

**Chapter  
11****Fish Stock Assessment Models- Evolution and Possibilities****J. Jayasankar***Fishery Resources Assessment Division  
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**Q**uantitative fish stock assessment has been a topic full of challenges and opportunities. It has held spellbound researchers, both biologists and statisticians alike ever since the first seeds were sown in the form of growth curves and biomass prediction, some seven decades back. The challenges were multi-faceted although the candidate resources remained almost stable on most of the angles from which they were assessed at. The life pattern or cycle of any given resource was well recorded with the distinct phases of arrival, survival, maturity and reproduction. In most of these models which were fancied at various points of time main focus was always on growth, mortality and reproduction. The epoch of an average fish was always kept in mind before any such paradigm was carved out. The life period was clearly interspersed with biological occurrences like larval stage, recruitment, predation, migration etc. The dynamics was what mostly targeted in the modeling initiatives and the jarring note in the smooth dynamics was the fishing related mortality, which is man-made. The populations were imagined to be a biomass pool with the eternal cycle of churning occurring incessantly. The period of study under most of the models is assumed to be based on blocks of a year or parts thereof and most of the defining parameters of these flux was measured per annum. When the annual biomass addition matches the natural depletion, the state and rate of various dynamic functions were most amenable for computational rigour. This stage referred to as "equilibrium" has spawned a whole set of models like the Schaffer production models, which are a class of exponential growth functions. Interestingly be it mass dynamics or average fish growth or for that matter depletion/ mortality the basic relationship assumed

between the change in biomass and the age or time was exponential. Thus a whole lot of models both working on the growth trajectory of average fish as well as the one used in macro-analytical situations, were exponential in nature. From the analytical point of view most of these models revolved around fitting the model by using observed time series of field data by way of estimating the parameters and trying to forecast the future scenario. Thus from a statisticians point of view, these were fitting of regression models to sample data with one or many responses to one or many causes, either non-linearly or pseudo linearly. The disturbance caused to the biomass dynamics by way of fishing efforts was also incorporated in the biomass dynamic models and an optimization function was carved out based on a pair of equations, one of Schaefer kind and the other linking the catch and effort. That optimization function is minimized or maximized as the cases may be and the parameters are estimated.

Such modeling-based efforts yield a comprehensive relationship function under given assumptions relating the various causal factors like effort, environment, oceanic factors with the biomass, yield or availability status of a given resource or set of resources. Once this relationship is modelled satisfactorily scenarios can be built to aid the fishery manager to foretell the direction towards which the fishery is leading or to explain the present status of the fishery. These tools will be the core to any management plan at any level, local at a state/ province level to national and large marine ecosystem level. Thus depending upon the canvas to which the planning is applicable suitable modelling tools are adopted from time to time by researchers and fishery resource planners.

### **Tropical fish stock assessment**

The stock dynamics and the modeling thereof are situation or state invariant in its core. This means that a model which relates present biomass of a stock with that of the previous year's biomass is bound to be similarly structured, whether applied for tropical fisheries or Mediterranean fisheries or temperate fisheries. The real difference between the models qualified to be applied in tropical fisheries and those which are not, happens to be those which take into account the complex nature of the tropical waters and the intertwined nature of phenology and dynamics of the resources which were coexisting in such waters more often than not. Also interfering in this picture is the nature of gears that get used in

such regions and their inherent and forced non-selectivity. So, the models when being discussed for tropical conditions must be the ones which give room for such multiplicity of species competing for common food resources and which get caught by more than one type of generic gears. Rest of the issues which hog the single species model, which were aplenty initially, are still valid with possible inclusion of the factors arising out of such scenarios. A most important collateral challenge that arises while dealing with such situations is the standardization of effort as applicable to each resource, which would lead to more dependable catch rates, which are often used as indices of stock wealth or biomass. Hence, we would see in subsequent sections those models and tools which incorporate these kinds of uniqueness tagged with tropical waters.

