Experimental Investigation of Water Leakages through a Longitudinal Crack due to Expansion of the Pipe Material under Pressure

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ABSTRACT: The effect of the pipe material plays a preeminent role in the overall water leakage behaviour of cracks, and specifically of longitudinal cracks for pipes composing Water Distribution Systems. Due to pipe material properties, a longitudinal crack on a pipe exhibits expansion behaviour under internal pressure increases, taken up by hoop stresses, which cause high stress concentration around the crack, provoking the expansion of cracked areas. With the aim of assessing the increment of water leakages through longitudinal cracks, caused by pressure increase, in this paper results from experimental tests of a longitudinal crack on a pipe wall were analysed and discussed. This is achieved by subjecting several plates with different length of cracks to tension and monitoring the opening of the cracks. A mathematical model for longitudinal crack opening is derived using the orifice equation, as a function of pressure, pipe material properties, pipe geometry and fluid properties for uni-axial stress state. Subsequently, an equation describing the increase of the leakage flow rate as function of the increase of the crack area in uni-axial stress state is determined. The derived model (Ilunga's Equation) contradicts the Torricelli's orifice equation which assumed that the orifice area is fixed, but variable with the change in pressure due to pipe material properties.

1 BACKGROUNDS

Physical losses (leaks, illegitimate use, unmetered use and faulty water meters) are the major component of losses in water distribution systems. Physical losses constitute more than 70% of the total water losses in water distribution systems (Farley, 2001). Water losses reduce the income of water service providers, which lead to poor customer satisfaction, subsequently reduces further the income due to non-payment by customers. Leakage is the major physical loss, sometimes it can reach up to 50 to 60% of the total water supply (Farley et al 2003). In Johannesburg, for instance the loss in water is around 40% on Non-Revenue-Water (DWS, 2016). Leaks occur for many reasons which include: age of pipes, water pressure, construction damage due to local construction activity, poor design construction, soils contracting or expanding due to rain, or drought, corrosion and many other issues.

Water pressure is one of the major factors influencing leakage in a water distribution system. It is important to manage the effect of pressure in water distribution networks in order to minimise excess pressure which is actually recognised as a fundamental aspect of leakage management strategy.

The general form for leakage equation through a round hole is described in the orifice equations 1 and 2:

$$Q = C_d A\sqrt{2g}.h^{\alpha} \quad (1)$$

$$Q = C_d A\sqrt{2gh} \quad (2)$$

where Q is the leakage flow rate, C_d the discharge coefficient, h the pressure head, α the leakage exponent (=0.5), g the acceleration due to gravity and A the area of the orifice.

In the orifice equation (1), the discharge is a function of the orifice area, discharge coefficient and square root of twice the gravity multiplied by the pressure head. The discharge coefficient values are available in the literature depending on the shape and contour of the orifice. Thus, the only variable in the orifice equation that has sensitive impact for increased leakage flow rate is the area of the orifice A. In this orifice equation (2) the leakage exponent is theoretically proportional to the square root of the pressure head, i.e. α = 0.5. However, several studies have found that the leakage exponent can be significantly greater than 0.5 (Greyvenstein, 2004).

Van Zyl and Clayton (Van Zyl, 2005) found different mechanisms that may be responsible for this pressure-leakage relation and concluded that the pipe material behaviour plays a major role in the observed behaviour, i.e. leak areas increase with increasing pressure. It is thus important to understand the pipe material behaviour well in order to effectively manage the problem of leakage. Cassa (Cassa, 2005) used the finite element procedure to analyse the relationship between pressure in the pipe and the behaviour of the pipe material containing a small hole, a longitudinal crack and a circumferential crack. The results of this study showed that round holes were very consistent with the theory for all kinds of pipe materials, whilst longitudinal cracks appear to increase losses in water as the crack expands and propagates. Circular cracks also showed an increase in water losses but not to as large extent.

