Identification method based on Zadeh filter models

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Abstract. Mathematical modeling which provides the description of objects and proper organization of control operations in future is an integral stage in the automation of production. One of the approaches to build a mathematical model of an object is to represent nonlinear systems as combinations of inertial and nonlinear inertialess elements. The models thus obtained are called block-oriented. In this paper, we consider nonlinear dynamic objects represented as the models of the Zadeh filter class. In the process of the method development the identification equations were derived for the case when the test signal is a single sinusoid. Then the case of two sinusoids was considered. Such investigations allowed us to identify the patterns and describe the general case for several test components in the signal. The results of digital modeling using the sum of harmonic signals confirm the feasibility and validity of the proposed approach for identifying nonlinear models of the Zadeh filter class.

1. Introduction

Various methods to build adequate mathematical models for control purposes are applied in automation of production [1-6]. One of the most widely developed approaches is the description of objects using block-oriented models, i.e. representation of nonlinear systems in the form of various combinations of inertial and nonlinear inertialess elements [7, 8]. The object of study is nonlinear dynamic objects, represented in the form of the models of the Zadeh filter class [9].

At present one of the promising trends of enhancing a set of service properties of such bronzes is alloying them with superdispersed powders. Introduction of their small amount into the melt before a crystallization process allows increasing strength propeties of castings. But a mechanism of interaction with lead-tin-based bronzes, as well as the process regularities of such modification, is not studied profoundly. However, such modification of copper alloys is promising from several points of view.

This paper presents an investigation of the influence of different content of additives of the pre-treated aluminium oxide powder on the structure of lead-tin-base bronze under formation.

2. Materials and methods

The output of the Zadeh filter model is given by the operator:

$$y(t)=\sum_{i=1}^{n} x_y \int_0^t h_i(\theta)\, x^j(t-\theta)\, d\theta + y(t), \qquad (1)$$

where $x(t)$ – the input signal of the object; $y(t)$ – the output signal of the object; a_i – the coefficients of the instantaneous polynomial; h_i – impulse responses of linear elements; $\theta(t)$ – input signal dependent stationary noise $x(t)$, but with a finite correlation interval.

The input signal of the Zadeh filter model is the input signal in the form of a sum of harmonic signals:

$$x(t)=\sum_{i=1}^{n} u_i \sin(\omega_i t),$$

where $x(t)$ – the input signal of nonlinear dynamic objects; u_i – amplitudes.

Nonlinear elements, included in the model, are described by polynomials of the form:

$$f_1\{x(t)\}=a_1 x(t); f_2\{x(t)\}=a_2 x^2(t);\ldots; f_n\{x(t)\}=a_n x^n(t). \quad (2)$$

To identify the model (1), we should:

1. Choose one model among the others (1) as a better candidate to describe adequately the object. The measures of adequacy are:

a) Fisher experimental criterion:

$$F^n = D^n_{ad} / D^n_{rpt},\ F^{fr} = D^{fr}_{ad} / D^{fr}_{rpt}, \quad (3)$$

where $D^{(.)}_{ad}$ – variance of adequacy, $D^{(.)}_{rpt}$ – variance of repeatability;

b) Time-frequency criteria:

$$\hat{L}_k(b_k)=\max_{\tau_k} L_k(b_k(\omega_n,\tau_k));\ \operatorname{sign} b_k = \operatorname{sign} b_k(\omega_n,\tau), \quad (4)$$

where $k=\overline{1,n}; L_k = 20\lg|b_k(\mathrm{d}\omega_n,\tau)|; d=1,2$.

2. Determine the static characteristics of the models (1).

3. Use the approximation for the impulse function of the form:

$$L^*\left[h_j(.)\right]^{-1} = k\prod_{i=1}^{d_1^j}(\theta_i^{*j}S+1)^{c_1^{(i)}} / \prod_{i=1}^{d_2^j}(T_i^{*j}S+1)^{c_2^{(i)}}, \quad (5)$$

where $\sum_{i=1}^{d_1^j} c_1^{(i)} = P_1^j, \sum_{i=1}^{d_2^j} c_2^{(i)} = P_2^j$, determine the values of time constants, for which the values of estimates are distributed $\hat{\theta}_i,(i=\overline{1,m})$ и $\hat{T}_i,(i=\overline{1,n})$, and also determine values P_1^j, P_2^j, $c_1^{(i)}$, $c_2^{(i)}$, θ_i^* and T_i^*.