### **Classical fish stock assessment approach**

The classical approach to fish stock assessment had been founded on the stages in the life cycle of a fish and some mathematical relationships that are most suited for describing a cause and effect relationship suited to these stages. Mostly the causes used to be those morphometric features that were visible and easily measurable and the effects used to be those which were of use as index to describe a particular stage of the animal's life. For example to describe the spawning ground, the cause used to be the total number of spawners and the effect used to be the recruits for surviving spawners. Similarly for the estimation of number of survivors at a particular stage of lifetime of cohorts, the starting population at the beginning of the year used to be the cause and the effect used to be the number of survivors at the end of the given age of the cohorts, thereby leading to the mortality. Thus similar relationships were defined for growth as well as biomass and these were all dependent on some established templates like exponential models that describe growth as well as depletion or decay, which were evolved in other branches of science. The best example is the classic Von Bertalanffy (1934) and that in fact spawned a plethora of modeling approaches, which were seminal in the context of fish stock assessment. By large the approaches to stock assessment can be broadly categorized into two viz. Analytical Methods and Holistic Methods. The Analytical Methods delve deep into the various life stages of the resource under focus and takes cue from applicable templates of models at each stage and combines all of them to arrive at the status of stock. This is more broad-

based and biologically sound leading to more precise estimation of stock health, but is heavy in data demand. These need time series of length frequency categorized count of animal at various time stamps and the type of gears and the effort expended on such gears with details of the mesh size etc. On the other hand holistic methods are simple optimization based models, wherein for a given stock/ sub-stock time series of catch rates and efforts/ catch are sufficient to arrive at the status of exploitation based upon the concept of surplus production, which again is templated on growth functions. Another set of methods falling in this category are the experimental cruise based data sets, like swept area method, which scales up the biomass based on the catch obtained during fixed duration trawling with fixed mouth width at randomly selected locations.

If the trail of evolution of these types of classical methods of fish stock assessment is studied in detail, one common phenomenon can be traced. All along the templates had been exponential growth or decay or depletion curves. But due to the difficulties faced in computationally estimating them, the all were attempted to be dealt with as linear relationship by means of appropriate transformations. The classic von Bertalanffy equation was converted into a linear type of relationship by logarithmic transformation, whose slope and intercept denoted the parametric estimates of relevance. Even in length frequency data, to convert them into age frequency, the method adopted was to treat each sample as a mixture of normally distributed values of animals of different age groups and to separate each one out of the mixture log transformation was again utilized whereby the parabolic normal functions got into linear forms and from them the probable distinct linear groups were culled out and their mean and standard deviations were computed. As the samples were taken on equal or known intervals the progress of the means of such culled out groups was traced and based on the rate of increment of mean growth for each cohort, the growth parameters were then estimated. Thus classic tools in fish stock assessment used a proper admixture of templates of non-linear models and suitable linearization antidotes for each of them. This itself gave the first possibility of improvement as computational facilities were improved- to estimate nonlinear relationship in its raw form. A lot of optimization and estimation tools like ADMB, Genetic Algorithm etc. were applied for this purpose. Also developed were Bayesian estimation tools which presumed known distributional vagaries for the parameters in a particular range, thus providing more realistic estimates of parameters of

growth, mortality and recruitment thereby leading to improved estimates of biological reference points (BRP) like maximum sustainable yield (MSY). All these methods were flawed on one key thing, that they targeted a single stock, which meant one species at a given location, without much information on the interaction of that focused species with its environment as well as its peer species and these analysis and inference were always carried out for “average fish” and with stringent assumptions like instantaneous and “knife edge” shifting of ages. These were dealt a more realistic smoothing in the later concepts on stock assessment methods.