The aim of this investigation is to understand and explain how the material behaves around a longitudinal crack in sidewall of pressurised pipes; and to provide a mathematical model which is an equation justifying the increase of leakage exponent related to the increase of longitudinal crack area due to pressure increase. To achieve this, rectangular thin and flat plates sections, made of steel with an artificial straight crack were studied through theoretical and experimental methods. A better understanding of the behaviour of this stress concentration around the longitudinal crack may help to explain how an increase in leakage exponent is related to the increase of the leak area within a pressurised pipe material.

2 MECHANICAL EXPLANATION OF THE MATERIAL BEHAVIOUR

2.1 *Pressurised water pipe with a longitudinal crack*

There are two different stresses that occur within a pressurised pipe or cylinder due to internal water pressure, namely longitudinal and circumferential stresses. The governing equations for the stress state in pipe walls show that the normal circumferential (or hoop) stresses are twice the size of the longitudinal stresses. This theory is only valid if the cylinder is straight with end caps and its walls away from any discontinuities that cause stress concentrations.

The stress distribution in a pipe material is affected by any discontinuity, such as a hole or crack, in the material. Areas of significantly increased stress, or stress concentration, occur at certain points adjacent to the discontinuity. The structural behaviour of a longitudinal crack in a pressurised pipe is known to be fairly affected more by the large hoop stresses in comparison to the longitudinal stress which can be eliminated as the cylinder is straight with no end caps. There is an appearance of stress concentration at the tips of the longitudinal crack, resulting in the propagation of the crack in longitudinal direction. This causes the pipe to fail catastrophically when the crack tip stresses exceed the critical level generated by the critical pressure as defined in equation 3 (Buckley, 2006). Because of the curvature of a pipe, a longitudinal crack bulges out under increasing pressure and this deformation remains when the material expansion exceeds its yield strength (See Figure 1).

$$h_{crit} = \frac{1}{\sqrt{\pi a_{crit}}}\left(\frac{K_{IC}t}{Y\rho g r}\right) \tag{3}$$

where h_{crit} is the critical pressure head, a_{crit} the critical crack length, K_{IC} the fracture toughness (material property), t pipe wall thickness, r the inner radius of the pipe, ρ the fluid density, g the acceleration due to gravity and Y the geometry factor.

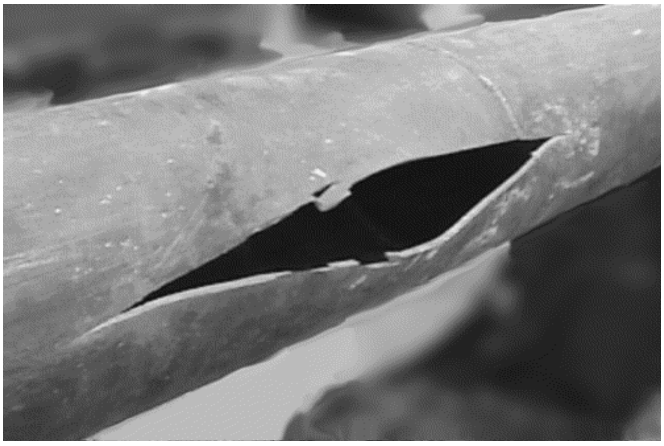

Figure 1. Expansion of a longitudinal crack on a pipe material due to internal pressure.

2.2 *Behaviour of a central crack in a rectangular plate under tension*

Consider a thin rectangular plate of length L, width W, thickness t, and having a central crack of length $2a$ as shown in Figure 2. The plate is assumed to be elastic, isotropic, homogenous and simply supported along all four edges. When a uniform tensile load N_y is applied transversely to the crack, stress concentrations occur at the crack tip. As the load increases, the plate stretches in length (in Y direction), and contracts laterally (in X direction) due to Poisson's ratio effect, causing the crack to open up until local buckling occurs around the crack. Thus the plate is said to have buckled, when the applied loading exceeds the elastic buckling load value. The critical stress corresponds to the maximum opening area of the crack and the maximum deflection. Any further increase in load causes the crack to grow in length until complete failure of the plate occurs. According to Griffith's criterion, for a particular plate material under consideration, crack propagation occurs when the ratio in the equation 4 exceeds the critical strain energy release rate G_c (Griffith, 1921) as follows:

$$\frac{\pi \sigma_{crit}^{2} a_{crit}}{E} > G_c \tag{4}$$

where σ_{crit} is the critical stress, a_{crit} the critical crack length and E the Young's modulus.