4. Determine the values of the object bandwidth estimates:

$$\Delta\omega_l = \int_0^\infty K_l(\omega,\overline{\hat{\theta}^*},\overline{\hat{T}^*}),\ l=\overline{1,n}. \quad (6)$$

5. Verify model compliance with the object based on the available information about the model structure. Fisher experimental criteria are used as a measure of adequacy.

6. Determine the values of the normalized impulse response taking into account information about the model structure h^H.

7. Use the values of static characteristics, determine $h(t)$.

8. Determine the values $\hat{\theta}_i,(i=\overline{1,m})$, $\hat{T}_i,(i=\overline{1,n})$ using information about the type of the approximating function for the impulse transient response.

To implement the procedures for the identification of this method, we used measuring and calculating systems presented in [10]. The identification equations are obtained by the inductive method, namely: first, the equations are derived for the case when the test signal is a single sinusoid, then - from two sinusoids, and so on until the obvious regularities revealed in the equations do not allow us to consider the general case for n harmonic test components.

For the Zadeh filter class model, the correlation equations have the form [10]:

$$\begin{cases} R_{x_i,y}(\tau) = \int_0^t h_i(\theta)\left[a_1 m_{1,1}(\tau-\theta) + \ldots + a_{2i-1} m_{1,2i-1}(\tau-\theta)\right] d\theta; \\ R_{x_i^2,y}(\tau) = \int_0^t h_i(\theta)\left[a_2 m_{2,2}(\tau-\theta) + \ldots + a_{2i} m_{1,2i}(\tau-\theta)\right] d\theta, \end{cases} \quad (7)$$

where $R_{x_i,y}(\tau)$ – the correlation function, calculated as a difference between the input and output signals of the object; $R_{x_i^2,y}(\tau)$ – the correlation function between the square of the input and output processes; $m_{(.)(.)}$ – the correlation moments of a sinusoidal signal.

The covariance equations for identification of the model under consideration are given in the form:

$$\begin{cases} K_{xy}^n(\pm\tau) = \sum_{l=1}^{l_1^2} \frac{(2^i-1)!}{2^{2i-1}(i_i-1)!i!} b_{2i-1}(\omega_n, \pm\tau) u^{2i}; \\ K_{x^2y}^n(\pm\tau) = \sum_{l=1}^{l_2^2} \frac{(2^i)!}{2^{2i+1}(i_i-1)!(i+1)!} b_{2i}(\omega_n, \pm\tau) u^{2i+2}, \end{cases} \quad (8)$$

where l_1^2 and l_2^2 – the numbers of odd and even coefficients, respectively $b_i(\omega_n, \pm\tau)$ –the time-frequency criteria of the nonlinear regression equations, dependent on the modulus and phase of the linear elements and the coefficients of the nonlinear elements of the model.

For frequency ω_n the time-frequency criteria $b_i(\omega_n, \pm\tau)$ will be:

$$\begin{cases} b_{2i}(\omega_n, \pm\tau) = a_{2i} K_{2i}(2\omega_n) \cos\{\pm 2\omega_n \tau - \psi_{2i}(2\omega_n)\}; \\ b_{2i-1}(\omega_n, \pm\tau) = a_{2i-1} K_{2i-1}(\omega_n) \cos\{\pm 2\omega_n \tau - \psi_{2i-1}(\omega_n)\}, \end{cases} \quad (9)$$

where a_i – the coefficients of nonlinear elements, $K_i(\omega_n)$ and $\psi_i(\omega_n)$ – the module and phase of the complex transfer coefficient of the linear element of the i-th branch in the Zadeh model.

To solve the systems of equations (8), the least squares method and all statistical procedures of regression analysis are used. The identification procedure is structured in a way that the evaluation criteria $\hat{b}_i(\omega_n, \pm\tau)$ tend to the theoretical $b_i(\omega_n, \pm\tau)$, then the method and algorithms for identifying the model class, structural elements and parameters can be developed based on the analysis of theoretical dependences of time-frequency criteria.

