astro-ph/0208282
HIP-2002-05
August 14, 2002

A primer on the ekpyrotic scenario

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Abstract

This is an introduction to the ekpyrotic scenario, with an emphasis on the two contexts of brane cosmology and primordial universe scenarios. A self-contained introduction to brane cosmology and a qualitative overview and comparison of the inflationary, pre-big bang and ekpyrotic scenarios are given as background. The ekpyrotic scenario is then presented in more detail, stressing various problems.

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Chapter 1

Introduction

1.1 An overview of the paper

This is intended to be a useful general level introduction to the *ekpyrotic scenario* [1-7]. Familiarity with brane cosmology topics such as the *Randall-Sundrum model* [8, 9] or other related fields is not assumed. I have attempted to keep the presentation reasonably self-contained, and it should be up-to-date on references as regards the ekpyrotic scenario (though not necessarily on tangential topics).

The ekpyrotic scenario was presented about a year and a half ago, in March 2001, as an alternative to the prominent scenarios of the primordial universe, inflation and pre-big bang. The aim of the scenario is to provide solutions to major cosmological problems, including the homogeneity and isotropy problem, the flatness problem and the problem of the seeds of large-scale structure, on the basis of fundamental physics.

The starting point of the ekpyrotic scenario is the unified theory known as *heterotic M-theory* [10-12]. This theory is eleven-dimensional, and the compact eleventh dimension is bounded at both ends by ten-dimensional slices known as *branes*[1]. These branes play a vital role in heterotic M-theory. However, in the context of cosmology such codimension one objects have mostly been investigated in phenomenological constructions, such as the Randall-Sundrum model, instead of the fundamental heterotic M-theory. The understanding gained by studying these phenomenological constructions can to a large degree be applied to the ekpyrotic scenario, so that phenomenological brane cosmology provides an important background for the ekpyrotic scenario in addition to heterotic M-theory.

Chapter 2 puts brane physics into the historical context of extra dimension theories and contains an overview of the basics of the brane scenario. It starts from the Randall-Sundrum model and proceeds to the general case of brane gravity and cosmology in the case of one extra dimension. The main result of studies of brane gravity is emphasised: it is possible to obtain approximately four-dimensional gravity independent of the size of

[1] These are not the same objects as the D-branes of string theory.

the extra dimension, in contrast to set-ups where the observers are not localised in the extra dimension.

Chapter 3 discusses the cosmological background. The main present cosmological problems are listed, and the solutions offered by the most studied comprehensive scenarios of the primordial universe –inflation and pre-big bang– as well as by the ekpyrotic scenario are presented, along with the problems of the well-studied scenarios.

Chapter 4 contains a more detailed presentation of the ekpyrotic scenario, including its problems. The heterotic M-theory set-up is presented and the construction and analysis of the four-dimensional effective theory is outlined. After briefly discussing the internal problems of the four-dimensional effective theory, the far more serious problems of the four-dimensional construction itself are addressed. Some problems faced by the five-dimensional approach are then discussed, and their relevance to the so-called "cyclic model of the universe" [13-15] –a spin-off of the ekpyrotic scenario– is commented upon.

I conclude that the ekpyrotic scenario is a welcome new idea but that most work done thus far is not solid. Careful analysis in the higher-dimensional setting is needed to promote the scenario from an interesting concept to a working model with testable predictions.

This paper is a revised version of the introductory part of my Ph.D. thesis at the Department of Physical Sciences at the University of Helsinki; the thesis consists of the introductory part and the publications [16, 17, 18] The original version of the introductory part, titled *Topics in brane cosmology,* is available at http://ethesis.helsinki.fi. I have added some references and comments, fixed a few typos in the text and made some other, mostly trivial, changes. The only significant change is that a mistake regarding the constraints on brane matter in boundary brane–boundary brane collisions in section 4.4.2 (and in [18]) has been corrected.

1.2 Acknowledgments

I wish to thank my thesis supervisors Kari Enqvist and Esko Keski-Vakkuri. Kari, for lending his physics intuition and for tutoring in the pragmatical aspects of reseach and publishing, as well as for encouraging (and arranging funding for) me to attend numerous conferences. Esko, for more involved guidance and many discussions.

Thanks are also extended to Fawad Hassan for explanations of various aspects of string theory and useful discussions, Hannu Kurki-Suonio for help with CMB and LSS references, Martin Sloth for discussions and explanations on the pre-big bang scenario as well as comments on the manuscript, Riccardo Sturani for answering my questions on the pre-big bang scenario and Jussi Väliviita for similarly clarifying things regarding CMB perturbations. Pikkukettu Hopeakettu deserves special mention for many discussions and comments on the manuscript.

It is a pleasure to acknowledge Robert Brandenberger's pleasant yet firm conduct as an opponent at the thesis defense, as well as his encouraging statements.

I wish to thank Andrei Linde, Burt Ovrut, Paul Steinhardt and Neil Turok for discussions related to the ekpyrotic scenario. The discussions played an important part in selecting both the subject of my second paper on the ekpyrotic scenario and the contents of the ekpyrotic parts of the thesis. Also, I wish to thank the referees of the thesis, Jorma Louko and Claus Montonen, for their useful comments and corrections.

Of course, all errors, omissions and opinions are mine and not to be blamed on others.

Conferences have been important to my research, and the trips have been supported by the Magnus Ehrnrooth Foundation and the Jenny and Antti Wihuri Fund, to which I express my gratitude. The stipend of the Waldemar von Frenckell foundation is also gratefully acknowledged.

I want to thank Christofer Cronström for supervising the latter stages of my Master's thesis in 1998-1999. Finally, I extend especially warm thanks to Jouni Niskanen for encouraging me and for offering me a position at the Theory Division of the Department of Physics after the behaviour of a professor at the Department had left me in a state of shock. It is quite possible that without Jouni's support I would not have continued doing physics after finishing my Master's thesis.

I am beyond gratitude to Heidi Hopeametsä for her continued love and support.

Chapter 2

The brane scenario

2.1 A brief overview of extra dimensions

2.1.1 A micro-history

Spacetime dimensions additional to the observed four were introduced to physics by Gunnar Nordström in 1914 [19], and became popular from the work of Kaluza and Klein in the 1920s [20-22]. The early work on Nordström-Kaluza-Klein theories stemmed from a desire to unify gravity and electromagnetism in a higher-dimensional, purely geometric framework. Since the theories attempted to unify the most fundamental interactions of the day, they were naturally phenomenological in character. For example, there was no principle to dictate the number of extra dimensions.

A more well-founded framework for Kaluza-Klein theories was provided by the advent of supergravity in 1976 [23-25]. First of all, supergravity sets a maximum limit of 11 on the number of spacetime dimensions. This limit arises in the following way. It is believed (though not proven) that there are no consistent interacting field theories with particles of spin more than two [26-28]. The maximum helicity range is therefore from -2 to 2, implying a maximum of 8 supersymmetry generators, since each changes helicity by one-half. This requirement on the amount of symmetry in turn sets the upper limit of 11 on the number of spacetime dimensions[1]. Second, besides guiding the choice of the number of spacetime dimensions, supergravity restricts the field content. The more symmetric a theory is, the more constrained it is; in the maximum eleven dimensions supergravity is unique, greatly increasing its appeal [25].

While supergravity theories provide a well-motivated and potentially realistic framework in which to implement Kaluza-Klein ideas, the extra dimensions are still something of a luxury rather than a necessity: aesthetic considerations may suggest more than four dimensions, but there is nothing in supergravity theories that would require them. In

[1] Assuming there is only one temporal direction.

this sense, string theories opened a new era in (among other things) the field of extra dimensions in 1974 [29]. Bosonic string theory is consistent only in 26 dimensions, and the introduction of supersymmetry brings the number down to 10 for superstring theory[2]. Different string theories are believed to be different limits of a fundamental theory known as M-theory that is formulated in one higher dimension, bringing the number to 11, perhaps not incidentally the maximum for supergravity theories.

There are different formulations and limits of M-theory. The ekpyrotic scenario is based on one known as heterotic M-theory [10-12]. Heterotic M-theory describes the strong coupling limit of heterotic string theory (which is a mixture of bosonic string theory and superstring theory). The theory is formulated on an orbifold $\mathcal{M}_{10} \times S_1/\mathbb{Z}_2$, where $\mathcal{M}_{10}$ is a smooth ten-dimensional manifold. There are two points on the circle S_1 that are left under invariant under the action of the group $\mathbb{Z}_2$. The smooth ten-dimensional manifolds at these *fixed points* are called branes and the space between the boundary branes is called the *bulk*.

The low-energy limit of heterotic M-theory is eleven-dimensional supergravity coupled to two ten-dimensional E_8 gauge theories, one on each brane. Compactifying six spatial dimensions on a Calabi-Yau manifold[3] leads to an effectively five-dimensional gauged N=1 supergravity theory [32, 33]. This five-dimensional theory with four-dimensional contributions at the boundary of the orbifold $\mathcal{M}_4 \times S_1/\mathbb{Z}_2$ serves as the framework of the ekpyrotic scenario. It will be presented in more detail in chapter 4.

2.1.2 Where are they?

An obvious problem in a physical theory with extra spacetime dimensions is to reconcile their existence with observations, which to date are all consistent with four spacetime dimensions. There are two kinds of observations: those that measure the spacetime geometry directly via gravity and those that measure it indirectly via observations of gauge interactions. The approach introduced by Klein [22], followed in the early Kaluza-Klein theories as well as in supergravity and string theories, is to take the extra dimensions to be compact and small. The idea is that probes with wavelength much bigger than the size of the extra dimension will not be able to resolve it, so that spacetime looks four-dimensional at the low energies presently accessible to observation.

Gravity has been directly tested to distances of about .1 mm [34], while gauge interactions have been probed to distances of the order of $(100\ \mathrm{GeV})^{-1} \sim 10^{-18}$ m [35]. No deviations from the four-dimensional predictions have been found. So, it seems that

[2]It is not impossible to formulate string theory in less than 26 or 10 dimensions, but the construction is less straightforward. For examples, see the covariant lattice approach in [30] and the Landau-Ginzburg models and the Gepner models in [31].

[3]Sometimes called a threefold in this context since it can be expressed in terms of three complex dimensions.

the size of extra dimensions in which fields with gauge interactions propagate must be smaller than 10^{-18} m.[4] On the other hand, this bound says nothing about dimensions in which fields with gauge interactions do not propagate. An example would be the eleventh dimension in heterotic M-theory, since gauge fields are confined to the ten-dimensional boundary branes. However, since gravity is an expression of spacetime geometry and thus by definition propagates in all dimensions, it might seem that gravity experiments set a definite limit of the order of .1 mm on the size of any extra dimensions. The main new contribution of the brane scenario to the field of extra dimensions is to demonstrate that this is not the case. When the observers are confined on a brane it is possible to obtain the correct lower-dimensional gravity (within the current observational limits) for large or even infinite extra dimensions.

2.2 The Randall-Sundrum model

2.2.1 The basic construction

Several papers discussing gravity in brane models appeared in 1999 [37-40]. However, two publications by Randall and Sundrum [8, 9] set off a voluminous exploration of brane physics. Sometimes the term "Randall-Sundrum model" (or "Randall-Sundrum scenario") is used to refer to brane set-ups generally, but I will take it to mean strictly the first model outlined by Randall and Sundrum in their initial papers.

