only as an observer and took no active role in student instruction.

Research Questions

This study was undertaken to answer the following research questions, with respect to an instructional intervention using strategies that specifically address the metacognitive and self levels of MNT ("the MNT intervention"):

1. What is the effect of the MNT intervention on student engagement in a Grade 10 Academic Mathematics classroom(s)?
2. What is the effect of the MNT intervention on student attitudes in a Grade 10 Academic Mathematics classroom(s)?
3. What is the effect of the MNT intervention on student achievement in a Grade 10 Academic Mathematics classroom(s)?

The hypothesis investigated in this study is that students in the MNT intervention classes will demonstrate significant increases across all three variables of interest compared to students in the control class.

Methodology and Methods

Engagement and attitude are latent variables (Reeve, 2013; Vandecandelaere et al., 2012). Most dimensions of engagement and attitude cannot be observed directly but rather must be infered from observations or student responses to surveys or interview questions; this makes measuring changes in engagement and attitude problematic. In order to make warranted assertions about these latent variables, multiple data sources are needed to provide sufficient evidence; therefore, a mixed methods methodology (Teddlie & Tashakkori, 2009) was employed.

Mixed methods methodology is defined as “the class of research where the researcher mixes or combines quantitative and qualitative research techniques, methods, approaches, concepts or language into a single study” (Johnson & Onwuegbuzie, 2004, p. 17). The fundamental principle of mixed research (Johnson, Onwuegbuzie, & Turner, 2007) is to employ multiple data collection methods using different strategies and approaches to obtain complementary strengths and nonoverlapping weaknesses. Thus, dimensions of the problem that may not be apparent using only one data collection method may be revealed when multiple methods are employed. Utilizing a mixed methods methodology also allows for triangulation, whereby results are supported by independent observations of the same result using different data collection methods (Teddlie & Tashakkori, 2009).

On the continuum of quantitative methods to qualitative methods, mixed methods methodology occupies the middle ground; however, quantitative methods and qualitative methods need not be equally represented in a study. The current study can be identified as QUAN+qual, meaning that quantitative methods (student surveys) are the dominant method, followed by qualitative methods (student interviews) which support and elaborate on the quantitative findings.

Theoretical Framework of Mixed Methods Methodology

Mixed methods methodology is grounded in pragmatism, “a philosophy that encourages us to seek out the processes and do the things that work best to help us achieve desirable ends” (Ozmon & Craver, 2003, p. 127). In this definition, we can see elements of the colloquial view of pragmatic, “relating to matters of fact or practical affairs often to the exclusion of intellectual or artistic matters: practical as opposed to idealistic” (“Pragmatic,” 2019, para. 1). On the continuum of idealism to realism, pragmatism certainly lies closer to realism. However, pragmatists like John Dewey would take issue with the phrase “often to the exclusion of intellectual or artistic matters.” While pragmatists claim consequences as the final test for thought, these consequences may be social, aesthetic, moral, or ethical (Ozmon & Craver, 2003).

Johnson and Onwuegbuzie (2004) state that pragmatism as a philosophical foundation for mixed methods methodology is not without significant debates. Quantitative methods are grounded in positivism, the view that there exists an objective truth which can be discerned using scientific methods, and that the observer is separate from the phenomenon that is being observed. In contrast, qualitative methods are based on constructivist or interpretivist views. This position holds that there are multiple versions of reality that are person-centred and that generalizations about phenomena are not possible since they are connected to the viewpoint of the observer. This dichotomy lead to the incompatibility thesis, which stated that the two methodologies cannot be part of the same study because of epistemological differences (Teddlie & Tashakkori, 2009).

However, subsequent discussion resulted in what Teddlie and Tashakkori (2009) call the dialectical thesis, which “assumes that all paradigms have something to offer and that the use of multiple paradigms contributes to greater understanding of the phenomenon under study” (p. 99). This position is supported by the results of the current study, which are elaborated upon in Chapter 5.

Research Design

Employing a mixed methods methodology was appropriate for this study since two of the variables of interest (namely, engagement and attitude) are latent variables that must be infered from observed or reported behaviours. The third variable of interest, achievement, is usually identified as observable, although some dimensions of achievement, such as understanding, must also be infered. Therefore, multiple data sources were needed in order to make warranted assertions about the results of the study. The quantitative phase was exploratory, to determine whether a relationship existed between the classroom interventions and changes in student engagement, student attitude, or student achievement. Because engagement and attitude are latent constructs, the qualitative phase was used to support conclusions drawn in the quantitative phase, through triangulation of data. The qualitative phase was also explanatory, to begin to construct relationships detailing how the classroom intervention affected the student variables, namely engagement, attitude, and achievement.

The independent variable in this study was the MNT instructional intervention. Dependent variables were student attitude, student engagement, and student achievement on the unit of study. Control classes received lessons on the same topics of study, but without a focus on the metacognitive and self domains of MNT. In this way, effects of the intervention on student engagement and student attitudes could be isolated from general classroom effects based on mathematics content and teacher instructional practices.

Selection of Site and Participants

Grade 10 Academic Mathematics (MPM2D; Ontario Ministry of Education, 2005) was identified as the course to be used for this research. By selecting this subject and level, the classes tend to be more homogeneous in background, thus reducing the impact of confounding variables not involved in the study. For example, since usually all students in MPM2D have attained a credit in Grade 9 Academic Mathematics, their background knowledge is somewhat more homogeneous. Socio-economic variables are usually dependent on school location. All classes in this study were selected from the same secondary school.

The board in which this research was carried out was situated in the Greater Toronto Area of Southern Ontario, Canada. It has a large student population and several large secondary schools that typically have at least four sections of Grade 10 Academic Mathematics in each semester. The school board had internal procedures for site selection. The school selected for this study had a student population of 1,914, with five sections of Grade 10 Academic Mathematics occurring in Semester 2 of the 2017-2018 school year.

The school operated on a five-period rotating schedule, although Period 3, which occurred in the middle of the day, did not rotate. On odd days of the calendar the schedule of periods was 1, 2, 3, 4, 5. On even days, this was reversed to 5, 4, 3, 2, 1.

There were four teachers teaching the Grade 10 Academic Mathematics course in Semester 2 of the 2017-2018 school year. Teachers received a letter of invitation

(Appendix B) and an informed consent form (Appendix C). Two teachers volunteered to participate. One teacher had two sections of the course which were designated as the treatment classes (T1 and T2). The second teacher had one class, which was designated as the control class.

Once the volunteer teachers were identified, the students in their classes received a letter of invitation (Appendix D) and an informed consent form (Appendix E). Since at least some of the students were under age 16, both of these documents were addressed to both the students and their parents/guardians, and outlined details of the study, the voluntary nature of the student's participation, the ability to withdraw from the study at any time, confidentiality considerations, and contact numbers for myself as the researcher and the Brock Research Ethics Board. The researcher had no control over the composition of the classes, which had already been assigned students. From a total of 81 students across the three classes, 73 signed consent forms were returned.

The control class (N=23) consisted of 16 female students and seven male students. The teacher had 10 years of experience teaching mathematics and held a mathematics specialist's certificate. This class occurred for 75 minutes at the same time every day (Period 3).

The teacher of the two treatment classes had 22 years of experience teaching mathematics and also held a specialist's certificate. This teacher was very involved in preparing for the study, and she frequently contributed suggestions for activities and sequencing of lessons, some of which were incorporated into the MNT intervention during the discussion phase prior to implementation.

The first treatment class (N=23) consisted of 14 female students and nine male students. The class timetable rotated, occurring Period 1 and Period 5 on a 2-day rotation. The second treatment class (N=22) had 13 females and nine male students. This class also rotated in the timetable, Period 2 and then Period 4. The combined treatment classes were 60% female. More details on class composition and individual student volunteers is found in Chapter 4.

Instrumentation

All quantitative student surveys regarding student engagement and student attitude were conducted using computer software (SurveyMonkey). Since the research was conducted in a bring your own device (BYOD) school board, this use of technology was both appropriate and potentially engaging to students. Classroom teachers were present during survey completion but took no part in the data collection. All student surveys were completed during class time using student personal computing devices. Each survey required approximately 15 minutes to complete. Absent students were asked to complete the surveys on their own time. There were very few missing data points in the student responses. Since all students had individual identification numbers assigned by the researcher, no student data were accessible to their classroom teachers. It was clearly explained by the researcher to all students that any student responses were confidential.

Engagement

Engagement was assessed with pre- and post student surveys, student reflections and interviews, and teacher interviews. To measure student engagement quantitatively, pre- and post surveys using Reeve's (2013) *Dimensions of Student Engagement Survey*© (*DSES*; see Appendix F) were employed. This instrument consisted of 39 five-point Likert scale questions. Written permission to use this survey was obtained from Dr. Johnmarshall Reeve, the copyright holder. This survey had four subscales: behavioural engagement, emotional engagement, cognitive engagement, and agentic engagement. No changes were made to the wording of any question. The survey consisted of blocks of 10 questions on each dimension of engagement (cognitive, emotional, behavioural, agentic). The first five questions in each block asked about engagement; the remaining five questions asked about disengagement and were reverse coded when analyzed. The only change made to this survey was to randomize the question order.

The *DSES* had an excellent Cronbach's α of 0.95 on 39 items. All four subscales (cognitive engagement, behavioral engagement, emotional engagement, and agentic engagement) had strong Cronbach's α values as well (Table 2). This is consistent with reliability values found in the literature (Reeve, 2013).

For qualitative measurements, interview protocols (Jacob & Furgerson, 2012; Whiting, 2008) for the student interviews (Appendix G) and separate protocols for the teacher interviews (Appendices H and I) were developed. These interview protocols were designed by the researcher based on a review of relevant literature and focused on the three research questions of this study.

Attitude

To measure student attitudes, pre- and post student surveys, student reflections, student interviews, and teacher reflections were employed. To measure student attitudes quantitatively, the *Attitudes Towards Mathematics Inventory*© (*ATMI*; Tapia & Marsh, 2005; see Appendix J) was used. This survey used a five-point Likert scale. The *ATMI* was a 40-item questionnaire, with four subscales: value, self confidence, enjoyment, and motivation. It should be noted that the motivation subscale refers to student motivation to take additional mathematics courses. Written permission to use this survey was obtained from Dr. Martha Tapia, the copyright holder. This instrument was used verbatim, with no changes to wording or order of questions.

Table 2

Reliability for Dimensions of Student Engagement Survey and Subscales

Scale	Number of items	Cronbach's α
Engagement (full scale)	39	0.950
Cognitive	12	0.855
Behavioural	10	0.853
Emotional	7	0.898
Agentic	10	0.867

The *ATMI* had an excellent Cronbach's α of 0.978. This is consistent with results in the literature (Asante, 2012). All four subscales, value, self confidence, enjoyment, and motivation had strong Cronbach's α values as well (Table 3); these values were consistent with reliability values found in the literature (Majeed et al., 2013).

The qualitative measurement of attitudes utilized student and teacher interviews, as outlined above for engagement. Therefore, for both engagement and attitude, multiple data sources were developed.

Achievement

The achievement instruments consisted of a rich assessment task (the painted cube task; Appendix K) and written summative assessments. The teachers cooperatively developed a scoring rubric for the rich assessment task (Appendix L). The written summative assessments and scoring guides were developed cooperatively by the teachers involved in the study and were vetted by the researcher. These assessments were scored using teacher-developed marking schemes, vetted by the researcher. All assessments (written tests and rich assessment task) were administered and scored by the classroom teachers. The researcher observed the administration of the rich assessment task but took no active role during the administration of the assessments. This ensured that all classes involved in the study were evaluated in a consistent manner. Thus, the rich assessment task and the summative assessment were quantitative, and the teacher observations were qualitative. Students' prior- achievement data were collected using students' self-reported final grade in their last math course taken, as a letter grade (Appendix Q). Since this self-report was on a five-point Likert scale, it is possible that students' self-reported grades contained an upward bias. Thus, the variety of data collection methods allowed triangulation of all variables of interest (engagement, attitude, achievement) to increase validity.

Table 3

Reliability of Attitudes Towards Mathematics Inventory and Subscales

Scale	Number of items	Cronbach’s α
Attitudes (full scale)	40	0.978
Value	10	0.938
Self confidence	11	0.927
Enjoyment	14	0.962
Motivation	5	0.910

Instructional Intervention

MNT was used as the theoretical framework to develop the instructional intervention. MNT consists of three domains or systems: self (which includes motivation), metacognitive, and cognitive. Figure 2 in Chapter 2 identifies the sublevels of MNT for each system and the literature review in Chapter 2 describes each level and sublevel in more detail. Since cognition is involved in all learning, the MNT intervention focused on the self system and the metacognitive system. Each activity was explicitly linked to an MNT sublevel, as shown in Appendix F. While each activity was linked to a sublevel of the self or metacognitive systems, the effectiveness of this instructional intervention was evaluated on a holistic basis without attempting to disentangle individual effects for each sublevel. The instructional intervention was designed by the researcher, based on instructional strategies outlined in Table 1 (Chapter 2) and explained in detail to the teachers involved prior to deployment in the classrooms. The instructional intervention took an active stance, using activities involving manipulatives, groups, and real-world problems. Technology was viewed as appropriate, since this was a BYOD school. The intervention consisted of a mixture of entire lessons activities interwoven with direct instruction; and short student surveys and goal-setting activities. Appendices M through O give examples of each of these.

Classroom Procedures

Prior to commencement of the study, volunteer teachers were interviewed to determine attributes such as their attitudes towards the teaching and learning of mathematics, knowledge of assessment and instructional strategies, and details of their teaching careers. An interview guide for the pre-interviews is presented in Appendix H.

Teachers were offered up to 2 days of professional development but chose instead to have a series of meetings with the researcher to become familiar with the control and treatment materials. The teachers provided several suggestions with respect to sequencing and strategies, most of which were incorporated into the MNT instructional intervention. This encouraged teacher buy-in and ownership of the intervention. Teachers delivered all lessons to their own classes. With respect to instruction, treatment classes received lessons with instructional strategies based on the self and metacognitive domains, comprising two classes, and the control class received lessons without a focus on metacognitive and self systems.

Throughout the intervention, the researcher was available as a resource but did not engage in any classroom teaching. The researcher observed approximately 25% of classes over the duration of the study, to support implementation fidelity. Observed classes were assessed for fidelity of implementation against eight criteria identifying the degree to which the lessons reflected the expectations of the MNT intervention: matching given sequencing of topics; inclusion of all elements of the MNT intervention; instructional strategies; responses to student questions; use of manipulatives; use of technology; responsiveness to student needs. This method of assessing fidelity of implementation was chosen over self-report surveys (O’Donnell, 2008) and was reinforced through data obtained from teacher post-intervention interviews.

The unit on quadratic functions and quadratic equations was identified by the researcher as the most appropriate for the study, based on an analysis of the units in the course as well as comparisons with other secondary mathematics courses. Grade 10 was selected based on the relative homogeneity of prior knowledge, since all students had completed the Grade 9 Academic mathematics course. In addition, confounding factors such as the transition from Grade 8 to Grade 9, and attending a new (and usually larger) school were minimized since the students had attended the same school in the prior academic year. The detailed Overall and Specific expectations for this unit are listed in Appendix P, and the expectations were explicitly linked to the activities and lessons in the MNT intervention (Appendix A). This unit is one of four units in the course, with the others being linear systems, analytic geometry, and trigonometry. The quadratics unit was the second unit taught in the semester, after linear systems.

As noted, before the treatment all students completed surveys on attitude and engagement (Appendices F and K), on computer, smartphone, or tablet. The surveys included students' basic demographic information (Appendix Q). Each student was given a unique identifier code, to enable post-treatment comparisons. The teachers then taught the first week of the unit. At the end of each week, students completed a brief reflection (Appendix R). Throughout the study, teachers completed brief daily reflections (Appendix S).

Teachers then delivered the next weeks of the lessons and students completed reflections at the end of each week. Two written summative assessments occurred; one assessment occurred part way through the unit and the other a final summative assessment. The summative assessments were created by the teachers involved in the survey. Both summative assessments consisted of written paper-and-pencil tests, scored with marking schemes. The researcher reviewed both assessments prior to their administration. Both written summative assessments were administered and scored by the teachers. At the end of the unit, students completed the rich assessment task (Appendix I), and again completed surveys on engagement and attitude.

After completion of the treatment, student volunteers were identified to participate in audiotaped interviews. The target was two students for each of the treatment classes excluding control classes. Permission forms were given for parent consent (Appendix T). Five students volunteered, and all were interviewed after receiving completed permission forms. An interview guide for these student interviews (Appendix F) was developed. All students were assigned pseudonyms when information was reported in the results section. At the conclusion of the study, both teachers participating in the research were interviewed again, using a separate targeted interview guide (Appendix I).

Data Processing and Analysis

The data consisted of student survey data, student and teacher interview data, student reflections, teacher reflections, and classroom observations by the researcher. With the exception of the researcher classroom observations, all the data were self-reported, which has implications for limitations, discussed later in this chapter. The quantitative data consisted of a mix of parametric and nonparametric data. Student achievement data was parametric. Student survey data were nonparametric, and so several comparisons involved a mixture of parametric and nonparametric data.

There is considerable debate concerning the most appropriate statistical methods to employ with Likert scale scores (Mircioiu & Atkinson, 2017). Jamieson (2004) points out that Likert scale scores are ordinal. They do not possess most of the characteristics associated with parametric statistical tests: interval scales, continuous variables, homoscedasticity, or normal distribution; therefore, Jamieson argues that nonparametric tests are the appropriate measures. However, Norman (2010) argues that parametric tests are robust with respect to violation of underlying assumptions, and therefore can be used to generate valid conclusions for Likert scale data. This position was supported by Poncet, Courvisier, Combescure, and Perneger (2016) using computer simulations of data drawn from various distributions. A third position taken by Peró-Cebollero and Guàrdia-Olmos (2013) specifies that if the sample size is sufficiently large (usually N>30 or N>50) and the distribution is approximately normal (not defined) then parametric tests are appropriate.

A second consideration is statistical power. Parametric tests are frequently cited as having greater statistical power due to their underlying distributions (Zimmerman & Zumbo, 1990); however, using computer simulations, Erceg-Hurn and Mirosevich (2008) and Larson-Hall (2012) claimed that nonparametric tests had equal or greater power, depending on the shape of the underlying distribution.

Both parametric and nonparametric methods were used in this study. Paired *t*-tests were used to analyze pre–post comparisons. One-way ANOVA was used for the treatment-control comparisons. Treatment-control comparisons were made for T1-control, T2-control, and T_{Total}-control. Levene's test of equal variances (Derrick, Ruck, Toher, & White, 2018) was used to ensure that results were not affected by differences in variances among the various classes. Results were confirmed using nonparametric methods (Mann-Whitney U tests, Wilcoxon signed rank tests). The significance level for all tests was set at $\alpha=0.05$.

Quantitative

Correlational analysis was used to investigate associations among engagement, attitude, achievement, and prior achievement for all students in each class participating in the study, including the control class. Since much of this data was nonparametric, Spearman's rho was the appropriate correlational measure (Naiman et al., 2000). The variables of interest were engagement and attitude (pre- and post), changes in engagement and attitude (pre- and post), incremental changes in engagement and attitude of the treatment classes with respect to the control class, and achievement. Control class and treatment classes were compared and individual students' pre- and post- responses in attitude and engagement, and effect sizes were estimated (Naiman et al., 2000).

Qualitative

All interview data were transcribed by the researcher, and member checking occurred to ensure the accuracy of the transcription. The student and teacher interviews were coded and vetted by the researcher using inductive content analysis (Krippendorff, 2013) to identify themes. An a priori coding table (Saldana, 2014) was developed for this purpose (see Table 4). The entries in the table were based on look-fors cited in the literature as well as additional look-fors identified by the researcher based on his extensive classroom experience. Subsequently, constructivist grounded theory (Charmaz, 2014) was employed to identify unanticipated themes that were not found using a priori coding. Student interview data and teacher interview data were dealt with separately. While five student interviews are insufficient to independently develop theories using constructivist grounded theory, the interview data were nonetheless used to support conclusions drawn from other sources through triangulation. The teacher interview data was used to provide supporting evidence of themes identified in the student data analysis.

Table 4

A Priori Coding Table

Construct	Look-fors
Engagement	Interesting Worked hard Slacked off Asked questions Did homework Didn’t do homework Exciting Useful Active Thought about it
Attitude	Interesting Useful Valuable Enjoyed Looked forward to class Confident Uninteresting Boring Not useful Uncomfortable Afraid Scared
Achievement	Did well Did better Didn’t do as well Understood Didn’t’ understand Was difficult Did OK Higher marks Lower marks

Limitations

Many of the possible limitations in this study relate to self-reporting of data. Mundia (2011) points out that "self-report questionnaires are never accurate instruments" (p. 207). They suffer from a number of possible biases, such as non-response bias and social desirability bias.

Non-response bias involves survey participants either refusing to participate in the survey entirely or refusing to answer some or all of the questions (Mundia, 2011). This can result in sample bias, where a segment of the population is underrepresented, or in missing data. Non-response bias was not found in this study; there was a high level of returned consent forms (90.1%) and the very low rate of omitted questions (0.14%).

Social desirability bias (SDB) involves survey respondents providing answers that place them in a favourable light with respect to the opinions of others (Caskie et al., 2014)— and particularly in this case their teacher, even though in this study the teachers had no role in the data collection. Thus, students may respond based on what they think their teacher (or the researcher) would like to hear, or what is socially acceptable in their society or peer group. Mundia (2011) found that SDB is most common in situations such as observations or interviews, or when responding to open-ended questions on surveys. Since the usual method for addressing social desirability bias is to ensure anonymity (Porter, 2013), in this study anonymity was supported by students being assigned identifier numbers that were not known to their teachers, and by teachers not being involved in either the survey completion or the student interviews. Further, student interview responses were compared to their survey responses (quantitative data), to identify any SDB that may have occurred in face-to-face interviews.

In investigating whether responses differ if surveys are administered using paper-and-pencil versus online surveys, Campos, Zucoloto, Bonafe, Jordani, and Maroco (2011) found no difference in validity or reliability and cited several advantages of on-line questionnaires: "higher perception of anonymity; absence of interviewer supervision; lower social interactions with interviewers and other respondents; lower social desirability pressure; and larger environmental variability" (p. 1881). Since this study was done in a BYOD board, use of online surveys was both appropriate and within normal classroom routines.

Another consideration was sample size. Cheung and Slavin (20106) found that smaller samples (less than 50 subjects) were able to maintain greater implementation fidelity. The cost for this implementation fidelity involved lower statistical power and hence less generalizability. This current study involved approximately 70 students and two teachers. The small number of volunteer teachers should have helped to maintain implementation fidelity, while the number of subject students provided a measure of statistical power for the findings.

In conclusion, while it was important to be aware of the possible biases such as SDB in self-report questionnaires, the potential impact of these biases was minimized in a number of ways: teachers and students were volunteers; questionnaires were relatively short, thus limiting fatigue; questionnaires were completed using technology, increasing interest and engagement; questionnaires asked about students' own experiences and emotions, and were related to students' current real life; and questionnaires had student identifier numbers and interviews were anonymized using pseudonyms, increasing the probability of accurate responses.

Ethical Considerations

Any research involving human subjects requires consideration of the ethics involved. Impacts on students from this study were minimized, since (a) all lessons were delivered by their regular classroom teacher; (b) the length of the study was approximately 4 weeks, approximately 20% of the total class time in a semester; and (c) the proportion of the students' final grades directly dependent on this unit of work was commensurate with the proportion of time spent in class on this topic, and the results of the study had no direct impact on students' marks.

Informed consent was obtained from the students as well as their parents or guardians. A letter outlining the intent of the study, its duration, and potential benefits was sent to all participants and their parents, as well as a separate letter to teachers who were considering participating in this study. The letter explained that participation in the study is voluntary, that students may choose to opt out of the study at any time, and that all student data were anonymous and confidential. The letter also contained a mechanism for asking questions. It stressed that the outcome of the study had no bearing on the students' final grades, and that the results of the study could lead to better instruction for students in the future. The researcher personally explained the study to the students and answered any questions. Students who opted out remained in their class, and received instruction from their teacher, but the students' data were excluded from the study.

The mechanism for identifying students who volunteered for follow-up interviews was outlined in a separate letter, again stressing that this participation was voluntary. Additional written consent was obtained for the students who were identified for follow-up interviews, signed by both the students and their parents.

All data collected from the teachers and students were securely stored electronically for the duration of the study, in a password-protected location. Dissemination of results will be anonymized and reported accordingly. At the end of the study, all student and teacher data will be destroyed. This study was approved by the Brock Research Ethics Board (file # FAXIO 17-096).

Summary

In summary, this study sought to examine whether instruction based on Marzano's New Taxonomy (MNT) that explicitly targeted dimensions of student metacognition and motivation had positive impacts on student engagement, attitude, and achievement. The MNT instructional intervention was implemented in a naturalistic way in all three Grade 10 Mathematics classrooms. The researcher was present only as an observer and the regular classroom teacher conducted all instruction. Student data for the study were completely confidential and the classroom teachers had no knowledge of any student responses or interview answers. There was enthusiastic buy-in from both the teachers and the students as evidenced from the high participation rate and very low number of questions omitted from the student surveys.

CHAPTER FOUR: RESULTS

This chapter presents the findings from the MNT intervention study. The findings are given in the order of the research questions (i.e., engagement, attitude, achievement). Within each section, quantitative findings are presented first (both pre-post and treatment-control), followed by qualitative findings. Quantitative data were analyzed using both parametric and nonparametric statistical analysis. No notable differences were found from using the two methods. Qualitative analysis consisted of both content analysis and constructivist grounded theory analysis to identify themes. Because only five student volunteers were interviewed, it is not possible to identify theories related to this MNT intervention. The quantitative and qualitative findings are followed by details on the classes involved as well as the two teachers (control and treatment), since some characteristics may have influenced the conclusions of the study.

This study was undertaken to examine the impact of a classroom intervention (the MNT intervention) on three aspects of student learning: engagement, attitude, and achievement. This was a mixed methods study consisting of student surveys, which were analyzed quantitatively; student post-intervention interviews, analyzed qualitatively; and teacher pre- and post interviews, as well as 20 classroom observations by the researcher, The study involved three classes of Grade 10 Academic Mathematics at one high school in Ontario, Canada. One class functioned as a control and did not receive the MNT intervention lessons. The two treatment classes received lessons that focused on motivation and metacognition while covering the same content as the control class. Table 5 provides details of the three classes.

Table 5

Classes Involved in the MNT Intervention Study

Class	N	Female	Male	Timetable period(s)	Teacher pseudonym
Control	23	16	7	3	Ms. Alford
Treatment (T1)	23	14	9	1, 5	Ms. Beckham
Treatment (T2)	22	13	9	2, 4	Ms. Beckham
Combined treatment classes (T_{Total})	45	27	18		Ms. Beckham

Engagement

Engagement was the first dependent variable to be examined. Both quantitative and qualitative data were collected and analyzed.

Quantitative Findings

All students completed pre- and post-surveys of engagement using the *Dimensions of Student Engagement Survey*© (*DSES*). The *DSES* is a 39 question five-point Likert scale survey (1=strongly disagree to 5=strongly agree; Appendix E).

