Doubly-fed inductor motor as the element of automatic control system

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Abstract. The presented linearized model of the dual-fed motor allows to synthesize the controller. The are many possible control system structures as well as the synthesis methods. This enables to provide a variety of the electric drive characteristics to meet the specific requirements.

1. The problem of the doubly-fed inductor motor control

The doubly-fed inductor motor is a promising element of electric servo drives. But its mathematical model contains eight nonlinear differential equations and, in this form, cannot be used for the synthesis of controllers in the process of designing the electric drives [1].

For the purposes of controllers' synthesis, it is advisable to use a linearized mathematical model.

In the paper the process of automatic formation of this motor linearized mathematical model in the form convenient for the controllers' synthesis has been considered. For this purpose, the program MathCAD is used as a tool of symbolic calculations.

2. Mathematical model in the form of nonlinear differential equations

The mathematical model of the motor is presented as two vector differential equations:

$$\left.\begin{aligned} \frac{d\underline{\psi}_1}{dt} &= -(\alpha_1 + j\cdot\omega_1)\cdot\underline{\psi}_1 + \alpha_1 k_2 \underline{\psi}_2 + \underline{u}_1; \\ \frac{d\underline{\psi}_2}{dt} &= -[\alpha_2 + j\cdot(\omega_1 - \omega_r)]\cdot\underline{\psi}_2 + \alpha_2 k_1 \underline{\psi}_1 + \underline{u}_2, \end{aligned}\right\}$$

where 1 and 2 — are the winding numbers; $\underline{\psi}$ — are the magnetic flux-linkage vectors; $\underline{u}$ — voltage vectors; ω_r — the rotor speed; k, σ, α — the coefficients which depend on the winding parameters; j — a unit imaginary number; ω_1 — winding voltage frequency 1.

Motor torque:

$$M = k_{\mathrm{M}} \operatorname{Im}\left[\psi_1 \tilde{\psi}_2\right],$$

where the symbol ~ denotes the complex conjugate vector; Im — the imaginary component; k_{M} — the coefficient which depends on the winding parameters.

Equations of motion:

$$\left.\begin{aligned} \frac{\mathrm{d}\omega_{\mathrm{r}}}{\mathrm{d}t} &= \frac{1}{J}\left[M + M_{\mathrm{L}}\right]; \\ \frac{\mathrm{d}\theta_{\mathrm{r}}}{\mathrm{d}t} &= \omega_{\mathrm{r}}, \end{aligned}\right\}$$

where J — is the moment of inertia; M_{L} — load torque; θ_{r} — the rotation angle.

Supply voltage:

$$\left.\begin{aligned} u_1 &= U_{1\mathrm{m}} e^{j\beta_1} = U_{1\mathrm{m}} e^{j\varepsilon_1}; \\ \bar{u}_2 &= U_{2\mathrm{m}} e^{j\beta_2} = U_{2\mathrm{m}} e^{j\cdot(\theta_2 + \varepsilon_2 - \varepsilon_1 + \theta_r)} = U_{2\mathrm{m}} e^{j\cdot(\theta_1 + \theta_{\mathrm{M}})}, \end{aligned}\right\}$$

where U_{m} — supply voltage amplitudes; ε_1, ε_2 — adjustable voltage phase shifts; θ_1, θ_2 — current phase voltage shifts 1 and 2:

$$\left.\begin{aligned} \frac{\mathrm{d}\theta_1}{\mathrm{d}t} &= \omega_1; \\ \frac{\mathrm{d}\theta_2}{\mathrm{d}t} &= \omega_2, \end{aligned}\right\}$$

where ω_1, ω_2 — angular voltage frequency 1 and 2.

The torque angle is:

$$\theta_{\mathrm{M}} = \beta_2 - \beta_1 = (\varepsilon_2 - \varepsilon_1) + (\theta_2 - \theta_1) + \theta_{\mathrm{r}}$$

For synthesis it is required to linearize the obtained equations.

3. Linearization of equations

For linearization the Taylor series is used.