The framework of the proposal of Randall and Sundrum is general relativity plus classical field theory. No quantum effects or unified theories are involved, and in particular there is no reference to heterotic M-theory, though the set-up is quite similar. The spacetime is taken to be a five-dimensional orbifold $\mathcal{M}_4 \times S_1/\mathbb{Z}_2$. One of the boundary branes is identified with the visible universe, also called the *visible brane,* and the other with a "hidden universe", or *hidden brane.* The branes are assumed to be parallel. Fields that feel gauge interactions are assumed to be confined to the branes, so that only gravity propagates in the extra dimension. There are cosmological constants on the branes and in the bulk. The action is

$$\begin{aligned} S_{\rm RS} &= \int_{\mathcal{M}_5} d^4x dy \sqrt{-g}\left(\frac{M_5^3}{2}R - \Lambda\right) \\ &\quad + \sum_{i=1}^{2}\int_{\mathcal{M}_4^{(i)}} d^4x \sqrt{-h^{(i)}}\left(\Lambda_i + \mathcal{L}_{\text{matter}(i)}\right) \ , \qquad (2.1) \end{aligned}$$

[4] In the context of string theory this requirement is easily fulfilled, since the natural scale of extra dimensions is set by the string scale, which is usually within a few orders of magnitude of the Planck scale $M_{Pl}^{-1} \sim (10^{18}\text{GeV})^{-1} \sim 10^{-34}$ m. There are also string models where the size of some extra dimensions is of the order of $(10^3\text{GeV})^{-1} \sim 10^{-19}$ m [36], placing them on the threshold of experimental detection.

where M_5 is the Planck mass in five dimensions, R is the scalar curvature in five dimensions, Λ, Λ_1 and Λ_2 are the cosmological constants in the bulk, on the visible brane and on the hidden brane, respectively and $\mathcal{L}_{\mathrm{matter}(i)}$ is the Lagrange density of matter on brane i, including the Standard Model fields on the visible brane. The coordinates x^μ and y are the coordinates parallel and perpendicular to the branes, respectively, and y covers the range from 0 to πr_c, where r_c is a constant. The visible brane is at $y_1 = \pi r_c$ and the hidden brane is at $y_2 = 0$. The tensor g_{AB} is the metric on $\mathcal{M}_5$ and $h^{(i)}_{\mu\nu} = g_{\mu\nu}(x^\mu, y_i)$ are the induced metrics on the branes $\mathcal{M}^{(i)}_4$.

The "vacuum state" of the above action, with four-dimensional Poincaré invariance with respect to the dimensions parallel to the brane (and $\mathcal{L}_{\mathrm{matter}(i)} = 0$) is a slice of anti-de Sitter space,

$$ds^2 = e^{-2ky}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2 \ , \tag{2.2}$$

where $\eta_{\mu\nu}$ is the four-dimensional Minkowski metric and k is a constant. The function e^{-ky}, and more generally (the square root of) a function of y multiplying the metric of the four-dimensional spacetime spanned by x^μ, is called the *warp factor*. The cosmological constants in the bulk and on the branes are related to each other and to k as follows:

$$\begin{aligned} \Lambda_1 &= -\Lambda_2 = -6M_5^3 k \\ \Lambda &= -6M_5^3 k^2 \ , \end{aligned} \tag{2.3}$$

where k is taken to be a few magnitudes below the Planck scale. The spacetime curvature is thus near the Planck scale, so that it is not clear how reliable the classical treatment of gravity is. Note that the cosmological constant in the bulk and the cosmological constant on the visible brane are negative, and that there is a fine-tuning between all three cosmological constants.

It should be emphasised that the "cosmological constants on the branes" are not the cosmological constants measured on the branes. In fact, the fine-tuning (2.3) is equivalent to setting the observed cosmological constants to zero, as is clear from the fact that the geometry on the branes is flat and not (anti-)de Sitter. This is in part because the relation between matter and curvature on the branes is not given by the usual four-dimensional Einstein equation, and in part because the bulk cosmological constant also contributes to brane gravity. These issues will be considered in detail in section 2.3. The appropriate interpretation of the quantities Λ_i is that they are tensions related to the embedding of the branes into the five-dimensional spacetime, so I will refer to them as "brane tensions" from now on, reserving the term "brane cosmological constant" for the effective cosmological constant actually measured on the brane[5].

[5] A useful convention also because in the ekpyrotic scenario, the main object of interest, the brane tensions are not necessarily constant but can vary in time; see chapter 4.

2.2.2 The hierarchy problem

Though brane scenarios have interesting cosmological applications, the motivation for the proposal of Randall and Sundrum did not stem from cosmology but from particle physics. The model was intended to address the so-called hierarchy problem, i.e. the origin of the vast difference between the scale of gravity $M_{Pl} \sim 10^{18}$ GeV and the scale of electroweak physics ~ 100 GeV $\sim$ TeV. The Randall-Sundrum model addresses the hierarchy problem in the following manner. Let us consider the action for the Higgs field H

$$\begin{aligned} S_{\text{matter}(1)} &= \int d^4x \mathcal{L}_{\text{matter}(1)} \\ &\supset \int d^4x \sqrt{-h^{(1)}} \left(h^{(1)\mu\nu} (D_\mu H)^\dagger D_\nu H - \lambda(|H|^2 - m_0^2)^2 \right) \ . \end{aligned} \tag{2.4}$$

Substituting the "vacuum" metric (2.2) into (2.4), we have

$$S_{\text{matter}(1)} \supset \int d^4x e^{-4k\pi r_c} \left(e^{2k\pi r_c} \eta^{\mu\nu} (D_\mu H)^\dagger D_\nu H - \lambda(|H|^2 - m_0^2)^2 \right) \ . \tag{2.5}$$

Making the field redefinition $H \to e^{k\pi r_c} H$, we obtain

$$S_{\text{matter}(1)} \supset \int d^4x \left(\eta^{\mu\nu} (D_\mu H)^\dagger D_\nu H - \lambda(|H|^2 - e^{-2k\pi r_c} m_0^2)^2 \right) \ . \tag{2.6}$$

The "bare" mass m_0 has been replaced by a mass suppressed by the warp factor, $m = e^{-k\pi r_c} m_0$. The same applies also to fields in other representations of the Poincaré group, such as fermions. Using the exponential warp factor one can generate an electroweak scale mass $m \sim 100$ GeV from a Planck scale mass $m_0 \sim 10^{18}$ GeV without the introduction of large parameters; only $\pi k r_c \approx 37$ is needed. From this point of view, the smallness of the electroweak scale does not have a particle physics origin but is an expression of the high curvature of spacetime, which is in turn related to the Planck scale bulk cosmological constant.

This resolution of the hierarchy problem relies on a small warp factor at the visible brane. However, the normalisation of the warp factor has no physical significance, and as pointed out by Randall and Sundrum, in the natural coordinate system of an observer on the visible brane the warp factor they measure is unity, $e^{-k(y-\pi r_c)}$. In this coordinate system there is no rescaling of the "bare" masses in the visible brane. However, one can reverse the view of hierarchy problem and say that it is not the Planck scale but the electroweak scale which is fundamental. Randall and Sundrum proposed that from this point of view the particle masses are naturally of the electroweak scale, whereas the large Planck mass arises from the exponential warp factor.

It is somewhat unclear how the emergence of Planck scale gravity from the fundamental constants of the TeV scale is supposed to come about, since the gravitational constant does not receive exponential suppression. The four-dimensional Planck mass given by Randall and Sundrum is

$$\begin{aligned} M_{Pl}^2 &= \frac{M_5^3}{k}(1 - e^{-2\pi k r_c}) \\ &\approx \frac{M_5^3}{k} \end{aligned} \tag{2.7}$$

If the fundamental scale of gravity M_5 is of the TeV range, one needs a large hierarchy, $k \approx M_5^3/M_{Pl}^2 \sim 10^{-30}$ TeV, to get the observed Planck mass. However, as a matter of fact the above "Planck mass" is not the Planck mass measured on the visible brane, but instead a quantity which has been integrated over the fifth dimension. The physical interpretation of such averaged quantities in a set-up with observers strictly localised on a brane is not clear[6]. Unfortunately, the physical Planck mass fares no better. The gravitational coupling on the visible brane is [41]

$$8\pi G_N \equiv \frac{1}{M_{Pl}^2} = \frac{\Lambda_1}{6M_5^6} \ , \tag{2.8}$$

so that one needs $\Lambda_1 \sim M_5^6 M_{Pl}^{-2} \sim 10^{-30}$ (TeV)^4, or $\sqrt[4]{\Lambda_1} \sim 10^{-8}$ TeV, still a large hierarchy. However, a more serious problem is visible in (2.8): according to (2.3) the tension of the visible brane, Λ_1, is negative, leading to a negative gravitational coupling. By switching the assignment of the visible and hidden branes one obtains a positive gravitational coupling, but then the warp factor is exponentially enhanced instead of suppressed at the visible brane.

2.3 Gravity on a brane

2.3.1 The general case

Even if the Randall-Sundrum model does not offer solutions to problems of particle physics, it may still lead to interesting modifications of gravity. There is a large literature on brane gravity, particularly brane cosmology: [1, 2, 4-9, 13-18, 37-67] offer some examples. Numerous publications and preprints could be added, some more relevant than others.

Kaluza-Klein recipes for gravity are sometimes applied to brane cosmology. However, apart from the questionable applicability of such treatment[7], the four-dimensional gravity in a brane setting is known exactly, so that there is little need for such approximate methods. The exact equations describing the four-dimensional gravity seen by an observer

[6]This issue will be discussed in more detail in section 4.3.5.
[7]See section 4.3.5 for discussion.

on an infinitely thin brane in a generic five-dimensional setting were derived in [41], assuming only the validity of general relativity. The starting point is the Einstein equation in five dimensions, written as

$$G_{AB} = \frac{1}{M_5^3} T_{AB} \ , \tag{2.9}$$

where the Latin capital letters cover all five directions, G_{AB} is the five-dimensional Einstein tensor and T_{AB} is the five-dimensional energy-momentum tensor; the five-dimensional metric is denoted by g_{AB} as before. Note that G_{AB} and T_{AB} include both bulk and brane quantities, the latter as delta function contributions. Given (2.9), the induced Einstein equation on a brane at a fixed point of $\mathbb{Z}_2$-symmetry is [41]

$$\begin{aligned} {}^{(4)}G_{\mu\nu} &= \frac{2}{3M_5^3}\left(T_{AB}\,{}^{(4)}g^{A}_{\ \mu}\,{}^{(4)}g^{B}_{\ \nu} + \left(T_{AB}n^A n^B - \frac{1}{4}T^{A}_{\ A} \right) {}^{(4)}g_{\mu\nu} \right) \\ &\quad + \frac{1}{24M_5^6}\bigg(- 6\,{}^{(4)}T_{\mu\alpha}\,{}^{(4)}T^{\alpha}_{\ \nu} + 2\,{}^{(4)}T^{\alpha}_{\ \alpha}\,{}^{(4)}T_{\mu\nu} \\ &\quad + \left(3\,{}^{(4)}T_{\alpha\beta}\,{}^{(4)}T^{\alpha\beta} - {}^{(4)}T^{\alpha}_{\ \alpha}\,{}^{(4)}T^{\beta}_{\ \beta} \right) {}^{(4)}g_{\mu\nu} \bigg) - E_{\mu\nu} \ , \end{aligned} \tag{2.10}$$

where the Greek letters cover the directions parallel to the brane, n^A is a unit vector normal to the brane, ${}^{(4)}g_{\mu\nu}$ is the metric induced on the brane (with ${}^{(4)}g_{AB} \equiv g_{AB} - n_A n_B$), ${}^{(4)}G_{\mu\nu}$ is the Einstein tensor formed from the metric ${}^{(4)}g_{\mu\nu}$, ${}^{(4)}T_{\mu\nu}$ is the energy-momentum tensor of the brane and $E_{\mu\nu}$ is a traceless contribution related to the bulk Weyl tensor C_{ABCD} via $E_{\mu\nu} = C_{ABCD} n^A n^{C}\,{}^{(4)}g^{B}_{\ \mu}\,{}^{(4)}g^{D}_{\ \nu}$. Note that in the derivation of this result there are no constraints on the energy-momentum tensors of the bulk or the visible brane, nor is there any limitation on the number and matter content of possible other branes.