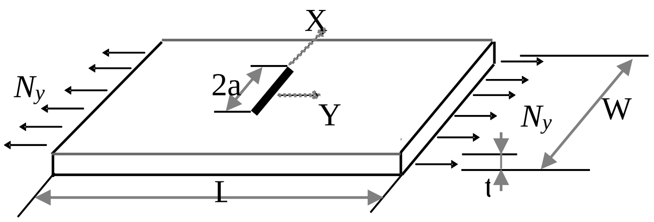

Figure 2. Geometry of the cracked plate under tensile stress.

2.3 *Experimental Verification of Rectangular Cracked Plate in Uniaxial Tensile Stress State*

Experimental tests were conducted on cold-rolled steel plate samples designed such that the plate to be pulled must absorb the energy necessary to produce strain in the region around the crack. The mechanical properties of these plate materials were as follows: (i) Yield strength: 175 MPa, (ii) Ultimate tensile strength: 302 MPa, (iii) Young's modulus of elasticity: 202 MPa. The tensile testing machine pulled gradually the plate sample and was measuring how much load was required to pull the plate apart and how much the plate stretches until such time as failure occurs. The load per unit of area that is required to initiate the fracture of the crack by extending the length of the crack in the plate material was defined as the critical stress during the test. This experimental test was performed at University of Johannesburg in Materials Laboratory. Figure 3 shows the buckling behaviour of the rectangular cracked plate.

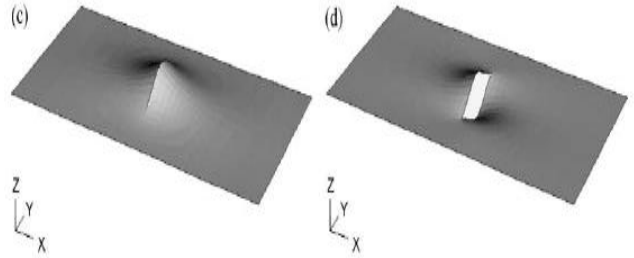

Figure 3. Behaviour of a rectangular cracked plate under uniaxial tensile stress in three-dimension.

Nine plate samples were tested to verify the behaviour of the material around the crack. The samples prepared were rectangular plates of 400mm of length and 200mm of width. Three various crack lengths were induced for this experimental set up which are: 50mm, 75mm and 100mm. As for plate thicknesses, these were selected according to the literature review relating to buckling of thin cracked plates as follows: 1.0 mm, 1.6 mm and 2.0 mm.

3 ANALYTICAL APPROACH

The analytical approach in this paper consists of deriving an equation for the longitudinal crack opening area as a function of the tensile load, plate thickness and plate material properties. If it is assumed that the plate material is elastic, thus due to stress-strain state, the straight crack in the plate will expand in size under a tensile load, resulting in a rhombus shape at the critical buckling stress of the plate material (Figure 3).

3.1 *Derivation of the Equation for longitudinal crack opening area in uniaxial stress state in elastic material*

As introduced previously, the derivation of the equation of longitudinal crack opening area started from an investigation on a flat cracked plate under uniaxial tension, the results obtained are therefore applied to a pressurised pipe. Consider the fact that hoop stress acts in pressurised pipe wall like tensile stress on plate wall. In mechanics, Hooke's law of elasticity is an approximation that states that the amount by which a structure material is deformed (strain) is linearly related to the load causing the deformation (stress).

$$\sigma = \varepsilon E \tag{5}$$

Strain and fracture of longitudinal crack in pressurised pipes are governed by the hoop stress in the absence of other external loads since it is the largest principal stress. Strain is the function of change of length over original length. Considering the original crack opening, in y direction as u_y; the change of crack opening can be calculated assuming Hooke's law for an elastic material. Due to stress-strain state in y direction, at the critical buckling stress, the longitudinal crack alters in size and its opening area is deformed to rhombus shape, (see Figure 3). The resulting equation will be function of membrane load, thickness of the plate and mechanical properties of the plate material.