The odd parameters $b_i(\omega_n, \pm\tau)$ of the nonlinear regression equations are not only functions of the lag τ, but also trigonometric functions of frequency ω_n, and even functions are trigonometric functions of frequency $2\omega_n$. The odd parameters of the nonlinear regression equations are not only functions of the lag τ, but also trigonometric functions of frequency, and even functions are trigonometric functions of frequency. Investigating the behavior of parameters $b_i(\omega_n, \pm\tau)$ in the frequency domain, it is possible to obtain information about the form of the functional series of the model, the order of linear elements, the distribution range of the impulse transient functions parameters, and the bandwidth of the object.

Static characteristics describe the absence of lag or advance phase in the harmonics of the output signal, while this property can manifest itself for one, several, or all harmonics. The static mode is characterized by: $\psi_1 = \psi_c$, $\psi_2 = \psi_c$, where ψ_c – the phase of the input harmonic effect, $\psi_1(\omega_n)$, $\psi_2(\omega_n)$ – the phases of the first and second harmonics, respectively. This condition is written for the first and second harmonics, since the proposed method is based on the use of these harmonics.

Static characteristics are determined according to the following algorithm:

$$\begin{cases} b_{2i}^{st}(\omega_n) = a_{2i-1} \max_{\omega_n} K_{2i-1}(\omega_n) \cos\{\omega_n \tau + \psi_{2i-1}(\omega_n)\}; \\ b_{2i}^{st}(\omega_n) = a_{2i} \max_{\omega_n} K_{2i}(2\omega_n) \cos\{2\omega_n \tau + \psi_{2i-1}(2\omega_n)\}. \end{cases} \quad (10)$$

When solving (10), mutual correlation functions $K_{xy}(\tau)$ are calculated where $\omega_n\tau = 0$ and $K_{x^2y}(\tau)$ where $2\omega_n\tau = 0$, the systems of equations (7) are solved and $\left|b_i^{st}(\omega_n)\right| = \left|\max_{\omega_n} b_i(\omega_n,\tau)\right|$ are determined.

The number of static characteristics determines the order of the kernels of the composed functions. Static characteristics possess the greatest noise immunity and are used in the procedures of regression analysis and impulse characteristics rationing.

The method of structural identification is based on the phase's compensation of the first and second harmonics of the object output signal; as a result, the structural identification reduces to finding extremes from (10). In this case, the time-frequency criteria $b_i(\omega_n,\pm\tau)$ will depend on the coefficients of the non-linear element polynomial and the modules of the complex transmission coefficients of the linear elements:

$$\begin{cases} \max\limits_{\tau}\left|b_{2i}(\omega_n,\pm\tau)\right| = \left|a_{2i}\right| K_{2i}(2\omega_n); \\ \max\limits_{\tau}\left|b_{2i-1}(\omega_n,\pm\tau)\right| = \left|a_{2i-1}\right| K_{2i-1}(\omega_n). \end{cases} \tag{11}$$

To determine the class of the model and its structural elements, the following criteria are used: logarithmic dependencies and logarithmic differences from $b_i(\omega_n,\pm\tau)$ – (11), as well as the sign functions, constructed using the coefficients $b_i(\omega_n,\pm\tau)$:

$$\begin{gathered} L_i(\omega_n) = 20\lg\left|b_i(\omega_n,\tau)\right|,\ \Delta L_{rm}(\omega_n) = L_r(\omega_n) - L_m(\omega_n), \\ \Delta L_i(\omega_n) = L_i(\omega_n') - L_m(\omega_n''),\ \lg(\omega_n''/\omega_n') = 1,\ r \neq m, r, k,\ m \in \overline{1,n},\ \operatorname{sign} b_i = \operatorname{sign} b_i(\omega_n,\tau). \end{gathered} \tag{12}$$

The class of the model, the degree of nonlinear transformation, the position of the linear element relative to the nonlinear element, the order of linear elements define the criteria (12). Based on criteria (12), we determine the order of the numerator and denominator of the fractional rational transfer function of linear elements, the approximating expression for the impulse transient functions, and the parameters of the impulse response.