Pre–post comparisons. Scores for students in the treatment classes (T_{Total}) are shown in Table 6. When pre- and post measures of overall engagement for T_{Total} were compared, a statistically significant difference and positive effect size of 0.54 was found (M=0.527, SD=0.694, $t(45)=5.29$, $p<0.001$). This effect size is considered medium (Cohen, 1992) and suggests that the MNT intervention had a positive impact on student engagement. All four of the engagement subscales of the *DSES* had statistically significant increases (Table 6).

The control class pre–post results were considerably different (Table 7). Neither the overall engagement scores nor any of the subscales showed significant differences.

For T_{Total} a comparison of pre-intervention scores and post-intervention scores shows that while engagement and all subscales increased significantly, the greatest increase occurred for the agentic engagement subscale (Figure 4). Agentic engagement represents student self-advocacy such as asking questions and telling the teacher which learning activities best fit the student.

For students in the treatment classes, 84% showed increases in self-reported overall engagement scores (M=0.44, SD=0.816, min=-1.46, max=3.48), as illustrated in Figure 5.

Table 6

Pre- and Post-DSES Scores for Treatment Students (T_{Total})

		Pre		Post					
	N	Mean	Std. deviation	Mean	Std. deviation	*t*	df	Sig.	Cohen d
Engagement	46	3.05	0.079	3.57	0.080	5.209	45	<0.001***	0.54
Emotional	46	2.96	0.756	3.68	0.659	6.216	45	<0.001***	0.65
Behavioural	46	3.39	0.093	3.73	0.618	2.868	45	0.006**	0.38
Agentic	46	2.78	0.087	3.60	0.836	6.991	45	<0.001***	0.73
Cognitive	46	3.12	0.084	3.44	0.947	2.936	45	0.005**	0.31

Note. ** significant at p=0.01; ***significant at p=0.001

Table 7

Pre- and Post-DSES Overall and Subscale Scores for Control Class

		Pre		Post				
	N	Mean	Std. deviation	Mean	Std. deviation	*t*	df	Sig.
Engagement	22	3.38	0.722	3.61	0.538	1.100	21	0.284
Emotional	22	3.70	0.693	3.84	0.590	0.873	21	0.392
Behavioural	22	3.56	0.600	3.63	0.651	0.380	21	0.708
Agentic	22	3.32	0.680	3.60	0.593	1.916	21	0.069
Cognitive	22	3.49	0.571	3.30	0.723	-0.998	21	0.329

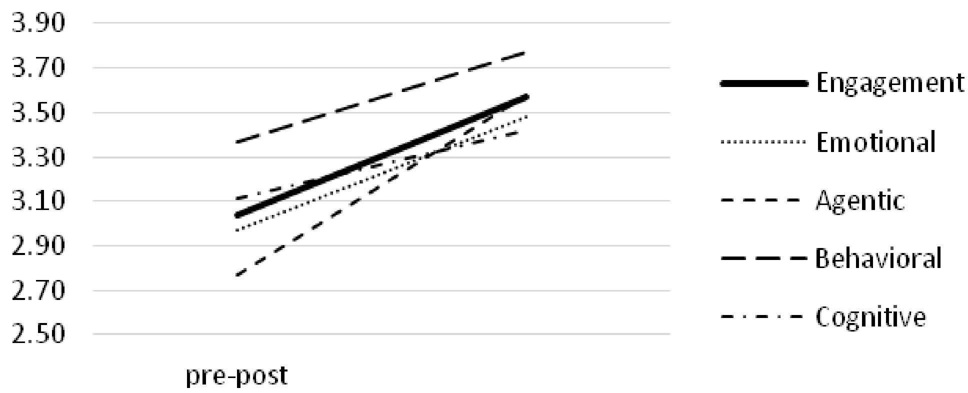

Figure 4. Changes in Engagement and subscales for treatment students pre-intervention and post-intervention. Overall and all subscales significant.

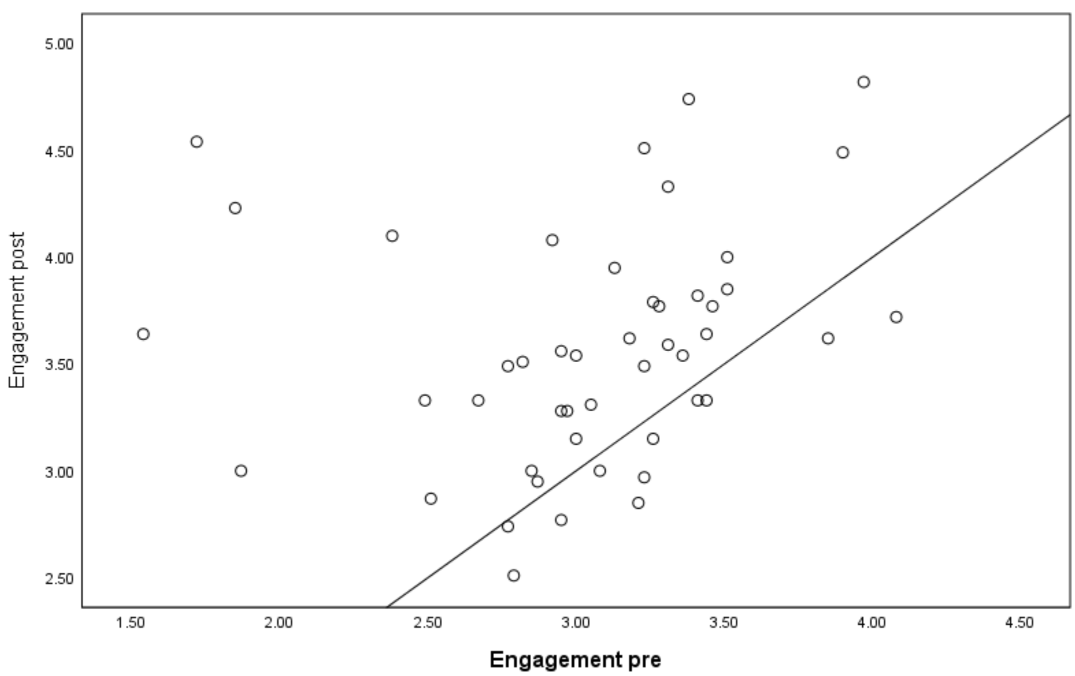

Figure 5. Comparison of Engagement scale post-intervention and pre-intervention for treatment students. The straight line represents engagement-post=engagement-pre (no change after the instructional intervention).

The straight line in Figure 5 indicates no change in pre- and post scores. The largest increases (Engagement post minus Engagement pre) were shown by students who had reported the lower scores in the pre-intervention survey (see Figure 6). In Figure 6 each data point gives the change (Engagement pre minus Engagement post) for a particular student in the treatment group.

Treatment-control comparisons. When treatment-control comparisons of engagement were made prior to the MNT intervention, the control class showed a significant differential advantage over T_{Total} (M= 0.34, SD= 0.158, t(66)=2.140, p=0.036). After the MNT intervention, no significant differences were found for the control class (M=-0.24, SD=1.024, t(21)=-1.100, p=0.284) compared to the treatment classes. Thus, after the MNT intervention, the level of engagement for both treatment classes had increased, while no change was found for the control class.

Qualitative Findings

A voluntary sample of five students from the treatment classes were interviewed after the MNT intervention. Three students were from T1 and two students were from T2. All were assigned pseudonyms for purposes of analysis. Table 8 provides further information on the students who were interviewed.

Below are some comments made by each student, based on the post-intervention interviews, May 7 to May 9, 2018.

Wendy:

- Likes to learn about new things
- Has strong work habits but doesn’t feel that she is especially good at mathematics
- Likes mathematics less than most other subjects
- Compared to other math units she feels that she did a little bit worse on the quadratics unit, but she understood the concepts

Lina:

- Feels that school is necessary for her future career
- Favourite subject is Drama
- She works had but sometimes other commitments interfere with completing homework
- Does not feel engaged in math class most of the time
- Wants to be an actor or a middle school teacher
- Was strongly influenced by a middle school teacher when she was in Grade 7

Dani:

- Enjoys school and doesn't really struggle with anything
- Favourite subject is Science
- Likes sports and plays on school teams
- Does not feel engaged in math class most of the time
- Planning to fast-track and go to university early

Rob:

- Likes the social aspect of school and friends
- Favourite subject is Computer Science
- Is a visual learner and doesn't feel that his learning style fits math class
- Feels pressure from family, who are both professionals
- Feels that he understands math concepts but his test scores do not reflect his understanding
- Plans to go into Software Engineering

Shelly:

- Likes school for both the learning and social dimensions
- Favourite subject is Visual Arts
- Feels pressure from family to perform well in school
- Plans a career in visual arts or architecture
- Is especially conscious of teachers who care about her

In addition, both teachers were interviewed pre- and post the MNT intervention. Teacher characteristics and pseudonyms are shown in Table 9.

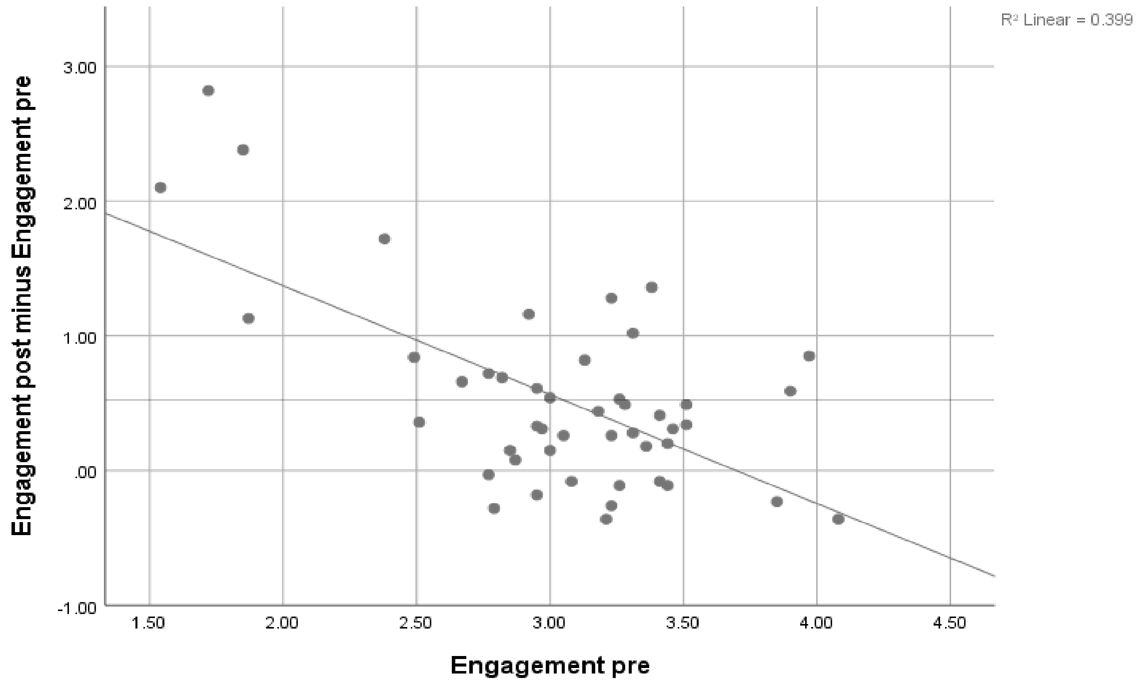

Figure 6. Comparison of magnitudes (Engagement post minus Engagement pre) for each student, showing that greatest changes occurred for students with initially low engagement pre scores. Trend line shown.

Table 8

Convenience Sample of Student Volunteers Who Were Interviewed

Pseudonym	Sex	Grade last year	Quad mark %	RAT %	Term XQuad %	Attitude		Engagement	
						Pre	Post	Pre	Post
Wendy	Female	80%+	75	68	87	2.67	3.33	4.10	4.60
Lina	Female	80%+	44	78	62	3.44	3.64	2.18	3.08
Dani	Female	80%+	88	68	90	3.13	3.95	2.62	3.23
Rob	Male	70%–79%	56	68	62	3.31	4.33	2.79	4.73
Shelly	Female	70%–79%	63	78	85	3.00	3.15	3.87	4.10

Table 9

Teacher Characteristics

Category	Sex	Pseudonym	Qualifications	Years of experience	Years in current school	Dominant teaching styles*
Treatment	Female	Ms. Beckham	Honours Specialist in Mathematics	22	<1	Command; Practice; Guided discovery
Control	Female	Ms. Alford	Honours Specialist in Mathematics	10	<1	Command; Practice; Guided discovery

Note. *Based on teaching styles classifications in Fernandez-Rivas & Espada-Mateos (2019).

The dominant teaching styles of both Ms. Beckham and Ms. Alford were teacher-directed, which would be classified by Fernandez-Rivas & Espada-Mateos (2019) as *command and practice*. However, both teachers also indicated that they sometimes engaged in guided discovery. Ms. Beckham stated "I do a lot of investigations" (Pre-intervention interview, February 21, 2018). However, the researcher observed that in classes that were rated as *low* on fidelity of implementation, Ms. Beckham utilized command and practice styles exclusively.

The quantitative and qualitative results are consistent with comments made by Ms. Beckham in her post-intervention interview: she indicated that she observed a noticeable difference in student engagement during the MNT intervention. She attributed this increased engagement to the active nature of the lessons; increased social interactions due to more group work; the use of manipulatives; and technology. She also noted that when she reverted to more traditional, teacher-lead lessons, the levels of engagement decreased.

Ms. Beckham's comments were consistent with comments of the five students who were interviewed. All students interviewed specifically mentioned the rich assessment task (Appendix I) as very engaging, since it was active, hands-on, employed manipulatives and technology, and student groups for the data collection stage. Another strategy specifically mentioned by the interviewees was the teacher's use of individual student whiteboards. Comments on this instructional strategy identified uniqueness, immediate feedback, and an active learning stance in the classroom. Grounded theory analysis of student interview comments identified several themes. Fun was a theme commonly cited in the interviews. The MNT intervention was considered significantly more fun than the students' usual math classes. A second theme was energy, since the interviewees felt that the classroom exhibited more energy and activity levels than normal. This energy was attributed to increased social activities and variety of learning opportunities. Increased use of student groupings was a third theme, identified by all five interviewees. Again, this was contrasted with the usual math classes, which tended to be teacher-directed and utilized students sitting in rows and working individually. The use of hands-on activities, manipulatives, and technology was a fourth theme. While the school was in a BYOD board and technology was routinely employed, activities using manipulatives such as algebra tiles and other hands-on materials (e.g., see Appendix M) was seen as a positive change by the student interviewees. Student comments identified this as a fourth theme that influenced engagement, variety of activities, and classroom organization changes. Finally, students identified positive changes in work habits based on interesting and relevant relationships between the mathematical content and the real world. Three of the students interviewed indicated that prior to the MNT intervention, their approach to classwork and homework was somewhat sporadic since they found the mathematical content to be uninteresting and sometimes boring. Figure 7 provides sample student comments on themes related to engagement.

Specific references were made to the increased social atmosphere in the class due to numerous group activities. When asked to identify one activity that was very engaging, Wendy stated

> When we did a bunch of activities in little groups and we passed them around the class. Because we got to work on the questions with other people. So, you got to put in your input but also have them. So, if you didn't know something, they might know it and if you complete the question the class you felt you were really pleased and tired. (Post-intervention interview, May 7, 2018)

As can be seen in Figure 7, Lina echoed this stance. This contrasted with the students' views that usually math classes were routine and teacher-centred.

At least three students felt that the active learning stance of the unit was a much better fit for their personal learning modality than the normal classroom routine which often employed a document camera and teacher-lead lessons; that their level of engagement was content and topic specific, in that they felt more engaged during some activities than others; and that there was an increased energy in the classroom compared to the normal classroom routine. Shelly (see Figure 7) commented on how this differed from the usual atmosphere in math class. Rob commented on this increased energy when he referred to his surprise that other students became more engaged: "I participate in class almost all the time. But what I've seen is more kids getting engaged. Kids around me that I don't expect to get engaged getting engaged. So that was interesting" (Post-intervention interview, May 8, 2018). This student's behaviour appeared to be contagious in that three students described other students in the class appearing to be more engaged than usual and that this behaviour seemed to influence additional students to become more engaged as well. Both the students and the teachers emphasized that engagement was task and context specific:

> I liked the activities where you had to work with your peers. Obviously, it's a very different model than you usually do, come with the whole equation together so that was obviously -- you're forced to work with everybody else. So, think about it. I like working with others. It's interesting to see what they say. So, it kind of breaks my thoughts. Usually math class is not like social. (Dani, post-intervention interview, May 8, 2018)

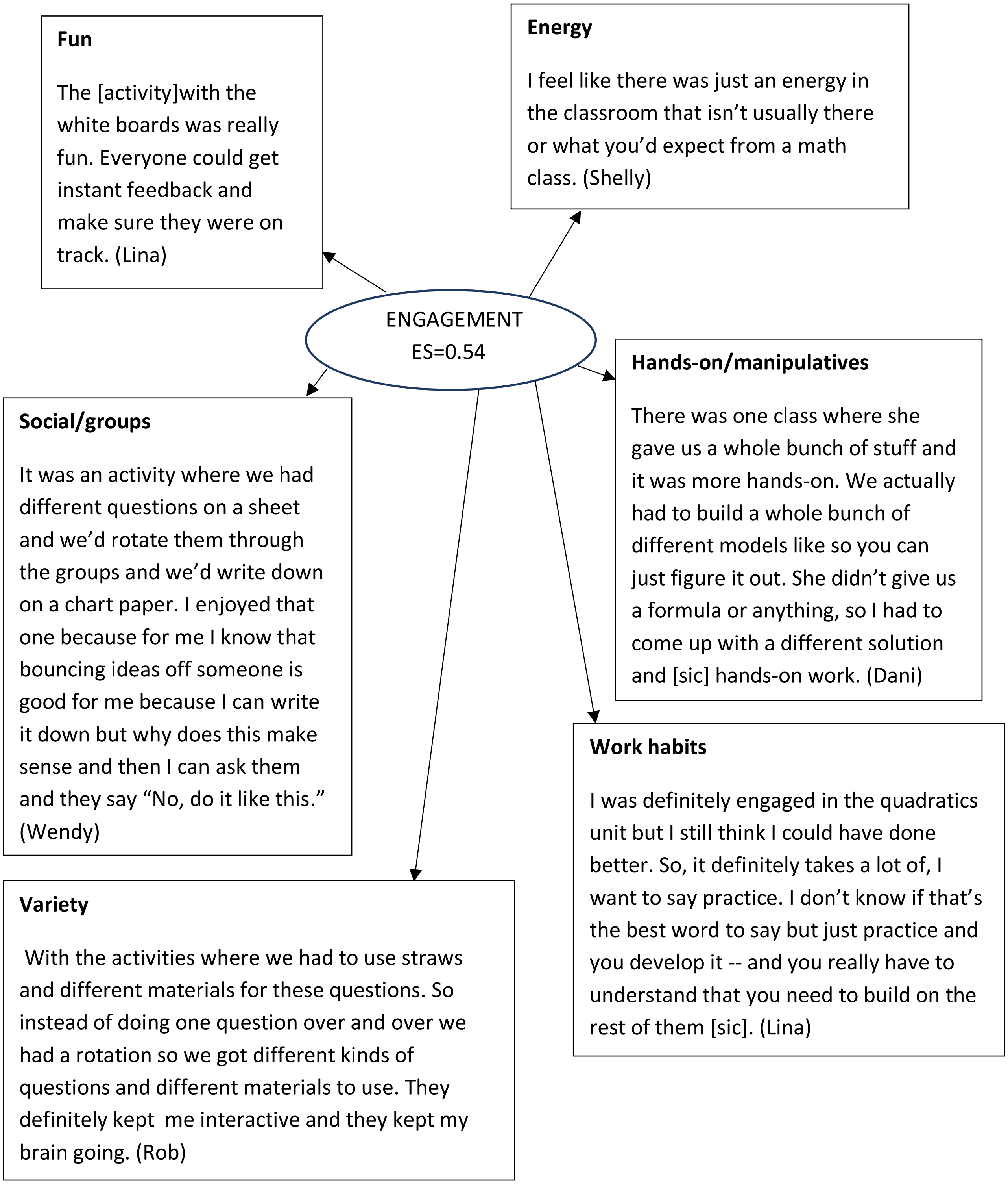

Figure 7. Student quotations reflecting aspects of MNT intervention that impacted on student engagement. Note: ES=effect size; All student quotations taken from post-intervention interviews, May 7 to May 9, 2018.

Ms. Beckham reinforced the context and task-specific stance:

> So, it's hard but I just found by doing different things you're hitting the different kids -- all is okay. This kid may not be engaged in this but now they're engaged in that area so that was pleasing. But I think the couple of conversation ones that we did worked really well, and there were some other ones. (Post-intervention interview, May 9, 2018)

The increased engagement in the treatment classes contrasted with the comments on Ms. Alford's control class:

> My students were sometimes engaged in the work, but I didn't see any dramatic differences compared to the earlier unit. Students talk to each other, and my students asked why we weren't doing the neat things that Ms. Beckham's classes were doing. So, I sometimes integrated some of the activities from the other classes into my class, and that seemed to get my students more engaged. (Post-intervention interview, May 9, 2018)

Attitude

The second dependent variable examined was student attitudes. Again, both quantitative and qualitative data were collected and analyzed.

Quantitative Findings

The *Attitude Towards Mathematics Inventory©* (*ATMI*) is a 40-question, five-point Likert scale survey (1=strongly disagree to 5=strongly agree; Appendix H). Table 7 shows statistics for the *ATMI* and subscales for T_{Total} (all treatment students). All students completed the *ATMI* both pre-and post the MNT intervention.

Pre–post comparisons. When pre-post comparisons were made for students in the treatment group, a significant positive effect size of 0.32 was found (M=0.270, SD=0.0870, t(45)=3.110, *p*=0.003) for overall student attitudes. While overall attitude and all subscales increased, the only statistically significant increase in the subscales occurred for the self-confidence subscale (Table 10 and Figure 8).

Analysis of Figure 9 found that 76% of students in T_{Total} showed a positive increase in their attitudes towards mathematics. The straight line in Figure 9 represents no change in attitudes (attitude post MNT intervention=attitude pre intervention).

In addition, the magnitudes of the average increases for the attitude scales (Attitude post minus Attitude pre) were smaller (M=0.15, SD=0.617, min=-1.66, max=1.94) than the magnitude of the average increases for the engagement scales (Engagement post minus Engagement pre), M=0.44, SD=0.817, min=-1.46, max=3.48 (see Figure 9). The differences in mean magnitudes between engagement changes and attitude changes was statistically significant (M=0.28, SD=0.932, t(68)=2.307, *p*=0.015). The pattern observed for the magnitudes of attitude changes was similar to the pattern for engagement, i.e., students who had the lowest attitude pre scores showed the largest increases in attitude post scores (Figure 10). In Figure 10 each data point gives the change (Attitude pre minus Attitude post) for a particular student in the treatment group.

For the control class (Table 11) the only significant change was in the self-confidence subscale (M=3.38, SD=0.660, t(21)=-2.608, *p*=0.016) and showed a negative change in attitudes toward mathematics over the time the unit was taught. This may reflect that the content of the quadratic relations unit was more difficult than the previous unit on linear systems.

Table 10

Pre- and Post ATMI Overall and Subscale Scores for T_{Total}

		Pre		Post					
Category	N	Mean	Std. deviation	Mean	Std. deviation	t	df	Sig.	Cohen d
Attitude	46	3.56	0.772	3.83	0.504	3.110	45	0.003**	0.32
Value	46	3.82	0.088	3.95	0.725	1.556	45	0.123	--
Enjoyment	46	3.67	0.105	3.80	0.090	1.243	45	0.220	--
Motivation	46	3.62	0.103	3.69	0.091	0.928	45	0.358	--
Self Confidence	46	3.56	0.101	3.84	0.087	3.138	45	0.003**	0.33

*** significant at p=0.01*

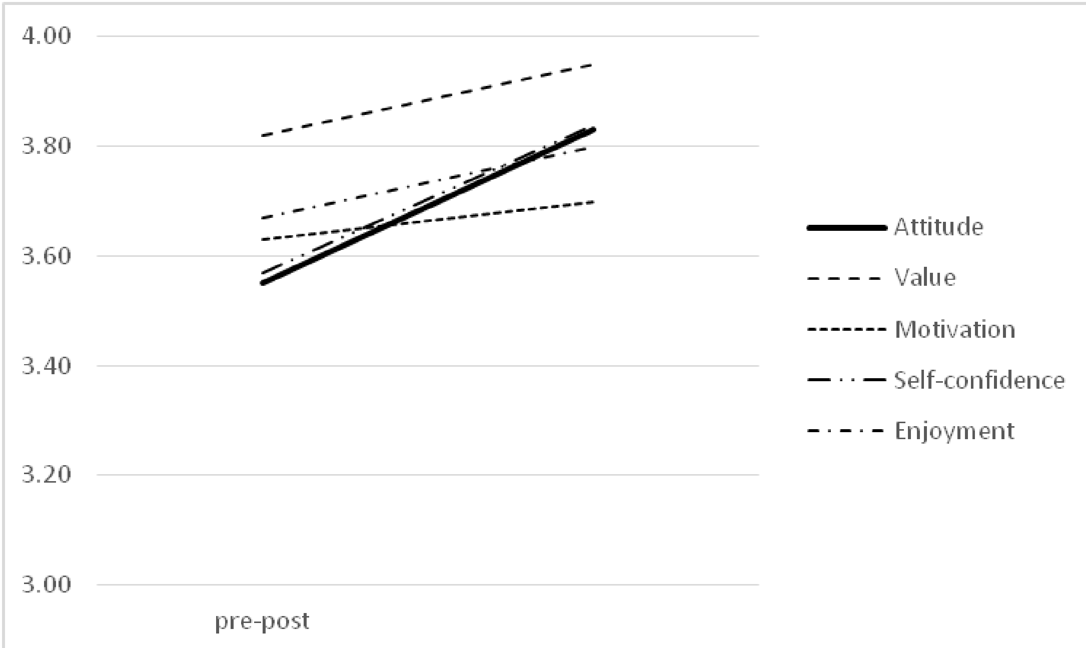

Figure 8. Changes in Attitude and subscales for T_{Total} pre-intervention and post-intervention. Note that only the overall attitude scale and the self-confidence subscale are statistically significant.

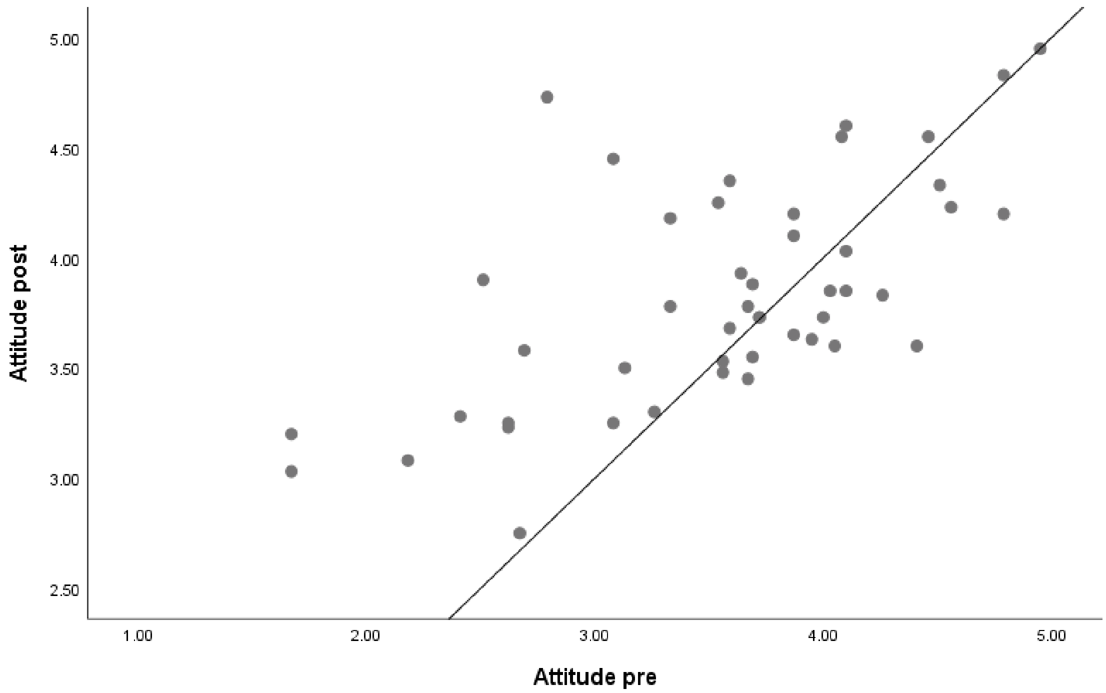

Figure 9. Comparison of attitude scale post-intervention and pre-intervention for treatment students. The straight line represents attitude-post=attitude-pre (no change after the instructional intervention).

