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Credit-based Distributed Real-time Energy Storage Sharing Management

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ABSTRACT In this paper, energy storage sharing among a group of cooperative households with integrated renewable generations in a grid-connected microgrid is studied. In such a microgrid, a group of households, who are willing to cooperatively operate a shared energy storage via a central coordinator, aims to minimize their long term time-averaged costs, by jointly taking into account the operational constraints of the shared energy storage, the stochastic solar power generations and the time-varying load demands from all households, as well as the fluctuating electricity prices. This energy management problem, which comprises storage management and load control, is first formulated as a constrained stochastic programming problem. Based on the Lyapunov optimization theory, a distributed real-time sharing control algorithm is proposed to solve the constrained stochastic programming problem without requiring any statistical knowledge of the stochastic renewable energy generations and the uncertain power loads. The credit-based distributed sharing algorithm, in which each household independently solves a simple convex optimization problem without requiring any statistics of the system, is designed to quickly adapt to the system dynamics while facilitating a fair allocation of the shared energy storage with respect to individual households' energy contributions. The performance gap of the proposed low-complexity distributed sharing algorithm is evaluated via theoretical analysis. Numerical simulations using a practical system setup are conducted to investigate the effectiveness of the proposed sharing control algorithm in terms of energy cost saving and fairness. The simulation results show that the proposed credit-based distributed sharing algorithm can not only save power consumption cost by coordinating the use the shared battery among households in a fair manner but also improve the utilization of renewable energy generation.

INDEX TERMS Energy Management, Lyapunov Optimization, Energy Storage Sharing, Smart Grid.

I. INTRODUCTION

THE fast-growing electric consumption and the concern of the carbon dioxide emission of traditional fossil fuel based power plants motivate a green power system with users deploying distributed renewable energy generators, e.g., wind turbines and solar photovoltaics (PVs). In South Africa, a strong solar radiation resource makes solar energy a particularly attractive option, where annual solar radiation ranges from 2400 to 2800kWh/m^2 [1]. Associated with global solar PV price reductions and spurred on by high annual grid power price increases, solar PV generator installation in South Africa has also been accelerating. However, the inherently intermittent and stochastic nature of renewable energy production owing to weather variability poses significant challenges to efficient utilization of renewable energy.

Increasing dynamics in power systems due to renewable integration and electricity demands have resulted in the exploration of energy storage systems (ESSs) for potential solutions [2]. From the perspective of power grid operation, the benefits of ESSs including generation backup, transmission support and voltage control have been well-recognized [3]. On the other hand, from the user's perspective, ESSs can be integrated with distributed renewable generations as a more practical solution to improve the energy efficiency and reliability by storing not only surplus energy generated from renewable resources but also cheaper energy at times of lower electricity prices for later use at times of renewable energy generation shortages and/or higher electricity prices. In this work, we mainly focus on the interplay between energy supply and energy storage at the user side.

There have been many studies on energy management in the context of renewable integration and energy storage from the perspective of residential energy storage management. Most of the previous studies assume that an ESS is owned by a user or one entity and analyze the energy storage management problem considering a single user single ESS scenario or a distributed ESSs scenario [4]–[10]. For instance, in recent studies [4], [5], the authors investigated single user optimal charging and discharging policies that balance cost savings with user convenience, such as activity delay, by exploiting the price variations without having to shift user demand to the low-price periods. Meanwhile, energy trading and exchanging in a grid-connected microgrid with distributed renewable generators (RGs) and ESSs owned by individual users, has gained lots of research attention recently [6]–[8]. For instance, the authors in [8] presented a multi-objective optimization method for a grid-connected community microgrid to reduce the electricity costs of all users, by sharing all renewable energy and storage resources in the system through local power exchange. Energy storage management also has been studied extensively in the literature for supply-demand balancing to counter the fluctuation of renewable generation for small-scale microgrids composed of multiple RGs and ESSs. The authors in [9], [10] proposed multiagent system-based control strategies to coordinate all components, i.e., distributed RGs, ESSs and loads, in a small-scale islanded microgrid to maintain the supply-demand balance within the system while maximizing the social welfare of all the participants [9] or minimizing the total power loss associated with charging/discharging inefficiency [10].

In recent years, the concept of ESS sharing, where the surplus energy of some users can be charged into a shared (common) ESS and then be discharged by other users with renewable energy deficit, has received increasing attention [11]–[22]. An ESS shared across a group of users, which can be energy consumers in an industrial park or neighboring households in a community, can benefit users not only through sharing the installation and maintenance costs of the ESS but also by exploiting the non-overlapping power consumption patterns of users. Nevertheless, along with the benefits, ESS sharing also brings increasing challenges in energy storage management and load management.

In this paper, we consider a microgrid of a group of households with their individual renewable energy generators, who are willing to operate a shared ESS (in the form of a battery) in a cooperative manner, aiming to minimize their long term time-averaged costs. In this sharing system, the main challenge of energy management is how to dynamically coordinate the households to optimally utilize the shared ESS, i.e., how to optimally charge/discharge energy to/from the shared ESS, so as to minimize their individual energy consumption costs while satisfying their individual preferences. This energy management problem can be viewed as an energy storage management problem combined with a demand side management problem. Due to the finite capacity, the ESS introduces correlation across time. Specifically, current charging/discharging action impacted by the previous charging/discharging action will impact the future charging/discharging. Given the inherent time-coupling feature of the ESS, the uncertainties in the multiple renewable generations and power demands from different households, as well as the electricity price variations, dynamically coordinating energy storage among a group of households is challenging when integrating energy storage management with demand side management.

Different mechanisms and approaches have been proposed to provide cost saving through a shared ESS. The authors in [11] discuss an energy storage managing method in a distribution network based on evenly dividing energy storage between customers and system operator, but does not optimize the division of energy storage. In [15], the authors proposed a reputation-based centralized energy management system (EMS) to jointly schedule households' appliance power consumption and allocate the available energy in the shared battery by considering households' reputations, which depend on the amount of renewable power they have shared. The proposed EMS runs a day-ahead optimization problem which is formulated as a Mixed Integer Linear Programming. In [16], the authors addressed the cost saving tradeoff problem of sharing an ESS among multiple users using a Markovian model and proposed a centralized control policy to dynamically allocate battery capacity among users. The policies for energy storage sharing using a predetermined time-of-use pricing scheme are studied in [17], in which, with a finite horizon formulation, an optimal centralized policy is proposed. In [18], a game theoretic approach is presented with a distributed algorithm to determine each user's energy production and storage a day-ahead. In [19], the authors studied a scenario where an aggregator owns a central storage unit and virtualizes the physical storage into separable virtual storage capacities that can be sold to users to store the energy purchased from the main grid, and modeled the interaction between the aggregator and users in each time slot as a two-stage problem. Assuming that users can perfectly predict their renewable generations and loads, a pricing-based virtual storage sharing scheme among a group of users was proposed and the solutions were characterized based on parametric linear programming under a day-ahead prediction on users renewable generations and loads. Similarly, using the same storage virtualization concept, the authors in [20] presented a shared ESS service architecture consisting of a virtually assigned ESS to each participating user and proposed a long-term service strategy, focusing on the selection of the shared ESS size and the service price, which is determined by the interactive decisions of the ESS service provider and users using Lagrangian relaxation. A centralized approach was introduced in [21], which consists of day-ahead ESS scheduling and real-time pricing for energy sharing among multiple PV prosumers with the assistance of an Energy Sharing Provider (ESP) equipped with an ESS. The ESP first decides the day-ahead scheduling strategy of the shared

ESS via stochastic programming based on day-ahead PV generation and load demand forecasts of individual prosumers. Then real-time prices are set via a Stackelberg game-based model to coordinate the energy consumption of the prosumers in each time slot. The authors in [22] developed a bi-level Stackelberg game-based discriminatory auction model to allocate and price energy for sharing a distributed energy resource (DER) consisting of a RG and an ESS within an apartment building. The bi-level auction based allocation model can be implemented in a distributed way where the DER owner determines the prices for DER energy based on the consumers demand curves, which are assumed to be known to the DER owner, while the consumers decide their consumption accordingly.

Most of these existing studies on ESS sharing assume that the renewable energy generations and loads of users are known ahead of time to a central agent, who optimizes the charging/discharging energy of each user, or assume perfect forecasting of the renewable generations, load demands and energy prices, which is practically unachievable. In practice, with unpredictable changes in renewable energy generations and demands, adaptive response to the dynamics of the system by utilizing the shared ESSs is an important requirement in the time-varying environment of the ESS sharing system.

Due to the time coupling constraints brought by the shared ESS, the optimization problem for energy management in this ESS sharing system turns out to be a time-coupling problem. Previously, such problems are usually solved using approaches based on Dynamic Programming [23], which not only require full statistical information of the random variables in the problem but also suffer from the "curse of dimensionality" problem [24].

Recently, the Lyapunov optimization theory [25], an effective method to deal with stochastic optimization and stability problems, has been widely adopted in the literature, such as [26]–[30], to develop online algorithms that require no a priori statistical knowledge of the underlying stochastic processes for real-time energy management in microgrids with renewable energy resources combined with ESSs. Using the Lyapunov optimization theory for event-driven queueing systems, an optimization problem with time-coupling constraints can be reformulated to a relaxed problem, which can be solved in each time slot based on the current system state and provides a suboptimal solution for the optimization problem. No information about the future or past system states is required. For instance, in [26], the optimal cost saving policies using the Lyapunove theory for a single storage system have been studied and a real time control algorithm was proposed to minimize one user's long term expected energy cost considering a renewable energy resource and a battery. The authors in [28] proposed a real-time distributed algorithm based on the Lyapunov optimization theory to coordinate a group of distributed storage units to provide power balancing service to a power grid through charging or discharging. The proposed algorithm accommodates a wide spectrum of vital system characteristics, including time-varying power imbalance amount and electricity price, finite battery size constraints, cost of using external energy sources, and battery degradation. However, most of the existing studies primarily consider a single user single ESS scenario or a distributed ESSs scenario.

In this paper, we focus on developing a real-time control algorithm for a battery sharing system. The main contributions of this paper are summarized as follows:

1) A real-time sharing energy management strategy based on the Lyapunov optimization theory is designed for storage control and load management for multiple households, requiring no statistical knowledge of the stochastic renewable energy generations and the uncertain power loads, so as to minimize the long term power consumption costs of all households.
2) A credit-based distributed implementation of the Lyapunov-based real-time sharing control strategy is proposed to coordinate the group of households in a distributed and fair manner, in which each household's energy management optimization problem is solved locally with almost all information obtained locally. Based only on the current system state, the proposed credit-based sharing control algorithm jointly optimizes energy charging/discharging and load management for all households while taking into account the energy contributions of individual households to the shared battery, and imposes load shedding and renewable energy curtailment if necessary.

The rest of the paper is structured as follows. The system model of a microgrid with a group of households sharing a common ESS is described in Section II. In Section III, we formulate the optimization problem for energy management in this sharing system. In Section IV, a Lyapunov-based real-time sharing control strategy is proposed to solve the optimization problem and its performance is analyzed. A credit-based algorithm is then proposed to implement the Lyapunov-based sharing control strategy in a distributed and fair manner. Numerical results obtained through simulation evaluations are presented in Section V. Finally, concluding remarks are provided in Section VI.

II. SYSTEM MODEL

We consider a smart community that consists of an energy storage sharing management (ESSM) system for a group of households $\mathcal{I} = \{1, \ldots, I\}$ whose load profiles are different and each of whom has an on-site RG. The ESSM system contains an energy storage battery with a finite capacity shared among all households who can charge energy harvested from their RGs as well as purchased from the main grid (MG) into the battery. The households' load demands can be supplied by their individual RGs, the shared battery and the MG. In this sharing system, the households cooperatively operate the shared battery via a central coordinator, who manages the shared battery to make sure its operational constraints are satisfied, so as to jointly minimize their electricity consumption costs by utilizing their renewable energy together with the MG combined with the finite-capacity energy storage. We assume that the ESSM system operates in slotted time t, i.e., $t \in \{0, 1, \ldots, T-1\}$.