Magnetic flux linkages are as follows:

$$\left.\begin{aligned} \frac{\mathrm{d}\bar{\psi}_{1\Delta}}{\mathrm{d}t} &= -(\alpha_1 + j\omega_{1.0})\cdot\psi_{1\Delta} + \alpha_1 k_2 \psi_{2\Delta} - \\ &\quad - j\bar{\psi}_{1.0}\omega_{1\Delta} + e^{j\beta_{1.0}} U_{1\mathrm{m}\Delta} + jU_{1\mathrm{m}0} e^{j\beta_{1.0}} \varepsilon_{1\Delta}; \\ \frac{\mathrm{d}\psi_{2\Delta}}{\mathrm{d}t} &= \alpha_2 k_1 \bar{\psi}_{1\Delta} - (\alpha_2 + j\omega_{2.0})\cdot\bar{\psi}_{2\Delta} - \\ &\quad - j\psi_{2.0}\omega_{1\Delta} + j\psi_{2.0}\omega_{\mathrm{r}\Delta} + jU_{2\mathrm{m}0} e^{j\beta_{2.0}} \theta_{r\Delta} - jU_{2\mathrm{m}0} e^{j\beta_{2.0}} \theta_{1\Delta} + \\ &\quad + jU_{2\mathrm{m}0} e^{j\beta_{2.0}} \theta_{2\Delta} + e^{j\beta_{2.0}} U_{2\mathrm{m}\Delta} + jU_{2\mathrm{m}0} e^{j\beta_{2.0}} \varepsilon_{2\Delta}, \end{aligned}\right\}$$

where Δ — are the variable increments; 0 — the variable values at linearization point.
Torque:

$$M_{\Delta} = k_{\mathrm{M}} \cdot \mathrm{Im}\left[\psi_{1\Delta}\tilde{\psi}_{2.0} + \psi_{1.0}\tilde{\psi}_{2\Delta}\right].$$

Equations of motion:

$$\left.\begin{aligned} \frac{\mathrm{d}\omega_{\mathrm{r}\Delta}}{\mathrm{d}t} &= \frac{1}{J}\left[M_{\Delta} + M_{\mathrm{L}\Delta}\right]; \\ \frac{\mathrm{d}\theta_{\mathrm{r}\Delta}}{\mathrm{d}t} &= \omega_{\mathrm{r}\Delta}. \end{aligned}\right\}$$

Equation for voltage equations:

$$\left.\begin{aligned} \frac{\mathrm{d}\theta_{1\Delta}}{\mathrm{d}t} &= \omega_{1\Delta}; \\ \frac{\mathrm{d}\theta_{2\Delta}}{\mathrm{d}t} &= \omega_{2\Delta}, \end{aligned}\right\}$$

The torque angle:

$$\theta_{\mathrm{M}\Delta} = \beta_{2\Delta} - \beta_{1\Delta} = \left(\varepsilon_{2\Delta} - \varepsilon_{1\Delta}\right) + \left(\theta_{2\Delta} - \theta_{1\Delta}\right) + \theta_{\mathrm{r}\Delta}.$$

Magnetic flux linkages $\overline{\psi}_{1.0}$ and $\overline{\psi}_{2.0}$ are calculated using the equations:

$$\begin{bmatrix} (\alpha_1 + j\omega_{1.0}) & -\alpha_1 k_2 \\ -\alpha_2 k_1 \psi_{1.0} & (\alpha_2 + j\omega_{2.0}) \end{bmatrix} \begin{bmatrix} \psi_{1.0} \\ \psi_{2.0} \end{bmatrix} = \begin{bmatrix} U_{1\mathrm{m}0} e^{j\beta_{1.0}} \\ U_{2\mathrm{m}0} e^{j\beta_{2.0}} \end{bmatrix},$$

whence:

