Measuring the terminal heights of bolides to understand the atmospheric flight of large asteroidal fragments

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Abstract The extent of penetration into the Earth's atmosphere of a meteoroid is defined by the point where its kinetic energy is no longer sufficient to produce luminosity. For most of the cases this is the point where the meteoroid disintegrates in the atmosphere due to ablation process and dynamic pressure during flight. However, some of these bodies have particular physical properties (bigger size, higher bulk strength, etc.) or favorable flight conditions (lower entry velocity or/and a convenient trajectory slope, etc.) that allow them to become a meteorite-dropper and reach the ground. In both cases, we define the end of the luminous path of the trajectory as the terminal height or end height. Thus, the end point shows the amount of deceleration till the final braking. We thus assume that the ability of a fireball to produce meteorites is directly related to its terminal height. Previous studies have discussed the likely relationship between fireball atmospheric flight properties and the terminal height. Most of these studies require the knowledge of a set of properties and physical variables which cannot be determined with sufficient accuracy from ground-based observations. The recently validated dimensionless methodology offers a new approach to this problem. All the unknowns can be reduced to only two parameters which are easily derived from observations. Despite the calculation of the analytic solution of the equations of motion is not trivial, some simplifications are admitted. Here, we describe the best performance range and the errors associated with these simplifications. We discuss how terminal heights depend on two or three variables that are easily retrieved from the recordings, provided at least three trajectory (h, v) points. Additionally, we review the importance of terminal heights, and the way they have been estimated in previous studies. Finally we

Introduction

This chapter is dedicated to estimating the terminal heights of fireballs using a dimensionless methodology. The key ideas of this methodology have been described in another chapter (Gritsevich et al., this volume), so we will focus on the simplifications and variables that are able to provide us with a detailed estimation of terminal heights.

We will first introduce the terminology that we are going to follow, defining the terms to be used in this chapter: meteoroid, meteor, fireball and meteorite. Briefly, a meteoroid is, in most cases, a detached part of an asteroid or a comet. The size of a meteoroid may range from few tens of microns to tens of meters in diameter. Meteoroids originating from Mars or Moon are also possible, but they represent a much smaller fraction. When a meteoroid enters into the Earth's atmosphere, it produces a bright luminous path produced by its ablation. The light emitting object is called a meteor. Due to its high entry velocity (up to 73 km/s), the meteor experiences intense aerodynamic forces that produce an abrupt braking. One of the immediate effects is the intense interaction of the meteoroid surface material with atmosphere which causes ionization and subsequent emission of light. This effect is quite helpful to track the meteor on the sky (which is colloquially known as shooting star). Depending on several factors (mass, trajectory slope, size, velocity, etc.) a meteoroid could penetrate into the atmosphere. The atmospheric density of particles is higher and the temperature around the meteor increases which, eventually, melts the external layers and provoke the meteor to lose mass due to the interaction with the surrounding flow of air particles. This is the physical process called ablation. Due to the meteoroid ablation and the aerodynamic pressure during its flight trough the atmosphere most meteoroids eventually disintegrate in the atmosphere. We refer to these deep-penetrating luminous meteors as either fireballs or bolides. The amount of light emitted due to the ablation enhances their visibility from the Earth's surface. Typically, the brightness magnitude of a fireball reaches or overcomes that of Venus (-4). Very bright fireballs are able to get to even lower altitudes, reaching a brightness over -16, being observable at distances of more than 700 km (see e.g. Trigo-Rodríguez et al. 2009). Such bright fireballs are named superbolides; they are usually meteorite-droppers, surviving to the ablation processes, reaching the ground as meteorites. Thus, a meteorite is produced by a meteoroid that survived partially its atmospheric flight and reached the ground. In general, it is estimated that less than a 3% of the incoming (preatmospheric) mass can survive as meteorites (Ceplecha et al. 1998)
Besides, small grains orbiting in space may interact with the atmosphere at low velocities, may survive to atmospheric deceleration with partial or no melting at all. These particles are deposited on the ground as micrometeorites. All these descriptions are illustrated in Fig.1.

Recovered meteorites are generally classified based on their composition (see e.g. Weisberg et al. 2006). This classification is globally accepted and it associates each meteorite with a particular class according to cosmochemical and mineralogical patterns. Similarly, some classifications for meteors and fireballs were introduced in the scientific literature (see e.g. Ceplecha 1967, Gritsevich et al. 2012). Since meteors and fireballs may disintegrate in the atmosphere, any possible classification may rely on some physical variable of the atmospheric flight. Ceplecha (1967, 1968 and 1988) initially discussed that a combination of the beginning height of the luminous trajectory (including the dependence on velocity) with a parameter describing the product of heat conductivity, density and specific heat would lead to classification of sporadic meteors, especially those photographed with a Super-Schmidt camera. Four groups were defined, A, B C and D, each related to a different range of fireball properties and heights. It was also suggested that members of each group show similar orbital dynamics. Alternatively, the work of Ceplecha and McCrosky (1976) suggested that meteors could be classified based on their atmospheric dynamical behavior. As we will see later on, they used the terminal height as a criterion to distinguish between different populations of meteorites. This criterion allowed them to determine the grade of ablation experienced by the body during its atmospheric flight. Four different groups were described: I, II, IIIA and IIIB. There is no reason for extending our review here because an extensive summary of meteor science and more characteristics of these classifications can be found in Ceplecha et al. (1998).

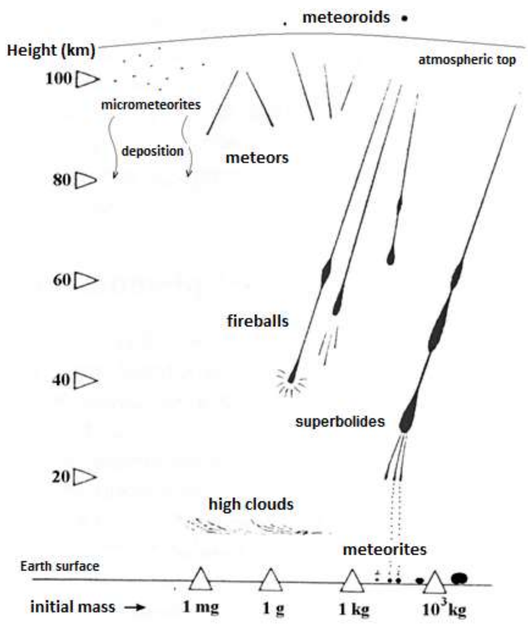

Fig.1. Graphical description of meteoroids, meteors, fireballs, superbolides, meteorites and micrometeorites. Adapted from Rendtel et al. (1995).