The bandwidth is determined on the basis of the obtained information about static characteristics, class of the model, type of approximating functions for impulse transient functions and initial values of time constants:

$$\Delta\omega^l = \int_0^\infty K_N^l(T_1,\ldots,T_k;\omega_n)\,\mathrm{d}\,\omega_n\,, \tag{13}$$

where $K_N(.)$ – the normalized linear modulus of the linear element, $l = 1,2,\ldots$

The equation systems to identify the method being developed are obtained by the inductive method, namely: first, the equations are derived for the case when the test signal is a single sinusoid; then, two sinusoids, and so on until the obvious regularities revealed in the equations make it possible to consider the general case for m test components in the signal.

3. Results and discussion

To implement the identification procedure, it is necessary to use the measuring and calculating system presented in figure 1. In this case we consider a situation when the analysis time is much longer than the transition process, i.e. wherein the output signal of a nonlinear dynamic object is stationary.

The functional diagrams provided below allow one to identify one-dimensional mutual correlation functions $K_{xy}(\tau)$ and $K_{x^2y}(\tau)$ – between the input test signal or its "square" and the output signal in the i-th branch of the Zadeh filter model. The experiments conducted on the basis of these functional diagrams enable one to solve the problem of identification.

The recorded input signal as a sum of harmonic signals in the general case, or a sinusoidal signal in a particular case is applied to the input of the object. The smaller the difference between the frequencies of the harmonic components of the test signal – the greater should be the sample size. Therefore, the frequencies of all sinusoidal components are multiple. Moreover, frequencies should be included in the bandwidth of the object.

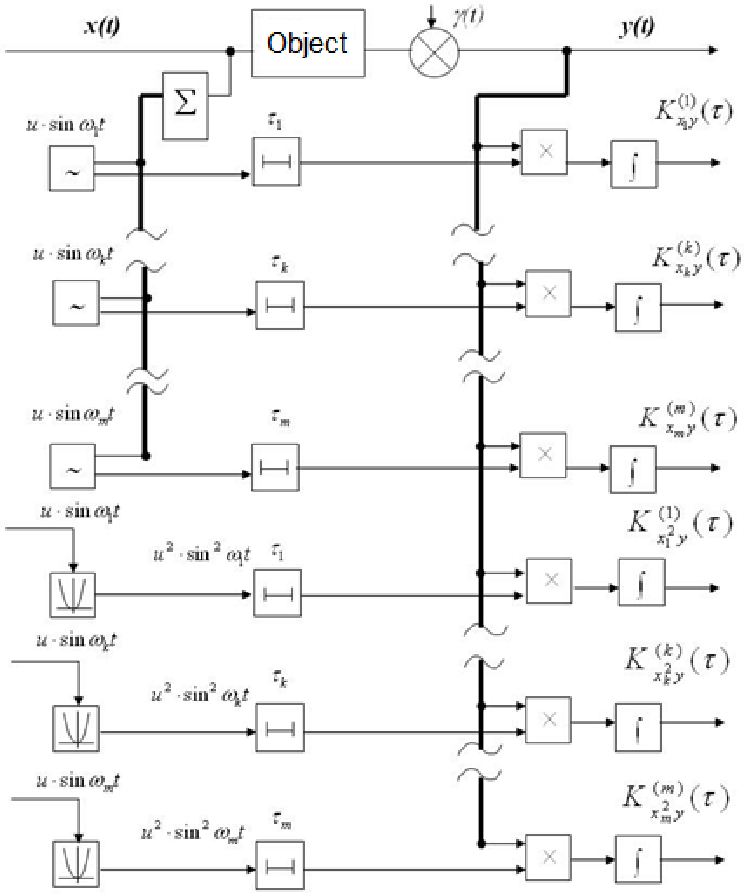

Figure 1. Block diagram of measuring and calculating system with delayed input signal.

When testing an object with harmonic signals, the analysis of the obtained characteristics of nonlinear dynamic objects is carried out in the space of frequency and lag. The time delay of the signals in the correlation identification methods should be used to find the structures of the models.

4. Conclusion

The results of digital modeling using the sum of harmonic signals confirm the efficiency of the proposed method and the algorithms for structural and parametric identification of nonlinear models in the Zadeh filter class.

