Standardizing the Calculation of the Lyapunov Exponent for Human Gait using Inertial Measurement Units

by

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A Dissertation Presented in Partial Fulfillment
of the Requirements for the Degree
Doctor of Philosophy

Approved November 2019 by the
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ARIZONA STATE UNIVERSITY

December 2019

ABSTRACT

There are many inconsistencies in the literature regarding how to estimate the Lyapunov Exponent (LyE) for gait. In the last decade, many papers have been published using Lyapunov Exponents to determine differences between young healthy and elderly adults and healthy and frail older adults. However, the differences in methodologies of data collection, input parameters, and algorithms used for the LyE calculation has led to conflicting numerical values for the literature to build upon. Without a unified methodology for calculating the LyE, researchers can only look at the trends found in studies. For instance, LyE is generally lower for young adults compared to elderly adults, but these values cannot be correlated across studies to create a classifier for individuals that are healthy or at-risk of falling. These issues could potentially be solved by standardizing the process of computing the LyE.

This dissertation examined several hurdles that must be overcome to create a standardized method of calculating the LyE for gait data when collected with an accelerometer. In each of the following investigations, both the Rosenstein *et al.* and Wolf *et al.* algorithms as well as three normalization methods were applied in order to understand the extent at which these factors affect the LyE. First, the *a priori* parameters of time delay and embedding dimension which are required for phase space reconstruction were investigated. This study found that the time delay can be standardized to a value of 10 and that an embedding dimension of 5 or 7 should be used for the Rosenstein and Wolf algorithm respectively. Next, the effect of data length on the LyE was examined using 30 to 1300 strides of gait data. This analysis found that comparisons across papers are only possible when similar amounts of data are used but comparing across normalization

methods is not recommended. And finally, the reliability and minimum required number of strides for each of the 6 algorithm-normalization method combinations in both young healthy and elderly adults was evaluated. This research found that the Rosenstein algorithm was more reliable and required fewer strides for the calculation of the LyE for an accelerometer.

DEDICATION

This dissertation is dedicated to my husband, Hyder, and my family for without their support I would have never been able to finish

ACKNOWLEDGMENTS

This dissertation would not have been possible without the guidance and support from my mentors, Dr. Thurmon Lockhart and Dr. Mark Spano. I would like to thank both of them for giving me so much of their time and energy over the years to advise me throughout my graduate school career. I also want to thank my committee – Dr. Clair Honeycutt, Dr. Daniel Peterson, and Dr. Hyunglae Lee – for all of their individual support and critical feedback and helping me grow as a scientist.

Additionally, I would like to express my gratitude to all of my supportive lab members over the years at the Locomotion Research Laboratory: Christopher Wayne Frames, Seong Moon, Rahul Soangra, Saba Rezvanian, Kaycee Glattke, and Markey Olson; for the laughter, the support, and all assistance.

Most importantly, I want to thank my family for all the strength, support, and encouragement for without which I would not have been able to persevere through this journey. To my best friend and husband, Hyder, thank you for your endless love, support, and patience. To my parents, thank you for your lifelong support of everything I do, exposing me to engineering from the very beginning, and being the greatest parents and role models a daughter could ever ask for. To my little sister, Tina, thank you for helping me write this dissertation with the constant readability feedback and catching all my silly grammar mistakes. And thanks for always being by my side and supporting me with love and friendship as only a sister can.

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CHAPTER 1: INTRODUCTION

MOTIVATION

Falls are a well-recognized risk factor for unintentional injuries among older adults, accounting for a large proportion of fractures, emergency department visits, and urgent hospitalizations. (Tinetti, 2003) According to the Centers for Disease Control and Prevention's web-based injury statistics query and reporting (WISQARS, 2010a, 2010b): in 2010, 3.7 million people over the age of 50 reported non-fatal fall-related injuries, and 24,000 people in this age bracket died from falling or from the resulting injuries. That year, non-fatal falling injuries resulted in 2.8 million emergency room visits costing $7.9 billion dollars across the country.

Every year, almost a third of people over the age of 65 fall at least once and 10-15% of those falls cause serious injuries or result in death. (Milat et al., 2011) Many of these falls could be prevented by screening people in this age group for fall risk and identifying "at-risk" fallers. Generally, a person's level of fall risk is determined by their musculoskeletal (e.g. the fitness of their lower limb muscles) and sensory (e.g. quality of their vision and proprioception) functions. (Pfortmueller et al., 2014) As we age, both systems naturally start to deteriorate over time. There are ways to limit this deterioration, such as exercise regimens (Freiberger et al., 2013; Gillespie et al., 2012; Sherrington et al., 2011), so individuals identified as "at-risk" fallers could begin to take more personal precautions or join an exercise program to reduce their fall risk.

Even though fall intervention programs have shown great success in reducing falls, enrolling and retaining people in these community-run programs has proven to be a challenge. (Day et al., 2002) One way to improve enrollment is to have general physicians screen their elderly patients for fall risk and suggest or prescribe exercise and fall risk prevention programs. It has been shown that the elderly visit their general physicians on a regular basis and view them as an important source of health-related information. In addition to this, elderly patients value their physicians' opinions and will thus be more likely to follow their recommendations. (Gardner et al., 2002) So, having a simple tool for physicians to utilize in their normal visits to assess changes in the severity of fall risk would be advantageous. Currently, clinical fall risk assessments are based on questionnaires and non-instrumented functional tests. Even though functional tests are more objective and quantitative than questionnaires about fall history, they lack the ability to discriminate between healthy and *at-risk* populations that need intervention programs. (Hamacher et al., 2011) Some of the most promising predictors of fall risk have sprouted from analyzing gait and postural stability with nonlinear dynamical tools such as Lyapunov Exponents (Dingwell and Cusumano, 2000; Lockhart and Liu, 2008), and various entropy measures (Borg and Laxåback, 2010; Busa and van Emmerik, 2016; Fino et al., 2015).

In biomechanics, the most commonly used nonlinear dynamical techniques are the Lyapunov exponent (LyE) and entropy measures which are used to quantify stability and complexity, respectively. For instance, in walking, we take very similar steps from right to left in terms of step size, walking velocity, etc. but these similar steps are not identical. These small changes are due to slightly different initial conditions before we take each step. LyE evaluates these changes (divergences) between initial conditions and is used to measure the stability of gait as a dynamical system. Quantitatively identifying people who have poor gait stability should help categorize individuals as healthy or at-risk.

Inertial measurement units (IMUs) have become widely used in assessing gait and other daily living activities as an alternative to traditional motion capture systems. They have also been used to calculate different parameters, include the LyE, as biomarkers for various ailments, e.g. patients with dementia (IJmker and Lamoth, 2012), multiple sclerosis (Huisinga et al., 2013), Parkinson's disease, (Fino et al., 2018) and concussions (Fino, 2016). Accelerometers are flexible, mobile, inexpensive, and have the advantage of recording gait in various environments with ease. (Tao et al., 2012) Thus, as IMUs become the prominent method of collecting gait data, it is important to standardize and tailor the protocol for calculating the LyE using this particular signal.

There are many inconsistencies and incongruities in the literature regarding how to estimate the Lyapunov exponent for gait. In the last decade, many papers have been published using Lyapunov Exponents to determine differences between young healthy and elderly adults and healthy and frail older adults. (Mehdizadeh, 2018) However, the differences in methodologies of data collection, input parameters, and algorithms used for the LyE calculation has led to conflicting numerical values for the literature to build upon. Without a unified methodology for calculating the LyE, researchers can only look at the trends found in studies. For instance, LyE is lower for young adults compared to elderly adults. (Granata and Lockhart, 2008a; Dennis Hamacher et al., 2015; Terrier and Reynard, 2015) But the values cannot be correlated across studies to create a classifier for individuals that are healthy or at-risk of falling. These issues could potentially be solved by standardizing the process of computing the LyE. There are several hurdles that must be overcome to create a standardized method. This includes but is not limited to the choice of algorithm, normalization of collected data, parameterization used in phase space reconstruction, and amount of data required. It is possible and likely that all of these factors must be tuned to how data is collected, i.e. motion capture data use of position, velocity, or joint angles or inertial measurement units (IMUs) use of linear and angular acceleration. Research has touched on different combinations of this problem. For example, using a group mean embedding dimension and time delay for the reconstruction of the phase space was found to improve reliability of the LyE when using IMUs (van Schooten et al., 2013) and motion capture data (Raffalt et al., 2018a). Other studies have looked at the effect of data length in both data collection methods using the Rosenstein *et al* algorithm and the Wolf *et al*. algorithm with various methods of data normalization. And one recent study (Raffalt et al., 2019) investigated how different normalization methods are more beneficial for specific algorithms when using motion capture data. It is impossible for a single paper to investigate the myriad of factors and implications of each one. Therefore, this dissertation investigates each of these hurdles in order to help create a standardized method of calculating the LyE for gait data when collected with an IMU (accelerometer).

SPECIFIC AIMS

The objective of this research was to develop a standardized methodology for calculating the LyE for human gait when using accelerometers. This will allow for biomechanical researchers to utilize LyE while understanding the implications of choosing various input variables associated with its calculation. We will analyze how a phase space is reconstructed and determine the minimum data needed to accurately calculate the LyE.

Aim 1: *Develop guidelines for phase space reconstruction for gait data by investigating how data length and preprocessing methods affect the methods used to determine these parameters and investigate if and/or how these parameters effect the value of the Lyapunov Exponent.*

Hypothesis 1a: Determining time delay and embedding dimension will not be affected by the amount of gait data provided to their calculation methods

Hypothesis 1b: Different combinations of time delay and embedding dimension will cause significant differences in the calculated LyE within a single subject and within a group of subjects.

Aim 2: *Evaluate the effect of data length on the estimation of the LyE for accelerometer data when different preprocessing methods and algorithms are utilized*

Hypothesis 2a: Larger data sets (greater than 150 strides) are not directly comparable to smaller data sets (50 strides or less), regardless of algorithm used to estimate the Lyapunov exponent

Hypothesis 2b: The required minimum number of strides, needed for precise and reliable estimation of the Lyapunov exponent, will be dependent on the algorithm and normalization method used.

Hypothesis 2c: The Rosenstein *et al.* algorithm will have better precision and reliability for calculating the Lyapunov exponent for accelerometer data

The culmination of these aims will create a standardized methodology for calculating LyE which will allow for the comparison of data and conclusions across all studies that employ their use. This, in turn, will hopefully, allow for better meta-analyses identifying the best measures for creating a precise and sensitive fall risk assessment tool.

ORGANIZATION

This dissertation has 6 chapters. Chapter 2 reviews the principles of nonlinear dynamics with respect to Lyapunov Exponents as well as how LyE have been used in clinical and community efforts and what standardization in the methodology has been researched. Chapter 3 investigates how data processing and amount of data affects the calculation of the time delay and embedding dimension. Chapter 4 indepthly investigates the effect of time delay and embedding dimension choices when different preprocessing or normalization methods are utilized for calculating the LyE. Chapter 5 studies the effect of data length on the LyE when using both the Rosenstein *et al* and Wolf *et al* algorithms. And finally, in Chapter 6, we investigate if elderly walking data can be processed using the same methods as young healthy adults without changing the reliability of the LyE.

CHAPTER 2: BACKGROUND

NONLINEAR DYNAMICS

Dynamical systems are defined as the deterministic mathematical equations that describe the evolving state of the system through time (Abarbanel, 1996; Hirsch, 1984; Ott, 2002). More simply, they are systems that change over time. Such a system will eventually (after some transient period) settle into either a periodic motion (i.e. a limit cycle) or into a steady state (i.e. a situation in which the motion has ceased). Common examples of such systems include pendula, chemical reactions, thermodynamics, astrological systems, and physiology.

Within dynamical systems, there is a class of systems that have nonlinear characteristics, in which a small subset of nonlinear systems is chaotic. Unlike regular dynamical systems, chaotic motions are neither periodic nor do they reach a steady state, rather, they are a state in between. They are complex signals that are often described as wild or random in nature. Classic examples of chaotic behavior include the double-well potential forced oscillator (Moon and Li, 1985) and a double pendulum (Richter and Scholz, 1984). Chaos is a dynamical system that exhibits aperiodic long-term behavior that depends sensitively on the initial conditions of that system. In particular, “aperiodic long-term behavior” delineates trajectories which do not settle down to fixed points, periodic orbits, or quasiperiodic orbits as time goes to infinity. The sensitive dependence on initial conditions describes the phenomenon of nearby trajectories separating exponentially fast, i.e. the system has a positive Lyapunov exponent (Nayfeh and Balachandran, 2004; Strogatz, 1994). In simplest terms, chaos is irregular in time, but has structure in the phase space (Abarbanel, 1996). To understand the underlying properties of a system, it is critical to understand the space it occupies.

Phase Space

A phase space is a finite-dimensional vector space $\mathbb{R}^m$ that contains all the possible states of a system. Each possible state corresponds to one unique point in the phase space and is used to identify the *attractors* in the system. An attractor draws (repels) nearby trajectories toward (away from) itself. Therefore, a set of initial conditions may be attracted to some subset of the phase space as time goes to infinity. There are three main types of attractors: point, limit cycle, and chaotic. A point attractor attracts nearby trajectories to a single point, while limit cycles attract periodic adjacent motions; refer to Figure 2-1. A chaotic attractor is defined by how trajectories diverge in time. If we take two points on the attractor that are only separated by a small distance at $t = 0$, then as t increases these points will move apart from one another exponentially fast. Therefore, a small uncertainty

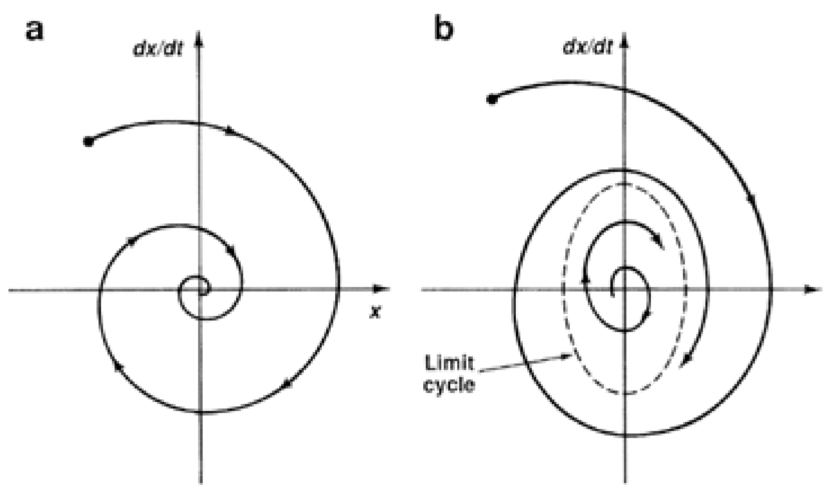

Figure 2-1: Examples of attractors. Fig. 1a is an example of a point attractor, pulling the trajectories around itself to a single point. Fig 1b shows an attracting limit cycle in the phase space. All of the trajectories within and outside the limit cycle are approaching the limit cycle. Figure was taken from (Grebogi et al., 1987)

in the initial conditions of the system will rapidly impede the ability of forecasting the system's future. Additionally, multiple attractors can combine these properties, repelling in one direction and attracting in another (Baker et al., 1996; Grebogi et al., 1987, 1983; Thompson and Stewart, 1986). This creates a unique pattern in the phase space. In classic examples of chaos theory, the phase space is usually a plot of position and momentum as a function of time. However, a phase space can also be reconstructed from a single continuously recorded variable, given that the sampling frequency and number of cycles of the system is sufficient. Nonlinear dynamics and chaos theory attempt to describe and extract features of these systems to understand their behavior and sensitivity to initial conditions (Baker et al., 1996).

Nonlinear Dynamical Analyses

The intrinsic dynamics of linear systems are governed by small causes that lead to small effects, whereas in a nonlinear system, a small cause can lead to disproportionate effects. Nonlinear dynamical analyses are a set of tools used when traditional linear methods fail to accurately represent and interpret data. The reconstruction of the phase space is the critical first step in determining the different features of dynamical systems, such as dimension of attractors, the maximum Lyapunov exponent, and entropy (Kantz and Schreiber, 2004). The phase space is reconstructed using the method of delays (Broomhead and King, 1986; Takens, 1981). For an N-point time series $x(n)$, the phase space can be reconstructed using the following equation:

$$y(n) = [x(n), x(n+\tau), \dots, x(n+(d_E-1)\tau)] \tag{2.1}$$

where τ is the time delay and d_E is the embedding dimension. Thus, creating the d_E-dimensional phase space as an $M \times d_E$ matrix where

$$M = N - (d_E - 1)\tau \tag{2.2}$$

Time delay (τ) is most commonly determined using the first minimum of the average mutual information (AMI) function. The AMI takes nonlinear correlations into account unlike the autocorrelation function. AMI evaluates the amount of information that is shared between data sets over a range of time delays (Fraser and Swinney, 1986). The first minimum of AMI marks the τ where the time shift, $s(t+\tau)$, adds maximal information to the knowledge we have from the original data set, $s(t)$ (Kantz and Schreiber, 2004). More simply the redundancy between the original signal and the time shifted signal is the smallest at that given τ. Embeddings with the same d_E but with different τ are equivalent mathematically when you have a noise free system. But in reality, a good choice in τ facilitates future analysis of the reconstructed phase space. If τ is too small, successive delay vectors are strongly correlated and all vectors, $y(n)$, will be clustered around the diagonal in the phase space. Alternatively, if τ is too large, neighboring elements will be independent, creating a large cloud of points in the phase space which cover the desired deterministic structures that are now confined to small scales (Kantz and Schreiber, 2004).

With time delay established, the embedding dimension is determined using global false nearest neighbors (FNN). FNN compares the distances between neighboring trajectories at increasing dimensions. False neighbors occur when trajectories overlap in a lower dimension but do not overlap in a larger dimension (Kennel et al., 1992). The total percentage of false neighbors decreases as embedding dimension increases, until the proper embedding dimension is reached. This is determined by the false nearest neighbor percentage as it approaches zero or reaches a plateau. An embedding dimension that isn't too small and not too large is ideal. If d_E is too small, trajectories will inevitably overlap in space. Likewise, a larger than necessary d_E is also avoided because the computational cost increases exponentially as d_E increases. And more importantly when noisy signals are used, these extra dimensions are not filled by the system dynamics but with noise (Abarbanel et al., 1993). Therefore, it is prudent to find a sufficient dimension that is not too small nor too large.

LYAPUNOV EXPONENTS

After the phase space has been reconstructed there are numerous analyses that can be performed to quantify stability, complexity, and amount of chaos. Here, we will focus on a measure of stability called the Lyapunov exponent (LyE). The LyE, or the largest LyE, quantifies the sensitivity of a dynamical system to initial conditions. Consider two trajectories with nearby initial conditions in the phase space of a dynamical system. If the attractor of this system is chaotic, then the trajectories will diverge at an exponential rate. This rate of divergence is the LyE. A positive LyE is sufficient for determining the presence of dynamical chaos and indicates local instability in a particular direction. (Bryant et al., 1990) There are several methods for calculating the LyE. (Rosenstein et al., 1993; Sato et al., 1987; Wolf et al., 1985) However, we will focus on the Wolf *et al.* (1985) and Rosenstein *et al.* (1993) algorithms, as they are the primary methodologies used in gait studies. In the following sections, we will detail each of these methods.

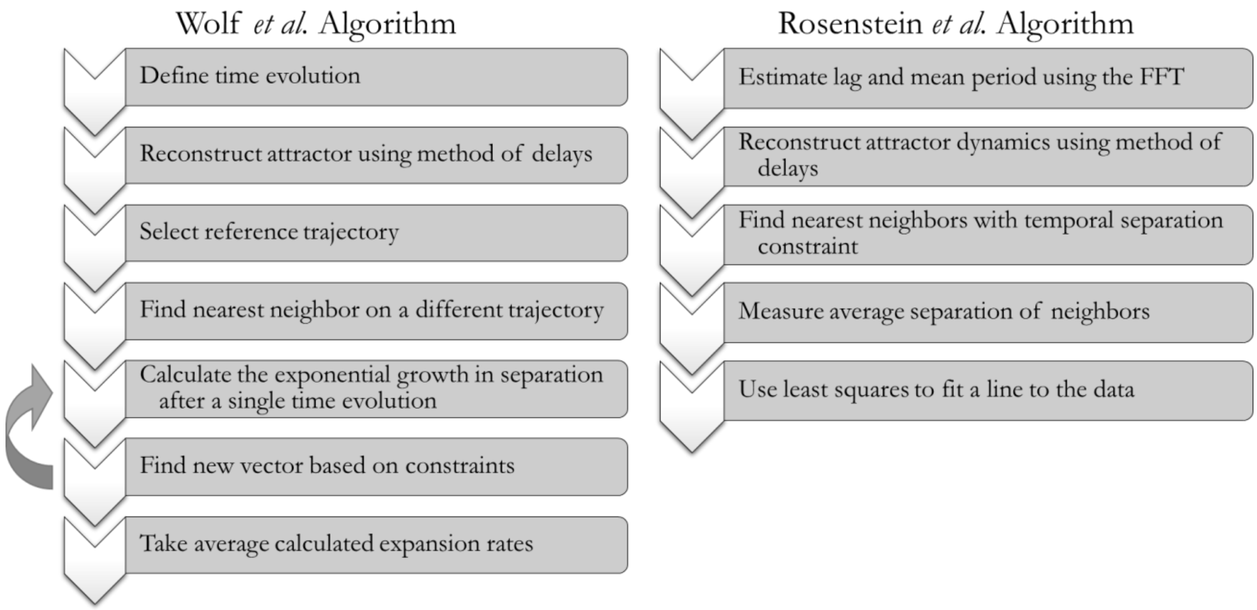

Figure 2-2: Flowchart for calculating the Lyapunov exponent summarizing both the Wolf *et al*. and Rosenstein *et al*. algorithms.

Wolf Algorithm

The first LyE algorithm for time series analysis was suggested by Wolf (1985) et al. This method tracks the average divergence of nearby trajectories in the phase space from a single reference trajectory (i.e. the original time series) to estimate the LyE. A simplified flowchart of this process is shown in Figure 2-2 for reference, while a detailed explanation will be in text. As in all methods for calculating the LyE, the first step is to reconstruct the phase space after selecting the appropriate embedding dimension (d_E) and time lag (τ). The first point of the time series is then chosen as the reference trajectory. Next, the nearest neighboring point on a different trajectory is determined by calculating the Euclidean distance between the reference point and all other points in the attractor. This initial distance between the two points is $L(t_0)$ and then after a time evolution of t_1 the distance becomes $L'(t_1)$. The exponential growth in the separation between these trajectories is calculated using Eq. (2.2)

$$Z_1 = \frac{1}{dt*n}\log_2\left(\frac{L'(t_1)}{L(t_0)}\right) \tag{2.3}$$

where dt is the inverse of the sampling frequency, n is the number of time points that the reference trajectory and the neighboring point are allowed to move through their respective trajectories together before this calculation occurs, and t_1 is equal to $dt * n$. The time evolution, t_1, is set *a priori*. Now a new neighboring vector must be chosen because if time evolution is too large then the distance between the two trajectories may shrink or rapidly expand if they go through a folding region of the attractor. This will lead to either an over or underestimation of the LyE. Let $L(t_1)$ be the distance between the evolved point on the reference trajectory (t_1) and a new vector. The new vector must satisfy two criteria to be chosen: the distance from the reference trajectory must be small and the angular separation between the reference trajectory and the replacement also needs to be small. This is depicted in Figure 2-3.

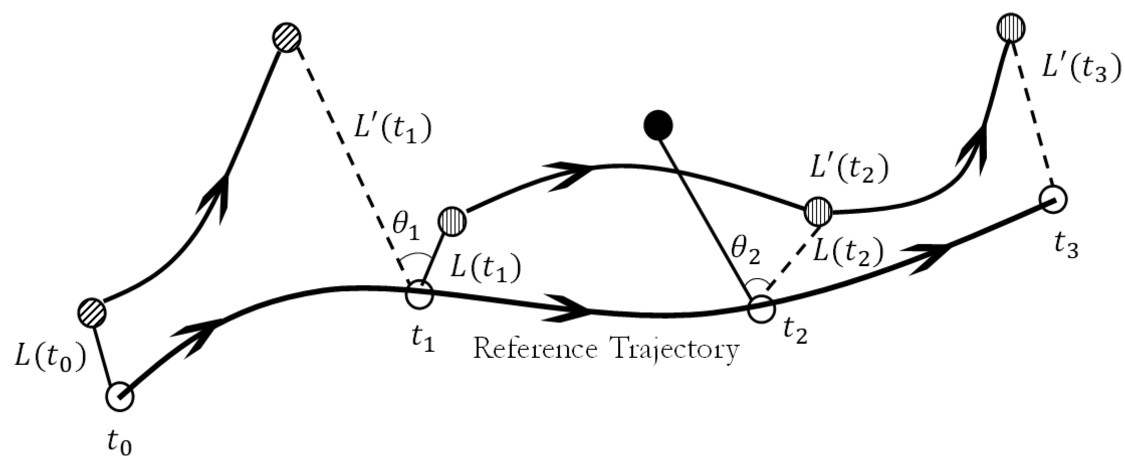

Figure 2-3: Schematic representing the evolution and replacement procedure used in the Wolf algorithm for estimating the LyE. When a new point is being chosen, the replacement length (L) and the orientation angle (θ) are being minimized.

In practice, the new point must be less than the distance SCALMX but still greater than the allowable minimum distance SCALMN between points. Additionally, the angular separation θ between the new point and the evolved point must be small. All of these variables are set *a priori*, SCALMN is usually set to 0.001 while θ is either 0.3 or 0.2 radians. SCALMX and θ are dynamic variables that will change throughout this process in case there is no nearest neighbor at these initial conditions (Wurdeman, 2016). If these conditions are not met by a new vector, the distance limit (SCALMX) is increased stepwise to the upper limit of five times the original limit. And if necessary, the direction limit is then repeatedly doubled to maximally π radians. This procedure is repeated until the reference trajectory has gone through all of the data samples. Then LyE is calculated from the average of the expansion and contraction rates (Z_M) from all time evolutions.

$$\lambda_1 = \frac{1}{M}\sum_{k=1}^{M} Z_k \tag{2.4}$$

where M is the total number of replacements (Wolf et al., 1985).

Rosenstein Algorithm

Rosenstein *et al.* (1993) created a new algorithm for computing the largest LyE. This method was introduced to improve the existing methods that suffered from at least one of the following setbacks: 1) reliability for small data sets; 2) computationally expensive; and 3) relatively difficult implementation. In the Rosenstein *et al.* algorithm, the LyE is calculated as the slope of the mean divergence curve which represents the temporal change of the average natural log-distance between two neighboring points on the attractor. Just as with the Wolf *et al.* algorithm, the phase space is first reconstructed and then the nearest neighbor (X_j) of every point on the reference trajectory (X_i) is found. Nearest neighbors are located by using the Euclidean norm (denoted below as $\| \quad \|$), with the additional constraint that each point must be on a separate trajectory.

$$d_j(0) = \min_{X_j} \left\| X_j - X_i \right\| \tag{2.5}$$

In order to ensure that each nearest neighbor lie on different trajectories, the neighbors must be separated in time by greater than the mean period of the time series.

$$|i - j| > mean\, period \tag{2.6}$$

The mean period of the time series is usually calculated as the inverse of the mean power frequency. This constraint allows for each pair of neighbors to be nearby initial conditions for separate trajectories. The average divergence distance of all possible nearest neighbor pairs is tracked through time creating a mean divergence curve (Figure 2-4). The LyE is then estimated using a least-squares fit to the linear slope of the divergence curve.

$$y(i) = \frac{1}{\Delta t} \left\langle \ln d_j(i) \right\rangle \tag{2.7}$$

where $\langle \quad \rangle$ denotes the average over all pairs of j (nearest neighbor pairs, $j = 1,2, \dots, M$).

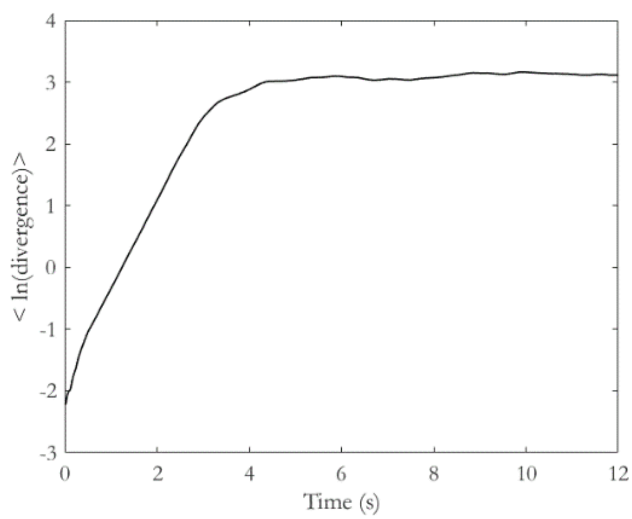

Figure 2-4: Example mean divergence curve of the Lorenz attractor. The slope of the initial linear portion of this graph (between 0.5 – 3s) is used in the Rosenstein algorithm to calculate the LyE.

APPLICATION OF LYAPUNOV EXPONENTS TO GAIT

In gait studies, the LyE is a direct measure of instability. Assessing the dynamic stability has become a prominent approach in human gait and posture research for understanding motor control (Ihlen et al., 2017; Terrier and Dériaz, 2011), as well as a biomarker for different pathologies (Chini et al., 2017; Dingwell et al., 2001; Fino et al., 2018) and adaptations in aging (Granata and Lockhart, 2008a; IJmker and Lamoth, 2012; Kang and Dingwell, 2009), particularly when it comes to fall risk.

The study of fall risk with nonlinear dynamical measures has been a driving force in the application of LyE in gait studies. The LyE, or local dynamic stability, was found to be significantly different between individuals who had fallen one or more times (fallers) and those that have never fallen (non-fallers). These differences extend to gait in different environmental contexts, including treadmill walking (Liu et al., 2012; Lockhart and Liu, 2008; Toebes et al., 2012) and over ground walking (Howcroft et al., 2018, 2016; Reynard et al., 2014; Rispens et al., 2015; Van Schooten et al., 2015). Furthermore, the LyE has been linked to fear of falling (Toebes et al., 2015) – a known risk factor for falls among older adults – and in assessing the effectiveness of various rehabilitation and exercise paradigms to reduce fall risks in this population. For example, Punt *et al.* (2015) utilized LyE to report improvements in gait stability by implementing excessive arm swings. Similarly, Hamacher *et al.* (2016a) assessed the combined cognitive and motor effects attributed to dance programs, and their positive influence on gait stability. Gait stability is measured by LyE and combined with other common fall risk tests (qualitative or quantitative) have shown to improve fall risk identification models (Rispens et al., 2015; Van Schooten et al., 2015). However, not all fall risk studies have reported significant differences between these two populations. And there are conflicting reports on which signal direction – vertical (VT), anteroposterior (AP), or mediolateral (ML) characterizes the largest difference in gait stability. Some researchers reported differences in all directions (Van Schooten et al., 2015), while others found differences only in the VT (Lockhart and Liu, 2008), AP (Howcroft et al., 2016), and ML (Bizovska et al., 2018a; Huijben et al., 2018) directions..

Accordingly, gait stability has proven useful in the early identification and prediction of neurological conditions, such as multiple sclerosis and Parkinson's disease. LyE have been used to identify fall risk patients within these cohorts (Fino et al., 2018; Huisinga et al., 2013; Tajali et al., 2019). Generally, fallers with these conditions have greater instability, and therefore, larger LyE compared to their non-faller counterparts. The application of LyE has also grown to identifying different pathologies within the general population. Patients with various gait disorders (e.g. stroke, multiple sclerosis, cerebral palsy, and traumatic brain injury) were shown to have greater instability. Additionally, people who have been diagnosed with dementia (IJmker and Lamoth, 2012), unilateral vestibular hypofunction (Liu et al., 2017), developmental coordination disorder (Speedtsberg et al., 2018), and degenerative cerebellar ataxia (Chini et al., 2017) have significantly different stability compared to healthy aged-match controls.

The application of LyE in gait has grown beyond just identifying fall risk individuals. The LyE is used to study how different environmental and physical conditions affect an individuals' ability to walk. Kibushi and colleagues studied how muscle synergies and coordination during gait respond to changes in gait speed (Kibushi et al., 2018). They determined that different muscle synergies have different LyE which might depend on the required motor output of specific subtasks within a gait cycle. And in a follow up study, they found that larger LyE correlated with fast walking speeds and very short stride lengths. Thus indicating that these conditions have more instability than walking at a slower pace and using smaller stride lengths (Kibushi et al., 2019). This could explain why as people age, they naturally adapt a slower pace and take smaller steps. Other outside influencers of gait such as types of flooring (Chang et al., 2010; Kim et al., 2018), walking while listening to music or television (Sejdić et al., 2013), and even walking while texting (Hamacher et al., 2016b) have found that LyE can be used to find differences in walking conditions. Physical exertion has also become an area of interest but has found conflicting results when using LyE. Hamacher *et al.* (2018) and Kao *et al.* (2018) did not find that exhaustion effected LyE during normal walking unlike previous literature that reported both an increase (Hamacher et al., 2016c) and a decrease (Vieira et al., 2016) in LyE after a fatiguing protocol. It is important to note that some studies used treadmills while one did not, and all studies used signals from different locations as well as various types of signals (velocity, acceleration, etc.).

Outside of intrinsic gait differences due to various patient populations and aging, LyEs are now being used to assess the recovery from various injuries and surgeries. Concussions affect 1.6 - 2.8 million people in the United States every year (Langlois et al., 2006). Concussed athletes had greater dual task costs when assessed using LyE than healthy athletes (Fino, 2016; Fino et al., 2016). This reflects as a reduced response by the neuromuscular control system to local perturbations. Additionally, this deficit persisted longer than the standard 1-2-week symptom directed return-to-play progression; raising the concern about the athlete’s well-being and risk of injury after players have returned to competition. Other injuries, such as anterior cruciate ligament (ACL) deficiency or ACL repairs have found LyE is useful for assessing various therapy and rehabilitation programs. Nazary-Moghadam *et al.* (2019) found that as gait speed increased in healthy subject the LyE showed no changes, but significantly decreased for subjects with ACL deficiency. This suggested that during rehabilitation, cognitive load task and high-speed walking should be used to challenge the knee. The LyE has also been used to assess the recovery of patients after ACL reconstruction surgery. One study found that six months of physiotherapy was effective in improving knee stability, but was not sufficient for a complete recovery (de Oliveira et al., 2019). And another, found that even after two years, knee stability was still reduced in surgical patients, regardless of graft type used in the surgery, compared to healthy controls (Moraiti et al., 2010). They postulated that the ACL reconstruction led to an altered gait variability instead of restoring it to its previous optimal variability and stability. Thus, the LyE as a measure of gait stability can be used to evaluate injury recoveries and the effectiveness of rehabilitation therapies.