Needless to say that these studies are of great relevance for planetary defense

purposes. Meter-sized meteoroids encountering the Earth produce meteorites that turn into hazardous projectiles like e.g. the recent Carancas or Chelyabinsk meteorite falls (Tancredi et al. 2009, Borovička et al. 2013). These falls are able to release a large amount of energy either via a final impact onto the Earth surface, creating a crater, triggering an earthquake or even a tsunami; or if they do not reach the surface, the energy transferred to the atmosphere may produce dangerous airblasts. However, there is not an easy way to carry out an accurate analysis of these phenomena. Lots of considerations shall be taken into account, besides the number of cases and our previous experience on this subject is quite poor due to the small number of hazardous events documented. In 1908, over Tunguska River, a violent event associated with a possible meteorite fall took place. No meteorite was recovered but the shock wave associated with the likely final explosion of the fireball devastated an area of 2150 km^2 approximately, of which, 100 km^2 resulted with burnt trees (Popova et al. 2013, Vasilyev 2008). Local inhabitants felt an Earthquake ranging from 4.5 to 5 on the Richter scale. The energy released by this event has been estimated to 10-50 Mt (Chyba et al. 1993, Collins et al. 2005). Considering different entry conditions and meteor origins Chyba et al. (1993) suggested that for a carbonaceous body with an entry angle ranging from horizontal to 45 degrees, the energetic explosion should have occurred at 14 km above earth's surface. There was no crater and no meteorite was recovered, so it was suggested that the energy was completely released to the atmosphere leading to a massive airblast. For many years, this event was taken as an isolated event which is not fully understood. More recently, in February 2013, over the Russian city of Chelyabinsk another spectacular event took place. This time numerous civilian cameras recorded the phenomenon, thus providing the first well documented hazardous event. It was an approximately 19 m body that entered the atmosphere at 19.03 km/s with a grazing angle of 18.5 degrees (Borovička et al. 2013). The energy released was estimated to be around 500 kt, causing glass damage in nearby towns (Brown et al. 2013). The Chelyabinsk meteoroid suffered intense fragmentation between heights of 45 and 30 km, and only an 8 m hole on ice was found (Borovička et al. 2013).
These two cases are examples of the hazardous potential of tens of meters sized objects. Unfortunately, the small size of these bodies makes it complicated to detect and timely identify those that may lead to future encounters, and therefore, to properly quantify the risk of future events. Conversely, we could, for example, predict the amount of energy released in the atmosphere through a dedicated study on terminal heights.

The sky is constantly observed by photographic and video cameras, devoted to record any fireball event taking place. Most of them are arranged locally under the same image acquisition and reduction software, and the same institution requirements. This set up is commonly known as a fireball network (hereafter FN). Although fireball networks are currently widespread, it was not until 1936 that the first organized couple of cameras were co-pointed to the sky ruled by F.L. Whipple at the Harvard Observatory, the Harvard Meteor Project (Jacchia and Whipple 1956). From that moment and on, researchers realized the relevancy of these observations. Their recordings are fundamental for any meteor research. For example, in this chapter we will later make use of the fireball data provided by the

Meteorite Observation and Recover Project in Canada, MORP (1970-1985).
In line with its contemporary FNs, the MORP (Halliday et al. 1978) was created with the idea of gaining knowledge on the origin and properties of fireballs and meteorites. It consisted of twelve observatories located mainly in the south of Canada. The control headquarters were located at the campus of the University of Saskatchewan in Saskatoon. Each of the observatories had five cameras each of which covered 54º of azimuth near the horizon. Neighbor stations covered the part of the unrecorded azimuth area. The cameras used photographic films (see more details in Halliday et al. 1978) which gives an idea of the relative complexity compared to the current digitalization era.

MORP was able to register more than 1010 fireballs, including a meteorite-dropper. MORP 285, known as Innisfree, was recorded on the 6th of February of 1977, and was recovered twelve days later (Halliday et al. 1977, 1978 and 1981). Since the fall was accurately observed by two stations, its orbital trajectory could be calculated. This is quite remarkable since the number of recovered meteorites for which we have been able to calculate their orbit is quite low (for a review see: Trigo-Rodríguez et al. 2015).

Current instrumentation combines the use of charged couple devices (CCDs) in photographic and video cameras along with more sophisticated software and optics. Photo and video images are often supported by spectrograph devices coupled to observation cameras which reveal significant information about the chemical composition of meteors. Video frames and pictures provide details on the meteor flight trajectory. In order to correctly deal with pictures, cameras are provided with a chopping shutter, which allows to sequentially splitting the meteor trajectory into shorter sections during the exposition time. Using astrometry techniques (see Ceplecha 1998), the exact position and time of these sections (the procedure is similar for video frames) in the sky is calculated; this is, at each moment we can accurately determine the altitude, latitude and longitude of the meteor. Thus, velocity and height values for the whole meteor trajectory are derived. It is also possible to obtain the light curve of the meteor atmospheric flight which provides alternative ways of estimating the kinetic energy released and any fragmentation occurred during the atmospheric entry. A good determination of the trajectory of the fireball is crucial to extract further flight characteristics and fireball properties using theoretical models.

Once a fireball is registered, the data extracted from these ground-based observations should undergo an analytical study. Thus, the reliability of the mathematical model used is essential. The *Single Body Theory* (Hoppe 1937), a.k.a. *classical model*, has been widely used to describe the dynamical laws of the atmospheric flight. Briefly, this theory considers the following coefficients as constant: drag, luminosity, heat transfer and ionization. The atmosphere is assumed to be isothermal and the fragmentation of the fireball cannot be modeled (except for separation of small particles). The major handicap of this theory is its accuracy. Normally, for the calculation of the value of certain variable we need to set beforehand the values of other variables. These values are not always known, so it

is common practice to use the generally accepted mean values. Each case requires careful analysis and high accuracy cannot be assured.
Alternatively, the dimensionless methodology presented by Gritsevich et al. (2016, this volume), overcomes this problem and offers a new point of view. It has been recently applied to the determination of the terminal heights and we will discuss in following lines its validity and any further improvements.