The induced Einstein equation (2.10) differs from the standard four-dimensional Einstein equation in three respects. The dependence of the four-dimensional Einstein tensor on the four-dimensional energy-momentum tensor is quadratic as opposed to linear, there is a contribution from the bulk energy-momentum tensor and there is a contribution from the bulk Weyl tensor (note that no such term is present in the five-dimensional Einstein equation).

The interpretation of the differences is straightforward. First, it is by dimensional analysis clear that the relationship between the induced Einstein tensor and the brane energy-momentum tensor cannot be linear. The Einstein tensor has the dimension m^2, the four-dimensional energy-momentum tensor m^4 and the five-dimensional gravitational coupling m^{-3}. The first combination of integer powers of the energy-momentum tensor and gravitational coupling that has the same dimension as the Einstein tensor is (e-m tensor)^2/(coupling)^2, the second is (e-m tensor)^5/(coupling)^6 and so on. Without analysing the five-dimensional Einstein equation and the Israel junction conditions that determine the embedding of the brane into the five-dimensional spacetime it is not obvious which possibility is actually realised. However, it is clear that obtaining the standard linear dependence is impossible. In a Kaluza-Klein setting where one integrates along the extra dimension, the ordinary Einstein equation emerges (as an approximation) because the size of the extra dimension provides a new dimensionful parameter. However, in a brane setting the metric in the vicinity of a brane is not sensitive to a global parameter such as the size of the extra dimension, and so it does not appear in the induced Einstein equation.

The terms proportional to the bulk energy-momentum tensor simply account for the fact that sources not confined to the brane can affect the geometry on the brane. On the other hand, the term related to the bulk Weyl tensor is somewhat surprising. The bulk Weyl tensor cannot be solved from the local matter contribution, only from the complete solution of the (five-dimensional) Einstein equation, and in this sense it may be called non-local. Note that $E_{\mu\nu}$ is the *only* non-local term in the induced equation, and thus the only one that may contain information about the global structure of the five-dimensional spacetime. All information that an observer localised on the brane can gravitationally obtain about the global structure of the fifth dimension, such as its possibly finite size or the presence of other branes, is included in $E_{\mu\nu}$.

2.3.2 Gravity in the Randall-Sundrum model

In the Randall-Sundrum model, the bulk contains only a cosmological constant Λ and the visible brane has tension Λ_1 in addition to its matter content. Then the induced equation (2.10) reduces to

$$\begin{aligned} {}^{(4)}G_{\mu\nu} \; = \; & \frac{\Lambda_1}{6M_5^6}\,{}^{(4)}T_{\mu\nu} - \frac{1}{12M_5^3}\left(6\Lambda + \frac{\Lambda_1^2}{M_5^3}\right){}^{(4)}g_{\mu\nu} \\ & + \frac{1}{24M_5^6}\Bigg(-6\,{}^{(4)}T_{\mu\alpha}\,{}^{(4)}T^{\alpha}_{\ \nu} + 2\,{}^{(4)}T^{\alpha}_{\ \alpha}\,{}^{(4)}T_{\mu\nu} \\ & + \left(3\,{}^{(4)}T_{\alpha\beta}\,{}^{(4)}T^{\alpha\beta} - {}^{(4)}T^{\alpha}_{\ \alpha}\,{}^{(4)}T^{\beta}_{\ \beta}\right){}^{(4)}g_{\mu\nu}\Bigg) - E_{\mu\nu} \; , \end{aligned} \tag{2.11}$$

where ${}^{(4)}T_{\mu\nu}$ now stands for the energy-momentum tensor of brane sources other than the tension, and the magnitudes of Λ and Λ_1 have been kept as free parameters not subject to the fine-tuning (2.3). The brane tension provides a local dimensionful parameter that makes it possible to obtain a linear dependence on the brane energy-momentum tensor. Remarkably, if the contribution of the Weyl tensor is small and the scale of the energy density is some orders of magnitude smaller than the five-dimensional Planck scale, one obtains (nearly) standard four-dimensional gravity on the brane.

Note that the flipside of possibly obtaining nearly standard gravity with a positive brane tension even with a large extra dimension is the impossibility of obtaining standard gravity without positive brane tension, even with a small extra dimension[8]. This irrelevance of the size of the extra dimension to the four-dimensional gravity is a distinctive feature of set-ups where observers are localised on a brane, and contrasts sharply with settings where observers are not localised in the extra dimensions.

From (2.11) we can identify the observed Newton's constant and the observed cosmological constant as

$$\begin{aligned} G_N \equiv \frac{1}{8\pi M_{Pl}^2} &= \frac{\Lambda_1}{48\pi M_5^6} \\ \Lambda_{eff} &= \frac{3M_5^3\Lambda}{\Lambda_1} + \frac{\Lambda_1}{2} \ . \end{aligned} \tag{2.12}$$

Two immediate observations from (2.12) are that in order to obtain a positive Newton's constant the brane tension has to be positive, as mentioned earlier, and that in order to obtain a zero effective cosmological constant on the brane there has to be a negative bulk cosmological constant. The second point was already apparent in the "vacuum" set-up of Randall and Sundrum: in order to obtain a static solution (which of course requires that the effective cosmological constants on the branes vanish) the bulk cosmological constant had to be negative and fine-tuned to both of the brane tensions, which is of course only possible if they have the same magnitude.

Without the fine-tuning of the Randall-Sundrum proposal, the effective cosmological constant can be adjusted to any desired value by choosing an appropriate Λ. Of some interest is the case $\Lambda = 0$,[9] since then the magnitude of the effective cosmological constant is fixed in terms of the brane tension and the five-dimensional Planck mass, or alternatively, the four- and five-dimensional Planck masses. Putting $\Lambda = 0$, the effective cosmological constant is, from (2.12),

$$\Lambda_{eff} = \frac{3M_5^6}{M_{Pl}^2} \ . \tag{2.13}$$

In order to obtain an effective cosmological constant in agreement with observations [68, 69, 70, 71] $\Lambda_{eff} \sim M_{Pl}^2 H_0^2 \sim 10^{-48}$ (GeV)^4, where H_0 is the present value of the Hubble parameter, the five-dimensional Planck mass would need to be $M_5 \sim \sqrt[3]{H_0 M_{Pl}^2} \sim 10^{-2}$ GeV. However, we will see in the next section that a value this small spoils cosmology at the time of light element nucleosynthesis, quite apart from other possible problems.

[8] Assuming that the bulk contains only a cosmological constant. It is clear from (2.10) that it is also possible to obtain standard gravity by putting an explicit dependence on the brane energy-momentum tensor in the bulk energy-momentum tensor. For an example, see [47, 53, 56, 60].

[9] For $\Lambda = 0$ there is of course no exponential warping of spacetime.

So, the Randall-Sundrum model does not offer a solution to the problem of a small non-zero cosmological constant. However, there is hope of explaining a zero cosmological constant, in other words the Randall-Sundrum fine-tuning, in a similar setting in terms of supersymmetry [72, 73].

While the effective cosmological constant, if different from zero, is in principle observable, there is nothing that would distinguish it from an ordinary four-dimensional cosmological constant. The second order terms in the energy-momentum tensor are suppressed by $\sim {}^{(4)}T_{\mu\nu} M_4^2/M_5^6$, so that unless the five-dimensional Planck mass is very small, they will be quite difficult to observe. The magnitude of the Weyl tensor-term $E_{\mu\nu}$ is not obviously suppressed, but it cannot be evaluated without knowing the bulk solution. Brane cosmology offers one clear way of assessing the observability of the second order terms and minimises the uncertainty due to $E_{\mu\nu}$.

2.3.3 Cosmology in the Randall-Sundrum model

Specialising the Einstein equation (2.11) to cosmology by assuming homogeneity and isotropy with respect to the visible spatial directions, we obtain [50]

$$\begin{aligned} \frac{\dot{a}^2}{a^2} + \frac{K}{a^2} &= \frac{\Lambda_1}{18 M_5^6}\rho + \frac{1}{36 M_5^3}\left(6\Lambda + \frac{\Lambda_1^2}{M_5^3}\right) + \frac{1}{36 M_5^6}\rho^2 + \frac{\mathcal{C}}{a^4} \\ \dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) &= 0 \ , \end{aligned} \tag{2.14}$$

where a is the scale factor of the brane, $6K$ is the constant spatial curvature on the brane, ρ and p are the energy density and pressure, respectively, of brane matter, $\mathcal{C}$ is a constant and a dot indicates a derivative with respect to proper time on the brane.

The second equation is the ordinary four-dimensional conservation law. A standard conservation law for brane matter is not a generic feature of brane cosmology, but simply a consequence of the assumption that there is no energy flow in the direction of the fifth dimension (since the bulk is empty apart from the cosmological constant).

The first equation is the Hubble law. As discussed in the previous section, the departures from standard cosmology are confined to the ρ^2-term and the Weyl tensor contribution $\mathcal{C}/a^4$. As long as the five-dimensional Planck scale M_5 is more than 10 TeV, the effect of the ρ^2-term will be negligible from the time of neutrino decoupling (at $\rho \sim 1$ MeV) onwards. Conversely, the value $M_5 \sim 10^{-2}$ GeV that could naturally explain the present-day cosmic acceleration is ruled out by the successful predictions of standard big bang nucleosynthesis [74]. The magnitude of the $\mathcal{C}$-term could only be known from a full bulk solution, but it can in any case affect only the early universe since its contribution declines faster than that of non-relativistic matter (for which $\rho \propto a^{-3}$).