1) Renewable generator
We assume that each household has a solar PV generator with different capacity and the amount of harvested energy in a time slot varies over time. Let $g_{pv,i}(t)$ denote the energy harvested from household i's solar PV generator in time slot t. Since $g_{pv,i}(t)$ is random due to the randomness of the solar source, we assume no prior knowledge of $g_{pv,i}(t)$ or its statistics.

2) Main grid power
Each household can purchase energy from the MG in time slot t at the unit price $p(t)$, which is time-varying and only known in time slot t. Let $g_{l,i}(t)$ denote the amount of energy purchased from the MG by household i in time slot t that directly supplies the household i's load, and $g_{s,i}(t)$ denote the amount of energy purchased from the MG by household i in time slot t that is stored into the shared battery to take advantage of time-varying electricity prices. Then the energy cost of household i in time slot t is

$$C_{MG,i}(t) = [g_{l,i}(t) + g_{s,i}(t)]p(t) \qquad \forall i \in \mathcal{I}. \quad (1)$$

3) Local power demand
Let $D_i(t)$ be the household i's load in time slot t. We assume a priority of using energy harvested from its solar PV generator $g_{pv,i}(t)$ to directly supply $D_i(t)$ and the excessive portion, if any, will be charged into the shared battery. When $D(t) \le g_{pv}(t)$, we denote the energy that household i charges into the shared battery in time slot t by

$$g_{ch,i}(t) \le g_{pv,i}(t) - D_i(t) \quad \forall i \in \mathcal{I}. \quad (2)$$

Note that, since the storage space of the shared battery is limited, not all the excessive portion can be charged into the battery if there is not enough storage space in the shared battery.
When $D_i(t) > g_{pv,i}(t)$, the residual $D_i(t) - g_{pv,i}(t)$ can be served with the power purchased from the MG $g_{l,i}(t)$ or the power drawn from the shared battery $g_{dis,i}(t)$. A balance between purchasing the power from the MG and drawing the power from the battery must be struck under the following feasibility condition:

$$g_{l,i}(t) + g_{dis,i}(t) = D_i(t) - g_{pv,i}(t) \quad \forall i \in \mathcal{I}. \quad (3)$$

The loads of each household can be classified into two categories:

- inelastic loads (in unit of kWh) represent the critical demands such as refrigerator and lights, which should not be shed or shifted over time.
- elastic loads (in unit of kWh) represent the controllable energy requests such as electric vehicles, air conditioners and other smart appliances, which can be flexibly curtailed or scheduled over time to minimize costs.

We consider a demand side management in the microgrid, where flexible loads can be shed in response to supply conditions. For each household, its demand is bounded by:

$$\overline{D}_i(t) \le D_i(t) \le \underline{D}_i(t) \qquad \forall i \in \mathcal{I} \quad (4)$$

where $\overline{D}_i(t)$ is the maximum power demanded by household i in time slot t, i.e., the most preferred power consumption of household i, and $\underline{D}_i(t)$ is the minimum power demanded by household i in time slot t that cannot be shed or rescheduled, i.e., the inelastic loads. Note that $\overline{D}_i(t)$ and $\underline{D}_i(t)$ are the demand requests decided by households based on the physical constraints and their willingness to shed their elastic loads. If a household refuses load shedding, the requested maximum and minimum power will be the same. The demand requests of each household in each time slot are assumed to be stochastic.
However, load shedding used for cost saving may cause discomfort to the households. When the shed power consumption $D_i(t)$ deviates from the preferred power consumption $\overline{D}_i(t)$, discomfort experienced by household i can be represented by a discomfort cost function

$$C_{COM,i}(t) = \alpha_i(t)[\overline{D}_i(t) - D_i(t))]^2 \qquad \forall i \in \mathcal{I}, \quad (5)$$

where weighted coefficient $\alpha_i(t)$ is a positive constant used to indicate the sensitivity of household i towards the power consumption deviation $\overline{D}_i(t) - D_i(t)$ in time slot i: the higher the value of $\alpha_i(t)$, the more sensitive the household i towards the power consumption deviation.
Meanwhile, in order to control the quality-of-service (QoS) [29] for each household, an upper bound is imposed on the portion of the unsatisfied flexible loads, which can be formally expressed by [31]

$$\lim_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T-1} \left[\frac{\overline{D}_i(t) - D_i(t)}{\overline{D}_i(t) - \underline{D}_i(t)} \right] \le \beta_i \qquad \forall i \in \mathcal{I}, \quad (6)$$

where $\overline{D}_i(t) - D_i(t)$ is the shed demand, $\overline{D}_i(t) - \underline{D}_i(t)$ is the total demand that can be shed in time slot t, and $\beta_i \in (0, 1]$ is a pre-designed threshold used to control the QoS. It reflects the tolerance of household i to the power consumption deviation. A smaller β_i indicates a tighter QoS control. Note that both $\alpha_i(t)$ and β_i are decided by household i based on its energy consumption preference and $\alpha_i(t)$ could vary over time in a stochastic manner.

4) Shared Energy Storage
The shared energy storage battery has a finite storage capacity S_{cap}. In practice, batteries are not ideal. There are energy conversion losses during the charging and

discharging processes. Let $\eta_{ch} \in (0,1]$ and $\eta_{dis} \in [1,\infty)$ be the charging efficiency coefficient and the discharging efficiency coefficient, respectively.

a) Denote $s(t)$ as the energy state of the battery at the beginning of time slot t. Then the energy state $s(t)$, known as state of charge (SOC), in kWh, fluctuates over time and evolves as follows:

$$\begin{aligned} s(t) = s(t-1) + \eta_{ch} \sum_{i\in\mathcal{I}} [g_{ch,i}(t) + g_{s,i}(t)] \\ - \eta_{dis} \sum_{i\in\mathcal{I}} g_{dis,i}(t) \triangleq s(t-1) + \sum_{i\in\mathcal{I}} b_i(t) \end{aligned} \tag{7}$$

where $b_i(t)$ is defined as the effective charging and discharging amounts in time slot t.

b) Because of limitation imposed by the charging and discharging circuits, the amounts of power charged into and discharged from the battery are upper bounded. Denote the maximum charging rate and discharging rate of the battery by R_{ch} and R_{dis}, respectively, so that

$$\begin{aligned} 0 \le \sum_{i\in\mathcal{I}} [g_{ch,i}(t) + g_{s,i}(t)] \le R_{ch} \\ 0 \le \sum_{i\in\mathcal{I}} g_{dis,i}(t) \le R_{dis} \end{aligned} \tag{8}$$

c) Charging a battery near its capacity or discharging it close to the zero energy state can significantly reduce battery lifetime [32]. Hence, lower and upper bounds on the battery energy state are usually imposed by its manufacturer or owner. Denote $[S_{min}, S_{max}]$ as the preferred energy range with $0 < S_{min} < S_{max} < S_{cap}$. Then the level of the shared battery in time slot i is bounded

$$S_{min} \le s(t) \le S_{max}. \tag{9}$$

d) Combining (7), (8) and (9), the boundaries of charging and discharging power in time slot t can be compactly represented as

$$\begin{aligned} 0 \le \sum_{i\in\mathcal{I}} [g_{ch,i}(t) + g_{s,i}(t)] \le \min\{R_{ch}, \frac{S_{max} - s(t-1)}{\eta_{ch}}\} \\ 0 \le \sum_{i\in\mathcal{I}} g_{dis,i}(t) \le \min\{R_{dis}, \frac{s(t-1) - S_{min}}{\eta_{dis}}\} \end{aligned} \tag{10}$$

The space-availability constraint and the energy-availability constraint in (10) must be satisfied at all time for the charging and discharging decisions to be feasible. In other words, the energy charged/discharged into/from the shared battery must not exceed the storage space/energy available for charging/discharging.

III. PROBLEM STATEMENT AND FORMULATION

A. PROBLEM STATEMENT

Solar power generations of multiple households bring more uncertainties to the energy management problem, making it challenging to balance supply and demand in real-time. In this paper, we study the problem of real-time energy storage and management in this microgrid aiming at achieving the long-term energy consumption objectives of the households while ensuring an acceptable level of the discomfort experienced by each household in real-time, taking into consideration the dynamics of the energy demands, renewable sources and energy prices as well as the operational constraints of the shared battery. Hence, the objective of the ESSM system is to jointly determine power consumption, power purchasing and energy charging/discharging actions of all households so as to minimize the long-term time-averaged costs of all households, subject to the operational constraints of the shared battery as well as time-varying solar power generations and load demands from households and electricity prices. Therefore, the control problem can be stated as follows: given the current random renewable supplies, the battery energy levels, the energy demand preferences of households and the electricity prices, design a control strategy that chooses the energy purchasing vectors, the battery charging and discharging vectors, as well as load serving vectors for all households such that the long-term time-averaged energy consumption costs of all households are minimized.

For the sake of clarity and ease of reading, we define the system state $\mathbf{X}(t)$ in time slot t using the renewable generations and demand preferences of households, the energy prices from the MG and the energy state of the shared battery

$$\mathbf{X}(t) \triangleq [\mathbf{g_{pv}}(t), \mathbf{d}(t), p(t), s(t)], \tag{11}$$

where $\mathbf{d}(t) \triangleq [\overline{D}_i(t), \underline{D}_i(t)]\ \forall i$ is the demand preference vector and $\mathbf{g_{pv}}(t) \triangleq (g_{pv,i}(t))\ \forall i$ is the renewable generation vector. We assume that $\mathbf{X}(t)$ is stochastic.

The control vector in time slot t is defined by

$$\mathbf{Y}(t) \triangleq [\mathbf{g_l}(t), \mathbf{g_s}(t), \mathbf{g_{ch}}(t), \mathbf{g_{dis}}(t), \mathbf{D}(t)]. \tag{12}$$

where $\mathbf{g_l}(t) \triangleq (g_{l,i}(t))\ \forall i$ and $\mathbf{g_s}(t) \triangleq (g_{s,i}(t))\ \forall i$ are the energy purchasing vectors for load serving and battery charging respectively, $\mathbf{g_{ch}}(t) \triangleq (g_{ch,i}(t))\ \forall i$ and $\mathbf{g_{dis}}(t) \triangleq (g_{dis,i}(t))\ \forall i$ are the battery charging and discharging vectors, respectively, and $\mathbf{D}(t) \triangleq (D_i(t))\ \forall i$ is the serving load vector. Note that, even though some of the components in $\mathbf{X}(t)$ can be arbitrarily correlated, the control decision in each time slot only depends on current system state $\mathbf{X}(t)$ without any previous system state information.

With the known information, i.e., the current system state $\mathbf{X}(t)$, the objective of the ESSM system is to make control decision to choose $\mathbf{Y}(t)$ in reaction to the current system state $\mathbf{X}(t)$ in each time slot in order to minimize the households' energy consumption costs, comprising the discomfort costs of load shedding and the costs of energy purchased from the MG, over a long-term T-slot period, while guaranteeing the QoS for each household, by jointly managing energy consumption, supply and storage. We define the instantaneous cost of all households by

$$C_{ToT}(t) = \sum_{i\in\mathcal{I}} C_{MG,i}(t) + \sum_{i\in\mathcal{I}} C_{COM,i}(t) = \sum_{i\in\mathcal{I}} [g_{l,i}(t) + g_{s,i}(t)]p(t) + \sum_{i\in\mathcal{I}} \alpha_i(t)[\overline{D_i}(t) - D_i(t)]^2 \tag{13}$$

Thus, the stochastic control optimization problem of the real-time energy management, called **P1**, can be formulated by

$$\mathbf{P1}: \quad \min_{\mathbf{Y}(t)} \quad \lim_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1} \mathbb{E}\left\{C_{ToT}(t)\right\}, \quad \text{s.t.} \quad (2)(3)(4)(6)(8)(9), \tag{14}$$

where $\mathbb{E}\{\cdot\}$ is taken with respect to $\mathbf{X}(t)$. Taking the randomness of the system state $\mathbf{X}(t)$ and the random control decision $\mathbf{Y}(t)$ in each time slot into account in the expectations of the objective function and constraints, **P1** seeks control decisions for the entire time horizon up till infinity taking the system dynamics into consideration. However, due to the time-coupling dynamics of (7), the current control action impacted by the previous control actions will impact the future control actions. Therefore, it is challenging to solve the stochastic optimization problem **P1** with the correlated control actions $\mathbf{Y}(t)$ over time.