We would like to remark that we will assume no sudden fragmentation in the analysis as described below. Only the main fragment is studied and all changes to its mass along the trajectory are approximated using a physically based approach. The inclusion of discrete fragmentation is only possible when dealing with very well-documented cases (with well-observed fragment trajectories) and it automatically increases the complexity of the study with an unknown number of free parameters, but a good handling of the problem shall improve the performance of the results. For instance, Revelle (2007), based on his previous work with Ceplecha (Ceplecha and Revelle 2005), explored the possibility of improving the *Single Body Theory* estimation of the terminal mass of any bolide. By including a fragmentation model called TPFM (triggered progressive fragmentation model), Revelle suggested that important atmospheric flight values like the end mass or the ablation should have a limited upper value defined by those of the *Single Body Theory*, being, in fact, smaller. The TPFM is mainly based on introducing the variation of the ratio defined by the cross-section area (participating in the deceleration of the body) and the mass of the body during the flight time: m(t)/A(t). In order to get a first approach to the particularities of each event, Revelle (2007) assumed two main subcases. On the one hand, the fragments of the bolide continue flying along with the main fragment; in this case the cross-sectional area increases (more drag) and the general mass has not changed. On the other case, the mass detached from (mainly the back face of) the remaining main fragment move away from it quite fast, the main fragment does not show a variation in its flight configuration and drag is still the same; now, the mass is reduced and the cross-section area remains the same. Though this methodology partially leads to good results, there is still more work to be done (for example other thermal effects should be considered). It is clear that the real advantage of the TPFM is including the meteor fragmentation in a consistent way, which could be crucial in some events.

The topic of this chapter is devoted to the terminal heights which are one of the most relevant and characteristic parameters of the atmospheric flight of fireballs. It corresponds to the final point of the luminous part of the trajectory. This is the point where a fireball disintegrates or, for meteorite-droppers, the last point where luminosity is present. Meteoroids typically disintegrate at a pretty well defined atmospheric height depending on their particular tensile strength (Trigo-Rodríguez and Llorca 2006, 2007). The terminal height also describes the amount of deceleration experienced by the meteor, which in turn means the degree of penetration into the atmosphere.

The study of the flight conditions and fireball properties leads to a better understanding of their terminal heights. And vice versa, we could gain better insight into the composition of the fireball, relate it to a parental body (i.e. a particular asteroid or comet), and, when the trajectory is accurately described, obtain the orbital elements that describe its space motion, analyzing their terminal heights.

Finally, by accurately calculating and observing the terminal height for any fireball, mathematical models can be adjusted and other atmospheric flight properties can be obtained. Both the calculated and the recorded values could differ sometimes, for several reasons. On the one hand, the calculated terminal heights depend on various parameters (entry mass, bulk density, shape coefficient, ablation coefficient, etc.). If the real values of these parameters have not been derived from observations, then, they are commonly assumed to be close to the accepted mean values. On the other hand, the recording of fireballs relies on the spatial resolution and instrumental limitations. Weather conditions do also affect the ability to record the whole flight trajectory.

Terminal heights in the literature

The chances to gather and extract a lot of information from FN have increased along the years. Previous studies did take advantage of this situation to gain knowledge in meteor science. As we have mentioned, the *Single Body Theory* was the most detailed mathematic model used to deal with calculations. Clear example of this is the work of Ceplecha and McCrosky (1976) on the Prairie Network (PN). Operated in USA between 1963 and 1975, the PN registered more than 2,700 fireballs, one of them being the Lost City meteorite (McCrosky et al. 1971). Being conscious of the importance of this database, Ceplecha and McCrosky (1976) undertook a deep analysis aiming to distinguish between ordinary and carbonaceous chondrites within recorded fireballs. Their research was based on the idea that carbonaceous chondrites are more fragile than ordinary chondrites. This means that the atmospheric flight of the carbonaceous chondrites should be shorter, mainly due to their higher ablation. In other words, the terminal heights shall be higher for carbonaceous than for ordinary chondrites. Generally, this should be true, but every meteor has its own peculiarities that modify its trajectory: the trajectory angle, the entry velocity, the shape, etc. Therefore, terminal heights cannot be considered as the sole classifying criterion. In order to account for these relevant parameters in any further classification based on terminal heights, Ceplecha and McCrosky (1976) suggested a new parameter, PE. This parameter is defined by the addition of the logarithms of the entry mass, velocity and zenith distance. The terminal height is included through the air density at that point. Note that all the parameters involved in defining PE express the atmospheric dynamic behavior of the meteor:

$$PE = log\rho_E + Alogm_\infty + BlogV_\infty + Clog(cosZ_R) \qquad (1)$$

Where the air density at terminal height is ρ_E [*g/cm*^{3}], the preatmospheric mass, m_∞ [*g*], preatmospheric velocity, V_∞ [*km/s*], and the zenith distance of the meteor radiant, Z_R [*degrees*]. Constants A, B and C are adjusted for all the meteor trajectories.

Ceplecha and McCrosky (1976) also realized that the observation of each fireball provided extra information that is not included via the PE parameter. For instance, the ablation coefficient (σ [s^2cm^{-2}]) and the geometrical coefficient depending on the shape of the object, the drag coefficient and the bulk density, called K [$cm^2g^{-2/3}$], were also determined from the trajectory when the observations and the theoretical equation were compared. The information of these two parameters was available for ninety meteors of the PN. The authors used this valuable information to set a second criterion which considered the sum of the average values of σ and K for the entire meteor trajectory. This SD criterion describes globally the physical changes that the meteor suffers during its atmospheric flight (change in mass, surface, ablation, etc.):

$$SD = \langle logK \rangle + \langle log\sigma \rangle \quad (2)$$

Despite the PE criterion was thought to be used as a unique classifying parameter, the combination with SD criterion could provide more accurate results and unambiguously characterize a fireball.

Ceplecha and McCrosky (1976) showed that (1) and (2) are theoretically related. This is important in three senses. First, (1) was initially intended to be empirical, but it has been proved that the *Single Body Theory* could explain it. Second, it is interesting to note that K is a function of the shape of the object and its density so it indirectly depends on the fireball fragmentation; therefore, one of the *Single Body Theory* weaknesses can be partially overcome including the SD criterion as well. Finally, it should be mentioned that since SD depends on the second derivative of observed measurements, it is less affected by observational errors than PE. Consequently, both parameters are supplementary.