Like the general induced Einstein equation in the Randall-Sundrum model, (2.11), the Hubble law (2.14) is not closed, and cannot be solved without specifying $\mathcal{C}$, which depends on the full bulk solution. Note that all possible effects due to other branes or the finite size of the extra dimension are contained in the value of the single constant $\mathcal{C}$. This feature, while surprising from a Kaluza-Klein point of view, is quite natural in a brane setting. As mentioned earlier, the five-dimensional Einstein equation and the Israel junction conditions which give the embedding of the brane into the five-dimensional spacetime are local, so that the four-dimensional Einstein equation can contain a non-local contribution only via the Weyl tensor. Since all contractions of the Weyl tensor with itself vanish, $E_{\mu\nu}$ is traceless, which means that its contribution to the Hubble law decays like radiation. Therefore the only degree of freedom is the magnitude given by the constant $\mathcal{C}$.[10]

Apart from non-closure, the Hubble law (2.14) has another shortcoming, also shared by the general Einstein equation (2.10): a solution of the induced equation is not necessarily a solution of the full bulk equation. In other words, the equation is a necessary but not a sufficient condition for the solution to exist. In practical terms, this means that even if one can solve the induced equation despite non-closure (for example, by neglecting the $\mathcal{C}$-term at late times), one still has to construct the bulk solution to know whether the cosmological solution is actually realised. One can turn the issue upside down and view this as a constraint imposed by the induced equation on the bulk solutions. This point of view can be illustrated with the *Israel junction conditions* which determine the embedding of the branes into the five-dimensional spacetime. For the isotropic and homogeneous cosmological case with the metric

$$ds^2 = -n(t,y)^2 dt^2 + \frac{a(t,y)^2}{(1+\frac{K}{4}r^2)^2} \sum_{j=1}^{3} (dx^j)^2 + b(t,y)^2 dy^2 \ , \tag{2.15}$$

where $r^2 = \sum_{j=1}^3 (x^j)^2$ and the constant $6K$ is the spatial curvature in the directions parallel to the branes, the junction conditions read [39]

$$\begin{aligned} \pm \frac{1}{b}\frac{a'}{a}\bigg|_{y=y_1} &= -\frac{1}{6M_5^3}(\rho + \Lambda_1) \\ \pm \frac{1}{b}\frac{n'}{n}\bigg|_{y=y_1} &= \frac{1}{6M_5^3}(2\rho + 3p - \Lambda_1) \ , \end{aligned} \tag{2.16}$$

where y_1 is the location of the brane, a prime denotes a derivative with respect to y, and the sign ambiguity is related to whether one takes the limit $y \to y_1$ from the right or from the left of y_1 (a' and n' are discontinuous at the brane).

To demonstrate how the junction conditions relate the bulk metric to the matter

[10] For this argument it is necessary that the second order contribution of the brane energy-momentum tensor is separately conserved. This is true in the homogeneous and isotropic case but not in general.

content of the brane, let us consider the following rather general ansatz

$$\begin{aligned} n(t,y) &= n(t,y) \\ a(t,y) &= a(t)n(t,y) \\ b(t,y) &= b(t,y) \ , \end{aligned} \tag{2.17}$$

where $n(t,y)$, $a(t)$ and $b(t,y)$ are arbitrary functions. Since $n'/n = a'/a$, the Israel junction conditions give the following constraint on brane matter

$$\rho + p = 0 \ , \tag{2.18}$$

which implies via (2.14) that $\dot{\rho} = 0$: only vacuum energy is allowed.

The above example shows that the four-dimensional part of the bulk metric cannot be factorisable so that we would have $g_{\mu\nu}(x^\mu, y) = f(y)^{(4)}g_{\mu\nu}(x^\mu)$ or even $g_{\mu\nu}(x^\mu, y) = f(x^\mu, y)^{(4)}g_{\mu\nu}(x^\mu)$. In particular, the Randall-Sundrum warp factor idea for solving the hierarchy problem is not possible in a cosmological setting.

Given the constraints, one may wonder whether for an arbitrary solution of the induced equation there exists a bulk solution that supports it. The answer is in the affirmative. In the case of a single brane, the explicit bulk solution for the case of a static fifth dimension, $\dot{b} = 0$, and arbitrary matter content $\rho(t)$ and $p(t)$ has been constructed [50]. The solution contains no free parameters (apart from possibly $\mathcal{C}$), so that it is clear that it will not be a solution if one includes a second brane with matter. The situation has been studied, keeping b constant but allowing the position of the second brane to change in time [65]. As expected, for general matter content on the two branes there is no solution. The interpretation is presumably that two branes with matter on them will inevitably cause the size of the fifth dimension to vary in time, not that solutions with matter on two branes and only a cosmological constant in the bulk do not exist[11]. As pointed out in [16], a similar result is obtained if one adds ideal fluid to the bulk: there are no solutions with generic matter in the bulk if one keeps the fifth dimension static.

Cosmology at the homogeneous and isotropic level gives little hope of detectable signals of the Randall-Sundrum model. However, the higher-dimensional setting of course modifies also the equations that describe departures from homogeneity and isotropy. Probably the most important among these are the perturbation equations that govern the behaviour of the cosmic microwave background [67], which may offer a possibility for detectable signals.

[11] If one keeps the bulk energy-momentum tensor as set of free parameters, solutions with a static fifth dimension may well exist. For an example (where b is time-independent to lowest order in a perturbative expansion), see [56, 60].

2.4 Summary

Even though the Randall-Sundrum model and other brane set-ups thus far presented do not solve the hierarchy problem, they offer an interesting new approach to particle physics and cosmology. In particular, they have challenged the old view of dimensional reduction via compactification by providing an alternative which demonstrates that it is possible to obtain nearly standard four-dimensional gravity with large or even infinite extra dimensions.

The greatest shortcoming of most brane gravity models, including the Randall-Sundrum model, is that they are not based on fundamental principles. In most cases, the number of spacetime dimensions and the brane structure are essentially unmotivated, as are the contents of the bulk, be it a cosmological constant, a scalar field or an ideal fluid. The confusion originating in uncertain foundations and aggravated by the misapplication of old Kaluza-Klein ideas[12] is readily appreciated by sampling the literature on Randall-Sundrum-type scenarios.

The ekpyrotic scenario is a realisation of the brane scenario that is based on fundamental physics. Heterotic M-theory offers a brane set-up grounded in a unified theory of gravity and particle physics that motivates the dimension of spacetime (and the codimension of the branes) and provides an explicit account of the contents of the bulk. The application of cosmological brane ideas in heterotic M-theory is is somewhat reminiscent of the application of the poorly motivated Kaluza-Klein ideas in the well-defined arena of supergravity, with the difference that the well-defined heterotic M-theory existed already *before* the Randall-Sundrum-inspired brane models.

While the origin of the ekpyrotic scenario is heterotic M-theory and the framework is brane physics, the main motivation comes from cosmology. Before proceeding to the ekpyrotic scenario, it is therefore appropriate to review the status of cosmological scenarios that serve as its backdrop.

[12] Such as trying to stabilise the extra dimension, which is not only overly restrictive [16, 59, 65] but also unnecessary, since the gravitational coupling constant on the brane does not depend on the size of the extra dimension, unlike in Kaluza-Klein settings.

Chapter 3

Scenarios of the primordial universe

3.1 Six cosmological problems

The ekpyrotic scenario is based on heterotic M-theory and aims to give a comprehensive description of the primordial universe. It was explicitly presented as an alternative to the prominent scenarios of the primordial universe, in particular inflation. I will therefore briefly review the current cosmological problems, present the scenarios of the primordial universe that have been proposed to address these problems and highlight the shortcomings of these proposals.

The cornerstone of modern cosmology is the big bang theory. The theoretical foundation of the big bang theory is very solid: one simply applies general relativity to a homogeneous and isotropic four-dimensional spacetime filled with matter that is treated as an ideal fluid, taking into account atomic, nuclear and possibly strong and electroweak physics in the early universe. The theory is also in excellent agreement with observations. The main support comes from the redshift of light emitted by distant objects, the temperature of the cosmic microwave background (CMB) and the abundance of the elements D, ^{3}He, ^{4}He and ^{7}Li [74].

However, the observational support for the big bang theory does not extend to eras before the decoupling of neutrinos from nucleons and electrons at about one second after the big bang. The only observables we at present have from times before neutrino decoupling –which I will refer to as the primordial era– are the anisotropies of the CMB [71, 75-78], large-scale structure [79, 80], baryon number, the amount and properties of dark matter and possibly the amount and properties of dark energy [81]. In addition, the spatial curvature of the universe is an observable presumably related to the primordial universe. None of the above observables have an explanation in the context of the big bang theory (with the possible exception of the baryon number via electroweak baryogenesis [82, 83]).

It is also clear from a theoretical point of view that the big bang theory is not the correct description of the primordial universe, since the high energy densities and curvatures near the big bang imply breakdown of the classical treatment of gravity on which the theory is based.

For a description of the primordial universe one has to turn to a something beyond general relativity plus ideal fluid. It is to be expected that this theory would explain not only the observables that the big bang theory cannot account for, but also the starting point of the big bang theory: the homogeneity and isotropy of the universe and the origin of matter. Specifically, a theory of the primordial universe should cover the following things.

1. The origin of matter

   The origin and relative amount of both visible and dark matter in the universe is a fundamental question confronting scenarios of the primordial universe. In the big bang theory, matter is always present, and there is no explanation as to the relative amount of visible and dark matter, or matter and antimatter. The amount and properties of dark energy, if it is not purely geometric, also falls in the same category.

   In considering the generation of matter, I will leave aside the question of the nature of the matter generated. Also, I will not address the question of dark energy, since no presently favoured scenario of the primordial universe sheds any light on it.

2. The homogeneity and isotropy of the universe

   The universe is spatially quite homogeneous and isotropic. In particular, the temperature of the cosmic microwave background is the same in all directions to an accuracy of 10^{-5}. This is a puzzling observation, since it means that parts of the universe that have never been in causal contact according to the big bang theory have nevertheless almost exactly the same conditions, in an apparent violation of locality.

3. The spatial flatness of the universe

   The universe appears to be nearly spatially flat. Unlike homogeneity and isotropy, this does not constitute a mystery or imply a violation of any physical principles within the confines of the big bang theory. There are three possibilities for the spatial geometry of a spatially homogeneous and isotropic universe: open, closed and flat. One would like to explain via some physical principle why the universe happens to possess one particular spatial geometry out of the three possibilities, but no fine-tuning is involved.

4. The seeds of large-scale structure

The problem of homogeneity and isotropy is somewhat vague due to the scarcity of observables, and it is the departures from homogeneity and isotropy that are the main quantifiable predictions for present scenarios of the primordial universe. The data on the inhomogeneities comes from observations of the cosmic microwave background [71, 75-78] and large-scale structure [79, 80]. In particular, any contender for a model of the primordial universe should explain the origin and nature of the anisotropies of the CMB with sufficient accuracy to be compared with the data.

The most important observationally confirmed aspects of the anisotropies of the CMB are that they are mostly *adiabatic* and their amplitude is almost scale-independent and about 10^{-5}. Precise definitions can be found in the literature [84, 85]. Roughly speaking, adiabaticity means that the perturbations are along the same direction in field space as the background (the alternative would be *isocurvature* perturbations), scale-invariance means that the amplitude does not depend on the wavelength, and the amplitude is simply the maximum relative difference between the perturbed cosmic microwave background temperature and the average temperature, $(T_{max} - T_{av})/T_{av}$.

The scale-dependence of the perturbations is usually expressed with the spectral index n. An index less than 1 means that the amplitude of large wavelengths is amplified with respect to small wavelengths, leading to a "red spectrum", an index of 1 means that all wavelengths have the same amplitude, leading to a scale-invariant spectrum, and an index of more than 1 means that small wavelengths are amplified with respect to large ones, leading to a "blue spectrum". According to observations, $n = 1.03^{+.10}_{-.09}$ [75].

5. The absence of topological defects

   Phase transitions at high energies in grand unified theories of particle physics are expected to produce monopoles, cosmic strings and domain walls. According to the big bang theory, the energy density of the primordial universe is high enough to produce an abundance of such relics, yet none are observed. Unlike the previous problems, the defect problem is one of non-observation, and therefore more vague. While the topological defect problem, also known as the relic problem, has been treated as a shortcoming of cosmology, it is not impossible that the issue might be resolved in the realm of particle physics instead.