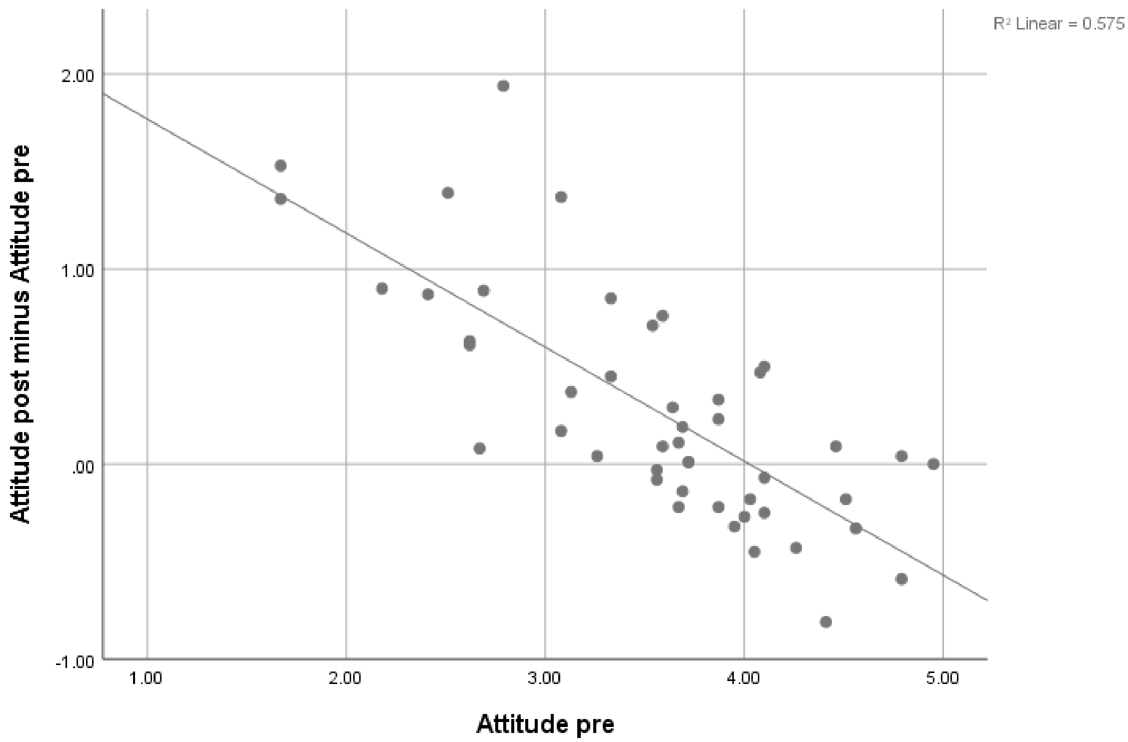

Figure 10. Magnitudes of changes (Attitude post minus Attitude pre) for each treatment student in T_{Total}. Trend line shown.

Table 11

Pre- and Post ATMI Overall and Subscale Scores for Control Class

		Pre		Post				
Category	N	Mean	Std. deviation	Mean	Std. deviation	t	df	Sig.
Attitude	22	3.55	0.651	3.45	0.569	-0.732	21	0.472
Value	22	3.63	0.513	3.63	0.513	#	21	#
Enjoyment	22	3.39	0.579	3.42	0.657	0.588	21	0.563
Motivation	22	3.16	0.734	3.15	0.743	-1.000	21	0.329
Self-confidence	22	3.45	0.679	3.38	0.660	-2.608	21	0.016*

Note. # t and significance cannot be computed since mean difference is 0; *significant at p=0.05

Treatment-control comparisons. Prior to the MNT intervention, no significant differences in attitudes were observed between the control class and T_{Total} (M=-0.007, SD=0.192, *t(*66)=-0.037, *p*=0.970). When treatment-control comparisons were made after the MNT intervention, T_{Total} showed a statistically significant increase in attitude scores compared to the control class (M=0.381, SD=0.1372, *t*(66)=2.781, *p*=0.007).

Qualitative Findings

Ms. Beckham indicated that she felt that the attitudes toward mathematics of some students in her classes had increased, although she provided only anecdotal evidence. All students who were interviewed indicated that the increased social culture of the classroom positively affected their attitude toward mathematics, even though two of the interviewees came into the unit with decidedly negative attitudes towards the subject.

In their interviews, the students indicated that the increased number of activities was different than the norm and that the uniqueness of this unit positively impacted their attitudes. The student responses reflected that attitude is a complex construct; interactions of attitude with other factors—such as prior and current achievement, peer and family effects, future plans, and other activities such as sports or the arts—all influenced the students' attitudes toward mathematics.

Additional themes identified in student interviews included student comfort levels (related to teacher or classroom culture) and frustration based on inadequate understanding of material. Perceived teacher care was also identified:

[Ms. Beckham] is a great teacher. You know that she cares about you. I feel like she teaches just the right way. A high school teacher like she doesn't hold your hand. But she doesn't push you off a cliff, you know. (Shelly, post-intervention interview, May 7, 2018)

Students often mentioned that classes in this unit were more "fun," although this may reflect a comingling of attitudes and engagement. Figure 11 provides additional aspects of the MNT intervention influence on attitudes based on student interviews. Grounded theory analysis resulted in attitude-related themes being identified in student interviews as self-confidence, teacher style, interest, persistence, and motivation to continue taking mathematics. Increased self-confidence was cited by four students, based on changes in classroom organization from teacher-centred to the use of small groups and activities that addressed different learning modalities.

A second theme was related to teacher style, with the teacher becoming more participatory with students in the learning process. Increased interest was a major theme identified. All five students identified the variety of activities and the linking of mathematical content to real-world situations as resulting in more interesting classes. An additional theme was persistence. Students stated that they tended to persist in solving problems related to the MNT intervention, and that they worked to achieve understanding of the mathematical content.

Finally, student interviewees stated that participating in the MNT intervention had increased their motivation to take additional mathematics courses beyond the compulsory third credit, both in the senior grades of high school and potentially at university. Figure 11 provides some selected student interview responses concerning these themes.

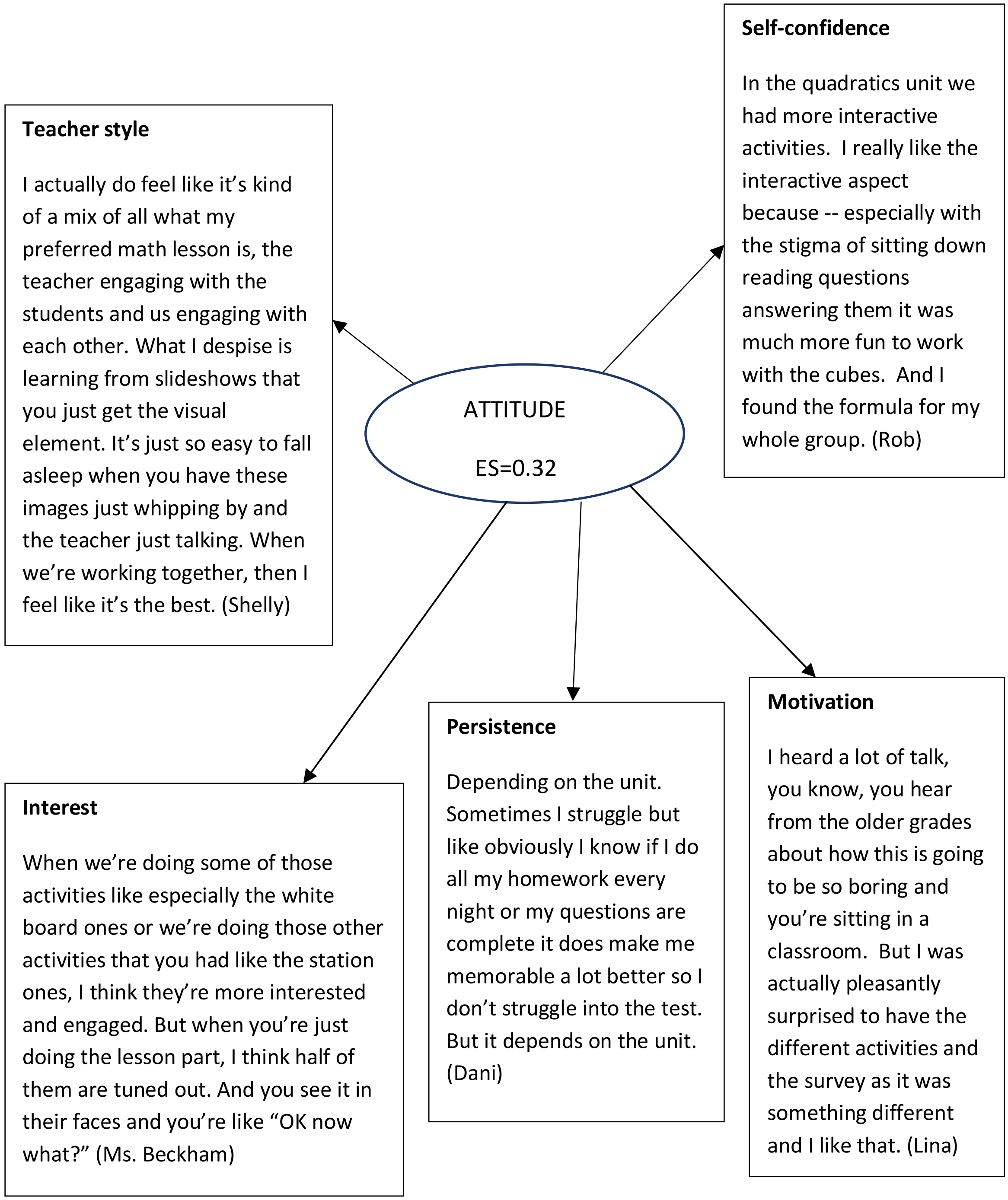

Figure 11. Quotations indicating different aspects of MNT intervention impact on student attitudes. Note: ES=effect size; All student quotations taken from post-intervention interviews, May 7 to May 9, 2018. Teacher quotations taken from post-intervention interviews May 9, 2018.

One repeated comment about the MNT intervention was the positive response to asking students' opinions. Four of the five students indicated that they really liked being asked about their levels of engagement and attitudes, and that no one had ever asked them about these attributes before:

> I liked a lot of your activities. I also liked the fact that you're interested in what the students think. I feel like sometimes our opinion gets overlooked and we kind of just get passed along to Grade 12. (Lina, post-intervention interview, May 8, 2018)

The students also liked specific personal goal-setting activities during the unit (see, e.g., Appendix V). Again, no one had ever asked them to set personal goals before this unit.

Achievement

The third independent variable in this study was student achievement. Both quantitative data, in the form of grades on the unit of study, and qualitative data were obtained and analyzed.

Quantitative Findings

The student achievement data available for examination included: last year's grade, self-reported on a five-point Likert scale as previously discussed; final term mark, as a percent; the quadratic unit summative mark (denoted Quad tests); the rich assessment task (RAT); and the term mark excluding the quadratics unit (TermXquad). The final term marks contained the quadratic summative mark plus the summative marks for three other units. These data would have had autocorrelation issues if compared to quadratic unit summative marks and therefore were discarded. Since the data, including engagement and attitude scores, were a mixture of parametric and non-parametric data, Spearman's ρ (r_s) was used for correlational analysis (Naiman et al., 2002).

The significant correlations are shown in Table 12. *Grade last year* was positively correlated with engagement scores prior to the intervention (r_s =0.330, p=0.027). This result echoes prior research on the reciprocal relationship between motivation and achievement (e.g., Koller et al., 2001). A surprising result was finding no significant correlation between the quadratic summative mark and the rich assessment task, given that both assessments are of content from the same unit. At times there was a high level of absenteeism in these classes, with 13 students out of 43 total treatment cases (30%) being absent for the rich assessment task. This may have affected the correlation analysis.

Pre–post comparisons. To compare quadratic summative marks with student self-reported grades for last year, the quadratic mark was converted to a Likert scale using the same scale as the self-reported grade. No significant relationship was found for students in the control class (z=-0.502, p=0.616, ns). However, a strong negative relationship was found for students in the treatment classes (z=-2.824, p=0.005). This implies that students' marks on the unit of study are significantly lower than marks reported for their prior year math course.

Treatment-control comparisons. No significant differences were found between T_{Total} and the control class for either grade last year or final term marks excluding the quadratic unit (TermXquad) for this year. However, a difference in grades on the quadratic unit tests was identified, with the control class (M=79.83, SD=11.664) higher than T_{Total} (M=65.58, SD=20.945), significant at p=0.01 (t(67)=3.024, p=0.004). A similar result was found for the rich assessment task (t(51)=3.216, p=0.002). So, for both the quadratic unit test and the RAT, the control class scored significantly higher than T_{Total}.

Table 12

Correlations Among Achievement-Related Variables for Treatment Students

	RAT	Quad tests	Term Xquad	Mark last year	Attitude post	Attitude pre	Engagement post	Engagement pre
RAT								
Quad tests	0.287							
TermXquad	0.379*	0.894**						
Mark last year	-0.132	-0.037	-0.011					
Attitude post	-0.098	-0.225	-0.153	0.141				
Attitude pre	-0.309	-0.134	-0.137	0.120	0.614**			
Engagement post	0.344	0.214	0.211	-0.095	0.048	0.050		
Engagement pre	0.301	0.424**	0.395**	0.330*	0.185	0.150	0.349*	

**Correlation significant at the 0.05 level; **Correlation significant at the 0.01 level*

Qualitative Findings

Both teachers involved in this study felt that the quadratics unit was the most difficult unit in the course, and noticeably more difficult than the linear systems unit that had preceded the MNT intervention. This was echoed by student interview comments. All students agreed that the current quadratics unit was more difficult than Grade 9 mathematics and also more difficult than the previous unit in this course, linear systems. However, Ms. Alford (control class) stated that her students were generally strong academically and she did not perceive that they struggled significantly with the content of the unit:

> My students right now are awesome. They are very very good for the most part. They're determined and they're very organized and they are very driven. I don't really have to keep on top of them, so I don't have to micromanage them for the most part. (Post-intervention interview May 9, 2018)

Student interview comments indicated that achievement was divided sharply between understanding material and demonstrating competence as indicated by student grades. Several students felt that their personal level of understanding was not adequately reflected in their marks on tests and evaluations:

> I did decently well but I felt like I know more than what I get on my tests and quiz marks. I feel kinda like when I see my grades especially when I work really hard. I see my grades not accurately representing what I know and it really hurts mentally, my confidence. (Rob, post-intervention interview, May 8, 2018)

All students indicated they felt that a strong link existed between achievement and work habits and homework completion:

> I struggle but obviously I know if I do all my homework every night or my questions are complete it does make me remember a lot better so I don't struggle, but it depends on the unit. (Dani, post-intervention interview, May 7, 2018)

Interestingly, while all those interviewed indicated increases in engagement, none of the students related this increased engagement to their personal achievement in mathematics. The students indicated that there was no perceived relationship between the students' increased engagement and attitude scores and their achievement on the unit as represented by the quadratics summative assessment marks reported by the students.

Class Characteristics

The structure and scheduling of the classes may also have impacted this study. While the control class occurred in Period 3 every day, the two treatment classes rotated (see Table 6). Classroom observations found that when T1 occurred in period one (first period of the day), the students were observed by the researcher to be very reserved and often arrived late or were absent. This class was also observed to be notably quiet and reserved. When asked about this situation, the teacher ascribed this to generally accepted research that teenagers function better later in the day as well as the high level of extracurricular activities in the school that frequently resulted in missed classes.

T2 was observed by the researcher to be more engaged in general but also was observed to demonstrate lower level of prior knowledge necessary for an academic mathematics class. While observing this class, the researcher noted that numerous student questions exhibited a lack of deep understanding of both current and prior mathematics content. This observation was supported statistically by a comparison of student self-reported final grades from the previous year, which showed a statistically significant difference (M=-0.664, SD=0.265, $t(43)=-2.510$, $p=0.016$) in favour of T1. Frequently student questions in T2 demonstrated a low level of comprehension both of current content and of prior knowledge necessary for progress at the Grade 10 level. Attendance in this class was also impacted by extracurricular absences.

The control class was reported by Ms. Alford to be very capable mathematics students: “My Grade 10 students right now are awesome. They are very good, they’re very determined, and they’re very driven. I don’t really have to be on top of them” (Ms. Alford, pre-intervention interview, February 21, 2018). However, Ms. Alford also noted a high level of student absences related to extracurricular activities.

Teacher Characteristics

In pre-intervention interviews, the two teachers in this study indicated very different reasons for becoming involved. These differing reasons informed their behaviours during the implementation of the MNT intervention. Ms. Alford was observed to demonstrate an outgoing, friendly demeanour that modeled interest in mathematics and teaching. She indicated that she joined the study because she was interested in learning new ways to engage her students and promote their success. Thus, Ms. Alford’s principal focus was on increased student engagement. She worked cooperatively with Ms. Beckham who regularly shared resources that she had created. When observing Ms. Alford’s classes, the researcher noted a high level of energy in the class and active participation by most of the students. Despite the researcher emphasizing that the control class must not utilize the same instructional techniques as the treatment classes, on several occasions Ms. Alford decided to use lessons or activities that were designed for the treatment classes, since she felt that these activities would provide better learning opportunities to engage her students. This may have potentially influenced treatment-control comparisons for the MNT study.

Ms. Beckham was observed to be more reserved than Ms. Alford but was highly organized and routinely developed resources that she shared with Ms. Alford. Ms. Beckham's classes were observed to be very organized and methodical, although the students clearly recognized that their teacher cared about their well-being, especially their academic well-being. In the pre-intervention interview, Ms. Beckham indicated that she volunteered for this study because she was unhappy with the impact of her current teaching strategies and wanted to learn new strategies that had the potential to be more engaging for her students. Thus, Ms. Beckham's focus was on student attitudes; she indicated that "I want them to enjoy math, not hate it" (pre-intervention interview, February 21, 2018). Ms. Beckham enthusiastically embraced the MNT intervention, offering several suggestions on lesson sequencing. She also conducted her own internet research to recommend three additional activities, which the researcher included in the intervention prior to its commencement.

Both teachers stated that they accepted a relatively fixed mindset (Dweck, 2006) with respect to student achievement, homework completion, attitudes toward mathematics and potential to become engaged:

> I think they come in primarily with more of a chip on their shoulders. It's maybe one of their least favourite subjects. They're just not as interested and they seem to equate math with homework and work and problem-solving and sort of not something fun. (Ms. Alford, pre-intervention interview, February 21, 2018)

Neither teacher was observed to "own the learning" of their students, accepting that some students would not understand fully the content that was being taught, although both teachers routinely solicited student questions and spent time with individual student assistance during class. Both teachers expressed some frustration related to students' lack of success:

> It's frustrating since there will always be some kids who get it and some who don't get it. Some of them don't really belong in academic, but their parents all want them to be doctors, so the kids struggle. No matter what you do, some of them won't understand. (Ms. Alford, pre-intervention interview, February 21, 2018).
>
> But some kids still can't get the conceptualization of what those little things mean. I think for the kids who can do the visual like the geometry, how the shapes connect, they can get it. But other kids still don't get it. It's still too abstract for them. (Ms. Beckham, pre-intervention interview, February 21, 2018)

The teachers also differed in offering students access to technology. Although a BYOD board, while Ms. Alford allowed free and ready access to technology at all times, Ms. Beckham required students to place their devices in a container at the front of the classroom, and only remove them for use at teacher-specified times, such as when calculators were required during the lesson.

Both teachers normally employed a traditional, teacher-directed teaching style (Table 6), although they sometimes employed guided discovery. Ms. Beckham noted that some activities in the MNT intervention were very successful for her students. She attributed this to the activities use of student groups, which were more social for students, as well as the variety and generally higher activity levels compared to her usual teaching style.

Fidelity of Implementation

Based on the criteria outlined in Chapter 3 (matching given sequencing of topics; inclusion of all elements of the MNT intervention; instructional strategies; responses to student questions; use of manipulatives; use of technology; responsiveness to student needs), a rating of a high degree of fidelity of implementation was assigned to classes that met more than five of the criteria, and a rating of moderate was assigned to classes that met at lteast four criteria. Classes meeting fewer than four of the criteria were rated as low fidelity of implementation. Of the 20 classes that were observed by the researcher, 14 (72%) were assessed as representing a high degree of implementation fidelity, and an additional three classes (14%) as having a moderate degree of implementation fidelity. Some adaptation was expected (Dane & Schneider, 1998; O'Donnell, 2008) and was observed as the teacher responded to the needs of individual students and differentiation based on class composition. However, the overall fidelity of implementation was good.

Summary Teacher Comments

At the conclusion of the study, during the post-intervention interviews Ms. Beckham stated that the MNT intervention had a positive impact on her students, particularly student engagement. She also felt that there had been a positive impact on homework completion. Ms. Beckham also observed a more collegial atmosphere among students, which she related to the increased use of student groups. She commented that there appeared to be an increased energy level in her classes during the MNT intervention compared to her usual classes. Ms. Beckham reinforced that engagement was task- and context-specific, with some students engaging more fully in certain activities and less so in others.

Ms. Alford, having observed the impact of the MNT intervention on Ms. Beckham's students, stated that this type of intervention appeared to be beneficial to students. Both teachers indicated that they planned to use portions of the MNT intervention in their other classes, particularly the Likert scales asking student opinions on the value, interest, self-efficacy, and degree of difficulty of content. They also planned to utilize more student-centred instruction in their classes, with more use of student groups, manipulatives, and technology.

When asked, both Ms. Alford and Ms. Beckham indicated that they would like to have similar classroom interventions for other units or grades, but were concerned about the time commitment to construct such interventions. Neither teacher stated concerns about their own self-efficacy or content knowledge for teaching that would be required to construct classroom interventions. However, neither teacher believed that other mathematics department members would participate in the development of these interventions; they both stated that a number of teachers in the department functioned in an insular fashion and generally did not share resources:

> I think some people are very set and have a negative attitude towards changing their practice. They're very set in their practices and I'm not sure they're really willing to explore different methods or different ideas or even using somebody else's work. So, if it's a collaborative thing maybe somebody else is in charge of creating something and I don't know if they're willing to let the reins go on that. (Ms. Alford, post-intervention interview, May 9, 2018).

For teachers involved in this study, their comments in the post-treatment interviews were more favourable towards an active, student-centred instructional stance than in the pre-treatment interviews. Both teachers indicated that they would incorporate a more active and social style into their teaching practices going forward.

> I really enjoyed it. I even shared it with [Principal] and I think he's already said it to you that I think people need to be exposed to these ideas so that it does not feel overwhelming. The little changes that you can do are important. I've said in the department there's just little things you can do in your lesson to get this huge mindshift. You get kids talking, you get the kids engaged versus just sitting there the whole period. (Ms. Beckham, post-intervention interview, May 9, 2018)

Summary

In summary, after the MNT intervention, significant positive effect sizes were found for both engagement and attitude, but not for student achievement. The MNT intervention functioned as an exemplar for the potential development of additional interventions in other courses or units. Both the teachers involved in this study indicated that they would incorporate similar activities into their professional practice. Implications of these results are discussed in Chapter 5.

CHAPTER FIVE: DISCUSSION AND IMPLICATIONS

This concluding chapter examines the results obtained in this study, and discusses the congruence of the results with the three research questions posed in Chapter 1. Implications for theory, implications for practice, and implications for future research are then discussed, followed by concluding remarks.

Discussion

The research questions addressed in this study were:

1. What is the effect of the MNT intervention on student engagement in a Grade 10 Academic Mathematics classroom(s)?
2. What is the effect of the MNT intervention on student attitudes in a Grade 10 Academic Mathematics classroom(s)?
3. What is the effect of the MNT intervention on student achievement in a Grade 10 Academic Mathematics classroom(s)?

The working hypotheses of this study were that the MNT intervention would positively impact student engagement, attitude, and achievement. Statistically significant positive effects were found for engagement and attitude, but not for achievement.

Engagement

The hypothesis that students in the MNT intervention classes would demonstrate significant increases in student engagement compared to students in the control class was supported. Paired *t*-tests found a significant effect size of 0.54 despite the relatively short duration of this intervention of approximately 30 instructional days. The validity of Marzano's ordering of self, metacognitive, and cognitive systems was somewhat supported, notably by student and teacher comments that when students engaged in a task, they were more likely to persist and make a sustained effort, especially if the task was viewed as interesting and engaging. This impact on student engagement was most pronounced with students who initially reported being less engaged in their normal classroom activities, but who indicated dramatically greater levels of engagement during the MNT intervention.

Both student and teacher comments reinforced literature that engagement is context- and task-specific and that engagement can be positively influenced in the short term (DeBellis & Golding, 2006). Features of the MNT intervention such as student social interactions, group activities, student-centred instructional strategies that employed hands-on activities, manipulatives, and technology were confirmed as positively impacting student engagement, consistent with existing research (Lassinantti, Stahlbrost, & Runardotter, 2019; Smith & Star, 2007). The instructional strategies in this study that employ an active, social stance for students have been found to have long-lasting effects on student engagement (Moyer et al., 2018; Smith & Star, 2007). The literature states that teacher practices must be intentional with respect to student engagement (Skilling et al., 2016); this current study's explicit focus on engagement and attitude provides additional research evidence of this instructional stance.

Another consideration is whether or not teacher style fosters student engagement. For example, Reeve and Jang (2006) identify autonomy-supportive behaviour by teachers as positively influencing student engagement. In the present study, Ms. Beckham was very strong on classroom structural dimensions, but less so on autonomy-supportive dimensions. Choice, a major determinant of autonomy support, was rarely provided except during the MNT intervention. While Ms. Beckham stated, "I do a lot of investigations," the use of the possessive pronoun "I" was telling. She did not say that her students did a lot of investigations, rather "I" did them, offering little or no choice to the students. This teacher's style was not congruent with the overall intent and many of the activities of the MNT intervention. However, Ms. Beckham made an effort to implement the MNT intervention with high fidelity of implementation.

