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## Investigating students' modelling styles in the process of scientific-mathematical modelling

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### Abstract

Modelling plays an important role for inquiry in science and mathematics education; therefore, a lot of research has been done in this field from both perspectives. However, an integrated view combining previous findings is rather limited. For the specific case of line graphs as a common representation that models relations between different variables in science, an integrated model of scientific-mathematical modelling was developed. The model integrates a scientific and mathematical perspective that describes line graphs as graphical representations of functional relationships. This model is used as a theoretical framework a) to describe cognitive processes that are necessary for modelling scientific phenomena with mathematical functions represented as line graphs and b) to analyse these processes empirically. In the presented study two modelling tasks were developed in which 10<sup>th</sup> grade students ( $N = 15$ ) are asked to model biological phenomena graphically as line graphs. Modelling processes are recorded using a SmartPen and concurrent think aloud. Results show that participants' modelling processes can be divided into sub-processes, related to the model of scientific-mathematical modelling. Furthermore, different individual modelling processes are reconstructed and graphically represented as graphical representations of individual modelling processes (GRIMPs). Based on a clustering process using GRIMPs, eight modelling styles are defined.

### Keywords

modelling process, modelling styles, scientific-mathematical modelling, line graphs

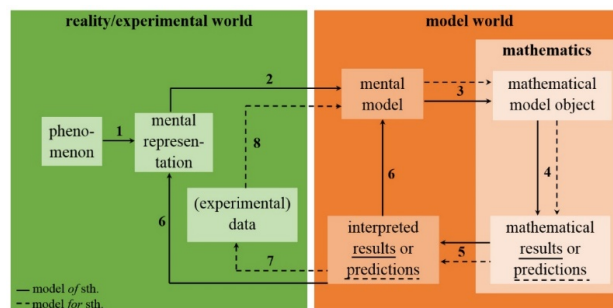
### 1. Introduction and theoretical background

In science as well as in mathematics education modelling plays an important role for inquiry and a lot of research has been done in this field from both perspectives (Blum, 1985; Borromeo Ferri, 2006; Krell, Walzer, Hergert, & Krüger, 2017; Nicolaou & Constantinou, 2014). While there is a wide range of models describing processes of mathematical modelling, studies focussing on relevant processes of modelling in scientific contexts are rather limited (Nicolaou & Constantinou, 2014). Moreover, it is argued that modelling serves as an important bridge between the STEM disciplines and, therefore, is of high relevance in all of these disciplines (Gilbert, 2004; Hallström & Schönborn, 2019). As science and mathematics are dealing with de-

pendencies of variables and their graphical representations (diagrams and line graphs in science, graphs of functions in mathematics) dealing with line graphs offers a valuable opportunity for describing both scientific and mathematical modelling processes as line graphs model quantitative and qualitative relations between different variables of living systems and, thus, model these relations by using mathematical considerations that involve dealing with functions. In order to describe cognitive processes involved in the modelling of biological phenomena with line graphs, scientific frameworks (scientific modelling: Krell et al., 2017; Passmore et al., 2014; Upmeier zu Belzen & Krüger, 2010; diagram competence: Kozma & Russell, 2005; Lachmayer, Nerdel, & Precht,

2007; Mevarech & Kramarsky, 1997) as well as mathematical frameworks (mathematical modelling: Blum & Leiß, 2005; Borromeo Ferri, 2006; functional thinking: Janvier, 1978; Malle, 2000; Nitsch et al., 2015; Vollrath, 1989) are integrated in a model of scientific-mathematical modelling (Meister & Upmeier zu Belzen, 2018; see figure 1). Moreover, the integrated model distinguishes between relevant stations (that are products in the scientific-mathematical modelling process, e.g. the mental model or the mathematical model object) and phases (mental processes in the sense of transitions between different stations, e.g. activating of experiences or mathematising). Hence, this model offers an approach to describe in detail the construction of a line graph as a stepwise modelling process of a scientific phenomenon using scientific and mathematical considerations by differentiating between relevant stations and phases (figure 1).

processes where several stations and/or phases are activated once or several times or be left out. Furthermore, Leuders and Prediger (2005) argue that dealing with qualitative line graphs in order to model real world phenomena fosters an epistemic understanding of such relationships (in the following, this is called *qualitative-graphical modelling*). From the perspective of scientific modelling, Göhner and Krell (2018) identify five different modelling strategies that describe typical modelling processes during a black box activity. As this black box activity doesn't focus on a specific scientific context, it is an open question whether individual modelling processes can be empirically described in a comparable way (with respect to modelling routes and modelling strategies) in the context of qualitative-graphical modelling processes of biological phenomena. Thus, the identification of respective patterns (that we call *modelling*



**Figure 1.** Model of scientific-mathematical modelling (Meister & Upmeier zu Belzen, 2018; based on Borromeo Ferri, 2006; Krell, Upmeier zu Belzen, & Krüger, 2016; Mahr, 2011). Boxes represent stations, arrows represent phases: **1** - perceiving phenomenon, **2** - activating experiences, **3** - mathematising, **4** - using mathematical competences, **5** - interpreting, **6** - validating, **7** - carrying out a scientific investigation, **8** - changing or retaining the model.

