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Humean Supervenience Rebugged

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Abstract This paper is a response to Lewis' 'Humean Supervenience Debugged' (1994). Lewis was in the business of defending Humean Supervenience, and the project seemed successful until the case of chance. Lewis thus originally named chance the 'big bad bug' for Humean Supervenience until the aforementioned paper in which he claims victory. Here I argue that he was unsuccessful and that Humean Supervenience remains bugged by chance. I will show how this bug remains due to a misdiagnosis of where the problem lies with regard to undermining. First, I define Humean Supervenience and chance, and state the bug in its original form, then secondly I describe Lewis' attempt to remove the bug. Thirdly, I explain why the bug persists, despite Lewis' efforts, and show the real source of the undermining problem to be due to the circularity of Humean Supervenient style accounts of chance. Finally I describe the situation this leaves chance in, and show how the incompatibility of chance and Humean Supervenience is evidence for the nonexistence of chance. I conclude that it is the circularity of the formation of Humean Supervenient laws of chance which continue to bug Humean Supervenience, leaving it untenable and resulting in little chance for chance.

Keywords David Lewis · Chance · Humean Supervenience · Undermining

Humean Supervenience and Chance

What Is Humean Supervenience?

Humean Supervenience (HS) is a contingent thesis that states that everything actual supervenes on the subvenient base: there cannot be a difference in anything in the world without a difference in the base. This subvenient base is the Humean mosaic, which is the spatiotemporal distribution or arrangement of all particular matters of fact (all of which have only perfectly natural properties). For Lewis it is certain that "truth is

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supervenient on being” (Lewis 1994 p473)¹ in this Humean way. Since we are working on the assumption that our world is chancy, the ‘truth’ and ‘everything actual’ includes chance, so chance too must be Humean Supervenient by supervening on the subvenient base. If chance does not supervene in this way then not everything is Humean Supervenient, and HS fails.

What Is Chance?

For Lewis, chances are single case objective probabilities, and in a chancy world there will be nontrivial nonextremal chances with probabilities between 0 and 1.² Lewis defines chance as being related to credence (the subjective degree of belief), in the way described by his a priori Principal Principle (PP)³:

$$PP : Cr(A|H_{tw}T_w) = Ch_{tw}(A)$$

The PP says that for all times t , worlds w and propositions A , at time t in world w the chance of A is equal to the reasonable initial credence in A conditional upon the total matters of particular fact up until time t in world w and the complete theory of chance for world w .⁴ The T specifies this complete theory of chance for the subscripted world w , where the theory of chance is meant to be a collection of hypothetical information about the chances of events in that world, in the form of history-to-chance conditionals: $H_{tw} \rightarrow Ch_{tw}(A)$. The laws of chance come from the complete theory of chance T_w so that particular matters of fact are given chances from the conjunction of H_{tw} with T_w . Lewis treats such a conjunction as being admissible evidence with regard to A as he states that it will attempt to provide the nontrivial nonextremal chance of A as opposed to whether A happens or not.⁵ Only admissible evidence regarding A is permitted in the PP, to ensure that the principle doesn’t tell us what actually happens in the future.⁶ So the PP relates two kinds of probability, namely how rational credence in A conforms to the chance of A , conditionalising on admissible evidence $H_{tw}T_w$.

¹ As I am referring repeatedly to this text, future references will be given by page number only.

² I will be assuming for the sake of argument that extremal chances will mean the occurrence or nonoccurrence of an event, such that 0 chance is to not occur, and having chance 1 is to occur. This has been worried over by those such as Mellor (2006). For example it may not be that having 0 chance means nonoccurrence in the infinite case—in a finite sequence of events each of nonzero (say $\frac{1}{2}$) chance of occurrence, the chance of them all occurring is greater than zero, but in an infinite sequence of such events it turns out that $\frac{1}{2}$ (for instance) to the power of infinity is zero. But this may not mean that infinite sequences cannot occur. This will not matter—see note 5.

³ This is a later and more important formulation called ‘the Principal Principle Reformulated’ (Lewis (1980) p97) which I believe to be a clearer version to work with than the equivalent original formulation: $C(A/XE) = x$ where C is the initial reasonable credence, A is the thing in question, X is the proposition that the chance of A is x , and E is the admissible evidence.