Jang, Reeve, and Deci (2010) further identify three teacher instructional behaviours that are autonomy supportive for students: use non-controlling informational language, acknowledgement of students' perspective and feelings, and support of students' inner motivations. Jang et al. (2010) also identify three dimensions of classroom structure that support student engagement: presenting clear, explicit and detailed instructions; guiding students' ongoing activities; and giving constructive feedback. Based on classroom observations and teacher interview comments, Ms. Beckham's teaching style demonstrated limited congruence with the autonomy supportive dimensions while showing greater congruence with the structural dimensions. It could be speculated that effect sizes for this study may have been significantly greater if the teacher's style was more congruent to the style advocated for in the MNT intervention.

Bodovski and Farkas (2007) identified student engagement as having positive effects on student attitudes in mathematics. This was observed in the current study, with students who reported larger levels of engagement also reporting positive gains in attitudes towards mathematics, although the magnitude of those gains was smaller than the corresponding increases in engagement. Collie and Martin (2007) found that increased gains in overall engagement also mitigated gains in student agency. Again, this was observed in the current study, as student agency showed that largest gains compared to other dimensions of student engagement.

Attitude

The hypothesis that students in the MNT intervention classes would demonstrate significant increases in student attitudes towards mathematics compared to students in the control class was supported. The positive effect size of 0.32 for attitude found from paired *t*-tests was approximately 60% of the effect size for engagement, demonstrating perhaps that while engagement appears to be linked to short-term emotions such as interest, attitude is less malleable. Again, this is consistent with existing research. According to McLeod (1992), repeated positive experiences over time would be required in order to result in significant changes in student attitudes. Further, attitude is quite resilient and difficult to modify in the short term (e.g., Di Martino & Zan, 2010). The results for changes in student attitude supported the MNT intervention's structure, paying attention to student affective dimensions and explicitly inquiring into student attitudes for both individual activities and lessons as well as longer term effects on student attitudes towards mathematics.

DeBellis and Golding (2006), expanding on the seminal work by McLeod (1992), identified a taxonomy of affect, consisting of four constructs: *emotions*, which are rapidly changing states of feeling, mild to very intense, usually local or embedded in context; *attitudes*, moderately stable predispositions toward ways of feeling in classes of situations, involving a balance of affect and cognition; *beliefs*, internal representations to which the believer attributes truth, validity, or applicability, usually stable and highly cognitive, sometimes highly structured; *values, ethics, and morals*, deeply held preferences, stable, highly affective as well as cognitive, may also be highly structured, sometimes characterized as "personal truth." In the current study, engagement falls in the emotions category, while attitude, characterized as "moderately stable," is considered somewhat malleable.

The student and teacher comments in this study also reaffirmed that attitude is a complex, multi-faceted construct with numerous modifier variables including: perceived teacher care, student personal comfort levels, level of difficulty of the content being studied, and active instructional strategies. These findings were consistent with existing research (Mata et al., 2012). The instructional strategies employed in this intervention had a positive impact on student attitudes, congruent with existing literature (Domino, 2009; Vandecandelaere et al., 2012). Domino (2009) also found that teacher care and ensuring student understanding positively influenced student attitudes, and this study echoed those results. Instructional strategies in this study, similar to those identified for encouraging student engagement, also positively influenced student attitudes (Elçi, 2017). Other factors identified in the literature that positively influence student attitude towards mathematics: perceived teacher care (Cooper & Miness, 2014); teachers' displays of enjoyment (Frenzel et al., 2009); teacher fairness (Mata et al., 2012); and classroom climate, especially social climate (Kunter et al., 2008) were all identified by students in this study as influencing their personal attitudes towards mathematics.

Student attitudes in this study were also influenced by the relative difficulty of the unit, which was perceived as being more difficult than the previous unit of study as well as the level of difficulty of the previous year's mathematics content. This is congruent with Elçi (2017) who found an inverse relationship between attitude and the level of difficulty of mathematical content studied, so that more difficult content would result in more negative attitudes towards mathematics.

Achievement

The hypothesis that students in the MNT intervention classes would demonstrate significant increases in student achievement compared to students in the control class was not supported. Student achievement in this unit of study, measured by grades, was lower in the treatment classes than in the control class. This led to a question concerning the link between engagement and achievement. While a medium positive effect size of 0.54 was found for engagement, no positive effect size was found for achievement. This contradicts a number of research studies claiming a positive association between these two constructs (e.g., Bodovski & Farkas, 2007), although the claim of association between engagement and achievement is not unanimous in the literature (Dotterer & Lowe, 2011; Marks, 2000).

There are a number of possible explanations for the lack of a positive effect size for achievement. One possibility is that the pre-intervention measure of achievement may have been flawed. Students were asked to record their marks from their last mathematics course taken, as a letter grade. This may have resulted in a "rose-coloured glasses" effect, with students overstating their previous marks. This was confirmed by Ms. Beckham for at least one of the student interviewees. A second possibility is that the treatment classes were less academically prepared than the control class. This possibility has some statistical support, as documented in Chapter 4. A third possibility is that the mathematical content of the unit under study (quadratics) is more difficult than other units in the course. This position also has some anecdotal support from student and teacher interviews.

In addition, student achievement may also have been impacted by the timetabling of the treatment classes. Both treatment classes occurred in different time periods each day (see Table 3, Chapter 4), while the control class occurred at the same time each day. This may have impacted student learning.

If this study were to be replicated, a more refined pre-post measure of achievement should be used. In addition, the results of this study point to the need to examine or control for the level of difficulty of the content when examining the relationship among achievement, attitude and engagement.

Because the MNT intervention was of relatively short duration (30 instructional days) and was perceived by students to be unique compared to their previous experiences in mathematics classes, achieving statistically significant changes across two of three variables of interest may not be surprising. Positive results in mathematics achievement may need to be considered a longer-term goal, since mathematics is a cumulative subject. Positively impacting student engagement and attitude, as found in the MNT intervention, may, if sustained, result in gains in achievement over the longer term. Further, Marks (2000) claims that the links between engagement and achievement are sparse, and that engagement stands as a goal of education, separate from any potential relationship to student achievement. This stance is consistent with Collie and Martin (2017) who cite engagement as a goal of education whether or not it impacts other educational variables.

Attribution of achievement to engagement/effort did not occur for the students interviewed in this study. These students focused more on the perceived dichotomy between achievement as measured by more traditional instruments such as written tests, and achievement as measured by understanding of concepts. None of the students interviewed felt that their achievement as measured by grades accurately reflected their understanding. This decoupling of engagement and achievement has important implications for life-long learning, as an espoused goal of education (Fredricks et al., 2004). Therefore, this study adds to the existing research on the importance of engagement in mathematics, as well as to the discussion relating engagement to mathematics achievement.

The MNT Intervention and Reform Mathematics

The MNT intervention was most effective for students who initially reported low levels of engagement Chapter 4, Figure 6), and for students who reported more negative attitudes towards mathematics (Chapter 4, Figure 10). This effect may be explained by the more active, reform mathematics instructional strategies employed by the MNT intervention. These instructional strategies involve real-world connections, hands-on activities, students working in groups, and using manipulatives to increase student involvement. Such strategies were found to increase and sustain student engagement (Moyer et al., 2018; Smith & Star, 2007). In the MNT intervention, these strategies had the greatest impact on the most disengaged students and students with negative attitudes. It could be argued that these students' learning styles were a much better fit for the instructional strategies in the MNT intervention than the dominant instructional strategies employed by Ms. Alford and Ms. Beckham, who employed traditional teacher-directed strategies to a large extent. This result provides supporting evidence of the efficacy of reform mathematics strategies and emphasizes that such strategies may have the greatest impact on students who most need to become more reengaged, as well as students with more negative attitudes towards mathematics.

Implications for Theory

The framework for this classroom intervention was Marzano's *New Taxonomy of Educational Objectives*©. This framework integrates the self (affective) system, the metacognitive system, and the cognitive system into a coherent whole (Figure 2, Chapter 2). This differs from other taxonomies which typically address only one system. For example, revised Bloom (Anderson & Krathwohl, 2001) addresses only the cognitive system and relegates metacognition to a passive information role. Further, Marzano postulates a hierarchical integration of self, metacognitive, and cognitive systems (Figure 3, Chapter 2) which emphasizes the sequential nature of system engagement, primacy being given to the self system, which encompasses student motivation; this is followed by engagement of the metacognitive system, an active system involving goal setting, planning, and monitoring; finally, the cognitive system engages to address and resolve the task. This study demonstrates that MNT is a viable framework for studies involving motivation (self system) and metacognition. The results of this study in supporting Marzano's sequencing of self, metacognitive, and cognitive systems and the primacy of the self system (motivation) are mixed. While gains in engagement and attitude were observed, the structure of the intervention did not specifically follow Marzano's sequencing, since each lesson included both self and metacognitive dimensions. However, the efficacy of such instructional features did mitigate the potential to modify student affective dimensions in a positive way.

As noted above, this study adds to the literature on the resilience of attitudes toward mathematics as well as the inverse relationship between attitudes and difficulty of content. This study also adds to the debate concerning the link between engagement and achievement, indicating that further studies need to be done to better determine the nature of this relationship. The study also confirms the importance of intentionally considering affective dimensions, and structuring classroom interventions that positively influence student engagement.

Implications for Practice

There are several audiences with respect to implications for practice. These audiences include policy makers, educational researchers, classroom teachers, and teacher educators.

Policy

First, for policy makers, this study supports a student-centred philosophy and demonstrates its effectiveness in engaging students as espoused by the Ontario Ministry of Education (2005) for mathematics. However, consideration must be given to additional professional learning for teachers in order to improve teacher understanding and assimilation of the principles of student-centred learning.

Secondly, this study emphasizes the need to consider student affective dimensions when setting policy for mathematics. Motivation and other affective dimensions have been shown to positively influence mathematics students’ performance, widely construed, especially when measuring student engagement and attitudes (Schoenfeld, 2015). This is an area that needs to be rectified and is especially important at this time since the mathematics policy documents are currently under revision based on the 7-year cyclic review established by the Liberal Government in 2005. A search of current mathematics curriculum documents (Ontario Ministry of Education, 2005, 2007) found no mentions of motivation, attitude, engagement, self-efficacy, self-confidence, enjoyment, persistence. One mention of value was found, but only in reference to credit value. There are opportunities here since the mathematics curriculum is also currently undergoing a 4-year revision by the Ontario Ministry of Education (Conservative government). First, there is an opportunity to broaden the definition of achievement beyond mathematics content, to include affective dimensions that have been shown to be important for student growth and well-being (Kuntze & Dreher, 2015). Secondly, explicitly including affective dimensions (such as positively changing student engagement and student attitudes) in Overall and Specific expectations in the mathematics curriculum policy documents will not only raise the profiles of these concepts with respect to teachers, as they become explicit expectations of instruction; it will signal to the broader education community that such affective dimensions are important and must be addressed. There are indications in the popular press that this current 4-year revision by the Progressive Conservative government will take student attitudes into account.

> Too many adults, including plenty of teachers (and parents) don’t like or feel comfortable with math. That sentiment is far too easily passed on to impressionable kids. And research shows that students’ attitudes toward math influences their outcomes. (“Let’s Be Smart on Math Fixes,” 2019, p. A12)

However, to date there has been no curriculum document released by the Ontario Ministry of Education that explicitly recognizes the connections between student attitudes towards mathematics and student competence in mathematics (Ontario Ministry of Education, 2019a, 2019b).

Schools and Teachers

For schools and classroom teachers, the MNT instructional intervention in this study provides an exemplar of theory to practice, where changes in student engagement and/or student attitudes are the goals; it also provides a template for developing similar complete units or activity packages. The instructional strategies utilized by the classroom intervention in this study provide evidence of the efficacy of *reform mathematics* principles (Smith & Star, 2007) such as the appropriate use of manipulatives; making real-world connections for students; supporting student autonomy through groups and choice; active rather than passive student participation. The results of this study also provide support for the principles of self-determination theory (Deci & Ryan, 2008): autonomy, competence, and relatedness: autonomy support through choice; competence through teaching for understanding and supporting students' different learning modalities; and relatedness through the use of student groups.

However, the difficulty in recruiting teachers willing to participate in this study points to the need to more clearly explain the intent of a study and its potential benefits to classroom teachers, as well as clearly delineate expectations of time and effort that volunteering for such a study would entail. Also needed is an indication by the Ontario Ministry of Education that such studies are valued, and that the results of these studies may be incorporated into ministry policy. This could influence more classroom teachers to become involved.

In organizational theory, teaching is identified as a professional bureaucracy (Mintzburg, 1989). This describes a structure in which overall policy and direction is given centrally but individual teachers experience a wide degree of independence in implementing policies. In such a structure, teachers need to be convinced of the efficacy of new policies before acceptance and true implementation will occur. To support change in this type of organization, dissemination of "success stories" and best practices based on actual classroom implementations of research-affirmed strategies is needed. Only when teachers see such strategies positively impacting students can teachers be expected to accept and implement changes in their own professional practice.

If development of a full unit or course based on the MNT structure is not feasible, teachers could consider simpler interventions. For example, the current study utilized brief student responses to questions such as "Was today's lesson interesting for you?" and "Rate how valuable today's topic was for you." Brief anonymous responses provide teachers with guidelines for modifying instructional strategies, while also motivating students to consider more fully what types of lessons are most productive for them personally. This increase in student agency has significant potential to increase student engagement (Deci & Ryan, 2008), at very little cost in teacher planning time.

Educational Researchers

Student attitudes towards mathematics is a burgeoning area of study that has attracted considerable interest in recent decades (e.g., Crano & Prislin, 2008; Pepin & Roesken-Winter, 2015). In addition, related constructs such as beliefs (McLeod & McLeod, 2002), interest (Renninger, Nieswandt, & Hidi, 2015), affect (Forgas, 2008), and emotions (Radford, 2015) have all received scrutiny. This study adds to the literature in the areas of student attitudes and engagement in mathematics and provides a classroom study which can be utilized as an exemplar.

When constructing their own classroom studies, researchers must also give consideration to logistical constraints, whereby some school boards receive multiple requests for classroom studies while other boards receive few or none. This could be alleviated if researchers were encouraged to consider the possibility of conducting their research in school districts that would be more open to accepting classroom studies and have the capacity to do so. Such a policy could be supported by the Ministry of Education, which could disseminate information about which research projects are currently underway in each board. Financial support could also be offered to researchers for conducting their studies in non-local or remote boards of education. The results would be a richer research palette as well as broader dissemination of theory-into-practice across the province.

A notable concern is that often teacher volunteers come only from the most enthusiastic and dedicated teachers. While this mitigates for stronger implementation fidelity, it also may bias results since these teachers are not necessarily representative of the teacher cohort as a whole. Coupled with the naturalistic setting in classrooms, this has implications for generalizability (Kruskal & Mosteller, 1979).

Finally, practitioners and researchers must recognize the need to control for, as much as possible, extraneous or confounding factors that may influence outcomes. For example, in the current study the level of difficulty of the mathematical content of the unit may have impacted student achievement and may have influenced engagement and attitude as well.

The MNT Framework

The MNT framework has the potential to enrich practice in a number of areas. One of the major implications for practice is to raise awareness of the linkages among the three systems of the MNT framework, self (motivation), metacognition, and cognition.

For current mathematics teachers, the framework provides a template to develop units or subunits of mathematics content that provide a specific focus on one or more systems, particularly student motivation and metacognition. Through teachers' awareness of the importance of these dimensions over and above the mathematics content, a more student-focused and student-engaged classroom climate will develop (see, for example, Irvine (in press-a, in press-b). In-service professional learning opportunities need to be provided for practicing teachers to become aware of the MNT framework and its implications.

Teacher educators would benefit from knowledge of the MNT framework and its relationships to HOTS and deep learning, as well as making explicit the roles of student motivation and metacognition in learning. These concepts could then be included in the curricula for preservice teachers of mathematics. Since these is now a significant body of research on student attitudes in mathematics (e.g., Pepin & Roesken-Winter, 2015), the MNT framework provides a structure for introducing these concepts into pre-service courses, as well as a viable framework for lesson planning with an emphasis on one or more MNT systems.

For educational researchers the MNT framework provides a structure for the construction of studies in one or more of the dimensions of the framework. The framework would be useful in structuring studies on student cognition in mathematics or in other subject areas, as well as multi-system studies linking two or more MNT systems. Having access to a rich and well-developed framework provides researchers with a structure that is understandable to the participants in a study and may be more easily communicable to any non-researchers involved.

Implications for Future Research

One implication for future research is consideration of the methodology employed. The appropriateness of employing a mixed methods methodology, especially when examining latent variables that are not readily observable, was strongly reinforced.

Mixed methods methodology identifies a methodology after research questions have been formulated, rather than specifying a methodology a priori (Johnson & Onwuegbuzie, 2004).

It is informative to consider what information would have been missed if only one methodology had been employed in this study. For instance, if only quantitative methodology had been used, since statistically significant effect sizes were found, the results would be generalizable, but no data on the reasons why the instructional intervention was effective could be discussed. Thus, the "what happened" could be found, but without any information on the "why it happened."

Adding the qualitative dimension to the methodology allowed for a deeper understanding of the phenomena; this allowed for more probing of the complexities of the attitude construct as well as consideration of dimensions such as the novelty effect of students being asked their opinions and feelings. This dimension is apparently rarely considered in mathematics classes, as no mention was found in the literature.

However, if only a qualitative methodology was employed, while the in-depth phenomena of student behaviour would have been accessible, it would not be possible to compute aggregate outcomes such as effect sizes, and the issue concerning the linkage between engagement and achievement may have been minimized or even missed altogether. This has important implications for policy development across broad jurisdictions such as Ontario and Canada. Education policy is influenced by aggregate measures that point the way forward to positively impact student learning for large numbers of students. Substantial changes in education policy can only be achieved through providing a body of evidence of the efficacy of the changes.

Therefore, educational researchers should consider a pragmatic approach to selecting methodologies. By first identifying the research questions for a study, and allowing the research questions to drive the choice of methodology, educational researchers will potentially find richer and more multi-dimensional answers to research questions than are accessible if the methodology drives the formulation of the research questions. This does not mean that all studies should employ mixed methods methodology; but once the research questions are specified, the appropriate methodology should be selected in order to provide fulsome answers to the research questions of the study.

Based on this study, there are a number of additional implications for future research. First, because of the lack of a significant link being found between engagement and achievement, this study, which occurred in a single secondary school, needs to be replicated in other venues, with different students, classes and school characteristics. These studies should employ more refined measures of student achievement and consider broader and more fulsome definitions of achievement beyond student grades. In addition, longitudinal studies utilizing similar instructional interventions need to be developed and implemented for longer periods of time. Will there be diminishing returns to engagement gains over a longer period, or will gains be proportional to the time period? Since attitude is more malleable in the intermediate and longer term, will greater positive gains in student attitudes be observed when the duration of interventions is increased? Initially, a classroom intervention based on MNT could be developed for the remaining three units of the Grade 10 Academic Mathematics course and implemented for a full semester. This would allow a more fulsome examination of the variables of interest as well as providing a substantial body of exemplar materials.

In the current study that focused on engagement and attitude, an interaction effect between the two variables was not examined. Implementation of a full-semester classroom intervention would enable investigation of whether there was an interaction effect, and if so, what form did this interaction take. The various moderator variables affecting attitude and whether there were interaction effects among the variables could also be examined.

In addition, the question whether positive student responses to self-reported surveys translate into changes in effort, persistence, self-efficacy, or achievement could be examined. Triangulation of such self-reported data with other measures would increase reliability and generalizability of findings.

Concluding Remarks

The MNT intervention was based on the framework given by *Marzano's New Taxonomy of Educational Objectives*©. It is noteworthy that the MNT intervention also satisfied the criteria for a powerful learning environment (PLE; Vandecandelaere et al., 2012), consisting of the four dimensions *motivate to exert learning*, *activate towards self-regulated learning*, *give feedback and coach*, and *structure and steer*. There are notable correlations among the two frameworks, with PLE's motivate to exert learning similar to Marzano's self system primacy, and PLE's activate towards self-regulated learning related to Marzano's placement of metacognition. Marzano does not explicitly address feedback, and PLE's structure and steer is assumed in the construction of the intervention. These similarities do demonstrate that alternate frameworks could be used to construct classroom interventions, although MNT provides additional structure around what elements constitute metacognitive dimensions, and provides much more detail on cognitive system attributes. MNT also provides a temporal sequencing of the three systems; self system (motivation) occurs first, followed by the metacognitive system, and ultimately the cognitive system. This temporal sequencing is absent from other theoretical frameworks that might be employed.

Mathematical Well-Being

The MNT framework is external to the students' locus of control, providing a framework for teachers and educators to develop instructional strategies to positively influence student behaviours. In 2010, Clarkson et al. proposed the concept of mathematical well-being (MWB), which provides a five-stage taxonomy based on an internal conception of students' locus of control. MWB consists of five stages beginning with *awareness and acceptance of mathematical activity*, and progressing through *positively responding to mathematical activity*, *valuing mathematical activity*, *having an integrated and conscious value structure for mathematics*, and finally, *independently competent and confident in mathematical activity*. The details of each level (Clarkson et al., 2010) describe student behaviours and motivation towards mathematical activity that delineates changes that occur in student beliefs (as indicated by student behaviours) towards the utility and value of mathematical activities. MWB provides an enlightening differentiation among the five levels of students' mathematical beliefs. However, MWB, in its current form, is not an effective framework for developing instructional strategies to support students' progression among the levels. Indeed, Clarkson et al. cite the need for developing and examining effective instructional techniques in their summary of future research required to further develop the MWB construct and move it from theory to practice.

In the *Renewed Mathematics Strategy* (Ontario Ministry of Education, 2016) reference was made to student well-being, but this document failed to make any mention of motivation, engagement, or related constructs. This document was only partially enacted before the fall of the Liberal government in 2018. However, the broad reference to student well-being appears quite dissimilar to MWB.

Final Thoughts

Awareness of student affective dimensions in mathematics teaching is important to benefiting both student learning of content as well as promoting more positive attitudes towards mathematics among students (Pepin & Roesken-Winter, 2015). Affect in mathematics learning has become a major research area over the last several years (Hannula, 2015; Schoenfeld, 2015). It is unacceptable that comments such as “I was never good at math” are socially acceptable. Therefore, paying attention to student motivational factors must be included alongside pedagogical strategies that are research-affirmed and involve students in their own learning. Supports must be provided to inform teachers of current research, encourage implementation of both motivational and instructional strategies, and structure job-embedded support systems in jurisdictions across Canada. The Ontario Ministry of Education has supported job-embedded professional learning for over a decade (Irvine & Telford, 2015). However, this job-embedded support needs to refocus on affective dimensions of student learning and provide teachers with instructional strategies that promote positive behaviours in student engagement and positive attitudes towards mathematics learning.

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Appendix A

Treatment Lessons

Additions to base problems unless indicated as replacement (R)

Expectations	Learning Goals			Metacognition Focus	Self Focus
– determine, through investigation with and without the use of technology, that a quadratic relation of the form $y = ax^2 + bx + c$ (a>0) can be graphically represented as a parabola, and that the table of values yields a constant second difference (***Sample problem:*** Graph the relation $y = x^2 - 4x$ by developing a table of values and plotting points. Observe the shape of the graph. Calculate first and second differences. Repeat for different quadratic relations. Describe your observations and make conclusions, using the appropriate terminology.); – identify the key features of a graph of a parabola (i.e., the equation of the axis of symmetry, the coordinates of the vertex, the y-intercept, the zeros, and the maximum or minimum value), and use the appropriate terminology to describe them;	*Students will learn the basic properties of parabolas and be able to describe these properties using appropriate mathematical language *Students will learn how to apply quadratic regressions to data sets *Students will learn how to use finite differences to determine equations of quadratic functions	Minds On	Carousel • crocodile river • handshake problem • pizza cuts • logpile	• Anticipation Guide	• Likert scale: interest • Groups • Placemat: Tell me everything you know about linear relations
		Action	Whole class • Use the method of finite differences to find equations for each pattern $y = ax + b$ (linear) $y = ax^2 + bx + c$ (quadratic) •	• Think Aloud • What do we want to know; what do we know; how can we connect these	• Likert scale: importance
		Consolidate/Debrief Homework: *Parabolas in Real Life*	• Extend the pattern to negative x's using your equations • Terminology (vertex, max/min, axis of symmetry, intercepts, domain, range)	• Journal entry • How well was your plan achieved? Did it require any modifications?	• (R) *Connecting Cube Quadratics* • Homework Crossword puzzle terminology + *Parabolas in Real Life*
– collect data that can be represented as a quadratic relation, from experiments using appropriate equipment and technology (e.g., concrete materials, scientific probes, graphing calculators), or from secondary sources (e.g., the Internet, Statistics	*Students will learn how to collect and model data that can be represented by a quadratic relation	Minds On	Groups Use technology to graph an example from *Curve Fitting* and discuss appropriate models	• Pairs • What/So What plan solution method	• Graphic organizer • Motivation
		Action	Groups Apply quadratic regressions to obtain equations for data given in *Curve Fitting*	• Groups • What/So What revisit	• Journal entry: How confident are you that you can solve problems involving quadratic relations • Choice

Canada); graph the data and draw a curve of best fit, if appropriate, with or without the use of technology (***Sample problem:*** Make a 1 m ramp that makes a 15° angle with the floor. Place a can 30 cm up the ramp. Record the time it takes for the can to roll to the bottom. Repeat by placing the can 40 cm, 50 cm, and 60 cm up the ramp, and so on. Graph the data and draw the curve of best fit.);					
		Consolidate/Debrief	Groups Debrief *Parabolas in Real Life*	• Journal entry: • How well was your plan achieved? Did it require any modifications?	• Emoji scales: • overall motivation • efficacy • interest • importance
– identify, through investigation using technology, the effect on the graph of $y = x^2$ of transformations (i.e., translations, reflections in the x-axis, vertical stretches or compressions) by considering separately each parameter a, h, and k [i.e., investigate the effect on the graph of $y = x^2$ of a, h, and k in $y = x^2 + k$, $y = (x - h)^2$, and $y = ax^2$];	*Students will learn the effect on the graph of a quadratic function of modifying a parameter in $y = a(x - h)^2 + \mathrm{k}$,	Minds On	Jigsaw Use technology to investigate the effect of various values of parameters • $y = ax^2$ • $y = -ax^2$ • $y = x^2 + \mathrm{q}$ • $y = (x - p)^2$	• What/So What • Why does each parameter change result in the transformation of the graph	• Choice • Choose group for jigsaw
		Action	Whole class Practice with various $y = a(x - p)^2 + \mathrm{q}$	• Use strategy of example, think-pair-share discussion, worked questions, then repeat	• On a scale of 1 to 10, identify how well you understand the impact of changing parameters
		Consolidate/Debrief	Whole class Summarize transformations Individual Journal entry: summarize the transformations of $y = a(x - p)^2 + \mathrm{q}$ and the impact of parameters	• Journal entry • Given a specific $y = a(x - h)^2 + \mathrm{k}$ describe the steps you would take to graph it	• (R) *Quadratic Aerobics*
-explain the roles of a,h, and k in $y = a(x - h)^2 + \mathrm{k}$, using appropriate terminology to describe the transformations, and identify the vertex and axis of symmetry -sketch, by hand, the graph of $y = a(x - h)^2 + \mathrm{k}$ by applying transformations to the graph of $y = x^2$	*Students will learn how to sketch and connect graphs and equations $y = a(x - h)^2 + \mathrm{k}$, using appropriate mathematical terminology	Minds On	Groups Matching graphs and equations	• Groups • Placemat • Sketch graphs from given equations and verify accuracy with technology	• Snowball PMI • Role of a, h, k in $y = a(x - h)^2 + \mathrm{k}$
		Action	Individual Sketch graphs for various values of parameters	Groups Matching graphs and equations	• (R) inside/outside circle: generate equation and explain impact of parameters

