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A mathematical modeling approach for high and new technology-project portfolio selection under uncertain environments

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Abstract. High and new technology-project as a tool to achieve productive forces through scientific and technological 9 knowledge is characterized as knowledge based with high risk and returns. Often conflicting objectives of these projects have 10 complicated their assessment and selection process. This paper offers a novel approach of high technology-project portfolio 11 selection in two main parts. In the first part, a new risk reduction compromise decision-making model is proposed that applies 12 a new approach in determining the weights of experts and in avoiding information loss. The objective function of a new 13 interval type-2 fuzzy sets (IT2Fs) based mathematical model of project portfolio selection is formed by the outcome. To 14 depict model's applicability, data from case study of high technology-project selection in the literature is used to present the 15 efficacy of the model. 16

Keywords: High and new technology-projects, project portfolio selection, compromise solutions, mathematical modeling,
 interval type-2 fuzzy sets

19 **1. Introduction**

Large high-tech mega-projects are referred to projects that require research and development and/or application of technology in addition to a substantial infrastructure and multi-million or even billion dollar budgets. Additionally, their time-horizons are measured in at least years [18]. Ability of decision makers (DMs)' to flawlessly analyze projects is weakened by high risk of uncertainty or inadequacy of project data

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[11, 21–24, 31]. This complication and vagueness is intensified in high-technology [16].

High-tech mega-projects have high levels of risk, vagueness and uncertainty. At the initial phases, uncertainty mostly affects performance expectations, political environments, goals, motivations, and potentials [25, 11]. Thomas and Mengel [10] stated that complex projects have vagueness and ambiguity of the not-yet-known that occur as events that crucially reframe meaning, interpretation, and social significance emerge.

Due to lack of adequate historical data, vagueness and high influence of experts' judgments on project selection problems, fuzzy sets theory has been referred to as a welcomed approach in considering

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project uncertainty [3, 27]. Most of the studies even 42 in the recent years are based on classic fuzzy sets. 43 Zadeh [13] expanded type-2 fuzzy sets (FSs). Type-44 2 FSs have fuzzy membership functions (MFs) also 45 called "membership of membership". In type-2 FSs 46 in contrast to type-1 FSs each membership value of 47 each element is expressed by fuzzy set in [0, 1], rather 48 than using a crisp number in [0, 1]. Despite all these 49 positive points, unfortunately using T2FSs to model 50 the environment of high-tech project is still new. 51

Some of the main literature gaps that motivated 52 proposing this paper are as follows: (1) literature of 53 project selection and projects portfolio selection is 54 very weak when it comes to high-tech projects (2) 55 this problem contains very high levels of uncertainty 56 and vagueness and they are not yet well addressed; (3) 57 the existing decision-making methods do not compre-58 hensively address risk of uncertainty and importance 50 of each DM's judgment. 60

In order to fill the gaps of this practical decision-61 making situation, this paper offers a novel two-part 62 model of high-tech project portfolio selection under 63 highly uncertain and vague conditions is proposed 64 that presents interval type-2 fuzzy sets (IT2FSs) 65 to model uncertainty. In the first part of the pre-66 sented approach, a new IT2FSs based-risk reduction 67 compromise decision-making process is presented 68 that avoids information loss in designating weights 69 to DMs. Employing IT2FSs gives the model with 70 high power of uncertainty modeling and calculat-71 ing. Moreover, each DM receives a weight based on 72 the judgments received in the process. In the second 73 part, a new mathematical model of project portfolio 74 selection with IT2F-constraints is proposed to find 75 the optimal portfolio of projects. Eventually, in this 76 paper the basic concept of IT2FSs is improved by 77 presenting a novel method of interval type-2 fuzzy 78 number (IT2FN)-ranking. 79

The following illustrates the remainder of this paper. In Section 2, the relevant literature on compromise decision making problems is reviewed. Section 3 displays the introduced model. Model's application is illustrated in Section 4 and eventually Section 5 presents the conclusion remarks.

86 **2.** Literature review

Most of the project selection related studies apply
the concept of multi-criteria decision-making and
multi-criteria analysis [1]. Actually, since project
evaluation and selection is a group decision-making

process that is affected by different project aspects, applying multi criteria decision-making methods could be a useful approach. On the other hand, one aspect that highly influences project evaluation and selection studies especially in case of high technology-projects is uncertainty. Over the years, a large number of fuzzy multi-criteria decision making (FMCDM) methods have been introduced. All approaches are mainly concerned with conducting the decision-making process better informed and more structured. Through reviewing previous studies, FMCDM can be categorized as a fuzzy multi-objective decision-making (FMODM) and fuzzy multi-attribute decision-making (FMADM) approach.

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A practical solution in highly uncertain environments is applying type-2 FSs. The development made by Wu and Mendel [2] was based on using words for interval type-2 fuzzy hierarchical MADM. The model was applied to assess a weapon system. Dereli and Alton [26] used IT2FSs to present a framework that evaluated technologies. Dereli and Alton [25] further investigated the problem of candidate technology assessment with the help of a fuzzy inference system that used type 2 fuzzy sets. Qin et al. [9] developed a decision model integrating VIKOR method and prospect theory. To illustrate the applicability of their method, they used case study of a high-tech risk evaluation.

Another approach in using IT2FSs in project environment is employing these sets in mathematical modeling and programming. To the best of our knowledge, this approach in project and project portfolio selection is new and only a small number of studies have used this approach. For instance, Mohagheghi et al. [28] presented a model of project cash flow prediction that could also be applied in project evaluation and appraisal. Mohagheghi et al. [29] applied IT2FSs to evaluate R&D project evaluation and project portfolio selection. As mentioned earlier employing type-2 FSs in mathematical modeling for project selection problems is new and most of the IT2FSbased approaches apply different MCDM techniques.