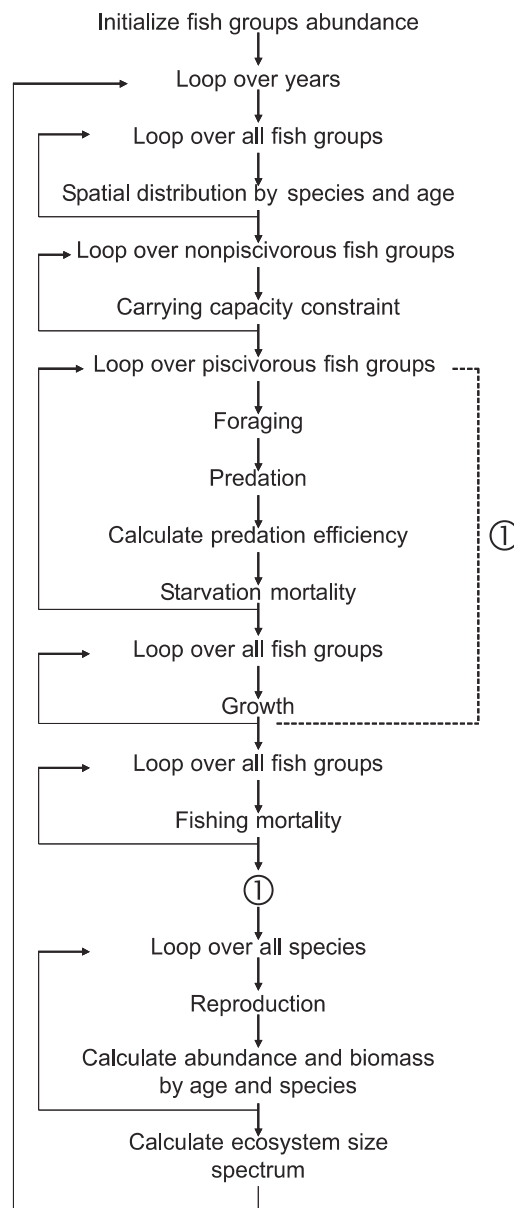
### **Individual based models (IBM)**

IBMs are models which are made up of a combination of relationships which contribute towards defining the current condition of the niche in which the resource under focus thrives comprehensively bundled with its feed, prey, predators, fishing intervention, physiological attributes. These types of models are mostly collection of modules which are best suited in predicting the crucial factors that proceed to impact the biomass of the species under focus. These types of hybrid prediction and simulation approaches are quite pliable for inclusion of many extraneous factors that can have cascading impact on the fish stock. With the advent of high power computational facilities such hybridization has been exploited to the hilt by researchers.

A typical IBM could be one including much broader habitat based components like availability of lower trophic level (LTL) biomass and the higher level foragers and their predators. The availability of food and the growth stage combination clearly heralding the status of larval mortality and the resultant niche based competitions between resources could also be included through IBM thereby scaling up to simulate regional ecosystems. One such comprehensive model is “Object oriented Simulator of Marine ecosystem Exploitation (OSMOSE)” (Shin and Cury, 2001, 2004). Herein the criterion for the selection of prey by a predator was considered to be firmly based on body sizes with opportunism applied at individual level with a localization principle based on the vicinity coming into picture. A cohort or super individual was made as pivot and the bio-ecological dynamics applied on that and replicated to the tune existing in the area and focus. Four model classes, which represent particular ecological entities, are used: the class “system”, the class “species”, the

class "age class", and the class "fish group" (Shin and Cury 2001). From each class, which is characterized by attributes and functions (e.g., growth, predation), a number of objects are created that are part of the simulated system. The architecture of OSMOSE is hierarchical, because a fish group belongs to an age class, which in turn belongs to a species. This structure enables the investigation of some key variables at different levels of aggregation, in particular the size spectrum of fish assemblages.

The process of implementation of OSMOSE can best be explained using the flow-chart given below:



As can be seen from the above figure, the dynamics associated with growth, mortality, reproduction (spawning) etc. could be modelled using the conceptualisation described in the previous case. But the new broad based habitat and trophism based components need some elaboration. The parameterization of the components is presented in the following table;

| <b>Stage/ Component</b> | <b>Model Definition</b>  |
|-------------------------|--|
| Foraging                | This is to be planned in such a way that the movement probabilities to the nearest spatial cell is highest and the availability of suitable prey/ LTL leading to feeding / starvation otherwise; It is a function of biomass and vicinity    |
| Predation               | This is functioned based on the spatio-temporal co-occurrence of prey- predator and the size of both; The prey- predator size ratio was subjected to a literature (FishBase) based threshold and the subsequent dynamics planned thereafter. |
| Starvation mortality    | This is depicted as a function of density dependent issue dependent on intra specific competition and is built upon predation efficiency as defined by Beverton and Holt (1957)  |

With these cardinal principles in place OSMOSE is rolled out to simulate regions under study but with two very important safeguards, first being the localised calibration and the second the sensitivity analysis. These are computationally intensive procedures leading to thousands of trial runs with various combinations of input parameters including crucial ones like larval mortality and plankton availability, whose sensitivity have been historically be recorded as delicate and hence crucial. Once validated with a decent strip of time step these calibrated tweaked models can be put to great use in estimating, simulating and forecasting marine fishery resources.