The optimization problem **P1** can be solved using approaches based on Dynamic Programming [24], provided that the system statistics, e.g., the distributions of the components of $\mathbf{X}(t)$, are known, which might be practically infeasible. In this study, given the stochastic system state $\mathbf{X}(t)$, we are interested in real-time energy management that not only requires no system statistics but also can quickly adapt to the system dynamics. Motivated by the recent studies, a real-time algorithm is developed to determine real-time control vector $\mathbf{Y}(t)$ over time, applying the general framework of Lyapunov optimization [25] to reformulate the optimization problem **P1** to handle the time-coupling constraint (9).

B. PROBLEM MODIFICATION

Time-average constrains can be transformed into queue stability constrains and simple real-time algorithms can be provided for complex dynamic systems using the Lyapunov optimization theory. Unfortunately, the time-coupling dynamics of $s(t)$ over time in (7) and the battery capacity constraint in (9), which require that no energy underflow and overflow happen for all time, impose a hard constraint on the charging and discharging decisions in each time slot. As a result, the charging and discharging decisions are correlated with each other over time. Therefore, **P1** cannot be directly solved using the standard Lyapunov optimization techniques.

To avoid such coupling, the hard constraint (9) in **P1** is relaxed to a softer constraint, which reflects the long-term time-averaged relationship among the charging and discharging decisions, given by

$$\lim_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1} \mathbb{E}\left\{\sum_{i\in\mathcal{I}} b_i(t)\right\} = 0. \tag{15}$$

The derivation of (15) is provided in Appendix A. Instead of bounding the energy state in each time slot in (9), the softer constraint in (15) requires that the mean rate of the effective charging and discharging amounts in the whole process is kept stable. Replacing the time-coupling constraint (9) with the time average queuing constraint (15), we relax **P1** to the following problem:

$$\mathbf{P2}: \quad \min_{\mathbf{Y}(t)} \quad \lim_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1} \mathbb{E}\left\{C_{ToT}(t)\right\}, \quad \text{s.t.} \quad (2)(3)(4)(6)(8)(15), \tag{16}$$

Through this relaxation transformation, the dependency of per time slot charging/discharging amount on the battery state $s(t)$ in constraint (9) is removed and the standard Lyapunov optimization techniques can be applied to obtain the optimal solution in a way that is independent of battery SOC level. This relaxation technique used to accommodate the type of time-coupling constraints such as (9) was first introduced in [33] for energy management in a data center equipped with an ideal battery, and then was widely adopted in the literature regarding energy storage management. However, with the relaxed constraint (15), the solution to **P2** may be infeasible to **P1**. Hence, in the next section, we will present a real-time control algorithm that can guarantee all constraints of **P1** are satisfied. We will show later in Section IV-C that the solution to **P2** obtained by the proposed real-time algorithm in fact also satisfies (9), so it is feasible for **P1**.

IV. LYAPUNOV-BASED REAL-TIME SHARING CONTROL ALGORITHM

In this section, we present a real-time control algorithm using the Lyapunov optimization techniques to solve **P2** and provide simple online solutions based on the current information of the system state.

A. VIRTUAL QUEUE DESIGN

According to the concept of virtual queues from the Lyapunov optimization theory [25], we first introduce virtual queues for the time-averaged inequality and equality constraints (6) and (15) in **P2** to transform them into queue stability constraints.

- Battery Queue: a virtual queue $K_b(t) = s(t) - \theta$ that accumulates the charging and discharging amounts, where θ is a perturbation parameter designed to ensure the constraint of the energy state in (9) is satisfied. The dynamic of $K_b(t)$ is given by

$$K_b(t+1) = K_b(t) + \sum_{i\in\mathcal{I}} b_i(t). \tag{17}$$

The intuition behind the battery queue $K_b(t)$ is to construct the decision making algorithm based on a quadratic Lyapunov function involving $K_b(t)$, then by keeping the quadratic Lyapunov function value small to push the value of $s(t)$ towards θ. Thus, it can be ensured that the battery queue always has enough energy by carefully choosing the value of θ. Note that $K_b(t)$ is a shifted version of the energy state $s(t)$ by a constant parameter θ and can be negative. We will show in Section IV-C the boundedness of $s(t)$ can be guaranteed through the design of the perturbation parameter θ and V_{max}.

- QoS-Aware Load Queue: a virtual queue $H_{l,i}(t)$ that is associated with the long-term constraint in (6). It evolves as follows:

$$H_{l,i}(t+1) = \max\{H_{l,i}(t) - \beta_i, 0\} + \frac{\overline{D}_i(t) - D_i(t)}{\overline{D}_i(t) - \underline{D}_i(t)} \tag{18}$$

Initialize $H_{l_i}(t)$ as $H_{l_i}(0) = 0$. With arrival rate being the shedding percentage and the departure rate being β_i in time slot t, the time averaged load shedding percentage must be less than or equal to β_i to ensure the queue $H_{l,i}(t)$ to be stable. Hence, maintaining the stability of $H_{l,i}(t)$ is equivalent to keeping the constraint (6) satisfied [25].

By introducing the virtual queues, the time-averaged constraints (6) and (15) are transformed into the mean rate stability constraints of virtual queues. Hence, an optimization problem which minimizes the cost $C_{ToT}(t)$ over time while ensuring that the mean rates of the two virtual queues are kept stable is feasible to **P2**.

Replacing the time-averaged constraints (6) and (15) with the mean rate stability constraints (17) and (18), we now relax **P2** to **P3**, which is suitable for the Lyapunov optimization framework, as follows:

$$\begin{aligned} \mathbf{P3}: \quad \min_{\mathbf{Y}(t)} \quad & \lim_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1} \mathbb{E}\{C_{ToT}(t)\}, \\ \text{s.t.} \quad & (2)(3)(4)(8)(17)(18). \end{aligned} \tag{19}$$

B. LYAPUNOV-BASED REAL-TIME SHARING CONTROL ALGORITHM DESIGN

In this section we apply the Lyapunov optimization techniques to solve **P3**. Define $\Theta(t) \triangleq (K_b(t), H_{l,i}(t), \forall i \in \mathcal{I})$ as the concatenated vector of the virtual queues. As a scalar measure of the virtual queues, a perturbed Lyapunov function is defined as $L(\Theta(t)) \triangleq \frac{1}{2}[K_b(t)^2 + \sum_{i\in\mathcal{I}} H_{l,i}(t)^2]$. The Lyapunov function $L(\Theta(t))$ is a scalar measure of queue stabilization. Intuitively, if $L(\Theta(t))$ is small then all queues are small; and if $L(\Theta(t))$ is large then at least one queue is large. Thus, by minimizing a drift in the Lyapunov function, i.e., by minimizing a difference in the Lyapunov function from one time slot to the next, the queues $K_b(t)$ and $H_{l,i}(t)$ can be stabilized. Define the conditional one-slot Lyapunov drift, which represents the expected change in the Lyapunov function from one time slot to the next, as $\Delta(t) \triangleq \mathbb{E}\{L(t+1) - L(t)|\Theta(t)\}$, where the expectation is taken over the random processes associated with the system, given the current queue states $K_b(t)$ and $H_{l,i}(t)$.

We now use the drift-plus-penalty minimization method introduced in the Lyapunov optimization theory [25] to solve **P3**. Firstly, adding the function of the expected cost at current time slot, i.e., the penalty function, to the conditional one-slot Lyapunov drift, we obtain the drift-plus-penalty term $\Delta(t) + V\,\mathbb{E}\{C_{ToT}(t)\}$, where V, a positive parameter, serves as a weight controlling the performance tradeoff between cost and (virtual) queueing delay, i.e., how much one cares about the cost compared with the queueing delay. Instead of minimizing the energy consumption cost objective in **P3**, in the Lyapunov optimization, the objective is to minimize the short term drift-plus-penalty function by controlling $\mathbf{Y}(t)$ in each time slot t.

The intuition behind this approach is as follows: Minimizing the Lyapunov drift term $\Delta(t)$ of the the drift-plus-penalty term alone pushes the queue-lengths of the virtual queues to lower values. The second term of the drift-plus-penalty term is a penalty term with the parameter V controlling the trade-off between minimizing the queue-length drift and minimizing the cost function. In other words, while **P3** is a problem of minimizing the time-averaged cost of energy consumption while maintaining the stability of the virtual energy queue and load queue, the drift-plus-penalty minimization method jointly considers the time-averaged constrains and the objective function through the introduction of the control parameter V. A larger value of V indicates a greater priority to minimizing the cost function at the cost of a greater size of the virtual queues and vice versa. Thus, by varying the parameter V, one can obtain a desired trade-off between the size of the queue backlogs and the cost of energy consumption. In our case, the maximum feasible V results in the minimized time-average cost of energy consumption.

Using the drift-plus-penalty minimization method, a control policy that solves problem **P3** is obtained by minimizing the drift-plus-penalty expression $\Delta(t) + V\,\mathbb{E}\{C_{ToT}(t)\}$. We examine the drift-plus-penalty term and obtain an upper bound on it in the following proposition.

Proposition 1. *In each time slot t, for all possible decisions and all possible values of $\Delta(t)$, the drift-plus-penalty term is upper bounded as follows:*

$$\begin{aligned} \Delta(t) + V\,\mathbb{E}\{C_{ToT}(t)\} \le B + K_b(t)\,\mathbb{E}\left\{\sum_{i\in\mathcal{I}} b_i(t)|\Theta(t)\right\} \\ + \sum_{i\in\mathcal{I}} H_{l,i}(t)\,\mathbb{E}\left\{\frac{\overline{D}_i(t) - D_i(t)}{\overline{D}_i(t) - \underline{D}_i(t)} - \beta_i|\Theta(t)\right\} \\ + V\,\mathbb{E}\left\{\sum_{i\in\mathcal{I}} C_{MG,i}(t) + \sum_{i\in\mathcal{I}} C_{COM,i}(t)\right\}, \end{aligned} \tag{20}$$

where V is a control parameter and $B \triangleq \frac{1}{2}\max\{R_{dis}^2, R_{ch}^2\}+$

$\frac{1}{2}(1+\beta_{max}^2)$.

Proof. See Appendix B □

Thus, with the drift-plus-penalty minimization method, the control decisions are chosen to minimize the upper bound on the Lyapunov drift-plus-penalty obtained in (20) instead of minimizing the drift-plus-penalty expression directly. It will be shown in Section IV-C that greedily minimizing the upper bound on the Lyapunov drift-plus-penalty obtained in (20) provides a bounded sub-optimal solution to **P3**. Hence, the real-time sharing control algorithm can be described as follows: in each time slot t, given the system state $\mathbf{X}(t)$ and the queue states $\Theta(t)$, the real-time sharing control algorithm determines the control decision $\mathbf{Y}(t)$ by solving the following linear programming problem **P4**:

$$\begin{aligned}\mathbf{P4}: \min_{\mathbf{Y}(t)} \quad & K_b(t)\sum_{i\in\mathcal{I}} b_i(t) + \sum_{i\in\mathcal{I}} H_{l,i}(t)\frac{\overline{D}_i(t)-D_i(t)}{\overline{D}_i(t)-\underline{D}_i(t)} \\ & + V\sum_{i\in\mathcal{I}} C_{MG,i}(t) + V\sum_{i\in\mathcal{I}} C_{COM,i}(t) \\ \text{s.t.} \quad & (2)(3)(4)(8)(17)(18).\end{aligned} \tag{21}$$

Although no statistical knowledge associated with the system state $\mathbf{X}(t)$ is required, the queue states $\Theta(t)$ carry sufficient statistical information needed to determine the control decision $\mathbf{Y}(t)$. We will show in Section IV-C that the design of the real-time problem **P4** can lead to some analytical performance guarantee.

C. ALGORITHM PERFORMANCE ANALYSIS

In this section, we analyze the performance of the real-time sharing control algorithm **P4** with respect to the original problem **P1**.

In the following proposition, we prove that the boundedness of the energy states (9) in **P1** can be satisfied by appropriately designing the perturbation parameter θ and the control parameter V. Therefore, the control decisions $\mathbf{Y}(t)$ derived from **P4** are feasible of to **P1**.