Assuming there is no change in crack length, the length of the larger diagonal of the rhombus can be taken as the length of the crack $2a$ .The smaller diagonal will represent the crack faces opening u_y at the centre of the crack, thus the change in crack faces opening is calculated adding the original crack faces opening with the change of crack faces opening due to increase of the membrane load in y direction.

This change results in an increase of a crack opening area, which is determined by the area of a rhombus equal to:

$$A = a.u_y \tag{6}$$

The relative displacement between crack faces opening u_y at any position x away from the crack tip under plane stress condition is given by Wells as follows (Wells, 1961):

$$u_y = \frac{4K}{E}\sqrt{\frac{a^2 - x^2}{\pi a}} \tag{7}$$

$$K = Y\sigma\sqrt{\pi a} \tag{8}$$

where; x is the variable taking into account the crack faces displacement in x-axis, (x varies from 0 to $+a$), a half length of the crack, E Young's modulus of the plate material and K stress intensity factor, K. Therefore, combining the change in crack faces opening of the central crack with the critical buckling stress in uniaxial state, the crack opening area in the plate wall was derived and obtained as follows:

$$A = u_y a\left(1 + \frac{N_y}{tE}\right) \tag{9}$$

Subsequently, equation 9 can be applied on a pressurised pipe presenting a longitudinal crack, to calculate the opening area caused by hoop stress in the pipe wall induced by internal pressure as shown on Figure 1. The resulting equation of the longitudinal crack opening area can be derived as a function of internal pressure, pipe geometry, fluid properties as well as pipe material properties. Pressure P is given by ρgh with ρ being the fluid density, g the gravitational acceleration and h pressure head of the fluid in meters.

The actual orifice area A_{act} (Equ. 10) of the longitudinal crack due to the deformation of the material around the crack in pressurised pipe is obtained by substituting N_y by $\Pr$ in equation 9.

$$A_{act} = u_y a\left(1 + \frac{v\rho ghr}{tE}\right) \tag{10}$$

3.2 *Derivation of the Equation for Leakage flow rate through a longitudinal crack in pressurised pipe (Ilunga's Longitudinal Crack Equation).*

The leakage flow rate behaviour can be predicted by substituting equation 10 into equation 2, to obtain a proper flow rate through the crack incorporating the mechanical properties of the pipe material in uniaxial stress conditions, the stress intensity factor K and the shell curvature parameter of the pipe λ .The shell curvature factor is due to the bulging strain of the crack . Taking into account of the exponential trend increase, Equation 2 can be written in the form of Equ. 11. Equation 11, is the mathematical derived equation showing the increase of the head pressure and expansion of the pipe material around the crack.

$$Q_{actual\max} = C_d\left[u_{y\max}.a_{crit}\left(h_{crit}^{\frac{1}{2}} + \frac{v\lambda K\rho gRh_{crit}^{\frac{3}{2}}}{tE} + \frac{v^2\lambda^2K^2\rho^2g^2R^2h^{\frac{5}{2}}{}_{crit}}{t^2E^2}\right)\right](2g)^{\frac{1}{2}} \tag{11}$$

Where: $Q_{act\max}$ is the maximum actual leakage flow rate, $u_{y\max}$ the maximum crack faces opening, C_d the discharge coefficient, a_{Crit} the critical crack length, h_{Crit} the critical pressure head of the fluid, K the stress intensity factor, ρ the fluid density, g the gravitational acceleration, R the internal radius of the pipe material, t the thickness of the pipe wall, λ the shell curvature factor due to bulging strain of the crack, v the Poisson's ratio of the pipe material and E the Young's modulus of the pipe material.