The breadth of application of the stability of gait using LyE to the study of gait, falls, and rehabilitation cements its importance in the literature. Although LyE has been used for studying gait *in*stability across multiple populations and in many different paradigms, there is a common theme that not all of these studies are comparable. Some studies use different data collection equipment, algorithms, and/or normalization methods. And even when publications research similar paradigms, some studies find significant differences while others do not. This could be due to sample and effect size within particular studies, but the inconsistency across publications could also be due to the lack of a universal methodology for calculating the LyE during gait. These variations in calculations also hinder comparisons across publications and populations, as well as, prevent meta-analyses.

VARIATIONS IN THE CALCULATION OF THE LYAPUNOV EXPONENT FOR GAIT DATA

To date, there has been several pivotal publications about the issues in calculating the LyE when using gait data from issues with how to reconstruct the phase space, which normalization methods to use, choice of algorithm, and amount of data length. Each of these factors can affect the final value of the LyE. However, these studies have not all been done using a single data collection method but under multiple, e.g. motion capture using position, velocity, or joint angles or accelerometers. Each of these issues will be discussed in detail in the following sections.

Data Collection Equipment

The effect of gait speed has been extensively studied but has conflicting conclusions based on different factors, e.g. data collection method, algorithm, and preprocessing methods used. This illustrates the importance of standardization when calculating the LyE when studying gait. A linear relationship between decreased instability (lower LyE) and lower gait speeds were found using the Rosenstein algorithm with trunk (Bruijn et al., 2010; Dingwell and Marin, 2006; Kang and Dingwell, 2008) and joint velocities (England and Granata, 2007) when recorded using motion capture systems. However, Bruijn *et al.* (2009a) only found this linear relationship in the AP direction, while the VT and ML directions had a quadratic (inverted U-shaped) effect as walking speed increased. When accelerometers are used, regardless of algorithm choice, the LyE decreased as gait speed increased (Bruijn et al., 2010; Huijben et al., 2018; Punt et al., 2015; Raffalt et al., 2017; Stenum et al., 2014). Raffalt *et al.* (2017) examined the effect of gait speed on the lumbar acceleration using the W-algorithm and found that the LyE was larger at gait speeds slower than the subjects' preferred walking pace in all directions. Stenum *et al.* (2014) found that the relationship between gait speed and the LyE when using the R-algorithm was dependent on if the original time-series was time-normalized or not and if the divergence curve was rescaled to time in seconds or left in units of stride-time. The study found that if gait data was not time-normalized and the divergence curve was rescaled based on the average stride duration, gait speed would have no effect on the LyE in the VT or ML direction.

It is important to note that the difference in relationships between the LyE and gait speed due to motion capture and accelerometer data found in both young healthy subjects (used in most studies) and healthy elderly adults (Huijben et al., 2018; Kang and Dingwell, 2008). But not all populations share this relationship as demonstrated by Craig *et al.* (2019) with patients diagnosed with multiple sclerosis. In this dissertation, we will be concentrating on the standardization of inertial measurement units (IMUs) because they have become widely used in assessing and monitoring gait and other daily living activities as an alternative to traditional motion capture systems. Even though modern motion capture laboratories collect precise data during walking and postural stability tasks they are prohibitively expensive, immobile, and require well trained technicians to collect and process experimental results. IMUs on the other hand are more flexible, mobile, and inexpensive. They also have the advantage of unlimited measurement volume and the opportunity of recording gait in various environments – e.g. clinical offices, community centers, or outdoor tracks – with ease. (Tao et al., 2012) The validation of gait assessments when using IMUs (Bruijn et al., 2010; Mundt et al., 2019) have made it possible to record average daily life activities for several days and up to a week at a time in large-scale studies. (Punt et al., 2016; Van Ancum et al., 2019) This has helped further establish the relationships between dynamic stability and fall prone individuals. (Bizovska et al., 2018a; Van Schooten et al., 2015) IMUs have also been used to calculate the LyE as a biomarker for various ailments, e.g. patients with dementia (IJmker and Lamoth, 2012), multiple sclerosis (Huisinga et al., 2013), Parkinson's disease, (Fino et al., 2018) and concussions (Fino, 2016). Thus, as IMUs become the more prominent method of collecting gait data, it is important to standardize the protocol for calculating the LyE using this signal.

Phase Space Reconstruction

When calculating the LyE, regardless of algorithm choice, the first step is to recreate the phase space. Phase space reconstruction requires *a priori* inputs of time delay (τ) and embedding dimension (d_E). In the literature, a range of time delays from 6 to 30 and embedding dimensions of 5 to 7 or more have been used. (Dennis Hamacher et al., 2015; Mehdizadeh, 2018) The first study to test if reconstruction had an impact on the LyE was van Schooten *et al.* (2013). They explored the intra- and inter-day reliability of four different reconstruction methods. They found that using the median τ and d_E calculated from average mutual information and global false nearest neighbors, respectively, for all subjects improved the within and between-session reliability of the LyE over individualized values. This relationship has been found for accelerometer and motion capture data, irrespective of algorithm choice. (Raffalt et al., 2017, 2018a; van Schooten et al., 2013) Since group median and/or mean values have shown to have better reliability, can an arbitrary (yet sufficient) τ or d_E be used as the standardized value for these parameters? To the author's knowledge, no study has investigated this or has systematically examined if time delay and embedding dimension affect the value of the LyE itself.

Algorithm Choice

One of the most prominent methodological divides in estimating the LyE in gait data is the algorithm that is used for its computation. As previously mention, the Wolf *et al.* (W-algorithm) and Rosenstein *et al.* (R-algorithm) algorithms are the two main algorithms used in the literature. The R-algorithm was utilized in 79% of publications, while the W-algorithm is only used in 15% (Mehdizadeh, 2018). There have only been a handful of studies that have used both algorithms. When comparing these algorithms again known nonlinear systems (i.e. Lorenz and Rossler systems), the R-algorithm had equal to or greater precision than the W-algorithm (Cignetti et al., 2012a; Rispens et al., 2014a), regardless of signal length. In gait studies, it has been found that the different algorithms perform better with specific normalization methods (Raffalt et al., 2019) and when different signal types (Raffalt et al., 2018a) (linear or angular displacement) are being investigated. When looking at studies that used IMUs in particular, the difference between algorithms were secondary comparisons. One study evaluated the effect of sensor placement and found that the LyE was robust against sensor misplacement or replacement when it was placed along the mid to lower back, regardless of which algorithm was used to calculate it (Rispens et al., 2014b). This study reported that the W-algorithm had better correlations between locations, but the R-algorithm had smaller standard deviations. In a separate study, the difference between laboratory-based gait assessments on a treadmill were compared to recording daily life activities over the course of a week. Significant differences were found between these gait assessments for both algorithms, but on the W-algorithm found correlations between these tests (Rispens et al., 2016). The correlation was interpreted as having more common information between the laboratory and daily life gait. But ultimately this study endorsed neither algorithm. In both of these IMU studies, the standard deviations of the R-algorithm were smaller (0.05-0.1) compared to the W-algorithm (0.14-0.32). This indicates that the R-algorithm still has greater precision, but more research is needed to evaluate which algorithm performs better when accelerometers are utilized.

Data Length

The first extensive look that the effect of measurement length on the precision and sensitivity was performed by (Bruijn et al., 2009b). They collected 20 minutes of gait data using a motion capture system and analyzed the velocity of the upper trunk. While varying the number of included strides from 30 to 300 strides, they found that the LyE increased as data length increased. They also found that the standard deviation of the LyE decreased at longer data lengths, implying better precision with longer data sets but the gain in precision is limited after 150 gait cycles are used. They concluded that a fixed number of strides should be used when comparing between subjects and across groups or treatment level because of the large effect of data length on the LyE. The increase in the LyE as data length increases has been confirmed by other studies (Cignetti et al., 2012b; Kang and Dingwell, 2006; Raffalt et al., 2018b; Reynard and Terrier, 2014). However, significant differences were not always found between the smaller data lengths studied (Kang and Dingwell, 2006; Raffalt et al., 2018b; Reynard and Terrier, 2014). Collecting large data sets of gait can be a challenge when the age, fitness, and health of different populations can limit an individuals' ability to walk for longer periods of time. Therefore, other data length research has focused on finding the minimum number of required strides (F. Riva et al., 2014) while still maintaining measurement reliability, as well as investigating if shorter but multiple trials of gait can be utilized instead of a single long continuous walk (Van Schooten et al., 2014).

However, there are still holes in literature with respect to the effect of data length when using IMUs for both the R- and W-algorithms. As the use of the Wolf algorithm with IMUs is increasing, it is prudent to determine the limits of its use with respect to data length and under different normalization methods. Different normalization methods have been used in each of the studies mentioned above; some have used a pure number of data points (Cignetti et al., 2012b; Kang and Dingwell, 2006) while others have time normalized their data to approximately 100 samples per stride (Bruijn et al., 2009b; Reynard and Terrier, 2014). It is currently unknown if preprocessing methods affect the relationship between the LyE and data length, as it has with gait speed (Stenum et al., 2014).

CHAPTER 3: EFFECT OF DATA LENGTH ON TIME DELAY AND EMBEDDING DIMENSION FOR CALCULATING THE LYAPUNOV EXPONENT IN WALKING

ABSTRACT

The Lyapunov Exponent (LyE) is a trending measure for characterizing gait stability. Previous studies have shown that data length has an effect on the resultant LyE but the origin of why it changes is unknown. This study investigates if data length affects the choice of time delay and embedding dimension when reconstructing the phase space, which is a requirement for calculating the LyE. The effect of three different preprocessing methods on reconstructing the gait attractor was also investigated. Lumbar accelerometer data were collected from ten healthy subjects walking on a treadmill at their preferred walking speed for 30 minutes. Our results show that time delay was not sensitive to the amount of data used during calculation. However, embedding dimension had minimum data requirements to determine the steady state value of the embedding dimension. This study also found that preprocessing the data using a fixed number of strides or a fixed number of data points had significantly different values for time delay compared to a time series that used a fixed number of normalized gait cycles, which have a fixed number of data points per stride.

INTRODUCTION

The Lyapunov exponent (LyE) is a nonlinear dynamical calculation that quantifies the rate of divergence or convergence of trajectories in an *n*-dimensional phase space. The phase space shows all of the possible trajectories for a dynamical system and is used to identify all of the possible attractors of the system. An attractor draws (repels) nearby trajectories toward (away) from itself, where multiple attractors can combine these properties, repelling in one direction and attracting in another (Baker et al., 1996; Grebogi et al., 1987, 1983). LyE, or local dynamic stability, is a popular approach to assess and enumerate an individual's ability to withstand small perturbations during gait. This nonlinear measure has been used to differentiate between healthy and fall prone elderly (Lockhart and Liu, 2008; Toebes et al., 2012), as well as, used to identify differences between healthy controls and patients with Parkinson's disease (Fino et al., 2018), and developmental disorders (Speedtsberg et al., 2018).

Multiple studies have found that the amount of gait data used when calculating the LyE affects the final outcome (Bruijn et al., 2009a; England and Granata, 2007; van Schooten et al., 2013). Previous studies on reliability of LyE have found different data minimum requirements; some required 54 and 150 strides [11,12] while others state a time duration minimum of 2-3 minutes of walking data (Cignetti et al., 2012c; Kang and Dingwell, 2009) is sufficient. However, no studies have investigated if data length plays a role in selecting the reconstruction parameters required for calculating LyE.

The first step in calculating the LyE is reconstructing the collected time series into the phase space so the gait attractor can be analyzed. The phase space is reconstructed using the method of delays (Broomhead and King, 1986) :

$$y(n) = [x(n), x(n+\tau), \ldots, x(n+(d_E-1)\tau)] \tag{1}$$

which requires a time delay, τ, and an embedding dimension, d_E. The time delay is most commonly (Fraser and Swinney, 1986) determined using the first minimum of average mutual information (AMI) function, which evaluates the amount of information that is shared between data sets over a range of time delays. With time delay established, the embedding dimension is then determined using global false nearest neighbors (FNN). FNN compares the distances between neighboring trajectories at increasing dimensions. False neighbors occur when trajectories overlap in a lower dimension but do not overlap in a larger dimension (Kennel et al., 1992). The total percentage of false neighbors declines as embedding dimensions increase until the proper embedding dimension is reached. This is usually determined by the false nearest neighbor percentage as it either approaches zero or plateaus out.

In addition to the varying data lengths being utilized, previous studies have also applied different preprocessing methods for gait time series normalization. This has also been found to have an effect on the calculation of LyE (Stenum et al., 2014). We have identified three major methods in the gait literature:

1) Fixed number of strides with variable number of total data points (Myers et al., 2011) – The time series will start and end on a heel contact, but each stride will contain a variable number of data points. This method maintains the distance between points on the attractor.
2) Fixed number of strides and data points per stride (Bruijn et al., 2009a; England and Granata, 2007; Van Schooten et al., 2014) – The time series is time-normalized to 100 samples per stride. This method alters the distance between data points within the phase

space but the number of points in each stride cycle is constant across subjects irrespective of gait speed.

3) Fixed number of data points, with a variable number of strides (Dingwell and Marin, 2006; Raffalt et al., 2019) – The time series starts at the same as methods 1 and 2 at a heel contact however, the end point is a fixed number of points regardless of the number of gait cycles it contains. This method also maintains the distance between points on the attractor but does not guarantee ending on a full cycle.

The aim of this study was to determine the effect of data length on the reconstruction parameters of the LyE, specifically the τ and d_E determined by AMI and FNN, respectively. We hypothesize that τ and d_E will not change with respect to data length given sufficient data is provided. Additionally, we investigated the effects of three data preprocessing methods on determining time delay and embedding dimension.

MATERIALS AND METHODS

Participants

Ten young health subjects (5 males and 5 females) with a mean $\pm$ standard deviation age of 24.5 $\pm$ 4.1 years, body height of 1.67 $\pm$ 0.10 meters, and body mass of 69.4 $\pm$ 11.6 kg were included in this study. All subjects were physically active and familiar with walking on a treadmill. Subjects reported no cardiovascular issues, neurological diseases, nor lower extremity surgeries in the last 3 months. Subjects provided written informed consent before participating in this study. This study was approved by the Institutional Review Board of Arizona State University.

Experimental Procedure

After subjects became familiar with the treadmill, each subject's preferred walking speed (PWS) was determined using a standardized protocol (Dingwell and Marin, 2006). The mean and standard deviation of PWS was 1.13 $\pm$ 0.1 m/s. After a short rest period, each subject walked on the treadmill for 30 minutes at their PWS. Participants wore three tri-axial acceleration sensors sampling at 128 Hz (APDM, Mobility Lab, APDM, Inc., Portland, OR) fitted with elastic bands and Velcro straps and were placed at each ankle and the lower lumbar around vertebrae L4 and L5. For this study, the ankle sensors were used to define heel contacts for truncating the gait data as necessary. A custom algorithm based on previously published algorithms (Norris et al., 2016; Pan and Tompkins, 1985) were used to define heel contacts. The lumbar sensor was used for reconstructing the phase space and calculating the LyE. The treadmill used in this experiment is a split-belt treadmill and is a part of the GRAIL system (Motekforce Link, Amsterdam, The Netherlands). Measurements were started after the treadmill and the subject were at a constant speed.

Three-dimensional acceleration data of the lumbar sensor were used for all of the calculations in this paper. The heel contacts for each step were determined and indexed and the time series was truncated to start and end on a heel contact (Dingwell et al., 2001; England and Granata, 2007). To investigate how different methods of preprocessing affect the calculation of time delay and embedding dimension, three different methods that are used in nonlinear dynamical calculations for gait were implemented:

1. Fixed number of strides with a variable number of points per stride
2. Fixed number of strides with 100 data points per stride
3. Fixed number of data points

These methods were applied to the vertical (VT), anterior posterior (AP), and mediolateral (ML) acceleration time-series and no other filtering/normalization methods were used. After the data was preprocessed, different sample lengths ranging from 30 to 500 strides were extracted from the same first heel contact of the time series. This was repeated for each acceleration direction. The data lengths selected for method 3 were based on 15, 30, and 60 seconds and 2, 3, 5 and 10 minutes of gait data. This range includes smaller and larger data collection times as well as very common data collection times of one to three minutes of data. All calculations were done using custom made MATLAB (version 2018b, Mathworks Inc., Natwick) programs.

Simulated Data

We simulated the Lorenz and Rössler attractors because they are well known dynamical systems and they are similar to human posture and gait data, respectively. The Lorenz system has a pronounce non-periodic behavior which may be considered representative for postural sway, while the Rössler system has a periodic behavior which is more comparable to gait. (Rispens et al., 2014) The systems, based on the differential equations and initial conditions outlined in Table 3-1, were simulated using MATLAB. Each nonlinear attractor was generated with 1×10^6 samples, where the first 8000 samples were discarded to avoid transient confounders with each time series. Each time series was then segmented into non-overlapping windows that each contained 5×10^4 samples. Ten of these windows were used in the subsequent analyses for both the Lorenz and Rössler attractors. To investigate the effect of data length, various data lengths was extracted from each window ranging from 2×10^3 to 7.7×10^4 samples. This range was used to mimic the data lengths extracted from the gait data using method 3 (data truncated based on a specific number of samples).

Table 3-1: Reference table for known chaotic dynamical systems. Values from (Rosenstein et al., 1993)

System	Equations	Parameters	Δt	Expected λ_1
Lorenz[a]	$\dot{x} = \sigma(y - x)$ $\dot{y} = x(R - z) - y$ $\dot{z} = xy - bz$	$\sigma = 16.0$ $R = 45.92$ $b = 4.0$	0.01	1.50
Rössler[b]	$\dot{x} = -y - z$ $\dot{y} = x + ay$ $\dot{z} = b + z(x - c)$	$a = 0.15$ $b = 0.20$ $c = 10.0$	0.10	0.090

[a] Wolf et al., 1985 [b]Rossler, 1976

Data Analysis

Time delay, τ, was determined as the first local minimum of the AMI function.(Fraser and Swinney, 1986) A time delay was determined for each directional acceleration as data length was varied for the simulated and collected data. The τ determined from AMI at 1×10^4 samples for known systems and 300 gait cycles or 1.5×10^4 data points for gait data. FNN(Abarbanel and Kennel, 1993; Kennel et al., 1992) was then used to determine the appropriate embedding dimension, d_E, using values of R_{tol} = 15 and A_{tol} = 4. These threshold values for within the FNN algorithm are within the suggested ranges set by Kennel et al.(Kennel et al., 1992) The final selection of the d_E is generally up to the discretion of the researcher where the FNN starts plateauing out. Therefore, to objectively select the d_E we added the following criteria: 1) the difference between subsequent dimensions must be less than 0.05; and 2) the actual percentage of FNN at that dimension must also be less than 10%. This method is depicted in Figure 3-1. These decision criteria were used for both the Lorenz system and all gait data collected. However, the second criterion had to be increased to 0.20 for the Rössler system because some subjects, at certain time epochs, never dropped below a 10% false nearest neighbors' rate.

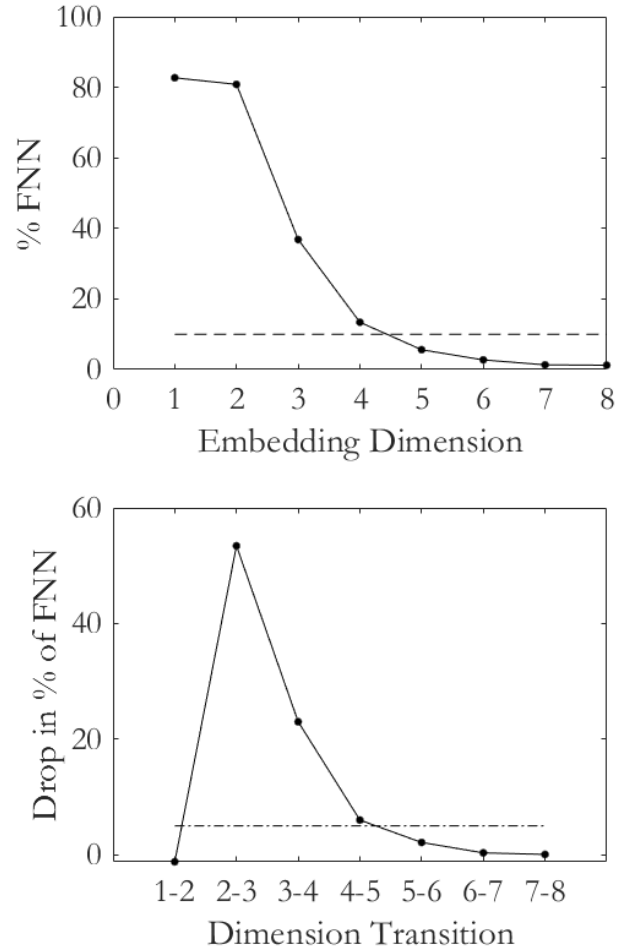

Figure 3-1: Methodology used to objectively select the embedding dimension. Top is the output of FNN. The Bottom figure was created by finding the difference between neighboring dimensions, each named for the transition they represent. The first criterion is found when the difference between dimensions is less than 0.05, displayed as the dash-dot line in the Bottom figure. For example, this point would be the 5-6 dimension transition. The second criterion then checks that dimension 5 has less than 10% FNN rate.

Statistical Analysis

To explore the effect of within-subject data length effects on τ and d_E, a one-way repeated measures ANOVA was done separately for all three gait methods and each simulated nonlinear system. A Wilcoxon Signed Rank test was used to determine the differences between preprocessing methods. For all statistical tests a p-value < 0.05 was considered significant. All statistical analysis was performed in JMP Pro (Version 14, SAS, Cary, NC).

RESULTS

Simulated Systems

There was no statistical effect of data length on τ for the Lorenz attractor in any direction. The Rössler attractor had significant differences only in the y-direction (F = 6.2509, p < 0.0001). However, a post-hoc Tukey test revealed no significant differences between any specific time epochs.

In the Lorenz (F = 29.22, p < 0.0001; F = 81.00, p < 0.0001; F = 21.00, p < 0.0001 for x, y, and z, respectively) and Rössler attractor (F = 39.86, p < 0.0001; F = 81.00, p < 0.0001; F = 2.81, p = 0.0130 for x, y, and z, respectively) data length did have an effect on the d_E. A post-hoc Tukey test was used to determine the minimum number of data points necessary to determine a consistent d_E for each directional vector in each known system.

Gait

When looking at the collected gait data, data length did not have an effect on τ in any direction, regardless of preprocessing methods. Just like with the known systems, data length did have an effect on the d_E. We found significant differences between d_E at different data lengths when using preprocessing method 1 (repeated measures ANOVA, F = 3.60, p = 0.0039; F = 16.12, p < 0.0001; F = 19.59, p < 0.0001 for VT, AP, and ML, respectively), method 2 (repeated measures ANOVA, F = 8.02, p < 0.0001; F = 10.84, p < 0.0001; F = 15.44, p < 0.0001 for VT, AP, and ML, respectively), and method 3 (repeated measures ANOVA, F = 7.01, p < 0.0001; F = 32.31, p < 0.0001; F = 17.55, p < 0.0001 for VT, AP, and ML, respectively).

Post hoc testing using a Tukey test revealed significant differences between the shorter and longer data set sizes. This was used to determine data minimums for selecting the steady state d_E.

Table 3-2: Statistical differences, p-values, in time delay values between different preprocessing at each comparable time epoch in gait cycles (GC) and number of data points (DP). The average number of data points in for each given number of gait cycles before processing was used to match the GC to its nearest DP length pair.

	VT	**AP**	**ML**
Method 1 vs Method 2			
30 GC	0.18	0.004	0.03
50 GC	0.06	0.02	0.22
100 GC	0.05	0.002	0.15
150 GC	0.05	0.006	0.23
200 GC	0.05	0.02	0.29
300 GC	0.06	0.002	0.06
500 GC	0.05	0.008	0.19
Method 2 vs Method 3			
30 GC vs 5 x 10^3 DP	0.32	0.004	0.03
50 GC vs 7.5 x 10^3 DP	0.05	0.02	0.25
100 GC vs 15 x 10^3 DP	0.05	0.002	0.18
200 GC vs 23 x 10^3 DP	0.05	0.04	0.30
300 GC vs 38 x 10^3 DP	0.06	0.002	0.06
500 GC vs 77 x 10^3 DP	0.002	0.01	0.19

p < 0.05 is significant

Preprocessing Methods

We found significant differences ($p < 0.001$) between the values of τ derived from preprocessing method 1 (fixed number of strides) and method 2 (fixed number of strides with 100 pts/stride). Significant differences were also found between preprocessing method 2 and method 3 (fixed number of data points). These differences were further broken down by data length and significant differences at each time epoch are shown in Table 3-2. Method 1 and 3 found similar τ values for each time epoch with no statistical differences between these methods.

There were significant differences comparing d_E values between preprocessing method 1 and 2 (Wilcoxon ranked: $p = 0.02$ and $p = 0.002$, for AP and ML directions, respectively) and methods 2 and 3 (Wilcoxon ranked: $p = 0.003$; $p = 0.01$ for AP and ML directions, respectively). However, the differences between these methods were at the group level only. No significant differences were found when time epochs were compared independently for both of the above comparisons between methods 1 – 2 and methods 2 – 3. This can be explained by the distribution of smaller time epochs having smaller d_E compared to larger time epochs in each preprocessing method. Therefore, we do not believe that preprocessing method has an effect on the embedding dimension itself.

DISCUSSION

The time delay and embedding dimension are critical inputs for reconstructing the phase space (Kantz and Schreiber, 2004) which is the first step in calculating the LyE. A previous study (Bruijn et al., 2009b) found that LyE increases as data length increases. The specific aspect of the LyE calculation that is sensitive to data length is still unknown. Therefore, this paper investigated the role of data length in the calculation of τ and d_E. Time delay and embedding dimension were calculated using AMI and FNN, respectively. We found that τ is not affected by data length, while d_E is underestimated without sufficient data for its calculation. Additionally, this paper found that stride normalization (method 2) has statistically different τ values compared to gait data that has not been normalized (method 1 & 3). Method 2 generally had smaller τ values in VT and ML directions but had larger values in the AP direction.

As hypothesized, the τ from the Lorenz and walking data does not change as data length increases, regardless of the directional vector. The Rössler system, however, was affected, but only in the y-directional time series. Of the simulated systems used in this study, the Lorenz attractor had an average coefficient of variation (CV) of 1.55% between simulated subjects across all directions and converged on a time delay of 11 points– the optimal value within a one-point range, as reported previously (Rosenstein et al., 1993). The Rössler attractor was highly variable subject to subject with average CV of 4.2%, 3.6%, and 9.63% in the x, y, and z direction, respectively. But once a data length of 7.5×10^3 points or greater was used, a τ of 15, 16, and 11 with a similar range, shown in Figure 3-2, was established in the x, y, and z direction.

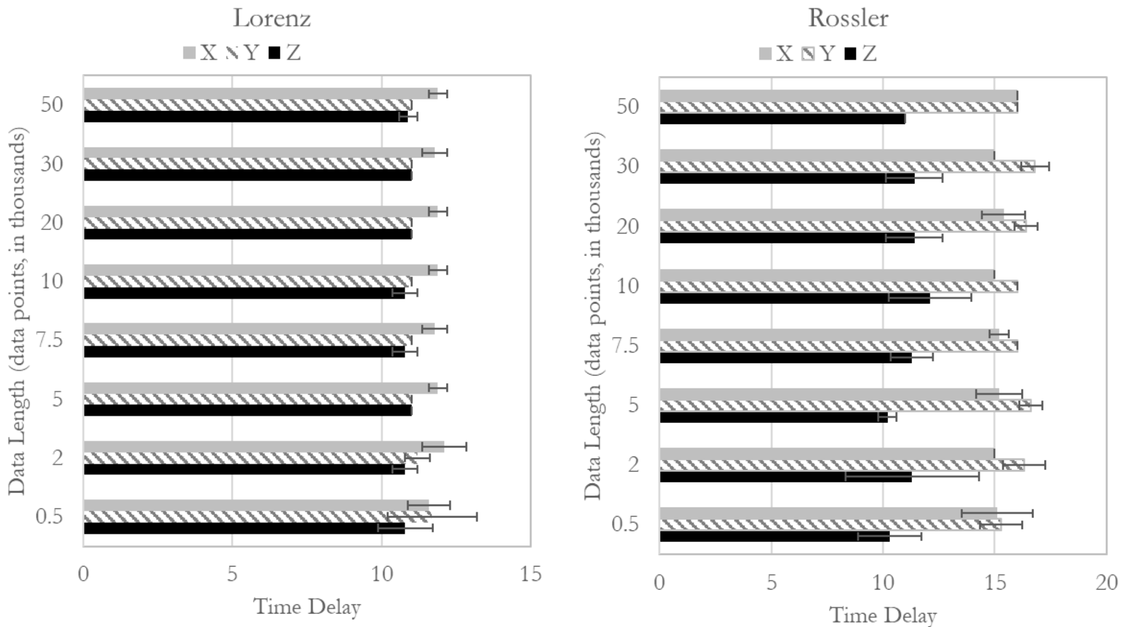

Figure 3-2: Mean (SD) of calculated time delay in known dynamical systems as data length is increased.

Accelerometer gait data maintains this small variance in τ as data length increases in the VT and ML directions, with median CV of 0% using method 1 and method 3 while a larger variance of 8.8% and 9.5% occurs in method 2 (fixed number of strides with 100 points per stride). The AP acceleration in method 1 (fixed number of strides) and method 3 (fixed number of points) had a 10.5% and 9.2% median CV as data length was changed, seen in

. Method 2 greatly reduced the median CV to 0% in the AP direction. It is possible the amount of variation in the AP direction may be an artifact of walking on a treadmill itself. This is because having a consistent time delay across data length is largely subject-dependent; 7 subjects revealed a consistent time delay, while 3 subjects didn't. This could be due to an individual subject's difficulty with finding a consistent pace on the treadmill; e.g., their strides change between different time epochs. Alternatively it may result from position changes, i.e., from the center to the top of the treadmill or vice versa. This irregularity in AP time delays is mitigated when preprocessing method 2 is applied, because every stride is normalized to 100 points per stride. Stride time normalization alters the time and distance relationships within the phase space. It is important to note that this preprocessing method does have significant effect on the value of τ. There is a 37%, 63%, and 31% difference between the median time delay values found between method 1 and method 2 for each direction respectively.

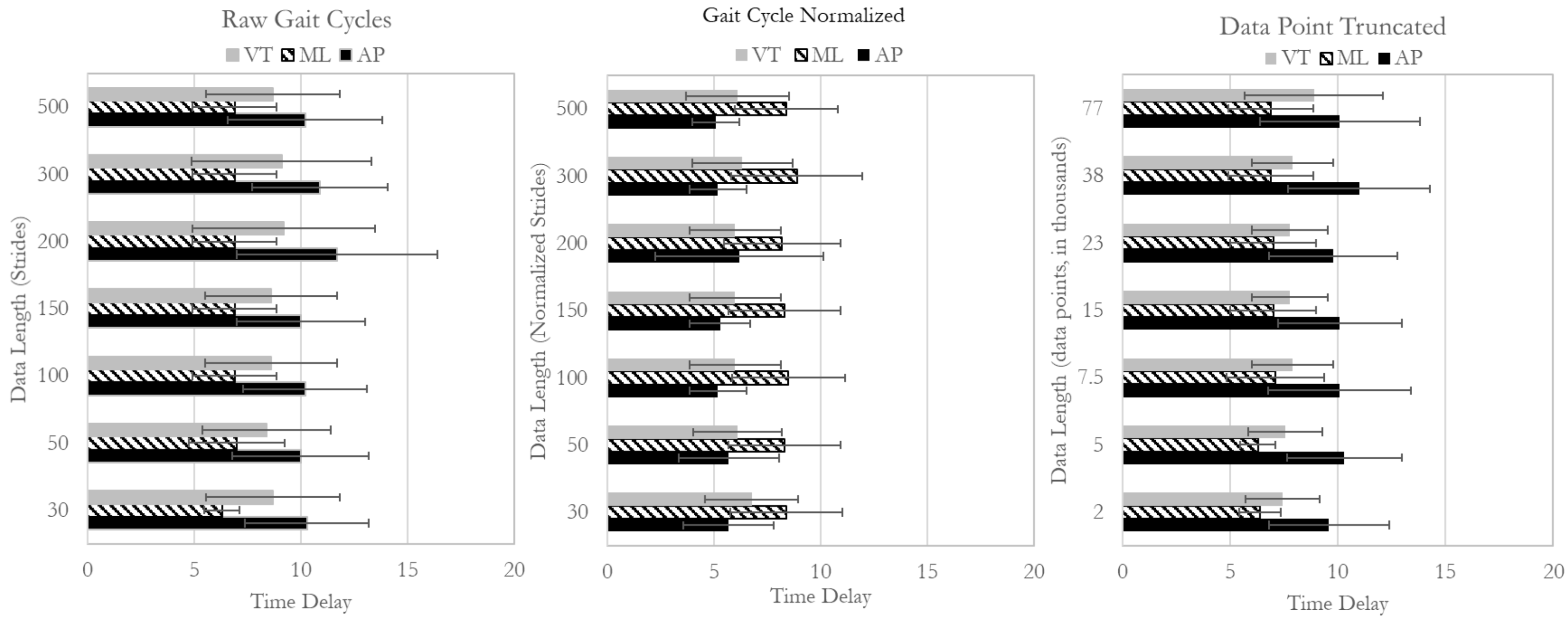

Figure 3-3: Mean (SD) of time delay when calculated with different data lengths and preprocessing methods for every signal direction: vertical (VT), mediolateral (ML), and anteroposterior (AP).

Time delay in gait is not as uniform as in the known dynamical systems. The known systems had a single point range about the mean τ, once a sufficient amount of data was used. In gait the τ ranged from 4 to 16 across all subjects while the Lorenz and Rössler simulated subjects' time delay ranged from 10 to 12 and 11 to 16, respectively. This larger range is expected due to the individual gait differences. However, this does beg the question, can the same time delay be used for every subject as well as for each acceleration direction? The majority of publications that calculate the LyE for gait use a single time delay for every subject (Mehdizadeh, 2018). Although one paper has looked at some of the differences between individualized and a pre-selected fixed time delay, the researchers were specifically investigating the intra patient reliability of LyE (van Schooten et al., 2013) and only in the ML direction. A more in-depth study into how underestimating or overestimating the τ in the LyE calculation is needed to understand its importance and contribution to the reliability of the LyE for gait.