The ablation and geometric coefficients in (2) along with the atmospheric density at the terminal height in (1), describe the meteor atmospheric flight dynamics. On the contrary, the classification suggested in Ceplecha (1967, 1968 and 1988) relies on the beginning height and pre-atmospheric orbit. Ceplecha and McCrosky (1976) discussed a possible relationship between both classifications but the only objects that could be compared in both studies are those with Taurid-like orbits. This leads to a small sample of nine bodies available, thus no strong conclusions could be derived. They also stated that results using the ablation and geometric coefficients values recorded at the last luminous point of the meteor trajectory instead of the average values seem to provide better results.
This new way of providing fireball classification complements the existing meteorite's composition based classification, and it is an alternative to the already mentioned classification suggested by Ceplecha (1967, 1968 and 1988).

Few years later, Wetherill and Revelle (1981) published a work where they also considered the possibility of distinguishing the ordinary chondrites present in the PN data. The authors suggested that a large number of ordinary chondrite falls may not have been found due to both, their small terminal masses and their small sizes.

The authors also discussed that previously accepted low bulk density values affected studies of meteorite falls. The authors claimed that, based on previous work from Revelle and Rajan (1979) and Revelle (1980), masses derived photometrically could be ten times higher than real masses, whereas dynamic masses could be two times smaller (just as a short review, the photometric mass is that derived from the light curve of the meteor; we assume the energy emitted represents up to a certain point the amount of kinetic energy of the body; as for the dynamic mass, it is the mass obtained from the deceleration equations of motion). Differences also were spotted regarding the cross-sectional area (about twice higher than for assumed spherical shape), which is related to any possible fragmentation occurred during the atmospheric flight (fragments flying very close to the main fragment increase the effective cross sectional area). Owing to this, chances that small ordinary chondrites could produce meteorite falls increased. However, not every ordinary chondrite meteor may survive its atmospheric flight, as it also depends on the initial mass and entrance geometry. Besides, meteorites with different compositions were present in the PN data set, so we cannot exclude that the terminal heights of these meteorites could be similar to those of ordinary chondrites. However, Wetherill and Revelle (1981) assumed that chances of this non ordinary chondrites presenting such terminal heights (within the PN data) were less than 16%.

In order to filter the dominant ordinary chondrites from other types, Wetherill and Revelle (1981) stated that any other ordinary chondrite present in the PN should show, scaled up to certain point, the same behavior as the Lost City meteorite. They expressed it mathematically through four criteria. A dedicated review of their work is recommended to any interested reader. We only remark here one of these criteria. Their third criteria stated:

> End height agrees with the Single-Body Theory theoretical value, calculated using dynamic mass, as well as with that of Lost City to within ± 1.5 km, when scaled for mass, velocity, and entry angle in accordance with classical meteor theory.

Once again, the terminal height appears as fundamental parameter of the analysis. Wetherill and Revelle (1981) considered the agreement between the observed and theoretical terminal heights as an indicator of a good adjusted meteorite atmospheric trajectory, this is, they showed similar deceleration, drag coefficient, ablation, etc. A normalization of this terminal height allowed it to be compared to the Lost City corresponding value, assuming a deviation of ± 1.5 km due to the errors in the calculation of the dynamic mass. As we have mentioned, they considered the dynamic mass to be more accurate than the photometric mass used in Ceplecha and McCrosky (1976). Anyway, they were aware of the fact that the derivation of both masses is affected by different errors and assumptions. All in all, as they stated, despite of the different methodologies applied, the final amount of ordinary chondrite fireballs identified within the PN is similar to the previous work of Ceplecha and McCrosky.

New way of calculating the terminal height

We have reviewed through a couple of very relevant bibliographic works the importance of the fireball terminal height to understand its atmospheric flight and to derive further properties. Up to now, the efforts to determine this value analytically relied on the classical theory. However, the large number of unknowns involved in the classic theory affects the accuracy of the results. Generally, the use of mean values for these variables is accepted. Nonetheless, there are cases where these values are far from realistic. The introduction of scaling laws and dimensionless variables helps to overcome these inaccuracies. The next lines will show how this new modelling can be applied to the analytical determination of the terminal height. Particularly we are going to focus on the simplifications of the exact solution achieved using this methodology.

Let us consider the dimensionless approach to describe the atmospheric trajectory developed by Stulov et al. (1995, 1997) and Gritsevich (2007). Since this methodology has been explained in Gritsevich et al. (2016, this volume) we will not delve into every detail but explain its basics. The dynamical behavior of a meteoroid that enters the Earth's atmosphere can be described using the Newton equations of motion. We are mainly interested in the variation with time of the velocity (deceleration) along its path, the height and the mass variation. If we project the meteoroid movement along its trajectory and consider the mass variation equation, we can easily derive the following expressions:

$$M\frac{dV}{dt} = -\frac{1}{2}c_d\rho_a V^2 S \tag{3}$$

$$\frac{dh}{dt} = -V \cdot \sin\gamma \tag{4}$$

$$H^*\frac{dM}{dt} = -\frac{1}{2}c_h\rho_a V^3 S \tag{5}$$

Were M is the mass, V the velocity, γ the slope between the trajectory and the horizon at each time, t is the time, h the height above the Earth's surface, S is the area of the middle section of the body, ρ_a is the density of the atmosphere, c_d is the drag coefficient, c_h is the heat exchange coefficient and H^* is the effective destruction enthalpy. Provided the high entry velocities, the effect of the drag is much higher than the gravity acceleration and this is usually not considered, that explains why we have not included it in equation (3).

Due to the large number of variables extra equations are required. Normally we accept the atmosphere as isothermal, which leads to an exponential equation for the atmospheric density $\rho/\rho_0 = exp(-h/h_0)$, where ρ_0 is the atmospheric density at sea level and h_0 =7.16 x 10^3 m is the scale height (note that Lyytinen and Gritsevich (2016) describe how to use more elaborate atmospheric models on the case-by-case basis). In addition to this, Levin (1956, 1961) suggested that the variation of the middle section and the mass of the body are related owing to $S/S_e = (M/M_e)^{\mu}$, where μ is a constant that indicates the spin velocity of the body during the flight (see also Bouquet et al. 2014), and the *e* subscript refers to the values of the variables when the body enters the atmosphere.