[Sample problem: Sketch the graph of $y = -(x-3)^2 + 4$, and verify using technology -determine the equation in the form $y = a(x-h)^2 + \mathrm{k}$ of a given graph of a parabola			$y = a(x-p)^2 + \mathrm{q}$	• Verify using technology	
		Consolidate/Debrief	Pairs Think-Pair-Share to construct questions matching graphs, equations, and information (domain, range, intercepts, vertex, axis of symmetry) Inside/Outside Circle to share with others	• Groups • What/So What • Effect of various parameter changes, how to recognize them, how to verify them	• Likert scale: interest
– expand and simplify second-degree polynomial expressions [e.g., $(2x + 5)^2$, $(2x - y)(x + 3y)$], using a variety of tools (e.g., algebra tiles, diagrams, computer algebra systems, paper and pencil) and strategies (e.g., patterning);	*Students will learn how to expand and simplify second degree expressions, with and without manipulatives	Minds On	Groups Use algebra tiles for some basic expansions	• Pairs • Order algebra tile pieces to show expansion and vice versa	• Groups • Discussion • Why is this/might this be important to me?
		Action	Whole Class Algebraic expansions Student practice	• Groups • Graffiti • Step by step expansion using algebra tiles, then algebraic expansions	• Journal entry • How useful is this to me?
		Consolidate/Debrief	Individual Inside/outside circle: Student generated examples of expansions Journal entry: Create an example of each type of expansion	• Journal entry • Give examples of expansions in both directions with and without algebra tiles	• Journal entry: my favourite expansion and why
– factor polynomial expressions involving common factors, trinomials, and differences of squares [e.g., $2x^2 + 4x$, $2x - 2y + ax - ay$, $x^2 - x - 6$, $2a^2 + 11a + 5$, $4x^2 - 25$], using a variety of tools (e.g., concrete materials, computer algebra systems, paper and pencil) and strategies (e.g., patterning);	*Students will learn how to factor polynomial expressions *Students will learn how to recognize and factor special cases	Minds On	Groups 1)Use algebra tiles for simple factoring *Factoring Using Algebra Tiles* 2) Whole class: Construct a decision tree for factoring	• 1)Verify factorizations by expanding • 2) Matching steps for an example	• 1) Likert scale How fun is algebraic manipulation • 2) Graphic organizer • Emotions
		Action	Whole class 1)Algebraic treatment of trinomials, perfect squares, difference of squares 2)Jigsaw practice	• 1) Recognition • What type of factoring is it • 2) Pairs • Timed retell • Given a card with a factorable expression on it, explain how to factor	• 1) Groups • cartoon placemat • different groups get different types of factoring • 2) Four corners • Different types of factoring at each corner (multiple questions on same type)
		Consolidate/Debrief	1) Individual practice Journal Entry: Explain the relationship between expanding and factoring	• 1) Pairs • One partner factors, the other partner	• 1)Emoji scales: • overall motivation • efficacy • interest • importance

			2) Groups: Write a script to explain to a classmate how to factor (various expressions)	verifies by expanding • 2) Game of *Facto*	• 2) Journal entry • How confident are you that given an expression to factor, you can factor it and verify your answer
– express $y = ax^2 + bx + c$ in the form $y = a(x - h)^2 + k$ by completing the square in situations involving no fractions, using a variety of tools (e.g. concrete materials, diagrams, paper and pencil);	*Students will learn how to complete the square, with and without manipulatives	Minds On	Groups Use algebra tiles to complete *Make a Square*	• Think Aloud • What do we know, what do we want to know, how are they related	• Graphic organizer • Emotions
		Action	Whole class Algebraic complete the square examples	• Pairs • Matching steps for a numerical example	• Groups • Choice apply completing the square to various expressions
		Consolidate/Debrief	Individual Practice completing the square Groups Think Aloud: What information can we obtain by completing the square	• Ticket to leave • Given a numerical example, outline the steps in completing the square	• Ticket to leave • Choose one of three expressions and complete the square
– determine, through investigation, and describe the connection between the factors of a quadratic expression and the *x*-intercepts (i.e., the zeros) of the graph of the corresponding quadratic relation, expressed in the form $y = a(x - r)(x - s)$;	*Students will learn how to determine the zeros of a quadratic relation and connect them to x-intercepts and equations expressed in the form $y = a(x - r)(x - s)$;	Minds On	Groups Matching zeros from graphs with zeros from algebra	• Groups • Outline a plan to convert to $y = a(x - r)(x - s)$	• Likert scale • How confident are you that you can convert among forms
		Action	Whole class Algebraic intercepts by factoring Intercepts using technology	• Pairs • Matching graphs and equations	• Likert scale • How interesting do you find these conversions
		Consolidate/Debrief	Individual Practice finding intercepts algebraically and writing quadratic functions in the form $y = a(x - r)(x - s)$; Ticket to leave: Given values in $y = a(x-p)^2 + \mathrm{q}$ Rewrite in form $y = a(x - r)(x - s)$;	• Journal entry • How can you be confident that you converted correctly	• Likert scale • How important do you think these conversions are to you
– determine the zeros and the maximum or minimum value of a quadratic relation from its graph (i.e., using graphing calculators or graphing software) or from its	*Students will learn how to determine features of a quadratic relation (x-intercepts, maximum/minimum) from its graph and from its equation	Minds On	Groups Michaela problem Watch Detroit Airport video 1 Brainstorm some questions that you	• Groups • Graphic organizer • Complete the Polya organizer	• Graphic organizer • Right angles for interest, efficacy, importance

defining equation (i.e., by applying algebraic techniques);	*Students will learn how to connect algebraic and graphical techniques to real life situations and identify restrictions		might ask about the fountains, and what information you would need to answer them Then watch video #2		
		Action	Groups Solve real world problems using a variety of techniques	• Groups • Solve, referring to Polya organizer	• Use computer software or graphing calculators to solve problems
		Consolidate/Debrief Homework	Whole class Polya plan Identify restrictions based on real life situation Watch Detroit Airport video #3	• Ticket to leave • Summarize plan, modification, restrictions, how to recognize	• Graphic organizer • motivation
– explore the algebraic development of the quadratic formula (e.g., given the algebraic development, connect the steps to a numerical example; follow a demonstration of the algebraic development [student reproduction of the development of the general case is not required]);	*Students will learn how to develop the quadratic formula and apply it ti find zeros of functions and x-intercepts of quadratic relations	Minds On	Whole class Sample algebraic solution by factoring	• Groups • Graph using technology, estimate zeros • How confident, accurate are zeros	• Groups • Graph using technology and estimate zeros • Discussion • How confident are you that the zeros are correct and accurate
		Action	Whole class Algebraic development of quadratic formula with values for a,b,c Algebraic development of quadratic formula with a,b,c Worked examples	• What/So what • Relate algebraic steps to numerical example	• Likert scale • importance
		Consolidate/Debrief	Individual practice	• Timed retell • Explain the steps for a numerical example	• Graphic organizer • Emotions
– solve problems arising from a realistic situation represented by a graph or an equation of a quadratic relation, with and without the use of technology (e.g., given the graph or the equation of a quadratic relation representing the height of a ball over elapsed time, answer questions such as the following: What is the maximum height of the ball? After what length of	*Students will learn how to model real life situations using quadratic functions *Students will learn how to solve quadratic models to answer real life questions	Minds On	Groups Dan Meyer basketball video	• Self-select jigsaw • Plan solution	• Open problems • Choice
		Action	Whole class Problems worked examples	• Execute plan • Gallery walk	• Likert scale • Confidence
		Consolidate/Debrief	Groups getthemath.org basketball problem	• Ticket to leave • Explain plan and execution, restrictions	• Likert scale • Importance

time will the ball hit the ground? Over what time interval is the height of the ball greater than 3 m?).					
– interpret real and non-real roots of quadratic equations, through investigation using graphing technology, and relate the roots to the *x*-intercepts of the corresponding relations; – sketch or graph a quadratic relation whose equation is given in the form $y = ax^2 + bx + c$, using a variety of methods (e.g., sketching $y = x^2 – 2x – 8$ using intercepts and symmetry; sketching $y = 3x^2 – 12x + 1$ by completing the square and applying transformations; graphing $h = –4.9t^2 + 50t + 1.5$ using technology);	*Students will learn how to interpret real and non-real roots of quadratic equations *Students will learn how to graph quadratic relations using a variety of methods	Minds On	Jigsaw Graph various quadratics with real integer, real decimal, non-real roots using technology	• Anticipation guide v2	• Emoji scales: • overall motivation • efficacy • interest • importance
		Action	Groups Find the zeros algebraically or explain why this is not possible	• Groups • Relate roots to graphs and identify patterns	• Likert scale • interest
		Consolidate/Debrief	Groups Sketch graphs using a variety of techniques (complete the square; factor to find roots; use technology to graph to find roots; table of values Gallery Walk to share solutions	• Journal entry • How can you tell how many real roots a quadratic equation will have?	• Graphic organizer • motivation
– solve quadratic equations that have real roots, using a variety of methods (i.e., factoring, using the quadratic formula, graphing) (***Sample problem:*** Solve $x^2 + 10x + 16 = 0$ by factoring, and verify algebraically. Solve $x^2 + x – 4 = 0$ using the quadratic formula, and verify graphically using technology. Solve $–4.9t^2 + 50t + 1.5 = 0$ by graphing $h = –4.9t^2 + 50t + 1.5$ using technology.).	*Students will learn how to solve quadratic equations that have real roots, using a variety of methods	Minds On	Groups Build a box	• Four corners • Choose solution method	• Groups • Discussion: • Importance • Efficacy • Interest • motivation
		Action	Whole Class Worked examples	• Groups • Solve a problem by at least two different methods	• Likert scale • efficacy
		Consolidate/Debrief	Groups Given a problem solving flowchart, identify the various features and then apply to problems *Problem Solving Flowchart v2*	• Timed retell • Explain at least one method to partner	• Ticket to leave • Choose one problem and present solution
– compare, through investigation using technology, the features of the graph of $y = x^2$ and the graph of $y = 2^x$, and determine the meaning of a negative exponent and of zero as an exponent (e.g., by examining patterns in a table of values for $y = 2^x$; by	*Students will learn to interpret the meaning of exponents of 0 and exponents of a negative integer *Students will learn how to extend the exponent rules to exponents of 0 or a negative integer	Minds On	Groups placemat Compare the graphs of $y = x^2$ and $y = 2^x$ • domain • range • intercepts • max/min	• Anticipation guide v3	• (R) Groups • *Money Maker*
		Action	Whole class	• Matching	• Groups • *Exponent Facto*

applying the exponent rules for multiplication and division).			Use Table feature of graphing calculator to develop values for exponents of 0 or negative integers Groups Practice evaluating, exponent laws	• Information to $y = x^2$ *or* $y = 2^x$	
		Consolidate/Debrief	Individual Journal entry: Give several examples of evaluating powers with integer exponents and the exponent laws	• Inside/outside circle • Information to $y = x^2$ *or* $y = 2^x$	• What/So What • List some examples and worked solutions
Review		Minds On	Groups Construct a summary page for quadratic functions		
		Action	Inside/Outside Circle Use Think-Pair-Share to each construct and confirm 3 questions involving quadratic functions Use Inside/Outside Circle to share with classmates	•	• These periods can be inserted as needed for consolidation, skill building, formative assessment. They do not have to be used as full classes, but a total of 75x2=150 minutes may be used in whole or in part.
		Consolidate/Debrief	Groups Solve max/min problems and quadratic equation problems	•	•
Consolidate periods (2)		Recommended: use pairs and groups: gallery walks, jigsaw, inside/outside circle, carousel, think-pair-share, create questions, open questions		• These periods can be inserted as needed for consolidation, skill building, formative assessment. They do not have to be used as full classes, but a total of 75x2=150 minutes may be used in whole or in part.	
RAT		Groups *The painted cube problem*		Groups *The painted cube problem v3*	
Test					
Total 26 classes				• Could include a mixed practice day prior to review/test	

Appendix B

Letter of Invitation to Teachers

Title of Study: *Investigating the Impact of Lessons Based on Marzano's Theory of Learning on Student Attitude, Engagement, and Achievement in Grade 10 Academic Mathematics*

Principal Investigator: Dr. Xavier Fazio, Associate Professor, Brock University

Principal Student Investigator: Jeff Irvine, Ph.D. Candidate, Brock University

I, Jeff Irvine from the Faculty of Education, Brock University, invite you to participate in a research study entitled *Investigating the Impact of Lessons Based on Marzano's Theory of Learning on Student Attitude, Engagement, and Achievement in Secondary School Mathematics.*

Motivation is recognized as a key influence on mathematics achievement. Among the theories of motivation are Robert Marzano's theory of behaviour. There are various components of motivation, and this study is designed to investigate the impact on two of these components, engagement and attitude, by developing, delivering, and evaluating lessons explicitly linked to Marzano's theory.

What Will Students Be Asked To Do
Students will be asked to complete on line surveys of engagement and attitude, before and after you teach the lessons designed for the Quadratic Functions strand. They will also be asked to complete a brief on line reflection at the end of each week. In Part Two of the study, student volunteers will be interviewed about their reaction to the lessons that have been taught.

What Will Teachers Be Asked To Do
You will be asked to collaboratively design a written summative assessment for the Quadratic Functions strand in Grade 10 Academic Mathematics, as well as a rubric for a rich assessment task, deliver these lessons, and reflect on their effectiveness. This will involve up to three days of professional development, daily on line reflections, and interviews before and after the study. You will not be asked to collect any student data, other than allowing time for students to complete two sets of on line surveys, and brief on line student reflections. The interviews will last approximately 30 minutes. Each interview will be audiotaped only to ensure that your answers are recorded correctly. Approximately 3 to 4 weeks after your interview, a copy of the transcript will be sent to you to give you an opportunity to confirm the accuracy of our conversation and to add or clarify any points that you wish. If no response has been received after a follow-up email, it will be assumed that no corrections or clarifications will be required, and the interview data will be included in the research study. No one other than the researcher will listen to the audiotapes. At the conclusion of the study the audiotapes of your interviews will be destroyed.

Your participation in this study is completely voluntary and you may withdraw from the study at any time. For teachers who complete the entire study, benefits will include establishment of a network of collaborative peers; a complete package of effective lessons for the Quadratic Functions strand; practice in and refinement of reflective practice in teaching; a certificate indicating that you have participated in lesson construction using Marzano's New Taxonomy; a nominal per diem to assist in covering travel and meals.

Anonymity cannot be guaranteed because of the existence of face-to-face contact between researchers and teacher participants during the lesson delivery and training, and the teacher interviews. However, all personal information will be kept confidential throughout the process. Furthermore, you will be assigned a pseudonym to respect your privacy and protect your identity. Your actual name will not appear in any publication or presentation resulting from this study. With your permission, anonymous quotations may be used in any publication or presentation resulting from this study.

This research has the potential to benefit student learning in mathematics by identifying effective lessons that positively influence student motivation.

If you have any pertinent questions about your rights as a research participant, please contact the Brock University Ethics Officer at (905) 688-5550, ext. 3035 and/or reb@brocku.ca.

If you have any questions, please feel free to contact me.

Thank you.

Jeff Irvine

Principal Student Investigator
Jeff Irvine
Tel: (905) 872-3345
Email: jeffrey.irvine@brocku.ca

Principal Investigator: Dr. Xavier Fazio, Associate Professor, Brock University
Tel: (905) 688-5550 X5209
Email: xfazio@brocku.ca

This study has been reviewed and received ethics clearance though Brock University's Research Ethics Board (File # xx-xxx-IRVINE)

Appendix C

Informed Consent Form for Teachers

Title of Study: *Investigating the Impact of Lessons Based on Marzano's Theory of Learning on Student Attitude, Engagement, and Achievement in Secondary School Mathematics*

Principal Investigator: Dr. Xavier Fazio, Associate Professor, Brock University

Principal Student Investigator: Jeff Irvine, Ph.D. Candidate, Brock University

Invitation

Motivation is recognized as a key influence on mathematics achievement. Among the theories of motivation are Robert Marzano's theory of behaviour. There are various components of motivation, and this study is designed to investigate the impact on two of these components, engagement and attitude, by developing and delivering lessons explicitly linked to Marzano's theory.

What Will Teachers Be Asked To Do

You will be asked to collaboratively design a written summative assessment for the Quadratic Functions strand in Grade 10 Academic Mathematics, as well as a rubric for a rich assessment task, deliver these lessons, and reflect on their effectiveness. This will involve up to three days of professional development, as a group with the researcher and other Grade 10 Academic teachers participating in the study. You will also be asked to complete daily on line reflections, and interviews before and after the study. You will not be asked to collect any student data, other than allowing time for students to complete two sets of on line surveys, and brief weekly on line student reflections. Daily reflections should take no more than five minutes per day. The teacher interviews will last approximately 30 minutes each. Each interview will be audiotaped only to ensure that your answers are recorded correctly. Approximately 3 to 4 weeks after your interview, a copy of the transcript will be sent to you to give you an opportunity to confirm the accuracy of our conversation and to add or clarify any points that you wish. If no response has been received after a follow-up email, it will be assumed that no corrections or clarifications will be required, and the interview data will be included in the research study. No one other than the researcher will listen to the audiotapes. At the conclusion of the study the audiotapes of your interviews will be destroyed. The researcher will observe approximately 25% of classes during the study, to support implementation fidelity.

Potential Benefits and Risks

This research has the potential to benefit student motivation and achievement in mathematics, by demonstrating that mathematics lessons designed to explicitly address aspects of Marzano's Taxonomy have a positive impact on student engagement and attitude. Findings from this study may result in a shift in teaching practices within Ontario that encourages educators to explicitly consider aspects of student motivation in designing and delivering lessons.

For teachers who participate in this study, benefits will include establishment of a network of collaborative peers; a complete package of effective lessons for the Quadratic Functions strand; and, practice in and refinement of reflective practice in teaching.

There are no foreseeable risks to participation in this study.

Confidentiality

Anonymity cannot be guaranteed because of the existence of face-to-face contact between researchers and teacher participants during the professional development and training, lesson observations, and the teacher interviews. However, all personal information will be kept confidential throughout the process. Furthermore, you will be assigned a pseudonym to respect your privacy and protect your identity, and no personal identifiers will be used in any publication or presentation. Your actual name will not appear in any publication or presentation resulting from this study. With your permission, anonymous quotations may be used in any publication or presentation resulting from this study. The name of your school will not appear in any publication or presentation. Further, since there are over 300 secondary mathematics teachers in Peel District School Board, it is extremely unlikely that anonymous comments concerning the teaching of Grade 10 Academic Mathematics could be attributed to you. All data related to the study will be securely stored in a password-protected computer. Only the researcher will have access to the data, and the data will be destroyed following the acceptance of the dissertation. Any audiotaped data will be securely stored in a locked cabinet, and audiotaped data will be deleted from the recording device upon acceptance of the dissertation.

Because Survey Monkey™ will be used for surveys and on line reflections, you should be aware that this data is located on an American server, and is subject to American Homeland Security laws. However, no risk is anticipated because of this. There is a link on survey monkey for Canada…https://www.surveymonkey.com/curiosity/patriot-act/

Voluntary Participation

Your participation in this study is completely voluntary and you may withdraw from the study at any time. If you wish, you may decline to answer any questions. If you decide to withdraw from this study during the period of data collection, every effort will be made to remove all data attributed to you from the dataset. After the data have been analyzed, your responses cannot be removed from the dataset. However, no part of your interview responses will be included in the final report. For teachers who complete the entire study, benefits will include establishment of a network of collaborative peers; a complete package of effective lessons for the Quadratic Functions strand; practice in and refinement of reflective practice in teaching; a certificate indicating that you have participated in lesson construction using Marzano's New Taxonomy; a nominal per diem to assist in covering travel and meals.

Publication of Results

Results of this study will be published as part of my doctoral dissertation. They may also be published in professional journals and presented at conferences. Feedback about this study will be available from the principal investigator who may be contacted at jeffrey.irvine@brocku.ca.

Contact Information and Ethics Clearance

If you have any questions about this study or require further information, please contact the principal investigator using the contact information provided above. This study has been reviewed and received ethics clearance though Brock University's Research Ethics Board (File # xx-xxx-IRVINE). If you have any pertinent questions about your rights as a research participant, please contact the Brock University Ethics Officer at (905) 688-5550, ext. 3035 and/or reb@brocku.ca.

Consent Form

I agree to participate in the study described above. I have made this decision based on the information I have read in the letter of invitation and the informed consent form. I had the opportunity to receive any additional details I wanted about the study and understand that I may ask questions in the future. I understand that I may withdraw this consent at any time during the data collection process. I understand that by signing this form I am indicating consent for my data to be used confidentially in this research project.

Participant Signature: ____________________________________ Date: ______________

Thank you for your assistance in this study. Please keep a copy of this form for your records.

Principal Student Investigator
Jeff Irvine
Tel: (905) 872-3345
Email: jeffrey.irvine@brocku.ca

Principal Investigator: Dr. Xavier Fazio, Associate Professor, Brock University
Tel: (905) 688-5550 X5209
Email: xfazio@brocku.ca

Appendix D

Letter of Invitation to Students

Title of Study: *Investigating the Impact of Lessons Based on Marzano's Theory of Learning on Student Attitude, Engagement, and Achievement in Secondary School Mathematics*

Principal Investigator: Dr. Xavier Fazio, Associate Professor, Brock University

Principal Student Investigator: Jeff Irvine, Ph.D. Candidate, Brock University

I, Jeff Irvine from the Faculty of Education, Brock University, invite you to participate in a research study entitled *Investigating the Impact of Lessons Based on Marzano's Theory of Learning on Student Attitude, Engagement, and Achievement in Secondary School Mathematics.*

Motivation is a really important aspect of math education. You have probably noticed that for some subjects and for some units in those subjects, you are much more interested and motivated to succeed than for other subjects or units. This study is looking for ways to make your math lessons more interesting and meaningful to you.

What Will Students Be Asked To Do
Students will be asked to complete on line surveys of engagement and attitude, before and after the Quadratic Functions unit in your child's Grade 10 math course. These surveys will take approximately 40 minutes each, and will occur during your child's regular mathematics class. Students will also be asked to complete a brief (5 minute) on line reflection at the end of each week. In Part Two of the study, if your child chooses to volunteer, your child may be interviewed for approximately 30 minutes about your child's reaction to the lessons that have been taught. These interviews will occur during the school day at a mutually agreeable time, such as lunch or after school, in your child's regular mathematics classroom. All the lessons in this unit will be taught by your child's regular teacher. The only difference from your child's regular classes is the surveys that your child will complete. Participation in this study will not affect your child's mark in any way.

Your child's participation in this study is completely voluntary and your child may withdraw from the study at any time.

Your child's survey answers are completely anonymous, and at no time will your child's teacher be able to view your answers or comments. Your child will be assigned a unique student number only so that the researcher can perform before and after comparisons of your child's survey answers. Once the study is complete, these student numbers will be destroyed. No one from the study will meet with your child at any time, unless your child is one of the students who volunteer to be interviewed. If your child does not volunteer to be interviewed, your child's participation in this study will end once you have completed the unit of study. If your child volunteers to be interviewed, the interview will be audiotaped only to ensure that answers are recorded correctly. Approximately 3 to 4 weeks after the interview, a copy of the transcript will be sent to your child via student email to provide an opportunity to confirm the accuracy of our conversation and to add or clarify any points that your child may wish. No one other than the researcher will listen to the audiotape. At the conclusion of the study the audiotape of the interview will be destroyed.

Anonymity is important and the researcher will have no personal contact with any student, other than students who volunteer to be interviewed. The researcher will observe some of the classes, but will not interact with students in any way. However, for all students participating in the study, whether or not they are interviewed, personal information will be kept confidential throughout the process. Furthermore, if they are interviewed, your child will be assigned a pseudonym to respect your child's privacy and protect their identity. Your child's actual name will not appear in any publication or presentation resulting from this study.

By participating in this research you can help to benefit student learning in mathematics, both for your child and for other students.

If you as parent/guardian have any questions about your rights as a research participant, please contact the Brock University Ethics Officer at (905) 688-5550, ext. 3035 and/or reb@brocku.ca.

If you as your parent/guardian have any questions, please feel free to contact me.

Thank you.

Jeff Irvine

Principal Student Investigator
Jeff Irvine
Tel: (905) 872-3345
Email: jeffrey.irvine@brocku.ca

Principal Investigator: Dr. Xavier Fazio, Associate Professor, Brock University
Tel: (905) 688-5550 X5209
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Appendix E

Informed Consent Form for Students

Title of Study: *Investigating the Impact of Lessons Based on Marzano's Theory of Learning on Student Attitude, Engagement, and Achievement in Grade 10 Academic Mathematics*

Principal Investigator: Dr. Xavier Fazio, Associate Professor, Brock University

Principal Student Investigator: Jeff Irvine, Ph.D. Candidate, Brock University

Invitation

Motivation is a really important aspect of math education. You have probably noticed that for some subjects and for some units in those subjects, your child is much more interested and motivated to succeed than for other subjects or units. This study is looking for ways to make your math lessons more interesting and meaningful to your child.

What Will Students Be Asked To Do

Students will be asked to complete on line surveys of engagement and attitude, before and after the Quadratic Functions unit in your Grade 10 math course. These surveys will take approximately 40 minutes each, and will occur during your child's regular mathematics class. Students will also be asked to complete a brief (5 minute) on line reflection at the end of each week. In Part Two of the study, if your child chooses to volunteer, your child may be interviewed about their reaction to the lessons that have been taught. If your child chooses to volunteer, you will receive a separate consent form providing the details of the interview process. This current consent form is only for the survey portion of the study and not for those students who volunteer to be interviewed. All the lessons in this unit will be taught by your child's regular teacher. The only difference from your child's regular classes is the surveys that your child will complete. Participation in this study will not affect your child's mark in any way.

Potential Benefits and Risks

By participating in this research your child can help to benefit student learning in mathematics, both for your child and for other students. Findings from this study may result in changing the way math is taught in Ontario, by encouraging teachers to pay more attention to students' motivation when designing their lessons.

There are no foreseeable risks to participation in this study. All your child's math classes will involve only your child's regular math teacher. Because Survey Monkey™ will be used for surveys and on line reflections, you should be aware that this data is located on an American server, and is subject to American Homeland Security laws. However, no risk is anticipated because of this. There is a link on survey monkey for Canada https://www.surveymonkey.com/curiosity/patriot-act/

Confidentiality

Your child's survey answers will be anonymized, and at no time will your child's teacher be able to view your answers or comments. Your child will be assigned a unique student number only so that the researcher can perform before and after comparisons of your child's survey answers. However, the researcher will not be able to link any response to the actual student's name. Once the study is complete, these student numbers will be destroyed. During the main part of the study, no one from the study will meet with your child at any time. If your child is one of the students who volunteer to be interviewed, you will receive a separate consent form that provides details of the interview process. If your child does not volunteer to be interviewed, your child's participation in this study will end once the unit of study is completed.