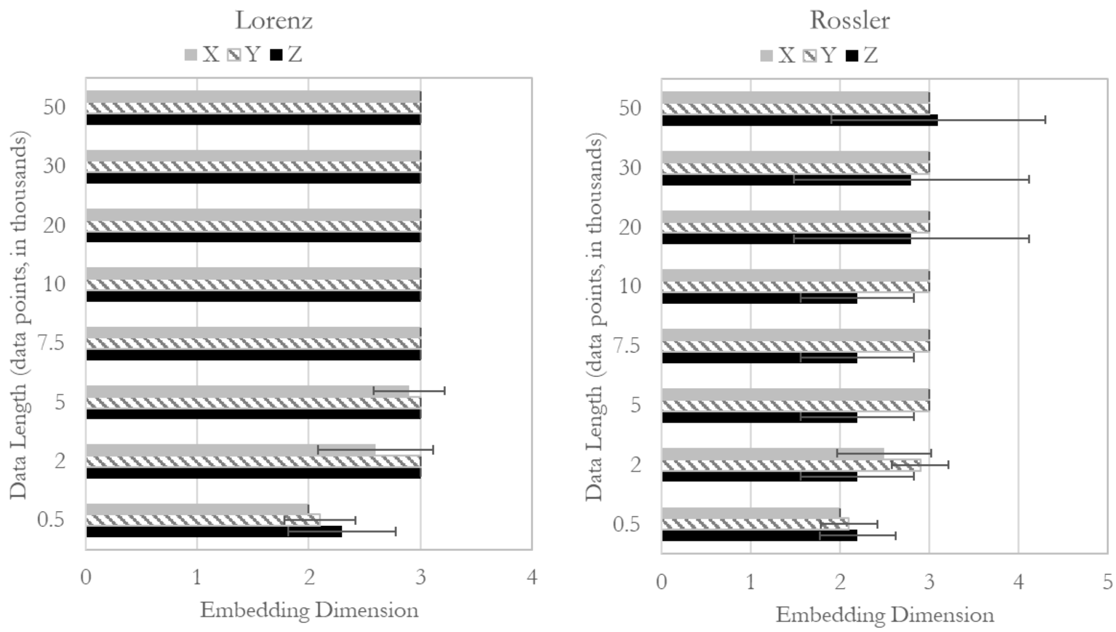

Figure 3-4: Mean (SD) of calculated embedding dimension in known dynamical systems as data length is increased.

We discovered that the calculated d_E varies with respect to data length. However, a steady state d_E can be reached as long as the minimum data requirement is met for the dynamical system. If we look at the simulated systems, shown in Figure 3-4, the calculated Lorenz quickly reaches a consistent d_E after 2×10^3 points. The Rössler system required 5×10^3 points to reach a steady state d_E in the x and y time series. The z time series did not always converge on to the same d_E as the other time series. This could be a sign that the z time series has insufficient information in its signal to be used for phase space reconstruction.

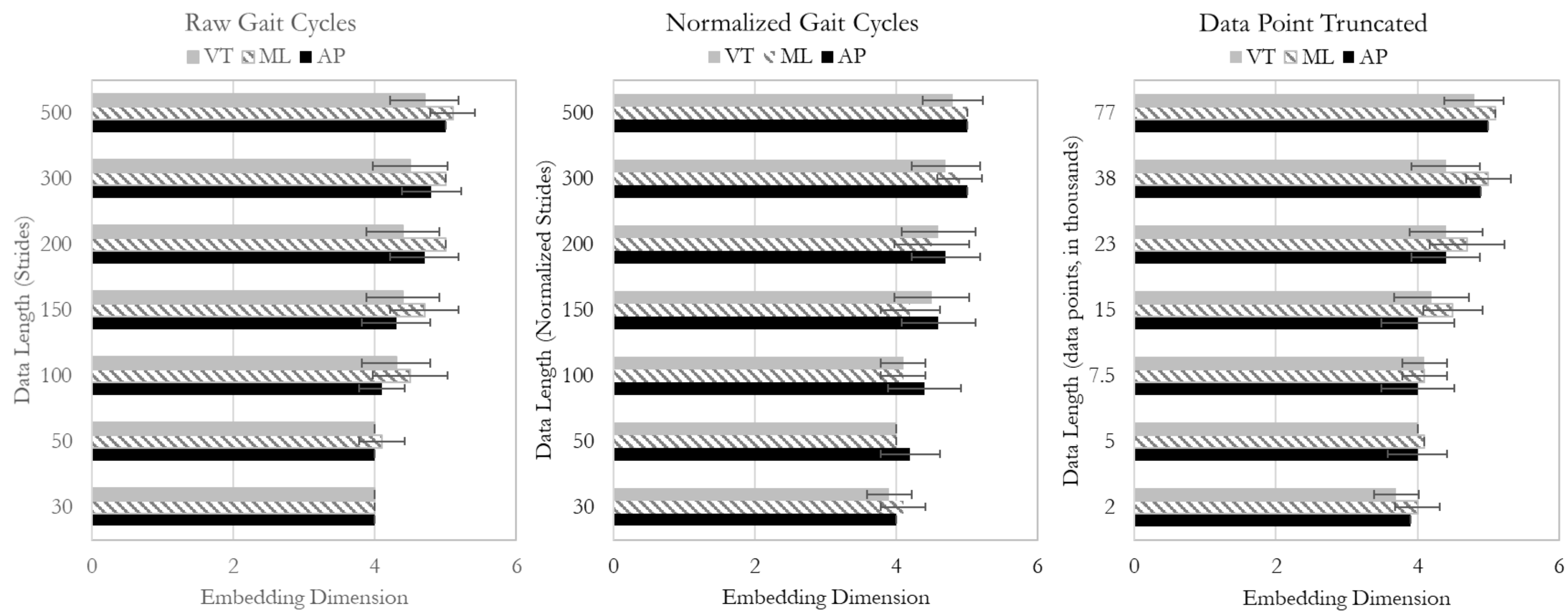

Figure 3-5: Mean (SD) of embedding dimension when calculated with different data lengths and preprocessing methods for every signal direction: vertical (VT), mediolateral (ML), and anteroposterior (AP).

The gait data performed similarly to the known systems, regardless of the preprocessing method used (Figure 3-5). Statistical differences were found between shorter and longer data lengths, but once a sufficient amount of data was used in the calculation, the d_E remained constant with increasing data length. Preprocessing method 1 allowed for the least amount of data (by strides and time) to be used with 100 gait cycles in the VT direction and 150 gait cycles for the AP and ML direction. Method 2 needed at least 150 normalized gait cycles and method 3 needed at least 2 minutes of walking data. Therefore, preprocessing method 1 is more advantageous for smaller gait datasets. It is important to note that all preprocessing methods, in every acceleration direction, did converge onto an d_E of 5 after 300 gait cycles. Therefore, we find that an embedding dimension of 5 is sufficient for processing young healthy adult gait data for LyE. Future research should look at how much d_E affects the final outcome of LyE using either algorithm for calculating local dynamic stability.

This study had some limitations with using only young healthy adults. We cannot assume that data length will have similar effects on τ and d_E when looking at different population groups, e.g., healthy or frail elderly. Although a treadmill was used for this experiment, this should not have an impact on the reconstruction of the phase space itself. The calculated LyE is believed to be different from treadmill and overground walking due to slightly different gait dynamics used to adapt to each situation (Dingwell et al., 2001; Terrier and Dériaz, 2011). However, this terrain difference has no influence on the method of phase space reconstruction.

CONCLUSION

The current study provided novel information by systematically investigating the effect of data length on time delay and embedding dimension in gait data. Data length does not play a large role in the calculation of τ using AMI, while a minimum data requirement must be first met when calculating the d_E using FNN. Therefore, the differences in the LyE at various data lengths is not due to the reconstruction of the gait attractor, but more likely due to the increasing signal to noise ratio as the data length increases. Additionally, we investigated the effect of three methods of gait data preprocessing. We found method choice significantly impacts the value of the τ but not the d_E when sufficient data is provided. Going forward it is clear that young healthy gait data can be processed using an d_E of 5 for any acceleration data regardless of how the data is preprocessed.

CHAPTER 4: CHOICE OF TIME DELAY AND EMBEDDING DIMENSION IMPACT THE LYAPUNOV EXPONENT OF GAIT

ABSTRACT

There is no universally accepted approach for calculating the Lyapunov Exponent (LyE) as a measure of gait. There is an imperative need to standardize this methodology in order for better comparisons across publications and populations. This study systematically investigated the effect of time delay, embedding dimension, and three pre-processing methods on the LyE using both the Rosenstein *et al.* (R) and Wolf *et al.* (W) algorithms. Three-dimensional acceleration of the lumbar was recorded from 17 healthy young adults during a thirty-minute walk. Time delay and embedding dimension had significant ($p < 0.005$) effects on the LyE regardless of direction, algorithm, and pre-processing method. The R-algorithm was robust against varying embedding dimensions for all preprocessing methods, while the Wolf algorithm was more robust against varying time delays. Neither the R- nor the W-algorithm outperformed the other. However in future studies, time delay should be standardized to 10 (in data points and percent gait cycle) and an embedding dimension of 5 and 7 should be used for the R and W- algorithms, respectively. We also found that comparing time delays within specific value ranges across publications can be done without statistical differences in the value of the LyE when comparing similar populations.

INTRODUCTION

The Lyapunov Exponent (LyE) is a popular approach to quantify gait stability. The LyE (also known as local dynamic stability and the maximum or largest Lyapunov exponent) describes the long-term behavior of a dynamical system. It quantifies the rate of divergence or convergence of trajectories in an *n*-dimensional phase space. The phase space of a system is a set of vectors that describes every point in time uniquely. In the study of gait, the LyE quantifies an individual's ability to withstand small perturbations while walking. An inability to properly react to such perturbations results in a larger divergence of the trajectories in the phase space, and thus it will result in a greater LyE. Therefore, a large LyE value is indicative of greater gait *in*stability. This nonlinear measures has been successfully used to determine differences in gait in the aging process (Terrier and Reynard, 2015), as well as between healthy controls and patients with Parkinson's disease (Fino et al., 2018), multiple sclerosis (Craig et al., 2019; Huisinga et al., 2013), developmental disorders (Speedtsberg et al., 2018), and the fall prone elderly (Lockhart and Liu, 2008; Toebes et al., 2012).

However, the literature is far from consistent with regards to how we calculate the LyE. This makes it difficult to compare results across publications and populations. Standardization of this measure is challenging because there are many factors involved at multiple levels of designing an experiment and during the execution of data analysis. The first decision begins with the type of instrumentation that is used to record gait, with 51% of published papers using motion capture systems and 38% using accelerometers or inertial measurement units (IMUs).(Mehdizadeh, 2018) There has been an increase in the utilization of IMUs for the assessment of clinical populations due to their portability and ease of use in comparison to the standard laboratory motion capture set up. In this paper, we will be focusing on the standardization of the LyE calculation in accelerometers because of the immediate translation of laboratory research into clinical protocols and applications. The next set of experimental decisions focus on how the phase space is reconstructed for the calculation of the LyE.

One of the most critical steps in estimating the LyE is the reconstruction of the phase space. Using the method of delays (Broomhead and King, 1986; Takens, 1981), the phase space can be reconstructed as follows:

$$y(n) = [x(n), x(n+\tau), \ldots, x(n+(d_E-1)\tau)] \tag{1}$$

which requires a time delay, τ, and an embedding dimension, d_E. Theoretically, the LyE is invariant under smooth transformations of the phase space, irrespective of the details of measurement process and the reconstruction of the state space. (Kantz and Schreiber, 2004) This is due to the fact that the LyE describes the long-term behavior of the system being investigated. Thus the average mutual information function (Fraser and Swinney, 1986) and global false nearest neighbors (Kennel et al., 1992) are generally used to determine τ and d_E, respectively. This method is employed in physics as well as most biomechanics papers calculating the LyE. While the LyE is generally invariant when the systems being investigated are built of first order differential equations, in experimentally collected data, even linear transformations of the phase space can affect the mean and standard deviation of the LyE. (Gates and Dingwell, 2010; Rosenstein et al., 1994) In gait, Van Schooten and colleagues (van Schooten et al., 2013) found that phase space reconstruction influences the test-retest reliability of the LyE when comparing intra and between-sessions. They concluded that the same fixed embedding dimension and time delay for all subjects yielded the most consistent results, which has been corroborated by Raffalt et al. (2018), and adopted as the prime methodology for reconstructing the phase space.

Additionally, different time series normalization methods have also been shown to affect the LyE and that different normalization methods work better for different LyE algorithms.(Raffalt et al., 2019; Stenum et al., 2014) In recent years, four gait data normalization methods have emerged that are common to use when calculating the LyE:

1) Fixed number of strides with a variable number of time series data points
2) Fixed number of data points with variable number of strides per time series
3) Fixed number of strides with a fixed number of points per stride
4) Fixed number of strides with a fixed number of points for the entire data series

Method 1 is raw gait data that is segmented by the number of strides that each subject must have in order to be included in the analysis. The second method is similar but uses time as the cut-off point, generally 2-3 minutes of data. Gait cycle normalization (method 3) is the most commonly seen in the literature, however the data point normalization method (method 4) is gaining popularity because it does not interfere with the temporal stride variation in gait. Each of these preprocessing methods is likely to have some kind of effect on the reconstruction of the gait attractor and perform better with either the Rosenstein *et al.* or Wolf *et al.* algorithm. For instance, having a fixed number of strides or a fixed number of data points for the time series is better for calculating the LyE with the Wolf algorithm. While normalizing the data to have a fixed number of strides with a fixed number of points per stride or with a fixed number of points for the entire time series performs better using the Rosenstein algorithm. (Raffalt et al., 2019) Therefore, it is also important standardize how gait data is preprocessed or normalized in order to improve reliability and repeatability of publications using LyE and IMUs.

Currently, it is assumed that time delay and embedding dimension do not play a large role in the final calculated LyE for gait. This assumption is built off of classical nonlinear systems like the Lorenz, shown in Figure 4-1. In this figure, we can see that time delay does not have a significant effect when using the Rosenstein algorithm, even if the LyE is underestimated with this algorithm. And once a sufficient time delay (approximately 10 based on the graph) is established, Wolf’s algorithm also has a plateau region where a more accurate LyE can be found. It is also evident that embedding dimension does not significantly affect the LyE value once a sufficient embedding dimension is chosen. This convincingly would lead us to believe that if a sufficient embedding dimension and time delay are chosen for a different nonlinear system, such as gait, then the LyE would be stable.

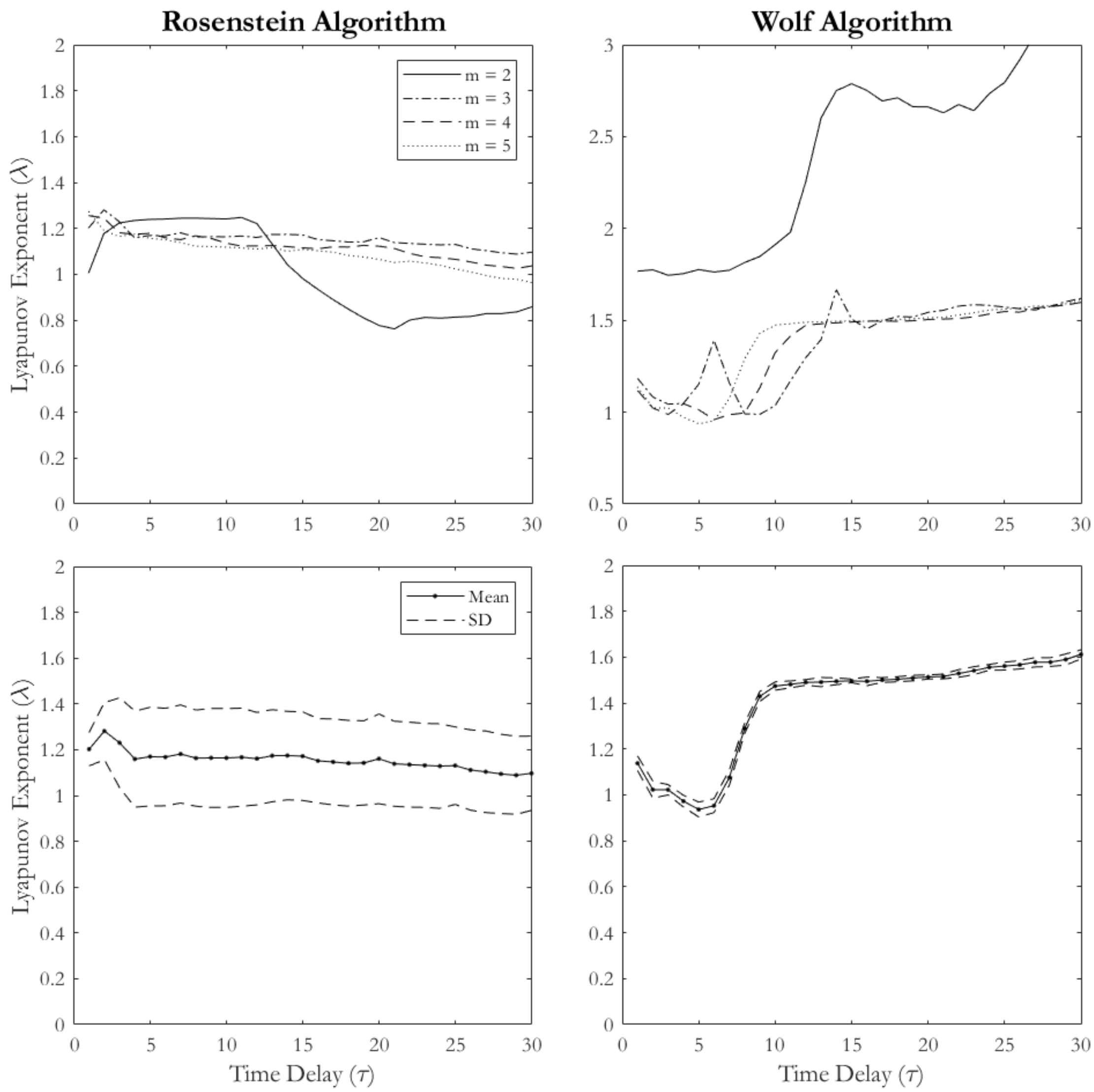

Figure 4-1: Lorenz Lyapunov Exponent calculated using the Rosenstein (left) and Wolf (right) algorithms. The x time series of the Lorenz equations was used. The top row compares the mean LyE when each combination of embedding dimensions ($m = 2 - 5$) and time delay ($\tau = 1 - 30$) values are used as input parameters. The bottom row focuses on a single embedding dimension displaying the mean and standard deviation (SD). An embedding dimension of 3 was used for the Rosenstein calculation while an embedding dimension of 5 was used for the Wolf algorithm.

Up until this point there has not been a systematic comparison of embedding dimension and time delay's effect on the LyE for walking. The LyE should be an invariant value for a given system, i.e. time delay and embedding dimension shouldn't play a large role in the calculation of the LyE. Experimentally, it is known that finding a good combination of these parameters does help in the calculation since experimental data is prone to noise and measurement error unlike simulated mathematical differential equations. A full factorial study of how τ and d_E affect the LyE in gait data has not been done to the authors' knowledge. Therefore, the objective of this study was to investigate the effect of τ and d_E on the LyE using both the Rosenstein's and Wolf's algorithm on gait. Gait is a noisy biological signal and therefore it is important to know if reconstructing the phase space under different conditions significantly changes the LyE. To fulfill this purpose, we used lumbar acceleration data from young healthy subjects who walked on a treadmill for thirty minutes. We then calculated the LyE after systematically reconstructing the phase space with different parameters using three normalization methods discussed above. The more advantageous and appropriate algorithm(s) with a specific preprocessing method(s) would be robust against changes in the LyE value when time delay and embedding dimension are varied.

METHODS

After this study was approved by the Institutional Review Board of Arizona State University, twenty young healthy subjects (12 males and 8 females) were recruited. All subjects were physically active and familiar with walking on a treadmill. Subjects reported no cardiovascular issues, neurological diseases, nor lower extremity surgeries in the last 3 months. Subjects proceeded written informed consent before participating in this study. Of the 20 subjects tested, only 17 subjects were used in the analyses due to sensor failure or signal anomalies (e.g. stumbling due to subject talking to researchers behind them). The final group of young healthy subjects (10 males and 7 females) had a mean $\pm$ standard deviation age of 23.9 $\pm$ 3.5 years, body height of 1.72 $\pm$ 0.11 meters, and body mass of 74.1 $\pm$ 18.6 kg.

Participants wore three tri-axial acceleration sensors sampling at 128 Hz (APDM, Mobility Lab, APDM, Inc., Portland, OR) fitted with elastic bands and Velcro straps. They were placed at each ankle and the lower lumbar, around vertebrae L4. The accelerometer (IMU) measured trunk accelerations along 3 axes: vertical (VT), anteroposterior (AP), and mediolateral (ML). After subjects became familiar with the treadmill in their own sneakers, each subject's preferred walking speed (PWS) was determined using a standardized protocol (Dingwell and Marin, 2006). The mean and standard deviation of PWS was 1.15 $\pm$ 0.09 m/s. After a short rest period, each subject walked on the treadmill for 30 minutes at their PWS. The treadmill used in this experiment was a split-belt treadmill and is a part of the GRAIL system (Motekforce Link, Amsterdam, The Netherlands). Measurements were started 30 seconds after the treadmill and the subject reached their individual PWS. Three-dimensional acceleration data of the lumbar sensor was used for all of the calculations in this paper.

Data Analysis

All data were analyzed using custom MATLAB (version 2018b, Mathworks Inc., Natwick) programs. The heel contacts for each step were determined and indexed and the time series was truncated to start and end on a heel contact. (Dingwell et al., 2001; England

and Granata, 2007) Using this data, the greatest number of strides shared by all subjects was determined to be 1300 gait cycles. Each subject time series data was then preprocessed using the following three methods with each containing the maximum number of gait cycles:

(1) Fixed number of strides with a variable number of data points per stride (gc)

(2) Fixed number of strides with a 100 data points per stride (gcNorm)

(3) Fixed number of strides with a total of 130,000 data points in the time series (dpNorm)

No other filtering or preprocessing was performed on the data. The LyE was calculated for every direction using each of the preprocessing methods and the Rosenstein et al. (1993) and Wolf et al. (1985) algorithms, which will be referred to R- and W-algorithms, respectively. And within these conditions each permutation of the embedding dimension ($d_E = 4,5,6,7$) and time delay ($\tau = 1,2, \ldots ,30$) were used to calculate the LyE.

In Rosenstein's algorithm, the LyE is the slope of the divergence curve. When normalized gait cycles are analyzed, the slope is taken over a span of 0 – 0.5 strides or the first 50 points of the divergence curve. In order to compare normalized and raw gait data, we found the average stride length for each subject and used the individualized half stride length as the bounds for taking the slope. For example, if a subject had an average stride length of 150 samples, then the slope of the mean divergence curve was taken from the first 75 points. And in the W-algorithm a time evolution step of seven was used.

Statistical Analysis

The performed analyses consisted of a systematic permutation of thirty time delays and four embedding dimensions. This was applied to 6 different LyE algorithm-time series normalization procedure combinations for each acceleration direction. The Friedman test, a nonparametric repeated measures ANOVA, was used to explore the effect of time delay and embedding dimension on the LyE. The nonparametric test was used for all analyses because the assumption of sphericity was violated, in addition to not all parameters were normally distributed. This test was performed independently for each acceleration direction, algorithm choice and preprocessing method.

Then, slices of the data set were taken for a more specific look at how time delay and embedding dimension independently played a role in the calculation of the LyE. First, a post-hoc pairwise comparison with a Bonferroni correction for multiple comparisons was used to determine the specific differences between each time delay when the embedding dimension of five or seven was chosen for the Rosenstein and Wolf algorithm, respectively. Then the same post-hoc comparison was used to determine the differences in embedding dimension for a set of time delays ($\tau = 5,8,10,12,15$). This range of time delays was selected because most time delays chosen in publications are within this range, based on the meta- and supplementary data from Mehdizadeh (2018). For all statistical tests, a p-value < 0.05 was considered significant. All statistical analysis was performed using SPSS Statistics (version 25, IBM, USA). The outcome of these statistical analyses is summarized in a result paragraph and presented in full in the appendix.

RESULTS

Overall, time delay and embedding dimension had a significant impact ($p < 0.005$, respectively) on the value of the LyE regardless of direction, algorithm, and preprocessing method. The differences between each direction and preprocessing methods are shown in Figure 4-2 for the R-algorithm and in Figure 4-3 for the W-algorithm.

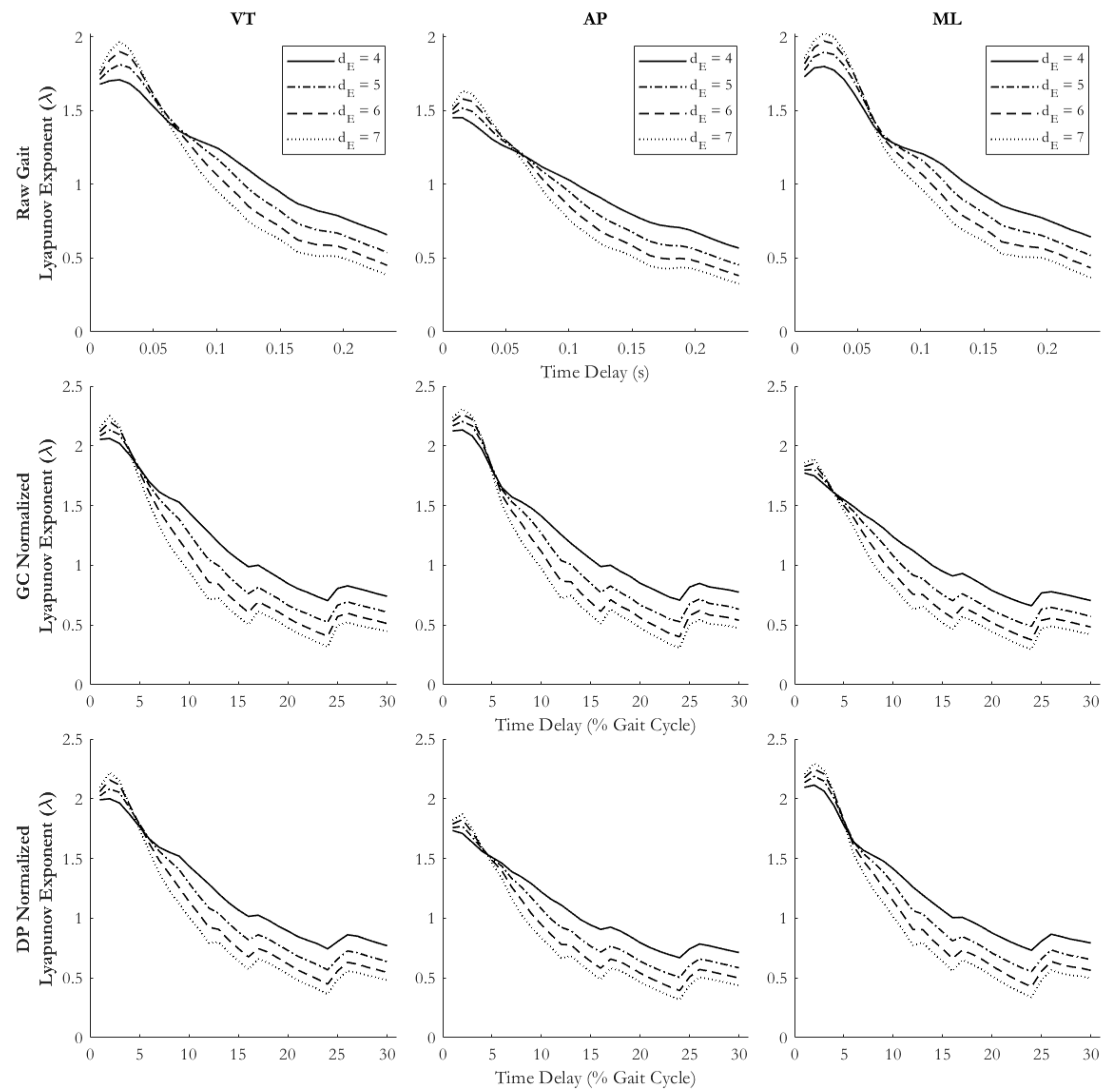

Figure 4-2: Effect of embedding dimension and time delay in the VT, AP, and ML direction using Rosenstein *et al* algorithm using three different preprocessing methods.

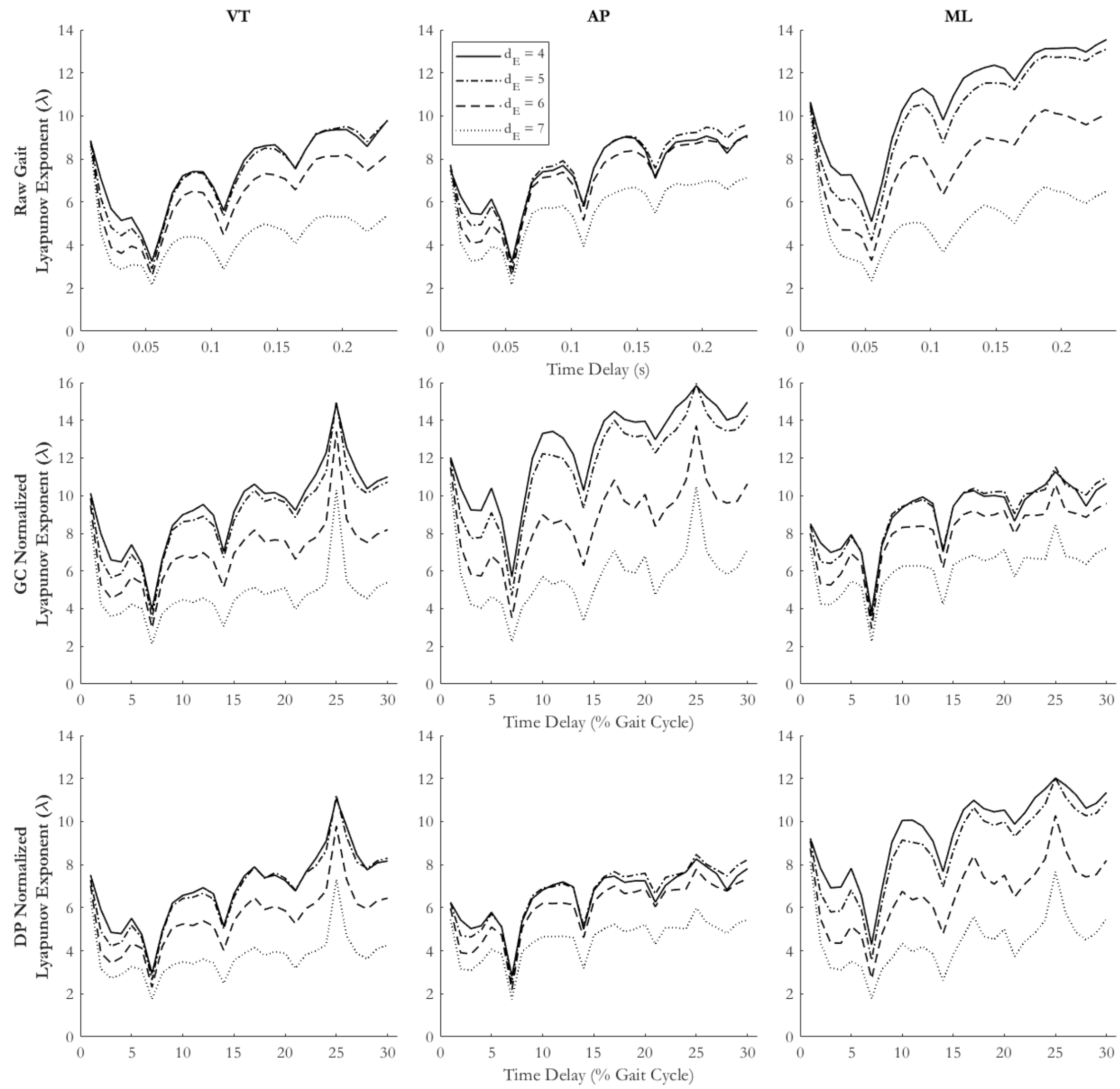

Figure 4-3: Effect of embedding dimension and time delay in the VT, AP, and ML direction using Wolf *et al* algorithm using three different preprocessing methods.

We found that embedding dimension, at particular time delays, had significant effects on the LyE calculated by the R- and W-algorithm. Table 4-1 shows the differences between different embedding dimensions at the selected time delays ($\tau = 5,8,10,12,15$) when gc, gcNorm, and dpNorm data was utilized with the R-algorithm, while Table 4-2 shows the results using the W-algorithm. Notably when the R-algorithm was used, there were no significant differences between embedding dimensions 5, 6, and 7 in any direction within this time delay selection regardless of the data set used. In gcNorm and dpNorm, most of the significant differences were between embedding dimension 4 and 7 for time delays greater than 8. The W-algorithm was more sensitive to differences in embedding dimension as seen in Figure 4-1. In all normalization methods, there were many significant differences between dimension pairs d4-d7, d5-d7, and d6-d7. Normalized gait data increased the number of significant differences between the embedding dimensions regardless of acceleration direction used. Although there were no significant differences between dimension pairs d4-d5 and d5-d6, these dimensions were found not to be sufficient for use with the W-algorithm. The W-algorithm overestimates the LyE when the dimension is not large enough, as exemplified in Figure 4-1 with the Lorenz system. The continuous decrease in the value of the LyE is also seen in Figure 4-3 with the smaller embedding dimensions. Therefore, we conclude that and embedding dimension of seven is more appropriate for the W-algorithm.

Table 4-1: Effect of embedding dimension on the LyE under select time delays using the R-algorithm. *p*-values > 0.5 are marked NS.

Normalization Method	Dir.	Tau	Dimension Pairwise Comparison (p-value)					
			d4-d5	d4-d6	d4-d7	d5-d6	d5-d7	d6-d7
Raw Gait Cycles	**VT**	5	NS	NS	NS	NS	NS	NS
		8	NS	NS	NS	NS	NS	NS
		10	NS	NS	NS	NS	NS	NS
		12	NS	NS	NS	NS	NS	NS
		15	NS	NS	NS	NS	NS	NS
	AP	5	NS	NS	NS	NS	NS	NS
		8	NS	NS	NS	NS	NS	NS
		10	NS	NS	NS	NS	NS	NS
		12	NS	NS	0.4439	NS	NS	NS
		15	NS	NS	NS	NS	NS	NS
	ML	5	NS	NS	NS	NS	NS	NS
		8	< 0.0005	< 0.0005	< 0.0005	NS	NS	NS
		10	NS	NS	NS	NS	NS	NS
		12	NS	NS	NS	NS	NS	NS
		15	NS	NS	NS	NS	NS	NS
Gait Cycle Normalization	**VT**	5	NS	NS	NS	NS	NS	NS
		8	NS	NS	NS	NS	NS	NS
		10	NS	NS	0.0094	NS	NS	NS
		12	NS	0.132	0.0026	NS	NS	NS
		15	NS	NS	0.1807	NS	NS	NS
	AP	5	NS	NS	NS	NS	NS	NS
		8	NS	NS	NS	NS	NS	NS
		10	NS	NS	0.0195	NS	NS	NS
		12	NS	0.1467	0.0034	NS	NS	NS
		15	NS	NS	0.1467	NS	NS	NS
	ML	5	NS	NS	NS	NS	NS	NS
		8	NS	NS	0.4439	NS	NS	NS
		10	NS	NS	0.0136	NS	NS	NS
		12	NS	0.0278	0.0007	NS	NS	NS
		15	NS	0.2003	0.0247	NS	NS	NS
Data Point Normalization	**VT**	5	NS	NS	NS	NS	NS	NS
		8	NS	NS	NS	NS	NS	NS
		10	NS	NS	0.0173	NS	NS	NS
		12	NS	0.2455	0.005	NS	NS	NS
		15	NS	NS	0.3655	NS	NS	NS
	AP	5	NS	NS	NS	NS	NS	NS
		8	NS	NS	0.4886	NS	NS	NS
		10	NS	NS	0.0153	NS	NS	NS
		12	NS	0.0552	0.0015	NS	NS	NS
		15	NS	0.3313	0.0493	NS	NS	NS
	ML	5	NS	NS	NS	NS	NS	NS
		8	NS	NS	NS	NS	NS	NS
		10	NS	NS	0.0312	NS	NS	NS
		12	NS	0.2715	0.0064	NS	NS	NS
		15	NS	NS	0.2003	NS	NS	NS

Table 4-2: Effect of embedding dimension on the LyE under select time delays using the W-algorithm. p-values > 0.5 are marked NS.

