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# Determining project characteristics and critical path by a new approach based on modified NWRT method and risk assessment under an interval type-2 fuzzy environment

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## Abstract

In this paper with respect to the importance of risks in real-world projects and ability of interval type-2 fuzzy sets (IT2FSs) to tackle the uncertainty, a new approach is introduced to consider risks and the correlation among risk factors by subjective judgments of experts on the probability and impact under IT2FSs. Furthermore, a new impact function for considering the correlation among the risk factors are extended under an IT2F environment. Moreover, a new subtraction operator is introduced for the critical path analysis. The node-weighted rooted tree (NWRT) method is modified based on the proposed new operator to avoid producing negative number for characteristics of each activity. Also, in order to cope with the uncertainty of the projects, NWRT method is developed under the IT2FSs. Eventually, to illustrate the validity and capability of the proposed method, two examples from the literature are solved and compared.

**Keywords:** Project scheduling, modified node-weighted rooted tree (NWRT) method, risk factors, interval type-2 fuzzy sets (IT2FSs), project characteristics, project critical path

## 1. Introduction

The critical path method (CPM) is a project management manner for the planning that determines critical and non-critical activities with the goal of preventing time-frame problems [1]. The longest path of project network is critical path. The CPM is a network-based method, designed to assist in evaluating project performance and to recognize bottlenecks in the network, and it is useful in practice for scheduling of complex projects [1]. Also, in the CPM the time of activities are certain and deterministic. In practice, this assumption always cannot be complied with the satisfying precision. Hence, program evaluation and review technique (PERT) has been introduced by using the random variable that the beta distribution has been applied to model the activity times [2]. In this method, many simplifying assumptions have been considered, therefore, it has been intensively developed in several directions under assumptions that the probability distributions of activity times are various to the beta distribution [3, 4]. In such situations, fuzzy sets theory is expressed by Zadeh [5] that do not

require posterior frequency distributions and can also cope with vague input information including feelings and emotions using subjective judgments of the decision makers (DMs).

One of the most important aspect of project management is the risk management. Risk management knows as one of the ten knowledge areas of project management. Project risk was defined by project management institute as, "*an uncertain event or condition that, if it occurs, has a positive or negative effect on a project's objectives*" [6]. Project risks exist owing to the uncertainty. There is evermore the probability that anything known or unknown could affect the achievement of project's goals. Risk management is concerning, being prepared to manage these risks [6].

Several studies have focused on project and risk management in the recent years. Espinoza [7] separated the project risk from the time value of money. Yim et al. [8] analyzed the impact of project classification on project risk indicators. Golini et al. [9] investigated the impact of international development projects of non-governmental organizations. Carvalho and Junior [10] expressed the relationship between the risk management and project success. Muriana and Vizzini [11] proposed a quantitative method for the assessment and mitigation of project risks.

Problems arising in construction projects are complex and are usually involving vast uncertainties and subjectivities. In comparison with other industries, the construction industry is known as a high-risk industry due to the unique features of construction activities [12-14]. The incidence of project risk may have positive or negative effects on one of the project objectives, such as time, cost, safety, quality or sustainability [15-20]. Construction projects have considerable risks due to the involvement of a large number of parties, such as owners, designers, supervision consultants, contractors, subcontractors, suppliers, manufacturers, and governments [21]. Uncertainty, vagueness and incomplete information are the inherent part of construction projects because there are a lot of risks in real-world projects. In this condition, fuzzy sets theory is a helpful manner to consider the uncertainty of projects. In recent years, several types of studies have used fuzzy sets theory to cope with uncertainty [22-24]. Taylan et al. [25] have presented fuzzy analytic hierarchy process (FAHP) and fuzzy technique for order of preference by similarity to ideal solution (FTOPSIS) for the risk assessment in construction projects. Kuchta and Ptaszyńska [26] have expressed a fuzzy based risk register for construction project risk assessment. Asan et al. [27] have introduced an IT2F-prioritization approach to project risk assessment.

In comparison to an ordinary type-1 fuzzy set (T1FS), type-2 fuzzy set (T2FS) has a degree of membership which is themselves fuzzy whereas ordinary fuzzy set has a crisp membership function [28]. A T2FS put out a measure of dispersion to better consider inherent uncertainties, which becomes especially useful in problems where it is hard to determine the deterministic membership function for a fuzzy set [29]. In fact, T2FSs are more appropriate and powerful than T1FSs and provide a suitable condition for considering subjective judgments of experts. In this paper, to address the uncertainty and vagueness of real-world projects, IT2FSs are used.

IT2FSs are very beneficial and pliable to address the uncertainties in comparison to classic fuzzy sets. The great discrepancy between T2FSs and T1FSs is membership degree which for IT2FSs is demonstrated by a fuzzy set in  $[0, 1]$ , instead of a crisp number in  $[0, 1]$  [30, 31]. In fact, T2FSs are three-dimensional and their membership degrees are determined by a fuzzy set in the interval  $[0, 1]$  and explained by both elementary and secondary membership to supply more degrees of freedoms to tackle the uncertainty. With considering the nature of the construction projects and inherent uncertainty of them, IT2FSs are suitable and powerful tools to tackle the project uncertainty.

In the recent years, many methods have been developed under IT2FSs [32-34]. Furthermore, the IT2FSs have been applied to the project management area successfully [35-37]. Bozdogan et al. [38] presented a method for risk prioritization under the IT2Fenvironment. Mohagheghi et al. [39] analyzed the project cash flow in construction industry based on the IT2FSs. Mohagheghi et al. [40] evaluated R&D project and portfolio selection by using an IT2F optimization approach.

In order to tackle the uncertainty of the real-project situations, fuzzy sets theory introduced by Zadeh [41]. But, classic fuzzy sets were criticized owing to the certain membership grade. In this situation, IT2FSs were proposed by [42]. As matter of fact, the IT2FSs are more powerful than T1FSs in coping with the uncertainty. With considering the nature of the projects and inherent uncertainty of them, the IT2FS is a suitable and powerful tool for coping with project uncertainty. Moreover, in the project management owing to the nature of projects and according to the matter that many activities of the project may do for the first time, planning faces difficulties because of the lack of information about activities. To tie this problem, the experts' opinions and judgments on activities are used. On the other hand, linguistic variables are mostly represented by fuzzy sets [43], and it makes more sense than certain numbers [41]. But, classic fuzzy sets have crisp membership grades in the interval  $[0, 1]$  which consequently cannot completely support various types of uncertainty appearing in linguistic explanations of numerical quantities or in the subjectively explained judgment of experts [44, 45]. In this case, IT2FSs can be applied. To date, IT2FSs have been used successfully for many project management problems [37, 40]

The backward recursion, explained in traditional fuzzy CPM, produces negative times because durations are explained by means of fuzzy sets and as soon as the latest starting date of the last task is set to its earliest date, but negative number for characteristics of activity has no physical meaning. To overcome this problem, Shanker et al. [46] introduced an analytical method for finding fuzzy critical path (FCP) in a project network. Kumar and Kaur [47] presented a methodology for fuzzy critical path analysis and introduced a new subtraction operator to avoid producing negative fuzzy number. Rao and Shanker [48] expressed a fuzzy critical path analysis based on the centroid of centroids of fuzzy numbers and new subtraction method. In all subtraction methods which have been introduced, the amount of fuzzy number has been increased. In fact, the crisp value of fuzzy numbers after applying the new methods are increased toward the traditional methods. In this situation, a new method is developed that the crisp value of this method is more suitable than others.

In the recent years, many researchers attempt to analysis of fuzzy project network and characteristics. Yakhchali and Ghodsypour [49] proposed a method for calculating latest starting times of activities in interval-valued networks. Yakhchali and Ghodsypour [50] also introduced an approach to compute latest starting times and activities' criticality in project network. Zareei et al. [4] presented an approach based on the analysis of events for solving FCP problem. Sireesha and Shankar [51] focused on a methodology to report project characteristics and multiple possible FCPs in a project network and introduced a graphical figure (rooted tree) to illustrate the fuzzy project network. Sireesha and Shankar [52] expressed a node-weighted rooted tree (NWRT) method based on the new graphical figure (rooted tree) to denote project characteristics and triangular FCPs. But, this methodology produces negative fuzzy number in backward recursion. To overcome this issue and use advantages of the new graphical figure and NWRT method, this method is extended in this paper by a new operator to avoid producing negative fuzzy number because the negative fuzzy number has no physical meaning.

Determining the critical path of project plays an important role in planning of real-world projects. With considering the nature of the projects and according to the matter that many activities of the project may do for the first time, the uncertainty of project is very high. In order to address this uncertainty, the IT2FSs are a useful tool because the IT2FSs are more powerful in coping with the uncertainty and provide more degree of freedom and flexibility. In this paper, the analysis of activities' risk is considered under the IT2F environment.

In this paper to address the uncertainty of real-world projects, IT2FSs are provided. Furthermore, a new operator is introduced to avoid producing negative fuzzy numbers for characteristics of project activities. Then, NWRT method is modified by this operator and updated in the IT2F-environment. Moreover, risks of any activity and its correlation are considered using subjective judgments of

experts. Also, a new impact function for considering the correlation of two kinds of risks for linguistic variables is developed under the IT2F environment. The novelties of this paper are summarized as follows:

- IT2FSs are used to address the uncertainty of real-world projects properly.
- A new operator is introduced to avoid producing negative fuzzy numbers under the IT2F environment.
- Risks and their correlation with each other for any activity is determined by subjective judgments under IT2FSs.
- A new impact function for considering the correlation among risk factors is developed under the IT2F environment.
- The NWRT method is introduced based on the new operator which is modified to avoid produce negative fuzzy number.
- The modified NWRT method is extended to the IT2F environment.

The rest of the study is organized as below: In section 2, the proposed operator and modified NWRT method are presented. In section 3, two examples are solved to illustrate the validity of the proposed method. In section 4, comparative analysis is given. In section 5, the conclusion is expressed.

## **2. Proposed methodology**

In this section, the risk assessment for each task or activity by subjective judgments of DMs is presented. Then, a new impact function for considering the correlation of risk factors is developed under IT2FSs. Moreover, the NWRT method is modified based on a new subtraction operator for avoiding the production of negative fuzzy numbers. In the presented approach, the total float (TF) of any tasks is specified by a new characteristic path float. Earliest time and latest time (ET and LT) of each task are determined using TF. Main advantage of NWRT amidst existing approaches is that it will be clear and can be properly comprehended by any less technically trained experts. The proposed framework is depicted in Figure 1.

### ***2.1. Proposed risk assessment***

In this sub-section, by considering the importance of risks in projects, risk assessment of each activity is done by the risk management group who are responsible for analyzing and determining the time of each activity. The important risks are categorized in four classes. First class is the risk of increasing costs; the second one is relevant to the reducing of quality; the third one is the reducing of safety; and finally the fourth class is the general risks. The expert's judgments on the probability and impact of each risk of any activities are gathered. Note that the expert's judgments are expressed using linguistic variables. Then, linguistic variables are converted to the equivalent IT2F-numbers. Also, the probability of correlation among the risks is expressed by expert and the impact between risks is determined by a new extended method. Moreover, the most important risk of any activities is determined by their measures. Finally, the important risks' effects are converted to the time of activity. The duration of the activity is the input of proposed NWRT method.