⁴ Lewis (1980) p98

⁵ For the notion of admissibility to have meaning and not be redundant there must be some values that turn out to entail the occurrence/nonoccurrence of A (and the obvious candidates for these values are 1 and 0). Without these extremal values having such significance there will never be inadmissible applications of the PP, as if values 0 and 1 do not entail non-occurrence and occurrence then the principles never give information about what happens in the world and as such will always be admissible. This is very problematic, and so I will be treating chances of values 0 and 1 to have their standard entailment of nonoccurrence and occurrence.

⁶ This admissibility will be discussed further later in this paper, to the effect that $H_{tw}T_w$ may turn out to be regarded as *inadmissible* evidence relative to A , otherwise it will provide no information about A at all and will thus be admissible at the cost of being useless and uninformative.

What Is the Big Bad Bug?

Lewis identifies and originally rejects what he considers to be an exhaustive list of methods for making chance Humean Supervenient. He thus names chance the ‘big bad bug’⁷ for HS, as if chance is not Humean Supervenient then HS fails. He later concedes and endorses the following method as successful:

T_w is contingent and chances are determined by the subvenient base.

This states that we should regard the theory of chance as contingent as opposed to necessary, so that it is dependant on how the world is and could be different accordingly. It further means that T_w is determined by the subvenient base, and as such the chances are determined by the subvenient base also. The complete spatiotemporal arrangement of particular matters of fact will provide the chances and determine T_w , therefore ensuring that the laws of chance are contingent and that chance is Humean Supervenient. However, now enters the problem of undermining. Undermining in itself is deemed merely and only ‘peculiar’ by Lewis: “This undermining is certainly very peculiar. But I think that, so far, it is no worse than peculiar.” (p483) What is unacceptable for Lewis about undermining is the fact that it is incompatible with the PP (in that the PP rules it out), and his original rejection of this method is purely due to this conflict. I shall now explain undermining and how it conflicts with the PP, and hope to motivate that it is problematic in itself rather than simply due to this conflict with the PP.

What Is Undermining?

If chance supervenes on the subvenient base, then T_w is determined by the total subvenient base. This means that the complete, actual arrangement of particular matters of fact over the past, present and future will determine the chances for any other arrangement of particular matters of fact. So, T_w is sensitive to the way things are not only in the history of a world w before a time t (H_{tw}) but also to the way things will be in the future of a world w after a time t . The problem with this is that T_w (when combined with H_{tw}) is said to provide us with the positive chance for non-actual undermining futures (call these futures F^*). But since F^* does not happen, it is not made up of particular matters of fact that are part of the subvenient base, thus it is not a future that determines T_w (because T_w is determined by the *actual* past, present and future).⁸ T_w would therefore be incompatible with F^* since if F^* were the actual future then it would determine a different theory of chance T_w^* . Suppose T_w gives F^* chance x , but then this chance is realised and F^* actually occurs, then T_w^* would be determined and T_w^* would give F^* chance y (where $x \neq y$). It changes its own chance and makes the present chance unstable.⁹ By determining a different theory of chance than the one that gives it its

⁷ Lewis (1986) pxiv

⁸ Lewis understands “it is because the chancemaking pattern lies partly in the future that we have some chance of getting a future that would undermine present chances. This problem would go away if we could assume that the chancemaking pattern lay entirely in the past” (p483) but Lewis rejects this method as it would make T_w necessary which he argues is false/problematic.

⁹ Lewis notes: “Different alternative futures would determine different present chances... It’s not that if this future came about, the truth about the present would change retrospectively. Rather it would never have been what it actually is, and would always have been different.” (p482–3)

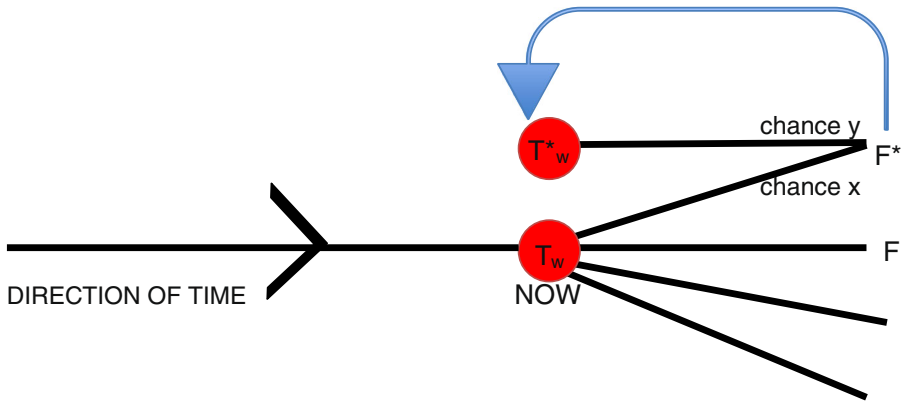


Fig. 1 Undermining

chance, F^* is said to undermine its own chances, and thus is called an undermining future. See Fig. 1 below¹⁰:

This undermining feature is a result of combining HS and chance, and is certainly disturbing in itself, but for Lewis it is only problematic due to its incompatibility with PP which occurs since a contradiction is derived when F^* is plugged into PP:

$$(LHS) \quad Cr(F^* | H_{tw}T_w) = Ch_{tw}(F^*) \quad (RHS)$$

$$0 = 0 < x < 1$$

The LHS equals 0 by $H_{tw}T_w$ being incompatible with F^* as it is not part of the subvenient base that makes up $H_{tw}T_w$,¹¹ and the RHS equals $0 < x < 1$ by T_w giving a positive chance to the possible future F^* in a chancy world. Thus, a contradiction.¹²

¹⁰ This diagram makes clear a parallel circularity problem to that of the paradox of time travel: If I were to travel back in time, I could do something back in the time I travelled *to* which would then result in a different future to that of the one I had travelled back *from*. The classic example of killing my grandfather makes the paradox vivid—if now, in 2014, I travelled back to 1959 (the year before my mother was born) and killed my grandfather before he and my grandmother had conceived my mother then my mother would not be born in order to give birth to me, and thus there would be no year 2014 in which I exist in order to travel back to 1959 to kill my grandfather. So similarly here is a circularity that entails a form of undermining.

¹¹ Thau notes: “It sounds a bit odd to say that the theory of chance [T_w] might rule out certain futures [F^*] which are nonetheless probabilistically possible, but only a bit odd.” (p495) This is along the same lines of Lewis’ dismissal of undermining being merely (and not worse than) peculiar. T_w rules out F^* by being incompatible with it, yet gives it a positive chance. But to be given chance 0 may not be to be ruled out (see note 2). However the contradiction remains as the LHS will still equal 0 (despite whether this means nonoccurrence) and the RHS will *not* equal 0. Furthermore F^* will need to be ruled out as not occurring if T_w is true, as that will mean F^* cannot occur (because if T_w is true then F^* *didn't* occur), and chance 0 will equal being ruled out.

¹² A contradiction can also arise completely on the *same side* (LHS) of the PP. The LHS can be argued to equal 0 (F^* being incompatible with T_w) yet also T_w combined with H_{tw} is meant to select the conditional giving F^* a positive chance and hence equals more than and not equal to 0. This neatly summarises the circularity in Humean Supervenient laws of chance I am arguing for.

Debugging Humean Supervenience

In order to rid of the conflict between the PP and undermining, Lewis makes a big concession, namely he drops his once considered a priori “all there is to know about chance”¹³ (Lewis 1980 p86) principle PP, and endorses a New Principle (NP) in place of it to be the new *actual* definition of chance. He claims the NP will not experience the undermining problem, simply by not being incompatible with it in the way the PP was (by deriving a contradiction when F* was plugged into it). This is Lewis’ answer to debugging HS (p487). The NP approximates PP and differs from the PP only with regard to the conditional probability on the RHS:

$$PP : Cr(A|H_{tw}T_w) = Ch_{tw}(A)$$

$$NP : Cr(A|H_{tw}T_w) = Ch_{tw}(A|T_w)$$

The NP states that reasonable initial credence in A should conform to the chance of A *conditional* on the theory of chance T_w. The NP is said to be able to accommodate undermining futures F* whereas the PP rules out undermining (p483). Lewis states, “by conditionalising credence or chance on T_w, we ignore undermining futures.” (p487) They are ignored because nothing is said of their chance in the NP, only their conditional chance. Thus the fact that all F* have a positive present chance is not mentioned in the NP and so ceases to be a problem. Before, a contradiction arose from the PP when F* was plugged in. Now, with the NP no such contradiction arises. This is because the RHS of the NP is the conditional chance of F*, and this conditional chance is 0, not between 0 and 1 as before. The conditional chance (in NP) of F* on T_w is 0 because F* (as an undermining future) is said to be incompatible with T_w, whereas the unconditional chance (in PP) of F* is between 0 and 1 because T_w gives F* a positive chance of occurring. Therefore the NP outputs 0 on both LHS and RHS for undermining futures F* thus avoiding contradiction. This completes Lewis’ purely technical cover-up solution as a method of debugging HS:

<p><i>PP:</i> Cr(A H_{tw}T_w)=Ch_{tw}(A)</p> <p><i>LHS:</i> Cr(F* H_{tw}T_w)=(x=0)</p> <p><i>RHS:</i> Ch_{tw}(F*)=(0<x<1)</p> <p style="text-align: center;">Contradiction</p>		<p><i>NP:</i> Cr(A H_{tw}T_w)=Ch_{tw}(A T_w)</p> <p><i>LHS:</i> Cr(F* H_{tw}T_w)=(x=0)</p> <p><i>RHS:</i> Ch_{tw}(F* T_w)=(x=0)</p> <p style="text-align: center;">No contradiction</p>
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Rebugging Humean Supervenience

In order to elucidate what I believe to be the real problem with undermining I will evaluate the NP more closely. Studying the role of T_w in the NP will expose the proper diagnosis of the undermining problem by showing that it is a problem due to the

¹³ He states that the PP will still be our working definition of chance, and that the NP will approximate the PP to a near enough degree, so that even though the NP is strictly the true principle the PP will approximate this closely enough to still be classed useful and a priori.

formation of T_w (as opposed to due to an incompatibility with the PP), and as such the NP is helpless in solving the problem. The incompatibility that bugs HS is not between undermining and the PP, but between HS and chance (regardless of the principle in use). Chance thus cannot be made Humean Supervenient, so either HS fails or there will be no such thing as chance.

How Do We Understand the NP?

For the NP to solve the undermining problem we must be able to make sense of it.¹⁴ There are two options for how to interpret the conditionals in the NP:

- (1) Treat the conditional on T_w as providing new information about A, such that it reads ‘the chance of A given what T_w says about A’;
- (2) Treat the conditional like a fraction where T_w itself has a chance, such that it reads ‘the chance of A conditional on the chance of T_w ’.

Therefore, the questions are: (1) what does T_w tell us of A; and (2) what is the chance of T_w ? I shall pursue these options in turn, arguing that their examination reveals Lewis’ HS account of chance to be circular and so still very much bugged.

Option 1: ‘The Chance of A Given What T_w Says About A’

T_w is the collection of hypothetical conditionals from possible histories to chance, so for all histories, T_w has a conditional from that history to the chance of any proposition A. So without information about the *actual* history H_{tw} , T_w has many conditionals for all the hypothetical histories which give different chances to A. T_w alone thus provides us with nothing concerning the *actual* chance of A. Therefore, $Ch_{tw}(A|T_w)$ interpreted as ‘the chance of A given what T_w says about A’ has no value, as T_w provides many possible values for the chance of A from various hypothetical histories but doesn’t provide the actual value for the chance of A since the actual history H_{tw} is not explicitly supplied to pick out the right conditional from T_w . However, despite H_{tw} not being *explicitly* supplied, it is indeed *implied*, as the actual history of the world is somehow stored in T_w as it is part of the subvenient base that formed T_w . So perhaps we would be able to select the conditional from history-chance of A. However with that Humean reasoning in mind, if the history could be somehow stored in T_w by being formed by a mosaic that contains that history then the future too could likewise be stored, as T_w is determined by the *complete* subvenient base that extends from the first moment in history to the last moment in future, and thus such a mosaic is in some way contained in T_w .¹⁵ T_w would then be able, *not* to tell us the chance of A, but the *actual* occurrence or

¹⁴ Both Briggs (2009) and Hall (1994) comment on the problems of interpreting the conditional $Ch_{tw}(A|T_w)$ and argue that this quantity is not user-friendly and worse is not well-defined. If this is true then the NP is rendered useless and unsuccessful in its function to debug HS.