Since this paper offers a new method of IT2Franking, a brief review of ranking methods is presented. Mitchell [4] presented one of the first type-2 fuzzy-ranking methods. The method was based on random inputs and the randomness involved in the process would affect the final results. Qin and Liu [9] used operators of arithmetic average, geometric average and harmonic average (HA) to rank IT2FNs. Kunda et al. [19] presented a model of interval type-2

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fuzzy-ranking. The method was based on the concept 143 of using relative preference index. Proposed rank-144 ing approaches are not totally satisfactory. Some of 145 the reasons are as follows: lack of enough discrim-146 ination while differentiating similar IT2FNs, having inconsistent and sometimes counter-intuitive results 148 under different situations, and requiring large com-149 putational effort under specific conditions. 150

As it was mentioned, any practical project eval-151 uation process requires sophisticated consideration 152 of uncertainty. Most of the existing literature of 153 the project and project portfolio selection is based 154 on classic fuzzy sets theory. In environments like 155 high technology-project environments that have a 156 very high level of uncertainty it is more practical 157 to use type-2 FSs. Therefore, in this paper, a new 158 model of project portfolio selection under an IT2F-159 environment that controls the risk of uncertainty in 160 addition to avoiding information loss when giving 161 weight to DMs is proposed. 162

3. Proposed approach 163

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In this section, first a new effective ranking method 164 is presented that is based on the concept of positive 165 and negative ideal solutions. The project portfolio 166 selection has two main parts. In the first part, a novel 167 decision-making approach is presented that avoids 168 information loss in addition to controlling uncertainty 169 of soft computing. This part of the model results 170 in ranking the candidate projects while consider-171 ing the selection criteria. The second part includes 172 a new mathematical model based on the concept of 173 IT2FSs that uses the results of the previous part of 174 the model to select the best portfolio of projects 175 while considering conflicting and practical limita-176 tions and considerations. It should be noted that the 177 applied IT2FS definitions and operators were taken 178 from [6–8, 12, 15, 20]. 179

3.1. Proposed ranking trapezoidal interval 180 *type-2 fuzzy numbers* 181

In this section, a novel approach for comparing and 182 ranking IT2FNs is presented. This approach is based 183 on sensible use of concept of ideal solutions. Also, a 184 distance-based similarity measure between IT2FNs 185 is appropriately developed for effectively obtaining 186 the overall performance for any given IT2FN ranking 187 and comparing process. This method is based on the 188 studies of Deng [5], Ren et al. [14], Mohagheghi et al. 189

[30] and Zhang and Zhang [34]. The step-by-step algorithm is introduced as follows:

- 1. Define the trapezoidal interval type-2 fuzzy positive ideal solution as $\tilde{\tilde{X}}_{max}$ and the negative ideal solution as \tilde{X}_{min} .
- 2. Calculate the distance-based degree of similarity between each interval type-2 fuzzy number $\tilde{A}_i (i = 1, 2, ..., n)$ and the positive interval type-2 fuzzy ideal solution (d_i^+) by applying Equation (1):

$$d_{i}^{+}\left(\tilde{\tilde{A}}_{i}, \tilde{\tilde{X}}_{max}\right)$$

$$= \sqrt{\sum_{i=1}^{4} (a_{i}^{U} - x_{i}^{U})^{2} + \sum_{i=1}^{4} (a_{i}^{L} - x_{i}^{L})^{2}} + \sum_{i=1}^{2} (H_{i}(\tilde{\tilde{A}}^{U}) - H_{i}(\tilde{\tilde{X}}^{U}))^{2}$$

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3. Calculate the distance-based degree of similarity between each interval type-2 fuzzy number \tilde{A}_i (i = 1, 2, ..., n) and the negative interval type-2 fuzzy ideal solution (d_i^-) by applying Equation (2):

$$= \left(\tilde{\tilde{A}}_i, \ \tilde{\tilde{X}}_{min}\right)$$
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$$= \sqrt{\sum_{i=1}^{4} (a_i^U - x_i^U)^2 + \sum_{i=1}^{4} (a_i^L - x_i^L)^2 + \sum_{i=1}^{2} (H_i(\tilde{\tilde{A}}^U) - H_i(\tilde{\tilde{X}}^U))^2 + \sum_{i=1}^{2} (H_i(\tilde{\tilde{X}}^L) - H_i(\tilde{\tilde{X}}^L))^2 + \sum_{i=1}^{2} (H_i(\tilde{\tilde{X}^L}) - H_i(\tilde{\tilde{X}}^L))^2 + \sum_{i=1}^{2} (H_i(\tilde{\tilde{X}}^L) - H_i(\tilde{\tilde{X}}^L))^2 + \sum_{i=1}^{2} (H_i(\tilde{\tilde{X}^L$$

4. Determine the point $E(\min(d_i^+), \max(d_i^-))$, which is referred to as the optimized ideal reference point.

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5. Calculate the distance from each alternative to point *E* by using the following:

$$ED_{i} = \sqrt{ \begin{bmatrix} d_{i}^{+} - \min(d_{i}^{+}) \end{bmatrix}^{2} + \begin{bmatrix} d_{i}^{-} - \max d_{i}^{-} \end{bmatrix}^{2} }, \qquad 214$$

$$i = 1, 2, \dots, n \qquad (3) \qquad 215$$

6. Rank the interval type-2 fuzzy numbers \tilde{A}_i (i = 1, 2, ..., n) in increasing order of ED_i . If two numbers happen to have the same value 205 206

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Table 1 Linguistic terms and trapezoidal interval type-2 fuzzy numbers

Linguistic variables	Trapezoidal interval type-2 fuzzy numbers
Extreme High (EH)	((8,9,9,10; 1,1),(8.5,9,9,9.5;0.9,0.9))
Very High (VH)	((6,7,7,8; 1,1),(6.5,7,7,7.5;0.9,0.9))
High (H)	((4,5,5,6; 1,1),(4.5,5,5,5.5;0.9,0.9))
Medium High (MH)	((2,3,3,4; 1,1),(2.5,3,3,4.5;0.9,0.9))
M (Medium)	((1,1,1,1; 1,1),(1,1,1,1;0.9,0.9))
Medium Low (ML)	((0.25, 0.33, 0.33, 0.5; 1, 1), (0.22, 0.33, 0.33, 0.4; 0.9, 0.9))
Low (L)	((0.17, 0.2, 0.2, 0.25; 1, 1), (0.18, 0.2, 0.2, 0.22; 0.9, 0.9))
Very Low (VL)	((0.13, 0.14, 0.14, 0.17; 1, 1), (0.13, 0.14, 0.14, 0.15; 0.9, 0.9))
Extreme Low (EL)	((0.1, 0.11, 0.11, 0.13; 1, 1), (0.11, 0.11, 0.11, 0.12; 0.9, 0.9))

of ED_i , determine their ED_i by the following Equation and rank them in increasing order of ED_i .