Proposition 2. *In each time slot t, set the perturbation parameter θ as*

$$\theta \triangleq S_{min} + \eta_{dis}R_{dis} + Vp_{max}, \tag{22}$$

where

$$0 \le V \le \frac{\eta_{ch}(S_{max} - S_{min} - \eta_{ch}R_{ch} - \eta_{dis}R_{dis})}{p_{max} - p_{min}}. \tag{23}$$

Then, under the real-time sharing control algorithm, given that the system state $\mathbf{X}(t)$ is i.i.d over time, we have

1) *All the control decisions $\mathbf{Y}(t)$ derived from **P4** are feasible to **P1**, i.e.,*

$$S_{min} \le s(t) \le S_{max}, \quad \forall t; \tag{24}$$

2) *The gap between the optimal cost of **P1** and the expected time-averaged cost under the proposed algorithm by solving **P4** is within bound B/V, i.e.,*

$$C_{P4}^* - C_{P1}^* \le \frac{B}{V} \tag{25}$$

*where C_{P4}^* is the expected time-averaged cost achieved by **P4**, C_{P1}^* is the optimal cost of **P1**, and $B \triangleq \frac{1}{2}\max\{R_{dis}^2, R_{ch}^2\} + \frac{1}{2}(1+\beta_{max}^2)$.*

Proof. See Appendix C □

While proposition 2.1 indicates that, under the real-time sharing control algorithm, the feasibility of the solutions is maintained, proposition 2.2 characterizes the gap between the resulting time-averaged cost and the optimal cost of **P1**, which is in the order of $O(1/V)$. To minimize this gap, the control parameter V should be set as $V_{max} \triangleq \frac{\eta_{ch}(S_{max}-S_{min}-\eta_{ch}R_{ch}-\eta_{dis}R_{dis})}{p_{max}-p_{min}}$. Since V_{max} increases with S_{max}, which depends on the shared battery capacity, the real-time sharing control algorithm is asymptotically equivalent to $P1$ as the shared battery capacity increases.

Summarily, the Lyapunov-based real-time sharing control algorithm provides a low-complexity alternative to achieve a similar performance to the original optimization problem **P1**. However, according to the definition of V_{max}, the proposed algorithm performs better for the shared battery with a larger capacity compared to the one with a smaller capacity.

D. DISTRIBUTED CREDIT-BASED SHARING ALGORITHM

In the previous section, we presented a Lyapunov-based real-time sharing control algorithm to coordinate all households' energy consumption and battery utilization. The real-time problem **P4** can be solved by a central agent in a centralized way, provided that the solar power generations and load demand preferences of all the households are all known to the central agent, i.e., all households have to report their renewable generations and demand preferences including the preferred power demands and the QoS control factors, to the central agent. However, this leads to privacy concerns since the households may not be willing to disclose their private information. In this section, we propose a credit-based distributed algorithm that allows the storage capacity/space of the shared battery to be allocated to the households according to their energy contributions to the shared battery. The credit-based control sharing algorithm solves the real-time energy management problem **P4** in a distributed manner, which is more implementable in practice.

Naturally, based on their solar power generations and loads, the group of households $\mathcal{I}$ can be divided into two groups: energy surplus group A, $\mathcal{I}_a$, in which $g_{gv,i} \ge D_i^*$ $\forall i \in \mathcal{I}_a$, and energy deficit group B, $\mathcal{I}_b$, in which $g_{gv,i} \le D_i^*$

$\forall i \in \mathcal{I}_b$. Hence, the optimization problem **P4** can be split into two sub-problems for group A and B, respectively:

$$\begin{aligned} \mathbf{P4-a}: \min_{\mathbf{Y}(t)} \ & K_b(t)\sum_{i\in\mathcal{I}_a} b_i(t) + \sum_{i\in\mathcal{I}_a} H_{l,i}(t)\frac{\overline{D}_i(t)-D_i(t)}{\overline{D}_i(t)-\underline{D}_i(t)} \\ & + V\sum_{i\in\mathcal{I}_a} C_{MG,i}(t) + V\sum_{i\in\mathcal{I}_a} C_{COM,i}(t) \\ \text{s.t.} \ & (2)(4)(8)(17)(18), \end{aligned} \tag{26}$$

and

$$\begin{aligned} \mathbf{P4-b}: \min_{\mathbf{Y}(t)} \ & K_b(t)\sum_{i\in\mathcal{I}_b} b_i(t) + \sum_{i\in\mathcal{I}_b} H_{l,i}(t)\frac{\overline{D}_i(t)-D_i(t)}{\overline{D}_i(t)-\underline{D}_i(t)} \\ & + V\sum_{i\in\mathcal{I}_b} C_{MG,i}(t) + V\sum_{i\in\mathcal{I}_b} C_{COM,i}(t) \\ \text{s.t.} \ & (3)(4)(8)(17)(18). \end{aligned} \tag{27}$$

It is noticed that the virtual queue state $K_b(t)$, which is determined by the battery charging and discharging amounts, $g_{s,i}$, $g_{ch,i}$ and $g_{dis,i}$ $\forall i \in \mathcal{I}$ in the previous time slot $t-1$, can be calculated at the central coordinator side. Thus, in time slot t, assuming $K_b(t)$ is known to all households, the optimization problems in **P4-a** and **P4-b** can be split into sub-problems for each household. Specifically, the sub-problem for each household is

$$\begin{aligned} & \mathbf{P4-a'} \ \text{for} \ i \in \mathcal{I}_a: \\ & \min_{\mathbf{Y_i}(t)} \ K_b(t)b_i(t) + H_{l,i}(t)\frac{\overline{D}_i(t)-D_i(t)}{\overline{D}_i(t)-\underline{D}_i(t)} + VC_i(t) \\ & \text{s.t.} \ (2)(4)(18), \\ & \qquad 0 \le g_{ch,i}(t) + g_{s,i}(t) \le \xi_{ch,i}(t)R_{ch}, \end{aligned} \tag{28}$$

and

$$\begin{aligned} & \mathbf{P4-b'} \ \text{for} \ i \in \mathcal{I}_b: \\ & \min_{\mathbf{Y_i}(t)} \ K_b(t)b_i(t) + H_{l,i}(t)\frac{\overline{D}_i(t)-D_i(t)}{\overline{D}_i(t)-\underline{D}_i(t)} + VC_i(t) \\ & \text{s.t.} \ (3)(4)(18), \\ & \qquad 0 \le g_{dis,i}(t) \le \xi_{dis,i}(t)R_{dis}, \end{aligned} \tag{29}$$

where $C_i(t) = C_{MG,i}(t) + C_{COM,i}(t)$, $\xi_{ch,i}(t)$ and $\xi_{dis,i}(t)$ represent the percentages of the maximum charging rate and discharging rate, R_{ch} and R_{dis}, taken by household i, respectively. Apparently, in each time slot, as long as $\sum_{i\in\mathcal{I}}\xi_{ch,i}(t) \le 1$ and $\sum_{i\in\mathcal{I}}\xi_{dis,i}(t) \le 1$, the constraint (8) is satisfied and the solutions to the sub-problems of individual households, **P4-a'** and **P4-b'**, are feasible to **P4**.

We now present a credit-based division scheme to divide the maximum charging/discharging rate, R_{ch}/R_{dis}, in (28) and (29) among households who request to charge/discharge. Firstly, in order to encourage cooperation among the households and avoid some households taking advantage of others by discharging much more than they contributed previously, we introduce a credit concept, in which the credit point of each household is determined by its contribution to the shared battery, i.e., the accumulated amount of energy it charged and discharged previously. Specifically, in each time slot t, the credit point of household i is given by

$$Cr_i(t) = \sum_{t=1}^{t-1} g_{ch,i}(t) + g_{s,i}(t) - g_{dis,i}(t). \tag{30}$$

The initial credit point $Cr_i(0) = 1/I$ and the central coordinator records the credit point of each household. At the beginning of each time slot t, each household first presumes that R_{ch}/R_{dis} is all taken by itself, i.e., $\xi_{ch,i}(t) = 1$ and $\xi_{dis,i}(t) = 1 \ \forall i \in \mathcal{I}$. Based on this presumption in addition to its solar power generation $g_{pv,i}$ and load demand $\mathbf{d}_i(t)$, each household with energy surplus/deficit calculates its optimal control vector, i.e., the optimal load $D_i(t)$, optimal energy purchasing request for load serving $g_{l,i}$, optimal energy purchasing request for battery charging $g_{s,i}$, optimal battery charging/discharging requests $g_{ch,i}/g_{dis,i}$ by solving the real-time optimization problem **P4-a'/P4-b'**. For both energy surplus and deficit groups, if the sum of charging/discharging requests obtained exceeds R_{ch}/R_{dis}, the central coordinator proportionally divides R_{ch}/R_{dis} among households who request charging/discharging based on their credit points, i.e., $\xi_{ch,i}(t) = \frac{Cr_i(t)}{\sum_{i\in\mathcal{I}_a} Cr_i(t)}$ and $\xi_{dis,i}(t) = \frac{Cr_i(t)}{\sum_{i\in\mathcal{I}_b} Cr_i(t)}$. Accordingly, each household redetermines its optimal control vector by solving the real-time problem **P4-a'/P4-b'** based on the adjusted value of $\xi_{ch,i}(t)/\xi_{dis,i}(t)$.

The credit-based distributed sharing control algorithm is summarized in Algorithm 1. With all information obtained locally or through simple communication, under the proposed credit-based distributed real-time sharing control algorithm, the optimization problem is solved locally without requiring any statistical information of the system. Thus, the proposed sharing control algorithm can be implemented more easily while avoiding disclosure of private information.

V. PERFORMANCE EVALUATION

A performance evaluation of the proposed real-time sharing control algorithm via numerical simulations is provided in this section.

A. SIMULATION SETUP

We consider a microgrid with 10 households in a neighborhood sharing one battery with capacity of S_{cap}, charging and discharging efficiencies of $\eta_{ch} = 0.8$ and $\eta_{dis} = 1.25$, respectively. For the sake of simplicity, we assume that $S_{max} = S_{cap}$ and $S_{min} = 0.1S_{max}$, respectively. In addition, the maximum charging and discharging rates are assumed to be of the same quantity, $R_{ch} = R_{ch} = 0.15S_{max}$. The initial battery energy level is set as S_{min}. Due to various living habits and some social factors such as the age and type of residence, the load demand of each household in the considered microgrid varies. We simply classify the households into three types: Type I low power consumption, Type II medium

Algorithm 1: Credit-based distributed real-time sharing algorithm

Initialize the virtual battery queue $K_b(0) = s(0) - \theta$, the control parameter $V = V_{max}$, the QoS-aware load queue of each household $H_{l,i}(0) = 0, \forall i$, and the credit point of each household $Cr_i = 1/I, \forall i$, respectively.

In each time slot, each household executes the following steps sequentially:

1. Receive $K_b(t)$;
2. Initialize $k = 1$ and assume $\xi^k_{ch,i}(t) = 1$ and $\xi^k_{dis,i}(t) = 1$, respectively;
3. Based on its current renewable generation $g_{pv,i}$, demand preferences $\mathbf{d}_i(t)$ and virtual QoS-aware load queue $H_{l,i}(t)$, as well as the virtual battery queue state $K_b(t)$, energy price $p(t)$ and upper bounds of charging and discharging rates $\xi^k_{ch,i}(t)R_{ch}$ and $\xi^k_{dis,i}(t)R_{dis}$, solve the real-time problem **P4-a'/P4-b'** to determine its optimal load $D^k_i(t)$, optimal energy purchasing request for load serving $g^k_{l,i}$, optimal energy purchasing request for battery charging $g^k_{s,i}$, optimal battery charging $g^k_{ch,i}$ and discharging requests $g^k_{dis,i}$;
4. Send its battery charging and discharging requests, $\sum g^k_{ch,i} + g^k_{s,i}$ and $g^k_{dis,i}$, to the central coordinator;
5. $D^*_i(t) \leftarrow D^k_i(t)$, $g^*_{l,i} \leftarrow g^k_{l,i}$, $g^*_{s,i} \leftarrow g^k_{s,i}$, $g^*_{ch,i} \leftarrow g^k_{ch,i}$ and $g^*_{dis,i} \leftarrow g^k_{dis,i}$;
6. Update $H_{l,i}(t)$ based on its evolution equation (18).