6. The singularity problem

   Unlike the previous five problems, which were observational (albeit, in the case of the topological defect problem, in a negative sense), the singularity problem is purely theoretical. According to the big bang theory, the universe began in a state of infinite curvature and energy density a finite time ago. While singularities as such may not necessarily be unphysical, it is clear that unbounded curvatures and energy densities imply that the classical treatment of gravity and matter cannot be trusted.

A consistent model of the primordial universe would either need to be non-singular or have a singularity that does not lie outside its domain of validity. An example of the latter would be an initial singularity which is not a curvature singularity and does not involve unbounded energy densities. A more modest requirement is to have a model where the singularity is observationally and theoretically irrelevant in the sense that the observables are not sensitive to the singularity and the internal consistency of the model is not degraded by the singularity. The big bang theory, for example, satisfies both of these requirements.

At the moment, there are two prominent scenarios of the primordial universe, the inflationary scenario and the pre-big bang scenario. I will now briefly review these scenarios and assess how they address the abovementioned problems.

3.2 The inflationary scenario

The currently favoured framework for addressing primordial cosmological problems is inflation. Inflation is not a firmly established theory but rather a scenario which finds its realisation in a number of different models. The scenario posits that a patch of the primordial universe started to undergo accelerating expansion, *inflation,* and that the presently observable universe originates from a small (in many models trans-Planck scale) volume of the primordial universe. In many models the universe starts with a big bang, after which the universe is supposed to be in an unordered state of high energy density and curvature, from which the inflationary patch emerges. For reviews on inflation, see [86, 87].

The set-up for most models of inflation is big bang theory modified in a simple way, by just adding a causative agent for inflation. In most models the source of inflation is the potential energy of one or more scalar fields. The scalar field(s) evolve slowly, so that the expansion lasts long (in units of the potential energy, which varies between different models, but is typically a few orders of magnitude below the Planck scale). There are other possibilities for the source of expansion, most notably vacuum energy [88, 89] and higher order curvature terms due to quantum fields [90, 91]. The action for most scalar models can be written as

$$S_{\rm scalar} = \frac{1}{16\pi G_N}\int d^4x\sqrt{-g}\left(R-\sum_{i=1}^{n}\frac{1}{2}\partial_\mu\phi^i\,\partial^\mu\phi^i - V(\phi^i)\right) , \qquad (3.1)$$

where G_N is Newton's constant, R is the scalar curvature, ϕ^i are some scalar fields and $V(\phi^i)$ consists of mass and interaction terms. Typical examples are chaotic inflation [92] with a single scalar field and $V(\phi) = \frac{1}{2}m^2\phi^2$ or $V(\phi) = \frac{1}{4}\lambda\phi^4$ and hybrid inflation [93] with two scalar fields and $V(\phi,\chi) = \frac{1}{2}m^2\phi^2 + \frac{1}{2}\lambda'\phi^2\chi^2 + \frac{1}{4}\lambda(M^2-\chi^2)^2$.

The action for the vacuum energy model is [88, 89]

$$S_{\text{vacuum}} = \frac{1}{16\pi G_N}\int d^4x\sqrt{-g}\,(R-\Lambda) \ , \tag{3.2}$$

where Λ is a cosmological constant. Treating general relativity as a quantum field theory makes the model considerably more complicated than the simplicity of the action would seem to indicate. (Counterterms needed to absorb the divergences of the quantised theory have been omitted from (3.2).)

A typical action for a higher order curvature model is [90, 91]

$$S_{\text{curvature}} = \frac{1}{16\pi G_N}\int d^4x\sqrt{-g}\left(R + aR^2 + bR_{\alpha\beta}R^{\alpha\beta} + cR_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}\right) \ , \tag{3.3}$$

where a, b and c are constants, $R_{\alpha\beta}$ is the Ricci tensor and $R_{\alpha\beta\gamma\delta}$ is the Riemann tensor. (The model outlined in [90, 91] also includes contributions that cannot be expressed in terms of a local action.)

Whatever the mechanism, in all models of inflation the universe expands by a huge factor, typically more than e^{60}, during the primordial era. The accelerating expansion eliminates almost all traces of the conditions before inflation. The six cosmological problems are addressed in the following manner.

1. The origin of matter

   In the inflationary scenario, the origin of possible matter in the primordial universe is not explained and is largely irrelevant, since it is diluted by the large expansion. The relevant question is the creation (and thermalisation) of the matter observed today, so-called "reheating" or "preheating". This problem is not yet entirely solved. The main proposed mechanisms are particle production due to a scalar field oscillating about the bottom of its potential [94, 95] and gravitational particle production in expanding spacetime [96, 97].

2. The homogeneity and isotropy of the universe

   Homogeneity and isotropy are supposed to be explained by the huge stretching of spacetime: any inhomogeneities and anisotropies are diluted by the expansion. However, it has been shown [98] that assuming the weak energy condition[1] it is impossible to start inflation unless there is already isotropy and homogeneity on

[1] The weak energy condition states that $T_{\mu\nu}u^\mu u^\nu \geq 0$ for all timelike vectors u^μ.

scales larger than the causal horizon. The weak energy condition is satisfied in all scalar field models with standard kinetic terms, as well as in the vacuum energy model, though it can be broken by higher order curvature terms. So, at least the scalar field and vacuum models of inflation can only ameliorate the homogeneity and isotropy problems, not solve them, though that is sometimes mentioned as their main motivation. The problem can be minimised by starting inflation as near the Planck scale as possible, so that the coincidence of initial homogeneity and isotropy is hopefully as small as possible.

3. The spatial flatness of the universe

   Spatial flatness is explained in the same way as homogeneity and isotropy: any possible spatial curvature of the universe is stretched to an unobservably small level due to the vast expansion, so that the universe looks spatially flat on presently observable scales.

4. The seeds of large-scale structure

   The origin of the large-scale structure of the universe is thought to lie in quantum fluctuations – in the scalar field case, the fluctuations of one or more scalar fields, in the vacuum energy case the fluctuations of the quantised spacetime metric, and in the higher order curvature term case in the spacetime curvature. These fluctuations leave a definite imprint on the homogeneous and isotropic background and grow to become the CMB anisotropies and the seeds of galaxies. The prediction of the definite type (mostly adiabatic) and shape (almost scale-invariant) of the CMB anisotropies is at present the most solid support of the inflationary scenario, particularly the simplest scalar field models. However, in these simplest scalar field models the amplitude of the perturbations is not naturally explained, since one has to generically tune some parameters to be quite small to obtain a small enough amplitude [99]. For example, for chaotic inflation with the potential $V(\phi) = \frac{1}{4}\lambda\phi^4$, one has to take $\lambda \sim 10^{-12}$ [86].

5. The absence of topological defects

   The monopoles, cosmic strings and other possible relics of the primordial high-curvature era are diluted to unobservable densities by inflation. One might then hope that if the reheating temperature is low enough, the relics would not reappear. However, it is possible to produce too many relics at reheating even if the temperature is several orders of magnitude lower than the energy scale at which the unwanted objects are typically produced [97]. So, the problem is not completely solved.

6. The singularity problem

It has been shown that any manifold on which the local Hubble parameter measured by an observer on a null or timelike geodesic is bounded from below by a positive constant in the past is singular [100]. No symmetry arguments or energy conditions are required, and most remarkably, the proof does not make use of the Einstein equation. This theorem implies that all scalar field models and the vacuum energy model as well as many higher order curvature models are singular. Furthermore, the modest requirements of observational and theoretical irrelevance are not quite satisfied, so that the singularity problem is in a sense worse than in the big bang theory. Since observables such as the CMB anisotropies are produced in the latter stages of inflation, they are insensitive to the initial singularity, satisfying the requirement of observational irrelevance. However, the requirement of theoretical irrelevance is not satisfied.

It is preferable to start inflation as near the singularity as possible in order to have to postulate as little homogeneity and isotropy as possible, as noted in connection with the homogeneity and isotropy problem. But one would have to know the distribution of fields and curvature near the singularity in order to evaluate how probable it is to have the desired homogeneous and isotropic volume larger than the causally connected volume. If the universe begins in a curvature singularity, a well-defined initial value problem that would allow for a rigorous treatment of this problem of course cannot be formulated. If the curvature is bounded at the singularity, it may be possible to formulate the initial value problem given the boundary conditions, for example by an instanton describing the birth of the universe [101]. However, the issue is complicated by higher order curvature terms, discussed below.

In addition to the open problems mentioned above, most prominently starting inflation and treating the singularity, there are two distinct but related problems which bear mentioning.

First, most models of inflation (the vacuum energy model being an interesting exception) are semiclassical theories of quantum gravity: the scalar fields (and the fluctuation modes of the metric) are treated quantum mechanically, while the background geometry is classical. In semiclassical quantum gravity, quantum fields induce higher orders of curvature such as those in (3.3) into the action [102]. The contribution of such terms is typically suppressed by the Planck mass, so that one might be tempted to argue that their effect is small, apart from considerations of the initial value problem as discussed above. However, since such terms generally contain fourth and higher order time derivatives, they can completely change the behaviour of the equations of motion, regardless of how small their coefficients are [103]. The situation is aggravated by the fact that inflation typically lasts long, allowing ample time for gravitational instabilities to develop. Usually such terms are simply ignored, but it should be understood that it is not mathematically consistent to do so, nor necessarily physically justified[2].

Second, in many models of inflation the perturbations that seed the large-scale structure seen today originate from physical wavelengths many orders of magnitude smaller than the Planck length. It is questionable whether one can apply semiclassical quantum field theory of free fields in the case of wavenumbers and frequencies many orders of magnitude larger than the Planck mass; even less applicable is the linear perturbation theory that is used to calculate the behaviour of the perturbations. In the light of string theory and other theories of quantum gravity [107], it may not make any sense at all to speak of distances shorter than the Planck length.

3.3 The pre-big bang scenario

The main alternative to inflation as a comprehensive scenario of the primordial universe is the pre-big bang scenario. As noted above, the approach in most models of inflation is to take general relativity and add an inflation-producing agent by hand, with the expectation that the set-up may later find justification in a more fundamental framework. There is usually no connection to a unified theory of gravity and particle physics, or to a theory of quantum gravity (the vacuum energy model being a remarkable exception). The pre-big bang scenario, like the ekpyrotic scenario, takes the opposite approach and attempts to descend from a promising unified theory of quantum gravity, namely string theory, down to phenomenology. For recent reviews of the pre-big bang scenario, see [108-110].

The framework of the pre-big bang scenario is ten-dimensional superstring theory compactified down to four dimensions. The simplest effective four-dimensional action usually considered is

$$S_{\rm PBB} = \frac{1}{16\pi G_N}\int d^4x\sqrt{-g}\left(R - \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - \frac{1}{2}\partial_\mu\beta\,\partial^\mu\beta - \frac{1}{2}e^{2\phi}\partial_\mu\sigma\,\partial^\mu\sigma\right)\ , \qquad (3.4)$$

where the scalar fields ϕ, β and σ are the dilaton, modulus and axion, respectively.

The four-dimensional universe described by the above action is supposed to start in the far past in a "trivial" state with low curvature and energy density. Due to an instability the universe starts collapsing and the curvature starts growing. The universe is supposed to "gracefully exit" (due to terms not present in the above action) from the collapsing phase to the usual expanding phase before reaching the "big crunch" singularity. The collapse in the pre-big bang scenario plays a role similar to expansion in the inflationary scenario. The six cosmological problems are addressed in the following manner.