{Please insert Figure 1 here.}

**Step 2.1.1:** Expert's judgments on the time of each activity are gathered. Furthermore, subjective judgments of an expert on ratings the probability and impact of each activity's risks are collected.

**Step 2.1.2:** Qualitative assessment of each risk factor is done based on expert's judgments. In other words, the risk with the very low probability and impact is removed from assessment and another risk factor is considered as the input of qualitative risk assessment stage.

**Step 2.1.3:** The expert's judgments on risks' probability and impact of each activity are converted to an equivalent IT2F-number by Table 1.

{Please insert Table 1 here.}

**Step 2.1.4:** A new function for obtaining the impact of two risks over each other or impact of three risks over each other, and so on, is developed under IT2FSs, adopted from Madhuri et al. [53].

If, there are two fuzzy numbers  $\tilde{A}_1$  and  $\tilde{A}_2$  as below:

$$\begin{aligned}\tilde{A}_1 &= (\tilde{A}_1^U, \tilde{A}_1^L) = ((a_{11}^U, a_{12}^U, a_{13}^U, a_{14}^U; H_1(\tilde{A}_1^U), H_2(\tilde{A}_1^U)), (a_{11}^L, a_{12}^L, a_{13}^L, a_{14}^L; H_1(\tilde{A}_1^L), H_2(\tilde{A}_1^L))) \\ \tilde{A}_2 &= (\tilde{A}_2^U, \tilde{A}_2^L) = ((a_{21}^U, a_{22}^U, a_{23}^U, a_{24}^U; H_1(\tilde{A}_2^U), H_2(\tilde{A}_2^U)), (a_{21}^L, a_{22}^L, a_{23}^L, a_{24}^L; H_1(\tilde{A}_2^L), H_2(\tilde{A}_2^L)))\end{aligned}\quad (1)$$

Then, the addition of these two numbers is calculated as follows:

$$\tilde{A}_3 = (\tilde{A}_3^U, \tilde{A}_3^L) = ((a_{31}^U, a_{32}^U, a_{33}^U, a_{34}^U; H_1(\tilde{A}_3^U), H_2(\tilde{A}_3^U)), (a_{31}^L, a_{32}^L, a_{33}^L, a_{34}^L; H_1(\tilde{A}_3^L), H_2(\tilde{A}_3^L)))\quad (2)$$

where,

$$\begin{aligned}a_{31}^L &= a_{11}^L + a_{21}^L - a_{11}^L * a_{21}^L \\ a_{32}^L &= a_{11}^L + a_{22}^L + \left[ (a_{12}^L - a_{11}^L) \frac{\min(H_1(\tilde{A}_1^L), H_1(\tilde{A}_2^L))}{H_1(\tilde{A}_1^L)} \right] + \\ &\left[ (a_{22}^L - a_{21}^L) \frac{\min(H_1(\tilde{A}_1^L), H_1(\tilde{A}_2^L))}{H_1(\tilde{A}_2^L)} \right] - a_{12}^L * a_{22}^L \\ a_{33}^L &= a_{14}^L + a_{24}^L - \\ &\left[ (a_{14}^L - a_{13}^L) \frac{\min(H_2(\tilde{A}_1^L), H_2(\tilde{A}_2^L))}{H_2(\tilde{A}_1^L)} \right] - \left[ (a_{24}^L - a_{23}^L) \frac{\min(H_2(\tilde{A}_1^L), H_2(\tilde{A}_2^L))}{H_2(\tilde{A}_2^L)} \right] - a_{13}^L * a_{23}^L \\ a_{34}^L &= a_{14}^L + a_{24}^L - a_{14}^L * a_{24}^L\end{aligned}\quad (3)$$

$$\begin{aligned}
a_{31}^U &= a_{11}^U + a_{21}^{LU} - a_{11}^U * a_{21}^U \\
a_{32}^U &= a_{11}^U + a_{22}^U + \left[ (a_{12}^U - a_{11}^U) \frac{\min(H_1(\tilde{A}_1^U), H_1(\tilde{A}_2^U))}{H_1(\tilde{A}_1^L)} \right] + \\
&\left[ (a_{22}^U - a_{21}^U) \frac{\min(H_1(\tilde{A}_1^U), H_1(\tilde{A}_2^U))}{H_1(\tilde{A}_2^L)} \right] - a_{12}^U * a_{22}^U \\
a_{33}^U &= a_{14}^U + a_{24}^U - \\
&\left[ (a_{14}^U - a_{13}^U) \frac{\min(H_2(\tilde{A}_1^U), H_2(\tilde{A}_2^U))}{H_2(\tilde{A}_1^L)} \right] - \left[ (a_{24}^U - a_{23}^U) \frac{\min(H_2(\tilde{A}_1^U), H_2(\tilde{A}_2^U))}{H_2(\tilde{A}_2^L)} \right] - a_{13}^U * a_{23}^U \\
a_{34}^U &= a_{14}^U + a_{24}^U - a_{14}^U * a_{24}^U
\end{aligned} \tag{4}$$

**Step 2.1.5:** The risk measurement is determined by multiplying the probability and impact in each other. Finally, the most important risk of each activity is determined and added to the time of each activity. In fact, the output of risks assessment in this step is the updated duration for activities of project network.

## 2.2. Introduced NWRT method

In the sub-section, NWRT method is expressed under IT2FSs. In fact, in this new graphical figure, (NWRT), initial node and ending node are considered as the root and leaf, respectively. The graphical figure illustrates all paths. Each vertex demonstrates the event and each edge illustrates the activity. The IT2F-activity times of the project are considered as corresponding edge weights in the rooted tree. The relative importance of root can be zero. The relative importance of ending node in each path will be the length of the path. Finally, the path with the max length will be critical path. In order to illustrate the rooted tree an example has been shown in Table 2, Figures 2 and 3.

{ Please insert Figure 2 here. }

{ Please insert Table 2 here. }

{ Please insert Figure 3 here. }

Path: 1-2-4-5

$$\omega(1) = \tilde{0}$$

$$\text{Relative importance of node 2, } \omega(2) = \omega(1) + \tilde{A} = \tilde{0} + \tilde{A} = \tilde{A}$$

$$\text{Relative importance of node 4, } \omega(4) = \omega(2) + \tilde{D} = \tilde{A} + \tilde{D}$$

$$\text{Relative importance of node 5, } \omega(5) = \omega(4) + \tilde{E} = \tilde{A} + \tilde{D} + \tilde{E}$$

Similarly, the weights of other nodes on other paths are calculated.

### 2.3. New subtraction operation

In this sub-section, a new operator for avoiding the production of negative numbers in critical path calculations is introduced.

If there are two IT2F-numbers  $\tilde{A}_1$  and  $\tilde{A}_2$  based on Eq. (A.4), then new subtraction operator ( $\bullet$ ) is defined as follows:

$$\begin{aligned} \tilde{A}_1 \bullet \tilde{A}_2 = & ((a_{11}^U - a_{21}^U, a_{12}^U - a_{22}^U, a_{13}^U - a_{23}^U, a_{14}^U - a_{24}^U; \min(H_1(\tilde{A}_1^U), H_1(\tilde{A}_2^U)), \\ & \min(H_2(\tilde{A}_1^U), H_2(\tilde{A}_2^U))), (a_{11}^L - a_{21}^L, a_{12}^L - a_{22}^L, a_{13}^L - a_{23}^L, a_{14}^L - a_{24}^L; \\ & \min(H_1(\tilde{A}_1^L), H_1(\tilde{A}_2^L)), \min(H_2(\tilde{A}_1^L), H_2(\tilde{A}_2^L)))) \end{aligned} \quad (5)$$

But, this operator has two problems as below:

1) Sometimes the trapezoidal or triangle structure of fuzzy numbers is disturbed. For example, if there are two fuzzy numbers as follows:

$$\begin{aligned} \tilde{A}_1 &= ((5, 7, 11, 12; 1, 1), (6, 8, 10, 11; 0.9, 0.9)) \\ \tilde{A}_2 &= ((2, 6, 9, 11; 1, 1), (3, 7, 8, 10; 0.9, 0.9)) \end{aligned}$$

Then,

$$\begin{aligned} \tilde{A}_1 - \tilde{A}_2 &= ((5 - 2, 7 - 6, 11 - 9, 12 - 11; 1, 1), (6 - 3, 8 - 7, 10 - 8, 11 - 10; 0.9, 0.9)) = \\ &((3, 1, 2, 1; 1, 1), (3, 1, 2, 1; 0.9, 0.9)) \end{aligned}$$

In this situation, the new subtraction operator is modified by using the following relation:

$$\begin{aligned} \tilde{A}_1 \tilde{A}_2 = & ((a_{(1)}^U, a_{(2)}^U, a_{(3)}^U, a_{(4)}^U; \min(H_1(\tilde{A}_1^U), H_1(\tilde{A}_2^U))), (a_{(1)}^L, a_{(2)}^L, a_{(3)}^L, a_{(4)}^L; \\ & \min(H_1(\tilde{A}_1^L), H_1(\tilde{A}_2^L)), \min(H_2(\tilde{A}_1^L), H_2(\tilde{A}_2^L)))) \end{aligned} \quad (6)$$

where,  $a_{(1)}^U \leq a_{(2)}^U \leq a_{(3)}^U \leq a_{(4)}^U$  and  $a_{(1)}^L \leq a_{(2)}^L \leq a_{(3)}^L \leq a_{(4)}^L$ . Note that  $a_{(1)}^U$  and  $a_{(1)}^L$  are defined as follows:

$$\begin{aligned} a_{(1)}^U &= \min \{ a_{11}^U - a_{21}^U, a_{12}^U - a_{22}^U, a_{13}^U - a_{23}^U, a_{14}^U - a_{24}^U \} \\ a_{(1)}^L &= \min \{ a_{11}^L - a_{21}^L, a_{12}^L - a_{22}^L, a_{13}^L - a_{23}^L, a_{14}^L - a_{24}^L \} \end{aligned} \quad (7)$$

2) When there are two fuzzy numbers as follows:

$$\begin{aligned} \tilde{A}_1 &= ((2, 5, 10, 12; 1, 1), (3, 6, 9, 10; 0.9, 0.9)) \\ \tilde{A}_2 &= ((3, 5, 8, 10; 1, 1), (4, 6, 7, 9; 0.9, 0.9)) \end{aligned}$$

Then, some of its elements will be negative as follows:



$$\begin{aligned} \tilde{A}_1 - \tilde{A} &= ((2-3, 5-5, 10-8, 12-10; 1, 1), (3-4, 6-6, 9-7, 10-9; 0.9, 0.9)) = \\ &((-1, 0, +2, +2; 1, 1), (-1, 0, +2, +1; 0.9, 0.9)) \end{aligned}$$

Negative numbers in the project scheduling have no physical meaning. In this state, the introduced operator is improved as below:

At first, like the first problem the structure of fuzzy numbers is modified, and then, negative numbers are removed by using the following relations:

$$\begin{aligned} a_{(1)}^U &= \max \{0, a_{11}^U, a_{21}^U\} \\ a_{(2)}^U &= \max \{0, \max \{0, a_{12}^U - a_{22}^U\} + \min \{0, a_{11}^U - a_{21}^U\}\} = C^U \\ a_{(3)}^U &= \max \{0, \max \{0, a_{13}^U - a_{23}^U\} + \min \{0, a_{12}^U - a_{22}^U\} + \min \{0, C^U\}\} = G^U \\ a_{(4)}^U &= \max \{0, [\max \{0, a_{14}^U - a_{24}^U\} + \min \{0, a_{13}^U - a_{23}^U\} + \min \{0, G^U\}]\} \end{aligned} \quad (8)$$

$$\begin{aligned} a_{(1)}^L &= \max \{0, a_{11}^L, a_{21}^L\} \\ a_{(2)}^L &= \max \{0, [\max \{0, a_{12}^L - a_{22}^L\} + \min \{0, a_{11}^L - a_{21}^L\}]\} = C^L \\ a_{(3)}^L &= \max \{0, \max \{0, a_{13}^L - a_{23}^L\} + \min \{0, a_{12}^L - a_{22}^L\} + \min \{0, C^L\}\} = G^L \\ a_{(4)}^L &= \max \{0, \max \{0, a_{14}^L - a_{24}^L\} + \min \{0, a_{13}^L - a_{23}^L\} + \min \{0, G^L\}\} \end{aligned} \quad (9)$$

In order to illustrate the ability of the proposed operator, the above example is solved as below:

$$\begin{aligned} a_{(1)}^U &= \max \{0, 2-3\} = 0 \\ a_{(2)}^U &= \max \{0, [\max \{0, 5-5\} + \min \{0, 2-3\} = -1]\} = 0 \\ a_{(3)}^U &= \max \{0, [\max \{0, 10-8\} + \min \{0, 5-5\} + \min \{0, -1\} = 1]\} = 1 \\ a_{(4)}^U &= \max \{0, [\max \{0, 12-10\} + \min \{0, 10-8\} + \min \{0, 1\}]\} = 2 \\ a_{(1)}^L &= \max \{0, 3-4\} = 0 \\ a_{(2)}^L &= \max \{0, [\max \{0, 6-6\} + \min \{0, 3-4\} = -1]\} = 0 \\ a_{(3)}^L &= \max \{0, [\max \{0, 9-7\} + \min \{0, 6-6\} + \min \{0, -1\} = 1]\} = 1 \\ a_{(4)}^L &= \max \{0, [\max \{0, 10-9\} + \min \{0, 9-7\} + \min \{0, 1\}]\} = 1 \end{aligned}$$

Generally, the new subtraction operator is defined by the following substeps:

**Step 2.3.1:** In this step, the subtraction operator is done as follows:

$$\begin{aligned} \tilde{A}_1 \bullet \tilde{A}_2 = & ((a_{11}^U - a_{21}^U, a_{12}^U - a_{22}^U, a_{13}^U - a_{23}^U, a_{14}^U - a_{24}^U; \min(H_1(\tilde{A}_1^U), H_1(\tilde{A}_2^U)), \\ & \min(H_2(\tilde{A}_1^U), H_2(\tilde{A}_2^U))), (a_{11}^L - a_{21}^L, a_{12}^L - a_{22}^L, a_{13}^L - a_{23}^L, a_{14}^L - a_{24}^L; \\ & \min(H_1(\tilde{A}_1^L), H_1(\tilde{A}_2^L)), \min(H_2(\tilde{A}_1^L), H_2(\tilde{A}_2^L)))) \end{aligned} \quad (10)$$

If the subtraction operator does not have any of the two problems mentioned above, the operation is done correctly. Otherwise, go to the next step.

**Step 2.3.2:** In this step, the subtraction operator is modified to keep the triangle and trapezoidal fuzzy structures as below:

$$\begin{aligned} \tilde{A}_1 \tilde{A}_2 = & ((a_{(1)}^U, a_{(2)}^U, a_{(3)}^U, a_{(4)}^U; \min(H_1(\tilde{A}_1^U), H_1(\tilde{A}_2^U))), (a_{(1)}^L, a_{(2)}^L, a_{(3)}^L, a_{(4)}^L; \\ & \min(H_1(\tilde{A}_1^L), H_1(\tilde{A}_2^L)), \min(H_2(\tilde{A}_1^L), H_2(\tilde{A}_2^L)))) \end{aligned} \quad (11)$$

Where,  $a_{(1)}^U \leq a_{(2)}^U \leq a_{(3)}^U \leq a_{(4)}^U, a_{(1)}^L \leq a_{(2)}^L \leq a_{(3)}^L \leq a_{(4)}^L$ . Also,  $a_{(1)}^U$  and  $a_{(1)}^L$  is defined as follows:

$$\begin{aligned} a_{(1)}^U &= \min \{ a_{11}^U - a_{21}^U, a_{12}^U - a_{22}^U, a_{13}^U - a_{23}^U, a_{14}^U - a_{24}^U \} \\ a_{(1)}^L &= \min \{ a_{11}^L - a_{21}^L, a_{12}^L - a_{22}^L, a_{13}^L - a_{23}^L, a_{14}^L - a_{24}^L \} \end{aligned} \quad (12)$$

If the results contain a negative number should be gone to the next step (step 3); otherwise, the final results have been obtained.

**Step 2.3.3:** In order to avoid producing negative numbers, unrealistic results of the proposed operator is improved as follows:

$$\begin{aligned} a_{(1)}^U &= \max \{ 0, a_{11}^U, a_{21}^U \} \\ a_{(2)}^U &= \max \{ 0, \max \{ 0, a_{12}^U - a_{22}^U \} + \min \{ 0, a_{11}^U - a_{21}^U \} \} = C^U \} \\ a_{(3)}^U &= \max \{ 0, \max \{ 0, a_{13}^U - a_{23}^U \} + \min \{ 0, a_{12}^U - a_{22}^U \} + \min \{ 0, C^U \} \} = G^U \} \\ a_{(4)}^U &= \max \{ 0, \left[ \max \{ 0, a_{14}^U - a_{24}^U \} + \min \{ 0, a_{13}^U - a_{23}^U \} + \min \{ 0, G^U \} \right] \} \end{aligned} \quad (13)$$

$$\begin{aligned} a_{(1)}^L &= \max \{ 0, a_{11}^L, a_{21}^L \} \\ a_{(2)}^L &= \max \{ 0, \left[ \max \{ 0, a_{12}^L - a_{22}^L \} + \min \{ 0, a_{11}^L - a_{21}^L \} \right] = C^L \} \\ a_{(3)}^L &= \max \{ 0, \max \{ 0, a_{13}^L - a_{23}^L \} + \min \{ 0, a_{12}^L - a_{22}^L \} + \min \{ 0, C^L \} \} = G^L \} \\ a_{(4)}^L &= \max \{ 0, \max \{ 0, a_{14}^L - a_{24}^L \} + \min \{ 0, a_{13}^L - a_{23}^L \} + \min \{ 0, G^L \} \} \end{aligned} \quad (14)$$

#### 2.4.T2F-total float of each activity

In this section, firstly, the TF of each path is obtained and proved that  $TF_{(i,j)}^{\tilde{F}}$   $\min_{s \in S_{1,n}} \{ PF_s^{\tilde{F}} \mid (i,j) \in s \}$  where  $TF_{(i,j)}^{\tilde{F}}$  represent TF of each activity,  $PF_s^{\tilde{F}}$  represent the TF of

each path, and  $S_{i,j}$  is a set of all paths from  $i$  to  $j$ . The IT2F-ES time ( $\tilde{EST}_{(i,j)}$ ) and IT2F-LS time ( $\tilde{LST}_{(i,j)}$ ) is calculated as follows:

$$\tilde{EST}_{(i,j)} = \max_{s \in S_{1,i}} \tilde{L}_s = \max_{s \in S_{1,i}} \sum_{(i,j) \in s} \tilde{t}_{(i,j)} = \max_{s \in S_{1,i}} \sum_{(i,j) \in s} ((t_{(1,i)1}^U, t_{(1,i)2}^U, t_{(1,i)3}^U, t_{(1,i)4}^U; H_1(\tilde{T}_{(1,i)1}^U), H_2(\tilde{T}_{(1,i)1}^U)), (t_{(1,i)1}^L, t_{(1,i)2}^L, t_{(1,i)3}^L, t_{(1,i)4}^L; H_1(\tilde{T}_{(1,i)1}^L), H_2(\tilde{T}_{(1,i)1}^L)))) \quad (15)$$

$$\begin{aligned} \tilde{LST}_{(i,j)} &= \max_{s \in S_{1,n}} \tilde{L}_s \bullet \max_{s \in S_{i,n}} \tilde{L}_s = \max_{s \in S_{1,n}} \sum_{(i,j) \in s} \tilde{t}_{(i,j)} \bullet \max_{s \in S_{i,n}} \sum_{(i,j) \in s} \tilde{t}_{(i,j)} = \max_{s \in S_{1,n}} \sum_{(i,j) \in s} ((t_{(1,n)1}^U, \\ &-t_{(i,n)1}^U, t_{(1,n)2}^U - t_{(i,n)2}^U, t_{(1,n)3}^U - t_{(i,n)3}^U, t_{(1,n)4}^U - t_{(i,n)4}^U; H_1(\tilde{T}_1^U), H_2(\tilde{T}_1^U)), (t_{(1,n)1}^L - t_{(i,n)1}^L, \\ &t_{(1,n)2}^L - t_{(i,n)2}^L, t_{(1,n)3}^L - t_{(i,n)3}^L, t_{(1,n)4}^L - t_{(i,n)4}^L; H_1(\tilde{T}_1^L), H_2(\tilde{T}_1^L))) = ((lst_{(i,j)(1)}^U, lst_{(i,j)(2)}^U, \\ &lst_{(i,j)(3)}^U, lst_{(i,j)(4)}^U; H_1(\tilde{T}_{(i,j)}^U), H_2(\tilde{T}_{(i,j)}^U)), (lst_{(i,j)(1)}^L, lst_{(i,j)(2)}^L, lst_{(i,j)(3)}^L, lst_{(i,j)(4)}^L, \\ &H_1(\tilde{T}_{(i,j)}^L), H_2(\tilde{T}_{(i,j)}^L))) \end{aligned} \quad (16)$$

Nevertheless, the IT2F-TF of each task is computed as follows:

$$\begin{aligned} \tilde{TF}_{(i,j)} &= \tilde{LST}_{(i,j)} \bullet \tilde{EST}_{(i,j)} = \max_{s \in S_{1,n}} \tilde{L}_s \bullet \max_{s \in S_{i,n}} \tilde{L}_s \bullet \max_{s \in S_{1,i}} \tilde{L}_s = \\ &\max_{s \in S_{1,n}} \sum_{(i,j) \in s} \tilde{t}_{(i,j)} \bullet \max_{s \in S_{i,n}} \sum_{(i,j) \in s} \tilde{t}_{(i,j)} \max_{s \in S_{1,i}} \sum_{(i,j) \in s} \tilde{t}_{(i,j)} \end{aligned} \quad (17)$$