¹⁵ T_w may be compatible with many mosaics, as the mapping from the arrangement of particular matters of fact to the laws need not be one-one, but could be many-one. So it could be that not only will the actual mosaic be somehow stored or compatible with T_w but also any mosaic that doesn’t contain in it an undermining event which would produce a different T_w^* . See note 18.

nonoccurrence of A , with regard to whether A was part of the pattern that determines, and is thus stored in, T_w .¹⁶

So, T_w is determined by the complete arrangement of particular matters of fact, including the future which we have assumed is chancy. This now poses a further problem for the role of T_w in telling us something about the chance of A —namely, if the future particular matters of fact have not yet occurred then T_w is not yet determined.¹⁷ Furthermore, if A concerned such a future event then by HS the facts wouldn't have yet attained in order to determine the laws T_w , so T_w will be neither compatible nor incompatible with A (and thus won't be able to tell us about A —chance or occurrence) since the future cannot somehow be stored in T_w as suggested above when the future (which includes A) hasn't happened yet. If the future is genuinely chancy (and has not occurred yet) then T_w remains to be defined, so to consult T_w when seeking to know the chance of future event A is pointless (because it is precisely such future events that include A which contribute to forming T_w to then tell us the chance of A). So T_w for A is undefined, as is $Ch_{T_w}(A|T_w)$, as T_w tells us nothing of A due to its being unformed. Since T_w cannot provide a value for A to understand $Ch_{T_w}(A|T_w)$, we move on to option 2.

Option 2: 'The Chance of A Conditional on the Chance of T_w '

We are now questioning the chance of T_w . Given HS, the laws supervene on particular matters of fact throughout the whole world (past, present and future), and the future particular matters of fact have chances thus conjunctions of these facts have chances, therefore the laws which supervene on the conjunctions of particular matters of facts must also have chances. The chance of T_w may then be considered undetermined, as what determines T_w may not yet have obtained, and only once they have obtained can T_w be formed, however by then T_w will equal 1. T_w will only have chance 1 if all the particular matters of fact have chance 1. This is because all the particular matters of fact need to occur (have chance 1) in order to define T_w (so that T_w has chance 1), otherwise both the particular matters of fact and T_w will have a nonextremal chance and will be as

¹⁶ This would make T_w by itself inadmissible evidence, as to be admissible requires that it doesn't say anything about how things actually turn out. If T_w were to be inadmissible in this way, then both the PP and the NP would never apply and would be regarded useless or incorrect. Lewis to some extent recognizes this aspect of HS and the inadmissibility of $H_{T_w}T_w$ relative to undermining futures (in that $H_{T_w}T_w$ rules out F^* by being incompatible with it) and concedes to allow for degrees of admissibility so that $H_{T_w}T_w$ can qualify as being an acceptable degree of admissible. It turns out though, that due to HS, T_w and $H_{T_w}T_w$ are always admissible by saying *nothing at all*. Interestingly then the PP and NP are useless either way: if $H_{T_w}T_w$ is admissible by saying nothing then they are completely uninformative, and if $H_{T_w}T_w$ is inadmissible then they cannot ever apply.

¹⁷ Thanks to John Horden for pointing out that the fact that the future is chancy and needs to be formed in order to determine the laws T_w may not pose a problem for Lewis who holds the difficult combination of eternalism with realism about chance. Under eternalism, the future will be as formed as the past, and so the complete subvenient base will already somehow *be there* ready to determine T_w , even when the future is chancy and hasn't happened yet. There seems to be a tension between this eternalist block universe view and its compatibility with nonextremal objective (nonepistemic) chance, but I shall consider it here as a possible escape for Lewis. Lewis would say the whole mosaic is out there, extending into the unforeseen future, regardless of what the chances are. So T_w would be determinately true even though it has some present chance of being false (by the future turning out differently having a positive chance). This is because being determinately true at a time and being nomologically determined to be true at that time (having chance 1) are not the same, and as such T_w needn't have chance 1 to be formed. Yet for the purposes of this paper, my target is not Lewis specifically, but rather any advocate of Humean chance. So unless every member of this target also endorses eternalism, there remains a problem of T_w being undefined due to being formed in part by an unformed chancy future.

uncertain as each other. We cannot give a value to the chances of particular matters of fact A or to T_w because they are defined in a circular way—the chance of T_w depends on the chance of all A and the chance of all A depend on T_w . This circularity prevents $Ch_{T_w}(A|T_w)$ providing any value and renders the NP useless.