1. The origin of matter

[2] For a mathematically consistent but not necessarily physically justified treatment of the higher order terms, relevant also for higher order curvature inflation, see [104-106].

The pre-big bang scenario is not yet definite enough to give an account of the generation of matter. However, matter production is expected to occur during the graceful exit phase, and may even play an important role in achieving the reversal of contraction to expansion.

2. The homogeneity and isotropy of the universe

   The homogeneity and isotropy of the universe is provided by a period of accelerating collapse which makes the universe homogeneous and isotropic in the same way as accelerating expansion. (Indeed, in the conformally related metric known as the "string frame", which is as physically relevant as the usual Einstein metric, the collapse looks like accelerating expansion.)

3. The spatial flatness of the universe

   During the collapsing phase, spatial curvature grows instead of decreasing as in inflation. However, the growth is slower than the growth of the energy density of the dilaton field, so that the contribution of spatial curvature to the dynamics of the universe becomes negligible. (In the string frame the increasing spatial flatness is more transparent: the accelerating expansion dilutes spatial curvature.)

4. The seeds of structure

   As in the inflationary scenario, the seeds of large-scale structure are quantum fluctuations about an isotropic and homogeneous background. In the original pre-big bang set-up, the fluctuations were those of the dilaton field which is the source for the collapse. They are adiabatic, since they are fluctuations of the quantity driving the background evolution. However, since the Hubble parameter is not constant but rapidly decreasing, the fluctuations are not scale-invariant but deeply blue, with $n \approx 4$. This problem was solved by considering the fluctuations of another field, the axion. It is possible to obtain nearly scale-invariant fluctuations for the axion field, but since it does not contribute to the background dynamics, these fluctuations will be of the isocurvature type instead of adiabatic. On a positive note, the amplitude of the fluctuations is set by the string scale and the spectral index, and it is possible to obtain the correct amplitude without the introduction of new small parameters.

   There have recently been two proposals to obtain a spectrum of scale-invariant adiabatic fluctuations in agreement with observation. The idea of the first proposal [111, 112] is that in the post-big bang era the axion field oscillates about a minimum, and eventually comes to dominate the energy density of the universe. The field then decays into photons, so that the nearly scale-invariant isocurvature fluctuations of the axion field are converted into nearly scale-invariant adiabatic fluctuations of the photon background. In this proposal, the amplitude of the fluctuations is set by the potential of the axion field, much as in inflation the amplitude is set by the potential of the scalar field driving inflation. However, unlike in inflation, no very small parameters are needed and it is possible to naturally obtain the correct amplitude of density perturbations.

In the second proposal [113] one adds an exponential potential for the dilaton, plus a new scalar field with a non-minimal coupling to gravity. With both of these fields contributing to the background, one can apparently obtain a scale-invariant spectrum of adiabatic fluctuations for the dilaton field, and possibly also for the axion field. However, the identification of the modes of the pre- and post-big bang phases has been criticised [114].

5. The absence of topological defects

   Since there is no exponential stretching of spacetime after the big bang, there seems to be no mechanism for diluting the abundance of dangerous relics. The problem might possibly be solved by having a low enough energy density at the big bang.

6. The singularity problem

   In the pre-big bang scenario the collapsing era during which the anisotropies of the CMB are produced and the current expanding era are separated by the big crunch/big bang curvature singularities. In this context, the singularity problem appears in the guise of joining these two eras in a non-singular manner, known as the *graceful exit*.

   On the one hand, the singularity problem in the pre-big bang scenario is more severe than in the big bang theory, or in the inflationary scenario, since the curvature singularity does not occur before but between eras of cosmology that produce observables.

   On the other hand, the pre-big bang scenario offers a solid framework for avoiding the singularity, unlike scalar field models of inflation. String and quantum corrections can provide various modifications to the equations of motion that might resolve the singularity. The work thus far seems to indicate that the graceful exit, if it can be achieved at all, is likely to occur in the strongly coupled regime of string theory [115-118]. Given that little is known about strongly coupled string theory, the assumption that the perturbations generated during the pre-big bang era can be transferred to the post-big bang era with simple matching conditions which are insensitive to the physics of the graceful exit seems questionable. It does not seem impossible that the largely unknown physics of the exit era could change the perturbations radically. Until this problem has been solved, all predictions of the pre-big bang scenario can only be considered preliminary.

In addition to the problems mentioned above, one could mention that the issue of initial conditions is not settled. The pre-big bang scenario does not suffer from a singularity problem in the era before graceful exit, so that a formulation of the initial value problem is possible, in contrast to scalar field inflation. The issue of initial conditions is related to the duration of collapse. Since the magnitude of the Hubble parameter increases during the collapse and the graceful exit should come into play before the Hubble parameter exceeds the string scale, its initial value provides a bound on the amount of collapse (or, in the string frame, expansion) possible. At the moment, this and other issues related to the initial conditions [119] remain open.

3.4 The ekpyrotic scenario

The next chapter will deal with the ekpyrotic scenario in detail, but I will here give a brief qualitative account of the scenario and outline the solutions it offers to the six cosmological problems.

Like the pre-big bang scenario, the ekpyrotic scenario starts from a fundamental, though speculative, unified theory. As mentioned in chapter 2, the starting point of the ekpyrotic scenario is five-dimensional heterotic M-theory, where the fifth dimension terminates at two boundary branes, one of which is identified with the visible universe. There are two different versions of the ekpyrotic scenario, the "old scenario", where there is a bulk brane between the boundary branes and the "new scenario", where only the boundary branes are present.

In both scenarios the initial state is supposed to be very near the vacuum state, where the branes are flat, parallel and empty. The vacuum is of course static, and dynamics follow from a small breaking of the supersymmetry, in the form of a very weak potential for interbrane distance. The potential is taken to be attractive so that it draws the branes –in the old scenario the bulk brane and the visible brane, in the new scenario the boundary branes– towards each other until they collide, an event called *ekpyrosis*. In the old ekpyrotic scenario, the bulk brane is absorbed into the visible brane in a *small instanton phase transition*, while in the new scenario the boundary branes bounce apart after the collision. Ekpyrosis is the defining feature of the ekpyrotic scenario, and most of the cosmological problems are explained in terms of this collision or in terms of symmetries related to the branes as follows.

1. The origin of matter

   A significant fraction of the kinetic energy of the moving brane is supposed to be converted into a thermal bath of radiation on the visible brane, providing the matter content of the universe.

2. The homogeneity and isotropy of the universe

   Because the branes are (almost) parallel they will collide at (almost) the same time at all their points, producing an energy density with an (almost) constant temperature, "ekpyrotic temperature", everywhere in the visible universe.

3. The spatial flatness of the universe

   Spatial flatness of the visible universe follows from the assumption of starting very near the vacuum, where the branes are flat.

4. The seeds of large-scale structure

   Though the branes start flat and parallel, they undergo quantum fluctuations during their journey across the fifth dimension. Due to these "brane ripples", some parts of the branes collide somewhat earlier or later than the average, resulting in slightly cooler or hotter regions, respectively. These primordial perturbations then grow to become the cosmic microwave background anisotropies and seed the large-scale structure seen in the universe today. The adiabaticity of the perturbations is easy to understand in the formalism where the interbrane distance appears as a scalar field[3]: then the perturbations are obviously in the same direction in field space as the background. The spectral index is supposed to be nearly scale-invariant since the conditions change very slowly during the journey of the brane(s). In the formalism there are a number of free parameters whose natural magnitude it is difficult to estimate, so that it is not clear how natural it is to obtain the correct small amplitude for the perturbations, but at least it is easy, for the same reason.

5. The absence of topological defects

   The production of unwanted relics is highly suppressed if the ekpyrotic temperature is lower than the energy scale at which such relics are produced. It should be noted that, in contrast to the inflationary scenario, the temperature of the universe is at no time higher than the ekpyrotic temperature.

6. The singularity problem

   In the ekpyrotic scenario the big bang is ignited at some finite temperature and there are no curvature singularities. Since the scenario is based on heterotic M-theory, the singularity theorems of general relativity, which is a low-energy approximation of M-theory, do not necessarily apply. However, the ekpyrotic scenario does not include a description of what happened before the start of brane movement. Any model where time does not extend infinitely far into the past (and that does not contain closed timelike curves) is of course geodesically incomplete and thus singular.

[3]This formalism will be considered in more detail in chapter 4.

In addition to solving cosmological problems, the ekpyrotic scenario also proposes to solve problems of particle physics. The small instanton phase transition in the brane collision may change the instanton number of the visible brane and break the gauge group from E_8 to some smaller group, for example $SU(3) \times SU(2) \times U(1)$ or $SU(5)$. It can also set the number of light families to three. These are interesting directions, but quite different from the cosmological issues, so they will not be discussed here further.

The initial conditions of the ekpyrotic scenario, like those of the inflationary scenario and the pre-big bang scenario, have been under debate. The initial conditions has been criticised for fine-tuning, since the dynamical evolution in the ekpyrotic scenario must start extremely near the vacuum state [2]. Starting the dynamical evolution nearly but not quite in some special symmetric state does seem unappealing. However, criticism along these lines is not terribly fruitful. First, we will never (or at least not in the foreseeable future) be able to measure the initial state of the universe. Second, the naturalness of the size of some parameters in a given theory cannot be properly assessed until it is known how these parameters arise. Thus far the symmetry breaking has been added by hand, and it seems premature to conclude anything one way or another until it has been actually derived from heterotic M-theory. (A similar argument could be fielded in defense of the parameters responsible for the amplitude of CMB perturbations in scalar field models of inflation.)

As an aside, let us note that the "cyclic model of the universe" [13-15] was in part motivated by a desire to obtain the highly symmetric initial conditions as a result of a dynamical process.

The ekpyrotic scenario has a number of problems. They will be considered in the next chapter after a more detailed account of how the scenario is supposed to work.

3.5 Summary

There are a few promising scenarios of the primordial universe and several well-studied models that at least partially realise these scenarios. However, at the present time there is no model that would give satisfactory answers to all six cosmological problems outlined. Also, all current models have some deep unsolved problems which are not merely technical. As noted, the ekpyrotic scenario was presented as an alternative to the inflationary and pre-big bang scenarios, in part motivated by these problems. The last chapter is devoted to a more detailed account of the ekpyrotic scenario and the problems that it in turn faces.

Chapter 4

The ekpyrotic scenario

4.1 The set-up

As mentioned in chapter 3, there are two versions of the ekpyrotic scenario: the old scenario, where there is a third brane in the bulk, and the new scenario, where there are only the boundary branes. In addition, there is a spin-off, the so-called "cyclic model of the universe" [13-15], which shares many features with the ekpyrotic scenario. This chapter will be devoted to a review of the old and new ekpyrotic scenarios and their shortcomings, with some comments on the cyclic model at the end, in section 4.5.

As mentioned in chapter 1, the ekpyrotic scenario is based on heterotic M-theory. The action for both the old and the new version of the ekpyrotic scenario consists of three parts:

$$S = S_{het} + S_{BI} + S_{matter} \ , \tag{4.1}$$

where S_{het} is the action of five-dimensional heterotic M-theory with minimal field content, S_{BI} describes the brane interaction responsible for brane movement and S_{matter} describes brane matter created in the brane collision.