Furthermore, the IT2F-TF of each path ( $\tilde{PF}_s$ ) is defined by:

$$\begin{aligned} \tilde{PF}_s &= \tilde{L}_s \bullet \tilde{L}_s = \max_{s \in S_{1,n}} \tilde{L}_s \bullet \tilde{L}_s = \max_{s \in S_{1,n}} \sum_{(i,j) \in s} \tilde{t}_{(i,j)} \bullet \sum_{(i,j) \in s} \tilde{t}_{(i,j)} = \sum_{(i,j) \in s} ((\max t_{(1,n)1}^U, \\ &-t_{(1,n)1}^U, \max t_{(1,n)2}^U - t_{(1,n)2}^U, \max t_{(1,n)3}^U - t_{(1,n)3}^U, \max t_{(1,n)4}^U - t_{(1,n)4}^U; H_1(\tilde{T}_1^U), H_2(\tilde{T}_1^U)), \\ &(\max t_{(1,n)1}^L - t_{(1,n)1}^L, \max t_{(1,n)2}^L - t_{(1,n)2}^L, \max t_{(1,n)3}^L - t_{(1,n)3}^L, \max t_{(1,n)4}^L - t_{(1,n)4}^L; H_1(\tilde{T}_1^L), \\ &H_2(\tilde{T}_1^L))) = ((pf_{s(1)}^U, pf_{s(2)}^U, pf_{s(3)}^U, pf_{s(4)}^U; H_1(\tilde{PF}_s^U), H_2(\tilde{PF}_s^U)), (pf_{s(1)}^L, pf_{s(2)}^L, pf_{s(3)}^L, \\ &pf_{s(4)}^L; H_1(\tilde{PF}_s^L), H_2(\tilde{PF}_s^L))) \end{aligned} \quad (18)$$

**Theorem:** The IT2F-TF of each task is obtained by:

$$\tilde{TF}_{(i,j)} = \min_{s \in S_{1,n}} \left\{ \tilde{PF}_s \mid (i,j) \in s \right\} \quad (19)$$

**Proof.** Consider

$$\begin{aligned}
\min_{s \in S_{1,n}} \left\{ P\tilde{F}_s \mid (i,j) \in s \right\} &= \min \left( \max_{s \in S_{1,n}} \sum_{(i,j) \in s} \tilde{t}_{(i,j)} \bullet \sum_{\substack{(i,j) \in s \\ s \in S_{1,n}}} \tilde{t}_{(i,j)} \right) = \\
\min \left( \max_{s \in S_{1,n}} \sum_{(i,j) \in s} \tilde{t}_{(i,j)} \oplus \left( \bullet \sum_{\substack{(i,j) \in s \\ s \in S_{1,n}}} \tilde{t}_{(i,j)} \right) \right) &= \min \left( \max_{s \in S_{1,n}} \sum_{(i,j) \in s} \tilde{t}_{(i,j)} \right) \\
\oplus \min \left( \bullet \sum_{\substack{(i,j) \in s \\ s \in S_{1,n}}} \tilde{t}_{(i,j)} \right) &= \max_{s \in S_{1,n}} \sum_{(i,j) \in s} \tilde{t}_{(i,j)} \bullet \max \left( \sum_{\substack{(i,j) \in s \\ s \in S_{1,n}}} \tilde{t}_{(i,j)} \right) = \\
\max_{s \in S_{1,n}} \sum_{(i,j) \in s} \tilde{t}_{(i,j)} \bullet \max \left( \max_{s \in S_{1,j}} \sum_{(i,j) \in s} \tilde{t}_{(i,j)} \oplus \max_{s \in S_{1,n}} \sum_{(i,j) \in s} \tilde{t}_{(i,j)} \right) &\quad \forall (i,j) \in S \\
= \max_{s \in S_{1,n}} \sum_{(i,j) \in s} \tilde{t}_{(i,j)} \bullet \max_{s \in S_{1,j}} \left( \sum_{(i,j) \in s} \tilde{t}_{(i,j)} \right) \bullet \max_{s \in S_{1,n}} \left( \sum_{(i,j) \in s} \tilde{t}_{(i,j)} \right) &\quad \forall (i,j) \in S \\
= T\tilde{F}_{(i,j)} \quad \forall (i,j) \in S &
\end{aligned} \tag{20}$$

In conclusion, the IT2F-TF of each activity is given as follows:

- 1) At first, the IT2F-TF of each path is determined by Eq. (18).
- 2) Then, The IT2F-TF of each task is reported by means of the above theorem and Eq. (19).

## 2.5. IT2F-earliest start and finish time

If  $\tilde{E}_i$  denotes the IT2F-earliest start occurrence of node  $i$ , then the  $\tilde{E}_i$  can be calculated as below:

$$\tilde{E}_i = E\tilde{S}T_{(i,j)} = \max \left\{ w_{n,q}(i) \mid n = 1, 2, \dots, m \right\} \tag{21}$$

Where  $n$  represents the level in the rooted tree,  $q$  denotes the position of the node  $i$  in the  $n$ th level from left to right and  $w_{n,q}(i)$  stands for the weight of the node  $i$  introduced in various levels.

Also, the fuzzy earliest finish (EF) time ( $E\tilde{F}T_{(i,j)}$ ) for each activity is computed as below:

$$E\tilde{F}T_{(i,j)} = \tilde{E}_i + \tilde{t}_{(i,j)} = \max \left\{ w_{k,q}(i) \mid k = 1, 2, \dots, m \right\} + \tilde{t}_{(i,j)} \tag{22}$$

Note that, the computation of IT2F-earliest start (ES) time in the proposed method is easier than the conventional approach and will be straightly gained from the rooted tree. But, the calculation of EF time for each activity is similar the conventional method, exactly.

## 2.6. IT2F-latest start and finish time

In this sub-section, with respect to IT2F-TF of each task, the IT2F-LS time ( $L\tilde{S}T_{(i,j)}$ ) can be calculated by:

$$\begin{aligned}
\tilde{LST}_{(i,j)} = T\tilde{F}_{(i,j)} \oplus E\tilde{ST}_{(i,j)} = & ((tf_{(i,j)1}^U + est_{(i,j)1}^U, tf_{(i,j)2}^U + est_{(i,j)2}^U, tf_{(i,j)3}^U + \\
est_{(i,j)3}^U, & tf_{(i,j)4}^U + est_{(i,j)4}^U; \min(H_1(T\tilde{F}^U), H_1(E\tilde{ST}^U)), \min(H_2(T\tilde{F}^U), \\
H_2(E\tilde{ST}^U))), & (tf_{(i,j)1}^L + est_{(i,j)1}^L, tf_{(i,j)2}^L + est_{(i,j)2}^L, tf_{(i,j)3}^L + est_{(i,j)3}^L, tf_{(i,j)4}^L + est_{(i,j)4}^L; \min(H_1(T\tilde{F}^L), H_1(E\tilde{ST}^L)), \min(H_2(T\tilde{F}^L), H_2(E\tilde{ST}^L)))
\end{aligned} \tag{23}$$

Furthermore, the IT2F-LF time of each task can be obtained by using the IT2F-LS time as follows:

$$\begin{aligned}
\tilde{L}_j = L\tilde{FT}_{(i,j)} = L\tilde{ST}_{(i,j)} \oplus \tilde{t}_{(i,j)} = & ((lst_{(i,j)1}^U + t_{(i,j)1}^U, lst_{(i,j)2}^U + t_{(i,j)2}^U, lst_{(i,j)3}^U + \\
t_{(i,j)3}^U, & lst_{(i,j)4}^U + t_{(i,j)4}^U; \min(H_1(L\tilde{ST}^U), H_1(\tilde{T}^U)), \min(H_2(L\tilde{ST}^U), H_2(\tilde{T}^U))), \\
lst_{(i,j)1}^L + t_{(i,j)1}^L, & lst_{(i,j)2}^L + t_{(i,j)2}^L, lst_{(i,j)3}^L + t_{(i,j)3}^L, lst_{(i,j)4}^L + t_{(i,j)4}^L; \min(H_1(L\tilde{ST}^L), \\
H_1(\tilde{T}^L)), & \min(H_2(L\tilde{ST}^L), H_2(\tilde{T}^L)))
\end{aligned} \tag{24}$$

Nevertheless, the computation of IT2F-latest finish and start times are easier and simpler than traditional one.

## 2.7. Overview of the proposed method

In this sub-section, an overview of the proposed method is explained to understand the new method, better. Also, the proposed framework is demonstrated in Figure 1. The main steps of the proposed method are defined as below:

**Step 1:** Gather expert's opinions on time of each activity.

**Step 2:** Identify the potential risks of each activity.

**Step 3:** Collect the expert's judgments on the probability and impact of potential risks

**Step 4:** Do primary refinement of risks and remove the risks with low probability and impacts for the qualitative risk analysis.

**Step 5:** Convert expert's judgments on the probability and impact of each risk to equivalent IT2F-numbers by using Table 1.

**Step 6:** Obtain the impact of  $k$  risks over each other by a new developed operator which is explained in Eqs. (1) to (4).

**Step 7:** Determine the most important risk of each activity by calculating the risk measurements of each activity and considering the impact of most important risk on the duration of activities based on step 2.1.5.

**Step 8:** Construct rooted tree project network based on Table 2 and Figure 3.

**Step 9:** Introduce a new subtraction operator using Eqs. (10) to (17) to avoid producing negative numbers.

**Step 10:** Modify the NWRT method by the new subtraction operator.

**Step 10-1:** Compute the IT2F-TF of each path by Eq. (18).

**Step 10-2:** Calculate the IT2F-TF of each activity via Eq. (19).

**Step 10-3:** Compute the IT2F-ES time of each task by means of Eq. (21).

**Step 10-4:** Calculate the IT2F-EF time of each task by Eq. (22).

**Step 10-5:** Obtain the IT2F-LS time of each task by means Eq. (23).

**Step 10-6:** Calculate the IT2F-LF time of each task via Eq. (24).

**Step 11:** Determine the criticality degree of each task and the project critical path by Eq. (A.8).

### 3. Application

In this section, to illustrate the applicability and ability of the proposed method, firstly, an example from literature is adopted and solved (Sireesha and Shankar, 2017). The project network is shown in Figure 4. Also, the expert's judgments on the time of each activity are illustrated in Table 3. Furthermore, the important risks of each activity and the probability and impact of each one are gathered from an expert and demonstrated in Tables 4 to 10. Generally, the probability and impact of each risk of activities are gathered from experts and the impact of each risk over each other are determined by using a new function that is developed under IT2FSs. Then, the most important risk of each activity are specified by using the amount of risk, and eventually, the impact of most important risk is considered as time of activities and added to the initial time of activities. Furthermore, the project network is updated by new activities' times. Moreover, the NWRT method that is originally proposed by Sireesha and Shankar (2017) is modified by means of new subtraction method for avoid producing negative numbers in determining the characteristics of each activity.