In each time slot, the central coordinator executes the following steps sequentially:

1. Initialize $k = 1$, broadcast the control parameter $V = V_{max}$, and virtual battery queue state $K_b(t)$;
2. After receiving the charging and discharging requests, $\sum g^k_{ch,i} + g^k_{s,i}$ and $g^k_{dis,i}$, from the households, evaluate $\sum_{i\in\mathcal{I}} g^k_{ch,i} + g^k_{s,i}$ and $\sum_{i\in\mathcal{I}} g^k_{dis,i}$, respectively.

if $\sum_{i\in\mathcal{I}} g^k_{ch,i} + g^k_{s,i} \le R_{ch}$ *and* $\sum_{i\in\mathcal{I}} g^k_{dis,i} \le R_{dis}$ **then**
- a. Inform all households to go to their Step 5;
- b. $\sum_{i\in\mathcal{I}} g^*_{ch,i} + g^*_{s,i} \leftarrow \sum_{i\in\mathcal{I}} g^k_{ch,i} + g^k_{s,i}$ and $\sum_{i\in\mathcal{I}} g^*_{dis,i} \leftarrow \sum_{i\in\mathcal{I}} g^k_{dis,i}$;
- c. Go to Step 3;

else
- a. $k \leftarrow k + 1$;
- b. $\xi^k_{ch,i} \leftarrow \frac{Cr_i(t)}{\sum_{i\in\mathcal{I}_a} Cr_i(t)}$ and $\xi^k_{dis,i} \leftarrow \frac{Cr_i(t)}{\sum_{i\in\mathcal{I}_b} Cr_i(t)}$;
- c. Send $\xi^k_{ch,i}$ and $\xi^k_{dis,i}$ to the respective households and inform all households to repeat their Step 3 and 4;

end

3. Update $K_b(t)$ and $Cr_i(t), \forall i$, based on their evolution equations (17) and (30), respectively.

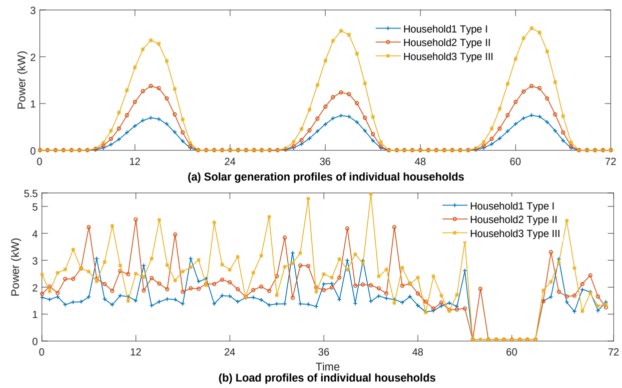

FIGURE 1. An example of solar generation profiles and load profiles of individual households.

power consumption and Type III high power consumption. Each household has a solar PV system with different capacity that generates a different amount of renewable energy everyday from 6am to 7pm. We assume households in each type have solar PV systems generating a similar amount of renewable energy everyday, which is selected from a uniform distribution with the mean value of 5kWh, 8kWh and 15kWh and a slight variance of 0.05kWh for Type I, Type II and Type III, respectively. Each day is divided into $T = 24$ time slots. As shown in the illustrative example in Fig.1(a), the renewable power generated by each household in each time slot is generated using a beta distribution with the mean value of 0.6kW and the standard deviation of 0.03kW.

The simulations are run over households with different appliance demand profiles of different types of households. An appliance demand profile generator is developed to simulate the time-varying power consumption of household appliances for each household in each time slot as shown in Fig.1(b). With this appliance demand profile generator, each appliance operates in a random time slot during a certain period per day and consumes a certain amount of power selected from a uniform distribution with a different mean for each household type to differentiate power consumption among different types of households, and a variance of 0.2-1kWh to differentiate power consumption among households in the same type. Note that, the main objective of the appliance demand profile generator and the solar power generation simulator is to simulate the differentiation in load demands and solar power generations of different households in each time slot to randomly construct the scenario, where some households have surplus solar energy to compete for the free storage space of the battery while others with energy deficit compete for the energy stored in the battery.

The total load demand generated by the appliance demand profile generator for each household in each time slot is used as its maximum power request $\overline{D}_i(t)$, while the minimum power demands $\underline{D}_i(t)$ that can not be shed is set randomly from $[0.3\overline{D}_i(t), 0.7\overline{D}_i(t)]$. The QoS related parameters $\alpha_i(t)$ for each household in each time slot and β_i for each household are chosen randomly from $[1.5, 3.5]$ and $[0.5, 0.7]$, re-

	Operation Duration	Price(R/kWh)
Weekdays		
Peak	7:00-9:00, 18:00-20:00	1.6257
Standard	6:00-7:00, 10:00-18:00 21:00-22:00	1.2860
Off-Peak	1:00-5:00, 23:00-24:00	1.0117
Weekends		
Standard	7:00-12:00, 18:00-20:00	1.2860
Off-Peak	1:00-6:00, 13:00-17:00, 21:00-24:00	1.0117

TABLE 1. Time-of-use (TOU) tariff

spectively.

In addition, the Time-of-Use tariff of Johannesburg city power as listed in Table 1 is used for simulation.

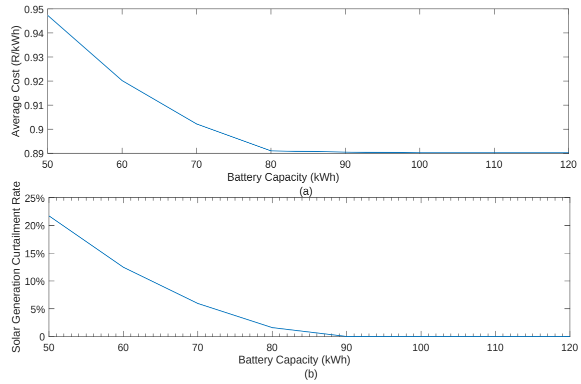

FIGURE 2. Average cost per kWh and solar generation curtailment rate under various battery capacities.

B. SIMULATION RESULTS

This section presents simulation results of the proposed distributed sharing algorithm. We consider a period of 90 days, where $T = 2160$ with each time slot representing 1 hour, and randomly generate 10 households consisting of 3 Type I households with an average daily load demand of 29.35kWh, 3 Type II households with an average daily load demand of 35.60kWh and 4 Type III households with an average daily load demand of 58.06kWh. In total, the 10 households have a daily average of 427.09kWh of load demand and a daily average of 103.85kWh of solar generation. The average monthly load demands and solar generations of individual households are listed in Table 2 for the sake of easy comparison. The real-time optimization problem in **P4** is solved using the CVX toolbox [34] for Matlab.

By varying the battery capacity, we investigate the effectiveness of the shared battery in cost saving in this battery sharing system. As can be observed in Fig.2(b), the solar generation curtailment rate drops with an increase in the battery capacity since there is more storage capacity to accommodate surplus solar generations and cheaper electricity from the MG. Accordingly, the average cost per kWh decreases with the increase in the battery capacity as shown in Fig.2(a). Obviously, considering the relatively high initial investment cost of batteries, which is expressed on a per kWh of energy capacity basis, a trade-off between battery capacity, solar generation curtailment, electricity consumption cost along with other factors should be made in sizing the shared battery, so that the battery sharing system can achieve optimal cost-benefit ratio. However, in this work, we mainly concentrate on how to utilize the shared storage given the dynamic behavior of the system to reduce electricity consumption cost and do not consider the optimal sizing of the shared battery. In the following, we further investigate the performance of the proposed sharing algorithm using a battery sharing system with the 10 households sharing a 90kWh battery, which is enough to accommodate the load demands with a relatively low cost and zero solar generation curtailment (shown in Fig.2), as an example.

Firstly, to evaluate the performance of the proposed distributed sharing algorithm in energy cost saving, a greedy sharing algorithm, where each household is myopic and only aims to minimize its current cost without taking the future and other households into account, is used for comparison. Specifically, under this myopic greedy sharing algorithm, assuming that all storage space and energy available in the shared battery can be used by itself, each household independently solves a simple cost minimization problem in (31) to derive its optimal charging (energy generated by its solar power generation and purchased from the MG) and discharging requests as well as the optimal energy consumption of its controllable loads. To ensure that the space-availability constraint and the energy-availability constraint in (10) are satisfied, if the sum of the amounts of charge/discharge from all households exceeds the storage space/energy available in the shared battery, the storage space/energy available for charging/discharging is proportionally divided among the households based on the amounts of their charging/discharging requests.

$$\begin{aligned} \min_{\mathbf{Y}(t)} \quad & [g_{l,i}(t) + g_{s,i}(t)]p(t) + \alpha_i(t)[\overline{D_i}(t) - D_i(t)]^2, \ \ \forall i \in \mathcal{I}, \\ \text{s.t.} \quad & (2)(3)(4)(7)(10) \quad \frac{\overline{D}_i(t) - D_i(t)}{\overline{D}_i(t) - \underline{D}_i(t)} \leq \beta_i, \ \ \forall i \in \mathcal{I}. \end{aligned} \tag{31}$$

Note that, since there is no load management mechanism in the greedy algorithm, the constraint $\frac{\overline{D}_i(t) - D_i(t)}{\overline{D}_i(t) - \underline{D}_i(t)} \leq \beta_i$ is added to make sure that the QoS of each household is satisfied. Moreover, to allow the greedy sharing algorithm to take advantage of the time-varying pricing, if there is storage space for charging, each household purchases energy from the MG to charge into the shared battery during off-peak periods.

As can be observed in Fig.4, under the greedy sharing algorithm, as long as there is storage space, energy is purchased to charge into the battery during off-peak periods, which results in a situation where there is less storage space for the generated solar energy. As a result, in total 30.80% generated solar energy can not be accommodated as shown in Table 2. In contrast, as shown in Fig.3, with the systematic optimization mechanism in the proposed sharing control

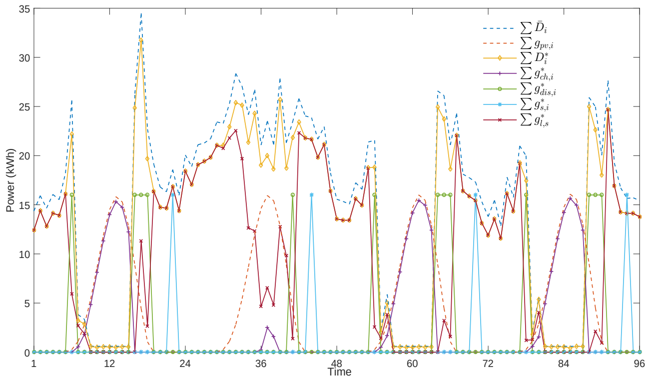

FIGURE 3. Real-time system states $\mathbf{X}(t)$ and control decisions $\mathbf{Y}(t)$ of the credit-based distributed sharing control algorithm.

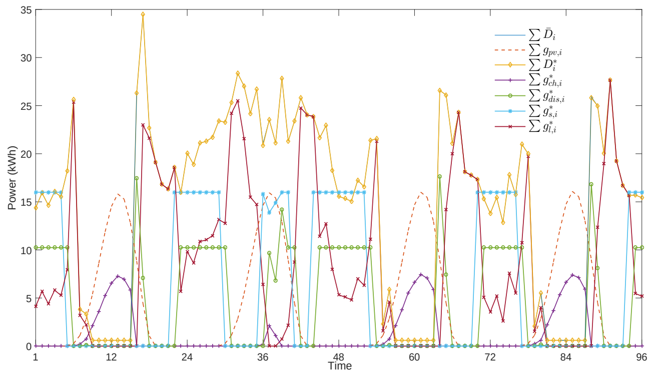

FIGURE 4. Real-time inputs and outputs of the greedy sharing algorithm.