Normalization Method	Dir.	Tau	Dimension Pairwise Comparison (p-value)					
			d4-d5	**d4-d6**	**d4-d7**	**d5-d6**	**d5-d7**	**d6-d7**
Raw Gait Cycles	**VT**	5	NS	NS	NS	NS	NS	NS
		8	NS	NS	NS	NS	NS	NS
		10	NS	NS	NS	NS	NS	NS
		12	NS	NS	NS	NS	NS	NS
		15	NS	NS	NS	NS	NS	NS
	AP	5	NS	NS	0.0153	NS	0.2455	NS
		8	NS	NS	NS	NS	NS	NS
		10	NS	NS	NS	NS	NS	NS
		12	NS	NS	NS	NS	0.1467	NS
		15	NS	NS	0.4439	NS	0.1628	0.4886
	ML	5	NS	0.2715	< 0.0005	NS	0.1187	NS
		8	NS	NS	0.0073	NS	NS	NS
		10	NS	NS	< 0.0005	NS	0.0026	NS
		12	NS	NS	< 0.0005	NS	< 0.0005	0.1067
		15	NS	0.0958	< 0.0005	NS	< 0.0005	0.1628
Gait Cycle Normalization	**VT**	5	NS	NS	0.0006	NS	0.069	NS
		8	NS	NS	0.003	NS	0.012	NS
		10	NS	NS	< 0.0005	NS	< 0.0005	0.0859
		12	NS	0.0859	< 0.0005	NS	< 0.0005	0.069
		15	NS	NS	< 0.0005	NS	< 0.0005	0.0173
	AP	5	NS	0.1628	< 0.0005	NS	0.0034	NS
		8	NS	NS	< 0.0005	NS	0.0247	NS
		10	NS	0.0136	< 0.0005	NS	< 0.0005	0.2715
		12	NS	0.1187	< 0.0005	NS	< 0.0005	0.1467
		15	NS	0.0195	< 0.0005	NS	< 0.0005	0.1628
	ML	5	NS	NS	0.1807	NS	0.069	NS
		8	NS	NS	NS	NS	NS	NS
		10	NS	NS	0.0044	NS	< 0.0005	0.0393
		12	NS	NS	< 0.0005	NS	< 0.0005	0.077
		15	NS	NS	0.069	NS	0.0057	0.0312
Data Point Normalization	**VT**	5	NS	NS	0.0015	NS	0.0552	NS
		8	NS	NS	0.0044	NS	0.0312	NS
		10	NS	NS	< 0.0005	NS	< 0.0005	0.0859
		12	NS	0.4439	< 0.0005	NS	< 0.0005	0.0247
		15	NS	NS	< 0.0005	NS	< 0.0005	0.0219
	AP	5	NS	NS	0.4029	NS	0.1467	NS
		8	NS	NS	NS	NS	NS	NS
		10	NS	NS	0.0064	NS	0.002	0.1067
		12	NS	NS	< 0.0005	NS	< 0.0005	0.0958
		15	NS	NS	0.1628	NS	0.0034	0.035
	ML	5	NS	0.132	< 0.0005	NS	0.0034	NS
		8	NS	NS	< 0.0005	NS	0.0278	NS
		10	NS	0.012	< 0.0005	NS	< 0.0005	0.3
		12	NS	0.0552	< 0.0005	NS	< 0.0005	0.3313
		15	NS	0.0493	< 0.0005	NS	< 0.0005	0.1467

For the Rosenstein *et al* algorithm, the effect of time delay when all embedding dimensions were included had significant differences (p < 0.05). These differences were also seen when only viewing time delays when $d_E = 5$, shown in Figure 4-4. In Figure 4-4, significant differences between time delays were separated by 10 or more steps in raw gait data and time-normalized gait data. The normalized gait data (gcNorm and dpNorm) had less significant differences when the time delay was between 20 and 30. There is a more consistent region of time delays that can be chosen without significant altering the value of the LyE when raw gait data is used with the R- algorithm.

We also found that time delay had significant effects on the LyE when calculated by the Wolf *et al.* algorithm. Time delay (when all embedding dimensions are included) created a complex pattern of time delay pairs that were significantly different ($p < 0.05$), shown in Figure 4-5. When only time delay effects with an embedding of 7 are extracted, there are far fewer significant differences between different time delays than compared to the R-algorithm.

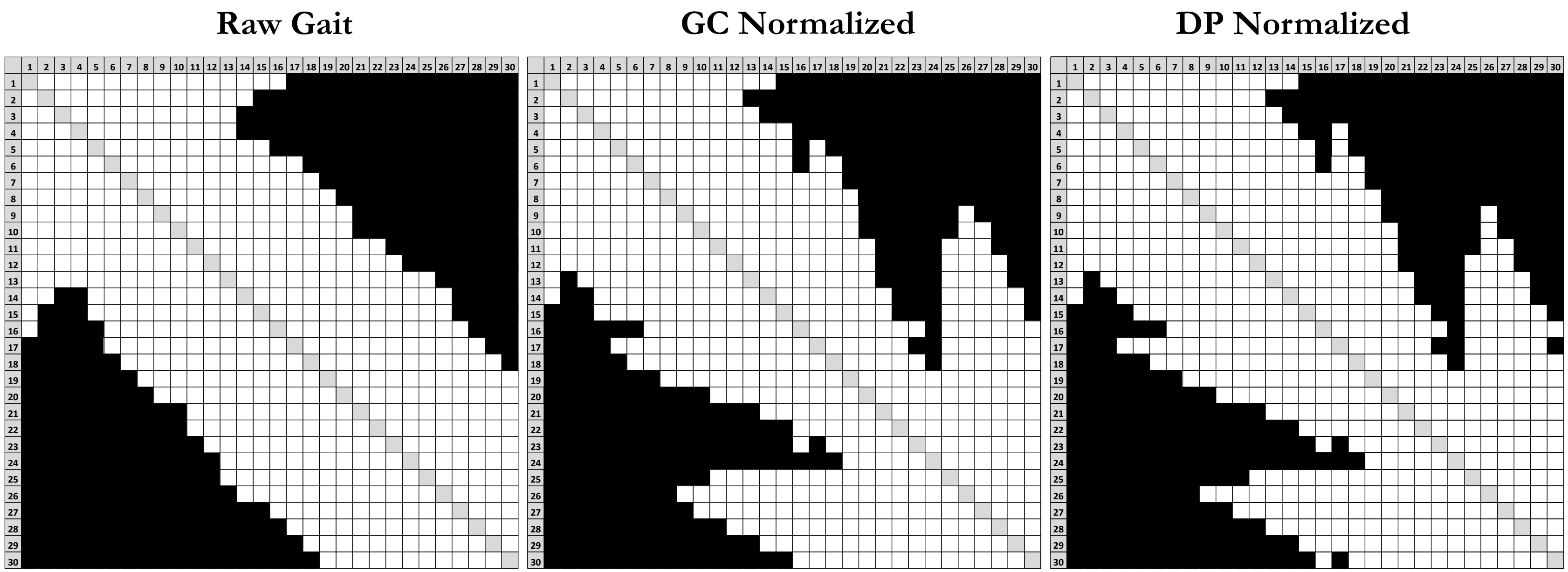

Figure 4-4: Effect of time delay on the LyE using the R-algorithm on 3 data pre-processing methods. This graphic shows the significant differences when two distinct time delays are compared when $d_E = 5$ in the VT direction. Filled in (black) boxes indicate significant differences and empty (white) boxes show where there are no significant differences between a pair of time delays.

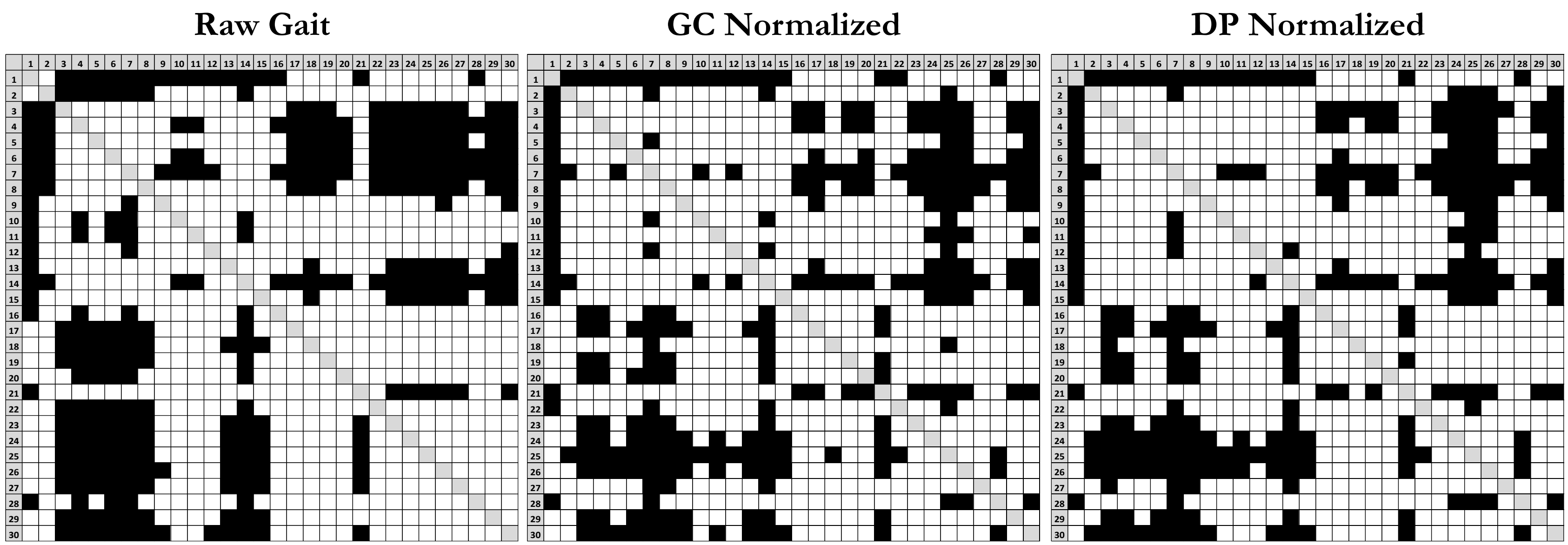

Figure 4-5: Effect of time delay on the LyE using the W-algorithm on 3 data pre-processing methods. This graphic shows the significant differences when two distinct time delays are compared when $d_E = 7$ in the VT direction. Filled in (black) boxes indicate significant differences and empty (white) boxes show where there are no significant differences between a pair of time delays.

DISCUSSION

This study investigated the effect of time delay and embedding dimension on the LyE calculated by the Rosenstein *et al* and the Wolf *et al* algorithms under different gait data preprocessing methods. Embedding dimension and time delay had a significant impact on the value of the LyE in each of the 6 algorithm-time series normalization methods combinations. We also found that when looking at an individual d_E, there exists windows of τ that are not statistically different from one another. The R-algorithm was more invariant to changes in the embedding dimension than the W-algorithm. While the W-algorithm was more invariant to the changes in time delay than the R-algorithm.

The objective of this study was to understand how different variations in calculating the LyE – from algorithm choice, various preprocessing methods, and to the parameters used for reconstructing the phase space – impact the final LyE values that are reported in the literature in order to standardize the procedure of calculating the LyE for accelerometers. In Figure 4-6, when the R- and W-algorithm results are overlaid on the same plot, it is evident that algorithm choice has drastic impact on the value of the LyE across all preprocessing methods and time delay choices. Therefore, we should not expect similar LyE values when comparing algorithms across papers. From this figure, we might also infer that for IMU data, R-algorithm has greater sensitivity than the W-algorithm. The average standard deviations for each direction and preprocessing method is shown in Table 4-3. However, deeper investigation into the sensitivity of each algorithm for IMU data is out of the scope of this paper.

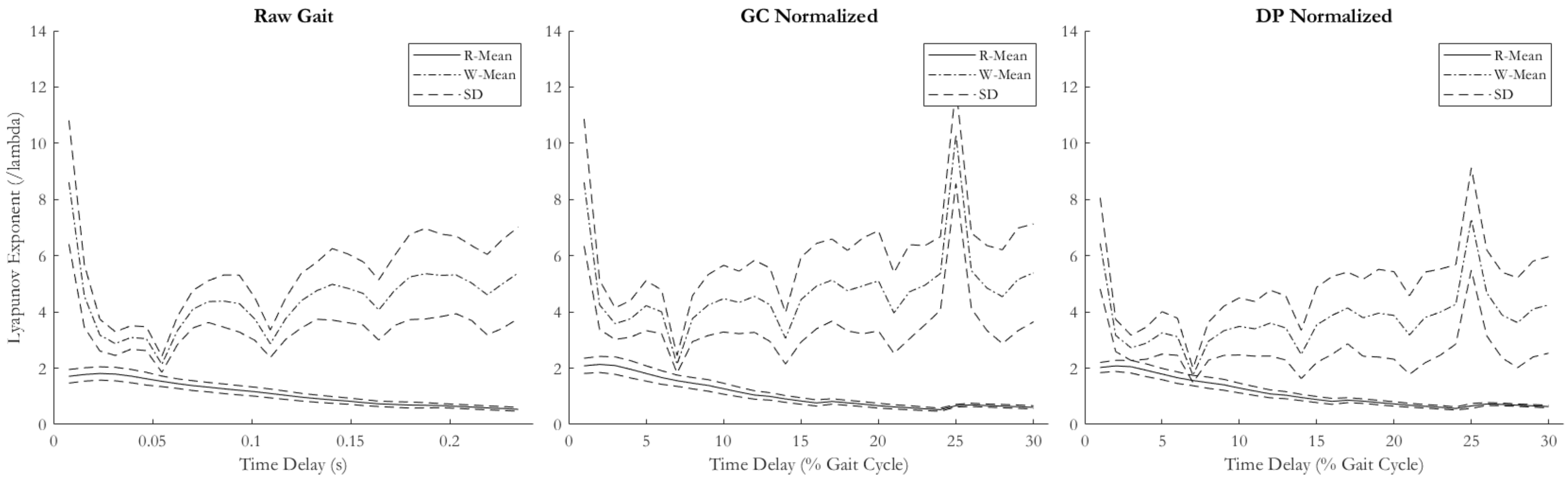

Figure 4-6: Comparing R- and W-algorithm across preprocessing methods

Table 4-3: Average standard deviations across preprocessing methods. An embedding dimension of 5 and 7 were used for R- and W-algorithm, respectively.

Normalization	Dir.	R-algorithm	W-algorithm
Raw Gait	VT	0.14	2.01
	AP	0.11	2.56
	ML	0.13	2.66
GC Normalized	VT	0.18	2.17
	AP	0.13	2.29
	ML	0.15	2.80
DP Normalized	VT	0.12	1.66
	AP	0.08	2.10
	ML	0.11	1.91

An important aspect on the discussion of reconstructing the gait attractor includes what filtering or preprocessing has been done to the experimental data. In this study, we did not filter any of our data due to the possible loss of information. It is generally not recommended to filter the signal when calculating the LyE (Dingwell and Marin, 2006), even though 23% of publications do apply some form of filtering. And even then, the filtering depends largely on what instrumentation was used for data collection. (Mehdizadeh, 2018) Time normalization was first introduced to remove the individual gait velocity variations from person to person (Dingwell et al., 2001) and to investigate how gait speed influenced the LyE (Dingwell and Marin, 2006; England and Granata, 2007) while still keeping the time series lengths equal across all walking velocities. These studies either used a set number of strides with 100 points in each stride (gcNorm) or a set number of strides within a fixed number of data points (dpNorm), e.g. 3000 data points for 30 strides. The first removes stride-to-stride temporal variables but provides and equal number of data points per stride. The latter method allows for stride-to-stride temporal variation while still allowing for a similar number of points per stride regardless of walking velocity.

Only one of the early studies (England and Granata, 2007) used both normalizations and found that walking velocity significantly influences the LyE in both techniques, but the two methods were not compared against each other, specifically.

In this study, three normalization procedures were evaluated: fixed number of strides (gc), fixed number of strides with 100 points per stride (gcNorm), and fixed number of strides with a fixed number of data points for the entire time series (dpNorm). The effect of each of these preprocessing methods in tandem with d_E and/or τ is discussed below.

Embedding Dimension

As previously mentioned, Table 4-1 and Table 4-2 show the results of the effect of changing the embedding dimension while time delay is kept constant for both the Rosenstein and Wolf algorithms, respectively. In general, the R-algorithm is invariant to d_E changes in the VT, AP, and ML directions for all preprocessing methods. The R-algorithm had no significant differences between the embedding dimensions of 5, 6, and 7 regardless of direction and normalization method used. Therefore, we recommend using an embedding dimension of 5 for the R-algorithm when using IMU acceleration data. We recommend an embedding dimension of 5 over the higher dimensions because larger dimensions are computationally more expensive. And more importantly, when a signal is reconstructed in larger than necessary dimensions the attractor lies in smaller and smaller regions of the created phase space. The “extra” dimensions in the phase space will not be populated by more of the dynamical system but will instead be filled with more signal contamination and higher dimensional noise. (Abarbanel et al., 1993) Both the gcNorm and dpNorm methods had more significant differences between d_E than when raw gait was used. However, most of these differences were between $d_E = 4$ and higher $d_E = 6$ or 7.

Overall, the preprocessing methods did not affect the invariance of the LyE when embedding dimension was varied and calculated by the R-algorithm.

The W-algorithm was less robust against the change of the embedding dimension when data is normalized compared to raw gait data, as seen in Figure 4-3. The raw gait data had a total of 9 significant differences out of the 90 permutations, while the gcNorm and dpNorm had 29. The most significant differences were between d4-d7 in all normalization methods. Thus, we can infer that an $d_E = 4$ is not a sufficiently large enough embedding dimension when using the W-algorithm to calculate the LyE using IMU data. This is also seen in the literature, where an embedding dimension of 7 is most commonly chosen when using the W-algorithm with IMU data. (Cignetti et al., 2012b; Huisinga et al., 2013; Rispens et al., 2015, 2014a) Additionally, parallels are seen when we compare how increasing the embedding dimension settles the LyE values in the W-algorithm in both the Lorenz (Figure 4-1, top right) and the gait attractor (Figure 4-3). It is likely that if we had investigated even larger embedding dimensions, the mean LyE would be more similar to $d_E = 7$ than the lower embedding dimensions. Both normalization methods had significant differences across multiple time delays and in all directions in all dimension comparisons except for d4-d5 and d5-6.

Time Delay

Up until this point, all the statistical analyses presented in this paper were performed using 1300 gait cycles of continuous walking on a treadmill. Bruijn et al. found that the duration of walking and/or the number of strides has a significant impact on the value of the LyE.(Bruijn et al., 2009b) This could be interpreted to mean that not enough data is being used to find the LyE or that only local rates or exponents of expansion are being found/calculated instead of the LyE. Therefore, we used an extreme amount of gait cycles to ensure that data length was sufficient for calculating the LyE. Now, as the data length does have an effect on the LyE, we also used an abbreviated data analysis protocol to validate our findings in smaller data sets using the first 150 gait cycles from the original dataset. The LyE was calculated using an embedding dimension of five and seven for the Rosenstein and Wolf algorithm, respectively, and with a range of time delays $\tau = 5,10,15, \ldots, 80$ under the three normalization methods used in the original analysis. The range of the time delays were expanded to investigate anomaly peaks seen at 25% of the gait cycle in gcNorm and dpNorm when the W-algorithm was used (Figure 4-7, left column). The results of these additional analyses are depicted in Figure 4-8 and the statistical results for the vertical direction (similar to Figure 4-4 and Figure 4-5) are shown in Figure 4-9. All statistical results and figures are presented in the appendix for all directions and preprocessing methods.

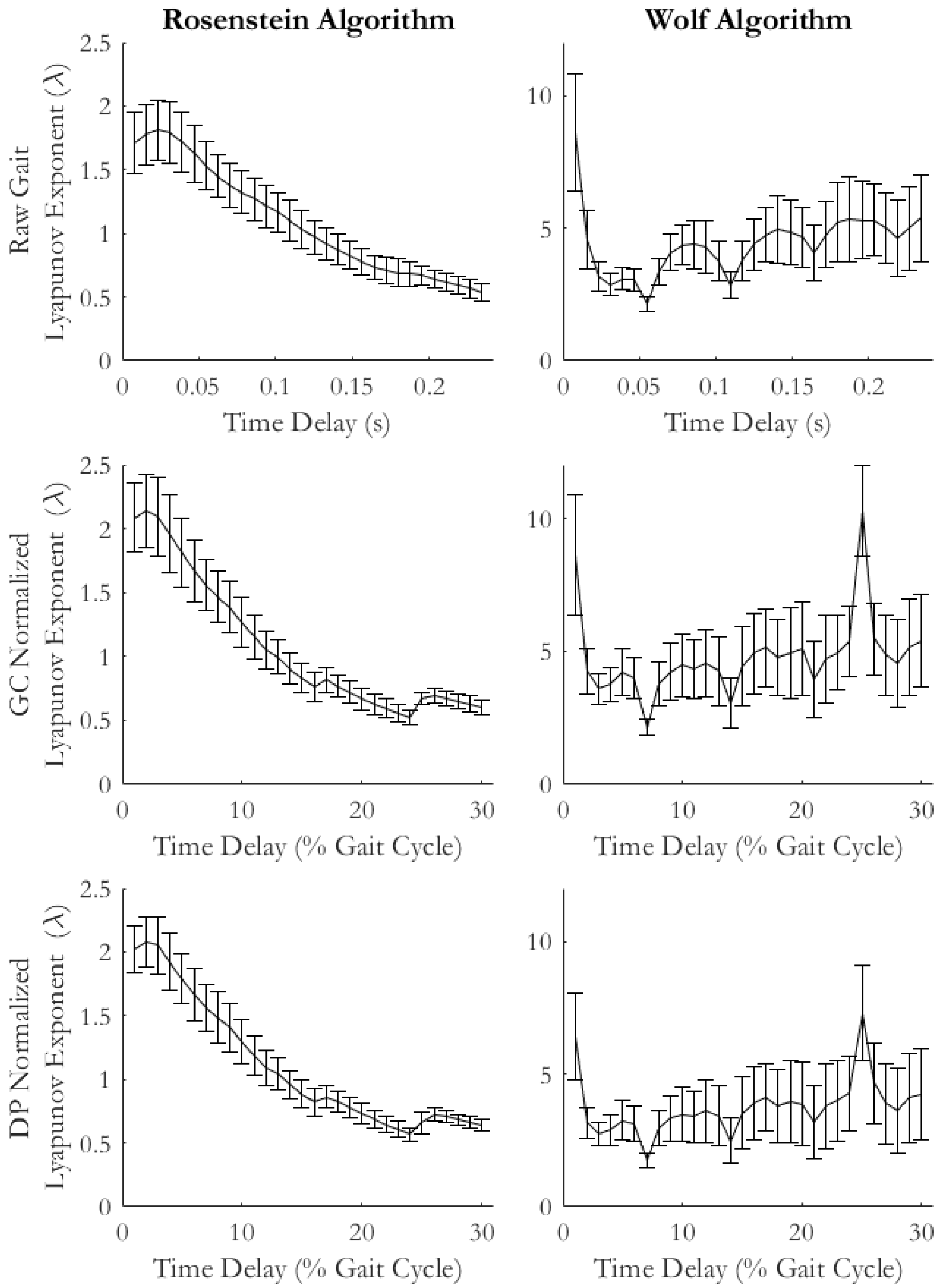

Figure 4-7: Mean and standard deviation of the LyE calculated from vertical acceleration data using 1300 gait cycles when the phase space was reconstructed with different time delays and an embedding dimension of 5 for Rosenstein *et al* algorithm (left) and 7 for Wolf *et al* algorithm (right). The phase space was also reconstructed using different data normalization methods for comparison – fixed number of strides or raw gait data (top), gait cycle normalized (middle), and data point normalization (bottom), and the LyE was calculated using both algorithms.

Time delay had a significant effect ($p < 0.005$) on the LyE regardless of algorithm choice and preprocessing method when 1300 and 150 gait cycles were used. When the R-algorithm and raw data is used, we found that a time delay of ± 5 points is not significantly different, but beyond that range the values of the LyE will be significantly different for the VT, AP, and ML direction. When gait is normalized using either gcNorm or dpNorm methods, a similar leeway of 5-point time delay range is used until it widens to a 10-point range when $\tau = 20$. This occurs in all three directional time series and for both 1300 and 150 strides. The W-algorithm had a more varied range of time delays that were not statistically different and was dependent on the acceleration direction. For AP and ML direction, the LyE were significantly different when the time delays were farther than 10 from each other in all preprocessing methods, except for when $\tau = 7$. In the VT direction this was also generally true, but it had exceptions when 1300 strides were used. In addition, $\tau = 10$ had significant differences when 150 strides was used, with a significant difference in the LyE between $\tau = 10$ and 15. When the W-algorithm is used with normalized data, there are almost no differences between time delays when τ is less than 15 in long and short data lengths. Additionally, when time delay is 25% of the gait cycle, it is significantly different from τ less than 20% of the gait cycle in all directions. When 150 strides are used, time delays greater than or equal to 20 are comparable in all directions, except for in the ML direction where time delays greater than 25 are similar.

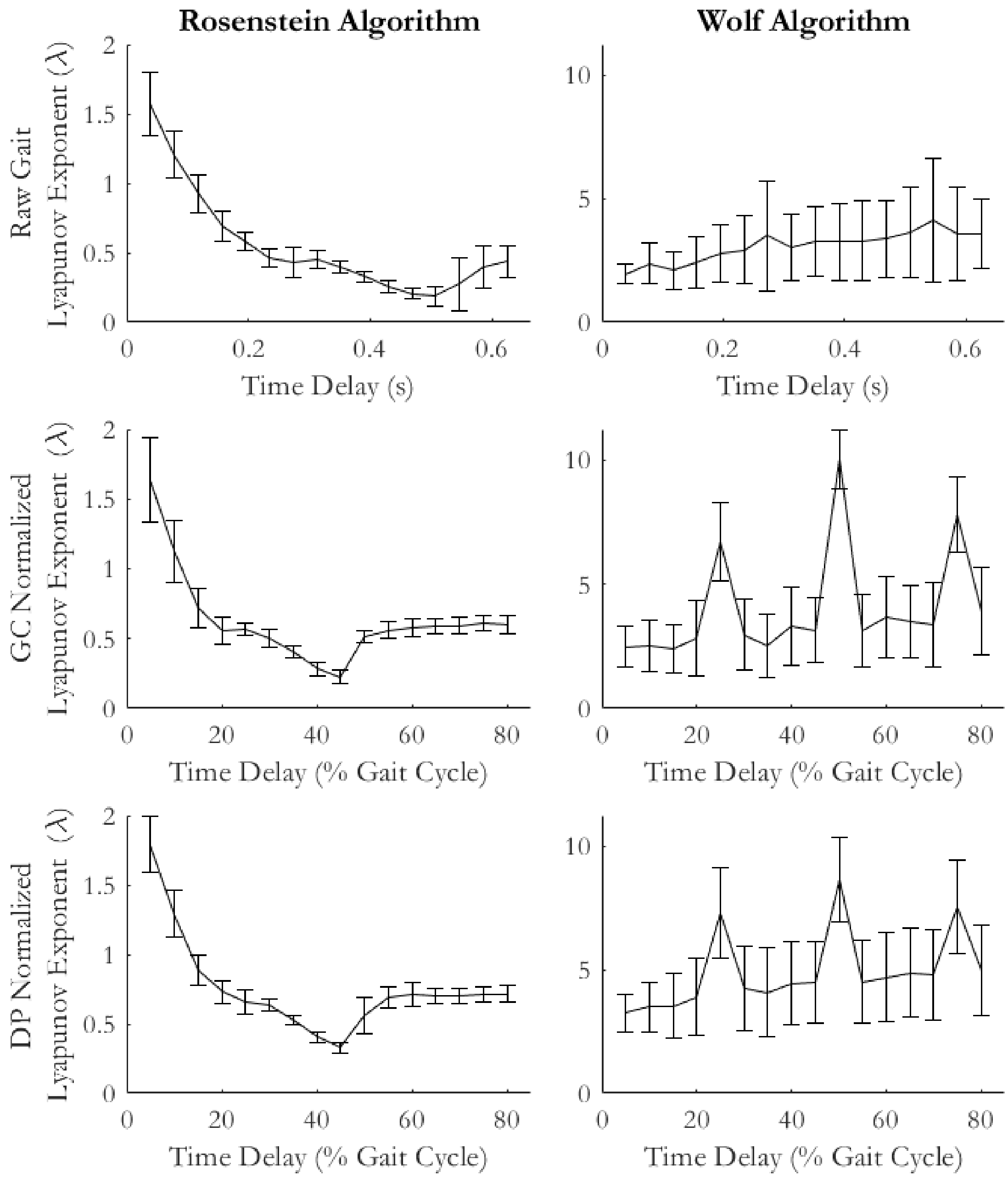

Figure 4-8: Abbreviated data analysis for 150 gait cycles using each of the preprocessing methods for Rosenstein et al (left) and Wolf et al (right) algorithms. The mean LyE for all subjects is depicted with the standard deviation as the error bars.

When the time series was gait cycle normalized or data point normalized, the Wolf algorithm had significantly higher LyE values, seen as peaks in Figure 4-7 and Figure 4-8. This peak did not appear when raw gait data was used. To test that this was not a sampling anomaly in our raw data ($F_s = 128\ Hz$), we resampled the 150 gait cycle data to 100 Hz. This peak was again not present. We hypothesized that this spike was caused by a harmonicity issue when the data is normalized to 100 pts/stride.

In order to test this theory, we calculated the LyE for time delays 35 to 80 by 5s using the 150 gait cycle data. Figure 4-8 shows that this harmonic peak is seen at 25%, 50%, and 75% of the gait cycle under both normalization methods but using raw data. We believe this peak also occurs in the data point normalization analysis because approximately 100 points are allocated to each stride, even though each stride is variable and the time series, as a whole, was resampled and not each individual stride. It is currently unknown if this phenomenon would also occur in motion capture data. However, we believe that if IMUs are being used and the LyE is calculated with W-algorith,m the data should not be preprocessed with either the gcNorm or dpNorm methods. This is consistent with Raffalt and colleagues' (2019) conclusion when investigating normalization procedures for each LyE algorithm. Additionally, researchers should be wary when comparing LyE calculated using W-algorithm when the time delay is 25 (points or % gait cycle).

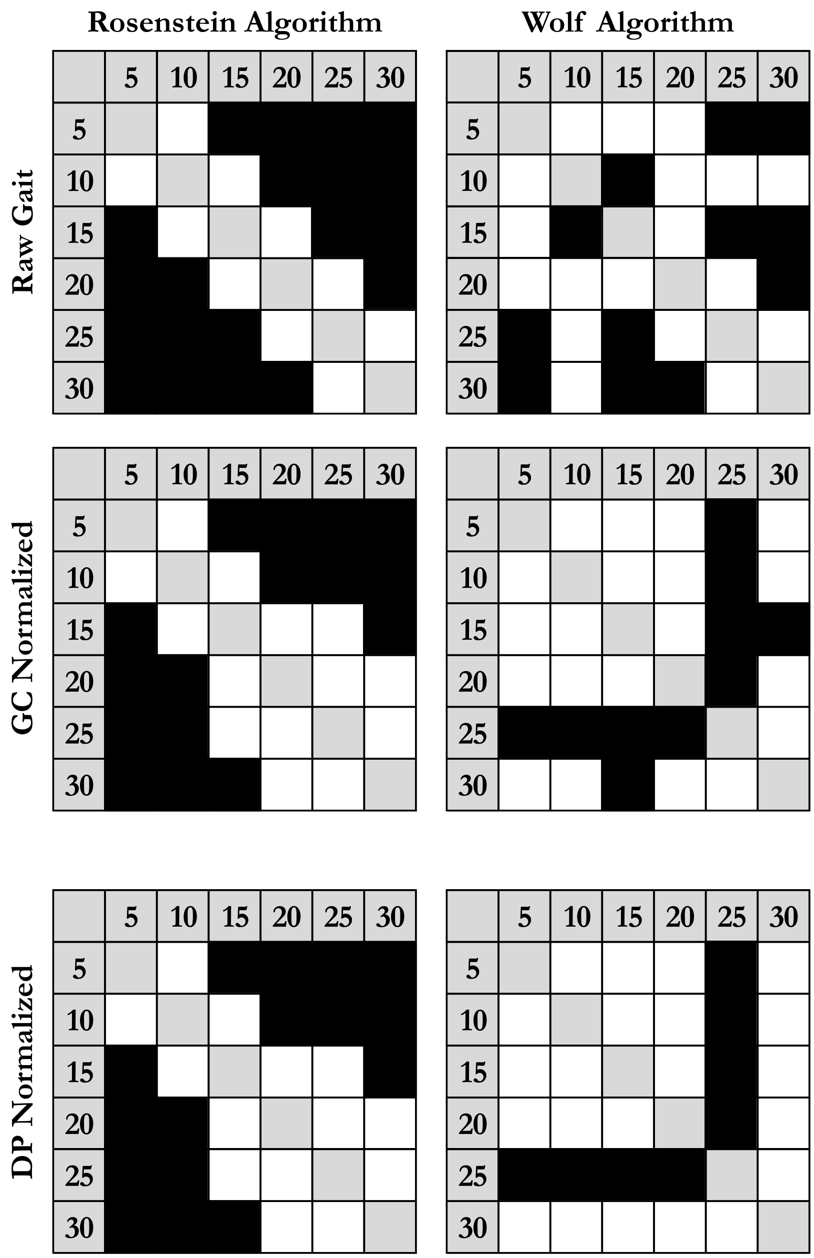

Figure 4-9: Abbreviated statistical analysis on the effect of time delay on the LyE using the R- and W-algorithm and all three data pre-processing methods for 150 gait cycles. This graphic shows the significant differences when two distinct time delays are compared when $d_E = 5$ for the R-algorithm and $d_E = 7$ for the W-algorithm in the VT direction. Filled in (black) boxes indicate significant differences and empty (white) boxes show where there are no significant differences between a pair of time delays.