This approach is more generally followed in the size spectrum models, where again the sub modular relationships are modeled simultaneously while arriving at the biomass at any given time step based on the feeder causes.

## **Stock Synthesis**

Yet another approach that has gained popularity in view of the fast paced developments in the field of modeling is the maximalist approach, wherein all kinds of data, which were not collected for a homogeneous goal, including the details on a routine log sheet of a fishing fleet and those like capture recapture type of highly research oriented ones could very well be utilized to arrive at parametric estimates which would have been obtained by all kinds of data pertinent spatially and temporally to the stock under study. Such an approach dwells heavily onto statistical approaches which deal with data deprived conditions like Bayesian inference etc. The latest version of stock synthesis model (SS3) revolves around fortran code based old efforts rehashed and strengthened with the power of optimization tools like Automatic Differentiation Model Builder (ADMB), where in partial derivatives of a suitable objective function would be used to zero in on the most efficient parametric estimates. The following are the key components of the species dynamics which are targeted in this method.

### **a) Stock structure**

This hovers around the concept of stock, which starts with a spatial delineation followed by the year of birth (cohorts) and finally leading to the seasonally delineated morphs. This conglomeration of biological entities is further categorized into fast growing or tardy groups called as platoons with sexual discrimination enshrined. Upon these basic units, platoons, the size/ age selective fishing vulnerability conditions are applied, which in turn leads to age specific mortality, which again is a factor of classification within platoons/ morphs. When a multi-site scenario is modeled under this arrangement it is presumed that each of these platoons is distributed across and has an opportunity to mingle with as per norms. Such arrangements ultimately end up mostly as one platoon per morph differentiated sex-wise. Even the phenomenon of hermaphroditism gets implanted in this model by means of a fraction of females to males at each time step.



## **b) Spawner- Recruitment**

The second major feature of SS3 is the definition options available to link reproductive potential with expected total number of newborns. The Beverton- Holt type or Ricker type of relationships are available as options and the output of this function is the expected mean number of age 0 animals of a kind. This module takes into account the seasonality vis a vis date of birth animals and that age grouping is retained till the next season occurs or recruitment takes place.

## **c) Life history/ biology**

SS3 follows a unique procedure to compute biomass from population. Departing from the oft used procedure of empirical body weight at age method, SS3 arrives at the weight directly from length and superimposing the gear selectivity aspect over it. It allocates differential weights to those retained and those discarded, thereby giving a larger emphasis on length-at-age, which is less prone to sampling errors. The body growth aspect is attempted in the usual fashion mostly by using VBGF. This has options for year wise and cohort wise specification of growth rates, which would be incorporated dynamically. This feature accounts for the improbable density dependent growth in marine ambience, too. These growth characteristics are tagged only with the morph/ platoon and are spatially invariant. SS3 also follows the reparametrized VBGF wherein the growth parameters get redefined as per reference ages.

## **d) Selectivity**

This methodology gives ample space for age-, size- and gender-selectivity for each fishing fleet being modeled for. A distinction is made between catch and survey here, by which fleets' are distinguished by their primary output, which is fed as input in population dynamics viz. catch, whilst a survey has a sense and role of observation only. This tool has a wide range of selection ogives to choose from, which can aptly describe any gear- resource scenario. This also paves way for a more practical modification of selection ogives as the fishery gets older and older. An option of using non-parametric smoothed selection curves is also available.

### **e) Fishing mortality**

SS3 framework revolves around the concept of absolute catch being known well enough to allow the model to compute fishing intensity required as a sort of reverse calculation. Thus this setup permits the forecast both in terms of numbers as well as weight. A seasonal, fleet-specific fishing intensity is directly derived to match the observed catch, which is almost on simulation lines. This tool gives three options for computing fishing mortality viz. effort needed to match midpoint of a season, continuous instantaneous rates of mortality and a hybrid one. Due to the existence of multiple fleets and differential rates of exploitation SS3 gives a sense of “fishing intensity” in one of the three output forms, viz. exploitation fraction, equilibrium reproductive output per recruit and annual numbers weighted F over a range of ages.