{ Please insert Figure 4 here. }

{ Please insert Table 3 here. }

{ Please insert Table 4 here. }

{ Please insert Table 5 here. }

{ Please insert Table 6 here. }

{ Please insert Table 7 here. }

{ Please insert Table 8 here. }

{ Please insert Table 9 here. }

{ Please insert Table 10 here. }

Then, the linguistic variables are converted to equivalent IT2F-numbers, and the impacts among all risks are determined by Eqs. (11) and (12). Moreover, the risk measurement based on multiple probability and impact is calculated and the most important risk of each activity is specified. Finally, the impact of most important risk of each activity is regarded as the time of activity and is added to the duration of activities. The new duration of activities is depicted in Table 11. Also, the IT2F node-weighted rooted tree is depicted in Figure 5.

{Please insert Table 11 here.}

In order to compare, the length of paths ( $LP$ ) is computed by Eq. (A.8) as follows:

$$\begin{aligned}
 LP_{1-2-3-5} &= ((136.1, 302.8, 426, 623.35; 1, 1), (206.87, 337.3, 383.15, 520.13; 0.9, 0.9)) = 367 \\
 LP_{1-2-5} &= ((87, 201.67, 228, 434.5; 1, 1), (136.8, 224.67, 257.83, 358.8; 0.9, 0.9)) = 248.74 \\
 LP_{1-3-5} &= ((119.3, 263.4, 368.93, 539.6; 1, 1), (183.53, 295.3, 328.7, 453.07; 0.9, 0.9)) = 319.05 \\
 LP_{1-4-5} &= ((88.38, 221.3, 314.3, 482.3; 1, 1), (145.44, 246.82, 284.1, 399; 0.9, 0.9)) = 272.79
 \end{aligned}$$

By comparing the results, it is clear that IT2F-critical path is 1-2-3-5. IT2F-TF of the paths is computed by means of Eq. (18) and illustrated in Table 12.

{Please insert Figure 5 here.}

{Please insert Table 12 here.}

Moreover, the IT2F-TF of each activity is computed by using Eq. (19) and presented in Table 13.

For instance,

$$\begin{aligned}
 TF_{(1,2)} &= \min \left\{ PF_{(1-2-3-5)}, PF_{(1-2-5)} \right\} = \min \left\{ \begin{array}{l} ((0, 0, 0, 0; 1, 1), (0, 0, 0, 0; 0.9, 0.9)), \\ (49.1, 101.1, 138, 188.85; 1, 1), \\ (70.07, 112.63, 125.32, 161.3; 0.9, 0.9) \end{array} \right\} \\
 &= ((0, 0, 0, 0; 1, 1), (0, 0, 0, 0; 0.9, 0.9))
 \end{aligned}$$

Furthermore, the IT2F-earliest occurrence time of each node  $i$  is calculated by means of Eq. (21).

For example,

$$\tilde{E}_3 = \max \{w_{2,2}(3), w_{3,1}(3)\} \max \left\{ \begin{array}{l} |((35,54.67,68.3,94;1,1),(44.3,61.5,61.5,80.67;0.9,0.9)), \\ ((51.9,94,125.4,177.75;1,1), \\ (67.67,103.5,115.95,147.73;0.90.9)) \end{array} \right\}$$

$$= ((51.9,94,125.4,177.75;1,1),(67.67,103.5,115.95,147.73;0.90.9))$$

Also, IT2F-ES time and IT2F-EF time of each node  $i$  are determined via Eqs. (21) and (22) and illustrated in Table 13.

For example,

$$E\tilde{S}T_{(3,5)} \quad \tilde{E}_3 = ((51.9,94,125.4,177.75;1,1),(67.67,103.5,115.95,147.73;0.90.9))$$

$$E\tilde{F}T_{(3,5)} = E\tilde{S}T_{(3,5)} + \tilde{T}_{(3,5)} = ((51.9,94,125.4,177.75;1,1),(67.67,103.5,115.95,147.73;0.90.9)) \oplus$$

$$((84.2, 208.75, 300.6, 445.6;1,1), (139.2, 233.8, 267.2, 372.4;0.9,0.9)) =$$

$$((136.1,302.8, 426, 623.35;1,1), (206.87, 337.3, 383.15, 520.13;0.9,0.9))$$

Finally, the IT2F-latest start and finish time are obtained by means of Eqs. (23) and (24), respectively. The results are demonstrated in Table 13.

For example,

$$L\tilde{S}T_{(3,5)} = T\tilde{F}_{(3,5)} + E\tilde{S}T_{(3,5)} = ((0,0,0,0;1,1), (0,0,0,0;0.9,0.9)) \oplus$$

$$((51.9,94,125.4,177.75;1,1),(67.67,103.5,115.95,147.73;0.90.9)) =$$

$$((51.9,94,125.4,177.75;1,1),(67.67,103.5,115.95,147.73;0.90.9))$$

and,

$$L\tilde{F}T_{(3,5)} = L\tilde{S}T_{(3,5)} + \tilde{T}_{(3,5)} = ((51.9,94,125.4,177.75;1,1),(67.67,103.5,115.95,147.73;0.90.9)) \oplus$$

$$((84.2, 208.75, 300.6, 445.6;1,1), (139.2, 233.8, 267.2, 372.4;0.9,0.9)) =$$

$$((136.1,302.8, 426, 3623.35;1,1), (206.87, 337.3, 383.15, 520.13;0.9,0.9))$$

The criticality degree of each activity and critical path is determined in Table 14. The IT2F-TF of each activity is converted to a crisp number by using Eq. (A.8). It is obvious that the critical activities are 1-2, 2-3 and 3-5; nevertheless, the critical path is the path 1-2-3-5.

{ Please insert Table 13 here. }

{ Please insert Table 14 here. }

In order to show the advantages of proposed method, a simple example is solved by two method of the literature. If  $\tilde{A}, \tilde{B}$  are defined as follows:



$$\tilde{A} = (10, 15, 30, 40)$$

$$\tilde{B} = (12, 13, 15, 20)$$

Then, the subtraction operation is done by using the Kumar and Kaur (2011) as below:

$$\tilde{A} \ominus \tilde{B} = (5, 10, 10, 11)$$

Also, the subtraction operation by means of Rao and Shankar (2013) method is computed as follows:

$$\tilde{A} \ominus \tilde{B} = (0, 0, 17, 28)$$

Finally, this operation is done by using the proposed method as follows:

$$\tilde{A} \ominus \tilde{B} = (0, 0, 15, 20)$$

The classic fuzzy subtraction operation is calculated by using the following:

$$\tilde{A} \ominus \tilde{B} = (10, 15, 30, 40) \ominus (12, 13, 15, 20) = (-10, 0, 17, 28) \xrightarrow{\text{crisp number}} = \frac{35}{4}$$

With this in mind, the crisp results of proposed method, Kumar and Kaur (2011), and Rao and Shankar (2013) are calculated as follows:

$$\left( \frac{5 + 10 + 10 + 11}{4} = \frac{36}{4} \right) \quad \text{Kumar and Kaur [48] method}$$

$$\left( \frac{0 + 0 + 17 + 28}{4} = \frac{45}{4} \right) \quad \text{Rao and Shankar [49] method}$$

$$\left( \frac{0 + 0 + 15 + 20}{4} = \frac{35}{4} \right) \quad \text{Proposed method}$$

As shown above, the previous methods compute the amounts larger than actual amount. But, the results of classic fuzzy subtraction and the proposed method is the same. In fact, the proposed method produces the real amounts in subtraction operation and avoids negative numbers.

#### 4. Comparative analysis

In this section, to validate the results of characteristics of each task and critical path, new proposed approach is applied to the two examples from recent literature. Note that these examples are solved by a the proposed method in the traditional fuzzy environment. The first example is related to Sireesha and Shankar (2013) that is solved in triangle fuzzy environment and the second one is relevant to Sireesha and Shankar (2017) that is solved in trapezoidal fuzzy environment. These two examples are depicted in Tables 14 and 15, respectively. In these two cases, it is observed that the criticality degree is the same, and it is shown that the validity of the proposed method. It is obvious that the proposed method avoids producing negative number for characteristics of each activity.

{Please insert Table 15 here.}

{Please insert Table 16 here.}

The proposed method compared with two methods of the literature, and the results confirmed the results of the proposed method. But, these two methods of the literature have produced negative numbers for characteristics of fuzzy project network. As it was seen in Tables 13 and 14, the proposed method avoided producing negative numbers for characteristics of fuzzy project network, and the results were the same. Nevertheless, the validity of the proposed method is confirmed based on Tables 13 and 14.

## 5. Conclusions

In this paper, IT2FSs have been used to consider the risks of each activity and determine the critical path of project network. A new method for determining the correlation among risk factors has been also extended under the IT2F environment to tackle the uncertainty of real-world projects better. Then, a new subtraction operator to avoid producing negative numbers in the project network has been introduced, and the NWRT method has been modified by this new operator. Furthermore, the NWRT method has been developed under IT2FSs. An example from literature has been solved to demonstrate the calculation procedure and capability of the proposed method. For the comparative analysis, the total float of each activity of two examples from the recent literature has been obtained, and finally, the results have been defuzzified to determine the criticality degree. It was observed that the criticality degree of two previous decision methods was the same with the results of the proposed method. The main advantage of the proposed method was to produce positive numbers because negative numbers have no physical meanings in a project network. The new subtraction operator for critical path computation did not generate larger values than real values in comparisons with previous methods. In fact, the new subtraction operator had two advantages. Firstly, it avoided producing negative numbers, and secondly, it avoided producing larger values than real values. Also, the presented approach was more simple and easier than traditional CPM calculation, and the critical path would be determined directly from the rooted tree. A mathematical modeling approach of the problem can be set for future research. The proposed methodology will require more inputs from the DMs, which can be time-consuming. To solve this problem, a new operator for considering the correlation between risk factors and obtaining probability can be added to the proposed method. Furthermore, the proposed method can be extended under the group decision environment for achieving more degree of accuracy.

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## Appendix

The membership degree of the T1FS is a crisp number in  $[0,1]$ . Usually, there are circumstances where it is hard to specify the exact membership function for a fuzzy set. It is not appropriate to use T1FSs in this situation. To overcome this issue, Zadeh [43] presented T2FSs which are the extension

of T1FSs. T2FSs are defined by both primary and secondary membership to put out more degrees of freedom and flexibility, and they are three-dimensional. Hence, T2FSs have the advantage of modeling uncertainty more accurately compared with T1FSs [54]. T2FSs are more appropriate, flexible and intelligent than T1FSs to illustrate uncertainties for handling fuzzy group decision problems [55-57]. Moreover, the computational burden is significant when using T2FSs to solve problems [58]. IT2FSs can be considered as a special case of general T2FS where all the amounts of secondary membership are equal to 1 [55]. Therefore, it not only demonstrates uncertainty better than T1FSs but also reduces the computation burden when compared with T2FSs.

A T2FS  $\tilde{A}$  in the universe of discourse  $X$  can be illustrated by a type-2 membership function  $\mu_{\tilde{A}}$  demonstrated as below [55]:

$$\tilde{A} = \left\{ ((x, u), \mu_{\tilde{A}}(x, u)) \mid \forall x \in X, \forall u \in J_x \subseteq [0, 1], 0 \leq \mu_{\tilde{A}}(x, u) \leq 1 \right\} \quad (\text{A.1})$$

Where  $J_x$  demonstrates an interval in  $[0, 1]$ . However, the T2FS  $\tilde{A}$  also can be shown by using the following:

$$\tilde{A} = \int_{x \in X} \int_{u \in J_x} \mu_{\tilde{A}}(x, u) / (x, u) \quad (\text{A.2})$$

Where  $J_x \subseteq [0, 1]$  and  $\int \int$  demonstrates union over all admissible  $x$  and  $u$ .