As previously mentioned, the time delay is usually determined by using the average mutual information (AMI). To put the factorial results into perspective, we also calculated τ for each preprocessing method in every direction using AMI, shown in Table 4-4. One hundred and fifty strides were used for calculating the AMI, which was determined to be sufficient as time delay calculated via AMI is invariant to data length in the previous chapter. All mean time delays across all subjects were between 6 and 10, while the individual subject time delays ranged between 4 and 18. This combined with our results in Figure 4-4, Figure 4-5, and Figure 4-9 implies that using multiple time delays for different directions does not play a large role in determining the LyE. Therefore, we suggest that a single time delay of 10 should be used as the standard τ to improve comparisons across publications and research groups. We chose $\tau = 10$ because of the effect of time delay on the mean divergence curve, shown in Figure 4-10. The mean divergence curvature becomes less pronounced and the "linear" portion of the curve, where the slope is supposed to be taken, becomes obscure as time delay increases. This pattern is seen regardless of normalization method.

Table 4-4: Time Delays calculated using Average Mutual Information

	VT			**AP**			**ML**		
	gc	**gcN**	**dpN**	**gc**	**gcN**	**dpN**	**gc**	**gcN**	**dpN**
Avg	10	6	7	10	6	7	8	8	6
SD	3.1	2.2	3.2	3.1	2.2	2.4	3.2	2.8	2.2
Min	5	4	4	5	4	4	5	4	4
Max	16	13	15	16	13	11	18	14	13
Median	10	5	6	10	5	7	7	7	5

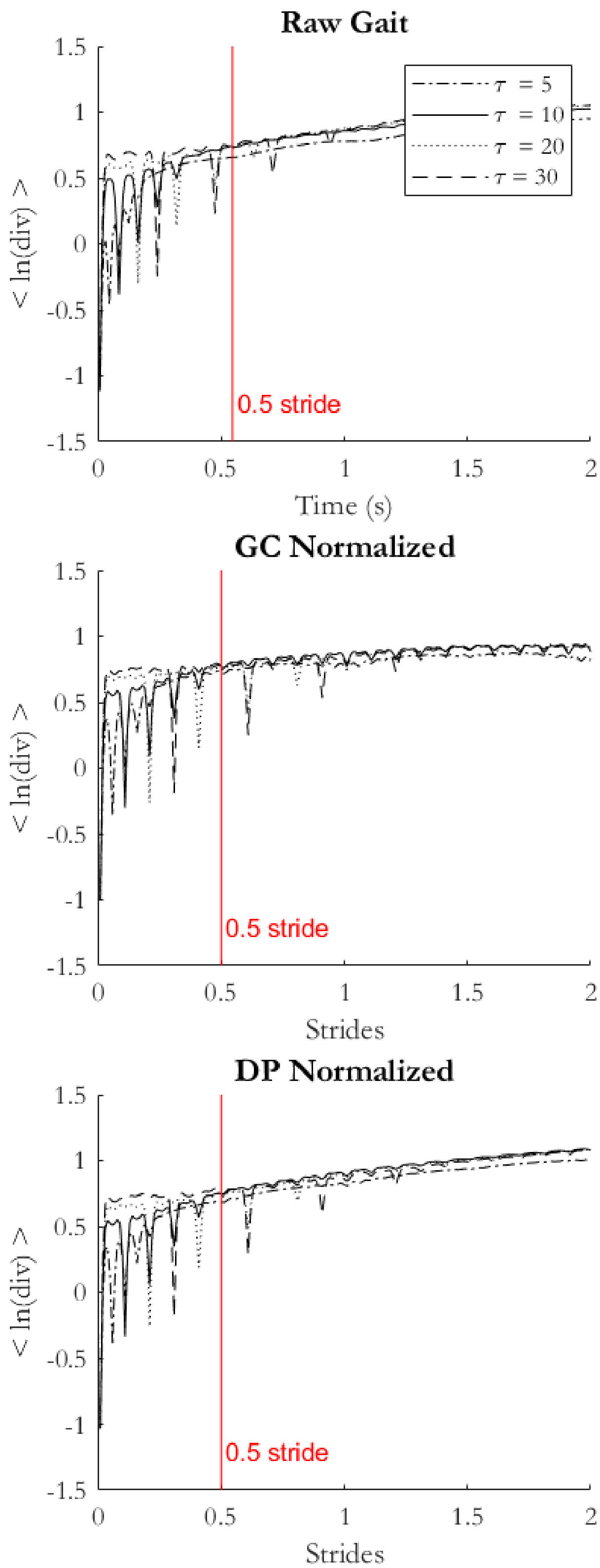

Figure 4-10: Average mean divergence curve across all subjects using 150 gait cycles.

The current study provides novel information by systematically studying the effect of embedding dimension and time delay on the LyE, but it also has its limitations. One of the major limitations of our study was that only young healthy adults were used. The objective of this investigation required a long continuous walk in order to assume the data used were sufficient in length for calculating the LyE. Secondly, the use of the treadmill may limit the generalizability of our results to over ground walking since treadmill walks tend to be more stable and less variable.(Eduardo Cofré Lizama et al., 2015; Rispens et al., 2016) However, a treadmill was necessary to collect uninterrupted thirty-minute gait data.

Overall, the R-algorithm was more robust against changes in the embedding dimension regardless of preprocessing method compared to the W-algorithm. And the W-algorithm was more invariant to changes in time delay. In terms of reliability and consistency, the Rosenstein algorithm might be better for IMU data than the Wolf algorithm. The Rosenstein algorithm had much smaller standard deviations of the mean LyE compared to the Wolf algorithm. The Wolf algorithm is also notably susceptible to data length when comparing 150 strides to 1300 gait cycles. Additionally, this investigation revealed a harmonics issue when gait data is gait cycle or data point normalized. For these reasons, we recommend that Rosenstein's might be more advantageous for processing IMU data. However, if studies are going to use the Wolf algorithm with IMU data, we recommend that data should be segmented using either a fixed number of strides or a fixed number of data points which is consistent with a recent study. (Raffalt et al., 2019) Future studies should further investigate the effect of data length on the LyE when an IMU is used, with respect to both algorithms. Currently, there are only a handful of studies (Bruijn et al., 2009b; Raffalt et al., 2018b; F. Riva et al., 2014) that have investigated the effect of data length on the LyE, but none have used both the R and W-algorithm with accelerometer data. As the use of accelerometers increases in the field of biomechanics, the limitations and data requirements for applying nonlinear dynamics with this data collection method must be defined.

CONCLUSION

To summarize, the purpose of this study was to begin the process of standardizing the protocol used to calculate the LyE from accelerometer data. These results did not provide enough information to definitively claim one algorithm is superior than the other when using IMUs. However, we can standardize the value of the embedding dimension for both the R-algorithm ($d_E = 5$) and the W-algorithm ($d_E = 7$) for all preprocessing methods. The R- algorithm was more robust against varying embedding dimensions than the W-algorithm which required the higher embedding dimension for all preprocessing methods. We also recommend that time delay should be standardized to $\tau = 10$ (in samples and % gait cycle), regardless of algorithm, pre-processing method, and acceleration direction. Although we found that the W-algorithm was more robust when time delay was varied, we also noted this robustness was not always a predictable pattern across directions and with respect to different normalization methods. The R-algorithm was more susceptible to significant differences in the LyE across time delays with an absolute distance greater than 5, but this pattern was seen regardless of acceleration direction and normalization method. Therefore, when comparing LyE values across publications, researchers must be cognizant of time delay differences between the different publication methodologies for both algorithms, as well as differences in the embedding dimension when specifically comparing studies using the W-algorithm.

CHAPTER 5: DATA LENGTH AFFECTS THE ROSENSTEIN'S AND WOLF'S ALGORITHMS DIFFERENTLY WHEN ESTIMATING THE LYAPUNOV EXPONENT FOR GAIT DATA

ABSTRACT

There are many inconsistencies in the literature regarding how to estimate the Lyapunov Exponent (LyE) for gait. These issues could be potentially solved by standardizing the process of calculating the LyE. In this paper, we explore how data length affects the value of the LyE when using both the Rosenstein *et al.* and the Wolf *et al.* algorithms. Additionally, how the gait time series is normalized before the reconstruction of the phase space has recently come under investigation, and thus we also looked at the effect of three different normalization methods with respect to each algorithm. We compared LyE values from a range of data lengths as well as calculated the minimum number of required strides for each of the 6 algorithm-normalization method combinations. Based on our results, we recommend that using between 50 and 100 gait cycles because this range is easily comparable across most published papers. We also found that the Rosenstein *et al.* algorithm requires less strides to estimate the LyE with greater reliability than the Wolf *et al.* algorithm. Therefore, we recommend that future studies use the Rosenstein *et al.* algorithm when using accelerometer data.

INTRODUCTION

There is an ever-increasing interest in quantifying gait dynamics using nonlinear methods. The calculation of the Lyapunov Exponents (LyE) is used as a method to assess the sensitivity of gait to small perturbations, also known as local dynamic stability. The LyE calculates the rate of divergences between neighboring trajectories in the reconstructed phase space which describes the overall dynamics of a system (Dingwell and Cusumano, 2000; Kantz and Schreiber, 2004). The ability of the LyE to quantify gait instability (Dingwell and Cusumano, 2000; Granata and Lockhart, 2008b) and be used determine fall risk (Daniel Hamacher et al., 2015; Lockhart and Liu, 2008) has been well established in the literature. However, there are many variations reported in the literature, primarily due to the lack of standardization in the methodology of its calculation.

Standardization is difficult to achieve because there are many parameters that need to be standardized, these range from algorithm choice and how data is normalized to the amount of data used in the final calculation. In practice, there are two main algorithms that have been used to calculate the LyE in gait: the Wolf *et al.* (1985) (W-algorithm) and the Rosenstein *et al.* (1993) (R-algorithm). The R-algorithm is generally favored because it is more robust against noise for small data sets, but there have been conflicting studies (Bruijn et al., 2009b; Cignetti et al., 2012b; Rispens et al., 2016, 2014a) about its precision and reliability. In addition to the difference in algorithm choice, previous studies have also applied various time series normalization procedures. The most common of which are:

1) Raw Gait Cycle data (gc): The time series is truncated to keep a fixed number of strides regardless of the total number of data points. This maintains the original

distance between points in the phase space but allows for individuals with a faster pace to have fewer data point available over all for the calculation.

2) Gait Cycle Normalized (gcNorm): As in the first method the time series is segmented to a include a fixed number of strides. Then each stride is resampled to have a fixed number of data points, usually 100. Therefore, all strides in this method will contain the same number of data points regardless of an individual's stride time.
3) Data Point Normalized (dpNorm): The time series is first truncated to include a fixed number of strides. Then the data is resampled to a specific number of total samples for the time series. This allows for fluctuations in data length for individual strides.

Recently, it has been found that different normalization methods might be more advantageous when used with either the R or the W-algorithms (Raffalt et al., 2019).

Another obstacle in standardizing the protocol for calculating LyE is deciding how many strides are required. A wide variety of stride lengths have been used in past studies, ranging from 10 strides or fewer strides in some studies (Chini et al., 2017; Eduardo Cofré Lizama et al., 2015; Huisinga et al., 2013; Rispens et al., 2016; Sloot et al., 2011; Van Schooten et al., 2014) to 200 strides or more strides in other studies (Liu et al., 2019; Terrier and Reynard, 2018; van Schooten et al., 2013). The median number of strides used by papers published before 2018 was 110 strides (Mehdizadeh, 2018). Many factors influence the data length requirements used for each study. As an example, some studies use shorter data lengths as it can be difficult for elderly or frail patients to perform extended walking tests. The effect of data length has been studied with respect to motion capture marker displacement (Bruijn et al., 2009b; Cignetti et al., 2012b) and joint angles (Raffalt et al., 2018b) using the R- and W-algorithm, respectively. However, the effect of data length using accelerometer data (Reynard and Terrier, 2014; F. Riva et al., 2014) has only been investigated with the R-algorithm. These studies found conflicting minimum number of strides from 150 (Bruijn et al., 2009b) to 90 (F. Riva et al., 2014) to 54 (Terrier and Reynard, 2014) strides while other studies claim a range of 2 to 3 minutes of data (Cignetti et al., 2012b) is sufficient. It's possible that these differences can be reconciled by looking at the combinations of algorithms and sensors used.

There is currently no literature on the effect of data length on the calculated LyE using both the Rosenstein and Wolf algorithms for accelerometer data. There have also been no studies to determine whether time normalization methods affect the minimum data length for accelerometers. The aim of this investigation was two-fold, (1) to assess the effect of data length on the LyE and (2) determine the minimum number of required strides using both the R- and the W-algorithm under three different time series normalization methods. To achieve this, we recorded three-dimensional accelerations of the lumbar from young healthy subjects who walked on a treadmill at their preferred walking speed for 30 minutes.

METHODS

Seventeen young healthy subjects (10 males and 7 females) were included with a mean $\pm$ standard deviation age of 23.9 $\pm$ 3.5 years, body height of 1.72 $\pm$ 0.11 meters, and body mass of 74.1 $\pm$ 18.6 kg. All subjects were physically active and familiar with walking on a treadmill. Subjects reported no cardiovascular issues, neurological diseases, nor lower extremity surgeries in the last 3 months. Subjects gave written informed consent before participating in this study, which was approved by the Institutional Review Board of Arizona State University.

Participants wore three tri-axial acceleration sensors (APDM, Mobility Lab, APDM, Inc., Portland, OR) with a sampling frequency of 128 Hz. The accelerometers were fitted with elastic bands and Velcro straps and placed at each ankle and the lower lumbar, around vertebrae L4. After subjects became familiar with the treadmill in their own sneakers, each subject's preferred walking speed (PWS) was determined using a standardized protocol (Dingwell and Marin, 2006). The mean and standard deviation of PWS was 1.15 ± 0.09 m/s. After a short rest period, each subject walked on the treadmill for 30 minutes at their PWS. Measurements were started 30 seconds after the treadmill and the subject reached the PWS. Three-dimensional acceleration data of the lumbar sensor were used for all of the calculations in this paper.

Data Analysis

Raw data was used to avoid problems associated with filtering nonlinear signals (Kantz and Schreiber, 2004). The heel contacts for each step were determined and indexed and the time series was truncated to start and end on a heel contact (Dingwell et al., 2001; England and Granata, 2007). From this, different data lengths, ranging from 30 to 1300 strides were extracted. Each data length was then processed using three normalization procedures: (1) Fixed number of strides with a variable number of data points per stride, gc; (2) Fixed number of strides with a 100 data points per stride, gcNorm; (3) Fixed number of strides with a fixed number of data point in the time series(100 points for each stride used), dpNorm.

The LyE was calculated for every direction using each of the preprocessing methods and both the R- and W-algorithms. Briefly, the R-algorithm calculates the average divergence distance of all possible nearest neighbor pairs in the phase space. This is tracked through time creating a mean divergence curve. The LyE is then estimated using a least-squares fit to the linear slope of the divergence curve. The slope was estimated from 0 – 0.5 strides when normalized (gcNorm and dpNorm) gait cycles were being analyzed as it was found to be more reliable than 0-1 strides (Reynard et al., 2014). In order to compare normalized and raw (gc) gait data, the average half stride length for each subject were used as the bounds for taking the slope. For example, if a subject had an average stride length of 150 samples, then the slope of the mean divergence curve was taken of the first 75 points. The W-algorithm, on the other hands, tracks a single reference trajectory in the phase space and its nearest neighbor until the separation between the two paths exceeds a specific limit. Then a new nearest neighbor is found, and the rate of expansion or contraction is calculated again. The final rate of divergence is calculated from the expansion and contraction rates.

The phase space was reconstructed from each acceleration direction using the method of delays. A constant time delay of 10 was used across all preprocessing methods and directions were used and an embedding dimension of $d_E = 5$ was chosen for the R-algorithm and an $d_E = 7$ was used for the W-algorithm. Additionally, a time evolution of 7 was used in the calculation of the W-algorithm. All calculations were performed using custom MATLAB programs (version 2018b, Mathworks Inc., Natwick).

Statistical Analysis

The Friedman test, a nonparametric repeated measures ANOVA, was used to explore the effect of data length on the LyE. The nonparametric test was used for all analyses because the assumption of sphericity was violated and because not all of the parameters were normally distributed. This test was performed independently for each acceleration direction, algorithm choice and preprocessing method. A post-hoc pairwise comparison with a Bonferroni correction for multiple comparisons was then used to determine the specific differences between select data lengths for each algorithm-normalization combination in all three accelerometer directions. For all statistical tests a p-value < 0.05 was considered significant. Statistical analysis was performed using SPSS (version 25, IBM, USA).

Additionally, to determine the minimum number of required strides, we calculated interquartile range/median ratio (*imr*) of the LyE for windows of decreasing length (from 300 to 30 strides, 1 stride resolution). The interquartile range and median value of the LyE was calculated for all data length windows starting at 300 strides and progressing backwards. This method was adapted from Riva et al. (2014). In this context percent *imr* indicates variations about the median with the lowest *imr* occurring at the largest window. As the LyEs from decreasing windows of strides are added to the pool, a new *imr* will be calculated. A consistently low *imr* as the number of included strides increases will indicate when a steady state value has been reached. Conversely, a high *imr* reveals that the measure undergoes large variations as the number of strides increases; this means that the measure it not fully reliable. A threshold of 10% was used to define the smallest required number of strides. This low *imr* threshold was set to ensure a reliable LyE would be calculated from the final minimum number of strides. The minimum number of strides was calculated per subject for each direction and algorithm-preprocessing method combination. Then the largest number of strides across all subjects was selected as the recommended minimum number of strides.

RESULTS

We found that data length significantly affects the value of the LyE calculated using both the R- and W-algorithm in every direction and for all preprocessing methods ($p < 0.0005$, for each). Figure 5-1 and Figure 5-2 show the results of how data length changes the LyE when the R-algorithm and the W-algorithm are used, respectively. As can be seen, the R-algorithm saturates between 300 and 500 strides, in each direction regardless of how the data was preprocessed. While W-algorithm has a less obvious saturation point due to the large standard deviations of the LyE in each direction. All statistical tables are presented in full in the appendix.

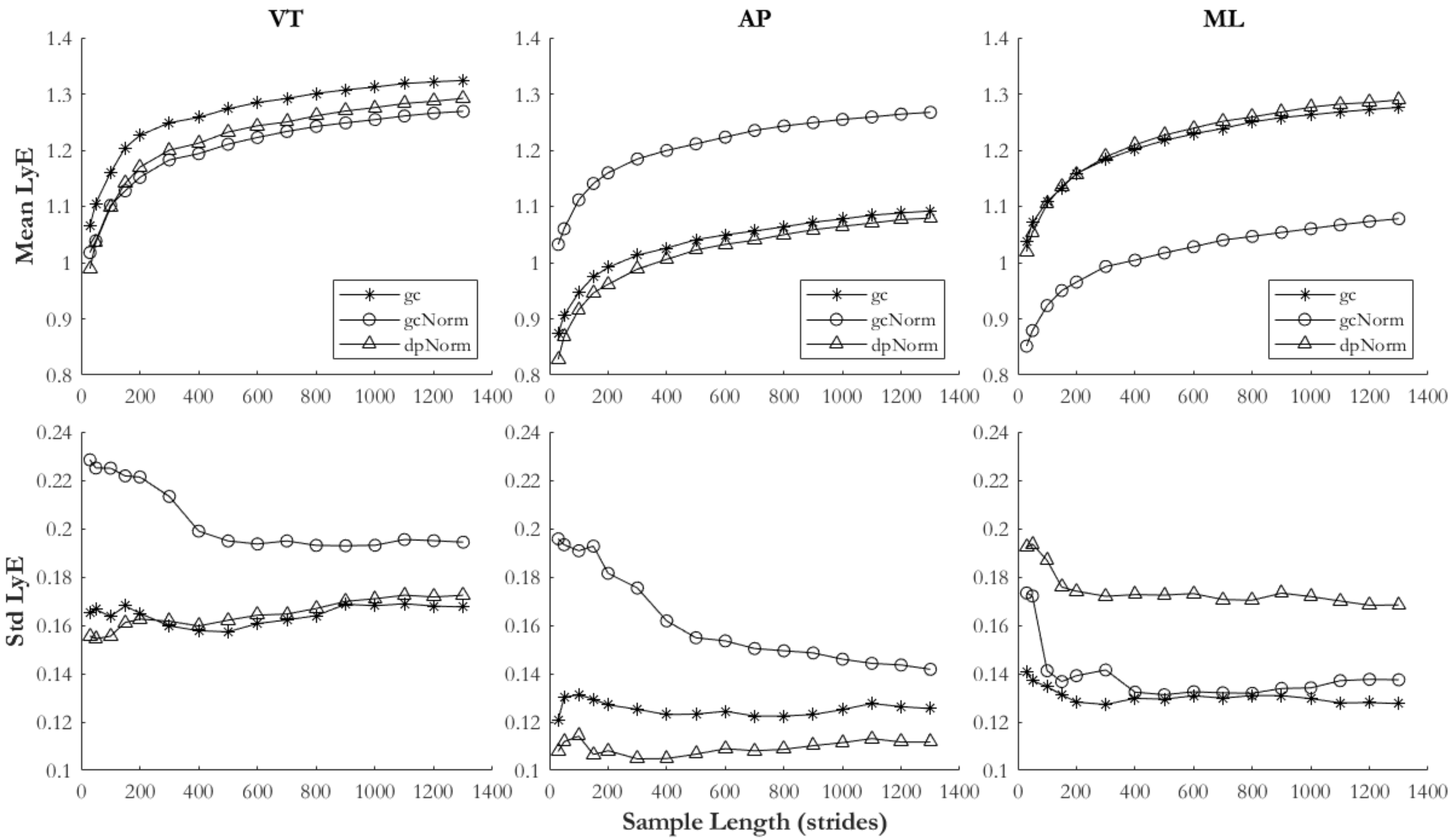

Figure 5-1: Data length effect on the LyE using the Rosenstein *et al.* algorithm. (*) Raw gait, (o) gait cycle normalization, and (Δ) data point normalization. Top panels show the mean for each of the different sample lengths; bottom panels show standard deviations.

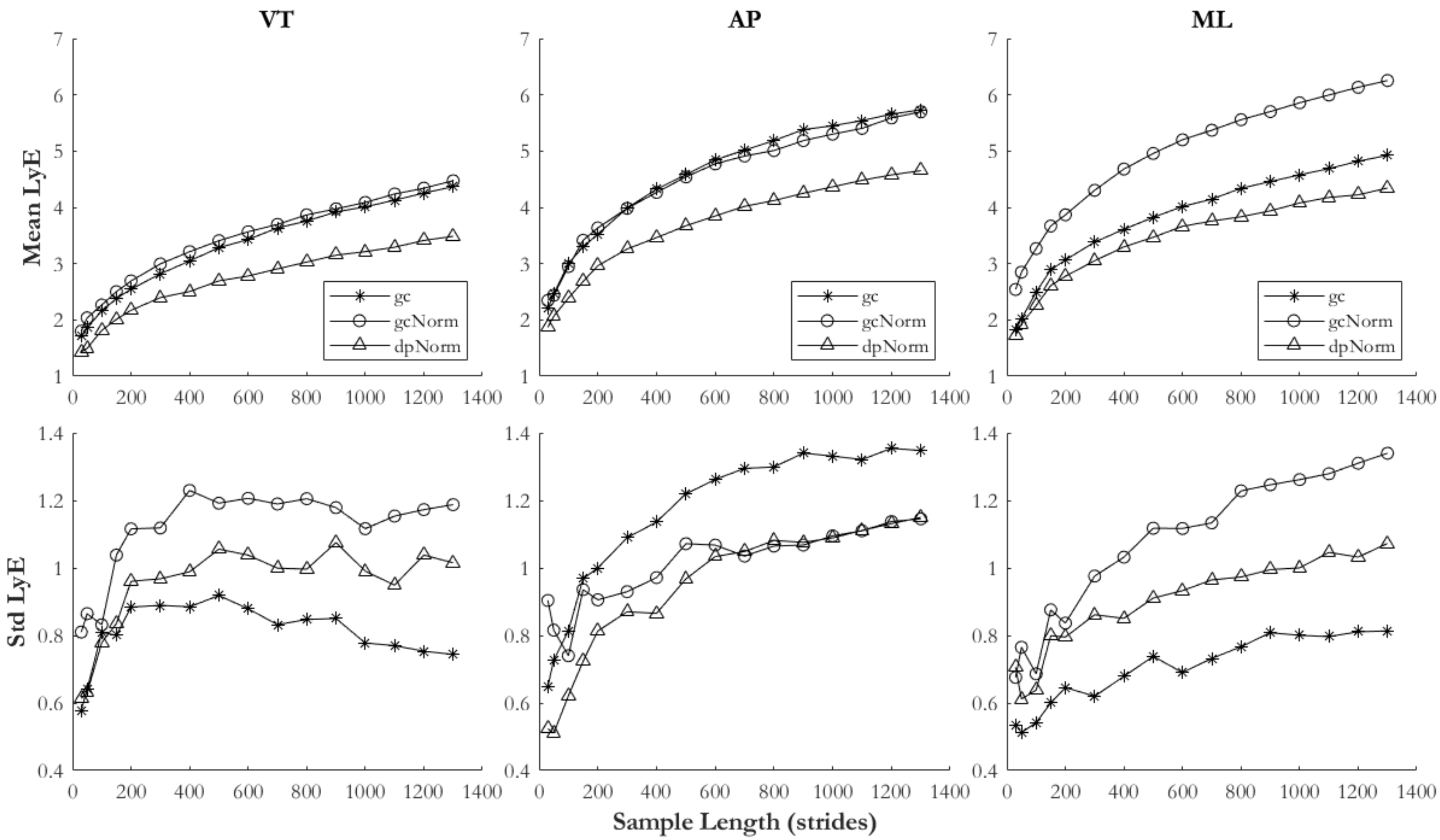

Figure 5-2: Data length effect on the LyE using the Wolf *et al.* algorithm. (*) Raw gait, (o) gait cycle normalization, and (Δ) data point normalization. Top panels show the mean for each of the different sample lengths; bottom panels show standard deviations.

Common data lengths (30, 50, 100, 150, and 200 strides) and longer than usual data lengths (300, 500, and 1000 strides) were compared to determine specifically how data length affects the LyE using an accelerometer. The statistical pairwise comparison across normalization methods and for both the R and W-algorithm when using VT acceleration data is shown in Figure 5-3. Across all algorithms, preprocessing methods, and signal directions, we found that there existed a sliding window of non-significance as best seen in Figure 5-3 (*top left*, R-algorithm used with raw gait). In the R-algorithm, this window increased to include no significant differences between 30 and 150 strides when raw gait was used in the ML direction, as well as, when gcNorm method was used in the VT and ML directions. Additionally, in the gcNorm method, no significant differences were found between 150 and 500 strides in the VT (Figure 5-3, *center left*) and between 200 and 1000 strides in the AP direction. When using the W-algorithm, the VT acceleration has the same significant differences across both the AP and ML direction when using gc and gcNorm processing methods. When dpNorm is used with AP and ML directions the non-significance window for 30 gait cycles expanded to include 150 gait cycles, while this window remained at 100 gait cycles when using the VT direction.

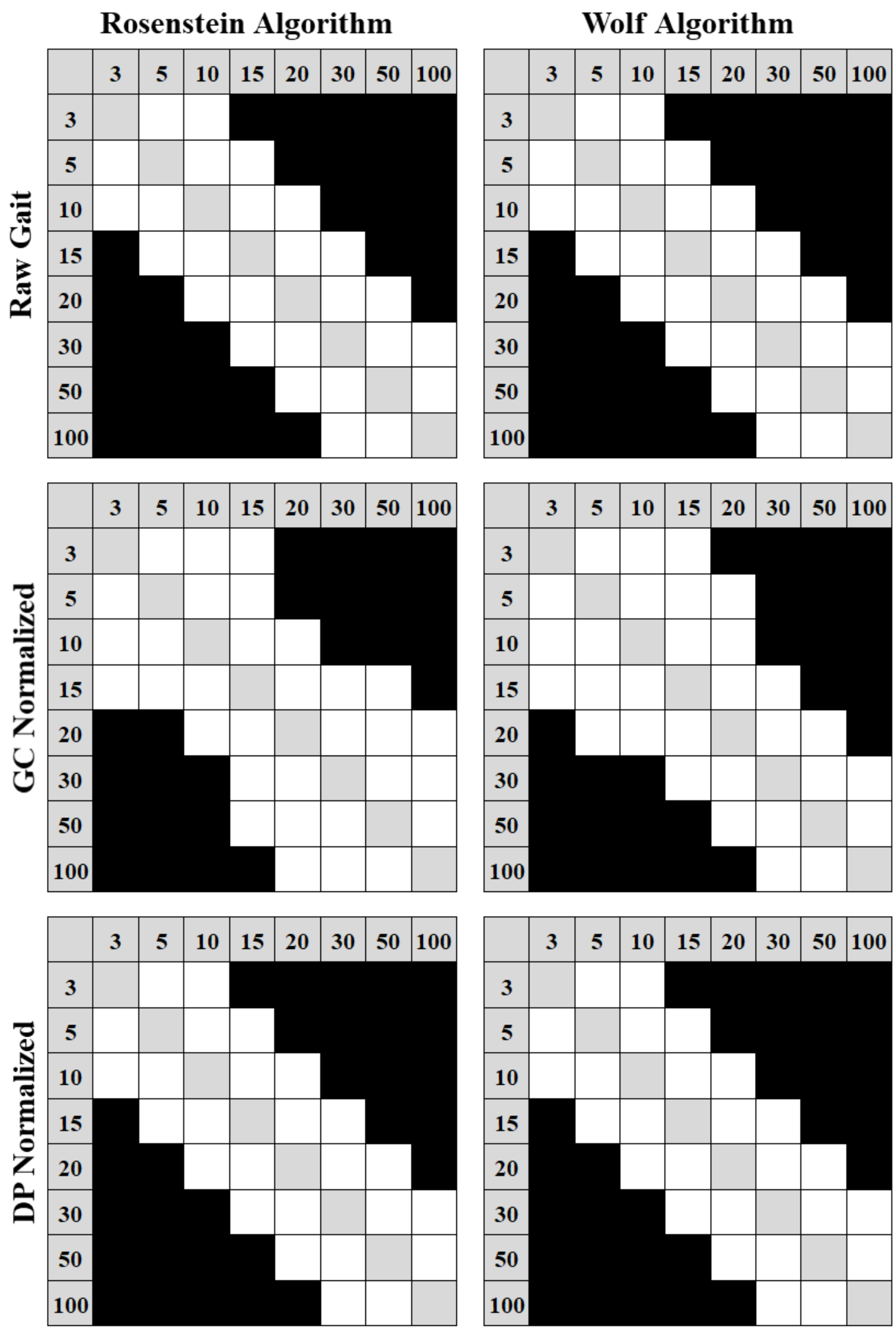

Figure 5-3: Statistical results on the effect of data length on the LyE using the R- and W-algorithm and all three data pre-processing methods. This graphic shows the significant differences between each data length. Data length in figure is reduce by a factor of ten. Only the VT direction results are shown above. Filled in (black) boxes indicate significant differences and empty (white) boxes show where there are no significant differences between a pair of data lengths.

Table 5-1: Number of required strides for each algorithm, preprocessing method, and acceleration direction at the 10% threshold. Additionally, reliability of each measure based on the maximum inter-subject *imr* is shown with the median values of the inter-subjects' medians for reference.

		Min. Number of Strides		**Max inter-subject *imr***		**Median inter-subject LyE**	
Normalization	**Dir.**	**R**	**W**	**R**	**W**	**R**	**W**
gc	VT	74	197	14%	39%	1.18	2.19
	AP	38	200	11%	33%	0.94	3.20
	ML	40	204	10%	36%	1.16	2.79
gcNorm	VT	69	222	11%	41%	1.09	2.00
	AP	66	196	13%	34%	1.11	2.92
	ML	30	174	10%	31%	0.95	3.12
dpNorm	VT	78	192	15%	30%	1.09	1.81
	AP	56	182	11%	32%	0.93	2.58
	ML	49	201	12%	32%	1.08	2.55

The minimum number of required strides for each of the 6 algorithm-normalization method combinations in the VT, AP, and ML direction are reported in Table 5-1. The R-algorithm had a lower minimum number of required strides and maximum variation about the median (*imr*) than the W-algorithm. The R-algorithm required between 30 and 78 strides across all directions and preprocessing methods, while the W-algorithm required a large number of strides of at least 174 strides and up to 222 strides.

DISCUSSION

While LyE is a commonly used nonlinear dynamical measures in assessing gait stability, there is still no single protocol for how to apply this measure. This includes decisions of which algorithm to use based on data collection methods (e.g. IMU vs motion capture), how much data is required (in number of strides or in time duration) and then finally how to process the data before reconstructing the phase space. The aim of this study was to understand how data length affects the LyE when the Rosenstein *et al.* and the Wolf *et al.* algorithms were used for each of the three pre-processing/normalization methods. We compared common data lengths (30, 50, 100, 150, and 200 strides) with longer than usual data lengths (300, 500, and 1000 strides) to determine specifically how data length affects the LyE using an accelerometer. We found that there existed a sliding window of non-significance across all algorithms, preprocessing methods, and signal directions. Figure 5-3 and additional figures provided in Appendix B can be utilized as a guide for appropriate and inappropriate comparisons of results across studies. For instance, if a study used 100 gait cycles it should have similar LyE results as a study that used as low as 30 strides and no more than 200 strides. However, different algorithms are not comparable, and caution should be used when comparing normalization methods.

Prior research has explored the effect of data length (Reynard and Terrier, 2014; F. Riva et al., 2014) for IMU data but each study used different preprocessing methods and only examined the R-algorithm. Other studies (Bruijn et al., 2009b; Raffalt et al., 2018b) have also used motion capture data to explore this question using both the R- and W-algorithm. Only recently has research begun to look at the effect of different algorithms with different normalization methods in motion capture data (Raffalt et al., 2019). In this study, we found that the reliability of each of the three normalization methods we tested had good to average (10-15%) reliability for the R-algorithm but poor to very poor (30-41%) reliability for the W-algorithm. None of the normalization methods outperformed the other when the R-algorithm was used. For the W-algorithm, the data point normalization method had better reliability (30-32%) than both the raw gait (33-39%) and gait cycle normalization method (31-41%). This is contrary to previous findings by Raffalt *et al.*

(2019), but this could be due to different experimental instrumentation; the current study used accelerometer data while motion capture data was used in the other study.