### **f) Expected values for data**

A powerful feature of SS3 is its ability to calculate expected values for a wide variety of data types. In each time step this tool tracks numbers at age as well as mean and distribution of time at age for each morph. Upon this matrix formed by these two dimensions age/ size or gender selectivity is applied to arrive at the most probable numbers and distribution of animals selected by a given fleet. This matrix formulation enables the tool to compute expected catch figures too. This is accomplished by a simulation process involving recruitment, growth and selectivity along with mortality. The catch weights thus arrived at are then assorted into weight bins thereby leading to an expected value of catch.

### **g) Fishery management targets and forecasts**

SS3 computes fishing intensity levels that would satisfy several common fishery management templates. This is done by a mechanism of computation of stock per recruit and yields per recruit that would occur at trial level of fishing intensity, uses the unfished stock per recruit rate to calculate the absolute level of recruitment, spawning biomass and yield that would occur if fishing intensity were maintained at that rate. It then proceeds to iteratively compute fishing intensities that would yield known stock per recruit as compared to virgin stock per recruit or target equilibrium spawning biomass or MSY, which are all user specified. This bolsters the armory of fishery managers with multi-dimensional assessment and targets, which are bound to be more robust than single index based management. This dovetails

many a harvest policy induced models and hence SS3 is the most preferred analytical tool in countries which utilize catch quotas. The software through which this methodology is rolled out has many features that integrate with quite a few established tool boxes and generic software as well as ensuring significant scalability. As SS3 has kept its constant tryst with evolution steadfastly quite frequent version upgrades signaling quantum leap on evolution are expected and hence this could very well be an option of multi gear tropical fishery managers.

### **Depletions based stock assessment**

As it had been recorded initially, dynamic depletion concepts were very much in vogue in fish stock assessment protocols for long. Hence there is little surprise in the vista of stock dynamic models which are broadly based on various combinations of stock population or biomass depletion owing to fishing mortality generated by various kinds and numbers of fleets. These kind of models involve estimation of population parameters which fall in two groups viz. stock abundance (initial biomass, periodic spikes of abundance and natural mortality) and secondly fishing operation (hyper stability/ hyper depletion, saturability, catchability). The bouquet of optional models is derived from number of fleets, various perturbations of distribution, and the status of migratory flux of the stock. The options also have a sense of probabilistic touch in the form of various likelihood options. Starting with knowing the pulses using simple exploratory plots to maximization of likelihood functions these set of tools have a palette in various shades of stock and effort combinations. The best model chosen from these analytical routines are used for inferential and management purposes. The computational process initiates with a plotting and observation of fitness based short listing of options followed by the generation of a wrapper function, which is put to test by way of optimization under exponential depletion models incorporating situations like immigration, emigration, stock in transit through fishing grounds. All these model formulations involve the nominal catch by the fleet/ gear, a parameter of saturability, perturbation index (which clearly defines the rate and state of depletion crucial to the selection of best suited model) and natural mortality. Here again as in the case of SS3 the instants are time steps. This procedure has provisions for short duration- high frequent time steps as well as longer annual time steps. The two- fleet scenario operates additively

exhibiting complementary information about stock abundance. The models based on which the catch are modeled are usually the ones of rare events or negative binomial counts with random normal term added as noise and a multiplicative exponential term to deal with underlying various forms of Gaussian curves. The main index observed in these type of models based on generalized depletion is the rate of depletion at each time step and their dynamics. The depletion index is the difference of the ratio of fish population under fishing assumption to the fish population under no fishing assumption from unity. Thus quite obviously more computational maneuvers may have to be done to arrive at forecast populations based on the time spaced depletion index rates.

One common thread running across all these approaches to quantitative fish stock assessment is the availability of computational options as ready made package, with just inputs to be supplied. This makes the lives of the researchers/ managers easy, as it serves the twin purpose of directly dishing out the decision provoking pointers like biological reference points, while flagging the appropriate kind, type and granularity of data to be collected from the field.

## **Conclusion**

With the common theme of linking common causes like effort with practically tangible effects like catch/ biomass the fish stock assessment models though falling under a few generic approaches, ultimately revolve around the fulcrum of how much is known and how much of what is known is dependable and precise. So as it can be noticed the later developments in these models always revolved around in built simulation of Bayesian type iterations rather than the single valued classical frequentist solutions, thereby making the solutions more broad based and less sensitive to violations of assumptions.

## **Suggested literature**

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