In order to study the variation of *M* and *V* with height, we combine equations (3) to (5) and the extra expressions. However, we introduce dimensionless variables ($M = M_e m$, $V = V_e v$, $h = h_0 y$, $S = S_e s$ and $\rho_a = \rho_0 \rho$) and solve the resulting equations with the conditions $y = \infty$ and $v = 1$ (for details see Gritsevich et al. this volume):

$$m = \exp[-(1 - v^2)\beta/(1 - \mu)] \tag{6}$$

$$y = ln2\alpha + \beta - ln\Delta, \quad \Delta = \overline{Ei}(\beta) - \overline{Ei}(\beta v^2) \tag{7}$$

Where $\quad \overline{Ei}(x) = \int_{-\infty}^{x} \frac{e^t}{t}.$

As a consequence of including dimensionless variables two new parameters appear in (6) and (7). The parameter α is a ballistic coefficient which characterizes the drag intensity (Eq. 8); and β is called the mass loss parameter which characterizes the ablation of the meteor body (Eq. 9), it is proportional to the fraction of kinetic energy of the unit mass of the body that is transferred to the body in the form of heat divided by the effective destruction enthalpy.

$$\alpha = \frac{1}{2} c_d \frac{\rho_0 h_0 S_e}{M_e sin\gamma} \tag{8}$$

$$\beta = (1 - \mu) \frac{c_h V_e^2}{2 c_d H^*} \tag{9}$$

The exact solution of the problem (Eqs. 6, 7) admits some simplifications. For fast meteors that show little deceleration during the luminous path, we can approximate $v = V/V_e = 1$. In these cases, β >> 1 given the high evaporation process that takes place. Stulov et al. (1995) suggested an alternative asymptotic solution of the system (6) and (7) for these cases:

$$v = 1, \quad m^{1-\mu} = 1 - 2\alpha\beta e^{-y}, \quad ln2\alpha\beta < y < \infty \tag{10}$$

However, (10) does not consider the final deceleration in the vicinity of $m = 0$. This would provide unrealistic results in some cases and it does not account for the drag process until that point. An appropriate way of solving this disadvantage consists of combining (10) with (6) which is suitable for arbitrary β values (Gritsevich 2008b):

$$v = \left(\frac{ln(1 - 2\alpha\beta e^{-y})}{\beta} + 1\right)^{1/2}, \qquad ln2\alpha\beta < y < \infty \tag{11}$$

These two equations (10) and (11) provide the first simplified solutions of the exact analytical solution expressed in (6 - 7). Analytically, the terminal height is the last point of the registered luminous path of the meteor atmospheric trajectory (when ablation processes are over). This point is reached when $m = 0$ in (10) (given that v remains constant) and it is calculated via $v_t = V_t/V_e$ for (11). Note that at this point, that the terminal velocity of a fireball is the velocity at its terminal height, and this velocity is represented as V_t.
For the sake of clarity in the following discussion we would use the dimensional height values for the mentioned points. We use the subscripts I, II, III, etc., to indicate the different ways of expressing the terminal height according to the simplifications made in (10) and (11). The resulting terminal height for fast meteors (Eq. 10) will be called hereafter h_I, and for the simplified solution where some deceleration is considered (Eq. 11) we will use subscript II, h_{II}:

$$h_I = h_0 y_t = h_0 \cdot ln 2\alpha\beta \tag{12}$$

$$h_{II} = h_0 y_t = h_0 \cdot ln \frac{2\alpha}{\left(1-e^{\beta(v_t^2-1)}\right)} \tag{13}$$

We shall remark here that, as explained in Gritsevich et al. (2016, this volume), for small β values (β < 2), we recommend the use of the asymptotic expression suggested by Kulakov and Stulov (1992) and Stulov et al. (1995), which provides very good results.
Given (12) and (13), we can point out that, for fast meteors, where deceleration is not accounted for, the terminal height (h_I) is a function of the dimensionless parameters α and β. As we have stated this simplification is not always true and may be only applied in some well-studied cases. Fireballs do decelerate before disintegrating or starting its dark flight when they are meteorite-droppers, therefore a second approximation is suggested for terminal heights (h_{II}). This new terminal height depends on α, β and the terminal velocity.
The terminal velocity is obtained from observations. Sometimes the final part of the trajectory could not be visible or even recorded, but the derivation of α and β only needs three observed (h, v) points, one of which should be the entry point (the entry velocity). Using these three points it is possible to obtain the remaining (h,v) trajectory points from the adjusted fireball trajectory (see Whipple and Jacchia 1957). Though the terminal point of the trajectory would not be exactly determined using this adjustment, a combination of this adjustment with other methodologies and/or hypotheses shall lead to a good estimation. Consequently, the dimensionless methodology allows us to calculate the terminal height depending only on two (α and β) parameters for $h_{I,}$ or three (α, β and V_e) parameters for h_{II}.

Regarding the entry velocity, which is required to obtain α and β, and to scale velocity values, in principle it is possible to consider it as another unknown and derive it along with α and β values as discussed by Gritsevich (2009). We foresee a future study on this subject in order to improve the methodology.

As for the derivation of α and β parameters, it is done via a least-squared method applied to the observed height and velocity values using (7). As mentioned, it requires at least three points of the trajectory, including the entry velocity. A detailed explanation of this derivation can be found in Gritsevich (2007) and Gritsevich (2008a, b). Although α and β derivation only requires three observed (h,v) values, since parameters α and β mainly describe the meteor deceleration, and this is remarkably present in the last part of the luminous trajectory of the meteor flight, it is highly recommendable to include (h,v) values of the part where the main deceleration is present. This will generally decrease the error in the results which, otherwise, may differ from the real behavior of the meteor; it is particularly convenient for meteors that penetrate deeper into the atmosphere due to the great amount of deceleration that they suffer.

In Moreno-Ibáñez et al. (2015) the accuracy of (12) and (13) was tested against the observed values of 143 meteoroids recorded by the MORP during atmospheric flight. It is important to recall that one of these bodies was recovered as a meteorite, thus proving the validity of the methodology for meteorites as well. The standard deviation of the results decreased from h_I (standard deviation is 4.11 km) to h_{II} (standard deviation is 1.52 km). Nonetheless, the results obtained with h_{II} showed a lack of agreement between observed and calculated terminal heights at low heights, which, on average, are related to low β values. This was assumed to be related to the combination of (6) and (10), which used simplified functions of the general solutions and are thought for high β values.