Lewis complains, however, that the PP allows for T_w to have a chance 1 only:

Replacing A with T_w in the PP: $Cr(T_w|H_{T_w}T_w) = Ch_{T_w}(T_w)$
 T_w is obviously compatible with itself T_w , so LHS: $Cr(T_w|H_{T_w}T_w)=1$

Since the LHS equals the RHS of the PP: $Ch_{T_w}(T_w)=1$

The PP therefore gives the conclusion that the chance of T_w is 1.

This can also be formalised as a *reductio ad absurdum* argument:

Assuming for *reductio* that $\sim T_w$ has a positive chance: $Ch_{T_w}(\sim T_w) = (0 < x < 1)$

$\sim T_w$ is incompatible with T_w , so the LHS of the PP: $Cr(\sim T_w|H_{T_w}T_w) = (x=0)$

Since the LHS = RHS: $Cr(\sim T_w|H_{T_w}T_w) = Ch_{T_w}(\sim T_w)$, so $(x=0)=(0 < x < 1)$

Contradiction! The probability x cannot be both 0 and between 0 and 1.

Therefore, $\sim T_w$ cannot have a positive chance so T_w must have chance 1.

As Lewis claims, our theory of chance is one that “never had any chance of being false.” (Lewis 1986 p84) But this *cannot* be correct, as when the chance of T_w is 1 then $Ch_{T_w}(A) = Ch_{T_w}(A|T_w)$, and this makes the NP and PP equivalent (which Lewis states is *not* the case as they must only approximate each other when HS is true). So on the other hand, if HS is true then the chance of T_w shouldn’t equal 1 as T_w will itself have a chance, thus $Ch_{T_w}(A) \neq Ch_{T_w}(A|T_w)$ meaning the NP and PP are not equivalent. $Ch_{T_w}(T_w) \neq 1$ is not only proved by HS, but also by our assumption of the world being chancy, as stated by Haddock: “From the assumption that our theory of chance never had any chance of being false, we’ve proved determinism; for any event A_i , our theory of chance ascribes to that event a probability of 1... Thus, $[Ch_{T_w}(T_w)]$ does not equal 1.” (Haddock 2011 p861–2, notation mine) Furthermore, we ought not look to the old PP for confirmation of whether T_w has a chance but instead look to the NP, and, as Haddock points out, on standard probability theory the conditional $P(A|T_w) = P(A \& T_w)/P(T_w)$, which means T_w must have a probability/chance. So if Lewis wants T_w to not be chancy then he must drop the NP, chance or HS as these all entail T_w *must* have a chance.

It could be contested that the laws cannot have a chance, such that any account of chance that permits the laws of chance themselves to have a chance is implausible. Laws can’t *really* have a *chance*, but they may have a subjective probability, like credence, assigned to them. The laws (theory) of chance can’t have a chance, since they are meant to be laws governing chance itself. How can the concept of chance apply to itself? One can ask, how likely is it

that T_w is correct, but the answer to this is an epistemic chance—it's what we *know* of whether T_w is correct rather than an actual metaphysical chance that T_w has of actually *being* correct. If chance is single case objective probabilities as Lewis states, then the laws cannot have a chance since laws are not single case things, so the concept of objective probability does not apply to laws. Theories of chance do not assign themselves probabilities as they are not themselves events, and simply fall outside the field over which chance is defined.

However it is clear that under HS the laws *will* have a chance, namely a chance of being *correct* (i.e. being the T_w that *does* describe the way things did *in fact* turn out, to correctly reflect the subvenient base that determines it). It can thus be shown that undermining bugs HS in another problematic form—it's not just futures that are undermining, but theories of chance too. There are many times t^* where the complete arrangement that makes up the total subvenient base is incomplete, in that the full future may not have occurred yet. Thus at t^* different future arrangements have a chance. Because of this, there will also be a chance at t^* for a different theory of chance T_w^* since the theory of chance is determined by the total arrangement of particular matters of fact and this arrangement has a chance of being different. Different arrangements determine a different theory of chance,¹⁸ and since different arrangements have a chance then different theories of chance have a chance. If T_w itself is chancy, then the chance distribution that T_w provides over particular matters of fact is chancy, thus undermining T_w itself.