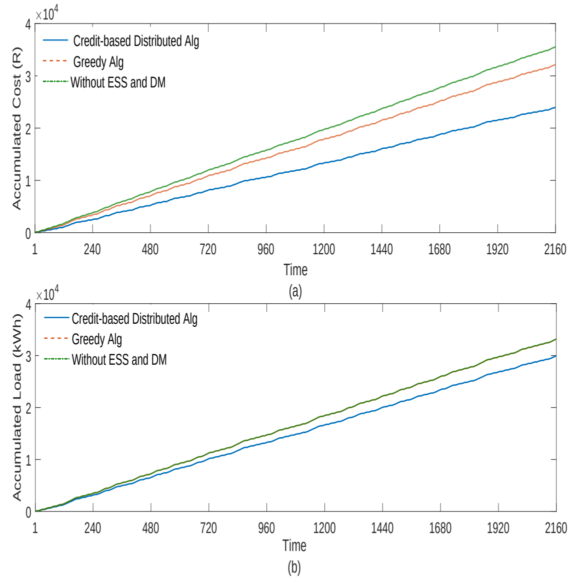

FIGURE 5. Comparison of accumulated cost and load between the credit-based distributed algorithm and the greedy algorithm.

algorithm that achieves zero solar generation curtailment, when necessary, energy is purchased to charge to the shared battery while taking into consideration the solar power that will be generated in the following time slots. Thus, the proposed sharing control algorithm can avoid solar generation curtailment more effectively.

Fig.5 provides a comparison of the accumulated served load and corresponding cost over time. For the sake of easy comparison, the original load demands and corresponding average monthly costs (with a system average monthly cost of R1185.52) without an ESS and a demand management mechanism are depicted in Fig.5 and listed in Table 2 as low benchmarks. It is illustrated that the proposed sharing control algorithm with an average monthly cost of R883.92 outperforms the myopic greedy algorithm with an average monthly cost of R1114.02. In addition, as shown in both Fig.5 and Fig.4, the greedy sharing algorithm without a proper load management mechanism has to serve more power consumption (with the average shed demand rate being 0.05%) with a higher average cost per kWh (R1.0072/kWh) compared to that of the proposed algorithm (R0.8850/kWh) with an average shed demand rate of 9.79%, as listed in Table 2. Table 2 also shows that, compared to the greedy algorithm, the proposed sharing control algorithm reduces the average monthly cost and solar generation curtailment of each individual household by 15.01%-25.17% and 29.13%-31.19%, respectively.

Secondly, to evaluate the fairness of the proposed distributed sharing algorithm that takes into account individual households' energy contributions, a proportional distributed sharing algorithm, in which the maximum charging rate R_{ch} in (28) and the maximum discharging rate R_{dis} in (29) are proportionally divided among the households based on the amounts of their charging/discharging requests, is compared with the proposed sharing algorithm. Specifically, under the proportional distributed sharing algorithm, for both energy surplus and deficit groups, if the sum of charging/discharging requests obtained exceeds R_{ch}/R_{dis}, R_{ch}/R_{dis} is proportionally divided among households based on the amount of their charging/discharging requests, i.e., $\xi^p_{ch,i}(t) = \frac{g_{ch,i}+g_{s,i}}{\sum_{i\in\mathcal{I}_a} g_{ch,i}+g_{s,i}}$ and $\xi^p_{dis,i}(t) = \frac{g_{dis,i}}{\sum_{i\in\mathcal{I}_b} g_{dis,i}}$. Accordingly, each household determines its optimal control vector by solving the the real-time problem **P4-a'/P4-b'** based on the value of $\xi^p_{ch,i}(t)/\xi^p_{dis,i}(t)$.

As illustrated in Fig.7, under the proportional sharing algorithm, available space/energy of the shared battery is allocated among households based on their charging/discharging requests, which results in a situation where the Type I household, who contributes less energy, discharges so much from the shared battery that the type III household, who contributes more energy, has to purchase energy when it needs, instead of benefiting from the energy it shared previously. In contrast, as illustrated in Fig.6, the credit-based sharing algorithm, which allows for the contribution of each household to the shared battery, enables the Type III household to benefit more from the shared battery, which accordingly reduces its energy cost as depicted in Fig.8. Meanwhile, it can also be observed in Fig.6 and Fig.8 that the utilization of the shared

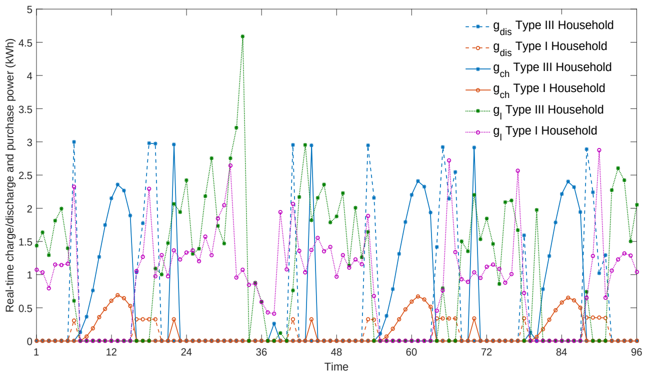

FIGURE 6. Real-time charge/discharge and purchase decisions of the credit-based distributed sharing control algorithm.

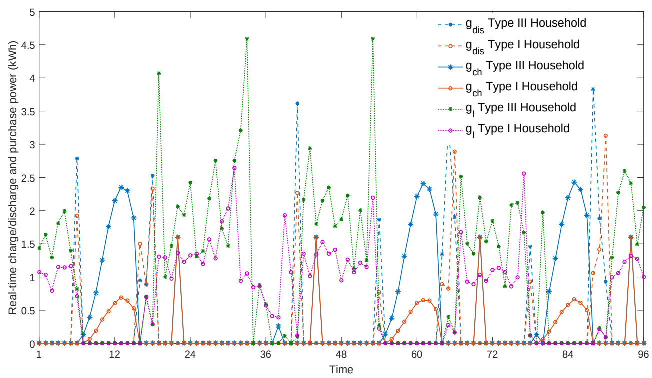

FIGURE 7. Real-time charge/discharge and purchase decisions of the proportional sharing control algorithm.

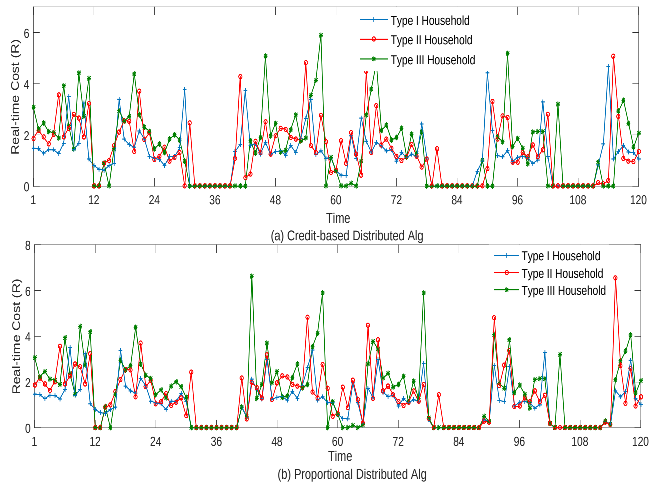

FIGURE 8. Comparison of real-time cost between the credit-based distributed algorithm and the proportional algorithm.

energy by the Type I household is limited by its contribution, which leads to an increase in its energy cost.

Furthermore, as can be observed in Table 2, from the system point of view, compared to the low benchmark case, both the credit-based sharing algorithm and the proportional sharing algorithm achieve significant cost saving by reducing system average monthly cost by 25.35% and 25.44%, respectively, with zero solar generation curtailment. However, in such a battery sharing system, individual households intend to save their own energy costs by storing their surplus solar energy and cheaper electricity purchased from the MG in the shared battery. Given that the average ratios between the solar generation and the load demand of Type I, II and III households are 16.17%, 23.13% and 37.26%, respectively, Type III households are expected to save more in energy cost compared to Type I and II households. According to Table 2, compared to the low benchmark case, the average monthly costs of Type I, II and III households incurred under the proportional sharing algorithm reduce by 26.89%, 25.38% and 24.27%, respectively, which indicates that households who contribute less energy free ride on the contributions of others. Meanwhile, the average monthly cost of Type I, II and III households incurred under the credit-based sharing algorithm reduce by 21.35%, 24.31% and 28.43%, respectively, which are in line with their average ratios of solar generation to load demand. Thus, from the individual households point of view, the credit-based sharing algorithm facilitates more fair allocation of the shared energy among households.

Thirdly, to investigate the effectiveness of the proposed sharing algorithm that is able to take advantage of the non-overlapping power consumption patterns of users, the proposed distributed sharing system is compared with a distributed ESSs case, where each household individually owns and operates an ESS with a similar control scheme using the Lyapunov optimization technique. For a fair comparison, in the distributed ESSs case, the battery capacity of individual household is set in proportion to its net load demand (the load demand minus the renewable generation that can be used to serve the load directly) while the overall capacity of all households is equivalent to the capacity of the shared battery in the sharing system. Specifically, the battery capacity of household i is equal to $\rho_i E_{max}$, where $\rho_i = \frac{\sum_{t=1}^{T} \overline{D}_i(t) - g_{pv,i}(t)}{\sum_{i=1}^{I} \sum_{t=1}^{T} \overline{D}_i(t) - g_{pv,i}(t)}$ and E_{max} is the capacity of the shared battery in the sharing system. As shown in Table 2, compared to the distributed ESSs case, households with a shared battery achieve 6.41%-9.76% lower average monthly costs while reducing the solar generation curtailment of each household to zero. This indicates that, by coordinating the utilization of the shared battery among households, the solar generation curtailment can be avoided more effectively, which in turn leads to energy cost reduction.

VI. CONCLUSIONS

In this work, a microgrid consisting of a group of households with renewable energy sources and controllable loads sharing a common battery is considered. An ESSM system, in which households cooperatively utilize the shared battery, is presented, aiming to minimize the long term time-averaged cost of the whole microgrid, i.e., the long-term time-averaged costs of all households, subject to the operational constraints of the shared battery as well as the arbitrary dynamics of

	System Average	H1	H2	H3	H4	H5	H6	H7	H8	H9	H10
Household Profile											
Average Monthly Load Demand (kWh)		1072.14	1062.03	1304.61	880.353	1305.55	1312.58	878.48	882.93	1302.85	1070.14
Average Monthly Solar Generation (kWh)		247.50	246.67	486.18	142.36	486.82	485.13	142.39	142.37	489.22	247.01
Average Monthly Cost (R)											
Without ESS and DM (Low Benchmark)	1185.52	1170.30	1161.14	1345.06	990.69	1346.01	1354.21	983.89	992.10	1344.57	1167.31
Credit-based Distributed Sharing Alg	883.92	887.53	873.69	954.52	779.31	968.39	973.34	773.16	780.79	961.42	887.05
Greedy Alg	1114.02	1092.92	1090.97	1275.50	920.57	1275.60	1286.78	909.57	920.32	1275.38	1092.58
Proportional Distributed Sharing Alg	886.18	876.71	862.93	1008.63	722.24	1017.44	1034.31	719.50	727.68	1021.16	871.19
Distributed ESSs	956.97	958.96	948.72	1029.80	860.42	1034.74	1044.26	856.35	865.20	1045.93	958.24
Average Cost per kWh (R/kWh)											
Without ESS and DM (Low Benchmark)	1.0708	1.0916	1.0933	1.0310	1.1253	1.0310	1.0317	1.1200	1.1236	1.0320	1.0908
Credit-based Distributed Sharing Alg	0.8850	0.9211	0.9191	0.8023	0.9989	0.8122	0.8112	0.9947	0.9985	0.8074	0.9230
Greedy Alg	1.0072	1.0199	1.0278	0.9796	1.0459	0.9784	0.9829	1.0356	1.0426	0.9804	1.0212
Proportional Distributed Sharing Alg	0.8905	0.9125	0.9101	0.8527	0.9302	0.8576	0.8669	0.9284	0.9298	0.8618	0.9061
Distributed ESSs	0.9235	0.9600	0.9608	0.8608	1.040	0.8617	0.8621	1.0346	1.0386	0.8707	0.9604
Solar Generation Curtailment Rate											
Without ESS and DM (low benchmark)	58.21%	56.78%	56.65%	59.51%	53.68%	60.01%	59.63%	53.62%	53.65%	60.11%	56.47%
Credit-based Distributed Sharing Alg	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
Greedy Alg	30.80%	30.14%	30.05%	31.19%	29.13%	31.54%	31.30%	29.27%	29.27%	31.63%	30.07%
Proportional Distributed Sharing Alg	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
Distributed ESSs	3.38%	0%	0%	8.22%	0%	8.38%	7.79%	0%	0%	9.42%	0%

TABLE 2. Comparison of costs and solar generation curtailment rates of the whole microgrid and individual households.

renewable generations, load demands, and electricity pricing. We study the energy management problem for such a sharing system and propose a credit-based distributed real-time sharing control algorithm, which takes into consideration energy contributions of individual households. Based on the Lyapunov theory, the proposed sharing control algorithm coordinates households to optimally charge/discharge energy to/from the shared battery, by jointly optimizing charging and discharging of the shared ESS as well as the power consumptions of all households' controllable loads in a distributed and fair manner without requiring any system statistics. Numerical simulations are presented to evaluate the performance of the proposed real-time sharing control algorithm in terms of energy cost saving and fairness. It is shown that, compared to the greedy sharing algorithm, the proportional distributed sharing algorithm and the distributed ESSs case, the proposed credit-based distributed sharing control algorithm outperforms in terms of both cost saving and renewable energy generation utilization.