Based on the framework of scientific modelling and modelling competence (Krell et al., 2017; Upmeier zu Belzen, van Driel, & Krüger, 2019) two ways of using line graphs are emphasised. On the one hand, line graphs are constructed and interpreted in order to describe a phenomenon (models of something, Mahr, 2011; Upmeier zu Belzen & Krüger, 2010). On the other hand, line graphs are used as an epistemic tool to generate new hypotheses concerning a phenomenon (models for something, Mahr, 2011; Upmeier zu Belzen & Krüger, 2010). Testing these hypotheses (e.g. in experiments) may lead to a change of the line graph (Krell et al., 2017).

From the perspective of mathematical modelling, Borromeo Ferri (2010) identifies individual modelling routes that describe observable individual modelling processes. In the context of scientific-mathematical modelling, these modelling routes can be described as modelling

styles) with respect to the amount of activated stations or the type of transitions between these stations might serve as a basis for identifying specific cognitive processes from a scientific and mathematical perspective while qualitative-graphically modelling biological phenomena and, therefore, as a basis for designing modelling activities that focus on these different modelling styles.

## 2. Research Questions

The aim of this explorative qualitative study is the empirical identification of specific modelling styles by describing biology related as well as mathematics related cognitive processes during the modelling of scientific phenomena using the model of scientific-mathematical modelling (fig. 1). More specific, we investigate individual modelling processes of 10<sup>th</sup> grade students while they model biological phenom-

ena by constructing qualitative line graphs, resulting in the following research question: *What styles of modelling processes related to the model of scientific-mathematical modelling can be described in qualitative-graphical modelling processes of biological phenomena of 10<sup>th</sup> grade students?*

### 3. Method

#### 3.1 Participants

German high school students from 10<sup>th</sup> grade ( $N = 15$ , mean age: 15.4 years,  $SD = 0.6$  years) participated voluntarily in this study after we asked for participation in several courses. With respect to the qualitative character of this study, this sample size was chosen in order to provide a satisfying range of individual modelling processes as well as to reach a state of saturation (Mason, 2010) as we didn't identify new patterns in the observed modelling processes of the last five participating students.

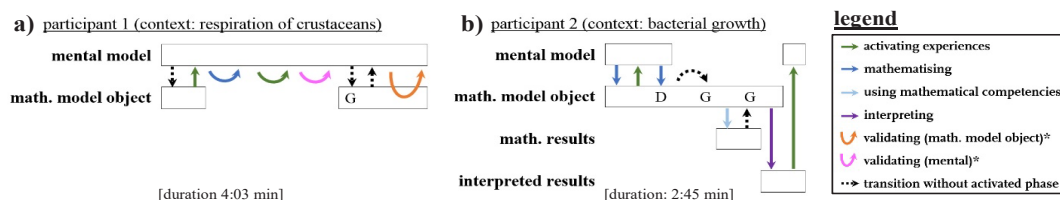
#### 3.2 Procedure

We developed an item-structured modelling task using the contexts of two different biological phenomena: bacterial growth and respiration of crustaceans. At the beginning of each modelling task, the phenomenon is presented in a short text (4 sentences, around 30 words) and in a video clip (around 20 sec. without audio). The first item assesses the individual mental representation by asking participants to verbalise their thoughts about the text they read and the video clip they saw. Item two gives instructions to model the phenomenon by constructing qualitative line graphs. The instruction addresses relevant variables in the context of bacterial growth: amount of bacteria and time; and in the context of respiration of crustaceans: speed of a ventilating organ and amount of oxygen. Participants are asked to work on both modelling tasks individually. Every participant started with the bacterial growth context. While

working on the modelling tasks participants were asked to verbalise their thoughts (concurrent think aloud: Sandmann, 2014, van Someren, Barnard, & Sandberg, 1994). Written and verbal data were recorded by using a SmartPen\*, which synchronises written and verbal data digitally (Angra & Gardner, 2017).