In order to avoid the circularity experienced from T_w being determined by a chancy future (and thus itself being chancy), one could appeal to a different sense of chance to provide a chance to the undetermined T_w , but this only leads to embarking on a regress. This is because T_w is meant to govern chance, but if chance applies to T_w as well then this chance needs to be governed by a higher order T_w' . To assign a chance to T_w being true the higher order T_w' is needed to provide the chance of T_w , leading to a regress as such: T_w gives a chance to A, then there is a higher order T_w' that gives a chance B to T_w giving the correct chance to A, then there is a higher higher order T_w'' that gives a chance C to the chance B being the correct chance of T_w giving the correct chance to A, etc. If there are two senses of chance (one that the particular matters of fact have and one that the laws have) then this

¹⁸ Thanks to Paul Noordhof who noted there needn't be a one-one mapping from arrangement-theory. It could be that multiple/variable realisation is true, such that there could be many different arrangements that all determine the same theory T_w . This implies that 'if A1 then T_w , or if A2 then T_w ' but not 'if T_w then A1'. T_w doesn't require that one particular arrangement occurred over others supporting (roughly speaking) the same frequencies. To illustrate, suppose that all future coin tosses have a 50 % chance of landing heads; and that this is just because throughout history (past, present and future), half of the coin tosses land heads. Suppose there are 100 coin tosses in the whole of history and that there have been 50 coin tosses so far, and 25 have landed heads. So there are 50 coin tosses yet to come, and 25 will land heads—otherwise the relevant chances would be different. But clearly there are many different ways in which there can be 25 heads out of 50 tosses. There could first be 25 heads, and then later 25 tails, or vice versa, or alternating heads and tails, and so on. The order doesn't affect the frequency, and by hypothesis the frequency determines the relevant chances. And even if actual chances have a (more sophisticated) best-system analysis of the sort advocated by Lewis, it still seems that, analogously, many different (but relevantly similar) patterns in the Humean mosaic would determine the very same chances that we have in the actual world. This gives T_w a certain degree of independence, loosening my circularity claim somewhat. However given that there still are arrangements (presumably the majority of the possible arrangements) that will be undermining for T_w , it still holds that T_w has a chance of being incorrect. The chance of T_w is now just the chance that the future wont be undermining. It is not the case that every nonactual future would determine the truth of a different theory of chance, but there are still some (again presumably most) that would. Given that they have a positive chance, and what gives them this chance therefore has a chance of being incorrect, the circularity from the undermining problem remains, as if there's a chance of undermining in a theory then that theory has a chance of being incorrect.

other sense would need to be governed¹⁹ by yet a further sense of chance. Without appealing to further higher order chances to govern the chance that laws have, we would have to explain the chance that laws give and the chance that laws have in terms of each other, making a circular account of chance. Either, then, the chances of T_w will be governed by a higher order of chance (embarking on a regress) or will be defined and governed by its own chance (becoming circular). Since neither of these options are appealing, chance is rendered incompatible with HS. The dilemma of regress or circularity occurs for all accounts of chance that maintain that the laws are contingent and determined by all particular matters of fact, and as such this is not a problem only for the NP but for all Humean (and frequency style) theories of chance.

To summarise, the two options we had for interpreting the conditional chance $Ch_{tw}(A|T_w)$ have proven to be unsuccessful, since the circular T_w entails that the chances T_w gives and the chance T_w has are all indeterminate, resulting in the conditional being undefined. The NP is thus useless, leaving HS bugged by chance without a principle to cover it up. I will now revisit undermining to see what this evaluation of the NP has revealed about the problem which fuelled the change of principle, and see where the problem really lies and why it was not solved.

Undermining Revisited

Undermining was regarded as merely peculiar, and only became a problem for Lewis due to its incompatibility with the PP.²⁰ The NP was supposed to solve this by ignoring the existence of undermining futures, avoiding the contradiction that the PP entailed. However, due to HS, T_w is chancy because it is determined by the chancy subvenient base, and thus neither the LHS $Cr(F^*|H_{tw}T_w)$ nor the RHS $Ch_{tw}(F^*|T_w)$ of the NP can equal 0. Both sides are undefined. This is because if T_w is chancy (in that there is a chance that it is not correct for world w) then there is a chance that F^* is compatible with $H_{tw}T_w$ and T_w by being part of the mosaic that determines it. There's a chance that F^* is part of the subvenient base that determines T_w and there's a chance that F^* is not. There is thus a chance that T_w will be compatible with F^* and a chance that T_w will not be. Therefore, the NP doesn't supply $Cr(F^*|H_{tw}T_w)=0$ or $Ch_{tw}(F^*|T_w)=0$, and so fails in its technical solution to the undermining problem. This further shows that no technical solution was required as there was no technical contradiction in the first place: the PP for the same reasons as above doesn't supply the contradictory $Cr(F^*|H_{tw}T_w)=0$ or $Ch_{tw}(F^*|T_w)=1$. The undermining problem was thus never a problem of incompatibility with the PP, but an incompatibility of chance with HS.