The R-algorithm had smaller standard deviations (SD) across all normalization methods and acceleration directions (Figure 5-1) compared to the W-algorithm. The gc method (0.16, 0.13, 0.13 in the VT, AP, ML directions, respectively) and the dpNorm method (0.16, 0.11, 0.18) had fairly similar SD in the VT and AP direction. The gcNorm method had the largest SD (0.21, 0.16, 0.14) in all directions except for the ML compared to the other two processing methods when using the R-algorithm. These standard deviations are larger than what has been reported in the literature (Rispens et al., 2014b) but not far from others (Terrier and Dériaz, 2011). When the W-algorithm was used the gcNorm method also had the largest SD (1.10, 1.00, 1.06, in the VT, AP, and ML directions). The gc method (0.80, 1.14, 0.69) had smaller SD in the VT and ML directions, while the dpNorm method (0.93, 0.91, 0.89) had the smallest SD in the AP direction, shown in Figure 5-2. From these standard deviations we can infer that the R-algorithm is more precise for accelerometer data. This is further supported by Table 3-1, where the minimum number of required strides and maximum variation about the median (*imr)* is much less for the R-algorithm than the W-algorithm.

We recommend that 50-100 strides should be used when computing the LyE with the R-algorithm based on the statistical comparison of data length and the minimum required number of strides calculated using *imr*. For the W-algorithm, we recommend at least 200 strides should be used when calculating the LyE from accelerometer data. The minimum number of required strides found in this study were lower than previously published values (F. Riva et al., 2014), however this is believed to be due to differences in how the slope of the mean divergence curve was calculated. In the previous study, the slope was taken over an entire stride, while this study took the slope of the divergence curve for a single step. A single step was used instead of a stride, because it has been shown to be more reliable. (Reynard et al., 2014) Reliability as measured by percent *imr* was less than 15% across all preprocessing methods with the R-algorithm, with gcNorm having the best reliability overall. The W-algorithm has poor reliability with a percent *imr* between 30%-41%, with dpNorm having the best reliability of 30-32%.

Several limitations existed in the current study. First, we only tested young healthy subjects. Second, the performance of walking on a treadmill is significantly different than walking over ground (Dingwell and Cusumano, 2000; Terrier and Dériaz, 2011). However, treadmill walking was necessary to collect 30 minutes of continuous and uninterrupted walking for this experiment. Additional studies will be necessary to confirm that similar results are found across subject populations and in over ground walking.

Overall, we conclude that the R-algorithm has better precision than the W-algorithm for accelerometer data. This is consistent with a previous algorithm comparison study (Rispens et al., 2014a) which found that the R-algorithm had equal to or greater precision than the W-algorithm when using known nonlinear systems. Notably, the R-algorithm may not have the same advantages over the W-algorithm when joint angles are used for calculating the LyE (Raffalt et al., 2018a). The previous study also found that the W-algorithm was more accurately calculate the "true" LyE as long as the signal length was sufficient the noise levels were low. Generally, biological data is on the noisier side of the spectrum of ideal to noisy data. Therefore, it is understandable that the W-algorithm gives less precise measurements for gait data. It can be argued that precision is more important in the estimation of the LyE for gait because the reproducibility of the LyE value is critical for creating diagnostic tools, etc. rather than the accuracy of the "true" LyE.

CONCLUSION

The importance of creating a standardized protocol for calculating the LyE based on the data collection instrumentation cannot be stressed enough. The standardization of this process will open the doors to the application of LyE in clinical trials and to create diagnostic tools which require an easily reproducible and precise measure. In this paper we investigate the implications of data length when calculating the LyE under 6 algorithm-preprocessing method combinations for accelerometer data. We found that different data lengths can be compared against a given range of data lengths across publications. For example, if one paper used 50 strides you can compare this to publications that used similar methodologies from 30 to 150 strides. We also contribute to the literature about the minimum data requirements for calculating the LyE based on algorithm and preprocessing method choices for future reference. Finally, we recommend using the R-algorithm over the W-algorithm for accelerometer data due to better precision (calculated by the SD) and reliability (calculated by % *imr*) found in this study.

CHAPTER 6: MINIMUM NUMBER OF STRIDES TO CALCULATE THE LYAPUNOV EXPONENT USING ROSENSTEIN'S AND WOLF'S ALGORITHMS FOR YOUNG AND ELDERLY ADULTS

ABSTRACT

Falls are the leading cause of disability in older adults with a third of adults over the age of 65 falling every year. Quantitative fall risk assessments using inertial measurement units and Lyapunov exponents (LyE) have shown that it is possible to identify at-risk individuals. However, there are inconsistencies in the literature on how to calculate LyE and how much data is required for a reliable result. This study investigates the reliability and minimum required strides for 6 algorithm-normalization method combinations when computing LyE using young healthy and community dwelling elderly individuals. Participants wore an accelerometer at the lower lumbar while they walked for three minutes up and down a long hallway. This study concluded that the Rosenstein *et al.* algorithm was successfully and reliably able to differentiate between both populations using only 50 strides. It was also found that normalizing the gait time series data by either truncating the data using a fixed number of strides or using a fixed number of strides and normalizing the entire time series to a fixed number of data points performed better when using the Rosenstein *et al.* algorithm.

INTRODUCTION

Falls are among the most common cause of decreased mobility and independence in older adults and rank as one of the most serious public health problems in the U.S., with costs exceeding $50 billion in 2015 (Ambrose et al., 2013; Bergen et al., 2016; Burns et al., 2016; Weisenfluh et al., 2012). Analogous to this reduction is the inherent decline in gait stability that impairs balance and predisposes older adults to falls and fall-related injuries. Dynamic stability, defined as the ability to maintain equilibrium despite the presence of small disturbances or control errors, is a fundamental motor task that must be rapidly adapted in the face of a dynamically varying environment (Dingwell et al., 2001; Dingwell and Cusumano, 2000; Wurdeman, 2016). Evidence suggests that older adults experience a gradual deterioration in these balance mechanisms and may require more task-dependent rehabilitative and training interventions. Quantitative assessment of gait has been shown to identify age-related decrements, fall risk and pathology (Bruijn et al., 2013; Hamacher et al., 2011; Toebes et al., 2012). In particular, gait measures derived from trunk acceleration signals can characterize trunk movement dynamics that regulate gait-related oscillations. However, aging may induce subtle impairments in gait without obvious detectable unsteadiness; therefore, nonlinear measures which are able to detect the hidden, subtle characteristics of aging in detrimental effects on locomotor control are used. In particular, Lyapunov exponents (LyE), also known as local dynamic stability, has become a popular approach for quantifying gait stability during continuous walking.

Modern motion capture laboratories collect precise data during walking and postural stability tasks; however, they are prohibitively expensive, immobile, and require well trained technicians to collect and process experimental results. Inertial measurement units (IMUs) or accelerometers have become widely used in assessing and monitoring gait and other daily living activities as an alternative to traditional motion capture. These sensors are more flexible, mobile, and inexpensive. They also have the advantage of unlimited measurement volume and the opportunity of recording gait in various environments – e.g. clinical offices, community centers, or outdoor tracks – with ease (Tao et al., 2012). Accelerometers and LyE have been used together as biomarkers for differentiating between healthy controls and various ailments, e.g. patient with dementia (IJmker and Lamoth, 2012), multiple sclerosis (Huisinga et al., 2013), and concussions (Fino, 2016). However, not all of these studies are comparable. Some studies use different data collection equipment, algorithms, and/or normalization methods. And even when publications research similar paradigms, some studies find significant differences while others do not. This could be due to sample and effect size within particular studies, but the inconsistency across publications could also be due to the lack of a universal methodology for calculating the LyE during gait.

To date, there has been several pivotal publications about the issues in calculating the LyE when using gait data and how various factors can impact the value of the LyE. In this study we will focus on the choice of algorithm and normalization methods used and examine their reliability and determine the minimum number of required strides for reliable computation in both young healthy and elderly adults. The most common algorithms for calculating LyE in gait are the Rosenstein *et al.* (R-algorithm) and Wolf *et al.* (W-algorithm) algorithms. We hypothesize that each algorithm will require significantly different number of strides for the calculation of LyE. Additionally, different time series normalization methods have also been shown to affect the LyE and that different normalization methods work better for different LyE algorithms (Raffalt et al., 2019; Stenum et al., 2014). Therefore, we will investigate three of the most common normalization methods with both the R- and W-algorithm. We hypothesize that normalization methods will affect the reliability of the calculated LyE. These findings augment wearable sensor's potential as an ambulatory fall risk identification tool in community-dwelling settings. Furthermore, they highlight the importance of gait features that rely less on step-detection methods, and more on time series analysis techniques in the community-dwelling elderly population.

METHODS

Seventeen young healthy adults participated in this study and eleven community dwelling older adult's data from an ongoing fall risk assessment study was used. All subjects reported no cardiovascular issues, neurological diseases, nor lower extremity surgeries in the last 3 months. Additionally, the elderly participants were required to be able to perform a 2-3-minute walk without the aid of a cane or a walker. Table 6-1 summarized each groups' subject characteristics. All subjects gave written informed consent before participating in this study, which was approved by the Institutional Review Board of Arizona State University.

Young healthy participants wore three tri-axial acceleration sensors (APDM, Mobility Lab, APDM, Inc., Portland, OR) with a sampling frequency of 128 Hz. The accelerometers were fitted with elastic bands and Velcro straps and placed at each ankle and the lower lumbar, around vertebrae L5. Elderly participants wore a single accelerometer (DynaPort, McRoberts, Den Haag, the Netherlands) at the lower lumbar attached to elastic bands with a sampling frequency of 100 Hz. All participants were asked to walk for 3 minutes on a makeshift walking track at their preferred walking speed. This track was secluded so no outside factors could interfere with or interrupt the data collection. Ten seconds were removed from the beginning and end of the acceleration measurements to avoid non-stationary periods. The trials from young healthy participants was down sampled to 100 Hz to match the elderly community dwelling data collection.

Table 6-1:Subject characteristics

	Young Adults	**Older Adults**
Gender (M/F)	11/7	2/9
Age (years)	23.9 ± 3.5	79.4 ± 7.9
Height (cm)	171.8 ± 11.4	169.7 ± 10.4
Weight (kg)	74.1 ± 18.6	77.3 ± 16.5
BMI	24.9 ± 4.4	26.9 ± 5.5

The following three preprocessing normalization methods were applied before calculating the LyE:

(1) Raw Gait Cycle data (gc): The time series is truncated to keep a fixed number of strides regardless of the total number of data points. This maintains the original distance between points in the phase space but allows for individuals with a faster pace to have fewer data point available over all for the calculation.

(2) Gait Cycle Normalized (gcNorm): As in the first method the time series is segmented to a include a fixed number of strides. Then each stride is resampled to have a fixed number of data points, usually 100. Therefore, all strides in this method will contain the same number of data points regardless of an individual's stride time.

(3) Data Point Normalized (dpNorm): The time series is first truncated to include a fixed number of strides. Then the data is resampled to a specific number oftotal samples for the time series. This allows for fluctuations in data length for individual strides.

For method (3), the total number of data points in the series was allocated 100 samples for every stride used. A time delay of 10 samples was used for all directions and all preprocessing methods. An embedding dimension of 5 was used when the LyE is calculated using the R-algorithm and a dimension of 7 was used for the W-algorithm. In a previous chapter, these values were proven acceptable for treadmill walking of young health adults when using 150 strides in the LyE calculation. In Appendix C we perform a similar analysis for both the young and elderly adults using 100 strides. Based on those results, we concluded that the same time delay and embedding dimensions are also sufficient for our current study. The LyE was calculated for all 6 algorithm-normalization method combinations since neither the R-algorithm nor the W-algorithm have been proven to outperform the other and both are widely used with gait data calculations (Mehdizadeh, 2018; Rosenstein et al., 1993; Wolf et al., 1985). The LyE was taken from 0 to 0.5 strides using the R-algorithm. Additionally, a time evolution of 7 was found to be appropriate for calculating the LyE with the W-algorithm.

To determine the minimum number of strides, we use the same procedure as Riva et al. (2014) using interquartile range/median ratio (*imr*). Briefly, the LyE was calculated using decreasing windows of strides, from 120 to 10 strides with a resolution of 1 stride. The imr is calculated starting from the largest window (which gives the smallest ratio) and proceeds to the smallest window. The minimum number of strides was calculated per index and per subject at an *imr* threshold of 10%. Then the largest number of strides required across all subjects was chosen. Percent *imr* is an indication of the variation around the median. When variations of the measure around the median value are small, *imr* percentage will be low. This is indicative of a steady state being reached.

Additionally, statistical differences between population groups were compared to test the effectiveness of algorithm and normalization method combinations. The groups were compared based on the found sufficient number of strides when using *imr*. A one-way ANCOVA was used for each directional signal –anteroposterior (AP), vertical (VT), and mediolateral (ML) – with respect to both algorithms, while population and normalization methods were used as model effects. A post-hoc Tukey was then used to determine differences between each of the model effects.

RESULTS

Algorithm and preprocessing method choice affected the number of strides required to reach a steady state using the 10% threshold. The minimum required strides for calculating the LyE are summarized in

Table 6-2 by subject group.

Table 6-2: Number of required strides for calculating the LyE using different normalization methods. Values used a 10% imr threshold for both young health (YH) and elderly adults (EA).

			Min. Number of Strides		
Group	**Algorithm**	**Dir.**	**gc**	**gcNorm**	**dpNorm**
YH	**Rosenstein**	VT	47	72	41
		AP	44	40	45
		ML	41	26	46
	Wolf	VT	96	109	99
		AP	98	112	108
		ML	117	113	113
EA	**Rosenstein**	VT	41	43	36
		AP	31	36	24
		ML	60	46	55
	Wolf	VT	92	105	89
		AP	101	75	81
		ML	98	114	120

For Rosenstein *et al.* algorithm, generally 50 strides was sufficient for the young healthy adults to calculate the LyE with any normalization method. The minimum number of strides for gc and dpNorm methods did not vary greatly when different acceleration directions were used. While the number of required strides for gcNorm method heavily depended on the acceleration direction. The elderly adults usually required less than 50 strides to calculate the LyE. Acceleration direction had more of an effect on the number of strides than any of the preprocessing methods. The required number of strides increased from the AP to the VT and then to ML direction, respectively.

The Wolf *et al.* algorithm required twice the number of strides compared to the Rosenstein algorithm. For the young healthy, gcNorm and dpNorm methods required approximately 110 strides for all directions, while gc required 98 strides for VT and AP directions and 117 for the ML direction. The required number of strides for the elderly were less consistent than the young healthy and heavily depended on the normalization method.

Table 6-3: Reliability of LyE calculated for young healthy (YH) and community dwelling elderly adults (EA). Reliability is based on the maximum inter-subject *imr*. The median values of inter-subjects' medians have been included for reference values.

			Maximum inter-subject *imr*			**Median inter-subject value of LyE**		
Group	**Algorithm**	**Dir.**	**gc**	**gcNorm**	**dpNorm**	**gc**	**gcNorm**	**dpNorm**
YH	**Rosenstein**	VT	17%	19%	16%	1.04	1.00	1.09
		AP	16%	15%	15%	0.89	1.06	0.93
		ML	19%	16%	18%	1.06	0.88	1.10
	Wolf	VT	36%	41%	35%	1.48	1.56	1.59
		AP	29%	51%	32%	2.04	2.08	2.20
		ML	35%	33%	39%	1.83	2.29	2.14
EA	**Rosenstein**	VT	19%	19%	19%	1.29	1.12	1.31
		AP	12%	19%	13%	1.13	1.05	1.18
		ML	20%	16%	19%	1.19	1.12	1.21
	Wolf	VT	32%	32%	32%	1.70	1.49	1.78
		AP	27%	28%	20%	2.64	2.44	2.60
		ML	23%	43%	21%	1.94	1.77	2.21

The reliability results are shown in Table 6-3. The maximum inter-subject *imr* was less than 20% for both young healthy and elderly adults when using the R-algorithm. The W-algorithm ranged from 29% to 51% for young healthy subjects and 20% to 43% for elderly adults. The median inter-subject value of the LyE is also provided as a reference for both young and community dwelling elderly adults.

Lastly, the two populations were compared when 50 and 75 strides were used with the R-algorithm and when 110 strides were used with the W-algorithm, shown in Table 6-4. Significant differences between the two population groups were found using the AP signal when both data lengths were used with the gc and dpNorm normalization methods. The normalization methods also found significant differences in the VT signal when 75 strides were used in the calculation. No significant differences between young healthy and community dwelling elderly adults were found when using the W-algorithm and any of the normalization methods.

Table 6-4: Significant differences between young health and elderly community dwelling adults

	Normalization	**VT**	**AP**	**ML**
R-algorithm 50 strides	gc	0.0942	0.0001*	NS
	gcNorm	NS	NS	NS
	dpNorm	0.1025	0.0001*	NS
R-algorithm 75 strides	gc	0.0344*	0.0001*	0.4890
	gcNorm	NS	NS	NS
	dpNorm	0.0273*	0.0001*	0.4867
W-algorithm 110 strides	gc	NS	NS	NS
	gcNorm	NS	NS	NS
	dpNorm	NS	NS	NS

NS represent no significance with $p > 0.5$

DISCUSSION

Gait stability is directly quantified by the LyE value. However, the implementation parameters are ill-defined and lack standardization procedures. Therefore, the aim of the present study was to investigate the reliability of the LyE and determine the minimum number of strides for its calculation using 6 algorithm-normalization method combinations. The Rosenstein *et al.* and the Wolf *et al.* algorithms were used along with three preprocessing methods: gc. gcNorm, and dpNorm. The R-algorithm required a significantly smaller number of steps with good reliability compared to the W-algorithm which only achieved average to poor reliability. And only the R-algorithm was able to differentiate the young healthy and elderly community-dwelling adults.

The minimum number of strides required for the R-algorithm were found to be much smaller than previously reported (F. Riva et al., 2014); this may be due to differences in methodology. The present study calculated the LyE using a single step, while Riva *et al*. calculated it from a stride. Even though our method requires less strides, it was deemed more reliable based on the maximum inter-subject *imr* values -- *imr* values rank reliability scores accordingly: excellent (*imr* < 10%), good (*imr* =10-20%), average (*imr* =20-30%), poor (*imr* =30-40%), and very poor (*imr* > 40%). The R-algorithm had good reliability in this study for both young healthy and community-dwelling older adults, while Riva *et al.* reported only average reliability for its young healthy subjects. This is the first paper, to the authors' knowledge, that has investigated the required minimum number of strides and reliability using *imr* with the W-algorithm. The W-algorithm required between 100 and 110 strides for all normalization methods and population groups which is almost double the number of strides required for the R-algorithm. Additionally, the W-algorithm had average to poor reliability across both populations with gc normalization method performing better for young healthy adults and dpNorm performing better for elderly adults.

The results of the present study also show that the R-algorithm was able to differentiate between both populations while the W-algorithm was unable. Significant differences between elderly and young healthy adults were found in the AP direction which is consistent with the literature (Liu et al., 2012; Lockhart and Liu, 2008). But interestingly, no significant differences were found in the ML direction, which is more commonly reported as significant. (Dennis Hamacher et al., 2015; Terrier and Reynard, 2015) This could be due to different data lengths and normalization methods used in those publications or even differences between over-ground and treadmill walking studies. It is also important to note that not all studies find significant differences between these populations like Bizovska *et al.* (2018). They found no differences in their young and elderly populations in both over-ground and treadmill walking trials.

Recent research has reported that raw gait data is ideal for the W-algorithm, i.e. just signal truncation, while both gcNorm and dpNorm normalization methods should be used for the R-algorithm (Raffalt et al., 2019). When the R-algorithm is used, dpNorm and the gc method had the lowest number of required strides and had good measurement reliability, as interpreted from percent *imr*. Both young healthy and elderly community dwelling participants required less than 60 strides to calculate the LyE. We recommend either the dpNorm or gc method of normalization over the gcNorm method for young healthy subject studies. The Wolf algorithm was more reliable for young healthy adults when raw gait was used than gcNorm or dpNorm methods. The gc method also required less strides for this group. For the community dwelling elderly adults, gc method was slightly less reliable compared to dpNorm method. Additionally, dpNorm required the least amount of data except for in the ML range. However, there isn't a large enough difference between gc and dpNorm to definitively state one normalization method is more advantageous than the other when using the W-algorithm.

The present study has a few key limitations. First, we only calculated the LyE starting from 120 gait cycles. This has been deemed a sufficient data length with limited gains in precision if more strides could have been included. (Bruijn et al., 2009b; Raffalt et al., 2018b; Reynard and Terrier, 2014; F. Riva et al., 2014; Terrier and Reynard, 2014) However, not all of these studies used accelerometers for data collection and there are a limited number of studies on the required number of strides for the W-algorithm. Secondly, there was a much larger proportion of females in the community dwelling elderly participants. This is largely due to participation in ongoing fall risk assessments that meet the criteria of this paper. In theory, the minimum number of strides is not gender based but this was out of scope to be tested in this paper. It should also be noted that the findings of this study were derived from a fairly small sample size, although similar studies have used as many or fewer subjects (Dennis Hamacher et al., 2015; Federico Riva et al., 2014) than the present study.

CONCLUSION

The present study investigated the reliability and minimum required number of strides to using to calculate LyE in young healthy and elderly community dwelling adults. As there is no universally accepted standard methodology for this calculation, 6 algorithm-normalization method combinations were used in order to help work towards creating a standardized process for accelerometers. We found that the Rosenstein *et al.* algorithm requires less strides for reliably calculating the LyE compared to the Wolf *et al.* algorithm. And the R-algorithm was able to differentiate between young healthy and elderly community-dwelling adults in the AP and VT direction using only 75 strides, while the W-algorithm was unable to differentiate these groups when using 110 strides. Our results show that either truncating the gait signal to a fixed number of strides or normalizing the signal to a fixed number of strides with a fixed number of total data points will compute a more reliable LyE when using the R-algorithm.

CHAPTER 7: CONCLUSION

Lyapunov exponents (LyE) calculated from accelerometers have been used as biomarkers for detecting fall risk (Bruijn et al., 2013; Dingwell and Cusumano, 2000; Granata and Lockhart, 2008b; Hamacher et al., 2011; Lockhart and Liu, 2008; Rispens et al., 2015; Toebes et al., 2012) and various ailments: dementia (IJmker and Lamoth, 2012), multiple sclerosis (Huisinga et al., 2013), Parkinson's disease, (Fino et al., 2018) and concussions (Fino, 2016). However, there are inconsistencies in the literature regarding how to compute the LyE for gait. LyE calculation has led to conflicting numerical values for the literature to build upon. Without a unified methodology for calculating the LyE, researchers can only look at the trends found in studies. As inertial measurement units become the prominent method of collecting gait data –due to their flexibility, mobility, and ability to record in any environment – it is important to standardize and tailor the protocol for calculating the LyE. Therefore, this dissertation was dedicated to developing a standardized methodology for calculating the LyE for human gait when using accelerometers. The effects of phase space reconstruction parameters (time delay and embedding dimension), algorithms (Rosenstein *et al.* (1993) and Wolf *et al.* (1985) algorithms), normalization methods (raw, gait cycle normalized, and data point normalized gait data), and amount of data used were investigated using young healthy and elderly community-dwelling adults. The result of this dissertation will allow for biomechanical researchers to utilize LyE while understanding the implications of choosing various input variables associated with its calculation.

The first aim of this dissertation was to develop guidelines for phase space reconstruction for gait data. We investigated the effects of calculating time delay (τ) and embedding dimension (d_E) at different data lengths (Chapter 3). We found that data length does not affect the calculation of time delay, when using average mutual information algorithm. However, calculating the embedding dimension using global false nearest neighbors required 100 strides to reach a steady state value. This study also found that preprocessing the data using a fixed number of strides or a fixed number of data points had significantly different values for time delay compared to a time series that used a fixed number of normalized gait cycles, which have a fixed number of data points per stride. Additionally, the impact of different time delay and embedding dimension combinations on the value of the LyE was investigated for young healthy adults (Chapter 4) and community-dwelling elderly adults (Chapter 6 & Appendix C). This study systematically investigated the effect of time delay, embedding dimension, and three pre-processing methods on the LyE using both the Rosenstein *et al.* (R) and Wolf *et al.* (W) algorithms. We concluded that the time delay can be standardized to $\tau = 10$ (in data points and % gait cycle) while the embedding dimension can be set to $d_E = 5$ for the R-algorithm or $d_E = 7$ for the W-algorithm when calculating the LyE from accelerometers regardless of the normalization method. These results did not provide enough information to definitively claim one algorithm or normalization method is superior than the other when using accelerometers.

The second aim of this dissertation evaluated the effect of data length on the computation of the LyE when using 6 algorithm-normalization method combinations for accelerometer data. We initially hypothesized that data lengths greater than 150 strides could not be directly comparable to smaller data sets with 50 strides or less, regardless of the algorithm and normalization method used to calculate the LyE. However, when studying young healthy adults (Chapter 5) we found that different data lengths can be compared against a given range of data lengths across publications. For example, if one paper used 50 strides, it can be compared to publications that used 30 to 150 strides with similar calculation methodologies. We also contributed to the literature regarding the minimum data requirements for calculating the LyE; based on algorithm and preprocessing method choices for future reference. Based on our results in young healthy and community-dwelling elderly adults, we recommend that 50 to 100 strides should be used when computing the LyE with the R-algorithm and at least 200 strides should be used when calculating with the W-algorithm (Chapter 5 & 6). We found that the reliability of the R-algorithm using each of the three normalization methods had good to average (10-15%) reliability, but poor to very poor (30-41%) reliability for the W-algorithm. The results show that either truncating the gait signal to a fixed number of strides (raw gait) or using data point normalization will compute a more reliable LyE when using the R-algorithm. For the W-algorithm, the data point normalization method had better reliability than the other methods. Lastly, we recommend using the R-algorithm over the W-algorithm for accelerometer data due to better precision and reliability found in this dissertation.

In conclusion, we recommend the following methodological choices for calculation of the LyE using accelerometer data:

1. *Time Delay*: 10 data points or 10% of the gait cycle
2. *Embedding Dimension*: 5
3. *Algorithm*: Rosenstein *et al.* algorithm
4. *Normalization Method*: raw gait or data point normalized data
5. *Data Length*: 50 to 100 strides

This standardized methodology for calculating LyE will allow for the comparison of data and conclusions across all studies that employ their use. Having a transparent protocol for calculating the LyE will enable study replication which will substantiate current and future findings with more confidence. This, in turn, will allow for better meta-analyses identifying optimal measures for more precise and sensitive fall risk assessment tools and biomarkers for various gait impairment diseases. Additionally, this dissertation outlines a methodology for other researchers to follow for determining their own standardized calculation of the LyE – and other nonlinear measures that are dependent on phase space reconstruction – regardless of the biomechanical system being evaluated.

While the outcomes from these investigations reveal that the LyE calculation methodology can be standardized, there are areas that require further investigation. Although we recommended using either truncated gait data or data point normalized data, more research is necessary to conclusively determine the optimal normalization method for accelerometer data. Additionally, this dissertation focused solely on the standardization of LyE calculation methodology for accelerometer data. Future research will be needed to standardize the computation of the LyE using motion capture data and potentially for different signal types, e.g. joint angles or marker displacement data.

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APPENDIX A

COMPLETE STATISTICAL ANALYSES OF TIME DELAY AND EMBEDDING DIMENSION EFFECT ON THE LYAPUNOV EXPONENT IN GAIT

[Consult Attached Files]

APPENDIX B

COMPLETE STATISTICAL ANALYSES OF DATA LENGTH EFFECT ON THE LYAPUNOV EXPONENT IN GAIT

In the following pages are the full statistical results for the full factorial comparison of data lengths investigated in the vertical (VT), anteroposterior (AP), and mediolateral (ML) directions of the lumbar accelerometer. There are tables for each of the three normalization methods – raw gat (gc), gait cycle normalized (gcNorm), and data point normalized (dpNorm) data – with respect to both the Rosenstein *et al.* and Wolf *et al.* algorithms. The final tables contain the mean and standard deviation of the LyE for all data lengths when using the 6 algorithm-normalization method combinations in the VT, AP, and ML directions.

Table B-1: Significant differences between data lengths when using R-algorithm with raw gait data in all directions. $p > 0.5$ marked as NS

Direction	Data Length	30	50	100	150	200	300	500	1000
VT	30		NS	NS	0.013	<0.0005	<0.0005	<0.0005	<0.0005
	50	NS		NS	0.177	0.0044	<0.0005	<0.0005	<0.0005
	100	NS	NS		NS	0.2683	0.0058	<0.0005	<0.0005
	150	0.013	0.177	NS		NS	NS	0.0456	<0.0005
	200	<0.0005	0.0044	0.2683	NS		NS	NS	0.0169
	300	<0.0005	<0.0005	0.0058	NS	NS		NS	NS
	500	<0.0005	<0.0005	<0.0005	0.0456	NS	NS		NS
	1000	<0.0005	<0.0005	<0.0005	<0.0005	0.0169	NS	NS	
AP	30		NS	NS	0.013	<0.0005	<0.0005	<0.0005	<0.0005
	50	NS		NS	0.3995	0.0169	<0.0005	<0.0005	<0.0005
	100	NS	NS		NS	NS	0.0076	<0.0005	<0.0005
	150	0.013	0.3995	NS		NS	NS	0.013	<0.0005
	200	<0.0005	0.0169	NS	NS		NS	0.3281	0.0058
	300	<0.0005	<0.0005	0.0076	NS	NS		NS	0.4841
	500	<0.0005	<0.0005	<0.0005	0.013	0.3281	NS		NS
	1000	<0.0005	<0.0005	<0.0005	<0.0005	0.0058	0.4841	NS	
ML	30		NS	NS	0.1428	0.0058	<0.0005	<0.0005	<0.0005
	50	NS		NS	NS	0.0358	<0.0005	<0.0005	<0.0005
	100	NS	NS		NS	NS	0.028	<0.0005	<0.0005
	150	0.1428	NS	NS		NS	NS	0.0058	<0.0005
	200	0.0058	0.0358	NS	NS		NS	0.1428	0.0033
	300	<0.0005	<0.0005	0.028	NS	NS		NS	0.2683
	500	<0.0005	<0.0005	<0.0005	0.0058	0.1428	NS		NS
	1000	<0.0005	<0.0005	<0.0005	<0.0005	0.0033	0.2683	NS	

Table B-2: Significant differences between data lengths when using W-algorithm with raw gait data in all directions. $p > 0.5$ marked as NS

Direction	Data Length	30	50	100	150	200	300	500	1000
VT	**30**		NS	NS	0.028	0.0007	<0.0005	<0.0005	<0.0005
	50	NS		NS	0.2683	0.013	<0.0005	<0.0005	<0.0005
	100	NS	NS		NS	NS	0.0076	<0.0005	<0.0005
	150	0.028	0.2683	NS		NS	0.4841	0.01	<0.0005
	200	0.0007	0.013	NS	NS		NS	0.2184	0.01
	300	<0.0005	<0.0005	0.0076	0.4841	NS		NS	NS
	500	<0.0005	<0.0005	<0.0005	0.01	0.2184	NS		NS
	1000	<0.0005	<0.0005	<0.0005	<0.0005	0.01	NS	NS	
AP	**30**		NS	NS	0.0358	0.0005	<0.0005	<0.0005	<0.0005
	50	NS		NS	0.3281	0.01	<0.0005	<0.0005	<0.0005
	100	NS	NS		NS	NS	0.0169	<0.0005	<0.0005
	150	0.0358	0.3281	NS		NS	0.3281	0.01	<0.0005
	200	0.0005	0.01	NS	NS		NS	0.3281	0.0044
	300	<0.0005	<0.0005	0.0169	0.3281	NS		NS	NS
	500	<0.0005	<0.0005	<0.0005	0.01	0.3281	NS		NS
	1000	<0.0005	<0.0005	<0.0005	<0.0005	0.0044	NS	NS	
ML	**30**		NS	NS	0.0358	0.001	<0.0005	<0.0005	<0.0005
	50	NS		NS	0.1147	0.0044	<0.0005	<0.0005	<0.0005
	100	NS	NS		NS	NS	0.01	<0.0005	<0.0005
	150	0.0358	0.1147	NS		NS	NS	0.0169	<0.0005
	200	0.001	0.0044	NS	NS		NS	0.3281	0.0058
	300	<0.0005	<0.0005	0.01	NS	NS		NS	0.3995
	500	<0.0005	<0.0005	<0.0005	0.0169	0.3281	NS		NS
	1000	<0.0005	<0.0005	<0.0005	<0.0005	0.0058	0.3995	NS	

Table B-3: Significant differences between data lengths when using R-algorithm with gait cycle normalized data in all directions. $p > 0.5$ marked as NS

Direction	Data Length	30	50	100	150	200	300	500	1000
VT	**30**		NS	NS	0.2683	0.0033	<0.0005	<0.0005	<0.0005
	50	NS		NS	NS	0.01	<0.0005	<0.0005	<0.0005
	100	NS	NS		NS	NS	0.0218	0.0044	<0.0005
	150	0.2683	NS	NS		NS	0.3281	0.0917	<0.0005
	200	0.0033	0.01	NS	NS		NS	NS	0.0578
	300	<0.0005	<0.0005	0.0218	0.3281	NS		NS	NS
	500	<0.0005	<0.0005	0.0044	0.0917	NS	NS		NS
	1000	<0.0005	<0.0005	<0.0005	<0.0005	0.0578	NS	NS	
AP	**30**		NS	NS	0.0456	<0.0005	<0.0005	<0.0005	<0.0005
	50	NS		NS	0.177	0.0025	<0.0005	<0.0005	<0.0005
	100	NS	NS		NS	0.3281	0.0058	<0.0005	<0.0005
	150	0.0456	0.177	NS		NS	0.4841	0.0456	0.001
	200	<0.0005	0.0025	0.3281	NS		NS	NS	0.0917
	300	<0.0005	<0.0005	0.0058	0.4841	NS		NS	NS
	500	<0.0005	<0.0005	<0.0005	0.0456	NS	NS		NS
	1000	<0.0005	<0.0005	<0.0005	0.001	0.0917	NS	NS	
ML	**30**		NS	NS	0.3281	0.013	<0.0005	<0.0005	<0.0005
	50	NS		NS	NS	0.2184	0.001	<0.0005	<0.0005
	100	NS	NS		NS	NS	0.0044	<0.0005	<0.0005
	150	0.3281	NS	NS		NS	0.3995	0.028	<0.0005
	200	0.013	0.2184	NS	NS		NS	NS	0.0058
	300	<0.0005	0.001	0.0044	0.3995	NS		NS	NS
	500	<0.0005	<0.0005	<0.0005	0.028	NS	NS		NS
	1000	<0.0005	<0.0005	<0.0005	<0.0005	0.0058	NS	NS	

Table B-4: Significant differences between data lengths when using W-algorithm with gait cycle normalized data in all directions. $p > 0.5$ marked as NS