$$ED_i = d_i^+ - \min\left(d_i^+\right). \tag{4}$$

3.2. Proposed type 2-risk reduction compromise 216 ratio model 217

In this section, a new risk reduction compromise 218 ratio method based on trapezoidal IT2FSs and foot-219 print of uncertainty (FOU) is developed that explores 220 the impacts of the criteria used in the decision-221 making process. Linguistic variables were converted 222 into trapezoidal interval type-2 fuzzy sets and are pre-223 sented in Table 1. This novel method can be described 224 in detail by means of the following. 225

First, decision information of each DM should be 226 gathered, therefore: 227

> $\tilde{\tilde{D}}_{K} = \left(\tilde{\tilde{D}}_{ij}^{K}\right)_{m \times n} = \begin{bmatrix} \tilde{\tilde{D}}_{11}^{K} & \cdots & \tilde{\tilde{D}}_{1n}^{K} \\ \vdots & \ddots & \vdots \\ \tilde{\tilde{D}}_{m1}^{K} & \cdots & \tilde{\tilde{D}}_{mn}^{K} \end{bmatrix}$ (5) $\tilde{\tilde{W}}_K = \left(\tilde{\tilde{w}}_1^k, \; \tilde{w}_2^k, \ldots, \tilde{\tilde{w}}_n^k\right), \; K \in T$ (6)

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Where $\tilde{\tilde{D}}_K$ is the decision matrix and $\tilde{\tilde{W}}_K$ is the weight vector of attributes, m is the number of criteria, n is the number of alternatives compared and Tdenotes the group of experts. $\tilde{\tilde{w}}_i$ is the weight vector of the criteria. Obviously, $\tilde{\tilde{D}}_{ii}^{K}$ and $\tilde{\tilde{W}}_{K}$ are trapezoidal IT2FSs.

The decision matrix should be normalized (\tilde{F}) using Equations (8 and 9).

$$\tilde{\tilde{F}} = \begin{bmatrix} \tilde{\tilde{F}}_{11} & \cdots & \tilde{\tilde{F}}_{1n} \\ \vdots & \ddots & \vdots \\ \tilde{\tilde{F}}_{m1} & \cdots & \tilde{\tilde{F}}_{mn} \end{bmatrix}$$
(7)

$$\tilde{F}_{ij} = (f_{i1}^U, f_{i2}^U, f_{i3}^U, f_{i4}^U; (\min H_{i1}(D_1^U), H_1(d^*))$$

$$(\min H_2(D_1^U), H_2(d^*)), f_{i1}^L, f_{i2}^L, f_{i3}^L, f_{i4}^L;$$
²³⁷

$$(\min H_1(D_1^L), H_1(d^*)), (\min H_2(D_1^L)),$$
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$$H_2(d^*)))$$
 (8) 239

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$$f_{1i}^{T} = \min\left(\frac{d_{ij_{1m}}^{T}}{d^{*}}, \frac{d_{ij_{1m}}^{T}}{d^{*}}, \frac{d_{ij_{1(5-m)}}^{T}}{d^{*}}, \frac{d_{ij_{1(5-m)}}^{T}}{d^{*}}\right), \qquad 242$$

$$T \in \{U, L\}, m \in \{1, 2\}$$

$$= \min\left(\frac{d_{ij_{1(5-n)}}^{T}}{d^{*}}, \ \frac{d_{ij_{1(5-n)}}^{T}}{d^{*}}, \ \frac{d_{ij_{1n}}^{T}}{d^{*}}, \ \frac{d_{ij_{1n}}^{T}}{d^{*}}\right), \qquad 245$$

$$T \in \{U, L\}, n \in \{3, 4\}$$
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$$= 1, 2, \ldots, n, j \in B$$

$$\tilde{F}_{ij} = (f_{i1}^U, f_{i2}^U, f_{i3}^U, f_{i4}^U; (\min H_{i1}(D_1^U), H_1(d^-)),$$

$$(\min H_2(D_1^U), H_2(d^-)), f_{i1}^L, f_{i2}^L, f_{i3}^L, f_{i4}^L;$$

$$(\min H_1(D_1^L), H_1(d^-)), (\min H_2(D_1^L),$$

$$H_2(d^-)))$$
(9) 252

where

$$f_{1i}^{T} = \min\left(\frac{d^{-}}{d_{ij_{2m}}^{T}}, \ \frac{d^{-}}{d_{ij_{2(5-m)}}^{T}}, \ \frac{d^{-}}{d_{ij_{2m}}^{T}}, \ \frac{d^{-}}{d_{ij_{2(5-m)}}^{T}}\right),$$
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$$f_{1j}^{T} = \min\left(\frac{d^{-}}{d_{ij_{2(5-n)}}^{T}}, \ \frac{d^{-}}{d_{ij_{2n}}^{T}}, \ \frac{d^{-}}{d_{ij_{2(5-n)}}^{T}}, \ \frac{d^{-}}{d_{ij_{2n}}^{T}}\right),$$
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$$T \in \{U, L\}, n \in \{3, 4\}$$
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$$i = 1, 2, \dots, n, j \in B$$
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Where B denotes the group of benefit criteria and 258 C represents the group of cost criteria. d^* and d^- are 259 also obtained as follows: 260

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$$d^* = \max_i \left(d_{ij} \right)_4^U$$
(10)

(11)

$$d^- = \min_i (d_{ij})_1^U$$

The normalized weighted decision matrix is calcu-263 lated by employing Equation (12). 264

$$\tilde{\tilde{G}} = \begin{bmatrix} \tilde{\tilde{G}}_{11} & \cdots & \tilde{\tilde{G}}_{1n} \\ \vdots & \ddots & \vdots \\ \tilde{\tilde{G}}_{m1} & \cdots & \tilde{\tilde{G}}_{mn} \end{bmatrix}$$
(12)