Anonymity is important and the researcher will have no personal contact with any student, other than students who volunteer to be interviewed. The researcher will observe some of the classes, but will not interact with students in any way. However, for all students participating in the study, personal information will be kept confidential throughout the process. Furthermore, your child will be assigned a pseudonym to respect their privacy and protect your child's identity. Your child's actual name will not appear in any publication or presentation resulting from this study. All data related to the study will be securely stored in a password-protected computer. Only the researcher will have access to the data, and the data will be destroyed following the completion of the study. Any audiotaped data will be securely stored in a locked cabinet, and audiotaped data will be deleted from the recording device upon completion of the study.

Voluntary Participation

Your child's participation in this study is completely voluntary and your child may withdraw from the study at any time. If your child wishes, they may decline to answer any questions. If your child decides to withdraw from this study during the period of data collection, every effort will be made to remove all data attributed to your child from the dataset. After the data have been analyzed, your child's responses cannot be removed from the dataset, but all responses will be anaonymous. If your child decides to withdraw from this study no part of your child's interview responses will be included in the final report.

Publication of Results

Results of this study will be published as part of my doctoral dissertation. They may also be published in professional journals and presented at conferences. Feedback about this study will be available from the principal investigator who may be contacted at jeffrey.irvine@brocku.ca.

Contact Information and Ethics Clearance

If you have any questions about this study or require further information, please contact the principal investigator using the contact information provided above. This study has been reviewed and received ethics clearance though Brock University's Research Ethics Board (File # xx-xxx-IRVINE). If you have any pertinent questions about your rights as a research participant, please contact the Brock University Ethics Officer at (905) 688-5550, ext. 3035 and/or reb@brocku.ca.

Consent Form

I agree to participate in the study described above. I have made this decision based on the information I have read in the letter of invitation and the informed consent form. I had the opportunity to receive any additional details I wanted about the study and understand that I may ask questions in the future. I understand that I may withdraw this consent at any time during the data collection process. I understand that by signing this form I am indicating consent for my data to be used confidentially in this research project.

Participant Signature: ______________________________________ Date: ______________

Parent/Guardian Signature: __________________________________ Date: ______________

Thank you for your assistance in this study. A copy of this form will be provided for your records.

Principal Student Investigator
Jeff Irvine
Tel: (905) 872-3345
Email: jeffrey.irvine@brocku.ca

Principal Investigator: Dr. Xavier Fazio, Associate Professor, Brock University
Tel: (905) 688-5550 X5209
Email: xfazio@brocku.ca

Appendix F

Dimensions of Student Engagement Survey

Directions: This inventory consists of statements about your attitude toward mathematics. There are no correct or incorrect responses. Read each item carefully. Please think about how you feel about each item. Choose the response that most closely corresponds to how the statement best describes your feelings.

1. When I'm in this class, I listen very carefully.
   Strongly Disagree Disagree Neutral Agree Strongly Agree

2. I enjoy learning new things in this class.
   Strongly Disagree Disagree Neutral Agree Strongly Agree

3. I don't try very hard in this class.
   Strongly Disagree Disagree Neutral Agree Strongly Agree

4. When I'm in this class, my mind wanders.
   Strongly Disagree Disagree Neutral Agree Strongly Agree

5. I let my teacher know what I need and want.
   Strongly Disagree Disagree Neutral Agree Strongly Agree

6. During class, I ask questions to help me learn.
   Strongly Disagree Disagree Neutral Agree Strongly Agree

7. I find it difficult to develop a study plan for this course.
   Strongly Disagree Disagree Neutral Agree Strongly Agree

8. When learning about a new topic in this course, I usually try to summarize it in my own words.
   Strongly Disagree Disagree Neutral Agree Strongly Agree

9. Most of the time in this class, I am passive.
   Strongly Disagree Disagree Neutral Agree Strongly Agree

10. This class is no fun for me.
    Strongly Disagree Disagree Neutral Agree Strongly Agree

11. I pay attention in this class.
    Strongly Disagree Disagree Neutral Agree Strongly Agree

12. In this class, I work as hard as I can.
    Strongly Disagree Disagree Neutral Agree Strongly Agree

13. When I'm in this class, I feel good.
    Strongly Disagree Disagree Neutral Agree Strongly Agree

14. When we work on something in this class, I feel discouraged.
    Strongly Disagree Disagree Neutral Agree Strongly Agree

15. In this class, I do only what I am told to do--nothing more.
    Strongly Disagree Disagree Neutral Agree Strongly Agree

16. I let my teacher know what I am interested in.
    Strongly Disagree Disagree Neutral Agree Strongly Agree

17. This class is fun.
    Strongly Disagree Disagree Neutral Agree Strongly Agree

18. When reading for this class, I try to explain the key concepts in my own words.
    Strongly Disagree Disagree Neutral Agree Strongly Agree

19. I'm not sure how to study for this course.
    Strongly Disagree Disagree Neutral Agree Strongly Agree

20. When thinking about the concepts in this class, I try to generate examples to help me understand them better.
    Strongly Disagree Disagree Neutral Agree Strongly Agree

21. When I study for this course, I have trouble figuring out what to do to learn the material.
    Strongly Disagree Disagree Neutral Agree Strongly Agree

22. In this course, I do just enough to get by.
    Strongly Disagree Disagree Neutral Agree Strongly Agree

23. When I'm in this class, I feel worried.
    Strongly Disagree Disagree Neutral Agree Strongly Agree

24. During this class, I express my preferences and opinions.
    Strongly Disagree Disagree Neutral Agree Strongly Agree

25. I try hard to do well in this class.
    Strongly Disagree Disagree Neutral Agree Strongly Agree

26. When I'm in this class, I just act like I'm working.
    Strongly Disagree Disagree Neutral Agree Strongly Agree

27. When I am in this class, I feel bad.
    Strongly Disagree Disagree Neutral Agree Strongly Agree

28. When I need something in this class, I'll ask the teacher for it.
    Strongly Disagree Disagree Neutral Agree Strongly Agree

29. When we work on something in this class, I get involved.
    Strongly Disagree Disagree Neutral Agree Strongly Agree

30. When I'm in this class, I think about other things.
    Strongly Disagree Disagree Neutral Agree Strongly Agree

31. Most of the time in this class, I am silent and unresponsive.
    Strongly Disagree Disagree Neutral Agree Strongly Agree

32. When reading for this class, I try to connect the ideas I am reading about with what I already know.
    Strongly Disagree Disagree Neutral Agree Strongly Agree

33. In this course, I find it difficult to organize my study time effectively.
    Strongly Disagree Disagree Neutral Agree Strongly Agree

34. During this class, I hide from the teacher what I am thinking about.
    Strongly Disagree Disagree Neutral Agree Strongly Agree

35. In this class, I avoid asking questions.
    Strongly Disagree Disagree Neutral Agree Strongly Agree

36. When I'm in this class, I participate in class discussions.
Strongly Disagree Disagree Neutral Agree Strongly Agree

37. When we work on something in this class, I feel bored.
Strongly Disagree Disagree Neutral Agree Strongly Agree

38. In this course, I often find that I don't know what to study or where to start.
Strongly Disagree Disagree Neutral Agree Strongly Agree

39. When we work on something in this class, I feel interested.
Strongly Disagree Disagree Neutral Agree Strongly Agree

Appendix G

Student Interview Protocol

Thank you for volunteering to be interviewed today. I'm Jeff Irvine, the researcher responsible for this study. I've been interested in the role of motivation in math class for a long time, and this survey will help me find some ways to make math class more interesting and enjoyable for students. When I was a classroom math teacher, I wanted students leaving my class to be able to say two things--I had fun today; and I learned something. I'm hoping that this current study will help.

There are no right or wrong answers to my questions. Please answer them honestly, with what you really feel. If you need me to repeat a question or if what I ask is confusing, I would be happy to repeat or rephrase the question. If you don't want to answer a question, that's OK as well.

1. HOW DO YOU FEEL ABOUT SCHOOL IN GENERAL? WHY?
2. WHAT IS YOUR FAVOURITE SUBJECT? WHY?
3. WHAT IS YOUR LEAST FAVOURITE SUBJECT? WHY?
4. HOW DO YOU USUALLY FEEL ABOUT MATH CLASS?
5. IN GENERAL DO YOU FEEL THAT YOU WORK HARD IN MATH CLASS?
6. WHO DO YOU THINK IS USUALLY BETTER AT MATH, GIRLS OR BOYS? WHY DO YOU THINK THAT?
7. HOW DO YOU FEEL THAT YOU LEARN MATH CONCEPTS BEST--BY SEEING IT, BY HEARING ABOUT IT, OR BY ACTUALLY DOING IT?
8. IN THE LAST MONTH YOUR MATH CLASSES HAVE BEEN SOMEWHAT DIFFERENT. IN THE LAST MONTH HAVE YOUR FEELINGS ABOUT MATH CLASS CHANGED IN ANY WAY? WHY DO YOU THINK THIS IS?
9. OVER THE LAST MONTH DO YOU FEEL THAT YOU HAVE BEEN MORE ENGAGED IN YOUR MATH CLASSES? WHY DO YOU THINK THAT?
10. DURING THE LAST MONTH DO YOU FEEL THAT YOU WORKED HARDER THAN YOU USUALLY DO IN MATH CLASS?
11. HAS YOUR ATTITUDE TOWARD MATH CLASS CHANGED OVER THE LAST MONTH?
12. WOULD YOU LIKE MORE OF YOUR MATH CLASSES TO BE LIKE THE LAST MONTH? WHY?
13. CAN YOU THINK OF ONE CLASS DURING THIS TIME THAT YOU REALLY LIKED? WHY/WHY NOT? WHAT MADE YOU LIKE THIS CLASS?
14. CAN YOU THINK OF ONE CLASS DURING THIS TIME WHEN YOU FELT YOU REALLY ENGAGED AND WORKED HARD? WHAT ABOUT THIS CLASS MADE YOU FEEL THAT WAY?

15. HOW WELL DO YOU THINK YOU UNDERSTAND THE TOPICS IN THIS UNIT? WHY DO YOU THINK THAT?
16. HOW WELL DO YOU THINK YOU DID ON THE EVALUATIONS FOR THIS UNIT? WHY DO YOU THINK THAT?
17. IS YOUR ACHIEVEMENT ON THIS UNIT DIFFERENT FROM YOUR USUAL ACHIEVEMENT IN MATH? WHY DO YOU THINK THAT IS?

Appendix H

Teacher Pre-Study Interview Guide

1. HOW MANY YEARS HAVE YOU BEEN TEACHING?
2. HOW MANY YEARS HAVE YOU TAUGHT AT (SCHOOL)?
3. HOW MANY YEARS HAVE YOU TAUGHT MATHEMATICS?
4. WHAT IS YOUR PERSONAL PHILOSOPHY OF TEACHING MATHEMATICS?
5. WHAT IS YOUR USUAL TEACHING STYLE IN YOUR MATH CLASSES?
6. DO YOUR STUDENTS GENERALLY DO WELL IN MATH? WHY DO YOU THINK THAT IS?
7. OVERALL, WHAT IS YOUR STUDENTS' ATTITUDE TOWARD LEARNING MATHEMATICS? TO WHAT DO YOU ATTRIBUTE THIS ATTITUDE?
8. DO YOU DO ANYTHING SPECIFICALLY TARGETED AT YOUR STUDENTS' ATTITUDES TOWARDS MATH? PLEASE EXPLAIN.
9. WHAT IS YOUR STUDENTS' TYPICAL LEVEL OF ENGAGEMENT IN LEARNING MATHEMATICS? TO WHAT DO YOU ATTRIBUTE THIS?
10. DO YOU DO ANYTHING SPECIFICALLY TARGETED AT YOUR STUDENTS' ENGAGEMENT IN MATH? PLEASE EXPLAIN.
11. HAVE YOU BEEN PART OF A OTHER RESEARCH STUDIES IN MATHEMATICS?
12. WHY DID YOU DECIDE TO VOLUNTEER FOR THIS RESEARCH STUDY?
13. IS THERE ANYTHING ELSE I SHOULD KNOW BEFORE WE BEGIN THIS RESEARCH STUDY?

Appendix I

Teacher Post-Study Interview Protocol

Thank you for participating in this study and agreeing to be interviewed today. I've been interested in the role of motivation in math class for a long time, and this survey will help me find some ways to make math class more interesting and enjoyable for students, and therefore increase their achievement in math. When I was a classroom math teacher, I wanted students leaving my class to be able to say two things--I had fun today; and I learned something. I'm hoping that this current study will help.

There are no right or wrong answers to my questions. Please answer them honestly, with what you really feel. If you need me to repeat a question or if what I ask is confusing, I would be happy to repeat or rephrase the question. If you don't want to answer a question, that's OK as well.

1. HOW DO YOU THINK YOUR STUDENTS USUALLY FEEL ABOUT MATH CLASS?
2. IS THIS DIFFERENT THAN THEIR FEELINGS ABOUT OTHER SUBJECTS? WHY DO YOU THINK THAT?
3. USUALLY HOW ENGAGED ARE YOUR STUDENTS?
4. WHAT ARE THEIR WORK HABITS LIKE?
5. DURING THIS STUDY, DID YOU SEE ANY CHANGES IN STUDENT ATTITUDES?
6. DURING THIS STUDY, DID YOU SEE ANY CHANGES IN STUDENT ENGAGEMENT?
7. DURING THIS STUDY, DID YOU SEE ANY CHANGES IN STUDENT WORK HABITS?
8. DURING THIS STUDY, DID YOU SEE ANY CHANGES IN PERSISTENCE?
9. WAS THERE A PARTICULAR LESSON THAT YOU FELT REALLY ENGAGED STUDENTS? WHICH ONE(S)?
10. DO YOU FEEL THAT THE LESSONS IN THIS STUDY WERE MORE EFFECTIVE OR LESS EFFECTIVE THAN THE WAY YOU USUALLY TEACH QUADRATIC FUNCTIONS? WHY?
11. WOULD YOU PARTICIPATE IN A SIMILAR STUDY TO DEVELOP LESSONS FOR ANOTHER STRAND IN GRADE 10 OR FOR ANOTHER COURSE? WHY/WHY NOT?
12. WHAT ELSE DO YOU THINK I SHOULD KNOW ABOUT HOW THIS STUDY IMPACTED YOUR MATH CLASSES?

Appendix J

Attitudess Toward Mathematics Inventory

Directions: This inventory consists of statements about your attitude toward mathematics. There are no correct or incorrect responses. Read each item carefully. Please think about how you feel about each item. Choose the response that most closely corresponds to how the statement best describes your feelings.

1. Mathematics is a very worthwhile and necessary subject.
   Strongly Disagree Disagree Neutral Agree Strongly Agree

2. I want to develop my mathematical skills.
   Strongly Disagree Disagree Neutral Agree Strongly Agree

3. I get a great deal of satisfaction out of solving a mathematics problem.
   Strongly Disagree Disagree Neutral Agree Strongly Agree

4. Mathematics helps develop the mind and teaches a person to think.
   Strongly Disagree Disagree Neutral Agree Strongly Agree

5. Mathematics is important in everyday life.
   Strongly Disagree Disagree Neutral Agree Strongly Agree

6. Mathematics is one of the most important subjects for people to study.
   Strongly Disagree Disagree Neutral Agree Strongly Agree

7. High school math courses would be very helpful no matter what I decide to study.
   Strongly Disagree Disagree Neutral Agree Strongly Agree

8. I can think of many ways that I use math outside of school.
   Strongly Disagree Disagree Neutral Agree Strongly Agree

9. Mathematics is one of my most dreaded subjects.
   Strongly Disagree Disagree Neutral Agree Strongly Agree

10. My mind goes blank and I am unable to think clearly when working with mathematics.
    Strongly Disagree Disagree Neutral Agree Strongly Agree

11. Studying mathematics makes me feel nervous.
    Strongly Disagree Disagree Neutral Agree Strongly Agree

12. Mathematics makes me feel uncomfortable.
    Strongly Disagree Disagree Neutral Agree Strongly Agree

13. I am always under a terrible strain in a math class.
    Strongly Disagree Disagree Neutral Agree Strongly Agree

14. When I hear the word mathematics, I have a feeling of dislike.
    Strongly Disagree Disagree Neutral Agree Strongly Agree

15. IT makes me nervous to even think about having to do a mathematics problem.
    Strongly Disagree Disagree Neutral Agree Strongly Agree

16. Mathematics does not scare me at all.
    Strongly Disagree Disagree Neutral Agree Strongly Agree

17. I have a lot of self-confidence when it comes to mathematics.
    Strongly Disagree Disagree Neutral Agree Strongly Agree

18. I am able to solve mathematics problems without too much difficulty.
    Strongly Disagree Disagree Neutral Agree Strongly Agree

19. I expect to do fairly well in any math class I take.
    Strongly Disagree Disagree Neutral Agree Strongly Agree

20. I am always confused in my mathematics class.
    Strongly Disagree Disagree Neutral Agree Strongly Agree

21. I feel a sense of insecurity when attempting mathematics.
    Strongly Disagree Disagree Neutral Agree Strongly Agree

22. I learn mathematics easily.
    Strongly Disagree Disagree Neutral Agree Strongly Agree

23. I am confident that I could learn advanced mathematics.
    Strongly Disagree Disagree Neutral Agree Strongly Agree

24. I have usually enjoyed studying mathematics in school.
Strongly Disagree Disagree Neutral Agree Strongly Agree

25. Mathematics is dull and boring.
Strongly Disagree Disagree Neutral Agree Strongly Agree

26. I like to solve new problems in mathematics.
Strongly Disagree Disagree Neutral Agree Strongly Agree

27. I would prefer to do an assignment in math than to write an essay.
Strongly Disagree Disagree Neutral Agree Strongly Agree

28. I would like to avoid using mathematics in college.
Strongly Disagree Disagree Neutral Agree Strongly Agree

29. I really like mathematics.
Strongly Disagree Disagree Neutral Agree Strongly Agree

30. I am happier in a math class than in any other class.
Strongly Disagree Disagree Neutral Agree Strongly Agree

31. Mathematics is a very interesting subject.
Strongly Disagree Disagree Neutral Agree Strongly Agree

32. I am willing to take more than the required amount of mathematics.
Strongly Disagree Disagree Neutral Agree Strongly Agree

33. I plan to take as much mathematics as I can during my education.
Strongly Disagree Disagree Neutral Agree Strongly Agree

34. The challenge of math appeals to me.
Strongly Disagree Disagree Neutral Agree Strongly Agree

35. I think studying advanced mathematics is useful.
Strongly Disagree Disagree Neutral Agree Strongly Agree

36. I believe studying math helps me with problem solving in other areas.
Strongly Disagree Disagree Neutral Agree Strongly Agree

37. I am comfortable expressing my own ideas on how to look for solutions to a difficult problem in math.
    Strongly Disagree Disagree Neutral Agree Strongly Agree

38. I am comfortable answering questions in math class.
    Strongly Disagree Disagree Neutral Agree Strongly Agree

39. A strong math background could help me in my professional life.
    Strongly Disagree Disagree Neutral Agree Strongly Agree

40. I believe I am good at solving math problems.
    Strongly Disagree Disagree Neutral Agree Strongly Agree

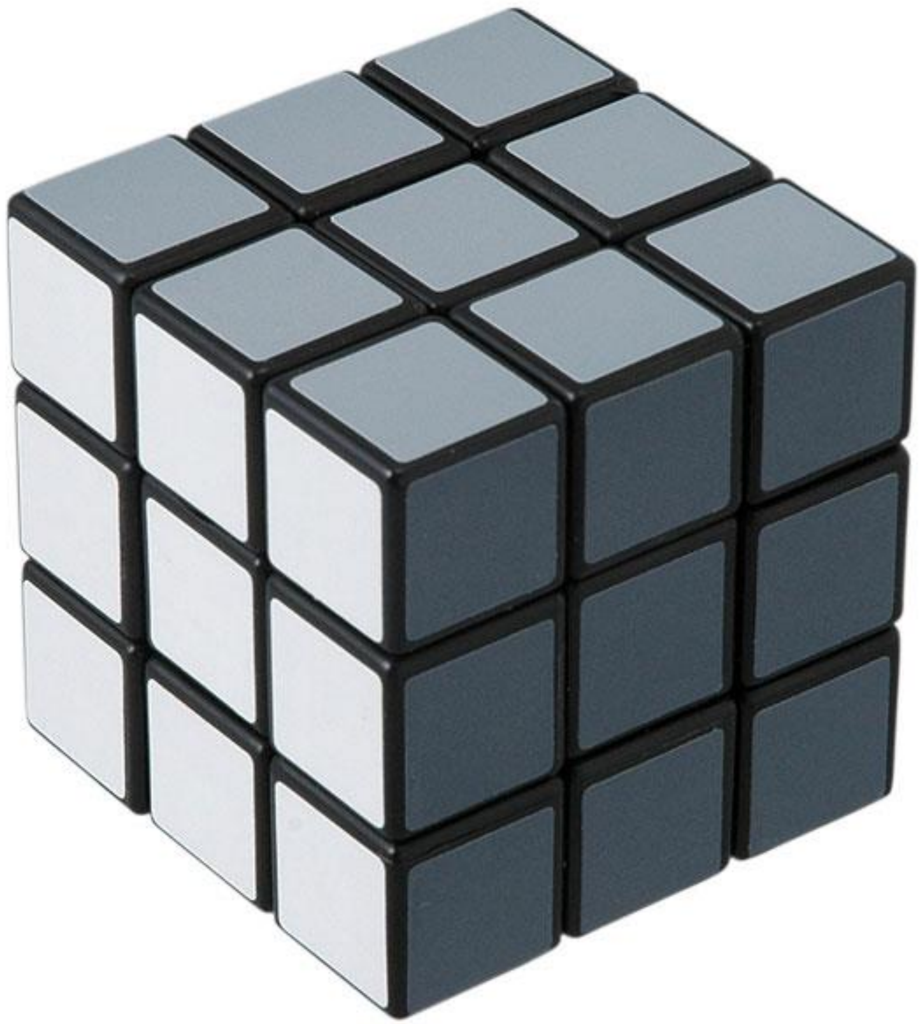
Appendix K

Rich Assessment Task

The Painted Cube Problem

- Using the linking cubes, construct a 3x3x3 cube.
- The large cube is immersed in paint. Answer the questions below.

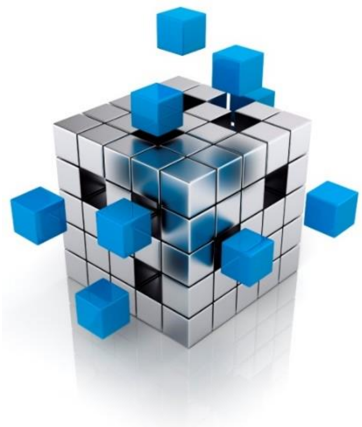
1. How many of the small cubes have no sides painted?
2. How many of the small cubes have exactly one side painted?
3. How many of the small cubes have exactly two sides painted?
4. How many of the small cubes have exactly three sides painted?



5. Complete the 10th row of this table.

Cube size	Total number of cubes	Number of small cubes that have exactly			
		0 sides painted	1 side painted	2 sides painted	3 sides painted
1x1x1	1				
2x2x2					
3x3x3					
4x4x4					
5x5x5					
6x6x6					
7x7x7					
8x8x8					
9x9x9					
10x10x10					

6. Generate formulas for each of the columns. Explain how you arrived at your formulas.



Appendix L

Rubric for Painted Cube Rich Assessment Task

	Level 1	Level 2	Level 3	Level 4
Exploration /Model Building	Unable to construct models with assistance	Constructs models with some assistance	Constructs models as required	Visualizes without full models
Patterning /Generalizing	With assistance, identifies some patterns	With assistance, identifies patterns for all situations	Identifies patterns for all situations	Identifies and extends patterns for all situations
Use of Technology	Requires significant assistance in use of technology	With assistance, uses technology appropriatel y	Uses technology appropriately	Uses technology to extend thinking
Conclusions /Forecasting	With assistance, craws some conclusions and generates some but not all formulas	With assistance, draws conclusions and generates formulas	Draws appropriate conclusions and generates formulas	Draws conclusions, generates formulas, and uses formulas to forecast for larger cases
Group Dynamics /Cooperation/Leadershi p	Makes limited contribution s to group processing	Makes some contributions to group processing	Contributes to group processing cooperativel y	Demonstrate s respectful leadership in group processing

Appendix M

Sample Complete Lesson Plan

Unit #: Day #: (max/min problems)		Grade
75 min	*Students will learn how to model real life situations using quadratic functions *Students will learn how to solve quadratic models to answer real life questions	**Materials** •
Minds On… 15 min	**→in groups** **Complete "build Trigger a Dog Pen" using individual white boards with grid lines**	Grid paper
Action! 15 min	**→whole class** **Teacher-lead algebraic solution of the warmup problem** **emphasize objective (maximize area), reduce area formula to one variable and then complete the square**	
Consolidate Debrief 45 min	**→four corners, students choose a corner** **In each corner, students complete a solution to the problem** **Students write solution on large chart paper** **Gallery walk to share solutions with other groups** **Corner problems are on handout "Max/min problems for four corners"**	4 problems on large chart paper Large chart paper for student solutions markers
	Home Activity or Further Classroom Consolidation **Teacher-assigned**	

Build Trigger a Dog Pen

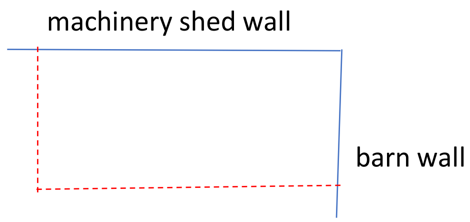
Sanjay was given a puppy as a birthday gift. He wants to make his new puppy, which he has named Trigger, a dog pen in the backyard. He has 24 m of fencing to work with and wants to make the biggest rectangular pen that he can.

Help Sanjay out:

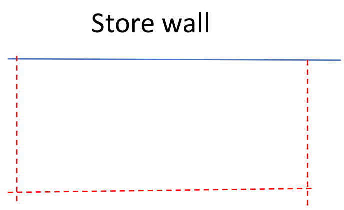
Use grid paper to draw all possible rectangles that have a total perimeter of 24 m. Then find the area of each rectangle and identify the dimensions that give the maximum area.

Max/min Problems for Four Corners Activity

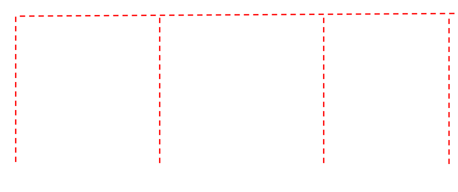
1. A farmer wants to build a livestock pen in the corner formed by her barn and her machinery shed. She has 40 m of fencing and wants the area of the pen to be as large as possible. What should be the dimensions of the pen and what is the maximum area?



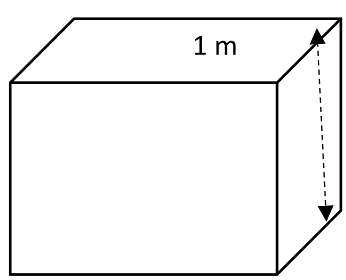
2. A hardware store wants to build a fenced area behind the store in which to store supplies. The fence will form three sides of the rectangular enclosure and the wall of the store will be the fourth side. The storage area should be as large as possible. There is 60 m of fencing available. What are the dimensions of the storage area and what is the maximum area?



A veterinary clinic wants to build three dog runs behind the clinic, as shown. They have 48 m of fencing available. What should the dimensions be to make the pens as large as possible?



3. A shipping company wants to make an open-topped box to ship video games. The box is to have a height of 1 m. The company has 12 linear metres of cardboard that is exactly 1 m high. The box bottom will be made later from reinforced cardboard. What dimensions for the box will maximize the volume? What is the maximum volume?