Direction	Data Length	30	50	100	150	200	300	500	1000
VT	**30**		NS	NS	0.2184	0.013	<0.0005	<0.0005	<0.0005
	50	NS		NS	NS	0.073	<0.0005	<0.0005	<0.0005
	100	NS	NS		NS	NS	0.0169	<0.0005	<0.0005
	150	0.2184	NS	NS		NS	NS	0.0218	<0.0005
	200	0.013	0.073	NS	NS		NS	0.3281	0.0025
	300	<0.0005	<0.0005	0.0169	NS	NS		NS	0.3995
	500	<0.0005	<0.0005	<0.0005	0.0218	0.3281	NS		NS
	1000	<0.0005	<0.0005	<0.0005	<0.0005	0.0025	0.3995	NS	
AP	**30**		NS	NS	0.2683	0.0169	<0.0005	<0.0005	<0.0005
	50	NS		NS	0.3995	0.028	<0.0005	<0.0005	<0.0005
	100	NS	NS		NS	0.4841	0.0033	<0.0005	<0.0005
	150	0.2683	0.3995	NS		NS	NS	0.013	<0.0005
	200	0.0169	0.028	0.4841	NS		NS	0.2184	0.0033
	300	<0.0005	<0.0005	0.0033	NS	NS		NS	0.4841
	500	<0.0005	<0.0005	<0.0005	0.013	0.2184	NS		NS
	1000	<0.0005	<0.0005	<0.0005	<0.0005	0.0033	0.4841	NS	
ML	**30**		NS	NS	0.073	0.0033	<0.0005	<0.0005	<0.0005
	50	NS		NS	0.4841	0.0358	<0.0005	<0.0005	<0.0005
	100	NS	NS		NS	NS	0.0033	<0.0005	<0.0005
	150	0.073	0.4841	NS		NS	0.3995	0.013	<0.0005
	200	0.0033	0.0358	NS	NS		NS	0.2184	0.0025
	300	<0.0005	<0.0005	0.0033	0.3995	NS		NS	NS
	500	<0.0005	<0.0005	<0.0005	0.013	0.2184	NS		NS
	1000	<0.0005	<0.0005	<0.0005	<0.0005	0.0025	NS	NS	

Table B-5: Significant differences between data lengths when using R-algorithm with data point normalized data in all directions

Direction	Data Length	30	50	100	150	200	300	500	1000
VT	**30**		NS	NS	0.0218	<0.0005	<0.0005	<0.0005	<0.0005
	50	NS		NS	0.3995	0.0076	<0.0005	<0.0005	<0.0005
	100	NS	NS		NS	NS	0.0218	<0.0005	<0.0005
	150	0.0218	0.3995	NS		NS	NS	0.01	<0.0005
	200	<0.0005	0.0076	NS	NS		NS	0.4841	0.0218
	300	<0.0005	<0.0005	0.0218	NS	NS		NS	NS
	500	<0.0005	<0.0005	<0.0005	0.01	0.4841	NS		NS
	1000	<0.0005	<0.0005	<0.0005	<0.0005	0.0218	NS	NS	
AP	**30**		NS	NS	0.0076	<0.0005	<0.0005	<0.0005	<0.0005
	50	NS		NS	0.4841	0.0578	<0.0005	<0.0005	<0.0005
	100	NS	NS		NS	NS	0.01	<0.0005	<0.0005
	150	0.0076	0.4841	NS		NS	NS	0.0169	<0.0005
	200	<0.0005	0.0578	NS	NS		NS	0.177	0.0025
	300	<0.0005	<0.0005	0.01	NS	NS		NS	0.4841
	500	<0.0005	<0.0005	<0.0005	0.0169	0.177	NS		NS
	1000	<0.0005	<0.0005	<0.0005	<0.0005	0.0025	0.4841	NS	
ML	**30**		NS	NS	0.0456	<0.0005	<0.0005	<0.0005	<0.0005
	50	NS		NS	NS	0.0058	<0.0005	<0.0005	<0.0005
	100	NS	NS		NS	NS	0.0169	<0.0005	<0.0005
	150	0.0456	NS	NS		NS	0.2683	0.0044	<0.0005
	200	<0.0005	0.0058	NS	NS		NS	0.4841	0.01
	300	<0.0005	<0.0005	0.0169	0.2683	NS		NS	0.4841
	500	<0.0005	<0.0005	<0.0005	0.0044	0.4841	NS		NS
	1000	<0.0005	<0.0005	<0.0005	<0.0005	0.01	0.4841	NS	

Table B-6: Significant differences between data lengths when using W-algorithm with data point normalized data in all directions. *p > 0.5* marked as NS

Direction	Data Length	30	50	100	150	200	300	500	1000
VT	**30**		NS	NS	0.028	0.0014	<0.0005	<0.0005	<0.0005
	50	NS		NS	0.1147	0.0076	<0.0005	<0.0005	<0.0005
	100	NS	NS		NS	NS	0.0076	<0.0005	<0.0005
	150	0.028	0.1147	NS		NS	NS	0.0169	<0.0005
	200	0.0014	0.0076	NS	NS		NS	0.2184	0.0058
	300	<0.0005	<0.0005	0.0076	NS	NS		NS	0.4841
	500	<0.0005	<0.0005	<0.0005	0.0169	0.2184	NS		NS
	1000	<0.0005	<0.0005	<0.0005	<0.0005	0.0058	0.4841	NS	
AP	**30**		NS	NS	0.0578	<0.0005	<0.0005	<0.0005	<0.0005
	50	NS		NS	NS	0.01	<0.0005	<0.0005	<0.0005
	100	NS	NS		NS	NS	0.013	<0.0005	<0.0005
	150	0.0578	NS	NS		NS	0.2184	0.0076	<0.0005
	200	<0.0005	0.01	NS	NS		NS	NS	0.0076
	300	<0.0005	<0.0005	0.013	0.2184	NS		NS	NS
	500	<0.0005	<0.0005	<0.0005	0.0076	NS	NS		NS
	1000	<0.0005	<0.0005	<0.0005	<0.0005	0.0076	NS	NS	
ML	**30**		NS	NS	0.1147	0.0044	<0.0005	<0.0005	<0.0005
	50	NS		NS	0.1428	0.0058	<0.0005	<0.0005	<0.0005
	100	NS	NS		NS	0.3995	0.0033	<0.0005	<0.0005
	150	0.1147	0.1428	NS		NS	NS	0.013	<0.0005
	200	0.0044	0.0058	0.3995	NS		NS	0.2683	0.0044
	300	<0.0005	<0.0005	0.0033	NS	NS		NS	0.4841
	500	<0.0005	<0.0005	<0.0005	0.013	0.2683	NS		NS
	1000	<0.0005	<0.0005	<0.0005	<0.0005	0.0044	0.4841	NS	

Table B-7: Means (SD) of the LyE for all data lengths calculated using the R-algorithm for each direction using raw gait data

	Raw Gait		
Strides	**VT**	**AP**	**ML**
30	1.07 (0.165)	0.87 (0.121)	1.04 (0.141)
50	1.11 (0.167)	0.91 (0.130)	1.07 (0.137)
100	1.16 (0.164)	0.95 (0.131)	1.11 (0.135)
150	1.20 (0.168)	0.97 (0.129)	1.13 (0.131)
200	1.23 (0.165)	0.99 (0.127)	1.16 (0.128)
300	1.25 (0.160)	1.01 (0.125)	1.18 (0.127)
400	1.26 (0.158)	1.03 (0.123)	1.20 (0.130)
500	1.27 (0.157)	1.04 (0.123)	1.22 (0.129)
600	1.28 (0.161)	1.05 (0.124)	1.23 (0.131)
700	1.29 (0.162)	1.06 (0.122)	1.24 (0.130)
800	1.30 (0.164)	1.06 (0.122)	1.25 (0.131)
900	1.31 (0.169)	1.07 (0.123)	1.26 (0.131)
1000	1.31 (0.168)	1.08 (0.125)	1.26 (0.130)
1100	1.32 (0.169)	1.08 (0.128)	1.27 (0.128)
1200	1.32 (0.168)	1.09 (0.126)	1.27 (0.128)
1300	1.32 (0.168)	1.09 (0.126)	1.28 (0.128)

Table B-8: Means (SD) of the LyE for all data lengths calculated using the R-algorithm for each direction using gait cycle normalized gait data

	Gait Cycle Normalized		
Strides	**VT**	**AP**	**ML**
30	1.02 (0.228)	1.03 (0.196)	0.85 (0.173)
50	1.04 (0.225)	1.06 (0.193)	0.88 (0.172)
100	1.10 (0.225)	1.11 (0.191)	0.92 (0.141)
150	1.13 (0.222)	1.14 (0.193)	0.95 (0.137)
200	1.15 (0.221)	1.16 (0.181)	0.97 (0.139)
300	1.18 (0.213)	1.18 (0.175)	0.99 (0.142)
400	1.19 (0.199)	1.20 (0.162)	1.00 (0.132)
500	1.21 (0.195)	1.21 (0.155)	1.02 (0.131)
600	1.22 (0.194)	1.22 (0.154)	1.03 (0.132)
700	1.23 (0.195)	1.24 (0.150)	1.04 (0.132)
800	1.24 (0.193)	1.24 (0.149)	1.05 (0.132)
900	1.25 (0.193)	1.25 (0.149)	1.05 (0.134)
1000	1.25 (0.193)	1.25 (0.146)	1.06 (0.134)
1100	1.26 (0.195)	1.26 (0.144)	1.07 (0.137)
1200	1.27 (0.195)	1.26 (0.144)	1.07 (0.138)
1300	1.27 (0.194)	1.27 (0.142)	1.08 (0.137)

Table B-9: Means (SD) of the LyE for all data lengths calculated using the R-algorithm for each direction using data normalized gait data

	Data Point Normalized		
Strides	**VT**	**AP**	**ML**
30	0.99 (0.156)	0.83 (0.108)	1.02 (0.192)
50	1.04 (0.154)	0.87 (0.112)	1.05 (0.194)
100	1.10 (0.155)	0.92 (0.114)	1.10 (0.187)
150	1.14 (0.161)	0.95 (0.107)	1.14 (0.176)
200	1.17 (0.163)	0.96 (0.108)	1.16 (0.174)
300	1.20 (0.162)	0.99 (0.105)	1.19 (0.172)
400	1.21 (0.160)	1.01 (0.105)	1.21 (0.173)
500	1.23 (0.162)	1.02 (0.107)	1.23 (0.172)
600	1.24 (0.164)	1.03 (0.109)	1.24 (0.173)
700	1.25 (0.165)	1.04 (0.108)	1.25 (0.171)
800	1.26 (0.167)	1.05 (0.109)	1.26 (0.170)
900	1.27 (0.170)	1.06 (0.110)	1.27 (0.173)
1000	1.28 (0.171)	1.06 (0.111)	1.28 (0.172)
1100	1.28 (0.172)	1.07 (0.113)	1.28 (0.170)
1200	1.29 (0.172)	1.08 (0.112)	1.28 (0.168)
1300	1.29 (0.172)	1.08 (0.112)	1.29 (0.168)

Table B-10: Means (SD) of the LyE for all data lengths calculated using the W-algorithm for each direction using raw gait data

	Raw Gait		
Strides	**VT**	**AP**	**ML**
30	1.71 (0.578)	2.21 (0.649)	1.83 (0.536)
50	1.86 (0.640)	2.46 (0.727)	2.02 (0.513)
100	2.16 (0.809)	3.00 (0.814)	2.49 (0.542)
150	2.38 (0.801)	3.30 (0.968)	2.89 (0.600)
200	2.55 (0.884)	3.52 (1.000)	3.07 (0.646)
300	2.82 (0.889)	3.99 (1.089)	3.38 (0.620)
400	3.05 (0.885)	4.32 (1.136)	3.61 (0.679)
500	3.28 (0.918)	4.58 (1.219)	3.82 (0.738)
600	3.43 (0.879)	4.84 (1.262)	4.02 (0.690)
700	3.63 (0.832)	5.02 (1.295)	4.15 (0.732)
800	3.76 (0.847)	5.19 (1.299)	4.34 (0.767)
900	3.92 (0.850)	5.38 (1.340)	4.46 (0.808)
1000	4.01 (0.778)	5.45 (1.331)	4.58 (0.800)
1100	4.13 (0.770)	5.54 (1.321)	4.69 (0.797)
1200	4.25 (0.753)	5.66 (1.354)	4.82 (0.811)
1300	4.37 (0.744)	5.73 (1.348)	4.92 (0.813)

Table B-11: Means (SD) of the LyE for all data lengths calculated using the W-algorithm for each direction using gait cycle normalized gait data

	Gait Cycle Normalized		
Strides	**VT**	**AP**	**ML**
30	1.80 (0.810)	2.34 (0.904)	2.54 (0.676)
50	2.04 (0.864)	2.44 (0.816)	2.85 (0.765)
100	2.27 (0.832)	2.95 (0.740)	3.27 (0.686)
150	2.50 (1.038)	3.41 (0.936)	3.67 (0.876)
200	2.69 (1.116)	3.64 (0.905)	3.87 (0.837)
300	2.99 (1.118)	3.98 (0.930)	4.30 (0.976)
400	3.21 (1.229)	4.26 (0.971)	4.68 (1.032)
500	3.41 (1.192)	4.54 (1.072)	4.96 (1.118)
600	3.57 (1.206)	4.77 (1.068)	5.20 (1.117)
700	3.70 (1.189)	4.91 (1.035)	5.37 (1.133)
800	3.87 (1.205)	5.01 (1.065)	5.56 (1.228)
900	3.97 (1.179)	5.19 (1.068)	5.70 (1.246)
1000	4.09 (1.116)	5.30 (1.095)	5.86 (1.261)
1100	4.24 (1.154)	5.40 (1.110)	6.00 (1.279)
1200	4.34 (1.173)	5.59 (1.137)	6.13 (1.310)
1300	4.47 (1.187)	5.70 (1.145)	6.25 (1.340)

Table B-12: Means (SD) of the LyE for all data lengths calculated using the W-algorithm for each direction using data point normalized gait data

	Data Point Normalized		
Strides	**VT**	**AP**	**ML**
30	1.43 (0.614)	1.87 (0.523)	1.73 (0.706)
50	1.50 (0.630)	2.07 (0.510)	1.92 (0.612)
100	1.81 (0.779)	2.39 (0.621)	2.25 (0.638)
150	2.01 (0.835)	2.68 (0.724)	2.61 (0.799)
200	2.17 (0.960)	2.96 (0.813)	2.78 (0.795)
300	2.40 (0.968)	3.26 (0.870)	3.05 (0.861)
400	2.50 (0.988)	3.46 (0.865)	3.30 (0.851)
500	2.69 (1.056)	3.67 (0.968)	3.47 (0.911)
600	2.78 (1.038)	3.85 (1.034)	3.66 (0.933)
700	2.91 (0.999)	4.02 (1.048)	3.76 (0.965)
800	3.03 (0.996)	4.12 (1.081)	3.83 (0.973)
900	3.15 (1.075)	4.25 (1.076)	3.94 (0.995)
1000	3.21 (0.990)	4.37 (1.089)	4.08 (1.000)
1100	3.29 (0.949)	4.48 (1.111)	4.18 (1.046)
1200	3.41 (1.038)	4.58 (1.131)	4.22 (1.032)
1300	3.49 (1.015)	4.65 (1.150)	4.34 (1.073)

APPENDIX C

INVESTIGATION OF THE EFFECT OF TIME DELAY AND EMBEDDING DIMENSION ON OVERGROUND WALKING IN YOUNG AND ELDERLY ADULTS

This supplementary material had two purposes:

1) To investigate the effect of time delay (τ) on the calculation of the Lyapunov exponent (LyE) using both the Rosenstein *et al*. (R-algorithm) and Wolf *et al.* (W-algorithm) algorithms and three normalization methods
2) To investigate the effect of embedding dimension (d_E) on the calculation of the LyE with respect to each of the 6 algorithm-normalization method combinations.

The present study used an equal number of 100 strides for all subjects, young healthy and elderly community dwelling adults. Each subject time series data was then preprocessed using the following three methods with each containing the maximum number of gait cycles:

(1) Fixed number of strides with a variable number of data points per stride (gc)
(2) Fixed number of strides with a 100 data points per stride (gcNorm)
(3) Fixed number of strides with a total of 10,000 data points in the time series (dpNorm)

No other filtering or preprocessing was performed on the data. The LyE was calculated for every direction using each of the preprocessing methods and the Rosenstein et al. (1993) and Wolf et al. (1985) algorithms, which will be referred to R- and W-algorithms, respectively. And within these conditions each permutation of the embedding dimension ($d_E = 4,5,6,7$) and time delay ($\tau = 2,4,6, \dots ,30$) were used to calculate the LyE.

In Rosenstein's algorithm, the LyE is the slope of the divergence curve. When normalized gait cycles are analyzed, the slope is taken over a span of 0 – 0.5 strides or the first 50 points of the divergence curve. In order to compare normalized and raw gait data, we found the average stride length for each subject and used the individualized half stride length as the bounds for taking the slope. For example, if a subject had an average stride length of 150 samples, then the slope of the mean divergence curve was taken from the first 75 points. And in the W-algorithm a time evolution step of seven was used.

The performed analyses consisted of a systematic permutation of fifteen time delays and four embedding dimensions. This was applied to 6 different LyE algorithm-time series normalization procedure combinations for each acceleration direction. The Friedman test, a nonparametric repeated measures ANOVA, was used to explore the effect of time delay and embedding dimension on the LyE. The nonparametric test was used for all analyses because the assumption of sphericity was violated, in addition to not all parameters were normally distributed. This test was performed independently for each population group, acceleration direction, algorithm choice and preprocessing method.

Then, slices of the data set were taken for a more specific look at how time delay and embedding dimension independently played a role in the calculation of the LyE. First, a post-hoc pairwise comparison with a Bonferroni correction for multiple comparisons was used to determine the specific differences between each time delay when the embedding dimension of five or seven was chosen for the R- and W-algorithm, respectively. Then the same post-hoc comparison was used to determine the differences in embedding dimension for a set of time delays ($\tau = 5,8,10,12,15$). This range of time delays was selected because most time delays chosen in publications are within this range, based on the meta- and supplementary data from Mehdizadeh (2018). For all statistical tests, a p-value < 0.05 was considered significant. All statistical analysis was performed using SPSS Statistics (version 25, IBM, USA).

Our results show that time delay and embedding dimension had a significant impact ($p < 0.005$, respectively) on the value of the LyE regardless of direction, algorithm, and preprocessing method. The differences between each direction and preprocessing methods are shown in Figure C- 1 for the R-algorithm and in Figure C- 2 for the W-algorithm for community dwelling elderly adults. These differences for young healthy adults are also shown in Figure C- 3 and Figure C- 4 for the R- and W-algorithms, respectively. We found that embedding dimension, at particular time delays, had significant effects on the LyE calculated by the R- and W-algorithm. Table C- 1 shows the differences between different embedding dimensions at the selected time delays ($\tau = 5,8,10,12,15$) when gc, gcNorm, and dpNorm data was utilized with the R-algorithm, while Table C- 2 shows the results using the W-algorithm in elderly adults.. Table C- 3 and Table C-4 show this effect in young healthy adults for the R- and W- algorithms, respectively.

Overall, the W-algorithm was more invariant to changes in time delay. This is shown in Figure C- 5 and Figure C- 6, which depict the statistical differences between two time delays when an embedding dimension of 5 and 7 were used for the R- and W-algorithms, respectively. And the R-algorithm was more robust against changes in the embedding dimension regardless of preprocessing method compared to the W-algorithm. In terms of reliability and consistency, the Rosenstein algorithm might be better for IMU data than the Wolf algorithm. The Rosenstein algorithm had much smaller standard deviations of the mean LyE compared to the Wolf algorithm. This is consistent with unpublished results using young healthy adults walking on a treadmill with 1300 and 150 gait cycles.

In conclusion, the data presented in this supplementary material validates the methodological choices made in the present study with respect to the choice of time delay and embedding dimension.

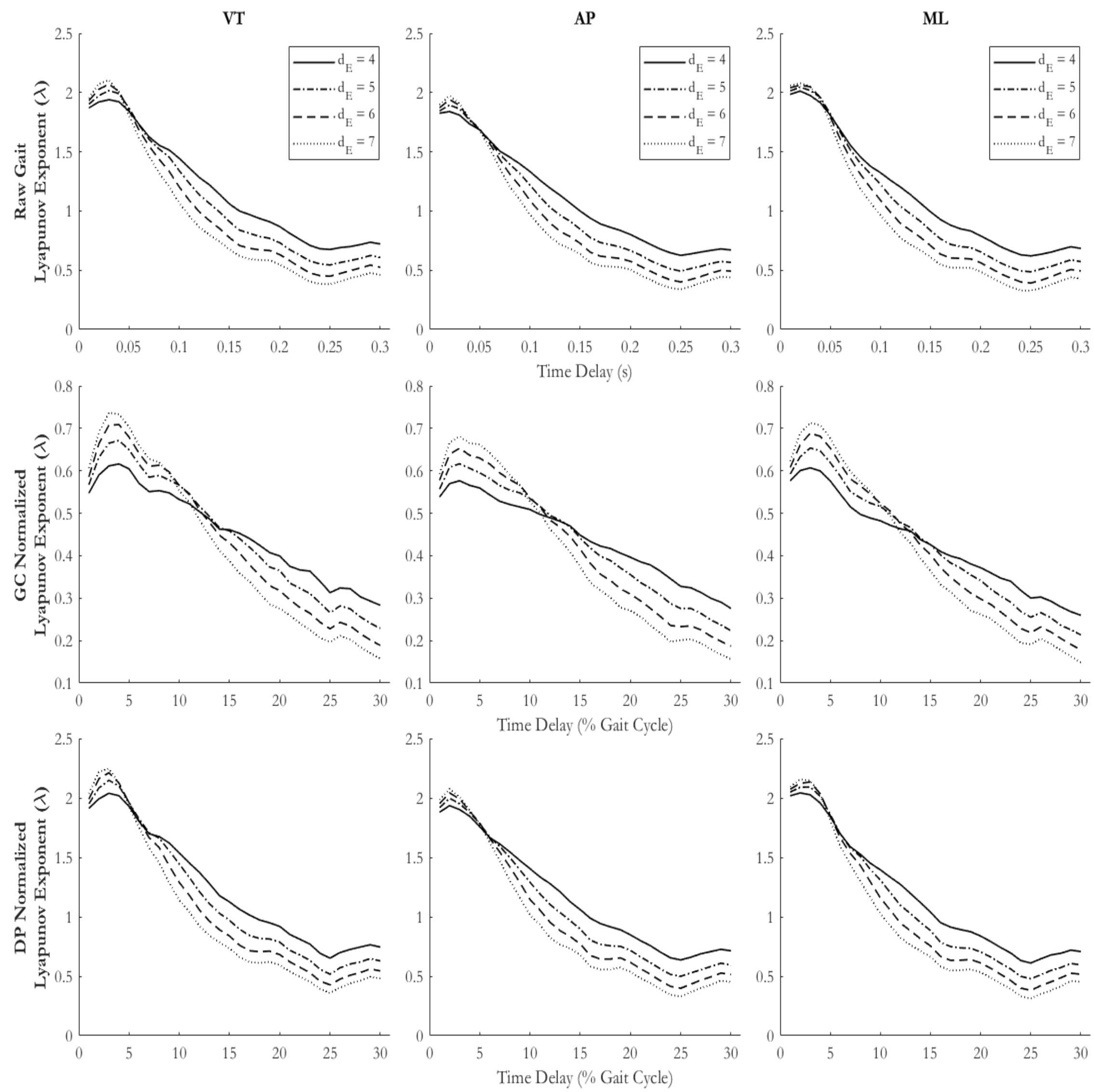

Figure C- 1: Effect of embedding dimension and time delay in the VT, AP, and ML direction using Rosenstein *et al* algorithm using three different preprocessing methods using elderly over-ground walking data.

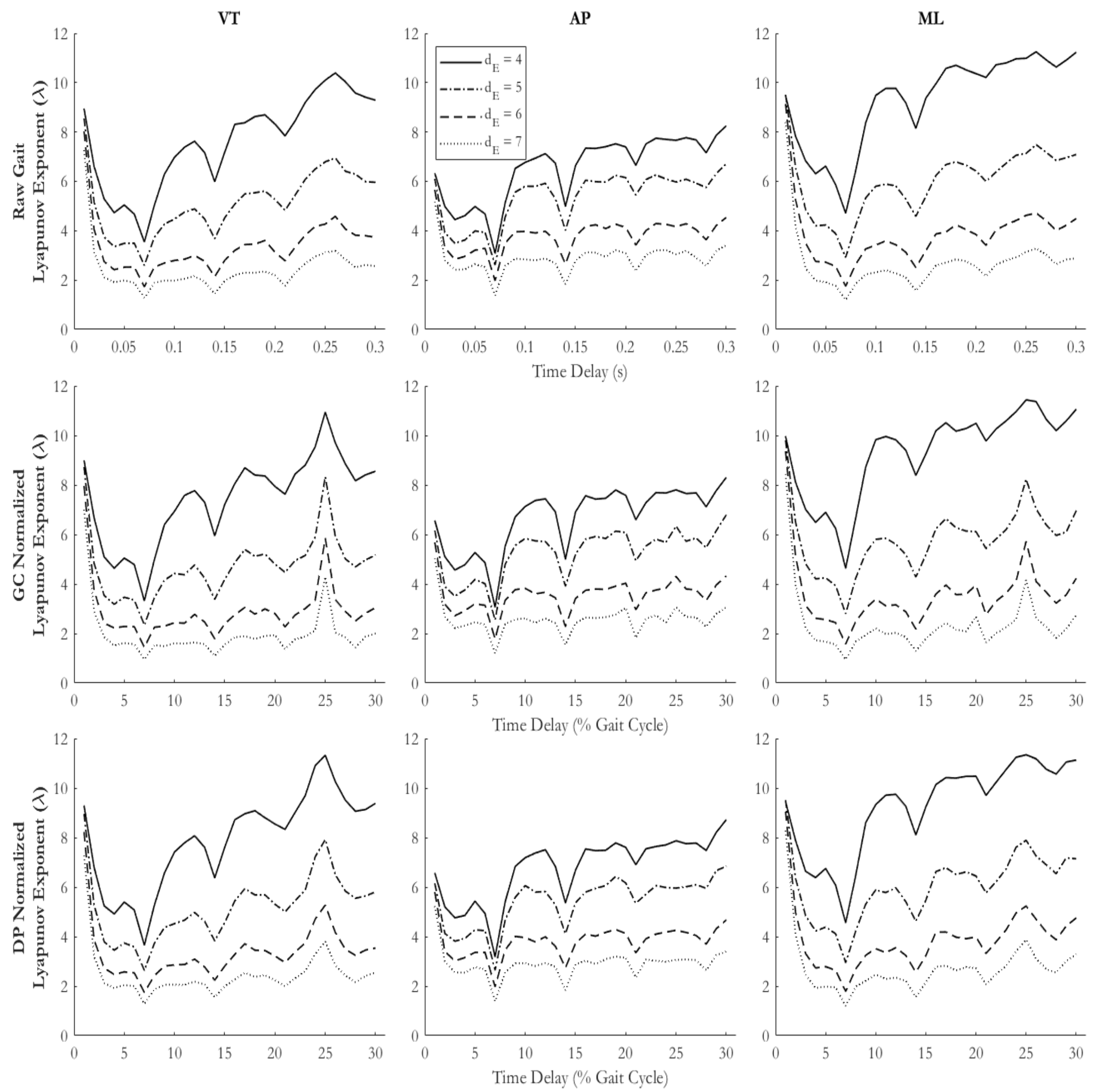

Figure C- 2: Effect of embedding dimension and time delay in the VT, AP, and ML direction using Wolf *et al* algorithm using three different preprocessing methods using elderly over-ground walking data.

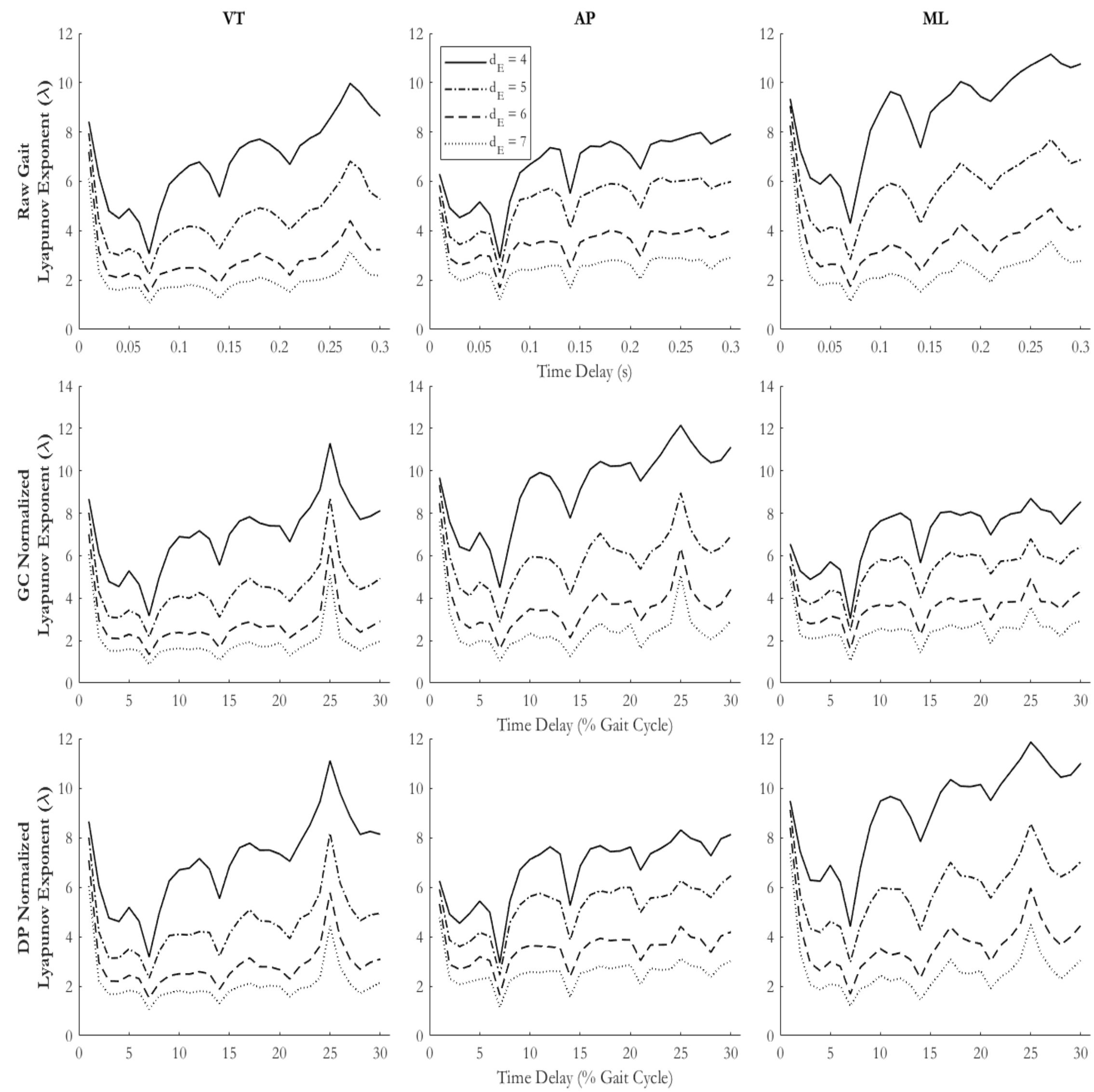

Figure C- 3: Effect of embedding dimension and time delay in the VT, AP, and ML direction using Rosenstein *et al* algorithm using three different preprocessing methods using young healthy adults over-ground walking data.

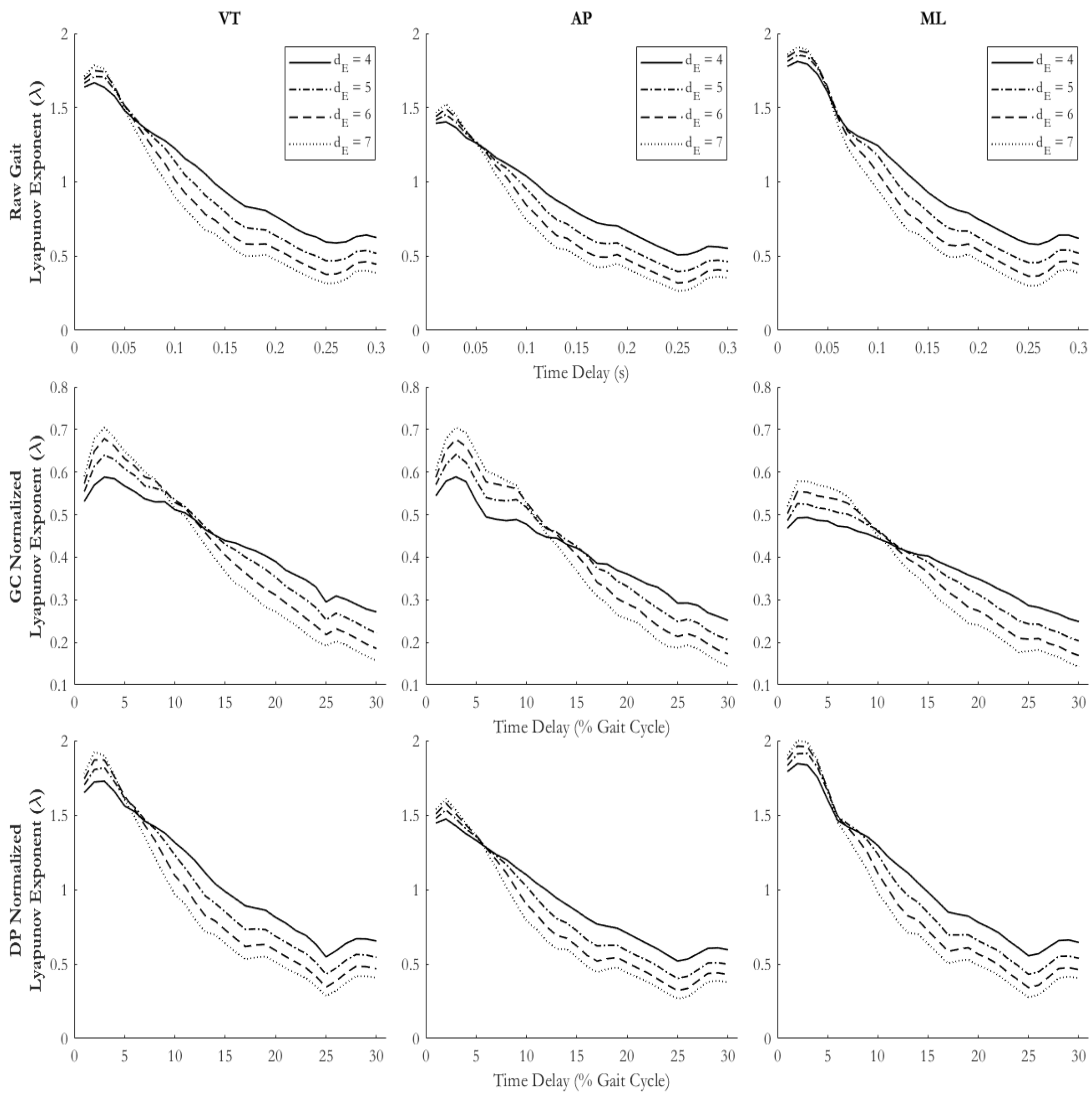

Figure C- 4: Effect of embedding dimension and time delay in the VT, AP, and ML direction using Wolf *et al* algorithm using three different preprocessing methods using young healthy adults over-ground walking data.