$$\left.\begin{aligned} \Psi_{1x0} &= \frac{[c \cdot \cos(\beta_{1.0}) - (\omega_{2.0} d + \alpha_2^2 \omega_{1.0}) \cdot \sin(\beta_{1.0})]U_{1\mathrm{m}0} - \alpha_1 k_2 [a \cdot \cos(\beta_{2.0}) - b \cdot \sin(\beta_{2.0})]U_{2\mathrm{m}0}}{a^2 + b^2}; \\ \Psi_{1y0} &= \frac{[c \cdot \sin(\beta_{1.0}) - (\omega_{2.0} d + \alpha_2^2 \omega_{1.0}) \cdot \cos(\beta_{1.0})]U_{1\mathrm{m}0} - \alpha_1 k_2 [a \cdot \sin(\beta_{2.0}) + b \cdot \cos(\beta_{2.0})]U_{2\mathrm{m}0}}{a^2 + b^2}; \\ \Psi_{2x0} &= \frac{[e \cdot \cos(\beta_{2.0}) + (\omega_{1.0} d + \alpha_1^2 \omega_{2.0}) \cdot \sin(\beta_{2.0})]U_{2\mathrm{m}0} - \alpha_2 k_1 [a \cdot \cos(\beta_{1.0}) - b \cdot \sin(\beta_{1.0})]U_{1\mathrm{m}0}}{a^2 + b^2}; \\ \Psi_{2y0} &= \frac{[e \cdot \sin(\beta_{2.0}) - (\omega_{1.0} d + \alpha_1^2 \omega_{2.0}) \cdot \cos(\beta_{2.0})]U_{2\mathrm{m}0} - \alpha_2 k_1 [a \cdot \sin(\beta_{1.0}) + b \cdot \cos(\beta_{1.0})]U_{1\mathrm{m}0}}{a^2 + b^2}, \end{aligned}\right\}$$

where $a = \omega_{1.0}\omega_{2.0} - \alpha_1\alpha_2\sigma$, $b = \alpha_1\omega_{2.0} + \alpha_2\omega_{1.0}$, $c = \alpha_1\left(\alpha_2^2\sigma + \omega_{2.0}^2\right)$, $d = \omega_{1.0}\omega_{2.0} + \alpha_1\alpha_2\left(1 - \sigma\right)$, $e = \alpha_2\left(\alpha_1^2\sigma + \omega_{1.0}^2\right)$.

The obtained expressions are used to get the mathematical motor model for the controller synthesis.

4. Motor model in matrix form

The linearized equations in a matrix form are as follows:

$$\left.\begin{aligned} \frac{\mathrm{d}\underline{\mathbf{x}}}{\mathrm{d}t} &= \mathbf{A}\underline{\mathbf{x}} + \mathbf{B}\underline{\mathbf{u}} + \mathbf{G}\underline{\mathbf{r}}; \\ \overline{\mathbf{y}} &= \mathbf{C}\overline{\mathbf{x}} + \mathbf{D}\overline{\mathbf{u}}, \end{aligned}\right\}$$

where $\bar{\mathbf{x}} = \begin{bmatrix} \psi_{1\mathrm{x}\Delta} \\ \psi_{1\mathrm{y}\Delta} \\ \psi_{2\,\mathrm{x}\Delta} \\ \psi_{2\,\mathrm{y}\Delta} \\ \omega_{\mathrm{r}\Delta} \\ \theta_{\mathrm{r}\Delta} \\ \theta_{1\Delta} \\ \theta_{2\Delta} \end{bmatrix}$; $\bar{\mathbf{u}} = \begin{bmatrix} \omega_{1\Delta} \\ \omega_{2\Delta} \\ U_{1\mathrm{m}\Delta} \\ U_{2\mathrm{m}\Delta} \\ \varepsilon_{1\Delta} \\ \varepsilon_{2\Delta} \end{bmatrix}$; $\bar{\mathbf{r}} = \begin{bmatrix} M_{\mathrm{L}\Delta} \end{bmatrix}$; $\bar{\mathbf{y}} = \begin{bmatrix} \theta_{\mathrm{r}\Delta} \\ \omega_{\mathrm{r}\Delta} \\ \theta_{\mathrm{M}\Delta} \\ i_{1\mathrm{x}\Delta} \\ i_{1\mathrm{y}\Delta} \\ i_{2\,\mathrm{x}\Delta} \\ i_{2\,\mathrm{y}\Delta} \\ M_{\Delta} \end{bmatrix}$; $\mathbf{G} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{J} \\ 0 \\ 0 \\ 0 \end{bmatrix}$; $\mathbf{D} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$;