In order to get a better performance of h_{II} compared to the analytical solution of the problem, Moreno-Ibáñez et al. (2015) made use of the mathematical analysis carried out in Gritsevich et al. (2016). This analysis sought for the possibility of including an approximation function which slightly modified this equation. Both, the analytical solution (7) and the simplified calculated height for decelerated meteors (11) have no singularities and are monotonous on the interval $0 < v < 1$. Besides, the dependency on α is the same (through $ln\alpha$) for both equations. Thus, the use of an approximation function is possible and it shall only affect parameter β. It shall be remarked that this approximation function is thought to improve accuracy in those cases where β > 3, otherwise the previous simplified solutions and the asymptotic solution are, in principle, more reliable.

By means of these approximation functions we try to adjust the mathematically derived results just in the range of meteor velocities values that we are usually interested in. This is $v \in [0.3, 1)$. These functions are meant to work for fixed α and β (although α is not strictly required in this analysis) values but the error analysis carried out by Gritsevich et al. (2016) prove that they work more efficiently for determined β values, depending on the approximated function used. The approximation function suggested by Gritsevich et al. (2016) is introduced through the β parameter where parameter β is substituted by β-A in (13); A represents the approximation function. We seek that expression (11) approaches, within the range of values mentioned, the analytical solution (7):

$$ln\left(\frac{2\alpha(\beta-A)}{1-e^{(\beta-a)(v^2-1)}}\right) \approx ln\alpha + \beta - ln\frac{\Delta}{2} \tag{14}$$

We will not describe the whole process here, but it is worth mention that function A is a function of both β and v, A = A(β,v), and its final shape is quite complex. The direct use of A(β,v) in (11) would stand up against the search of simplification we are looking for. Nonetheless, after analyzing different efficient simplifications for function A(β,v) and their attached errors, two reliable possibilities came out (Gritsevich et al. 2016):

- A_0=1.1
- A_1=1.0+(1.0-v)·(2.5)/β

We shall recall here, that we are approximating mathematically (11) to the exact solution (7). Hence, the new expression of the terminal heights that we will introduce are still simplified solutions of (7) and their results should be considered in terms of fast meteors where deceleration has been accounted for.

The error analysis performed when using A_0 and A_1 in (14) suggests that the optimal performance of these approximations (given 20-30 meteor trajectory points observed) occurs at $\beta \approx 2.89$ for A_0, and at $\beta \approx 2.1$ for A_1. The average statistical deviation for any derived parameters using A_0 is 5-10% and 1-2% for A_1 (Gritsevich et al. 2016).

Then, if we apply this function approximation to the derivation of the terminal height (13), our accuracy should increase. We first start using $A = A_0$, we decided to call this new terminal height h_{III}:

$$h_{III} = h_0 \cdot ln\left(\frac{2\alpha(\beta-1.1)}{1-e^{(\beta-1.1)(v_t^2-1)}}\right) \tag{15}$$

Its validity has been tested by means of the fireball data gathered by the MORP network (Moreno-Ibáñez et al. 2015). The graphical representation of these results is plotted in Fig.2.

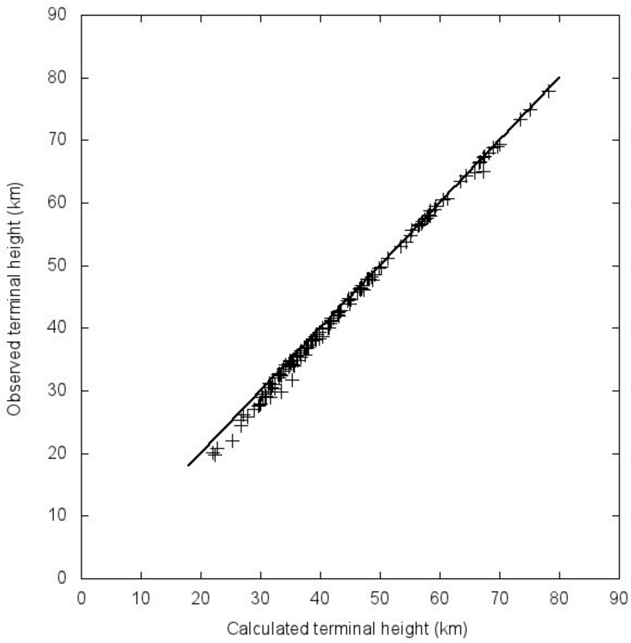

Fig.2. Observerd terminal height values (MORP) vs. calculated values (h_{III}). The line $h_{observed}=h_{calculated}$ is also plotted (from Moreno-Ibáñez et al. 2015).

The resulting accuracy is quite good. The standard deviation is reduced down to a value of 0.75 km. It can be pointed out that most of the error come from the lowest height values, which are again mainly associated with small β values. Since the approximation A_0 is supposed to show a better performance for higher values of the mass-loss parameter, these differences were expected to appear. Note that h_{III} is the result of a simplification made on the analytic solution of the equations of motion. Thus, despite including the mathematical modification suggested by Gritsevich et al. (2016) we may still appreciate a residual error due to the original simplification assumed. All in all, the adjustment proved to be good and promising.

Alternatively, we present here the analysis of the terminal height (13) for the MORP database using A= A_1. Let's call the new expression for the terminal height h_{IV}:

$$h_{IV} = h_0 \cdot ln\left(\frac{2\alpha\left(\beta-\left(1.0+(1.0-v_t)\right)\frac{2.5}{\beta}\right)}{1-e^{\left(\beta-\left(1.0+(1.0-v_t)\frac{2.5}{\beta}\right)\right)(v_t^2-1)}}\right) \tag{16}$$

Results of this new analysis are plotted in Fig.3.