If all  $\mu_{\tilde{A}}(x, u) = 1$ , then  $\tilde{A}$  is called IT2FS. An IT2FS  $\tilde{A}$  can be viewed as a special case of T2FSs, demonstrated:

$$\tilde{A} = \int_{x \in X} \int_{u \in J_x} 1 / (x, u) \quad (\text{A.3})$$

{Please insert Figure A.1 here.}

The upper membership grade and the lower membership grade of an IT2FS are type-1 membership grades, respectively. Fig. A.1 illustrates a trapezoidal IT2F  $\tilde{A}_i = (\tilde{A}_i^U, \tilde{A}_i^L)$   $((a_{i1}^U, a_{i2}^U, a_{i3}^U, a_{i4}^U; H_1(\tilde{A}_i^U), H_1(\tilde{A}_i^U)), (a_{i1}^L, a_{i2}^L, a_{i3}^L, a_{i4}^L; H_1(\tilde{A}_i^L), H_1(\tilde{A}_i^L)))$ , where  $\tilde{A}_i^U$  and  $\tilde{A}_i^L$  are T1FSs,  $a_{i1}^U, a_{i2}^U, a_{i3}^U, a_{i4}^U, a_{i1}^L, a_{i2}^L, a_{i3}^L, a_{i4}^L$  are the reference points of the IT2F  $\tilde{A}_i$ ;  $H_j(\tilde{A}_i^U)$  denotes the membership value of the element  $a_{(j+1)}^U$  in the upper trapezoidal membership function  $\tilde{A}_i^U$ ;  $1 \leq j \leq 2, H_j(\tilde{A}_i^L)$  denotes the membership value of the element  $a_{(j+1)}^L$  in the lower trapezoidal membership function  $\tilde{A}_i^L$ ;  $1 \leq j \leq 2$  [55].

The basic algebraic operations between trapezoidal IT2FSs are described by the following [56]:

$$\begin{aligned}
\tilde{F}_1 &= (\tilde{F}_1^U, \tilde{F}_1^L) = ((f_{11}^U, f_{12}^U, f_{13}^U, f_{14}^U; H_1(\tilde{F}_1^U), H_2(\tilde{F}_1^U)), (f_{11}^L, f_{12}^L, f_{13}^L, f_{14}^L; \\
&H_1(\tilde{F}_1^L), H_2(\tilde{F}_1^L))) \\
\tilde{F}_2 &= (\tilde{F}_2^U, \tilde{F}_2^L) = ((f_{21}^U, f_{22}^U, f_{23}^U, f_{24}^U; H_1(\tilde{F}_2^U), H_2(\tilde{F}_2^U)), (f_{21}^L, f_{22}^L, f_{23}^L, f_{24}^L; \\
&H_1(\tilde{F}_2^L), H_2(\tilde{F}_2^L)))
\end{aligned} \tag{A.4}$$

The addition operation:

$$\begin{aligned}
\tilde{F}_1 \oplus \tilde{F}_2 &= (\tilde{F}_1^U, \tilde{F}_1^L) + (\tilde{F}_2^U, \tilde{F}_2^L) = ((f_{11}^U + f_{21}^U, f_{12}^U + f_{22}^U, f_{13}^U + f_{23}^U, f_{14}^U + f_{24}^U; \\
&\min(H_1(\tilde{F}_1^U), H_1(\tilde{F}_2^U)), \min(H_2(\tilde{F}_1^U), H_2(\tilde{F}_2^U))), f_{11}^L + f_{21}^L, f_{12}^L + f_{22}^L, f_{13}^L + \\
&f_{23}^L, f_{14}^L + f_{24}^L; \min(H_1(\tilde{F}_1^L), H_1(\tilde{F}_2^L)), \min(H_2(\tilde{F}_1^L), H_2(\tilde{F}_2^L))).
\end{aligned} \tag{A.5}$$

The subtraction operation:

$$\begin{aligned}
\tilde{F}_1 \ominus \tilde{F}_2 &= (\tilde{F}_1^U, \tilde{F}_1^L) - (\tilde{F}_2^U, \tilde{F}_2^L) = ((f_{11}^U - f_{24}^U, f_{12}^U - f_{23}^U, f_{13}^U - f_{22}^U, f_{14}^U - f_{21}^U; \\
&\min(H_1(\tilde{F}_1^U), H_1(\tilde{F}_2^U)), \min(H_2(\tilde{F}_1^U), H_2(\tilde{F}_2^U))), f_{11}^L - f_{24}^L, f_{12}^L - f_{23}^L, f_{13}^L - f_{22}^L, \\
&f_{14}^L - f_{21}^L; \min(H_1(\tilde{F}_1^L), H_1(\tilde{F}_2^L)), \min(H_2(\tilde{F}_1^L), H_2(\tilde{F}_2^L))).
\end{aligned} \tag{A.6}$$

The multiplication operation:

$$\begin{aligned}
\tilde{F}_1 \otimes \tilde{F}_2 &= (\tilde{F}_1^U, \tilde{F}_1^L) \times (\tilde{F}_2^U, \tilde{F}_2^L) = ((f_{11}^U \times f_{21}^U, f_{12}^U \times f_{22}^U, f_{13}^U \times f_{23}^U, f_{14}^U \times f_{24}^U; \\
&\min(H_1(\tilde{F}_1^U), H_1(\tilde{F}_2^U)), \min(H_2(\tilde{F}_1^U), H_2(\tilde{F}_2^U))), f_{11}^L \times f_{21}^L, f_{12}^L \times f_{22}^L, f_{13}^L \times f_{23}^L, \\
&f_{14}^L \times f_{24}^L; \min(H_1(\tilde{F}_1^L), H_1(\tilde{F}_2^L)), \min(H_2(\tilde{F}_1^L), H_2(\tilde{F}_2^L))).
\end{aligned} \tag{A.7}$$

The defuzzified value of a trapezoidal IT2FN is defined as follows [36]:

$$Def(\tilde{F}_1) = \frac{1}{2} \left( \sum_{T \in \{U, L\}} \frac{f_{11}^T + H_1(\tilde{F}_1^T) f_{12}^T + H_2(\tilde{F}_1^T) f_{13}^T + f_{14}^T}{2 + H_1(\tilde{F}_1^T) + H_2(\tilde{F}_1^T)} \right) \tag{A.8}$$

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## **Figures' captions**

**Figure 1.** Proposed framework

**Figure 2.** Fuzzy project network

**Figure 3.** Rooted tree project network

**Figure 4.** Project network

**Figure 5.** IT2F node-weighted rooted tree

**Figure A.1.** Trapezoidal interval type-2 fuzzy set



## **Tables' captions**

**Table 1.** Linguistic variables and their corresponding interval type-2 fuzzy numbers

**Table 2.** Fuzzy activities' times

**Table 3.** IT2F-activities time

**Table 4.** Probability and impact of risks' activity 1-2

**Table 5.** Probability and impact of risks' activity 1-3

**Table 6.** Probability and impact of risks' activity 1-4

**Table 7.** Probability and impact of risks' activity 2-3

**Table 8.** Probability and impact of risks' activity 2-5

**Table 9.** Probability and impact of risks' activity 3-5

**Table 10.** Probability and impact of risks' activity 4-5

**Table 11.** New durations for activities

**Table 12.** Total float of each path

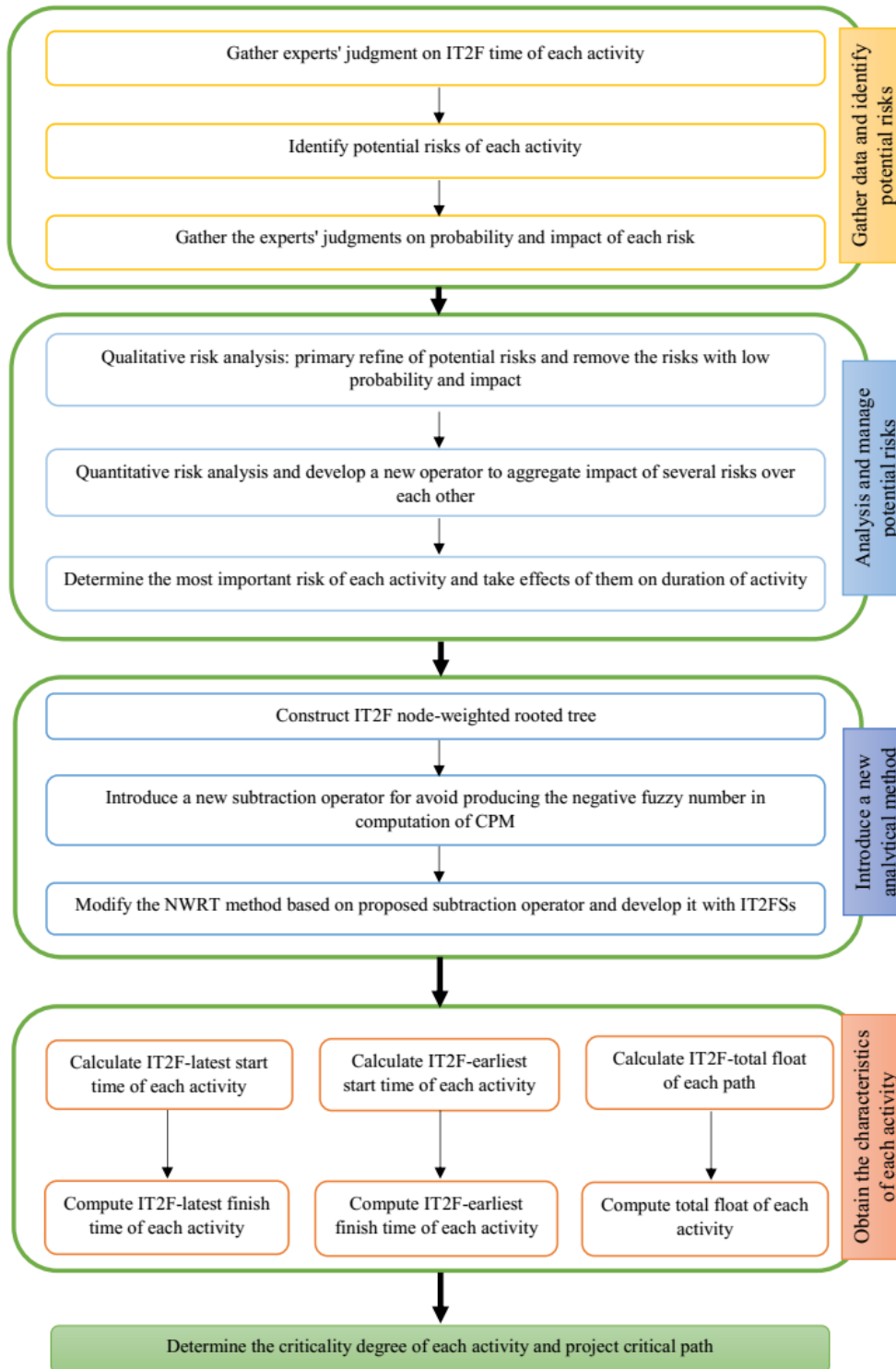
**Table 13.** Results of IT2F-earliest, latest times