 $\tilde{\tilde{G}}_{ii} = \tilde{\tilde{F}}_{ii} \times \tilde{\tilde{w}}_i$ 266 $= (g_{i1}^{U}, g_{i2}^{U}, g_{i3}^{U}, g_{i4}^{U}; (\min H_{i1} (G_{1}^{U}), H_{1} (F_{1}^{U})),$ 267 $(\min H_{i1}(G_1^U), H_1(F_1^U)),$ 268 $g_{i1}^{L}, g_{i2}^{L}, g_{i3}^{L}, g_{i4}^{L}; (\min H_1(G_1^L), H_1(F_1^L)),$ 269 $(\min H_2(G_1^L), H_2(F_1^L))$ (13)

where 271

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$$g_{1i}^{T} = \min \left(\begin{array}{c} f_{ij_{1m}}^{T} w_{j_{2m}}^{T}, f_{ij_{1m}}^{T} w_{j_{2(5-m)}}^{T}, \\ f_{ij_{1(5-m)}}^{T} w_{j_{2m}}^{T}, f_{ij_{1(5-m)}}^{T} w_{j_{2(5-m)}}^{T} \end{array} \right)$$

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$$T \in \{U, L\}, m \in \{1, 2\}$$

$$g_{1j}^{T} = \min \begin{pmatrix} f_{ij_{1(5-n)}}^{T} w_{j2(5-n)}^{T}, \\ f_{ij_{1(5-n)}}^{T} w_{j2n}^{T}, f_{ij_{1n}}^{T} w_{j2(5-n)}^{T}, f_{ij_{1n}}^{T} w_{j2n}^{T} \end{pmatrix},$$

$$T \in \{U, L\}, n \in \{3, 4\}$$

The ideal decisions of all individual decisions in mean sense should be the average of all individual decisions. A negative ideal decision should be of the maximum separation from the positive ideal decision [32]. Therefore, the best decision (G^*) , the left negative ideal decision (G_l^-) and the right negative ideal decision (G_r^-) are calculated by applying the following equations:

$$\tilde{\tilde{G}}^* = \begin{bmatrix} \tilde{\tilde{g}}_{11}^* & \cdots & \tilde{\tilde{g}}_{1n}^* \\ \vdots & \ddots & \vdots \\ \tilde{\tilde{g}}_{m1}^* & \cdots & \tilde{\tilde{g}}_{mn}^* \end{bmatrix}$$
(14)

where

$$\tilde{\tilde{g}}_{ij}^* = \left(\frac{1}{t}\sum_{k=1}^t g_{ij1}^U, \frac{1}{t}\sum_{k=1}^t g_{ij2}^U, \frac{1}{t}\sum_{k=1}^t g_{ij3}^U\right)^{-276}$$

$$1/t \sum_{k=1}^{t} g_{ij4}^{U}; \ (H_{i1}(G_1^U)), \ (H_{i2}(G_1^U)),$$
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$$1/t \sum_{k=1}^{t} g_{ij1}^{L}, \ 1/t \sum_{k=1}^{t} g_{ij2}^{L}, \ 1/t \sum_{k=1}^{t} g_{ij3}^{L}, \ 276$$

$$\frac{1}{t}\sum_{k=1}^{t}g_{ij4}^{L}; \text{ (min } H_1(G_1^L), H_1(F_1^L)),$$
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$$(\min H_2(G_1^L), H_2(F_1^L)))$$
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$$\tilde{\tilde{G}}_{L}^{-} = \begin{bmatrix} \tilde{\tilde{g}}_{l_{11}} & \cdots & \tilde{\tilde{g}}_{l_{1n}} \\ \vdots & \ddots & \vdots \\ \tilde{\tilde{g}}_{l_{m1}} & \cdots & \tilde{\tilde{g}}_{l_{mn}} \end{bmatrix}$$
(15)

where $\tilde{\tilde{g}}_{l_{ii}}^{-}$ =

$$\tilde{\tilde{G}}_{R}^{-} = \begin{bmatrix} \tilde{\tilde{g}}_{r_{11}}^{-} \cdots \tilde{\tilde{g}}_{r_{1n}}^{-} \\ \vdots & \ddots & \vdots \\ \tilde{\tilde{g}}_{r_{m1}}^{-} \cdots \tilde{\tilde{g}}_{r_{mn}}^{-} \end{bmatrix}$$
(16)

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where $\tilde{\tilde{g}}_{r_{ij}}^- = \max_{1 \le k \le t} \{ \tilde{\tilde{g}}_{ij}^k \}$

The difference of each individual judgment from the ideal judgments including positive ideal decision, the left negative ideal decision and the right negative ideal decision are respectively denoted by d_k^* , d_l^- and d_r^- and are determined by the following Equations:

$$d_{l}^{*} = \begin{cases} \sum_{p=1}^{4} (g_{ijp}^{U\ k} - g_{ijp}^{U\ *})^{2} + \sum_{p=1}^{4} (g_{ijp}^{L\ k} - g_{ijp}^{L\ *})^{2} \\ + \sum_{p=1}^{2} (H_{p}(\tilde{\tilde{G}}_{l}^{U^{K}}) - H_{p}(\tilde{\tilde{G}}_{l}^{U\ *}))^{2} \\ + \sum_{p=1}^{2} (H_{p}(\tilde{\tilde{G}}_{l}^{L^{K}}) - H_{p}(\tilde{\tilde{G}}_{l}^{L\ *}))^{2} \\ k \in T \end{cases}$$
(17)
$$z_{87}$$

$$d_{l}^{-} = \sqrt{\sum_{p=1}^{4} (g_{ljp}^{U\ k} - g_{ljp}^{U\ l})^{2} + \sum_{p=1}^{4} (g_{ljp}^{L\ k} - g_{ljp}^{L\ l})^{2}} + \sum_{p=1}^{2} (H_{p}(\tilde{G}_{l}^{U^{K}}) - H_{p}(\tilde{G}_{l}^{U^{l}}))^{2} \qquad (18)$$

$$\sqrt{\sum_{p=1}^{2} (H_{p}(\tilde{G}_{l}^{L^{K}}) - H_{p}(\tilde{G}_{l}^{L^{l}}))^{2}}$$