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APPENDIX A

Derivation of (15)

Summing both sides of the energy state equation (7) over $t = \{0, 1, \cdots, T-1\}$ and dividing them by T yields

$$\frac{1}{T}\sum_{t=0}^{T-1}\sum_{i\in\mathcal{I}} b_i(t) = \frac{s(T)}{T} - \frac{s(0)}{T}. \tag{32}$$

where $\sum_{i\in\mathcal{I}} b_i(t)$ is defined as the effective charging and discharging amount of in time slot t in (7). Taking expectations on both sides of (32) and taking limits over T to infinity gives

$$\lim_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\mathbb{E}\left\{\sum_{i\in\mathcal{I}}b_i(t)\right\}=\lim_{T\to\infty}\frac{s(T)}{T}-\lim_{T\to\infty}\frac{s(0)}{T}. \tag{33}$$

Since both $s(T)$ and $s(0)$ are finite due to (9), the right hand side of (33) is equal to zero.

APPENDIX B

Proof of Proposition 1

According to the definition of $L(\Theta(t))$, the difference

$$\begin{aligned}&L(\Theta(t+1))-L(\Theta(t))\\&=\frac{1}{2}[K_b(t+1)^2-K_b(t)^2]+\sum_{i\in\mathcal{I}}\frac{1}{2}[H_{l,i}(t+1)^2-H_{l,i}(t)^2]\end{aligned} \tag{34}$$

Based on the queue update of $K_b(t)$ in (17), the term $K_b(t+1)^2-K_b(t)^2$ in (34) can be upper bounded by

$$K_b(t+1)^2-K_b(t)^2\le 2K_b(t)\sum_{i\in\mathcal{I}}b_i(t)+\max\{R_{dis}^2,R_{ch}^2\}. \tag{35}$$

Similarly, based on the queue update of $H_{l,i}(t)$ in (18), the term $H_{l,i}(t+1)^2-H_{l,i}(t)^2$ in (34) can be upper bounded by

$$H_{l,i}(t+1)^2-H_{l,i}(t)^2\le 2H_{l,i}(t)\left[\frac{\overline{D}_i(t)-D_i(t)}{\overline{D}_i(t)-\underline{D}_i(t)}-\beta_i\right]+1+\beta_i^2. \tag{36}$$

Applying inequalities (35) and (36) to (34), taking the conditional expectation over $L(\Theta(t+1))-L(\Theta(t))$ given $\Theta(t)$ and adding the term $V\,\mathbb{E}\{C_{ToT}(t)\}$ yield the upper bound in (20).

APPENDIX C

Proof of Proposition 2

Proof of Proposition 2.1:

The per-slot problem **P4** includes all constraints of the original problem **P1** except for the energy state constraint (9). Hence, to prove the solution derived from **P4** are feasible to **P1** is to show the energy state $s(t)$ is bounded within $[S_{min}, S_{max}]$. The optimization problem **P4** can be rearranged to **P5**

$$\begin{aligned}\mathbf{P5}:\ \min_{\mathbf{Y}(t)}\ &[Vp(t)+K_b(t)\eta_{ch}]\sum_{i\in\mathcal{I}}g_{s,i}(t)+K_b(t)\eta_{ch}\sum_{i\in\mathcal{I}}g_{ch,i}(t)\\&+Vp(t)\sum_{i\in\mathcal{I}}g_{l,i}(t)-K_b(t)\eta_{dis}\sum_{i\in\mathcal{I}}g_{dis,i}(t)\\&+V\sum_{i\in\mathcal{I}}\alpha_i[\overline{D}_i(t)-D_i(t)]^2+\sum_{i\in\mathcal{I}}H_{l,i}(t)\frac{\overline{D}_i(t)-D_i(t)}{\overline{D}_i(t)-\underline{D}_i(t)},\\ \text{s.t.}\quad &(2)(3)(4)(8)(17)(18).\end{aligned} \tag{37}$$

Let $\mathbf{D}^*(t)\triangleq D_i^*(t)$, $\mathbf{g_{ch}}^*(t)\triangleq g_{ch,i}^*(t)$, $\mathbf{g_{dis}}^*(t)\triangleq g_{dis,i}^*(t)$, $\mathbf{g_l}^*(t)\triangleq g_{l,i}^*(t)$ and $\mathbf{g_s}^*(t)\triangleq g_{s,i}^*(t)\ \forall i\in\mathcal{I}$ be the optimal solution to (37). It is noticed that $\mathbf{D}^*(t)$ will not directly affect the battery queue $K_b(t)$. Hence, $\mathbf{D}^*(t)$ can be treated as a given load. As mentioned previously, we consider two cases in determining how to utilize the solar energy generation: **Case 1:** energy surplus where $g_{gv,i}>D_i^*$ and **Case 2:** energy deficit where $g_{gv,i}\le D_i^*$. Thus, the user group I can be naturally divided into two groups: group A, $\mathcal{I}_a$, where $g_{gv,i}>D_i^*\ \forall i\in\mathcal{I}_a$, and group B, $\mathcal{I}_b$, where $g_{gv,i}\le D_i^*\ \forall i\in\mathcal{I}_b$. Correspondingly, the optimization problem **P5** can be split into two per-slot sub-problems for group A and B, respectively, as follows:

- **Case 1:** when $g_{gv,i}>D_i^*$, we have $g_{l,i}^*(t)=0$. Then, the optimization problem for group A **P5-a** is written as:

$$\begin{aligned}&\mathbf{P5\!-\!a}:\ \text{Energy Surplus}\\&\min_{\mathbf{Y}(t)}[Vp(t)+K_b(t)\eta_{ch}]\sum_{i\in\mathcal{I}_a}g_{s,i}(t)+K_b(t)\eta_{ch}\sum_{i\in\mathcal{I}_a}g_{ch,i}(t)\\&\quad-K_b(t)\eta_{dis}\sum_{i\in\mathcal{I}_a}g_{dis,i}(t)+V\sum_{i\in\mathcal{I}_a}\alpha_i[\overline{D}_i(t)-D_i(t)]^2\\&\quad+\sum_{i\in\mathcal{I}_a}H_{l,i}(t)\frac{\overline{D}_i(t)-D_i(t)}{\overline{D}_i(t)-\underline{D}_i(t)}\\&\text{s.t.}\quad(4)(8)(17)(18).\end{aligned} \tag{38}$$

- **Case 2:** when $g_{gv,i}\le D_i^*$, according to (3), we have $g_{ch,i}^*(t)=0$ and $g_{l,i}(t)=D_i^*(t)-g_{dis,i}^*(t)-g_{pv,i}(t)$. Then, the optimization problem for group B **P5-b** is written as:

$$\begin{aligned}&\mathbf{P5\!-\!b}:\ \text{Energy Deficit}\\&\min_{\mathbf{Y}(t)}[Vp(t)+K_b(t)\eta_{ch}]\sum_{i\in\mathcal{I}_b}g_{s,i}(t)-K_b(t)\eta_{dis}\sum_{i\in\mathcal{I}_b}g_{dis,i}(t)\\&\quad+Vp(t)\sum_{i\in\mathcal{I}_b}[D_i(t)-g_{dis,i}(t)-g_{pv,i}(t)]\\&\quad+V\sum_{i\in\mathcal{I}_b}\alpha_i[\overline{D}_i(t)-D_i(t)]^2+\sum_{i\in\mathcal{I}_b}H_{l,i}(t)\frac{\overline{D}_i(t)-D_i(t)}{\overline{D}_i(t)-\underline{D}_i(t)}\\&=[Vp(t)+K_b(t)\eta_{ch}]\sum_{i\in\mathcal{I}_b}g_{s,i}(t)-Vp(t)\sum_{i\in\mathcal{I}_b}g_{pv,i}(t)\\&\quad-[Vp(t)+K_b(t)\eta_{dis}]\sum_{i\in\mathcal{I}_b}g_{dis,i}(t)+Vp(t)\sum_{i\in\mathcal{I}_b}D_i(t)\\&\quad+V\sum_{i\in\mathcal{I}_b}\alpha_i[\overline{D}_i(t)-D_i(t)]^2+\sum_{i\in\mathcal{I}_b}H_{l,i}(t)\frac{\overline{D}_i(t)-D_i(t)}{\overline{D}_i(t)-\underline{D}_i(t)}\\&\text{s.t.}\quad(4)(8)(17)(18).\end{aligned} \tag{39}$$

By combining **P5-a** and **P5-b** together, the optimization problem **P5** is transformed into the following optimization problem:

$$\begin{aligned}\textbf{P6}: \min_{\mathbf{Y}(t)} \ & [Vp(t)+K_b(t)\eta_{ch}]\sum_{i\in\mathcal{I}} g_{s,i}(t) + K_b(t)\eta_{ch}\sum_{i\in\mathcal{I}_a} g_{ch,i}(t) \\ & -[Vp(t)+K_b(t)\eta_{dis}]\sum_{i\in\mathcal{I}_b} g_{dis,i}(t) - K_b(t)\eta_{dis}\sum_{i\in\mathcal{I}_a} g_{dis,i}(t) \\ & + Vp(t)\sum_{i\in\mathcal{I}_b} D_i(t) + V\sum_{i\in\mathcal{I}}\alpha_i[\overline{D}_i(t)-D_i(t)]^2 \\ & + \sum_{i\in\mathcal{I}} H_{l,i}(t)\frac{\overline{D}_i(t)-D_i(t)}{\overline{D}_i(t)-\underline{D}_i(t)} \\ \text{s.t.} \quad & (4)(8)(17)(18). \end{aligned} \tag{40}$$

Note that the optimal solution to **P6** has the following properties:

- If $K_b(t) > -Vp_{min}/\eta_{ch}$, $\sum_{i\in\mathcal{I}} g^*_{s,i} = 0$;
- If $K_b(t) < -Vp_{max}/\eta_{ch}$, $\sum_{i\in\mathcal{I}} g^*_{s,i} + g^*_{ch,i} = R_{ch}$.

We now prove the boundary of $s(t)$ in (24) using induction. First it is obvious that the lower and upper bounds hold for $t=0$. Now suppose that the boundary holds for time slot t, i.e., $S_{min} \le s(t) \le S_{max}$. This in turn indicates $S_{min}-\theta \le K_b(t) \le S_{max}-\theta$, i.e., $-Vp_{max}/\eta_{ch} - \eta_{dis}R_{dis} \le K_b(t) \le S_{max}-S_{min}-Vp_{max}/\eta_{ch}-\eta_{dis}R_{dis}$. Hence, to prove the boundary of $s(t)$ in (24) also holds for time slot $t+1$, we need to prove the boundary of $K_b(t)$, i.e., $[-Vp_{max}/\eta_{ch} - \eta_{dis}R_{dis}, S_{max}-S_{min}-Vp_{max}/\eta_{ch}-\eta_{dis}R_{dis}]$, holds for time slot $t+1$. We consider the following cases:

1) First suppose $-Vp_{max}/\eta_{ch} - \eta_{dis}R_{dis} \le K_b(t) < -Vp_{max}\eta_{ch}$, we have $\sum_{i\in\mathcal{I}}(g^*_{s,i}+g^*_{ch,i}) = R_{ch}$, $\sum_{i\in\mathcal{I}_b} g^*_{dis,i} = 0$ and $\sum_{i\in\mathcal{I}_a} g^*_{dis,i} = 0$. Then, based on (17), the battery queue $K_b(t)$ updates as follows:

$$\begin{aligned} K_b(t+1) &= K_b(t) + \eta_{ch}\sum_{i\in\mathcal{I}}(g^*_{s,i}+g^*_{ch,i}) - \eta_{dis}\sum_{i\in\mathcal{I}} g^*_{dis,i} \\ &= K_b(t)+\eta_{ch}R_{ch} \\ &> K_b(t) \ge -Vp_{max}/\eta_{ch} - \eta_{dis}R_{dis}. \end{aligned}$$

In addition, as $K_b(t) < -Vp_{max}\eta_{ch}$, we have

$$\begin{aligned} K_b(t+1) &< -Vp_{max}/\eta_{ch} + \eta_{ch}R_{ch} \\ &\le S_{max}-S_{min}-Vp_{max}/\eta_{ch}-\eta_{dis}R_{dis}, \end{aligned}$$

as long as $S_{max}-S_{min}-\eta_{dis}R_{dis}-\eta_{ch}R_{ch} \ge 0$ holds.