The developed method of structural-parametric identification is relevant for nonlinear dynamic objects that are adequately described by block-oriented models of the Zadeh filter class and starts with collecting a priori information about the controlled object.

Then, a test signal is sent to the input of the object as a sum of m sinusoids with different frequencies in accordance with the block diagram of the measuring and calculating system, which significantly simplifies the experimental technique and result in reduction of time spent on the experiment, as well as computational operations.

Further, the values of one-dimensional mutual correlation functions $K_{xy}(\tau)$ and $K_{x^2y}(\tau)$ are determined for different levels of the input signal and lags determined by the formulas: $\omega_m\tau = 0$ for $K_{xy}(\tau)$ and $2\omega_m\tau = 0$ for $K_{x^2y}(\tau)$, and two systems of correlation identification equations are solved, the values of the coefficients $b_i(\omega_m, \tau)$ are determined. The last three steps are repeated changing the test signal until $| b_i(\omega_m, \tau) |$ reach their maximum values.

At the stage of non-parametric (structural) identification, the number of elements in the model of the Zadeh filter class is determined by the number of calculated information coefficients b_{ist} (static characteristics). The adequacy of the obtained model is tested according to the Fisher criterion. The logarithmic amplitude-frequency characteristics and logarithmic differences are calculated, as well as the sign functions of the information coefficients $b_i(\omega_m, \tau)$ (the extension of the structural identification stage).

Based on the information obtained at the stage of structural identification, the order of the numerator and denominator of the fractional rational transfer function of linear elements (ultimately, the approximation expression for the impulse transfer function) and the parameters of the impulse characteristics (the values of time constants) are determined with logarithm dependencies on information coefficients $b_i(\omega_m, \tau)$, as well as the values of the coefficients of nonlinear inertialess elements (parametric identification stage). Based on the information obtained, the bandwidth of non-linear object in the Zadeh models class (parametric identification stage) is estimated.

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References

[1] Albalawi F, Durand H, Alanqar A and Christofides P D 2018 Achieving operational process safety via model predictive control *J. Loss Prevent. Proc.* **53** 74–88

[2] Indrawati S, Helia V N and Andhityara K 2018 Model of continuous review system with backorder case for inventory planning and control: A methanol industry case application *Proc. of the Int. Conf. on Industrial Engineering and Operations Management* 2173–79

[3] Maxim A, Copot D, De Keyser R and Ionescu C M 2018 An industrially relevant formulation of a distributed model predictive control algorithm based on minimal process information *J. Process Contr.* **68** 240–253

[4] Stopakevych O and Ulitska O 2017 Design of precise control systems of industrial plants *Eastern-European J. of Enterprise Technologies* **2(2-86)** 56-62

[5] Shumikhin A G and Aleksandrova A S 2017 Identification of a controlled object using frequency responses obtained from a dynamic neural network model of a control system *Computer Research and Modeling* [in Russian – Compiuternye issledovaniya y modelirovanie] **9(5)** 729–40

[6] Zanoli S M, Cocchioni F and Pepe C 2018 Model predictive control with horizons online adaptation: A steel industry case study *European Control Conf.* (*ECC 2018*) 2332–37

[7] Pupkov A N, Evdokimov I V, Yamskikh T N and Tsarev R Y 2018 Synthesis of multidimensional nonlinear dynamic systems multichannel regulator for a class of block-oriented models *Int. Multidisciplinary Scientific GeoConference Surveying Geology and Mining Ecology Management* (*SGEM*) **18(2.1)** 731–42

[8] Schoukens J, Vaes M, Esfahani A F and Relan R 2015 Challenges in the identification of discrete time block oriented models for continuous time nonlinear systems *IFAC-PapersOnLine* **48(28)** 596–601

[9] Bushtruk A D and Bushtruk T N 2001 Structural identification of nonlinear dynamic objects in the conditions of a passive experiment *Automat. Rem. Contr.* **62(8)** 1278–83

[10] Evdokimov I V 2006 *Mathematical Support and Software for Identifying Nonlinear Dynamic Objects Using the Sum of Harmonic Signals* PhD Thesis, Bratsk, Russia p 135