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$$d_{r}^{-} = \begin{cases} k \in T \\ \sum_{p=1}^{4} (g_{ijp}^{U^{-k}} - g_{ijp}^{U^{-r}})^{2} + \sum_{p=1}^{4} (g_{ijp}^{L^{-k}} - g_{ijp}^{L^{-l}})^{2} \\ + \sum_{p=1}^{2} (H_{p}(\tilde{\tilde{G}}_{l}^{U^{K}}) - H_{p}(\tilde{\tilde{G}}_{l}^{U^{r}}))^{2} + (19) \\ \sqrt{\sum_{p=1}^{2} (H_{p}(\tilde{\tilde{G}}_{l}^{L^{K}}) - H_{p}(\tilde{\tilde{G}}_{l}^{L^{r}}))^{2}} \\ k \in T \end{cases}$$

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The closeness coefficient of the individual decision (R_k) with respect to ideal decisions denoted by (CC_k) is achieved as follows:

$$CC_{k} = \frac{d_{l}^{r} + d_{l}^{l}}{d_{l}^{l} + d_{l}^{r} + d_{l}^{*}}, K \in T$$
(20)

It is considered that larger value of CC_k determines more importance on kth DM opinion, and bigger value of weight for kth DM [33]. The importance of an expert in his/her area of expertise is referred to as the individual importance and denoted by IM_k . Combination of the two DM importance considerations can be obtained as follows:

$$\pi_k = \alpha I M_k + (1 - \alpha) C C_k, K \in T$$
(21)

where α ($0 \le \alpha \le 1$) is the optimistic coefficient that indicates whose value can be chosen according to group's opinions, IM_k ($0 \le IM_k \le 1$) is the measure of importance of kth DM as an expert in his/her own area of expertise.

Eventually, the weights of DMs are obtained as follows:

$$\mu_k = \frac{\pi_k}{\sum_{k=1}^t \pi_k}, K \in T \tag{22}$$

The weighted (on attributes and DMs) decision 208 matrix (S) for each DM is calculated by the following: 299

$$\tilde{\tilde{S}}_{k} = \left(s_{ij}\right)_{m \times n} = \left(\mu_{k} \times g_{ij}^{k}\right)_{m \times n}$$
³⁰⁰

$$= \begin{bmatrix} \tilde{s}_{11}^{k} & \cdots & \tilde{s}_{1n}^{k} \\ \vdots & \ddots & \vdots \\ \tilde{s}_{m1}^{k} & \cdots & \tilde{s}_{mn}^{k} \end{bmatrix}$$
(23) 30

where

$$\check{\tilde{s}}_{ij} = (\mu_k g^U_{ij1}, \ \mu_k g^U_{ij2}, \ \mu_k g^U_{ij3}, \ \mu_k g^U_{ij4};$$

$$(H_{i1}(G_1^U), H_{i2}(G_1^U)), \mu_k g_{ij1}^L, \mu_k g_{ij2}^L,$$
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$$\mu_k g_{ij3}^L, \ \mu_k g_{ij4}^L; \ (H_{i1}(G_1^L), \ H_{i2}(G_1^L))$$
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The individual decision, which is weighted on attributes and DMs, is converted into the group decision, for each alternative. This is done by the following Equation:

$$\widetilde{\tilde{S}}_{i} = (s_{kj})_{j \times n} = \begin{bmatrix} \widetilde{\tilde{s}}_{11}^{i} & \cdots & \widetilde{\tilde{s}}_{1n}^{i} \\ \vdots & \ddots & \vdots \\ \widetilde{\tilde{s}}_{t1}^{i} & \cdots & \widetilde{\tilde{s}}_{tn}^{i} \end{bmatrix}, i \in M, \quad (24)$$

To manage the risk of uncertainty in the process the 306 following mathematical model for each alternative is 307 presented. 308

$$H_{i} = \max\left(\sum_{i=1}^{M} \tilde{q}_{Bi} - \sum_{i=1}^{M} \tilde{q}_{Ci}\right)$$
(25) 309

Subject to :

$$\tilde{q}_{Bi} = \sum_{j \in B} \sqrt{\frac{1}{4} \begin{pmatrix} \left((s_{ij})_1 \right)^2 + \left((s_{ij})_2 \right)^2 + \left((s_{ij})_3 \right)^2 \\ + \left((s_{ij})_4 \right)^2 \end{pmatrix}}$$
(26) 311

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$$\tilde{q}_{Ci} = \sum_{j \in C} \sqrt{\frac{1}{4} \begin{pmatrix} ((s_{ij})_1)^2 + ((s_{ij})_2)^2 + \\ ((s_{ij})_3)^2 + ((s_{ij})_4)^2 \end{pmatrix}}$$
(27) 31

$$\hat{S}_{ij} = (s_{ij_1}, s_{ij_2}, s_{ij_3}, s_{ij_4})$$
 (28) 31

$$S_{ij_1}^U \le s_{ij_1} \le s_{ij_1}^L j = 1, \dots, m, \ i = 1, \dots, n$$
 (29) and

$$s_{ij_2}^{U} \le s_{ij_2} \le s_{ij_2}^{L} j = 1, \dots, m, \ i = 1, \dots, n$$
 (30) 310

$$s_{ij_3}^L \le s_{ij_3} \le s_{ij_3}^U j = 1, \dots, m, \ i = 1, \dots, n \quad (31)$$

$$s_{ij_4}^L \le s_{ij_4} \le s_{ij_4}^U j = 1, \dots, m, \quad i = 1, \dots, n \quad (32)$$
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$$\left[\frac{((s_{ij})_4 + (s_{ij})_1^L) - ((s_{ij})_1 + (s_{ij})_4^L)}{((s_{ii})_4^U + (s_{ii})_4^L) - ((s_{ij})_4^U + (s_{ii})_4^L)}\right] \le \varepsilon$$
(33)

$$_{320} \quad (s_{ij})_k > 0 \ j = 1, \dots, m, \ K = 1, 2, 3, 4 \quad (34)$$

Where ε denotes the maximum amount of acceptable uncertainty. This amount is imposed on the mathematical problem by Equation (33). In this step 323 based on the concept of FOU, the IT2FNs are converted to type-1 fuzzy sets. These new fuzzy numbers are made in the limits of the initial IT2FNs by Equations (29–32). The area between the lower and upper limits of an IT2FS is known as FOU. The presented approach aims at controlling and reducing the risk of this uncertainty that exists in IT2FNs by using FOU. 330