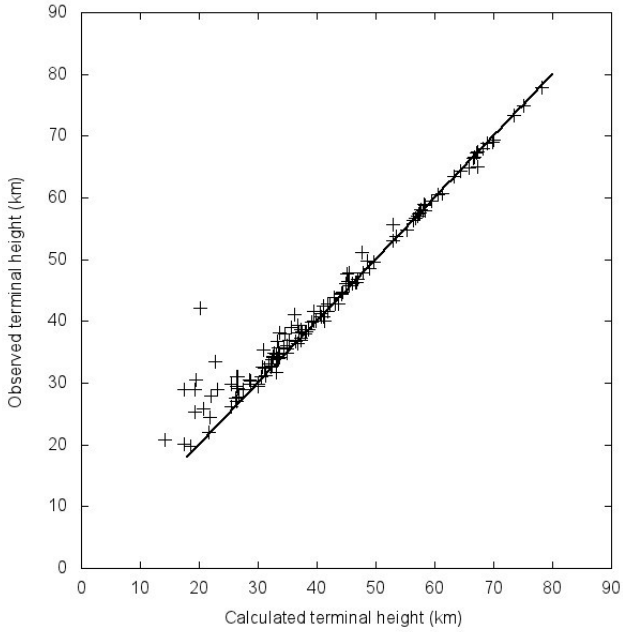

Fig.3. Observerd terminal height values (MORP) vs. calculated values (h_{IV}). The line $h_{observed}=h_{calculated}$ is also plotted.

This case should be analyzed in a more careful way. But, as we can observe in Fig.3, the correlation between the observed and calculated terminal heights is broken again for low height values (low β values). Besides, now the differences between the calculated and the observed terminal heights are negative. The explanation can be found in β values. As we have discussed, these approximations were thought to work efficiently for β > 3, and we should only consider its accuracy for that range of results. Graphically, Fig.4 and Fig 5 show the relationship between the mass-loss parameter and the h_{III} and h_{IV} respectively. The sudden change in accuracy is quite clear at the right side of the dashed line indicating β = 3. Lower β values show different levels of terminal height accuracy.

Additionally, it is quite interesting to note that, for the MORP database, the use of h_{III} lead to better global results (including those fireballs with β < 3) than h_{IV}; conversely, h_{IV} shows better adjustment if consider only meteors with β > 3. The global results for h_{IV} are biased by five cases at very low β values; hence, avoiding the contribution of these events to the global accuracy the global accuracy enhances dramatically.

To summarize, the approximation functions proposed in Gritsevich et al. (2016) are apparently capable to improve the general behavior of the dimensionless methodology when we consider the simplifications to the analytic solution described (10) and (11). In particular, the main objective is to solve the problems with the accuracy derived for high β values. We have used them here for the specific case of the terminal height and focusing on the problems which arise at low β values of h_I and h_{II}. In these last cases, further study should be addressed. However, we have proved that the use of A_0 enhances the global accuracy of (11). This is explained by the improved accuracy at moderated β values, which may include some of the meteors that are able to penetrate to lower heights with such moderate β values.

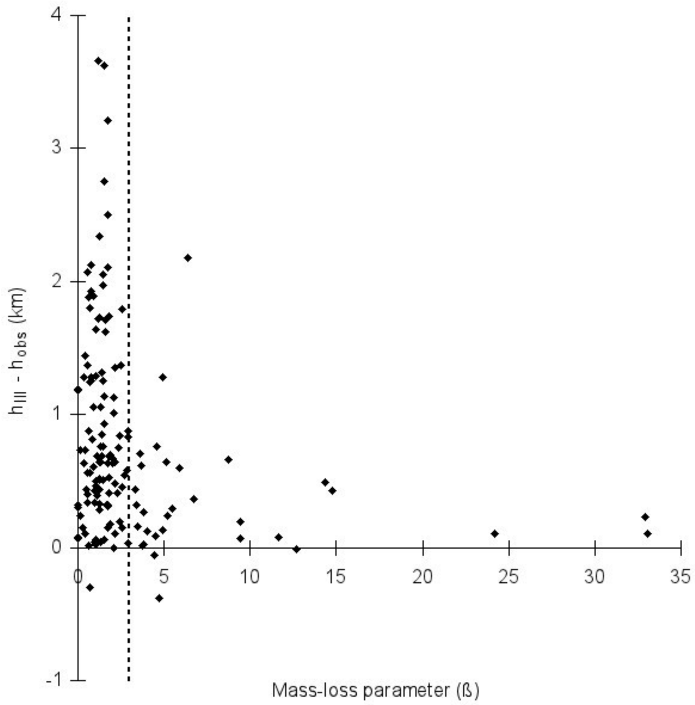

Fig. 4. Mass-loss parameter (β) against h_{III}-h_{obs}. The dashed line indicates β =3.

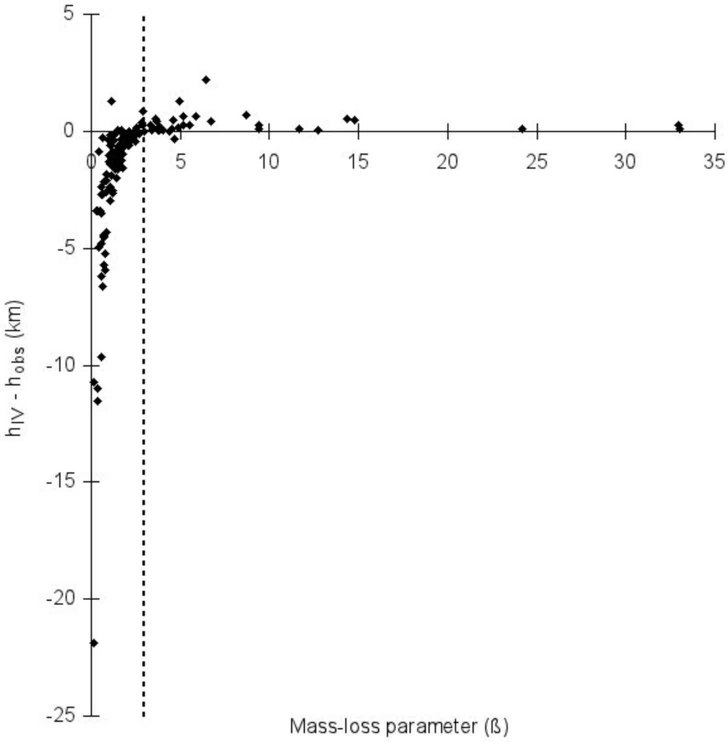

Fig. 5. Mass-loss parameter (β) against h_{IV}-h_{obs}. The dashed line indicates β =3.