#### 3.3 Data analyses

Written and verbal data were transcribed for each modelling task representing one individual modelling process ( $N_{transcripts} = 30$ ). These processes were analysed applying qualitative content analysis (Mayring, 2010; Schreier, 2012) by using a category system that was derived deductively from the model of scientific-mathematical modelling focussing on the predefined six stations and eight phases and was refined inductively by two more phases (validating the mental model and validating the mathematical model object). Recoding 13 % of the transcripts ( $n = 4$ ), the intra-rater agreement ( $\kappa = .93$ ) was very good and the inter-rater agreement ( $\kappa = .66$ ) was good (Fleiss & Cohen, 1973) indicating a valid use of the category system. Based on the coding of each modelling task individual modelling processes were reconstructed by means of a sequence of coded stations and phases. These sequences were graphically represented as graphical representations of individual modelling processes (GRIMPs). As shown in figure 2, in a GRIMP activated stations are represented as boxes respectively the timespan in which they are activated during the modelling process. Arrows represent transitions between two stations (*inter-stational*) or within one station (*intra-stational*). Coloured arrows represent activated phases (each colour represents a specific phase) and broken arrows represent transitions without an activated phase. As activated stations and transitions are represented with respect to their occurrence and not to their duration, time is represented relatively in a GRIMP.



**Figure 2.** Graphical representation of individual modelling processes (GRIMPs). **a)** GRIMP of participant 1 and the context respiration of crustaceans, **b)** GRIMP of participant 2 and the context bacterial growth. Coloured arrows represent activated phases (see legend), broken arrows represent transitions without an activated phase, **D** – hypothetical data included, **G** – construction of the graph. \*: inductively derived

\* In this study we used the MOLESKINE+ Smart Writing Set.

All 30 GRIMPs were analysed on a macro level focussing on general characteristics with respect to activated stations and phases. Based on this, all GRIMPs were clustered into eight modelling styles.

#### 4. Results

As the reconstructed individual modelling processes were very diverse with respect to the sequence of activated stations and phases we developed a meaningful graphical representation in order to represent the individual modelling process adequately (*GRIMP*: graphical representation of an individual modelling process).

##### 4.1 Description of modelling styles

As every of the 15 participants modelled two phenomena we reconstructed 30 GRIMPs. We identified three aspects in which they differed on a macro level. First, we identified two groups concerning the amount of activated stations: either GRIMPs just included the mental model and the mathematical model object or they included additionally at least one more activated station (see second column in table 1). Second, we were able to divide both of these groups into two subgroups focussing on the dominant type of transitions (inter- or intra-stational, see third column in table 1). Third, we analysed whether GRIMPs included a validation of the constructed model object (see fourth column in table 1).

emathical model object are activated, the type of transitions is mainly inter-stational and the constructed model object is not validated. Furthermore, table 1 shows the relation of the modelling styles based on the amount of aspects in which they differ.

Moreover, GRIMPs which are integrated in a modelling style with mainly intra-stational transitions (modelling styles 3, 4, 7 and 8) can be described as modelling processes in which participants tend to model the phenomenon mentally before externalising their mental model as a mathematical model object (the mental modelling is represented as intra-stational transitions in a GRIMP, see figure 2 a). In contrast, GRIMPs which are integrated in a modelling style with mainly inter-stational transitions (modelling styles 1, 2, 5 and 6) can be described as modelling processes in which participants tend to iteratively verbalise an aspect of their mental model, externalise this aspect in the mathematical model object, verbalise another aspect of their mental model and so on. In addition, GRIMPs which are integrated in modelling style 5, 6, 7 or 8 can be described in contrast to those integrated in modelling style 1, 2, 3 or 4 as modelling processes in which participants work with their constructed model object (e.g. by deriving mathematical or interpreted results as shown in figure 2 b).

**Table 1.** Modelling styles based on three aspects of the respective GRIMPs (amount of activated stations, type of transitions and validation of the mathematical model object). Integrated GRIMPs are shown based on the participant and the modelled context as “participant number\_context”; **B**: context bacterial growth, **C**: context respiration of crustaceans, **MM**: mental model, **MMO**: mathematical model object.

modelling style	activated stations		type of transitions		validation (MMO)		integrated GRIMPs
	MM + MMO	MM + MMO + more	mainly inter-stational	mainly intra-stational	no	yes	
# 1	X		X		X		3 <sub>B</sub> , 4 <sub>B</sub> , 5 <sub>B</sub> , 6 <sub>B</sub> , 7 <sub>B</sub> , 9 <sub>B</sub> , 8 <sub>B</sub> , 10 <sub>C</sub>
# 2	X		X			X	3 <sub>C</sub> , 13 <sub>B</sub> , 13 <sub>C</sub> , 14 <sub>B</sub>
# 3	X			X	X		4 <sub>C</sub> , 11 <sub>B</sub>
# 4	X			X		X	1 <sub>C</sub> , 5 <sub>C</sub>
# 5		X	X		X		1 <sub>B</sub> , 2 <sub>B</sub> , 8 <sub>C</sub> , 10 <sub>B</sub> , 12 <sub>B</sub>
# 6		X	X			X	6 <sub>C</sub> , 7 <sub>C</sub> , 9 <sub>C</sub> , 14 <sub>C</sub>
# 7		X		X	X		12 <sub>C</sub>
# 8		X		X		X	2 <sub>C</sub> , 11 <sub>C</sub> , 15 <sub>B</sub> , 15 <sub>C</sub>