The quantitative utility (QU) for each alternative should be calculated. The degree of each alternative's utility is directly related to its obtained H value. The degree of an alternative's utility can be computed as below:

$$QU_i = \left[\frac{H_i}{H_{max}}\right] \times 100\% \tag{35}$$

At the end of this process, each alternative gains a 331 score which is presented by QU_i . This score demon-332 strates the desirability of each alternative considering 333 its benefit and cost criteria. 334

3.3. Proposed mathematical model 335

In this section, a model is presented that is aiming 336 at obtaining a portfolio of projects that suits all the 337 existing criteria of the process in the best possible 338 way. Notations used in this section are described as 339 follows: 340

$$\begin{array}{ll} {}_{341} & \left(it2\,fi_1^U, it2\,fi_2^U, it2\,f_3^U, it2\,f_4^U; H_1\left(it2\,fi_1^U\right)\right.\\ {}_{342} & H_2\left(it2\,fi_1^U\right)\right),\\ {}_{343} & \left(it2\,fi_1^L, it2\,fi_2^L, it2\,fi_3^L, it2\,fi_4^L; H_1\left(it2\,fi_1^L\right)\right.\\ {}_{344} & H_2\left(it2\,fi_1^L\right)\right), \end{array}$$

IT2F investment project *i*, 345

Min₁, minimum amount if acceptable investment, 346 Max_I , maximum amount of acceptable invest-347 ment, 348

 QU_i , Score of project *i* obtained in Section 3.2,

$$\begin{pmatrix} IT2FHRi_{1}^{U}, IT2FHRi_{2}^{U}, IT2FHRi_{3}^{U}, IT2FHRi_{4}^{U}; \\ H_{1}\left(IT2FHRi_{1}^{U}\right), H_{2}\left(IT2FHRi_{1}^{U}\right) \end{pmatrix},$$

$$\begin{pmatrix} IT2FHRi_{1}^{L}, IT2FHRi_{2}^{L}, IT2FHRi_{3}^{L}, IT2FHRi_{4}^{L}; \\ H_{1}\left(IT2FHRi_{1}^{L}\right), H_{2}\left(IT2FHRi_{1}^{L}\right) \end{pmatrix},$$

IT2F human resource requirement of project i 350 Max_{HR} , maximum level of available human 351 resource. 352

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 x_i , decision variable which is defined by:

$$x_i = \begin{cases} 0 \text{ if project } i \text{ is rejected} \\ 1 \text{ if project } i \text{ is selected} \end{cases}$$

$$Z_2 = \max \sum_{i=1}^{m} x_i Q U_i$$
 (36) 355

Subject to :

$$Min_{I} \leq \sum_{i=1}^{n} \begin{pmatrix} it2fi_{1}^{U}, it2fi_{2}^{U}, it2f_{3}^{U}, it2f_{4}^{U}; \\ H_{1}\left(it2fi_{1}^{U}\right), H_{2}\left(it2fi_{1}^{U}\right) \end{pmatrix}, \\ Min_{I} \leq \sum_{i=1}^{n} \begin{pmatrix} it2fi_{1}^{L}, it2fi_{2}^{L}, it2fi_{3}^{L}, it2fi_{4}^{L}; \\ H_{1}\left(it2fi_{1}^{L}\right), H_{2}\left(it2fi_{1}^{L}\right) \end{pmatrix} \leq Max_{I} \qquad 357$$

$$(37) \qquad (37) \qquad$$

$$\sum_{\substack{i \in short - term}} \begin{pmatrix} it2 fi_{1}^{U}, it2 fi_{2}^{U}, it2 fi_{3}^{U}, \\ it2 fi_{4}^{U}; H_{1}(it2 fi_{1}^{U}), H_{2}(it2 fi_{1}^{U}) \end{pmatrix}, \\ it2 fi_{4}^{L}; H_{1}(it2 fi_{1}^{U}), H_{2}(it2 fi_{1}^{U}) \end{pmatrix}, \\ \leq \frac{\alpha}{\mu} \sum_{i=1}^{N} \begin{pmatrix} it2 fi_{1}^{U}, it2 fi_{2}^{U}, it2 fi_{3}^{U}, it2 fi_{4}^{U}; \\ H_{1}(it2 fi_{1}^{U}), H_{2}(it2 fi_{1}^{U}) \end{pmatrix}, \\ (it2 fi_{1}^{L}, it2 fi_{2}^{L}, it2 fi_{3}^{L}, it2 fi_{4}^{U}; \\ H_{1}(it2 fi_{1}^{U}), H_{2}(it2 fi_{1}^{U}) \end{pmatrix}, \\ (it2 fi_{1}^{U}, it2 fi_{2}^{U}, it2 fi_{3}^{U}, it2 fi_{4}^{U}; \\ H_{1}(it2 fi_{1}^{U}), H_{2}(it2 fi_{1}^{U}) \end{pmatrix}, \\ \sum_{\substack{i=1\\ \in mid-term}} \begin{pmatrix} it2 fi_{1}^{L}, it2 fi_{2}^{L}, it2 fi_{3}^{L}, it2 fi_{4}^{U}; \\ H_{1}(it2 fi_{1}^{U}), H_{2}(it2 fi_{1}^{U}) \end{pmatrix}, \\ (it2 fi_{1}^{L}, it2 fi_{2}^{L}, it2 fi_{3}^{L}, it2 fi_{4}^{U}; \\ H_{1}(it2 fi_{1}^{U}), H_{2}(it2 fi_{1}^{U}) \end{pmatrix}, \\ x_{i} \end{pmatrix}$$