Conclusions

Along this chapter we have presented the utility of the terminal height for meteor science and the mathematical adjustment provided by the values derived from the dimensionless methodology. The results shown in this chapter are summarized in following discussion:

1.- The dimensionless terminal height expressions presented in this chapter provide useful tools to tackle previously analyzed problems. The methodology discussed in Ceplecha and McCrosky (1976) in order to distinguish between ordinary and carbonaceous chondrite fireballs recordings, could be alternatively approached by using dimensional analysis. Furthermore, the terminal heights introduced here largely resemble the PE criterion suggested by Ceplecha and McCrosky (1976). The mathematical definition of α depends on the ratio of preatmospheric cross-section to preatmospheric mass (a ratio easily convertible to bulk density, preatmospheric mass and shape coefficient, all of these parameters are used in the PE criterion), and on the trajectory slope γ related to Z_R of the PE criterion. Respectively, the mass-loss parameter is proportional to preatmospheric velocity with a power of two and inversely proportional to the effective destruction enthalpy. Thus, the degree of penetration of fireballs into the Earth's atmosphere is correctly described with the definition of terminal heights discussed in this chapter.

2.- The dimensionless methodology is able to describe in a simple way the physical event. For example, the ablation coefficient is easily derived from the mass-loss parameter, $\sigma = 2\beta/(1-\mu)V_e^2$. Note that generally $\mu = 0$ or $2/3$ (see Bouquet et al. 2014), this constraints the value of μ when deriving the ablation parameter and, hence, the derivation is quite straightforward. Given the difficulties of deriving the exact values for some physical properties (i.e. bulk density, shape, etc.) from the observation, the reduction of unknowns achieved with this methodology could be used as a powerful tool to pursue a classification based on α and β parameters (see previous chapter by Gritsevich et al., this volume). In some cases it could be quite convenient to use h_{III} or h_{IV} and β to characterize different events instead of using a combination of α and β. Particularly, members of meteor showers (generally carbonaceous chondrites) can be classified using these two parameters, given the excellent behavior of h_{III} and h_{IV} for high β values.

3.- The discrepancies between observed and calculated terminal heights found at low β values have to be studied in more detail. Typically, meteorite-droppers have low β values, which mean low ablation and thus, higher chances of survival. In other words, tough bodies (such as ordinary chondrites) may be affected by this error, and any further study should be aware of it. We already mentioned that these discrepancies could be due to simplifications arising from the analytical solution. Though the mathematical modification introduced by means of the approximation function A_0 is able to correct the global accuracy of the results, the local deviation at low β is still present. This is also of particular relevance for any planetary defense study. At low β values the suggested calculated terminal heights have lower values than observed values. This would mean that any prediction about the atmospheric penetration of fireballs based on h_{II} or h_{III} would indicate higher values that the observed ones. On the contrary, the values suggested by h_{IV} would be lower than the real recordings. Anyway, given that observations also involve various errors (atmospheric conditions, whole trajectory recording, resolution of the camera, etc.), this subject should be studied in more detail.

4.- For significantly decelerated bolides and a few well-studied cases, such as the Innisfree meteorite, published terminal heights may differ depending on the data-reduction approach used (e.g. 21 km in Halliday et al. (1981); 19.8 km in Halliday et al. (1996)). This not only affects the accuracy between calculated and observed terminal heights for any particular fireball, but also the global accuracy of the methodology described here. Nonetheless, this could be taken as an opportunity. The dimensionless methodology could set constraints on terminal heights and fireball flight duration values, which may help to put adequate restrictions on the recorded values.

5.- It is worth noticing that a good estimation of terminal heights opens new fields of studies. First, it is possible to forecast terminal heights when the last part of the fireball trajectory has not been recorded, which happens quite often. The more number of recorded points, the better the accuracy (α and β do strongly depend on the deceleration, and this is better described with an increasing number of (h, v) trajectory observations), but it is still possible to obtain α and β with only three recorded points. Depending on each event this may have little influence on their derivation. This is quite advantageous if we consider a fast meteoroid because, no more parameters are required (see Eq. 6). On the contrary, for decelerated bodies with high β values, the missing (h,v) at the end of the trajectory could be adjusted based on the rest of the trajectory data (Whipple and Jacchia 1957). Thus, as discussed in this chapter, V_t (the terminal velocity) could be obtained (provided some assumptions or extra data from other observational techniques) for most of the registered fireballs and h_{III} or h_{IV} could be derived.

Secondly, it is also notable, that meteor height may be expressed as a function of time. Thus, the ability of predicting terminal heights may be directly linked with the forecast of a total duration of meteor phase. This leads to a new class of problems, such as, for example, insights into determination of luminous efficiency based on meteor duration and calculation of critical kinetic energy needed to produce luminosity.

6.- For the MORP data studied here, the use of the new implemented h_{III} provides more accurate global results than previous h_I and h_{II} terminal heights. Conversely, the use of h_{IV} shows some unexpected discrepancy that can be explained due to five cases with low β values. However, in this case, the adjustment using the approximated function A_I is more precise for values of $\beta > 3$ (see Fig. 5). This is in agreement with the results discussed in Gritsevich et al. (2016). Mathematically, these authors concluded that close to $\beta \approx 2.89$ for A_0, and at $\beta \approx 2.1$ for A_I (provided $v \in [0.3, 1)$) the difference between (13) and the resulting expression using approximated functions is optimized. This statement has been tested with a large amount of real cases in this chapter, supporting the analytical study.

7.- Direct comparison between h_{III} and h_{IV} could be used for other purposes. The results presented here correspond only to one FN. It is still difficult to conclude whether h_{III} or h_{IV} would provide better general results for other FN data. It could be interesting to find out whether h_{III} is able to absorb better the widespread in results for different β values. According to MORP results, h_{III} achieves a better global accuracy. This is in part due to the five cases that bias the global accuracy achieved with h_{IV}. Nonetheless, it seems that terminal heights of fireballs showing moderate β values are more accurately determined using h_{III}. Resolving whether better global results are obtained either with h_{III} or h_{IV} might be quite useful in two senses, to detect and avoid systematic errors in database recordings, and to derive fast accurate results for large sets of data.

Globally, the terminal heights studied in this chapter have proved the dimensionless methodology to adequately describe the atmospheric flight of fireballs using three variables (α, β and V_e). Thus, it could be very interesting to use it with other FN. We foresee its application to the Finnish Fireball Network and the Spanish Meteor

Network.

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