**Table 14.** Criticality degree of each activity

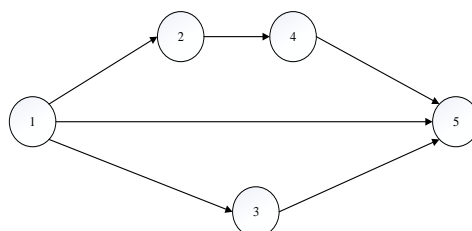
**Table 15.** Comparative analysis of the proposed method and Sireesha and Shankar (2017) on the first example

**Table 16.** Comparative analysis of the proposed method and Sireesha and Shankar (2013) on the second example

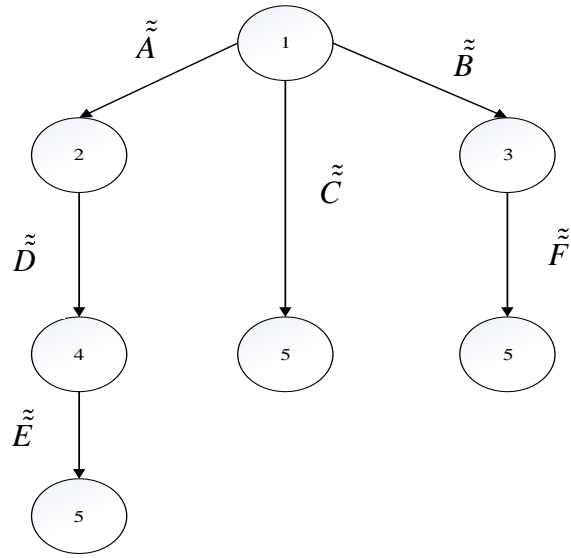
**Figures:**



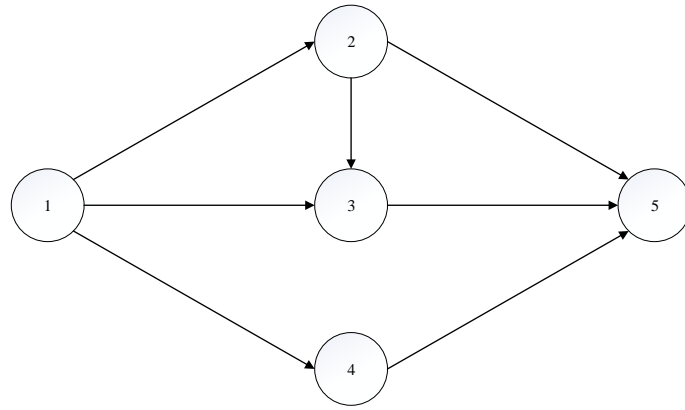
**Figure 1.**



**Figure 2.**



**Figure 3.**



**Figure 4.**

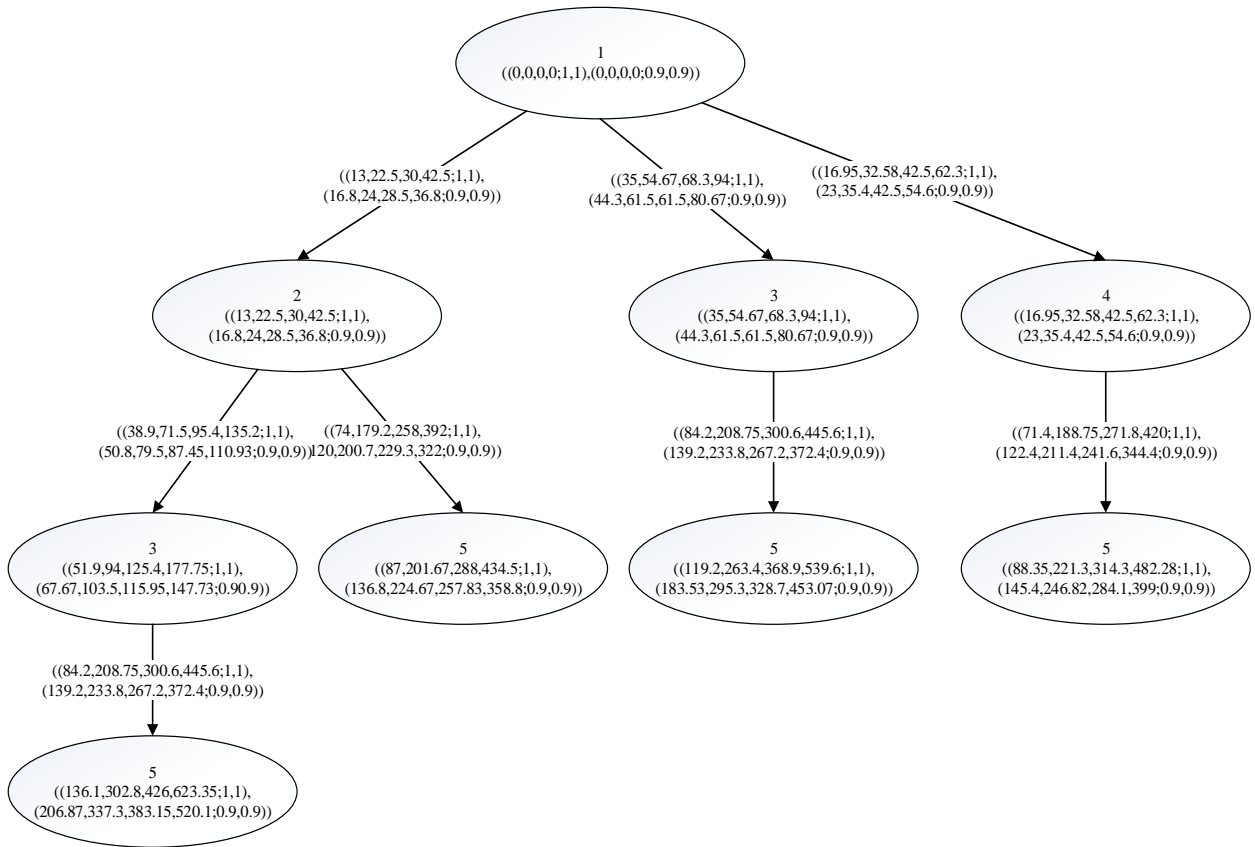


Figure 5.

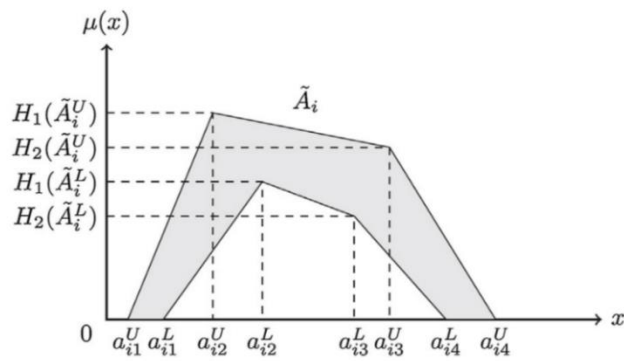


Figure A.1.

**Tables:**

**Table 1.**

Linguistic variables	Interval type-2 fuzzy numbers
Very Low (VL)	((0,0,0,0.1;1,1),(0,0,0,0.05;0.9,0.9))
Low (L)	((0,0.1,0.1,0.3;1,1),(0.05,0.1,0.1,0.2;0.9,0.9))
Medium Low (ML)	((0.1,0.3,0.3,0.5;1,1),(0.2,0.3,0.3,0.4;0.9,0.9))
Medium (M)	((0.3,0.5,0.5,0.7;1,1),(0.4,0.5,0.5,0.6;0.9,0.9))
Medium High (MH)	((0.5,0.7,0.7,0.9;1,1),(0.6,0.7,0.7,0.8;0.9,0.9))
High (H)	((0.7,0.9,0.9,1;1,1),(0.8,0.9,0.9,0.95;0.9,0.9))
Very High (VH)	((0.9,1,1,1;1,1),(0.95,1,1,1;0.9,0.9))

**Table 2.**

Activities	Fuzzy activities' times
1-2	$\tilde{A} = ((a_1^U, a_2^U, a_3^U, a_4^U; H_1(A^U), H_2(A^U)), (a_1^L, a_2^L, a_3^L, a_4^L; H_1(A^L), H_2(A^L)))$
1-3	$\tilde{B} = ((b_1^U, b_2^U, b_3^U, b_4^U; H_1(B^U), H_2(B^U)), (b_1^L, b_2^L, b_3^L, b_4^L; H_1(B^L), H_2(B^L)))$
1-5	$\tilde{C} = ((c_1^U, c_2^U, c_3^U, c_4^U; H_1(C^U), H_2(C^U)), (c_1^L, c_2^L, c_3^L, c_4^L; H_1(C^L), H_2(C^L)))$
2-4	$\tilde{D} = ((d_1^U, d_2^U, d_3^U, d_4^U; H_1(D^U), H_2(D^U)), (d_1^L, d_2^L, d_3^L, d_4^L; H_1(D^L), H_2(D^L)))$
4-5	$\tilde{E} = ((e_1^U, e_2^U, e_3^U, e_4^U; H_1(E^U), H_2(E^U)), (e_1^L, e_2^L, e_3^L, e_4^L; H_1(E^L), H_2(E^L)))$
3-5	$\tilde{F} = ((f_1^U, f_2^U, f_3^U, f_4^U; H_1(F^U), H_2(F^U)), (f_1^L, f_2^L, f_3^L, f_4^L; H_1(F^L), H_2(F^L)))$

**Table 3.**

Activities	Expert
1-2	((10,15,20,25;1,1),(12,16,19,23;0.9,0.9))
1-3	((30,40,50,60;1,1),(35,45,45,55;0.9,0.9))
1-4	((15,23,30,37;1,1),(18,25,30,35;0.9,0.9))
2-3	((30,45,60,75;1,1),(35,50,55,65;0.9,0.9))
2-5	((60,125,180,240;1,1),(90,140,160,210;0.9,0.9))
3-5	((60,125,180,240;1,1),(90,140,160,210;0.9,0.9))
4-5	((60,125,180,240;1,1),(90,140,160,210;0.9,0.9))

**Table 4.**

Risks	Expert	
	Probability	Impact
Risk of increasing cost = $R_1$	ML	M
Risk of reducing quality = $R_2$	M	ML
Risk of reducing safety = $R_3$	MH	M
Correlation between $R_1, R_2$	L	Calculate via Eqs. (3) and (4)
Correlation between $R_1, R_3$	ML	Calculate via Eqs. (3) and (4)
Correlation between $R_2, R_3$	ML	Calculate via Eqs. (3) and (4)
Correlation among $R_1, R_2, R_3$	L	Calculate via Eqs. (3) and (4)