Appendix N

Sample HOTS Activity

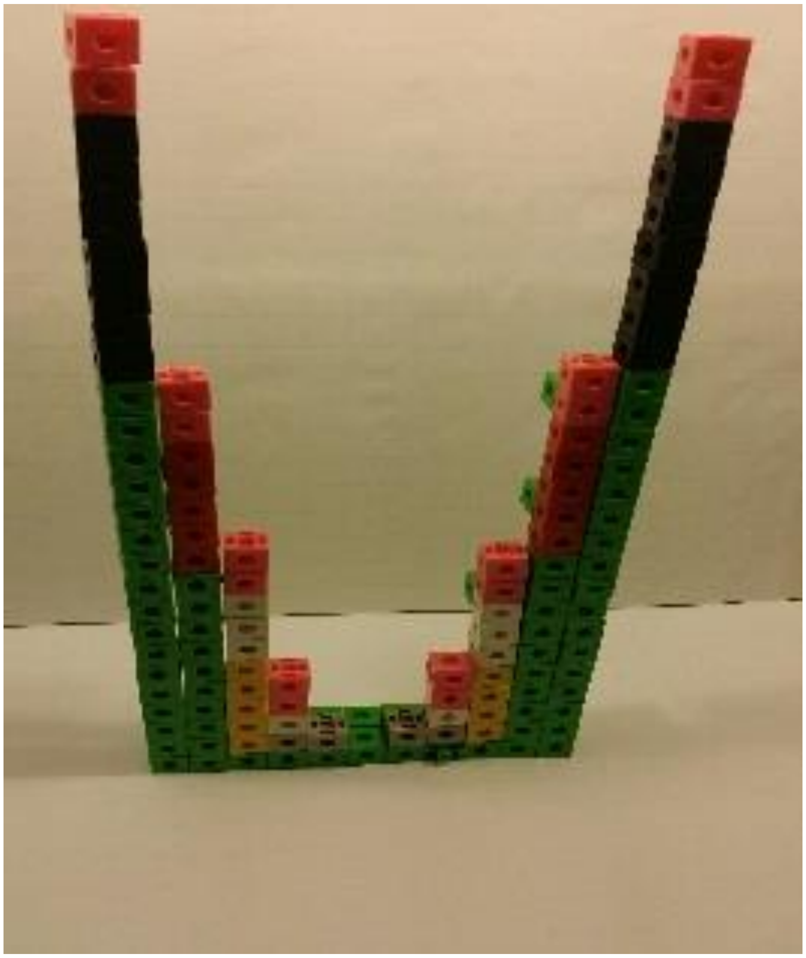
Connecting Cube Quadratics

1. Build a representation of $y = x^2$ for $-5 \leq x \leq 5$, using 3 colours of connecting cubes. The representation must show first and second differences.

2. How would you modify your representation for each of the following:

$$y = 2x^2$$
$$y = 0.5x^2$$
$$y = x^2 + 3$$
$$y = (x - 2)^2$$
$$y = (x + 3)^2 + 2$$

3. Justify your answers for #2. Describe how to build a representation for the general quadratic $y = a(x - h)^2 + k$



Appendix O

Sample Goal-Setting Activity

MY GOAL FOR QUADRATIC WORD PROBLEMS

The next part of this unit involves solving word problems using quadratic relations.

3. Identify your personal goal for this part of the unit. Such as:
   - Master the material and apply it to new as well as routine problems
   - Competently apply the material to routine problems
   - Sufficiently understand the material to achieve at least a B or better on this material
   - Other

My goal for this section of the unit is ...

To be able to use past knowledge in conjunction with my new learning and Problem solving skills to find a solution to given problems. I want to be able to solve and understand what the numbers in a word problem mean.

4. What do you plan to do to achieve your goal? Some examples are:
   - Complete all assigned homework and correct mistakes
   - Ask questions of the teacher when I don't understand
   - Ask for help in class when I don't understand, or I get stuck
   - Ask for help outside of class when I don't understand or get stuck
   - Work with a friend to complete and understand my work
   - Other

To achieve my goal I will ...

go over lessons after school, complete all homework, ask for extra help if I need it and more. I will look over tests and correct any mista to make sure I understand past lessons / problems before learning new ones.

MY GOAL FOR QUADRATIC WORD PROBLEMS

My goal for this section of the unit is ... to get a 50% !

To achieve my goal I will ... Teach myself how to do Math

MY GOAL FOR QUADRATIC WORD PROBLEMS

My goal for this section of the unit is ...

My goal is to thoroughly understand this unit and ~~to~~ get 90s for this unit

To achieve my goal I will ...

To achieve my goal I will do all the homework, and create a study guide for the ~~test~~ to make sense of understand everything

MY GOAL FOR QUADRATIC WORD PROBLEMS

My goal for this section of the unit is ...

To master this material and be able to apply it to new problems.

To achieve my goal I will ...

Ask for help outside of class and complete and understand my work.

MY GOAL FOR QUADRATIC WORD PROBLEMS

My goal for this section of the unit is ...

understand the material
achieve atleast an 80

To achieve my goal I will ...

try harder

MY GOAL FOR QUADRATIC WORD PROBLEMS

The next part of this unit involves solving word problems using quadratic relation

3. Identify your personal goal for this part of the unit. Such as:
   - Master the material and apply it to new as well as routine problems
   - Competently apply the material to routine problems
   - Sufficiently understand the material to achieve at least a B or better on this material
   - Other

My goal for this section of the unit is ...

understand the entire concept so it's easier to do, I also want to higher my mark to a B

4. What do you plan to do to achieve your goal? Some examples are:
   - Complete all assigned homework and correct mistakes
   - Ask questions of the teacher when I don't understand
   - Ask for help in class when I don't understand, or I get stuck
   - Ask for help outside of class when I don't understand or get stuck
   - Work with a friend to complete and understand my work
   - Other

To achieve my goal I will ...

not procrastinate and work hard by Seeking extra help and doing the work well.

Appendix P

Overall and Specific Expectations for the Quadratic Relations Unit

(*The Ontario Curriculum Grades 9 and 10 Mathematics Revised.*, 2005)

Overall Expectations

By the end of this course, students will:

QR.OV1 determine the basic properties of quadratic relations;

QR.OV2 relate transformations of the graph of $y = x^2$ to the algebraic representation $y = a(x - h)^2 + k$;
QR.OV.3 solve quadratic equations and interpret the solutions with respect to the corresponding relations;
QR.OV4 solve problems involving quadratic relations.

Specific Expectations

Investigating the Basic Properties of Quadratic Relations

By the end of this course, students will:

QR 1.01– collect data that can be represented as a quadratic relation, from experiments using appropriate equipment and technology (e.g., concrete materials, scientific probes, graphing calculators), or from secondary sources (e.g., the Internet, Statistics Canada); graph the data and draw a curve of best fit, if appropriate, with or without the use of technology (*Sample problem:* Make a 1 m ramp that makes a 15° angle with the floor. Place a can 30 cm up the ramp. Record the time it takes for the can to roll to the bottom. Repeat by placing the can 40 cm, 50 cm, and 60 cm up the ramp, and so on. Graph the data and draw the curve of best fit.);

QR.1.02– determine, through investigation with and without the use of technology, that a quadratic relation of the form $y = ax^2 + bx + c$ can be graphically represented as a parabola, and that the table of values yields a constant second difference (*Sample problem:* Graph the relation $y = x^2 - 4x$ by developing a table of values and plotting points. Observe the shape of the graph. Calculate first and second differences. Repeat for different quadratic relations. Describe your observations and make conclusions, using the appropriate terminology.);

QR 1.03– identify the key features of a graph of a parabola (i.e., the equation of the axis of symmetry, the coordinates of the vertex, the y-intercept, the zeros, and the maximum or minimum value), and use the appropriate terminology to describe them;

QR 1.04– compare, through investigation using technology, the features of the graph of $y = x^2$ and the graph of $y = 2^x$, and determine the meaning of a negative exponent and of zero as an exponent (e.g., by examining patterns in a table of values for $y = 2x$; by applying the exponent rules for multiplication and division).

Relating the Graph of $y = x^2$ and Its Transformations

QR 2.01– identify, through investigation using technology, the effect on the graph of $y = x^2$ of transformations (i.e., translations, reflections in the x-axis, vertical stretches or compressions) by considering separately each parameter a, h, and k [i.e., investigate the effect on the graph of $y = x^2$ of a, h, and k in $y = x^2 + k$, $y = (x – h)^2$, and $y = ax^2$];

QR 2.02– explain the roles of a, h, and k in $y = a(x – h)^2 + k$, using the appropriate terminology to describe the transformations, and identify the vertex and the equation of the axis of symmetry;

QR 2.03– sketch, by hand, the graph of $y = a(x – h)^2 + k$ by applying transformations to the graph of $y = x^2$ [*Sample problem:* Sketch the graph of $y =– (x – 3)^2 + 4$, and verify using technology.];

QR 2.04– determine the equation, in the form $y = a(x – h)^2 + k$, of a given graph of a parabola.

Also, every unit is expected to involve the seven mathematical process expectations: problem solving, reasoning and proving, reflecting, selecting tools and computational strategies, connecting, representing, and communicating.

Appendix Q

Student Demographic Questions

Student Number: ## ###

Class Student

Sex: Male Female

What was your final mark in the last math course you took?

A (80%-100%)

B (70%-79%)

C (60%-69%)

D (50%-59%)

F (below 50%)

Appendix R

Student Weekly Reflection

Choose the response that best matches your feelings about math this week.

- I enjoyed this week in math.

  Strongly Disagree Disagree Neutral Agree Strongly Agree

- I worked hard this week in math.

  Strongly Disagree Disagree Neutral Agree Strongly Agree

- I would like more math classes to be like this week's math classes.

  Strongly Disagree Disagree Neutral Agree Strongly Agree

- Comments (optional)

Appendix S

Teacher Daily Reflections

Overall, the lesson went well.

Strongly Disagree	Disagree	Neutral	Agree	Strongly Agree

Most students were engaged most of the time.

Strongly Disagree	Disagree	Neutral	Agree	Strongly Agree

Most students enjoyed this lesson most of the time.

Strongly Disagree	Disagree	Neutral	Agree	Strongly Agree

Rate the effectiveness of each part of the lesson.

Minds On

very ineffective	ineffective	neutral	effective	very effective

Action

very ineffective	ineffective	neutral	effective	very effective

Consolidate/Debrief

very ineffective	ineffective	neutral	effective	very effective

DO NOT USE STUDENTS" ACTUAL NAMES FOR THE FOLLOWING

These students were particularly engaged today (How do you know):

These students were particularly disengaged today (How do you know):

What recommendations do you have for making this lesson better:

Appendix T

Informed Consent Form for Students Who Volunteer to Be Interviewed

Title of Study: *Investigating the Impact of Lessons Based on Marzano's Theory of Learning on Student Attitude, Engagement, and Achievement in Secondary School Mathematics*

Principal Investigator: Dr. Xavier Fazio, Associate Professor, Brock University

Principal Student Investigator: Jeff Irvine, Ph.D. Candidate, Brock University

Invitation

Motivation is a really important aspect of math education. You have probably noticed that for some subjects and for some units in those subjects, you are much more interested and motivated to succeed than for other subjects or units. This study is looking for ways to make your math lessons more interesting and meaningful to you.

What Will Students Be Asked To Do

In this part of the study, if your child chooses to volunteer, your child may be interviewed about their reaction to the lessons that have been taught. The interview will last approximately 30 minutes and will be held at if your child's school at a mutually agreeable time, like a lunch period or after school. Participation in this study will not affect your child's mark in any way. If your child volunteers to be interviewed, the interview will be audiotaped only to ensure that answers are recorded correctly. Approximately 3 to 4 weeks after the interview, a copy of the transcript will be sent to your child via student email to provide an opportunity to confirm the accuracy of our conversation and to add or clarify any points that your child may wish. If no response has been received after a follow-up email, it will be assumed that no corrections or clarifications will be required, and the interview data will be included in the research study. No one other than the researcher will listen to the audiotape. At the conclusion of the study the audiotape of the interview will be destroyed.

Potential Benefits and Risks

By agreeing to be interviewed, your child will have the opportunity to tell me whether they found the math lessons in this unit to be more engaging and interesting than math lessons your child has experienced in the past.

By participating in this research your child can help to benefit student learning in mathematics, both for your child and for other students. Findings from this study may result in changing the way math is taught in Ontario, by encouraging teachers to pay more attention to students' motivation when designing their lessons.

There are no foreseeable risks to participation in this study. All your child's math classes will involve only your regular math teacher.

Confidentiality

If your child volunteers to be interviewed, the interview will be audiotaped only to ensure that answers are recorded correctly. Approximately 3 to 4 weeks after your interview, a copy of the transcript will be sent to your child's student email to provide an opportunity to confirm the accuracy of our conversation and to add or clarify any points that your child may wish. No one other than the researcher will listen to the audiotape.

Anonymity is important and for all students participating in the study, personal information will be kept confidential throughout the process. Furthermore, your child will be assigned a pseudonym to respect their privacy and protect your child's identity. Your child's actual name will not appear in any publication or presentation resulting from this study. All data related to the study will be securely stored in a password-protected computer. Only the researcher will have access to the data, and the data will be destroyed following the completion of the study. Any audiotaped data will be securely stored in a locked cabinet, and audiotaped data will be deleted from the recording device upon completion of the study.

Voluntary Participation

Your child's participation in this study is completely voluntary and your child may withdraw from the study at any time. If your child wishes, they may decline to answer any questions. If your child decides to withdraw from this study, no part of your child's interview responses will be included in the final report.

Publication of Results

Results of this study will be published as part of my doctoral dissertation. They may also be published in professional journals and presented at conferences. Feedback about this study will be available from the principal student investigator who may be contacted at jeffrey.irvine@brocku.ca.

Contact Information and Ethics Clearance

If you have any questions about this study or require further information, please contact the principal investigator using the contact information provided above. This study has been reviewed and received ethics clearance though Brock University's Research Ethics Board (File # xx-xxx-IRVINE). If you have any pertinent questions about your rights as a research participant, please contact the Brock University Ethics Officer at (905) 688-5550, ext. 3035 and/or reb@brocku.ca.

Consent Form

I agree to participate in the interview portion of the study described above. I have made this decision based on the information I have read in the letter of invitation and the informed consent form. I had the opportunity to receive any additional details I wanted about the study and understand that I may ask questions in the future. I understand that I may withdraw this consent at any time during the data collection process. I understand that by signing this form I am indicating consent for my data to be used confidentially in this research project.

Participant Signature: ____ Date: _______________

Parent/Guardian Signature: ___________________________________ Date: _______________

Thank you for your assistance in this study. A copy of this form will be provided for your records.

Principal Student Investigator
Jeff Irvine
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Principal Investigator: Dr. Xavier Fazio, Associate Professor, Brock University
Tel: (905) 688-5550 X5209
Email: xfazio@brocku.ca

Appendix U

Activities and Instructional Strategies Employed in the MNT Intervention

Instructional Strategies

Four corners

Grafitti

Inside/outside circle

Jigsaw

Journals

Open questions

Placemat

Plus minus interesting

Snowball

Think aloud

Think-pair-share

Timed retell

What/so what double entry

What/why/source

Manipulatives

Algebra tiles

Linking cubes

Straws

Spaghetti

Desmos.com

Surveys

This New Unit in Math

When I think of starting this new unit in math

- I feel
  very confident | confident | OK | not confident | really not confident
- I feel
  really excited | excited | neutral | not excited | really not excited
- I feel that the new unit will be
  really interesting | interesting | OK | not interesting | really not interesting
- I feel that the new unit will be
  really useful to me | useful to me | neutral | not useful to me | really not useful to me
- I feel motivated to do my best on this unit
  strongly agree | agree | so-so | disagree | strongly disagree
- What one word sums up your feelings about this unit:___________________________

Circle the emoji that best represents your feelings for today's class

How interested were you in today's class?

😢 😣 😔 😏 😊

How confident are you about what you learned in today's class?

😢 😣 😔 😏 😊

How useful do you think what you learned today is to you?

😢 😣 😔 😏 😊

Are you looking forward to tomorrow's math class?

😢 😣 😔 😏 😊

How interested were you in today's class? PLACE YOURSELF ON EACH SCALE

Really interested □□□□□□□□□□ Really uninterested

How confident are you that you understand the material in today's class?

Very confident □□□□□□□□□□ Not at all confident

How useful do you think what you learned in today's class is to you?

Very useful □□□□□□□□□□ Not at all useful

Do you want to learn more about this topic in tomorrow's math class?

Definitely want to learn more □□□□□□□□□□ Not at all interested in learning

Circle how you feel about what you learned in math class today.

Today's class was interesting.

Strongly agree   Agree   Neutral   Disagree   Strongly disagree

Today's class was useful to me.

Strongly agree   Agree   Neutral   Disagree   Strongly disagree

I feel confident that I understand the material from today's class.

Strongly agree   Agree   Neutral   Disagree   Strongly disagree

I will complete all the assigned homework for today's class.

Strongly agree   Agree   Neutral   Disagree   Strongly disagree

I would like to learn more about the material from today's class.

Strongly agree   Agree   Neutral   Disagree   Strongly disagree

How do you feel about?

Circle a word in each row that shows how you feel about ____________________

- Like   Dislike   ____
- Excited   Bored   ____
- Afraid   Comfortable   ____
- Worried   Calm   ____
- Clear   Confused   ____
- Happy   Sad   ____
- Love   Hate   ____
- Confident   Anxious   ____

Now fill in a reason for each of your choices on the blank lines.

How confident are you that you can convert among the different forms of a quadratic relation?

Very confident   Confident   Not sure   Not that confident   Really not confident

How interesting do you find these conversions?

Very interesting   Interesting   Neutral   Not interesting   Really not interesting

Given your answers to the questions above, what action will you take regarding these conversions?

____

____

Student Weekly Reflection

Choose the response that best matches your feelings about math this week.

- I enjoyed this week in math.

  Strongly Disagree   Disagree   Neutral   Agree   Strongly Agree

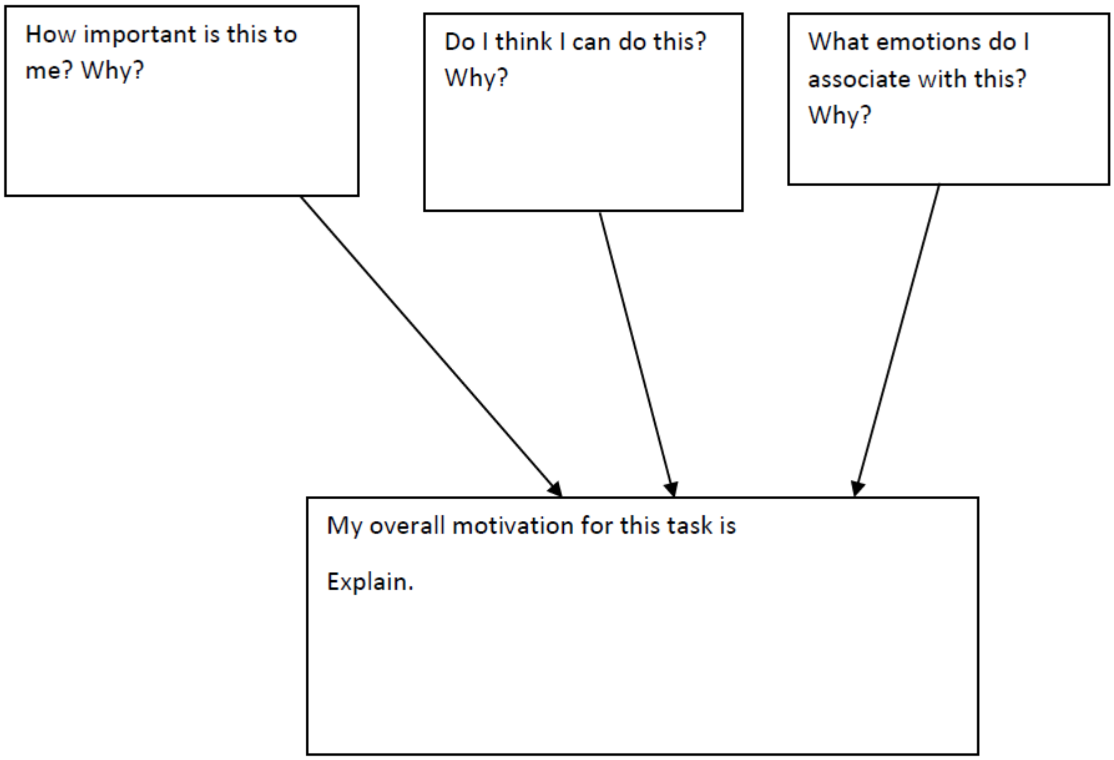
- I worked hard this week in math.

  Strongly Disagree   Disagree   Neutral   Agree   Strongly Agree

- I would like more math classes to be like this week's math classes.

  Strongly Disagree   Disagree   Neutral   Agree   Strongly Agree

- Comments (optional)


How important is this to me? Why?
Do I think I can do this? Why?
What emotions do I associate with this? Why?
My overall motivation for this task is
Explain.

How interested were you in today's class?

PLACE YOURSELF ON EACH SCALE

Really interested

Really uninterested

How confident are you that you understand the material in today's class?

Very confident

Not at all confident

How useful do you think what you learned in today's class is to you?

Very useful

Not at all useful

Do you want to learn more about this topic in tomorrow's math class?

Definitely want to learn more

Not at all interested in learning more

Circle how you feel about what you learned in math class today.

Today’s class was interesting.

Strongly agree Agree Neutral Disagree Strongly disagree

Today’s class was useful to me.

Strongly agree Agree Neutral Disagree Strongly disagree

I feel confident that I understand the material from today’s class.

Strongly agree Agree Neutral Disagree Strongly disagree

I will complete all the assigned homework for today’s class.

Strongly agree Agree Neutral Disagree Strongly disagree

I would like to learn more about the material from today’s class.

Strongly agree Agree Neutral Disagree Strongly disagree

My goal for word problems involving quadratic functions

The next part of this unit involves solving word problems using quadratic functions.

Identify your personal goal for this part of the unit:

Examples:

- Master the material and apply it to new as well as routine problems
- Competently apply the material to routine problems
- Sufficiently understand the material to achieve at least a B or better on this material
- Other

My goal for this section of the unit is to

____
____
____

What do you plan to do to achieve your goal?

Examples:

- Complete all assigned homework and correct mistakes
- Ask questions of the teacher when I don't understand
- Ask for help in class when I don't understand, or I get stuck
- Ask for help outside of class when I don't understand or get stuck
- Work with a friend to complete and understand my work
- Other

To achieve my goal I will

____
____
____
____

Activities

How Factorable Are Trinomials

- Choose three numbers from 1 through 9
- Substitute your numbers for a, b, c in $ax^2 + bx + c$
- Is your trinomial factorable using integers?
- Try all possible permutations (arrangements) of your three numbers for a, b, c and factor as many of the trinomials as possible
- What percentage of your trinomials were factorable using integers?
- Compare your results with others in your group
- Conjecture how your results might be different if you were allowed to use negative integers as well as positive ones
- Conjecture how your results might be different if you were allowed to use zero as well as negative and positive integers
- What do you think would happen if you were allowed to use numbers bigger than 9? Explain your thinking.

Anticipation Guide: Graphs of Quadratic Relations
[for example, $y = 2x^2 - 12x + 18$]

	Before the unit starts		After the unit is over	
1. The graph of a quadratic relation will be a straight line.	AGREE	DISAGREE	AGREE	DISAGREE
2. The graph of a quadratic relation will be a curve.	AGREE	DISAGREE	AGREE	DISAGREE
3. The graph of a quadratic relation will always have one y-intercept.	AGREE	DISAGREE	AGREE	DISAGREE
4. The graph of a quadratic relation will always have two x-intercepts.	AGREE	DISAGREE	AGREE	DISAGREE
5. The graph of a quadratic relation will always be symmetric.	AGREE	DISAGREE	AGREE	DISAGREE
6. The graph of a quadratic relation will always have a minimum value.	AGREE	DISAGREE	AGREE	DISAGREE
7. The graph of a quadratic relation will always have a maximum value.	AGREE	DISAGREE	AGREE	DISAGREE

Anticipation Guide: The Graphs of $y = x^2$ and $y = 2^x$

1. The domain of both functions is unrestricted True/False

2. The y-intercept of both functions is 0 True/False

3. The x-intercept of both functions is 0 True/False

4. Both functions will always be positive, i.e. the range of both functions is $y > 0$
   True/False

5. The graph of $y = 2^x$ will always be above the graph of $y = x^2$ True/False

Anticipation Guide: Intercepts and the Quadratic Formula

1. Does the quadratic formula always result in two solutions?
   YES: Explain
   NO: Give at least one counterexample

2. Can you tell the number of x-intercepts of a parabola just by looking at its equation?
   YES: Give some examples
   NO: Explain

3. Are the quadratic formula and the number of x-intercepts of a parabola related? Explain.

Anticipation Guide: Graphs of Quadratics
[for example, $y = 2x^2 - 12x + 18$]

1. The graph of a quadratic relation will be a straight line. AGREE/DISAGREE

2. The graph of a quadratic relation will be a curve. AGREE/DISAGREE

3. The graph of a quadratic relation will always have one y-intercept. AGREE/DISAGREE

4. The graph of a quadratic relation will always have two x-intercepts. AGREE/DISAGREE

5. The graph of a quadratic relation will always be symmetric. AGREE/DISAGREE

6. The graph of a quadratic relation will always have a minimum value. AGREE/DISAGREE

7. The graph of a quadratic relation will always have a maximum value. AGREE/DISAGREE

Completing the Square

Work with a partner to role play the following situation: A student asks their tutor to check some work they have done. One of you will play the role of tutor, and the other will be the student. After analyzing 4 questions, switch roles. Your task: For each question

- What did the student do right? (Tutor)
- What if anything did the student do wrong? (Tutor)
- Discuss how to fix any errors. (Tutor, Student)
- Fix any errors. (Student).

Here are the questions:

$$y = x^2 - 14x + 3$$

$$y = x^2 - 14x + 49 + 3$$

$$y = (x-7)^2 + 3$$

$$y = x^2 - 10x + 7$$

$$y = x^2 - 10x + 25 - 25 + 7$$

$$y = (x-5)^2 - 18$$

$$y = x^2 + 6x - 5$$

$$y = x^2 + 6x + 36 - 36 - 5$$

$$y = (x+6)^2 - 41$$

$$y = 2x^2 + 8x + 4$$

$$y = 2x^2 + 8x + 16 - 16 + 4$$

$$y = 2(x+4)^2 - 12$$

$$y = 3x^2 + 12x + 6$$

$$y = 3(x^2 + 4x + 2)$$

$$y = 3(x^2 + 4x + 4 - 4 + 2)$$

$$y = 3(x+2)^2 - 6$$

$$y = 4x^2 - 16x + 12$$

$$y = 4(x^2 - 4x + 3)$$

$$y = 4(x^2 - 4x + 4 - 4 + 3)$$

$$y = 4(x - 2)^2 - 1$$

$$y = 2x^2 + 12x - 6$$

$$\frac{y}{2} = x^2 + 6x - 3$$

$$\frac{y}{2} = x^2 + 6x + 9 - 9 - 3$$

$$\frac{y}{2} = (x + 3)^2 - 12$$

$$y = 2(x + 3)^2 - 12$$

$$y = 3x^2 - 12x - 9$$

$$\frac{y}{3} = x^2 - 4x - 3$$

$$\frac{y}{3} = x^2 - 4x + 4 - 4 - 3$$

$$\frac{y}{3} = (x - 2)^2 - 7$$

$$y = 3(x - 2)^2 - 21$$

Curve Fitting

For each set of data below, conduct the following analysis:

- ***Look for patterns in the data that suggest that a quadratic function might be an appropriate model for the data.***
- ***Suggest reasons why the data have these patterns.***
- ***Use technology to fit a quadratic model to the data. How good a fit is each model?***
- ***Identify a relevant range for each model.***
- ***Are there other models that might produce a more realistic result for each situation.***

1.

speed (km/h)	fuel economy (Km/L)
24	10.19
32	11.66
40	12.57
48	13.26
56	13.17
64	13.71
72	13.67
80	13.81
88	13.90
96	13.17
104	12.53
112	11.57

Source: Transportation Energy Data Book.