2) Secondly, suppose $-Vp_{max}/\eta_{ch} \le K_b(t) \le -Vp_{min}/\eta_{ch}$,

a) if $K_b(t) \ge -Vp(t)/\eta_{ch}$, we have $\sum_{i\in\mathcal{I}} g^*_{s,i} = 0$. There are two possibilities to study:

i) when $-Vp(t)/\eta_{ch} \le K_b(t) < -Vp(t)/\eta_{dis}$, we have $\sum_{i\in\mathcal{I}} g^*_{ch,i} = \min\{R_{ch}, \sum_{i\in\mathcal{I}_a}(g_{pv,i}-D^*_i)\}$ and $\sum_{i\in\mathcal{I}} g^*_{dis,i} = 0$. Thus,

$$\begin{aligned} K_b(t+1) &= K_b(t) + \eta_{ch}\min\{R_{ch}, \sum_{i\in\mathcal{I}_a}(g_{pv,i}-D^*_i)\} \\ &> K_b(t) \ge -Vp_{max}/\eta_{ch} - \eta_{dis}R_{dis}. \end{aligned}$$

In addition, as $K_b(t) < -Vp_{min}/\eta_{ch}$, we have

$$\begin{aligned} K_b(t+1) &< -Vp_{min}/\eta_{ch} + \eta_{ch}R_{ch} \\ &\le S_{max}-S_{min}-Vp_{max}/\eta_{ch}-\eta_{dis}R_{dis}, \end{aligned}$$

based on the definition $V \le \frac{\eta_{ch}(S_{max}-S_{min}-\eta_{ch}R_{ch}-\eta_{dis}R_{dis})}{p_{max}-p_{min}}$ in (23).

ii) when $-Vp(t)/\eta_{dis} \le K_b(t) \le -Vp_{min}/\eta_{ch}$, we have $\sum_{i\in\mathcal{I}} g^*_{ch,i} = \min\{R_{ch}, \sum_{i\in\mathcal{I}_a}(g_{pv,i}-D^*_i)\}$ and $\sum_{i\in\mathcal{I}} g^*_{dis,i} = \min\{R_{dis}, \sum_{i\in\mathcal{I}_b}(D^*_i - g_{pv,i})\}$. In other words, the maximum possible increase is $\eta_{ch}R_{ch}$ and the the maximum possible decrease is $\eta_{dis}R_{dis}$. Thus, using the upper bound of V as the case above, we have

$$\begin{aligned} K_b(t+1) &< K_b(t) + \eta_{ch}R_{ch} \\ &\le -Vp_{min}/\eta_{ch} + \eta_{ch}R_{ch} \\ &\le S_{max}-S_{min}-Vp_{max}/\eta_{ch}-\eta_{dis}R_{dis}, \end{aligned}$$

while

$$\begin{aligned} K_b(t+1) &> K_b(t) - \eta_{dis}R_{dis} \\ &\ge -Vp_{max}/\eta_{ch} - \eta_{dis}R_{dis}. \end{aligned}$$

b) if $K_b(t) < -Vp(t)/\eta_{ch}$, we have $\sum_{i\in\mathcal{I}} g^*_{s,i} + \sum_{i\in\mathcal{I}_a} g^*_{ch,i} = R_{ch}$ and $\sum_{i\in\mathcal{I}} g^*_{dis,i} = 0$. Thus, using the upper bound of V as the case above, we have

$$\begin{aligned} K_b(t+1) &= K_b(t) + \eta_{ch}R_{ch} \\ &\le -Vp_{min}/\eta_{ch} + \eta_{ch}R_{ch} \\ &\le S_{max}-S_{min}-Vp_{max}/\eta_{ch}-\eta_{dis}R_{dis}, \end{aligned}$$

while

$$\begin{aligned} K_b(t+1) &= K_b(t) + \eta_{ch}R_{ch} \\ &\ge -Vp_{max}/\eta_{ch} + \eta_{ch}R_{ch} \\ &> -Vp_{max}/\eta_{ch} - \eta_{dis}R_{dis}, \end{aligned}$$

3) Thirdly, suppose $-Vp_{min}/\eta_{ch} < K_b(t) \le 0$, we have $\sum_{i\in\mathcal{I}} g^*_{s,i} = 0$. As the case 2.a above, we consider two possibilities as follows:

a) when $-Vp_{min}/\eta_{ch} < K_b(t) \le -Vp_{min}/\eta_{dis}$, we have $\sum_{i\in\mathcal{I}} g^*_{ch,i} = \min\{R_{ch}, \sum_{i\in\mathcal{I}_a} g_{pv,i} - D^*_i\}$ and $\sum_{i\in\mathcal{I}} g^*_{dis,i} = 0$. Thus,

$$\begin{aligned} K_b(t+1) &= K_b(t) + \eta_{ch}\min\{R_{ch}, \sum_{i\in\mathcal{I}_a} g_{pv,i} - D^*_i\} \\ &> K_b(t) > -Vp_{min}/\eta_{ch} \\ &> -Vp_{max}/\eta_{ch} - \eta_{dis}R_{dis}. \end{aligned}$$

In addition, as $K_b(t) > -Vp_{min}/\eta_{dis}$, we have

$$\begin{aligned} K_b(t+1) &< -Vp_{min}/\eta_{dis} + \eta_{ch}R_{ch} \\ &\le S_{max}-S_{min}-Vp_{max}/\eta_{ch}-\eta_{dis}R_{dis}, \end{aligned}$$

b) when $K_b(t) \ge -Vp_{min}\eta_{dis}$, we have $\sum_{i\in\mathcal{I}} g^*_{ch,i} = \min\{R_{ch}, \sum_{i\in\mathcal{I}_a}(g_{pv,i}-D^*_i)\}$ and $\sum_{i\in\mathcal{I}} g^*_{dis,i} = \min\{R_{dis}, \sum_{i\in\mathcal{I}_b}(D^*_i - g_{pv,i})\}$. In other words, the maximum possible increase is

R_{ch} and the the maximum possible decrease is R_{dis}. Thus, using the upper bound of V as the case above, we have

$$\begin{aligned} K_b(t+1) &< K_b(t) + \eta_{ch} R_{ch} \\ &\leq -V p_{min}/\eta_{dis} + \eta_{ch} R_{ch} \\ &\leq S_{max} - S_{min} - V p_{max}/\eta_{ch} - \eta_{dis} R_{dis}, \end{aligned}$$

while

$$\begin{aligned} K_b(t+1) &> K_b(t) - \eta_{dis} R_{dis} \\ &\geq -\eta_{dis} R_{dis} > -V p_{max}/\eta_{ch} - \eta_{dis} R_{dis}. \end{aligned}$$

4) Finally, suppose $0 < K_b(t) \leq S_{max} - S_{min} - V p_{max}/\eta_{ch} - \eta_{dis} R_{dis}$, we have

$$\sum_{i\in\mathcal{I}} g^*_{s,i} = 0, \quad \sum_{i\in\mathcal{I}} g^*_{ch,i} = 0$$
$$\text{and} \quad \sum_{i\in\mathcal{I}} g^*_{dis,i} = \min\{R_{dis}, \sum_{i\in\mathcal{I}} (D^*_i - g_{pv,i})\}.$$

Hence,

$$\begin{aligned} K_b(t+1) &= K_b(t) - \eta_{dis} \min\{R_{dis}, \sum_{i\in\mathcal{I}} D^*_i - g_{pv,i}\} \\ &< K_b(t) \leq S_{max} - S_{min} - V p_{max}/\eta_{ch} - \eta_{dis} R_{dis}. \end{aligned}$$

In addition, as $K_b(t) > 0$, we have

$$\begin{aligned} K_b(t+1) &= K_b(t) - \eta_{dis} \min\{R_{dis}, \sum_{i\in\mathcal{I}} D^*_i - g_{pv,i}\} \\ &> -\eta_{dis} \min\{R_{dis}, \sum_{i\in\mathcal{I}} D^*_i - g_{pv,i}\} \\ &> -\eta_{dis} R_{dis} > -V p_{max}/\eta_{ch} - \eta_{dis} R_{dis}. \end{aligned}$$

From the induction, the boundary of $s(t)$ in (24) holds for any time slot with any control decisions derived from **P4**, which indicates that all constraints of **P1** are satisfied. Hence, all control decisions derived from **P4** are feasible to **P1**.

Proof of Proposition 2.2:

To prove Proposition 2.2, we first give the following lemma, which can be derived from Theorem 4.5 in [25].

Lemma 1. *There exists a stationary and randomized control policy Π that achieves the following:*

$$\mathbb{E}\{C^{\Pi}(t)\} = C^*_{P1}; \tag{41}$$

$$\mathbb{E}\{\eta_{ch} \sum_{i\in\mathcal{I}} [g^{\Pi}_{ch,i}(t) + g^{\Pi}_{s,i}(t)] - \eta_{dis} \sum_{i\in\mathcal{I}} g^{\Pi}_{dis,i}(t)\} \, \mathbb{E}\{\sum_{i\in\mathcal{I}} b^{\Pi}_i(t)\} = 0, \tag{42}$$

$$\mathbb{E}\left\{\frac{\overline{D}_i(t) - D^{\Pi}_i(t)}{\overline{D}_i(t) - \underline{D}_i(t)}\right\} \leq \beta_i, \tag{43}$$

where all expectations are taken over the randomness of the system state and the possible randomness of the energy charging/discharging and purchasing decisions.

Since the proposed algorithm is to minimize the RHS of (20), the value of the RHS should be smaller than that under the policy Π, which yield:

$$\begin{aligned} &\Delta(t) + V\,\mathbb{E}\{C^*_{P4}(t)\} \leq B + K_b(t)\,\mathbb{E}\{\sum_{i\in\mathcal{I}} b^{\Pi}_i(t)|\Theta(t)\} \\ &+ \sum_{i\in\mathcal{I}} H_{l,i}(t)\,\mathbb{E}\left\{\frac{\overline{D}_i(t) - D^{\Pi}_i(t)}{\overline{D}_i(t) - \underline{D}_i(t)} - \beta_i|\Theta(t)\right\} \\ &+ V\,\mathbb{E}\{\sum_{i\in\mathcal{I}} C^{\Pi}(t)\} \\ &= B + V C^*_{P1}, \end{aligned} \tag{44}$$

where (42) and (43) in Lemma 1 have been used. Taking an expectation over $\Theta(t)$ on both sides and summing over $t \in \{0, 1, 2, \cdots, T-1\}$, we obtain

$$V \sum_{t=0}^{T-1} \mathbb{E}\{C^*_{P4}(t)\} \leq TB + TVC^*_{P1} - \mathbb{E}\{G(T) - G(0)\} \tag{45}$$

Dividing both sides by TV yields:

$$\frac{1}{T}\sum_{t=0}^{T-1} \mathbb{E}\{C^*_{P4}(t)\} \leq \frac{B}{V} + C^*_{P1} - \frac{\mathbb{E}\{G(T) - G(0)\}}{VT} \tag{46}$$

Since $\mathbb{E}\{G(T)\}$ and $\mathbb{E}\{G(0)\}$ are finite, taking limits over T to infinity gives:

$$\lim_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1} \mathbb{E}\{C^*_{P4}(t)\} \leq \frac{B}{V} + C^*_{P1}. \tag{47}$$

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