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$$\begin{split} {}_{364} & \leq \frac{\beta}{\mu} \sum_{i=1}^{N} \begin{pmatrix} it2fi_{1}^{U}, it2fi_{2}^{U}, it2fi_{3}^{U}, it2fi_{4}^{U}; \\ H_{1}(it2fi_{1}^{U}), H_{2}(it2fi_{1}^{U}) \end{pmatrix}, \\ & \left(it2fi_{1}^{L}, it2fi_{2}^{L}, it2fi_{3}^{L}, it2fi_{4}^{L}; \\ H_{1}(it2fi_{1}^{L}), H_{2}(it2fi_{1}^{U}) \end{pmatrix}, \\ & \left(it2fi_{1}^{U}, it2fi_{2}^{U}, it2fi_{3}^{U}, it2fi_{4}^{U}; \\ H_{1}(it2fi_{1}^{U}), H_{2}(it2fi_{1}^{U}) \end{pmatrix}, \\ & \sum_{i \in long-term} \begin{pmatrix} it2fi_{1}^{L}, it2fi_{2}^{L}, it2fi_{3}^{L}, it2fi_{4}^{L}; \\ H_{1}(it2fi_{1}^{U}), H_{2}(it2fi_{1}^{U}) \end{pmatrix}, \\ & \left(it2fi_{1}^{L}, it2fi_{2}^{L}, it2fi_{3}^{L}, it2fi_{4}^{L}; \\ H_{1}(it2fi_{1}^{U}), H_{2}(it2fi_{1}^{U}) \end{pmatrix}, \\ & \leq \frac{\gamma}{\mu} \sum_{i=1}^{N} \begin{pmatrix} it2fi_{1}^{U}, it2fi_{2}^{U}, it2fi_{3}^{U}, it2f_{4}^{U}; \\ H_{1}(it2fi_{1}^{U}), H_{2}(it2fi_{1}^{U}) \end{pmatrix}, \\ & \left(it2fi_{1}^{L}, it2fi_{2}^{L}, it2fi_{3}^{L}, it2fi_{4}^{L}; \\ H_{1}(it2fi_{1}^{L}), H_{2}(it2fi_{1}^{U}) \end{pmatrix}, \\ & x_{i} \end{pmatrix} . \\ & x_{i} \end{pmatrix} . \\ & x_{i} \end{pmatrix} . \\ & (41) \\ & \alpha + \beta + \gamma = \mu \end{split}$$

 $\alpha + \beta + \gamma = \mu$ 367

$$x_i \neq x_i^{''}$$
 for $i = 1, 2, ..., n; (i, i^{''}) \in K$ (43)

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$$x_i = 1$$
 for $i = 1, 2, ..., n; \forall i \in L$ (44)

Equation (37) keeps the amount of investment in the feasible region. Equation (38) keeps the number 372 of human resource of the entire selected portfo-373 lio in the practical area. Equations (39-41) can be 374 added to the model to plan short, mid and long-term 375 time horizons. Equation (43) indicates the mutual 376 exclusiveness relationship of projects. Equation (44) 377 makes inclusion of a certain project in the portfolio 378 compulsory. 379

To solve the mathematical model with IT2FSs 380 embedded in the constraints, the concept of expected 381 value defined by Hu et al. [6] was used. In this 382 approach, each IT2FN used in the model is trans-383 formed to crisp value. The following presents the 384 applied approach of transformation: 385

$$E(A) = \frac{1}{2} \left(\frac{1}{3} \sum_{i=1}^{3} a_i^L + a_i^U \right) \\ \times \frac{1}{4} \left(\sum_{i=1}^{2} \left(H_i(A^L) + H_i(A^U) \right) \right)$$
(45)

3.4. Procedure of the proposed project portfolio 388 selection approach 389

In sum, the algorithm is provided by means of the 390 following steps:

Step 1. Provide individual decision information for each DM. Each DM expresses his/her decision matrix. Their decision matrixes are gathered as expressed in Equations (5 and 6).

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Step 2. Normalize the gathered decision matrixes by Equations (8 and 9).

Step 3. Construct the weighted (on attributes) individual decision by Equation (13).

Step 4. Determine the ideal decisions of all individual decisions. The best decision (G^*) , the left negative ideal decision (G_1^-) and the right negative ideal decision (G_r^-) are calculated by Equations (14–16), respectively.

Step 5. Compute the separations of each individual judgment from the best judgment (G^*) , the left negative ideal decision (G_1) and the right negative ideal decision (G_r^-) applying Equations (17–19), respectively.

Step 6. Decide the closeness coefficient of each individual judgment to supreme judgments by using Equation (20).

Step 7. Find the comprehensive closeness coefficient of each DM by employing Equation (21).

Step 8. Obtain the weights of DMs by using Equation (22).

Step 9. Create a decision matrix that is weighted on attributes and DMs for each DM by Equation (23).

Step 10. Convert the individual decision that is weighted on attributes and DMs into the group decision for each alternative by using Equation (24).

Step 11. Solve the mathematical model presented in Equations (25–34) for each alternative.

Step 12. Calculate the quantitative utility of each alternative by using Equation (35).

Step 13. Form the final objective function of the project portfolio selection model by using the obtained quantitative utility.

Step 14. Gather the data concerning the constraints and the limitations and form the final model.

Step 15. Solve the mathematical model to achieve the optimal portfolio of projects.

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In this part, an existing problem in the recent literature is adopted and solved using the proposed
approach. Furthermore, the model is presented in two
parts and each part is illustratively dealt with by the
model.

438 4.1. First part of the proposed model

In this section, to display model's applicability in
real-world problems, the data from the case study of
Tavana et al. [16] is applied. The main objective of
the studied organization is to find the most suitable
projects for funding depending on the annual budget
constraints.

The following criteria were considered in the problem: Total cost (C_1) , Production time (C_2) , System safety (C_3) , System reliability (C_4) , Feasibility (C_5) and eventually, reusability (C_6) . 5 projects $(P_1) - (P_5)$ from the studied case are selected to be used in the proposed method. A group consisting of 5 experts have expressed their ideas.