Simplified heterotic M-theory. Five-dimensional heterotic M-theory contains a large number of fields [32, 33], so that dynamical analysis is quite difficult. In the ekpyrotic scenario a pruned version of the theory is obtained by considering the minimal field content, that is, by putting to zero all fields whose equation of motion allows it. The resulting simplified action is [1, 5, 32, 33]

$$
\begin{aligned}
S_{\text{het}} &= \frac{M_5^3}{2}\int_{\mathcal{M}_5} d^5x\sqrt{-g}\left(R-\frac{1}{2}\partial_A\phi\,\partial^A\phi-\frac{3}{2}\frac{1}{5!}e^{2\phi}\mathcal{F}_{ABCDE}\mathcal{F}^{ABCDE}\right)\\
&\quad -\sum_{i=1}^{3}3\alpha_i M_5^3\int_{\mathcal{M}_4^{(i)}} d^4\xi_{(i)}\bigg(\sqrt{-h_{(i)}}e^{-\phi}\\
&\quad -\frac{1}{4!}\epsilon^{\mu\nu\kappa\lambda}\mathcal{A}_{ABCD}\partial_\mu X^A_{(i)}\partial_\nu X^B_{(i)}\partial_\kappa X^C_{(i)}\partial_\lambda X^D_{(i)}\bigg)\ , \qquad (4.2)
\end{aligned}
$$

where M_5 is the Planck mass in five dimensions, R is the scalar curvature in five dimensions, e^ϕ is the "breathing modulus", which describes the size of the Calabi-Yau threefold and $\mathcal{A}_{ABCD}$ is a four-form gauge field, with field strength $\mathcal{F}=d\mathcal{A}$. The Latin indices run from 0 to 4 and the Greek indices run from 0 to 3. The four-dimensional manifolds $\mathcal{M}_4^{(i)}$, $i=1,2,3$, are the visible, hidden and bulk branes respectively, with internal coordinates $\xi^\mu_{(i)}$ and tensions $3\alpha_i M_5^3 e^{-\phi}$. Note that the brane tensions can vary in time (and space) since they depend on the breathing modulus. The coefficients α_i have to sum to zero [3], and are parametrised as $\alpha_1=-\alpha$, $\alpha_2=\alpha-\beta$ and $\alpha_3=\beta$, with $\beta>0$. The tensor g_{AB} is the metric on $\mathcal{M}_5$ and $h^{(i)}_{\mu\nu}$ are the induced metrics on $\mathcal{M}_4^{(i)}$. The functions $X^A_{(i)}(\xi^\mu_{(i)})$ are the coordinates in $\mathcal{M}_5$ of a point on $\mathcal{M}_4^{(i)}$ with coordinates $\xi^\mu_{(i)}$, in other words they give the embedding of the branes into the five-dimensional spacetime.

The "vacuum" of the above action is a BPS state, which is invariant under Poincaré transformations in the directions parallel to the brane and preserves one-half of the eight supersymmetries of the five-dimensional theory. In the vacuum state the branes are flat, parallel and static, so that their embedding is simply given by ($t\equiv x^0$, $y\equiv x^4$)

$$
X^A_{(i)}(\xi^\mu_{(i)}) = (t, x^1, x^2, x^3, y_i)\ , \qquad (4.3)
$$

with $y_1=0$, $y_2=R$ and $y_3=Y$, where R and $0<Y<R$ are constants. The metric and the fields in the vacuum state are given by

$$
\begin{aligned}
ds^2 &= -N^2D(y)dt^2 + A^2D(y)\sum_{j=1}^{3}(dx^j)^2 + B^2D(y)^4dy^2\\
e^{\phi(y)} &= BD(y)^3\\
\mathcal{F}_{0123y}(y) &= -\alpha A^3NB^{-1}D(y)^{-2} \qquad y\le Y\\
&\quad -(\alpha-\beta)A^3NB^{-1}D(y)^{-2} \qquad y\ge Y\ , \qquad (4.4)
\end{aligned}
$$

where $D(y)=\alpha y-\beta(y-Y)\theta(y-Y)+C$ and N, A, B and C are constants. In the new ekpyrotic scenario, where there is no bulk brane, the above equations hold with $\beta=0$.

Brane interaction. The vacuum is of course static. Dynamics are provided by a small breaking of the supersymmetry in the form of non-perturbative M-theory interactions between the branes, mediated by the exchange of M2-branes. In principle it should be possible to obtain the resulting effective potential for the brane distances from M-theory, but so far a potential with the desired properties has not been derived. At present the potential has been added by hand to a four-dimensional effective theory and even an effective treatment of the brane interaction in five dimensions is lacking.

However, whatever the genesis or the exact form of the interaction, it presumably approaches zero as the boundary branes approach each other, at least in the new ekpyrotic scenario. The "fifth dimension" is the eleventh dimension of the full theory, so that its size is given by the value of the dilaton, in other words the string coupling constant[1]. As the size goes to zero, the coupling constant vanishes. The vanishing of the interaction between the bulk brane and the visible brane at their collision, which is assumed in the old scenario, is less obvious.

The brane interaction is supposed to be attractive, so that in the old scenario the bulk brane is attracted to the visible brane, while in the new scenario the boundary branes are attracted to each other. The branes are assumed to remain flat and parallel (apart from quantum fluctuations), so that in the old ekpyrotic scenario their embedding differs from that of the BPS state (4.3) only via the time-dependence of Y. In the new scenario the embedding is the same as in the BPS state (apart from quantum fluctuations).

The spatial homogeneity and isotropy in the directions parallel to the branes is also assumed to be maintained during their journey (again, apart from quantum fluctuations). The metric can be without loss of generality written as

$$ds^2 = -n(t,y)^2 dt^2 + a(t,y)^2 \sum_{j=1}^{3} (dx^j)^2 + b(t,y)^2 dy^2 \ . \tag{4.5}$$

Any time-dependence of the size of the fifth dimension is contained in $b(t,y)$, since the boundary branes stay at constant coordinate position.

Brane matter. All brane matter is assumed to be created in a brane collision, so that in the old scenario the hidden brane remains empty, while in the new scenario the hidden brane may also contain matter. The brane matter action is

$$S_{\text{matter}} = \sum_{i=1}^{2} \int_{\mathcal{M}_4^{(i)}} d^4\xi_{(i)} \sqrt{-h_{(i)}} \mathcal{L}_{\text{matter}(i)} \ . \tag{4.6}$$

[1] The matter may not be so simple. The Newton's constant on the brane is not given by the usual heterotic result [17], so that it is not clear whether the usual identifications for the other constants are correct either. The issue is discussed in section 4.4.1.

The detailed form of the matter Lagrange density is unimportant, since under the assumptions of homogeneity and isotropy the energy-momentum tensor of brane matter in any case has the ideal fluid form. Since the treatment of brane matter in a cosmological context has thus far been phenomenological, the brane matter term has not been included in the action (4.2), even though five-dimensional heterotic M-theory does provide a description of brane matter.

The calculational problem of the ekpyrotic scenario consists of obtaining the brane interaction from M-theory, solving the equations of motion obtained from (4.1) for the homogeneous and isotropic background using initial conditions very near the BPS state, calculating the quantum fluctuations of the branes as a perturbation of this background and finally transferring the brane ripples into perturbations of brane energy density. Even if the first step had been completed and the resulting terms would be simple, solving the five-dimensional equations for the background would be difficult. Therefore, the ekpyrotic scenario has mostly been discussed in the framework of an effective four-dimensional theory.

4.2 The four-dimensional effective theory

The four-dimensional effective theory of the ekpyrotic scenario is motivated by the complexity of the five-dimensional equations, the ease at which one can implement an effective treatment of the brane interaction in four dimensions and the idea that at low energies there should exist an effective covariant four-dimensional description of the locally observable physics. I will first present the effective theory in this section and then discuss its shortcomings in section 4.3.

4.2.1 The homogeneous and isotropic background

The procedure of obtaining the four-dimensional effective theory from the five-dimensional theory consists of two different approximations. The first is the “moduli space approximation”. The idea is that as long as the system evolves slowly, the evolution can be described as movement in the space of vacua spanned by the integration constants of the BPS solution. So, one takes the BPS solution (4.4) and promotes the integration constants N, A, B, C and Y, known as “moduli”, to functions which depend on coordinates parallel to the branes, so that for the homogeneous and isotropic background they depend only on time. In the moduli space approximation, the metric and the fields are

$$\begin{aligned} ds^2 &= -N(t)^2 D(t,y) dt^2 + A(t)^2 D(t,y) \sum_{j=1}^{3} (dx^j)^2 + B(t)^2 D(t,y)^4 dy^2 \\ e^{\phi(t,y)} &= B(t) D(t,y)^3 \\ \mathcal{F}_{0123y}(t,y) &= -\alpha A(t)^3 N(t) B(t)^{-1} D(t,y)^{-2} \qquad y \leq Y(t) \\ & \quad -(\alpha-\beta) A(t)^3 N(t) B(t)^{-1} D(t,y)^{-2} \qquad y \geq Y(t) \ , \end{aligned} \tag{4.7}$$

where $D(t,y) = \alpha y - \beta(y - Y(t))\theta(y - Y(t)) + C(t)$; $\theta(y - Y)$ is the step function.

The second approximation is to substitute the moduli metric and fields (4.7) back into the action (4.1) and integrate over the fifth dimension to obtain a four-dimensional action. The idea behind this procedure is that as one cannot resolve the fifth dimension at low energies due to its small size, one can integrate over it, a standard prescription in the Kaluza-Klein approach to extra dimensions. Since the dependence of the moduli metric and fields on the transverse coordinate y factorises, the integration is trivial. To the resulting four-dimensional action one then adds a potential term to support the movement in the space spanned by the moduli. In the old ekpyrotic scenario the potential is for the modulus Y, whereas in the new scenario it is presumably for B. As mentioned earlier, these terms are hoped to be eventually computable from heterotic M-theory.

The resulting four-dimensional action for the old ekpyrotic scenario is, with the approximations $B =$ constant, $C =$ constant and with a small bulk brane tension, $\beta \ll |\alpha|$ [1],

$$S_{4\mathrm{d}} \approx 3M_5^2 \int d^4x \tilde{n} \tilde{a}^3 \left(-\frac{1}{\tilde{n}^2} \frac{\dot{\tilde{a}}^2}{\tilde{a}^2} + \frac{\beta}{I_3} \left[\frac{1}{2} \frac{1}{\tilde{n}^2} D(Y)^2 \dot{Y}^2 - \frac{V(Y)}{B I_3 M_5} \right] \right) \ , \tag{4.8}$$

where $I_3(t)$ is a positive function which is constant to zeroth order in β/α, $V(Y)$ is the potential added by hand and $\tilde{n}$ and $\tilde{a}$ are defined in terms of the moduli N, A and B as

$$\begin{aligned} \tilde{n} &\equiv N\sqrt{B I_3 M_5} \\ \tilde{a} &\equiv A\sqrt{B I_3 M_5} \ . \end{aligned} \tag{4.9}$$

The relation of the effective four-dimensional action of the new ekpyrotic scenario to the five-dimensional action has not been presented, but is presumably similar. The authors of the ekpyrotic scenario identify (4.8) as the action of a four-dimensional homogeneous, isotropic and spatially flat universe containing a scalar field Y that is minimally coupled to gravity. Apart from the appearance of $D(Y)$ in the kinetic term of Y, the action has the standard form in the approximation where one keeps only the leading terms in β/α (in other words, neglects the time-dependence of I_3).