This 'peculiar' aspect of HS will remain as long as T_w is determined by a pattern that extends into the chancy future. Any Humean Supervenient account of chance where T_w

¹⁹ Perhaps 'govern' is inaccurate, as a strict Humean/Lewisian does not regard laws as really governing events but rather merely *reflecting* them. Under HS in a chancy world w the chance of A at t in w will be x simply because the frequency of A -type events (in the circumstances of the type which hold at t in w) are those that would make the Best System Analysis of Laws give chance x to A -type events as part of the best (simple, informative and most fitting) description of the patterns of events which hold in the world w . So really the laws just reflect and describe the frequencies. Nevertheless, since the frequencies have an overall chance of coming about, the laws themselves have a chance to correctly reflect those frequencies or not, and so we must question how we understand these chances that laws have, and I propose it is either circular or regressive.

²⁰ I argue that undermining is much more severe than mere peculiarity, and occurs due to the circularity in all contingent Humean Supervenient accounts of chance, which the NP can't resolve.

is contingent and determined by all matters of particular fact in the subvenient base that covers the past, present, and future, will encounter undermining problems, regardless of the credence-chance principle in use. This is a result of the circularity they experience—in that T_w provides the chances for the particular matters of fact yet the chances of those very same particular matters of fact determine T_w —so any chancy future will be undermining if it is not a future that determines T_w since T_w dictates those futures chances. Thus the contradiction with the PP was *not* the heart of the undermining problem. The real essence of the undermining problem is the circular T_w , which is not solved by the concession to the NP, and shows HS and chance to be incompatible. Lewis acknowledges the devastating effects this may have for a successful account of chance: “The big bad bug bites a range of different Humean analyses of chance. Simple frequentism falls in that range; so does the Best System Analysis.” (p482) Thau (1994) makes a similar discovery, that this circularity undermining problem is experienced by all frequency and ‘Justified Certainty’ theories. Because of this it seems unlikely that there can be a successful analysis of chance, as arguably the best accounts involve frequencies and the PP/NP. If we are to agree with this that our best accounts of chance are Humean/Lewisian in style, then this result is all the worse for chance. We thus are forced down one horn of a dilemma—stick with our Humean style laws (and do away with objective chance), as we cannot keep chance and dispense of HS as our best account of chance is Humean.

Conclusion

I have argued that HS remains bugged by chance, despite Lewis’ attempt to free it from the undermining problem. I claimed that the real essence of this problem went unnoticed, and it was because of this failure of recognition that Lewis’ debugging failed. Undermining is worse than peculiar, *it is threatening to any account of chance*, and so cannot be solved by covering up the issue with a principle that ignores it. I showed the real essence of the undermining problem to be due to Humean Supervenient laws of chance being circular: what gives the laws a chance is what the laws give a chance to. This proper diagnosis of the undermining problem shows the cause to be the circularity of HS laws of chance, and so it is this formation of HS laws that need to be changed, not (as Lewis attempted) any credence-chance principle. The incompatibility between HS, chance, and the original chance principle—the PP—cannot be removed by a change in chance principle (as Lewis attempts by changing to the NP) as the real incompatibility exists simply between HS and chance. Therefore it turns out that one must be dropped. HS is a thesis that ranges over everything actual, and so if we are to insist that there are chances (and chose not to drop chance) then HS will not be able to account for such chances. As such, HS will be left philosophically untenable due to the bug of chance. I thus conclude that HS is *rebugged* by chance, leaving HS untenable and potentially ruining our chance of having chance, therefore making Lewis’ project unsuccessful.²¹

²¹ I would like to thank Stephen Barker, Paul Noordhof, Barry Lee, John Horden and Dave Ingram for useful discussion and comments relating to earlier drafts of this paper, and to the audiences of the Graduate conference at the University of Stockholm and the Mind and Reason research group at the University of York at which I presented versions of this paper.

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