Since the calculations are too large to be fully dis-452 played, partial calculations are presented as follows. 453 The closeness coefficient of the individual judgment 454 with respect to supreme judgments is obtained and 455 displayed in Table 2. π_k is then calculated. It should 456 be noticed that each DM was given the IM_k of 0.2 and 457 α was equal to 0.5. π_k is also displayed in Table 2. 458 Finally, the weights of DMs are calculated. They also 459 are displayed in Table 2. The initial judgments are 460 weighted by using Equation (23). 461

The weighted (on attributes and DMs) decision matrix (S) for each DM is aggregated before being used in the mathematical model. The aggregation is carried by applying the following:

$$\begin{pmatrix} \frac{\sum_{k=1}^{K} s_{kij1}^{L}}{K}, \frac{\sum_{k=1}^{K} s_{kij2}^{L}}{K}, \frac{\sum_{k=1}^{K} s_{kij3}^{L}}{K} \frac{\sum_{k=1}^{K} s_{kij4}^{L}}{K}; \\ \min H_{1}(\tilde{S}_{kij}^{L}), H_{2}(\tilde{S}_{kij}^{L}), \\ \frac{\sum_{k=1}^{K} s_{kij1}^{L}}{K}, \frac{\sum_{k=1}^{K} s_{kij2}^{L}}{K}, \frac{\sum_{k=1}^{K} s_{kij3}^{L}}{K} \frac{\sum_{k=1}^{K} s_{kij4}^{L}}{K}; \\ \min H_{1}(\tilde{S}_{kij}^{L}), H_{2}(\tilde{S}_{kij}^{L}), \end{pmatrix}$$
(46)

462 463 464 It should be mentioned that the aforementioned steps are carried out for all the gathered judgments. Eventually, the mathematical model for each alternative is solved. It should be noted that maximum level of uncertainty is set equal to 0.5. H_i , QU_i and the results of the existing literature are displayed

Table 2 The closeness coefficient, π_k and μ_k

Decision Mak	ter	CC_k	π_k	μ_k
DM_1		0.65	0.42	0.18
DM_2		0.83	0.51	0.22
DM_3		0.67	0.43	0.18
DM_4		0.82	0.51	0.22
DM_5		0.67	0.43	0.18
	Final c	Table 3 computational re	esults	
Projects	H_i	QU_i	Proposed approach	Tavana et al.
			ranking	[16]
$\overline{P_1}$	7.122591	100	11	
•	7.122591 6.102921	100 85.684	ranking	[16]
P_2			ranking 1	[16] 1
P_2 P_3	6.102921	85.684	ranking 1 2	[16] 1 2

in Table 3. The results show the reliability if the proposed model in addition to its novelty in giving weights to each DM depending on the achieved judgments.

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4.2. The second part of the proposed approach

Since the provided case study lacked the required data for this part of the model, in order to display application of this part, the problem is adapted and the required data is added. Table 4 displays the adapted data for each project. To demonstrate model's ability to handle problems under different scenarios, different levels of constraints are considered, and the model is solved under those different constraints. Table 5 displays the achieved results.

4.3. Model's advantages over similar studies

Applying the proposed model in the existing literature demonstrated several advantages. The most important implications of the model's application are as follows: (1) the model is based on type 2 FSs. This uncertainty-modeling tool gives the model a practical edge over the existing classic fuzzy models; (2) the model is in two main parts, and it means that the DM can easily observe the results of judgments on projects before choosing the portfolio. Furthermore, uncertain data concerning both quantifiable and unquantifiable can be applied in each part of the model; (3) each DM is given a weight that is based on the expertise and importance of the expert in any studying field, in addition to the data gathered from

Projects	IT2F investment (million dollars)	IT2F human resource (persons)
P_1	((160,180,210,230;1,1),(170,190,200,220;0.9,0.9))	((20,30,45,55;1,1),(25,35,40,50;0.9,0.9))
P_2	((260,280,310,330;1,1),(270,290,300,320;0.9,0.9))	((15,25,40,50;1,1),(20,30,35,45;0.9,0.9))
P_3	((110,130,160,180;1,1),(120,140,150,170;0.9,0.9))	((0,10,25,35;1,1),(5,15,20,30;0.9,0.9))
P_4	((60,80,110,130;1,1),(70,90,100,120;0.9,0.9))	((7,12,27,37;1,1),(7,17,22,32;0.9,0.9))
P_5	((210,230,260,280;1,1),(220,240,250,270;0.9,0.9))	((10,20,35,45;1,1),(15,25,30,40;0.9,0.9))
- 5	((210),200,200,11),((220),210,200,210,000,00)) Table 5	((,,,,-,-,),(10)=0,00,10,00,00,00

Table 4 Adapted data of the studied projects

Table 5 Results of the second part of the model						
Projects	Budget 0-100	Budget 100-200	Budget 200-300	Budget 0-500		
	Human resource 0-30	Human resource 30-50	Human resource 50-70	Human resource 0-120		
P_1	0	1	1	1		
P_2	0	0	0	0		
P_3	0	0	0	1		
P_4	1	0	1	1		
P_5	0	0	0	0		
Objective	62.3	100	162.3	264.2		

other experts; (4) the approach avoids information
loss in the decision-making process.

499 5. Conclusions

New technology-project selection is one of the 500 most important tasks of many organizations. Since 501 high technology-projects are nowadays very crucial 502 to advancements of science and technology, and they 503 have not been comprehensively addressed in project 504 selection literature, this paper proposed a novel 505 approach of high technology-project selection. More-506 over, the presented approach was in two main parts. 507 In the first part, a new multi criteria decision-making 508 model that avoids information loss was presented that 509 was able to review and rank the projects. In the second 510 part, a model of project portfolio selection was pre-511 sented that simultaneously considered investments 512 requirements and human resource requirements in 513 finding the optimum portfolio of high technology-514 projects. To displays the model's application, a case 515 study for the high technology-project selection prob-516 lem from the existing literature was chosen and 517 adopted properly to be solved by the model. Applying 518 the approach provided several implications that were 519 discussed. Finally, for further researches, integrating 520 the proposed model in decision support systems could 521 be a practical and interesting work. 522

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