Use your model to predict fuel economy at a speed of 10 km/h; 120 km/h; and 150 km/h.

2. In a portion of the GTA, home prices have fluctuated significantly. The table below gives some information about a detached two story home in that area. *Source: MLS Listing Service.*

years after 2008	home prices ($000s)
0	$1,110
2	$1,039
4	$838
6	$774
8	$861
10	$1,110

Use your model to predict home prices in this area in the year 2020 and for the year 2050.

3. Farmers frequently use mathematics to enhance their production. Below is some data about pig weights that can be used to modify the feed mix for pigs. *Source: Livestock Research for Rural Development.*

protein intake (g/day)	195	238	297	341	401	427

shoulder weight (g)	8130	8740	9680	9690	9810	8990
Kidney weight (g)	239	287	287	334	379	373

Use your model to predict shoulder weights for protein intakes of 320 g/day and 500 g/day. Then predict kidney weights for the same two protein intakes. What if the protein intake was changed to 1000 g/day?

4. Running records are often adjusted for wind speed, to provide fairness. *Source: The Physics of Sport.*

wind speed (m/s)	change in finishing time (s)
-6	2.28
-4	1.42
-2	0.67
0	0.00
2	-0.57
4	-1.05
6	-1.42

Use your model to predict changes in finishing times for a headwind of 10 m/s and a tailwind of 12 m/s. If the runner faced a headwind of 20 m/s, what effect would that have on his/her finishing time?

5.

year	AIDS cases (US)
1999	41356
2000	41267
2001	40833
2002	41269
2003	43171

Use your model to predict the number of AIDS cases in the US in 2018. Discuss the validity of this model.

6.

Age	% divorced
22	0.9
27	3.6
32	7.4
37	10.4
42	12.7
50	15.7
60	16.2
70	13.1
80	6.5

Use your model to predict the divorce rate for 25 year olds and for 65 year olds.

7. For maximum effect, a shot put is thrown at an angle of 45 degrees. Two variables of interest are the horizontal distance and the height. *Source: The Physics of Sport.*

distance (m)	height (m)
5.90	7.37
11.80	11.80
17.70	16.22
23.60	19.17
29.50	20.94
35.40	22.71
41.30	22.71
47.20	22.12
53.10	20.94
59.00	18.88

Use your model to predict the distance for a shot that is thrown to a height of 18 m and a height of 10 m.

8. The following table represents birth rate per thousand of population, for women whose age is given in the left column.

Age (yr)	Birth Rate per thousand population
12	0.6
17	41.5
22	103.0
27	115.1
32	99.3
37	46.9
42	9.8

Use your model to predict birth rates for 20 year old women and for 50 year old women. What factors affect the relevant range for this model?

9. A baseball thrown through the air will travel different distances depending on the angle at which the ball is thrown and the amount of spin on the ball. *Source: The Physics of Sport.*

Angle (degrees)	10	15	30	36	42	45	48	54	60
Distance with backspin (ft)	61.2	83.0	130.4	139.4	143.2	142.7	140.7	132.8	119.7
Distance with no spin (ft)	58.3	79.7	126.9	136.6	140.6	140.9	139.3	132.5	120.5
Distance with topspin (ft)	56.1	76.3	122.8	133.2	138.3	139.0	137.8	132.1	120.9

Use your models to predict the distance for a ball thrown at 6 degrees and a ball thrown at 75 degrees. How should a baseball be thrown to achieve maximum distance?

10. Twitter net profit for the last five years is shown below.

Year	Net Profit ($millions)
2011	$44.51
2012	$188.17
2013	$398.17
2014	$956.69
2015	$1,490.00

Use your model to predict Twitter net profit in the year 2020. Discuss the validity of your model.

11. Income typically changes over time. This is called the *Life Cycle Hypothesis*.

Age(yr)	Midpoint	Median Income
15-24	19.5	$10,518
25-34	29.5	$32,581
35-44	39.5	$43,967
45-54	49.5	$45,950
55-64	59.5	$41,550
65 and over	69.5	$27,707

Use your model to predict median incomes for people aged 23 and people aged 53.

Crocodile River

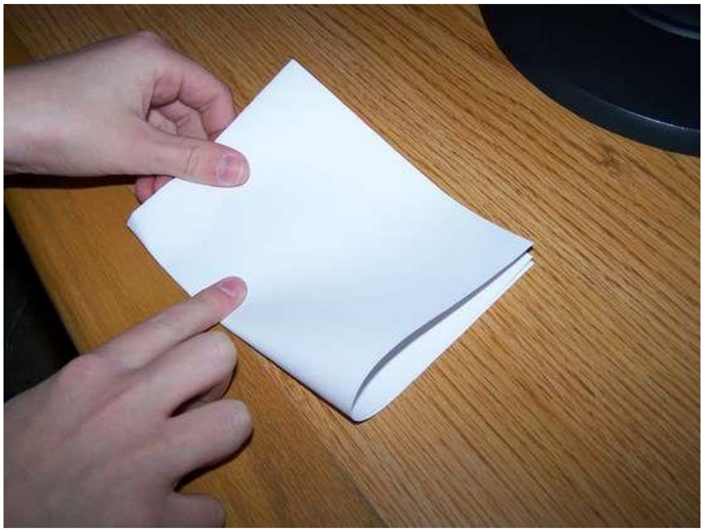
To cross a crocodile infested river, there is a small boat. The boat will hold either one adult or two children. How many trips will it take to get 10 adults and two children across the river?

Paper Folding

Materials: One piece of computer paper.

By folding the paper complete the table. Then find a formula relating number of folds and number of layers of paper.

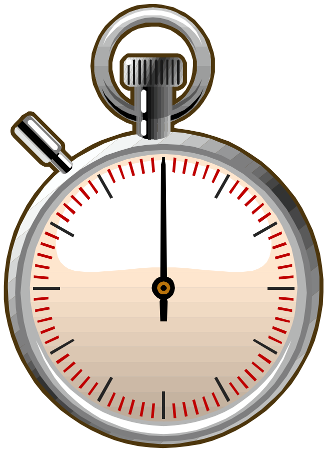
Number of folds	Number of layers
0	1
1	
2	
3	
4	
5	
6	
7	



Rules For FACTO

The card deck consists of 40 cards, 20 cards with polynomials in expanded form, and 20 cards with the same polynomials in factored form.

- The object of the game is to accumulate the most cards
- Each player is dealt four cards, one at a time
- Four cards are placed face-up on the table
- Players immediately check to see if, from the four up cards, there is a match [factored form plus expanded form]. A player who finds a match yells “FACTO” and claims the two cards for him/herself, placing them facedown in front of him/her
- After the initial FACTO round, the player to the left of the dealer attempts to find a match between a card still on the table and a card in his/her hand. If a match is found, the player yells “FACTO” and claims both cards for his face-down pile. **Time must be given for the other players to verify that the two cards are in fact a match [factored form plus expanded form]
- The player who claimed the pair now draws another card from the deck, so that everyone still has four cards.
- Play moves to the next player (only one FACTO claim per round)
- If a player cannot match any of the upcards, he/she lays down one card face-up and draws another card from the deck.
- If no up-cards remain on the table, a player must lay down a card face-up and draw another card from the deck.
- The game ends when all cards have been matched [or teacher-imposed time limit is reached]
- The winner is the player with the most cards in his/her face-down pile.



Timed Retell

- Students work individually on the same question
- Once both have completed the question, Student A outlines their solution to Student B, with a time limit
- Student B then asks clarifying questions of Student A, or adds more information to the solution.
- Can be used with THINK ALOUD strategy

■

▪ What	Why	Source
e.g. Perimeter is 40 m	40 m of fencing	Problem
$2l + 2w = 40$	$P = 2l + 2w$	Assumption: field is rectangular Prior knowledge
$A = lw$	Area of a rectangle is length X width	Prior knowledge
Substitute into area equation	Reduce to one variable	Problem asks for maximum area

Think Aloud

? Teacher verbalizes his/her thinking

? Includes all steps/mis-steps

? Emphasis on What-Why

? Can be used with Think-Pair-Share

In the THINK ALOUD strategy, the teacher vocalizes his/her thought processes while solving a problem.

PROBLEM: Find the equation of the line perpendicular to the line $y = -2x + 6$, and having an x-intercept of 8.

"What am I asked to find?

The equation of a line.

The equation of a line has the form $y = mx + b$

So I need the value of the slope and the y-intercept.

I don't have either one given to me.

I'll try to find the slope first.

What information do I have?

The line I want is perpendicular to $y = -2x + 6$

I know the slope of this line is –2.

I know that the line I want is perpendicular to this one.

Perpendicular lines have slopes that are negative reciprocals of each other.

So the slope of the line I want must be ½

Now I know that my line is $y = \frac{1}{2}x + b$

I still need to find the value of *b*

What other information do I have in the problem?

The x-intercept of the line I want is 8

But I need the y-intercept

An x-intercept is a point, the point where the line crosses the x-axis.

Any point on the x-axis can be written as (something, 0)

So this x-intercept of 8 is the same as the point (8,0)

How can I use this point to find the value of b?

The point (8,0) is on the line I want.

So if I substitute 8 for x and 0 for y in $y = \frac{1}{2}x + b$ I can figure out the value of *b*

$$0 = \frac{1}{2}(8) + b$$

Solve this equation for *b*

$$0 = 4 + b$$
$$-4 = b$$

So the y-intercept of my line is –4

The equation of the line I want must be $y = \frac{1}{2}x - 4$

How can I check to make sure I'm right?

I could graph both this line and the line $y = -2x + 6$ to make sure they're parallel**.**

Then I can verify that the x-intercept of my line is 8.

It would be easiest to do this on a graphing calculator.

If I don't have one, I could find the x-intercept of my line by substituting 0 for y and solving, and make sure that the product of the slopes of the two lines is –1.

This would make sure the two lines are perpendicular and that my line satisfies both conditions in the problem.

Log Pile: Logs are usually piled so that the ends form triangles. For example, 3 logs are piled into a triangle 2 rows high. How many logs are there in a triangular pile 10 rows high? How many logs are there in a triangular pile ***n*** rows high? (drinking straws make great logs)

What's My Concept?

Listed below, in the YES column, are examples of my concept. In the NO column are examples that are not my concept. Identify the common elements in the YES column, and what distinguishes them from the NO examples. Conjecture (educated guess) the concept that the YES examples represent. Then classify the TESTER questions as YES or NO, and give reasons. Finally, complete the Guess What? section.

YES	NO
$2^5 \times 3^2 \times 5$	1440
$(a-4)^2$	$a^2 - 16a + 16$
$(x+5)(x-3)$	$(x^2 + 2x - 15)$
$5x(2x-7)$	$5x^2 - 35x$
$(m-6)(m+6)$	$m^2 - 36$
$(2k+5)(k-8)$	$2k^2 - 11k - 40$
$(y-2)(y^2+2y+4)$	$(y^3 - 8)$

Conjecture: The concept illustrated by the YES examples is

_______________________ because ____________________________

____

____

Classify each TESTER and give reasons:

	YES	NO	because
$(w+5)(w-5)$	[ ]	[ ]	__________________

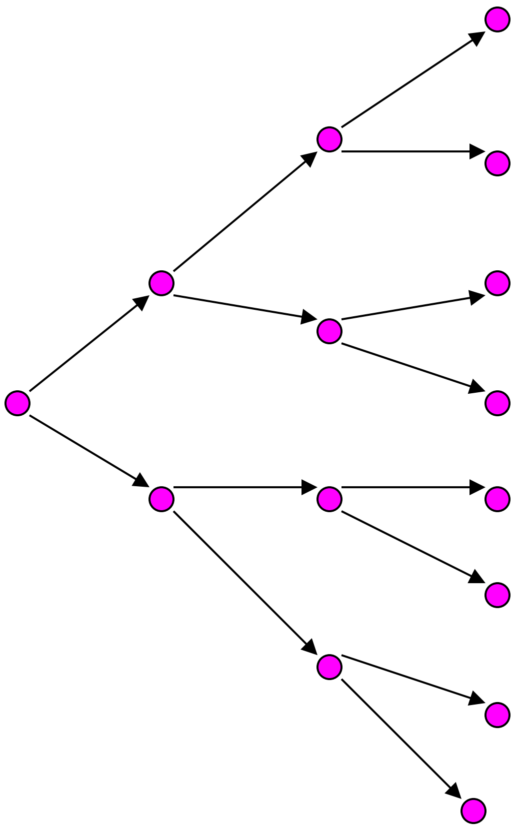
$\left(x^2 + 5x + 6\right)$ [ ] [ ] ____________________

$3a^2\left(2a + 7\right)$ [ ] [ ] ____________________

$\left(z^2 + 10\right)$ [ ] [ ] ____________________

<u>Guess What?</u> In the original YES/NO examples, each YES example is equal to the NO example beside it. PROVE IT!!

DECISION TREE



Crocodile River: A group of explorers has reached a very long, wide river infested with crocodiles. On their side of the river is a small boat and two children. The boat will hold ***either*** one adult or the two children. How many trips will be needed to take the entire party of 10 adults and the two children across the river? Write a formula for the number of trips needed with ***n*** adults and two children.

Materials: linking cubes to represent the adults, 2 small cubes for the children, something for a boat.

How Many Intercepts?

For each parabola $y = ax^2 + bx + c$

1. Predict the number of x-intercepts
2. Verify your prediction by graphing using technology
3. Compute the value of $b^2 - 4ac$ for each equation

Equation	Predicted number of x-intercepts	Actual number of x-intercepts	Value of $b^2 - 4ac$
$y = 3x^2 + 6x - 1$			
$y = 3x^2 - 6x + 27$			
y= $3x^2 - 12x + 36$			
y= $x^2 - 4x + 9$			
$y = -3x^2 + 6x + 2$			
$y = 2x^2 - 12x + 18$			
$y = 4x^2 - 8x + 1$			
$y = -2x^2 - 20x - 53$			
$y = -2x^2 - 16x - 3$			
$y = \frac{1}{2}x^2 - 4x + 5$			

Draw some conclusions about how the value of $b^2 - 4ac$ (called the *discriminant*) can help you predict the number of x-intercepts:

____

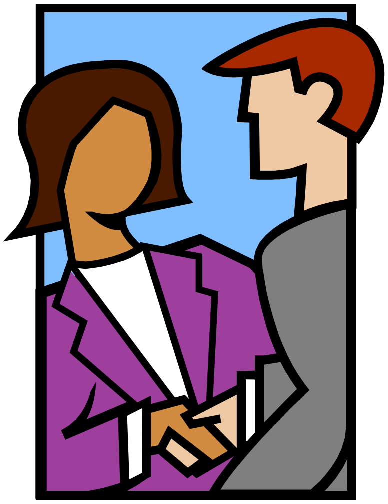
____

For each of the following parabolas, use the value of the discriminant to predict the number of x-intercepts. Then verify by graphing with technology.

$y = x^2 - 8x - 11$ $\qquad$ $y = 5x^2 - 10x + 9$ $\qquad$ $y = x^2 - 14x + 45$

$y = -4x^2 - 8x - 4$ $\qquad$ $y = 3x^2 - 12x + 8$ $\qquad$ $y = -2x^2 - 16x - 32$

$y = x^2 - 12x + 40$



At a party, everyone shakes hands with everyone else. If there are 12 people at the party, how many handshakes occur? Generalize for *n* people at the party.

Instructions for Placemat Comic Completion

1. Individually, in your work area, write instructions on how to factor the type of question that your group has been given.
2. The instructions must fit the three bubbles on the comic.
3. After everyone has completed their instructions, the group decides on the clearest set of instructions. These clear instructions are printed in the comic bubbles.
4. Print the type of factoring question that your group worked with on the top of the placemat.
5. Post your placemat on the wall.
6. Do a gallery walk to read the instructions created by the other groups.
7. In your notebook, write down the instructions on how to factor the different types that your class has worked on.

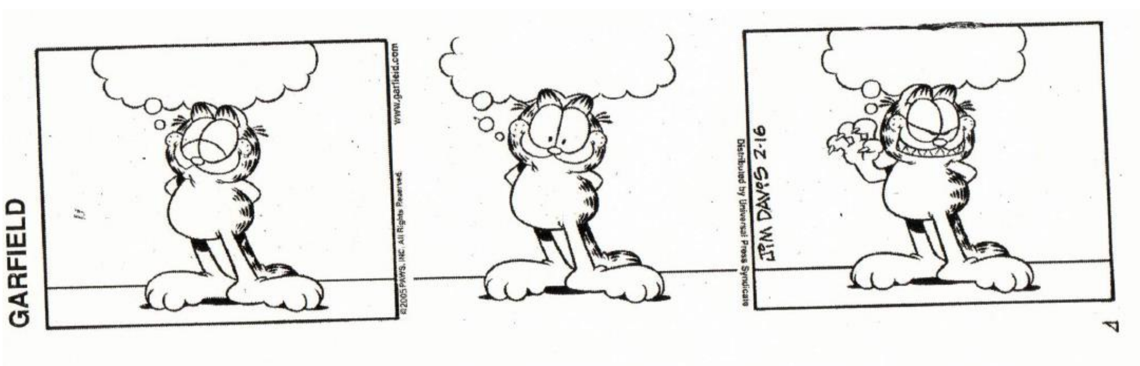
Note to teacher:

Form groups of 4 or fewer.

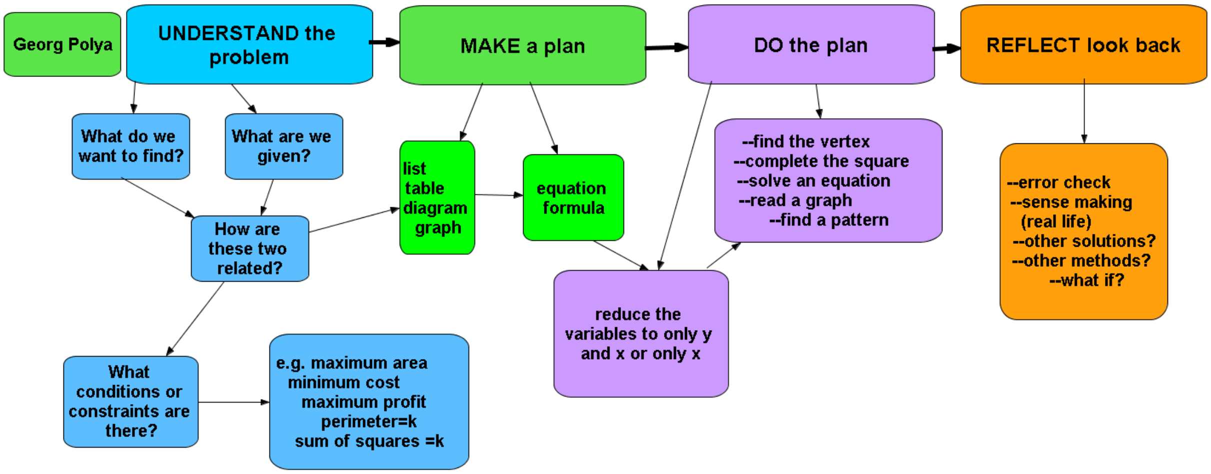
Give each group a placemat and an index card with a type of factoring.

Once groups have finished, have them post their placemats for a gallery walk.

Students can/shold take their notebooks on the gallery walk sot that they can write down the instructions for how to factor each of the types.



If you have two minutes to solve a problem, the first 90 seconds should be spent in thought. Walter W. Sawyer



Problem Solving Flowchart

$Money Maker$

You have been offered a part time job stocking shelves in a phone store. The manager is somewhat eccentric, and has offered you the choice between two ways of getting paid.

Choice A: You will be paid $ x^2 where x is the number of days you work.

Choice B: You will be paid $ 2^x where x is the number of days you work.

Decide which payment method you should take.

Verify your decision by comparing your paycheques if you work a total of

I. 10 days
II. 20 days
III. 30 days

Is there any range of days worked for which the two plans result in the same pay?

Is one plan always better than the other plan? Explain.

$$$$$$$$$$$$$$$$$$$$$$$$$$$$$

Parabola Battleship

- Pairs
- Works like the game Battleship
- One student has a graph of a parabola with information labeled (vertex, intercepts, etc.)
- Student does not show the graph to the other partner
- Partner asks questions of first student
- Aim is to reproduce the labeled graph
- Once accurate graph is produced, partners switch roles

Parabolas in Real Life

Part A:

- Do an internet search for images of parabolas
- Either print or bookmark 6 to 10 images to share with your group tomorrow

Part B:

- Select one image per person
- Impose a grid and find an equation for your parabola
- Find the coordinates of a point 2/3 of the height of the maximum or minimum of your parabola.

Cognitive

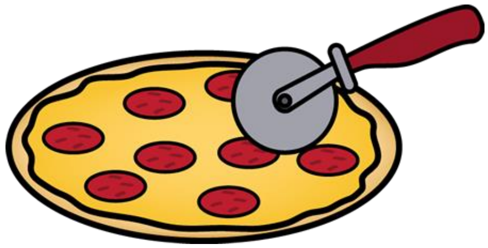
- Using sorting rules agreed on by your group, sort all the images of parabolas into groups
- Choose a suitable scale and then find the actual coordinates of a point 2/3 of the height of the maximum or minimum of your parabola.

Metacognitive

- Design a plan for finding an equation for your parabola
- Carry out your plan
- Use your equation to find the coordinates of a point 2/3 of the height of the maximum or minimum of your parabola.

Self

- Rate your interest in learning more about parabolas (Likert 1-5)
- After carrying out this activity, how important do you think knowing about parabolas is (Likert 1-5)
- Overall, how motivated are you to find out more about parabolas (Likert 1-5)

-  **Pizza Cuts**: What is the maximum number of pieces of pizza resulting from 6 cuts? Cuts must be straight but do not need to pass through the centre. Find a formula for the maximum number of pieces for *n* cuts.
- Materials: Pizza-sized circle, spaghetti for the cuts.

$$ax^2 + bx + c = 0$$

$$a\left(x^2 + \frac{b}{a}x + \frac{c}{a}\right) = 0$$

$$a\left(x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2 + \frac{c}{a}\right) = 0$$

$$a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} - \frac{b^2}{4a^2} + \frac{c}{a}\right) = 0$$

$$a\left(\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2} + \frac{c}{a}\right) = 0$$

$$a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a} + c = 0$$

$$a\left(x + \frac{b}{2a}\right)^2 + \frac{b^2 - 4ac}{4a} = 0$$

$$a\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

$$x + \frac{b}{2a} = \frac{\pm\sqrt{(b^2 - 4ac)}}{2a}$$

$$x = -\frac{b}{2a}\frac{\pm\sqrt{(b^2 - 4ac)}}{2a}$$

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

Unit #: Day #: (quadratic formula)

Grade

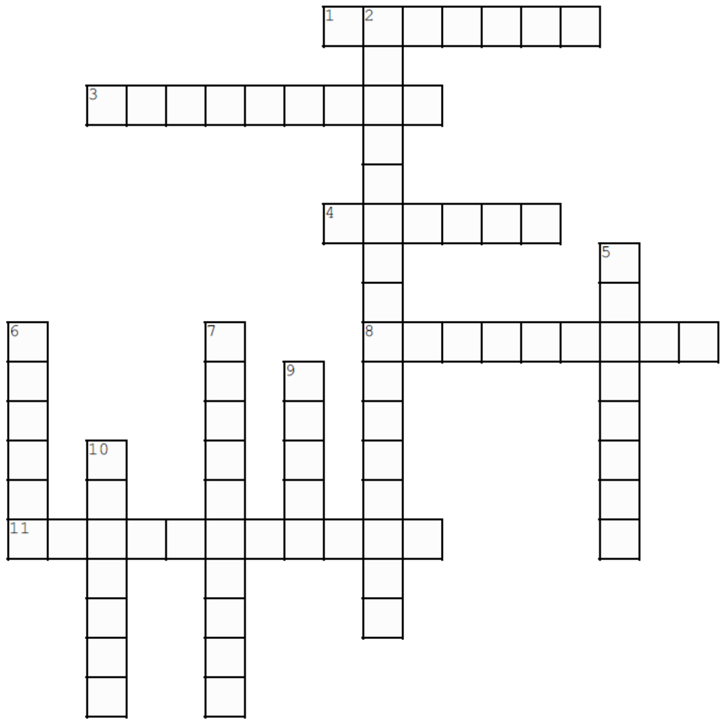
75 min	*Students will learn how to develop the quadratic formula and apply it ti find zeros of functions and x-intercepts of quadratic relations *Students will learn how to solve quadratic equations that have real roots, using a variety of methods	**Materials** •
Minds On… 15 min	→**whole class** **Teacher lead model quadratic formula using a numerical example**	Grid paper
Action! 15 min	→**groups** **Put pieces of quadratic formula in order using warmup example as a template**	Quadratic formula pieces
Consolidate Debrief 45 min	→**whole class** **Teacher-lead examples of using quadratic formula to solve equations**	

Home Activity or Further Classroom Consolidation

Teacher-assigned

Quadratic Functions

Complete the crossword below



Across

1. highest point of a parabola that opens down
3. function of degree two
4. set of acceptable x values
8. curve that is the same on both sides of a reflecting line
11. ppoints where the function crosses the x axis

Down

2. reflecting line for a symmetric function
5. shape of the graph of a quadratic function
6. highest or lowest point on a parabola
7. point where the function crosses the y axis
9. set of possible y values
10. lowest point of a parabola that opens up

Unit #: Day #: (introduction to quadratic relations)

Grade

75 min	*Students will learn how to identify patterns that represent quadratic relations, and generate equations for these patterns using a variety of methods Ntroduction to quadratics	**Materials** •
Minds On… 40-45 min	**→groups, carousel** **Each group will rotate through 5 activities, and generate a table of values for each situation. They will then use difference tables to identify whether the relation is linear, quadratic, or other (exponential)** **The five stations are: handshake problem, log pile, pizza cuts, paper folding, and crocodile river**	
Action! 10 min	**→whole class** **Teacher-lead Desmos quadratic regression (use the data from the handshake problem). Use the PowerPoint : Using Desmos to do quadratic regressions**	
Consolidate Debrief 20min	**→individual** **Students complete regressions for the remaining four data sets from the activities**	

Home Activity or Further Classroom Consolidation

Could include a data set for a cubic. For example, marble mountain. Data set: Marble Mountain.

OR, could have students generate a couple of quadratic regressions using real-world data: Curve Fitting.

Quadratic Aerobics

BE the Function

$$y = x^2$$

$$y = -x^2 - 2$$

$$y = 2(x-3)^2$$

$$y = -\frac{1}{2}(x+4)^2 + 1$$