$$\mathbf{B} = \begin{bmatrix} \psi_{1\mathrm{y}0} & 0 & \cos(\beta_{1.0}) & 0 & -U_{1\mathrm{m}0}\sin(\beta_{1.0}) & 0 \\ -\psi_{1\mathrm{x}\,0} & 0 & \sin(\beta_{1.0}) & 0 & U_{1\mathrm{m}0}\sin(\beta_{1.0}) & 0 \\ \psi_{2\,\mathrm{y}0} & 0 & 0 & \cos(\beta_{2.0}) & 0 & -U_{2\mathrm{m}0}\sin(\beta_{2.0}) \\ -\psi_{2\,\mathrm{x}\,0} & 0 & 0 & \sin(\beta_{2.0}) & 0 & U_{2\mathrm{m}0}\cos(\beta_{2.0}) \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix};$$

$$\mathbf{A} = \begin{bmatrix} -\alpha_1 & \omega_{1.0} & \alpha_1 k_2 & 0 & 0 & 0 & 0 & 0 \\ -\omega_{1.0} & -\alpha_1 & 0 & \alpha_1 k_2 & 0 & 0 & 0 & 0 \\ \alpha_2 k_1 & 0 & -\alpha_2 & \omega_{2.0} & -\psi_{2\,\mathrm{y}0} & -U_{2\mathrm{m}0}\sin(\beta_{2.0}) & U_{2\mathrm{m}0}\sin(\beta_{2.0}) & -U_{2\mathrm{m}0}\sin(\beta_{2.0}) \\ 0 & \alpha_2 k_1 & -\omega_{2.0} & -\alpha_2 & \psi_{2\,\mathrm{x}\,0} & U_{2\mathrm{m}0}\cos(\beta_{2.0}) & -U_{2\mathrm{m}0}\cos(\beta_{2.0}) & U_{2\mathrm{m}0}\cos(\beta_{2.0}) \\ -\frac{L_\mathrm{m}\psi_{2\mathrm{y}0}}{J\sigma L_1 L_2} & \frac{L_\mathrm{m}\psi_{2\mathrm{x}\,0}}{J\sigma L_1 L_2} & \frac{L_\mathrm{m}\psi_{1\mathrm{y}0}}{J\sigma L_1 L_2} & -\frac{L_\mathrm{m}\psi_{1\mathrm{x}\,0}}{J\sigma L_1 L_2} & -\frac{k_\omega}{J} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

For the controller synthesis these matrix expressions or the transfer functions on their basis can be used.

5. Transfer functions

The transfer functions are obtained as follows [2]:

$$\left.\begin{aligned} s\mathbf{x}(t) &= \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{G}\mathbf{r}(t); \\ \bar{\mathbf{y}}(t) &= \mathbf{C}\bar{\mathbf{x}}(t) + \mathbf{D}\bar{\mathbf{u}}(t), \end{aligned}\right\}$$

where $s = \frac{\mathrm{d}}{\mathrm{d}t}$ — is Laplace operator.

The matrix transfer functions of the motor are:

$$\bar{\mathbf{x}}(s) = \mathbf{W}_\mathrm{x}(s) \cdot \bar{\mathbf{u}}(s), \qquad \bar{\mathbf{y}}(s) = \mathbf{W}_\mathrm{y}(s) \cdot \bar{\mathbf{u}}(s)$$

where $\mathbf{W}_\mathrm{x}$ and $\mathbf{W}_\mathrm{y}$ contain the elementary transfer functions between separate input and output variables.

6. Conclusion

The presented linearized model of the dual-fed motor allows to synthesize the controller. There are many possible control system structures as well as the synthesis methods. This enables to provide a variety of the electric drive characteristics to meet the specific requirements.

References

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[2] Doyle J C 2015 *Feedback control theoty* (New York) p 214