**Table 5.**

Risks	Expert	
	Probability	Impact
Risk of increasing cost = $R_1$	M	L
Risk of reducing quality = $R_2$	ML	ML
General risks = $R_4$	L	M
Correlation between $R_1, R_2$	L	Calculate via Eqs. (3) and (4)
Correlation between $R_1, R_4$	VL	Calculate via Eqs. (3) and (4)
Correlation between $R_2, R_4$	VL	Calculate via Eqs. (3) and (4)
Correlation among $R_1, R_2, R_4$	VL	Calculate via Eqs. (3) and (4)

**Table 6.**

Risks	Expert	
	Probability	Impact
Risk of reducing quality = $R_2$	M	ML
Risk of reducing safety = $R_3$	MH	ML
General risks = $R_4$	MH	L
Correlation between $R_2, R_3$	M	Calculate via Eqs. (3) and (4)
Correlation between $R_2, R_4$	ML	Calculate via Eqs. (3) and (4)
Correlation between $R_3, R_4$	M	Calculate via Eqs. (3) and (4)
Correlation among $R_2, R_3, R_4$	ML	Calculate via Eqs. (3) and (4)

**Table 7.**

Risks	Expert	
	Probability	Impact
Risk of increasing cost = $R_1$	M	ML
General risks = $R_4$	M	ML
Correlation between $R_1, R_4$	ML	Calculate via Eqs. (3) and (4)

**Table 8.**

Risks	Expert	
	Probability	Impact
Risk of increasing cost = $R_1$	ML	M
Risk of reducing safety = $R_3$	L	MH
Correlation between $R_1, R_3$	VL	Calculate via Eqs. (3) and (4)

**Table 9.**

Risks	Expert	
	Probability	Impact
Risk of reducing quality = $R_2$	M	M
Risk of reducing safety = $R_3$	MH	ML
Correlation between $R_2, R_3$	M	Calculate via Eqs. (3) and (4)

**Table 10.**

Risks	Expert	
	Probability	Impact
Risk of increasing cost = $R_1$	M	ML
General risks = $R_4$	M	ML
Risk of reducing safety = $R_3$	ML	M
Correlation between $R_1, R_4$	ML	Calculate via Eqs. (3) and (4)
Correlation between $R_1, R_3$	L	Calculate via Eqs. (3) and (4)
Correlation between $R_3, R_4$	L	Calculate via Eqs. (3) and (4)
Correlation among $R_1, R_3, R_4$	L	Calculate via Eqs. (3) and (4)

**Table 11.**

Activities	Expert
1-2	((13,22.5,30,42.5;1,1),(16.8,24,28.5,36.8;0.9,0.9))
1-3	((35,54.67,68.3,94;1,1),(44.3,61.5,61.5,80.67;0.9,0.9))
1-4	((16.95,32.58,42.5,62.28;1,1),(23.04,35.42,42.5,54.6;0.9,0.9))
2-3	((38.9,71.55,95.4,135.25;1,1),(50.87,79.5,87.45,110.93;0.9,0.9))
2-5	((74,179.17,258,392;1,1),(120,200.67,229.3,322;0.9,0.9))
3-5	((84.2,208.75,300.6,445.6;1,1),(139.2,233.8,267.2,372.4;0.9,0.9))
4-5	((71.4,188.75,271.8,420;1,1),(122.4,211.4,241.6,344.4;0.9,0.9))

**Table 12.**

Path	Length of the path	Total float of each path
1-2-3-5	((136.1,302.8,426,623.35;1,1), (206.87,337.3,383.15,520.13;0.9,0.9))	((0,0,0;1,1), (0,0,0;0.9,0.9))
1-2-5	((87,201.67,228,434.5;1,1), (136.8,224.67,257.83,358.8;0.9,0.9))	((49.1,101.13,138,188.85;1,1), (70.07,112.63,125.32,161.3;0.9,0.9))
1-3-5	((119.3,263.4,368.93,539.6;1,1), (183.53,295.3,328.7,453.07;0.9,0.9))	((16.9,39.38,57.07,83.75;1,1), (23.3,42.54,45.67,67.07;0.9,0.9))
1-4-5	((88.38,221.3,314.3,482.3;1,1), (145.44,246.82,284.1,399;0.9,0.9))	((47.75,81.47,111.7,141.07;1,1), (61.43,90.48,9.05,121.13;0.9,0.9))



**Table 13.**

Act.	Activity time	Total float of each activity	Earliest start time
1-2	((13,22.5,30,42.5;1,1), (16.8,24,28.5,36.8;0.9,0.9))	((0,0,0;1,1), (0,0,0;0.9,0.9))	((0,0,0;1,1), (0,0,0;0.9,0.9))
1-3	((35,54.67,68.3,94;1,1), (44.3,61.5,61.5,80.67;0.9,0.9))	((16.9,39.38,57.06,83.75;1,1), (23.3,42,54.45,67.07;0.9,0.9))	((0,0,0;1,1), (0,0,0;0.9,0.9))
1-4	((16.95,32.58,42.5,62.28;1,1), (23.04,35.42,42.5,54.6;0.9,0.9))	((47.75,81.47,111.7,141.07;1,1), (61.4,90.48,99.05,121.13;0.9,0.9))	((0,0,0;1,1), (0,0,0;0.9,0.9))
2-3	((38.9,71.55,95.4,135.25;1,1), (50.87,79.5,87.45,110.93;0.9,0.9))	((0,0,0;1,1), (0,0,0;0.9,0.9))	((13,22.5,30,42.5;1,1), (16.8,24,28.5,36.8;0.9,0.9))
2-5	((74,179.17,258,392;1,1), (120,200.67,229.3,322;0.9,0.9))	((49.1,101.13,138,188.85;1,1), (70.07,112.63,125.32,161.3;0.9,0.9))	((13,22.5,30,42.5;1,1), (16.8,24,28.5,36.8;0.9,0.9))
3-5	((84.2,208.75,300.6,445.6;1,1), (139.2,233.8,267.2,372.4;0.9,0.9))	((0,0,0;1,1), (0,0,0;0.9,0.9))	((51.9,94.05,125.4,177.75;1,1), (67.67,103.5,115.95,147.73;0.9,0.9))
4-5	((71.4,188.75,271.8,420;1,1), (122.4,211.4,241.6,344.4;0.9,0.9))	((47.75,81.47,111.7,141.07;1,1), (61.4,90.48,99.05,121.13;0.9,0.9))	((16.95,32.58,42.5,62.28;1,1), (23.04,35.42,42.5,54.6;0.9,0.9))

**Table 13. Continued**

Earliest finish time	Latest start time	Latest finish time
((13,22.5,30,42.5;1,1), (16.8,24,28.5,36.8;0.9,0.9))	((0,0,0;1,1), (0,0,0;0.9,0.9))	((13,22.5,30,42.5;1,1), (16.8,24,28.5,36.8;0.9,0.9))
((35,54.67,68.3,94;1,1), (44.3,61.5,61.5,80.67;0.9,0.9))	((16.9,39.38,57.06,83.75;1,1), (23.3,42,54.45,67.07;0.9,0.9))	((51.9,94.05,125.4,177.75;1,1), (67.67,103.5,115.95,147.73;0.9,0.9))
((16.95,32.58,42.5,62.28;1,1), (23.04,35.42,42.5,54.6;0.9,0.9))	((47.75,81.47,111.7,141.07;1,1), (61.4,90.48,99.05,121.13;0.9,0.9))	((64.7,114.05,154.2,203.35;1,1), (84.47,125.9,141.55,175.73;0.9,0.9))
((51.9,94.05,125.4,177.75;1,1), (67.67,103.5,115.95,147.73;0.9,0.9))	((13,22.5,30,42.5;1,1), (16.8,24,28.5,36.8;0.9,0.9))	((51.9,94.05,125.4,177.75;1,1), (67.67,103.5,115.95,147.73;0.9,0.9))
((87,201.67,288,434.5;1,1), (136.8,224.67,257.83,358.8;0.9,0.9))	((62.1,123.63,168,231.35;1,1), (86.87,136.63,153.82,198.13;0.9,0.9))	((136.1,302.8,426,3623.35;1,1), (206.87,337.3,383.15,520.13;0.9,0.9))
((136.1,302.8,426,623.35;1,1), (206.87,337.3,383.15,520.13;0.9,0.9))	((51.9,94.05,125.4,177.75;1,1), (67.67,103.5,115.95,147.73;0.9,0.9))	((136.1,302.8,426,3623.35;1,1), (206.87,337.3,383.15,520.13;0.9,0.9))
((88.35,221.33,314.3,482.3;1,1), (145.4,246.82,284.1,399;0.9,0.9))	((64.7,114.05,154.2,203.35;1,1), (84.47,125.9,141.55,175.73;0.9,0.9))	((136.1,302.8,426,3623.35;1,1), (206.87,337.3,383.15,520.13;0.9,0.9))

**Table 14.**

Act.	Total float of each activity	Criticality degree
1-2	((0,0,0;1,1),(0,0,0;0.9,0.9))	0
1-3	((16.9,39.38,57.06,83.75;1,1),(23.3,42,54.45,67.07;0.9,0.9))	47.95
1-4	((47.75,81.47,111.7,141.07;1,1),(61.4,90.48,99.05,121.13;0.9,0.9))	94.21
2-3	((0,0,0;1,1),(0,0,0;0.9,0.9))	0
2-5	((49.1,101.13,138,188.85;1,1),(70.07,112.63,125.32,161.3;0.9,0.9))	118.26
3-5	((0,0,0;1,1),(0,0,0;0.9,0.9))	0
4-5	((47.75,81.47,111.7,141.07;1,1),(61.4,90.48,99.05,121.13;0.9,0.9))	94.21

**Table 15.**

Activity	Sireesha and Shankar (2017) $TF_{ij}^{\sim}$	Criticality degree	Proposed method	Criticality degree
1-2	(-160,0,160)	0	(0,0,0)	0
1-3	(-130,20,170)	20	(10,20,30)	20
1-4	(-110,37,185)	37.3	(25,37,50)	37.3
2-3	(-160,0,160)	0	(0,0,0)	0
2-5	(-100,45,190)	45	(30,45,60)	45
3-5	(-160,0,160)	0	(0,0,0)	0
4-5	(-110,37,185)	37.3	(25,37,50)	37.3

**Table 16.**

Activity	Sireesha and Shankar (2013) $TF_{ij}^{\sim}$	Criticality degree	Proposed method	Criticality degree
1-2	(-7,0,6,14)	3.25	(2,3,3,5)	3.25
1-3	(-12,-3,3,12)	0	(0,0,0,0)	0
1-5	(2,6,10,18)	9	(6,7,9,14)	9
2-4	(-7,0,6,14)	3.25	(2,3,3,5)	3.25
2-5	(-5,2,7,16)	5	(4,4,5,7)	5
3-4	(-6,2,6,15)	4.25	(3,3,5,6)	4.25
3-6	(-12,-3,3,12)	0	(0,0,0,0)	0
4-5	(-7,0,6,14)	3.25	(2,3,3,5)	3.25
4-6	(-6,2,6,15)	3.75	(2,3,4,6)	3.75
5-6	(-7,0,6,14)	3.25	(2,3,3,5)	3.25

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