Table 1 shows all eight derived modelling styles as well as the respective integrated GRIMPs (see appendix for representative GRIMPs for each modelling style). Hence, every modelling style can be described via a combination of the three aspects introduced above, e.g. modelling style 1 is characterised by a modelling process in which only the mental model and the math-

#### 5. Discussion

Results show that students' modelling processes correspond with the proposed stations and phases of the model of scientific-mathematical modelling (Meister & Upmeyer zu Belzen, 2018). Moreover, reconstructed GRIMPs represent individual modelling processes by focusing on both relevant stations as products in the

individual modelling process and phases as transitions between different stations. Based on the description of modelling routes (Borromeo Ferri, 2006) GRIMPs offer a detailed insight into individual sub-processes by representing different activated phases.

By focussing on the three aspects (amount of activated stations, type of transitions and validation of model object), eight modelling styles have been derived from the reconstructed GRIMPs. Modelling styles 1, 2, 3 and 4 describe modelling processes in which only the mental model and the mathematical model object are activated. Therefore, they go in line with expressive modelling processes (Campbell, Oh, & Neilson, 2013; Göhner & Krell, 2018; Oh & Oh, 2010) in which model objects are constructed in order to describe the phenomenon. As modelling styles 3, 4, 7 and 8 include those modelling processes which mainly contain intra-stational transitions they offer a more detailed description of mental modelling processes (Göhner & Krell, 2018).

As only two of the fifteen participants show the same modelling style for both contexts, we conclude that modelling styles might be context related. Moreover, verbal data from the first task shows that all students were familiar with the context bacterial growth and none of the students was familiar with the context respiration of crustaceans. As most of the GRIMPs related to modelling styles 2, 4, 6 or 8 (with validation) represent modelling processes of an individually unknown context (respiration of crustaceans) we hypothesize that the qualitative-graphical modelling of unknown contexts might go in line with a tendency to validate constructed model objects and therefore a tendency to use the model object as a model for the phenomenon (Krell et al., 2017; Mahr, 2011). Thus, we will conduct further analyses in order to identify the potential influence of the context as a source of variance on the modelling styles.

In order to get a more detailed description of the eight modelling styles all 30 transcripts will be recoded with the focus on the relation between context related and mathematics related considerations as well as on aspects of functional thinking (Nitsch et al., 2015; Vollrath, 1989). We assume that these findings provide a basis for further research in order to foster scientific modelling on an elaborative level as the diagnosis of both a specific modelling style and the representation of the individual modelling process as a GRIMP provides the opportunity to describe potential difficulties in the individu-

al modelling process as well as the development of specific scaffolds.

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## Appendix

**Table 2.** Modelling styles with one respective integrated GRIMP. Coloured arrows represent activated phases (see legend in figure 2), broken arrows represent transitions without an activated phase, numbers under the mathematical model object represent the respective number of the constructed line graph, **D** – hypothetical data included, **G** – construction of the graph.

modelling style	representative integrated GRIMP
# 1	<p>participant 8 (context: bacterial growth) [duration 1:23 min]</p> <p>mental model </p> <p>math. model object</p>
# 2	<p>participant 3 (context: respiration of crustaceans) [duration 2:23 min]</p> <p>mental model </p> <p>math. model object</p>
# 3	<p>participant 11 (context: bacterial growth) [duration 2:35 min]</p> <p>mental model </p> <p>math. model object</p>
# 4	<p>participant 1 (context: respiration of crustaceans) [duration 4:03 min]</p> <p>mental model </p> <p>math. model object</p>
# 5	<p>participant 2 (context: bacterial growth) [duration 2:45 min]</p> <p>mental model </p> <p>math. model object</p> <p>math. results</p> <p>interpreted results</p>
# 6	<p>participant 9 (context: respiration of crustaceans) [duration 5:01 min]</p> <p>phenomenon </p> <p>mental model</p> <p>math. model object</p> <p>math. results</p>
# 7	<p>participant 12 (context: respiration of crustaceans) [duration 4:29 min]</p> <p>mental model </p> <p>math. model object</p> <p>math. results</p>
# 8	<p>participant 15 (context: bacterial growth) [duration 7:09 min]</p> <p>mental model </p> <p>mental model</p> <p>math. model object</p> <p>math. results</p> <p>interpreted results</p>