3.3 *Predicting of the Leakage Exponent through a Longitudinal Crack due to Changes in Pressure*

The graph in Figure 4 is obtained by applying Equation 11 to experimental data prepared by Buckley (2006). This graph shows that there is a significant increase in leakage flow rate through a longitudinal crack-area which expands as a result of the increase in pressure. Equation 11 confirms that the expansion of the crack area influences the increase of the leakage exponent α from 0.5 up to 0.903; and explains the linear relationship between the pressure and the leakage flow rate for low pressures then exponential behaviour for high pressures. This equation proves that the leakage exponent increase in a pipe material is directly proportional to the radius of the pipe material, the density of the fluid flowing inside the pipe, and inversely proportional to the wall thickness and Young's modulus.

The linear relationship in Figure 4 demonstrates the linear elastic behaviour of the pipe material in the elastic zone. The exponential relationship results in the fact that the material expands until it passes its yield point and moves into the plastic zone, where deformations become permanent.

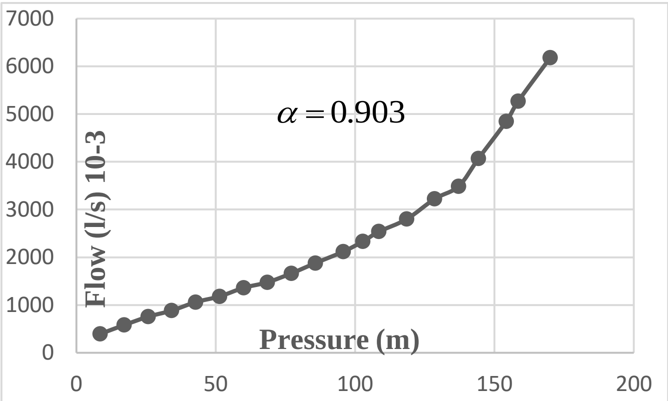

Figure 4. Behaviour of a leakage exponent through a 40mm longitudinal crack in uPVC class 6 Pipe as function of pressure head and crack expansion.

4 CONCLUSION

Material around the crack exhibits elastic expansion behaviour under pressure increase taking up by hoop stress inside the pipe. The stress resulting from hoop stress generates strain in pipe wall which opens up the crack in the pipe wall, due to occurrence of high stress concentration that arise around crack tips. When hoop stress exceeds the material's yield strength, the material around the crack bulges out due to plastic strain. For a given pressurised pipe and wall thickness to diameter ratio, the effect of the opening is more important for the longer the crack is. Therefore, there is an increase in leakage area due to the expansion of the crack as the pressure increases in the pipe wall. Subsequently, leakage exponents increase significantly, exceeding the theoretical value of 0.5.

The major result obtained from this investigation was the derived theoretical equation for flow leakage increase through a longitudinal crack, in a uniaxial stress state due to the expansion of the crack. The derived equation is a function of material properties, pipe geometry and fluid properties; it can be used as a mathematical model to predict flow leakage through longitudinal cracks in pressurised water pipes.

The derived mathematical model for the expansion of longitudinal cracks (Equation 11) compares well with previous experimental and finite element models of the same pipe materials. The derived model (Ilunga's longitudinal crack Equation) contradicts the orifice equation (Equation 2) which assumed that the orifice area is fixed. Experimental results have proven that the crack area can expand with increasing pressure and subsequently the leakage exponent through the crack area increases as well. The results from this investigation have confirmed a leakage exponent increase α from 0.5 up to 0.9 (Ilunga, 2016).

It is recommended that when one is designing a pipe material such cracks should be avoided because they may reduce the strength of the pipe structure or could lead to a catastrophic failure of the pipe material. Consequently, when manufacturing cylindrical pressure vessels from rolled-formed plates, the longitudinal joints must be designed to carry twice as much stress as the circumferential joints. It is important to note that the "buckling" mode of the crack in the plate is similar in form and nature to the "bulging" mode of a longitudinal crack in thin walled-pressurised vessels. The leakage flow behaviour through a longitudinal crack varies linearly for low pressure then exponentially for high pressures.

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