Table C- 1: Effect of embedding dimension on the LyE under select time delays using the R-algorithm calculated from over-ground walking of elderly adults. NS: *p*-values > 0.5

Normalization Method	Dir.	τ	d4-d5	d4-d6	d4-d7	d5-d6	d5-d7	d6-d7
Raw Gait Cycles	**VT**	5	NS	NS	NS	NS	NS	NS
		8	NS	NS	NS	NS	NS	NS
		10	NS	NS	NS	NS	NS	NS
		12	NS	NS	0.3895	NS	NS	NS
		15	NS	NS	NS	NS	NS	NS
	AP	5	NS	NS	NS	NS	NS	NS
		8	NS	NS	NS	NS	NS	NS
		10	NS	NS	0.2775	NS	NS	NS
		12	NS	NS	0.2474	NS	NS	NS
		15	NS	NS	NS	NS	NS	NS
	ML	5	NS	NS	NS	NS	NS	NS
		8	NS	NS	NS	NS	NS	NS
		10	NS	NS	0.4857	NS	NS	NS
		12	NS	NS	0.1075	NS	NS	NS
		15	NS	NS	NS	NS	NS	NS
Gait Cycle Normalization	**VT**	5	NS	NS	NS	NS	NS	NS
		8	NS	NS	NS	NS	NS	NS
		10	NS	NS	0.1371	NS	NS	NS
		12	NS	NS	0.0740	NS	NS	NS
		15	NS	NS	NS	NS	NS	NS
	AP	5	NS	NS	NS	**< 0.0005**	NS	NS
		8	NS	NS	NS	NS	NS	NS
		10	NS	NS	0.1214	NS	NS	NS
		12	NS	NS	**0.0444**	NS	NS	NS
		15	NS	NS	NS	NS	NS	NS
	ML	5	NS	NS	NS	NS	NS	NS
		8	NS	NS	NS	NS	NS	NS
		10	NS	NS	0.095	NS	NS	NS
		12	NS	0.4857	**0.0200**	NS	NS	NS
		15	NS	NS	NS	NS	NS	NS
Data Point Normalization	**VT**	5	NS	NS	NS	NS	NS	NS
		8	NS	NS	NS	NS	NS	NS
		10	NS	NS	NS	NS	NS	NS
		12	NS	NS	0.4857	NS	NS	NS
		15	NS	NS	NS	NS	NS	NS
	AP	5	NS	NS	NS	NS	NS	NS
		8	NS	NS	NS	NS	NS	NS
		10	NS	NS	0.3895	NS	NS	NS
		12	NS	NS	0.1371	NS	NS	NS
		15	NS	NS	NS	NS	NS	NS
	ML	5	NS	NS	NS	NS	NS	NS
		8	NS	NS	NS	NS	NS	NS
		10	NS	NS	NS	NS	NS	NS
		12	NS	NS	0.1075	NS	NS	NS
		15	NS	NS	NS	NS	NS	NS

Table C- 2: Effect of embedding dimension on the LyE under select time delays using the W-algorithm calculated from over-ground walking of elderly adults. NS: p-values > 0.5

Normalization Method	**Dir.**	τ	**Dimension Pairwise Comparison (p-value)**					
			d4-d5	**d4-d6**	**d4-d7**	**d5-d6**	**d5-d7**	**d6-d7**
Raw Gait Cycles	**VT**	5	**0.02**	NS	**< 0.0005**	**< 0.0005**	0.1546	NS
		8	NS	**0.0342**	**< 0.0005**	NS	**0.0049**	NS
		10	NS	**0.0031**	**< 0.0005**	NS	**< 0.0005**	NS
		12	NS	**0.0031**	**< 0.0005**	NS	**< 0.0005**	NS
		15	NS	**0.0015**	**< 0.0005**	NS	**< 0.0005**	NS
	AP	5	0.1546	NS	**< 0.0005**	**< 0.0005**	0.4857	NS
		8	NS	NS	**< 0.0005**	NS	**0.0020**	NS
		10	NS	**0.0152**	**< 0.0005**	NS	**< 0.0005**	NS
		12	NS	**0.0049**	**< 0.0005**	NS	**< 0.0005**	NS
		15	NS	**0.0200**	**< 0.0005**	NS	**< 0.0005**	NS
	ML	5	0.1546	NS	**< 0.0005**	**< 0.0005**	0.4857	NS
		8	NS	NS	**< 0.0005**	NS	**0.0020**	NS
		10	NS	**0.0152**	**< 0.0005**	NS	**< 0.0005**	NS
		12	NS	**0.0049**	**< 0.0005**	NS	**< 0.0005**	NS
		15	NS	**0.0200**	**< 0.0005**	NS	**< 0.0005**	NS
Gait Cycle Normalization	**VT**	5	NS	**0.0444**	**< 0.0005**	NS	0.1075	NS
		8	NS	0.0505	**< 0.0005**	NS	**0.0200**	NS
		10	NS	**0.002**	**< 0.0005**	NS	**0.0008**	NS
		12	NS	**0.0027**	**< 0.0005**	NS	**< 0.0005**	NS
		15	NS	**< 0.0005**	**< 0.0005**	NS	**< 0.0005**	NS
	AP	5	NS	0.3110	**< 0.0005**	NS	0.3482	NS
		8	NS	0.2775	**< 0.0005**	NS	**0.0017**	NS
		10	NS	**0.0262**	**< 0.0005**	NS	**< 0.0005**	NS
		12	NS	**0.0023**	**< 0.0005**	NS	**< 0.0005**	NS
		15	NS	**0.0006**	**< 0.0005**	NS	**< 0.0005**	NS
	ML	5	NS	**0.0132**	**< 0.0005**	NS	**0.0262**	NS
		8	NS	**0.0075**	**< 0.0005**	NS	**0.0065**	NS
		10	NS	**0.0049**	**< 0.0005**	NS	**0.0042**	NS
		12	NS	**0.0011**	**< 0.0005**	NS	**0.0009**	NS
		15	NS	**0.0009**	**< 0.0005**	NS	**< 0.0005**	NS
Data Point Normalization	**VT**	5	NS	**0.0262**	**< 0.0005**	NS	0.1214	NS
		8	NS	**0.0100**	**< 0.0005**	NS	**0.0017**	NS
		10	NS	**0.0017**	**< 0.0005**	NS	**0.0015**	NS
		12	NS	**0.0031**	**< 0.0005**	NS	**< 0.0005**	NS
		15	NS	**0.0011**	**< 0.0005**	NS	**< 0.0005**	NS
	AP	5	NS	0.1371	**< 0.0005**	NS	0.2474	NS
		8	NS	0.4857	**< 0.0005**	NS	**0.0031**	NS
		10	NS	**0.0075**	**< 0.0005**	0.311	**< 0.0005**	NS
		12	NS	**0.0012**	**< 0.0005**	NS	**< 0.0005**	NS
		15	NS	**0.0262**	**< 0.0005**	NS	**< 0.0005**	NS
	ML	5	NS	**0.0087**	**< 0.0005**	NS	**0.0087**	NS
		8	NS	**0.0200**	**< 0.0005**	NS	**0.0132**	NS
		10	NS	**0.0042**	**< 0.0005**	NS	**0.0031**	NS
		12	NS	**0.0049**	**< 0.0005**	NS	**0.0012**	NS
		15	NS	**< 0.0005**	**< 0.0005**	NS	**0.0007**	NS

Table C- 3: Effect embedding dimension on the LyE under select time delays using the R-algorithm calculated from over-ground walking of young health adults. NS: *p*-values > 0.5

Normalization Method	Dir.	τ	Dimension Pairwise Comparison (p-value) d4-d5	d4-d6	d4-d7	d5-d6	d5-d7	d6-d7
Raw Gait Cycles	**VT**	5	NS	NS	NS	NS	NS	NS
		8	NS	NS	NS	NS	NS	NS
		10	NS	NS	0.0401	NS	NS	NS
		12	NS	NS	**0.0281**	NS	NS	NS
		15	NS	NS	NS	NS	NS	NS
	AP	5	NS	NS	NS	NS	NS	NS
		8	NS	NS	NS	NS	NS	NS
		10	NS	NS	0.0716	NS	NS	NS
		12	NS	NS	**0.0401**	NS	NS	NS
		15	NS	NS	0.2636	NS	NS	NS
	ML	5	NS	NS	NS	NS	NS	NS
		8	**< 0.0005**	NS	NS	NS	NS	NS
		10	NS	NS	0.0802	NS	NS	NS
		12	NS	NS	0.0897	NS	NS	NS
		15	NS	NS	NS	NS	NS	NS
Gait Cycle Normalization	**VT**	5	NS	NS	NS	NS	NS	NS
		8	NS	NS	NS	NS	NS	NS
		10	NS	NS	**0.0195**	NS	NS	NS
		12	NS	0.3238	**0.0081**	NS	NS	NS
		15	NS	NS	0.4835	NS	NS	NS
	AP	5	NS	NS	NS	NS	NS	NS
		8	NS	NS	NS	NS	NS	NS
		10	NS	NS	**0.0451**	NS	NS	NS
		12	NS	0.3584	**0.0118**	NS	NS	NS
		15	NS	NS	0.4379	NS	NS	NS
	ML	5	NS	NS	NS	NS	NS	NS
		8	NS	NS	NS	NS	NS	NS
		10	NS	NS	**0.0249**	NS	NS	NS
		12	NS	0.1395	**0.0037**	NS	NS	NS
		15	NS	NS	0.0897	NS	NS	NS
Data Point Normalization	**VT**	5	NS	NS	NS	NS	NS	NS
		8	NS	NS	NS	NS	NS	NS
		10	NS	NS	0.1395	NS	NS	NS
		12	NS	NS	0.0639	NS	NS	NS
		15	NS	NS	NS	NS	NS	NS
	AP	5	NS	NS	NS	NS	NS	NS
		8	NS	NS	NS	NS	NS	NS
		10	NS	NS	0.0716	NS	NS	NS
		12	NS	NS	**0.0316**	NS	NS	NS
		15	NS	NS	0.4379	NS	NS	NS
	ML	5	NS	NS	NS	NS	NS	NS
		8	**< 0.0005**	NS	NS	NS	NS	NS
		10	NS	NS	0.2923	NS	NS	NS
		12	NS	NS	0.0716	NS	NS	NS
		15	NS	NS	NS	NS	NS	NS

Table C- 4: Effect embedding dimension on the LyE under select time delays using the W-algorithm calculated from over-ground walking of young health adults. NS: *p*-values > 0.5

Normalization Method	Dir.	Tau	Dimension Pairwise Comparison (p-value)					
			d4-d5	**d4-d6**	**d4-d7**	**d5-d6**	**d5-d7**	**d6-d7**
Raw Gait Cycles	**VT**	5	NS	**0.0071**	**< 0.0005**	NS	**0.0316**	NS
		8	NS	**0.0118**	**< 0.0005**	NS	**0.0016**	NS
		10	NS	**0.0016**	**< 0.0005**	NS	**< 0.0005**	NS
		12	NS	**< 0.0005**	**< 0.0005**	NS	**< 0.0005**	NS
		15	NS	**< 0.0005**	**< 0.0005**	NS	**< 0.0005**	NS
	AP	5	NS	0.0802	**< 0.0005**	NS	0.1121	NS
		8	NS	0.0639	**< 0.0005**	NS	**0.0014**	NS
		10	NS	**0.0249**	**< 0.0005**	NS	**< 0.0005**	0.4379
		12	NS	**0.0008**	**< 0.0005**	0.1925	**< 0.0005**	NS
		15	NS	**0.0071**	**< 0.0005**	NS	**< 0.0005**	NS
	ML	5	NS	**0.0037**	**< 0.0005**	NS	**0.0092**	NS
		8	NS	**0.0092**	**< 0.0005**	NS	**0.0012**	NS
		10	NS	**< 0.0005**	**< 0.0005**	NS	**< 0.0005**	NS
		12	NS	**< 0.0005**	**< 0.0005**	NS	**< 0.0005**	NS
		15	NS	**< 0.0005**	**< 0.0005**	NS	**< 0.0005**	NS
Gait Cycle Normalization	**VT**	5	NS	**0.0118**	**< 0.0005**	NS	**0.0152**	NS
		8	NS	**0.0134**	**< 0.0005**	NS	**0.0016**	NS
		10	NS	**< 0.0005**	**< 0.0005**	NS	**< 0.0005**	NS
		12	NS	**< 0.0005**	**< 0.0005**	NS	**< 0.0005**	NS
		15	NS	**< 0.0005**	**< 0.0005**	NS	**< 0.0005**	NS
	AP	5	NS	**0.0021**	**< 0.0005**	NS	**0.0048**	NS
		8	NS	**< 0.0005**	**< 0.0005**	NS	**0.0007**	NS
		10	NS	**< 0.0005**	**< 0.0005**	NS	**0.0009**	NS
		12	NS	**< 0.0005**	**< 0.0005**	NS	**< 0.0005**	NS
		15	NS	**< 0.0005**	**< 0.0005**	NS	**< 0.0005**	NS
	ML	5	NS	**0.0316**	**< 0.0005**	NS	**0.0172**	NS
		8	NS	**0.022**	**< 0.0005**	NS	**0.0006**	NS
		10	NS	**0.0005**	**< 0.0005**	0.3963	**< 0.0005**	NS
		12	NS	**0.0009**	**< 0.0005**	NS	**< 0.0005**	NS
		15	NS	**0.0016**	**< 0.0005**	NS	**< 0.0005**	NS
Data Point Normalization	**VT**	5	NS	**0.0134**	**< 0.0005**	NS	**0.0172**	NS
		8	NS	**0.0014**	**< 0.0005**	NS	**0.0009**	NS
		10	NS	**< 0.0005**	**< 0.0005**	NS	**< 0.0005**	NS
		12	NS	**< 0.0005**	**< 0.0005**	NS	**< 0.0005**	NS
		15	NS	**0.0007**	**< 0.0005**	NS	**< 0.0005**	NS
	AP	5	NS	0.1395	**< 0.0005**	NS	**0.0104**	NS
		8	NS	**0.0281**	**< 0.0005**	NS	**< 0.0005**	NS
		10	< 0.0005	NS	**< 0.0005**	**< 0.0005**	NS	**< 0.0005**
		12	< 0.0005	NS	**< 0.0005**	**< 0.0005**	NS	**< 0.0005**
		15	< 0.0005	NS	**< 0.0005**	**< 0.0005**	NS	**< 0.0005**
	ML	5	NS	**0.0062**	**< 0.0005**	NS	**0.0062**	NS
		8	NS	**< 0.0005**	**< 0.0005**	NS	**0.0006**	NS
		10	NS	**< 0.0005**	**< 0.0005**	NS	**0.0005**	NS
		12	NS	**< 0.0005**	**< 0.0005**	NS	**< 0.0005**	NS
		15	NS	**< 0.0005**	**< 0.0005**	NS	**< 0.0005**	NS

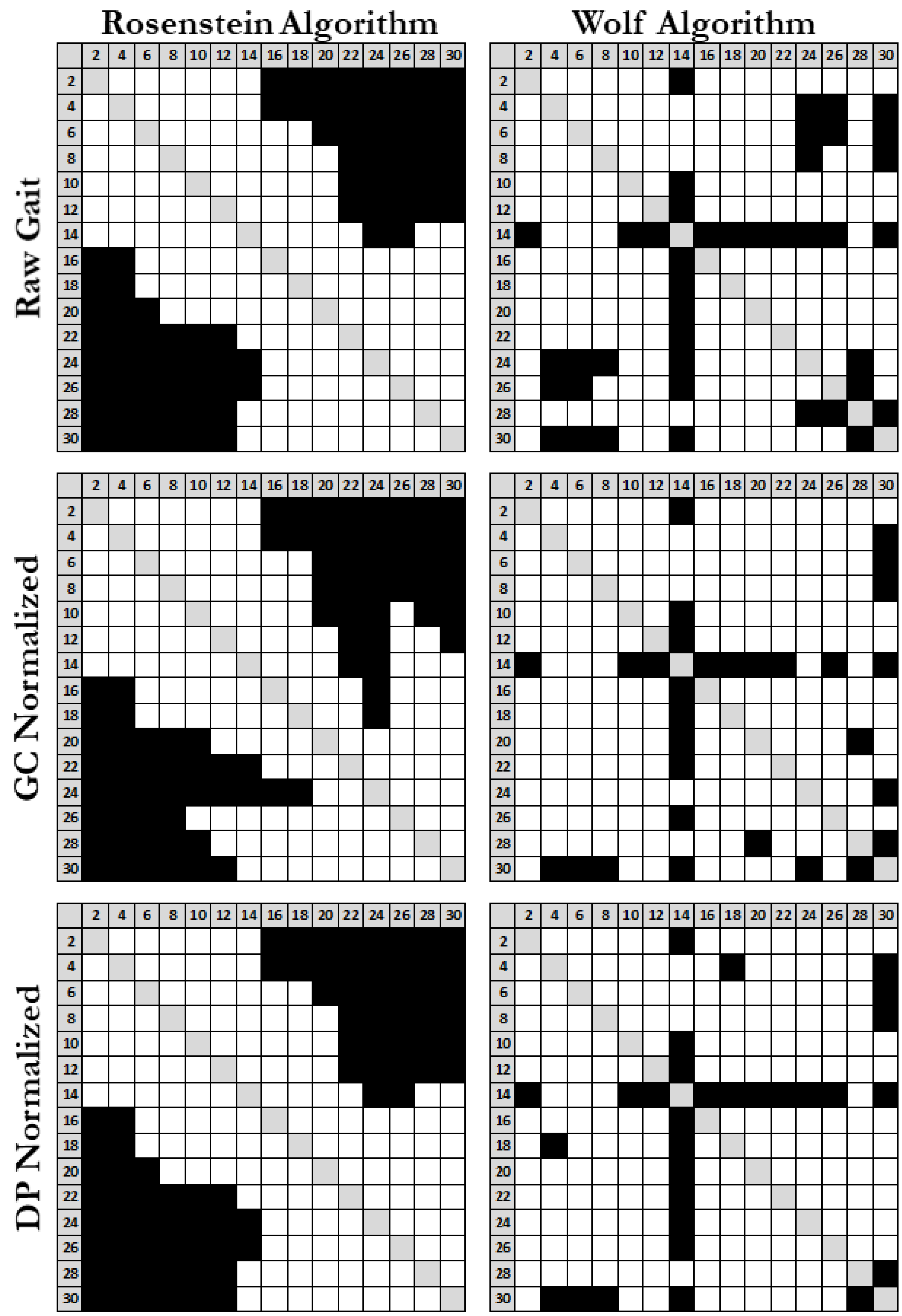

Figure C- 5: Effect of time delay on the LyE in elderly adults using 6 algorithm-normalization method combinations. This graphic shows the significant differences when two distinct time delays are compared in the AP direction. Filled in (black) boxes indicate significant differences and empty (white) boxes show where there are no significant differences between a pair of time delays.

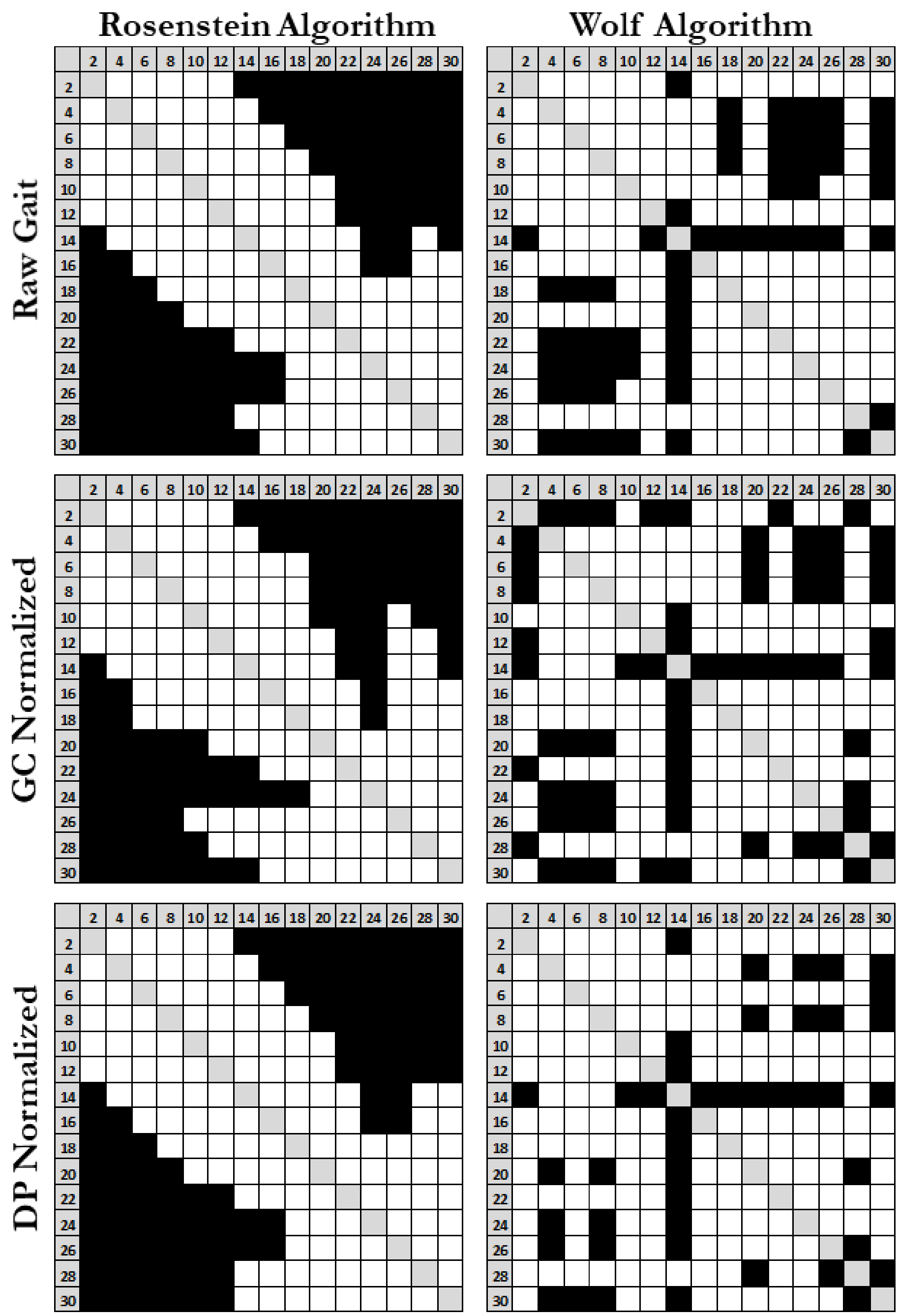

Figure C- 6: Effect of time delay on the LyE in young healthy adults using 6 algorithm-normalization method combinations. This graphic shows the significant differences when two distinct time delays are compared in the AP direction. Filled in (black) boxes indicate significant differences and empty (white) boxes show where there are no significant differences between a pair of time delays

APPENDIX D

INSTITUTE REVIEW BOARD APPROVAL FROM ARIZONA STATE UNIVERSITY

APPROVAL: EXPEDITED REVIEW

Thurmon Lockhart
Biological and Health Systems Engineering, School of (BHSE)
-
Thurmon.Lockhart@asu.edu

Dear Thurmon Lockhart:

On 7/22/2017 the ASU IRB reviewed the following protocol:

Type of Review:	Initial Study
Title:	Application of Nonlinear Dynamics in Gait and Balance
Investigator:	Thurmon Lockhart
IRB ID:	STUDY00006518
Category of review:	(4) Noninvasive procedures, (7)(a) Behavioral research
Funding:	None
Grant Title:	None
Grant ID:	None
Documents Reviewed:	• Consent, Category: Consent Form; • Protocol, Category: IRB Protocol; • Medical History Form, Category: Measures (Survey questions/Interview questions /interview guides/focus group questions); • Recruitment, Category: Recruitment Materials;

The IRB approved the protocol from 7/22/2017 to 7/21/2018 inclusive. Three weeks before 7/21/2018 you are to submit a completed Continuing Review application and required attachments to request continuing approval or closure.

If continuing review approval is not granted before the expiration date of 7/21/2018 approval of this protocol expires on that date. When consent is appropriate, you must use final, watermarked versions available under the “Documents” tab in ERA-IRB.

In conducting this protocol you are required to follow the requirements listed in the INVESTIGATOR MANUAL (HRP-103).

Sincerely,

IRB Administrator

cc: Victoria Smith
Seong Hyun Moon
Thurmon Lockhart
Markey Olson
Christopher Frames
Saba Rezvanian
Victoria Smith

APPROVAL:CONTINUATION

Thurmon Lockhart
Biological and Health Systems Engineering, School of (BHSE)
-
Thurmon.Lockhart@asu.edu

Dear Thurmon Lockhart:

On 6/28/2018 the ASU IRB reviewed the following protocol:

Type of Review:	Continuing Review
Title:	Application of Nonlinear Dynamics in Gait and Balance
Investigator:	Thurmon Lockhart
IRB ID:	STUDY00006518
Category of review:	(4) Noninvasive procedures, (7)(a) Behavioral research
Funding:	None
Grant Title:	None
Grant ID:	None
Documents Reviewed:	• Consent, Category: Consent Form; • Medical History Form, Category: Measures (Survey questions/Interview questions /interview guides/focus group questions); • Recruitment, Category: Recruitment Materials; • Protocol-perturbations, Category: IRB Protocol;

The IRB approved the protocol from 6/28/2018 to 7/20/2019 inclusive. Three weeks before 7/20/2019 you are to submit a completed Continuing Review application and required attachments to request continuing approval or closure.

If continuing review approval is not granted before the expiration date of 7/20/2019 approval of this protocol expires on that date. When consent is appropriate, you must use final, watermarked versions available under the "Documents" tab in ERA-IRB.

In conducting this protocol you are required to follow the requirements listed in the INVESTIGATOR MANUAL (HRP-103).

Sincerely,

IRB Administrator

cc: Victoria Smith
Seong Hyun Moon
Thurmon Lockhart
Markey Olson
Christopher Frames
Saba Rezvanian
Victoria Smith

APPROVAL: MODIFICATION

Thurmon Lockhart
BHSE: Biological and Health Systems Engineering, School of
480/965-1499
Thurmon.Lockhart@asu.edu

Dear Thurmon Lockhart:

On 5/8/2019 the ASU IRB reviewed the following protocol:

Type of Review:	Modification
Title:	Application of Nonlinear Dynamics in Gait and Balance
Investigator:	Thurmon Lockhart
IRB ID:	STUDY00006518
Funding:	None
Grant Title:	None
Grant ID:	None
Documents Reviewed:	• Consent_subStudy1, Category: Consent Form; • Protocol-perturbations, Category: IRB Protocol; • Medical History Form, Category: Measures (Survey questions/Interview questions /interview guides/focus group questions); • Volunteer Ad 2, Category: Recruitment Materials; • Recruitment, Category: Recruitment Materials; • Consent, Category: Consent Form;

The IRB approved the modification.

When consent is appropriate, you must use final, watermarked versions available under the “Documents” tab in ERA-IRB.

In conducting this protocol you are required to follow the requirements listed in the INVESTIGATOR MANUAL (HRP-103).

Sincerely,

IRB Administrator

cc: Victoria Smith
Seong Hyun Moon
Thurmon Lockhart
Markey Olson
Christopher Frames
Saba Rezvanian
Victoria Smith

Consent Form: Bioscience

Title of research study: Application of Nonlinear Dynamics in Gait and Balance

Investigator: Thurmon E. Lockhart, Ph.D., Professor, School of Biological and Health Systems Engineering

Why am I being invited to take part in a research study?

We invite you to take part in a research study because you may be eligible for the research concerning the fall risk for healthy individuals.

Why is this research being done?

Balance is vital to being able to perform basic everyday activities such as sitting down, standing up, and walking. Just about everything an individual does physically requires balance control and most of the time this is done automatically without conscious attention. Posture and gait stability is a vital indicator of fall risk assessment.

1. This study is being done for two reasons:
2. To apply a new math technique to better understand the dynamics of how people balance and walk

To see how balance while standing and while walking changes when a perturbation is given

How long will the research last?

We expect that individuals will spend on average 2 hours participating in the proposed activities. However, testing can take between one and five hours. The proposed activities includes walking on a treadmill and may or may not include wearing a headset.

How many people will be studied?

We expect between 25 and 500 people will participate in this research study.

What if I say yes, I want to be in this research?
It is up to you to decide whether or not to participate. Participation is voluntary.

The proposed activities in this research includes walking and standing on an instrumented treadmill that may or may not move unexpected, as well as, you may or may not be asked to wear a headset during these activities. Prior to any testing, you will first sign this document and then fill out a medical history exam to determine your eligibility for the study. The survey will take about 10-15 minutes to complete. If you are not eligible to participate in the study, we will thank you for your time and you will be free to go. If you are eligible, then we will measure your height and weight and proceed with testing. You will be outfitted with one or more sensors and reflective markers which will be used to collect data. You will then perform walking and standing balance trials with and without perturbations. This part of the protocol and take from an hour to five hours in duration depending on the detailed protocol that the research is using. The researcher will provide you with a more detailed timeline at the time of signing this consent form.

What happens if I say yes, but I change my mind later?

You can leave the research at any time it will not be held against you. If you stop being in the research, already collected data may not be removed from the study database.

Is there any way being in this study could be bad for me?

Perturbations occur in normal daily walking and by itself is not dangerous, however, a fall due to perturbations have a negative consequences. Since in this study, you will wear a fall arresting harness system that prevents you from falling to the ground, the risk of this experiment is very low. Although rare, bruising from the harness is possible. You will be given rest at regular intervals and you can always ask to take additional breaks.

What happens to the information collected for the research?

Efforts will be made to limit the use and disclosure of your personal information to people who have a need to review this information. We cannot promise complete secrecy. Organizations that may inspect and copy your information include the IRB and other representatives of this organization.

What else do I need to know?

Your participation in this study is voluntary. You must be at least 18 years old and we ask that you wear athletic, non-reflective clothing and athletic sneakers for this experiment. If you are not wearing the appropriate clothing, we will provide a change of clothes for you to wear for the duration of the experiment.

If you agree to participate in the study, then consent does not waive any of your legal rights. However, no funds have been set aside to compensate you in the event of injury.

Who can I talk to?

If you have questions, concerns, or complaints, or think the research has hurt you, talk to the research team at team at Arizona State University – Dr. Thurmon E. Lockhart, thurmon.lockhart@asu.edu, or 480-965-1499.

This research has been reviewed and approved by the Bioscience IRB (“IRB”). You may talk to them at (480) 965-6788 or research.integrity@asu.edu if:

- Your questions, concerns, or complaints are not being answered by the research team.
- You cannot reach the research team.
- You want to talk to someone besides the research team.
- You have questions about your rights as a research participant.
- You want to get information or provide input about this research.

Signature Block for Capable Adult

Your signature documents your permission to take part in this research.

____ ____________________
Signature of participant Date

____
Printed name of participant

____ ____________________
Signature of person obtaining consent Date

____
Printed name of person obtaining consent

AFTER MODIFICATION APPROVAL:

Consent Form: Bioscience

Title of research study: Application of Nonlinear Dynamics in Gait and Balance

Investigator: Thurmon E. Lockhart, Ph.D., Professor, School of Biological and Health Systems Engineering

Why am I being invited to take part in a research study?

We invite you to take part in a research study because you may be eligible for the research concerning the fall risk for healthy individuals.

Why is this research being done?

Balance is vital to being able to perform basic everyday activities such as sitting down, standing up, and walking. Just about everything an individual does physically requires balance control and most of the time this is done automatically without conscious attention. Posture and gait stability is a vital indicator of fall risk assessment. This study is being done for two reasons to apply a new math technique to better understand the dynamics of how people walk.

How long will the research last?

We expect that individuals will spend on average 1 hour participating in the proposed activities. However, testing can take between 60 and 90 minutes. The proposed activities includes walking on a treadmill and in a hallway.

How many people will be studied?

We expect between 10 and 30 people will participate in this research study.

What happens if I say yes, I want to be in this research?

It is up to you to decide whether or not to participate. Participation is voluntary.

The proposed activities in this research includes walking on an instrumented treadmill. Prior to any testing, you will first sign this document and then fill out a medical history exam to determine your eligibility for the study. The survey will take about 5 minutes to complete. If you are not eligible to participate in the study, we will thank you for your time and you will be free to go. If you are eligible, then we will measure your height and weight and proceed with testing. You will be outfitted with one or more sensors and reflective markers which will be used to collect data. You will then walk on the treadmill for 30 minutes and then a short break 2-5 minutes, as needed, will be given. Then the final task of walking over a level plane (hallway) for 3 minutes will be performed.

What happens if I say yes, but I change my mind later?

You can leave the research at any time it will not be held against you. If you stop being in the research, already collected data may not be removed from the study database.

Is there any way being in this study could be bad for me?

This study does not have any added risks than if the participant was walking on a regular treadmill. This treadmill has additional safety measures, such as stopping if an individual walks too close to the front or back of the treadmill.

What happens to the information collected for the research?

Efforts will be made to limit the use and disclosure of your personal information to people who have a need to review this information. We cannot promise complete secrecy. Organizations that may inspect and copy your information include the IRB and other representatives of this organization.

What else do I need to know?

Your participation in this study is voluntary. You must be at least 18 years old and we ask that you wear athletic, non-reflective clothing and athletic sneakers for this experiment. If you are not wearing the appropriate clothing, we will provide a change of clothes for you to wear for the duration of the experiment or we will reschedule your participation to a later date when you are wearing appropriate clothing.

If you agree to participate in the study, then consent does not waive any of your legal rights. However, no funds have been set aside to compensate you in the event of injury.

Who can I talk to?

If you have questions, concerns, or complaints, or think the research has hurt you, talk to the research team at team at Arizona State University – Dr. Thurmon E. Lockhart, thurmon.lockhart@asu.edu, or 480-965-1499.

This research has been reviewed and approved by the Bioscience IRB ("IRB"). You may talk to them at (480) 965-6788 or research.integrity@asu.edu if:

- Your questions, concerns, or complaints are not being answered by the research team.
- You cannot reach the research team.

- You want to talk to someone besides the research team.
- You have questions about your rights as a research participant.
- You want to get information or provide input about this research.

Signature Block for Capable Adult

Your signature documents your permission to take part in this research.

____ ____________________

Signature of participant Date

____

Printed name of participant

____ ____________________

Signature of person obtaining consent Date

____

Printed name of person obtaining consent