It would seem that a four-dimensional covariant low energy effective theory has been obtained, though it should be immediately emphasised that the lapse function $\tilde{n}$ and scale factor $\tilde{a}$ are not the lapse function and scale factor measured at the visible brane. The scenario can then be analysed using the standard methods of general relativity plus scalar fields, in the same manner as scalar field inflation. Work on the ekpyrotic scenario has concentrated on such analysis, especially on analysis of the perturbations around the homogeneous and isotropic background. The analysis of the background proceeds as follows.

The effective potential for the brane distance is taken to be

$$V(Y) = -F(Y) V_0 e^{-cY} \ , \tag{4.10}$$

where V_0 and c are positive constants and $F(Y)$ takes into account that the potential vanishes when the bulk brane collides with the visible brane; it is assumed that $F(Y) = 0$ for $Y = 0$ and $F(Y) \approx 1$ everywhere else. It is possible to use a potential that is not exponential, as long the it satisfies $VV''/V'^2 \approx 1$. For example, a power law potential $V(Y) \propto Y^q$ with a large $q \gtrsim 40$ will also do [1, 2].

The behaviour of a scalar field with the potential (4.10) and minimally coupled to gravity is simple to analyse in the homogeneous and isotropic case. There is a solution where the scale factor contracts obeying a power law, $\tilde{a} \propto t^p$, with $p = 2/(M_4^2 c^2)$. The contraction of $\tilde{a}$ was interpreted as the contraction of the fifth dimension, which led to the problem of stabilising the collapse or dealing with the boundary brane collision, which in turn led to the new ekpyrotic scenario.

The new ekpyrotic scenario sprung from the idea that if the fifth dimension is not stabilised and the boundary branes will eventually collide, this collision can be used to ignite ekpyrosis, rendering the bulk brane unnecessary. From the point of view of the four-dimensional effective theory, the scenarios are quite similar. The main difference is that in the old scenario, ekpyrosis occurs before the effective scale factor vanishes, while in the new scenario the branes collide at that very moment. The branes are then supposed to bounce apart, and the scale factor is supposed to start expanding from zero. The interbrane distance is supposed to be eventually stabilised via some as of yet unknown mechanism.

4.2.2 Perturbations around the background

As stressed in chapter 3, a model of the primordial universe should give quantitative predictions about the anisotropies of the CMB. In order to be in agreement with observation, the temperature fluctuations should be mostly adiabatic, nearly scale-invariant and have an amplitude of about 10^{-5}. In the ekpyrotic scenario the origin of the CMB anisotropies is the quantum fluctuations of the interbrane distance, which in the four-dimensional effective theory are treated like the fluctuations of a scalar field. Perturbations have been extensively discussed in the four-dimensional effective theory and in similar settings [1, 2, 7, 113, 114, 120-134].

The perturbation theory around the collapsing background is well-known. The perturbations are adiabatic since they are fluctuations of the same quantity that is responsible for the collapsing background, and getting the correct amplitude is simply a question of tuning some parameters. The crucial question is whether the perturbations are scale-invariant. At first sight this might seem not to be the case.

In inflation the perturbations are scale-invariant because both the Hubble parameter and the scalar field are almost constant. In the original version of the pre-big bang scenario with only one field, the perturbations have a large spectral index, $n \approx 4$, due to the rapidly decreasing Hubble parameter. The effective four-dimensional theory of the ekpyrotic scenario is much like the pre-big bang scenario, so one would expect a blue spectrum. It seems that one does obtain a large spectral index, $n \approx 3$, for some perturbations. However, it is apparently also possible to get a spectral index close to unity for some perturbation variables, by tuning the potential (4.10) to be very flat, $c \gg 1$, so that the collapse is very slow, $p \ll 1$. The question is then: what is the perturbation variable whose spectrum is inherited by the CMB fluctuations?

The only ambiguity in the evolution of the perturbations is due to the curvature singularity where the scale factor vanishes. Since the background is not well-defined at this point, it is clear that perturbation theory around the background makes no sense either. However, the approach taken by the authors of the ekpyrotic scenario is to find perturbation theory variables which remain finite and match these across the collision. Such a prescription requires that the singularity is resolved in some manner which leaves the perturbation theory unaffected. This problem is somewhat similar to the graceful exit problem of the pre-big bang scenario, not least because it is also unresolved. The resolution has been suggested to happen in the five-dimensional context [6, 7, 135, 136], but no consistent and detailed account has been given. The proposal in [6, 7, 135, 136] is formulated in flat spacetime, and does not apply to the ekpyrotic setting where brane tension will always curve spacetime [18].

The matching across the "bounce" has been much debated [7, 113, 114, 121, 122, 124-130, 133, 134]. It seems that, first, there is no unambiguous way to choose how to match across the bounce, and second, that the impact on the CMB depends sensitively on the matching conditions chosen. Furthermore, it has been demonstrated that the treatment of the bounce as a sharp transition to match across may not necessarily be justified [126], not a surprising result. It has also been argued that regardless of the matching, a consistent large-scale treatment of the perturbations within the context of general relativity will never yield the desired scale-invariant spectrum [114, 122], and that perturbation theory breaks down even before the singularity [120]. At best, the result of a scale-invariant spectrum within the four-dimensional effective theory rests on matching conditions which are, to quote one of the authors, "a guess" [137].

The ekpyrotic scenario has been much criticised for the ambiguities associated with the spectral index and for the prescription used in joining the collapsing phase to an expanding one. However, a more fundamental issue is that the whole framework of the four-dimensional effective theory is highly questionable.

4.3 Problems of the four-dimensional approach

Some have pointed out [121, 122, 131] that the issues of bounce and perturbations should be properly handled in the context of the five-(or higher)dimensional theory due to the singularity and the associated ambiguities of the effective theory. However, the four-dimensional description is problematic already at the homogeneous and isotropic level, even without considering the singularity. I will now briefly go through the problems of the four-dimensional approach. Some of the issues were first brought up in [5], and others were highlighted in [18].

4.3.1 The five-dimensional equations of motion

The basis of the four-dimensional effective theory, the moduli space ansatz (4.7) along with the potential for the interbrane distance, does not satisfy the five-dimensional equations of motion derived from the action (4.1) [5, 18]. As noted before, the potential was added directly to the four-dimensional effective theory, and it was implied that in the five-dimensional theory it would be a delta function source located on the moving brane. However, such a delta-function potential cannot support time-dependent movement of the bulk brane with the moduli metric: quite simply, the brane does not move.

Even if one adds an arbitrary energy-momentum tensor in the bulk, the movement of the bulk brane is so limited as to render the ekpyrotic scenario unworkable. The bulk brane can only move if the brane interaction is coupled to the Calabi-Yau breathing modulus ϕ, and even then the velocity $\dot{Y}$ will vanish at the brane collision, leading to zero ekpyrotic temperature, according to the original formulae [1].

The new ekpyrotic scenario does not necessarily suffer from such problems, but since the curvature and energy density given by the moduli ansatz diverge at the collision of boundary branes, it is clear that the moduli space approximation does not work in the new ekpyrotic scenario, at least near the collision[2].

These problems are simply a consequence of the overly constrained form of the moduli metric (4.7). In particular, the factorisation of the y-dependence of the metric in the directions parallel to the brane is known to be a severely constraining condition [57, 58].

[2]Note that it is possible to have non-singular boundary brane collisions, given a less symmetric metric; see section 4.4.2.

In this connection it may also be noted that the moduli metric cannot support matter on the branes [17]. The factorisable form of the metric is so constraining that the Israel junction conditions which relate the embedding of the brane to its energy density and pressure do not permit any matter, as illustrated in section 2.3.3.

According to the authors of the ekpyrotic scenario, the moduli ansatz does not need to satisfy the equations of motion. It is difficult to understand how the moduli space approximation is supposed to work without satisfying the dynamical equations of the theory even at some approximate level. Also, it is unclear how one could evaluate the validity of the effective theory except by comparing with the full theory, given that the parameters that are supposed to remain small in order for the approximation to be valid (time derivatives of physical quantities, presumably) can take any value without degrading the internal consistency of the effective theory. There is, for example, no expansion in terms of these parameters, and it is not clear what the corrections to the moduli space approximation would be.

But even if we took for granted that the moduli space approximation does not need to satisfy the equations of motion and can describe brane movement, and that the curvature singularity can be ignored, the constraining form of the metric is responsible for other severe problems.

4.3.2 Flow of energy off spacetime

A reasonable condition for a theory formulated on a compact manifold or orbifold is that no energy should flow away across the boundary of spacetime. If the manifold or orbifold has no boundary, this condition is trivially satisfied; otherwise it provides a non-trivial boundary condition.

In the new ekpyrotic scenario, the "no-flow condition" for the moduli metric (4.7) is violated whenever the hidden brane moves [18]. Or, more reasonably, the no-flow condition prevents the hidden brane from moving within the confines of the moduli space ansatz.

The old ekpyrotic scenario fares little better. In the approximation $B =$ constant, $C =$ constant used in [1], movement of the bulk brane (as well as the boundary branes) is prohibited by the no-flow condition. Relaxing these conditions on B and C, it is not impossible for the bulk brane to move without energy flowing away across the boundary of spacetime. However, due to the constrained form of the moduli metric the no-flow condition relates the bulk brane movement to the size of the fifth dimension, making another problem apparent.

4.3.3 The expansion of the fifth dimension

The main problem of the old ekpyrotic scenario was considered to be that the fifth dimension contracted as the bulk brane travelled from the hidden brane to the visible brane. As noted above, the moduli metric is so constrained that the no-flow condition relates the size of the fifth dimension to the movement of the bulk brane. Setting aside the problem that the bulk brane cannot actually move at all with the potential used, or with B and C constant, let us see what this relation shows. The velocity of the size of the fifth dimension $L(t) \equiv \int_0^R dy\, b(t,y) = \int_0^R dy B(t) D(t,y)^2$ is [18]

$$\begin{aligned} \dot{L}(t) &= \int_0^R dy\, \dot{b}(t,y) \\ &= -\frac{\beta \dot{Y}}{\beta Y + (\alpha - \beta) R} BR(\alpha Y + C)^2 \; . \end{aligned} \tag{4.11}$$

Recall that $|\alpha| > \beta > 0$, and that the bulk brane travels from $y = R$ to $y = 0$, so that $\dot{Y} < 0$. Therefore, the sign of the velocity $\dot{L}$ is opposite to the sign of the tension of the visible brane, $-\alpha$. In the set-up analysed in [1], the tension of the visible brane is negative, so that the fifth dimension expands, in contradiction with the result of the four-dimensional effective theory that it collapses. Let us recall that this collapse was regarded as the most severe problem of the old ekpyrotic scenario.

4.3.4 The Hubble law

The contradictions between the results of the five-dimensional theory and the four-dimensional effective theory are not confined to the bulk equations of motion which are not satisfied or to the boundary conditions which lead to expansion of the fifth dimension. A comparison between the Hubble law observed on the visible brane obtained from the effective four-dimensional equations and the real Hubble law from the exact five-dimensional equations provides further illustration of the status of the effective theory.

The equations of the four-dimensional effective theory are those of Einstein gravity plus a scalar field. However, as emphasised in section 4.2, the parameters of the effective theory are not the parameters seen by an observer on the visible brane (for example, the contraction of the effective scale factor $\tilde{a}$ does not imply contraction of the scale factor of the visible brane). The Hubble law resulting from the action (4.8) is given in [1] as

$$\frac{1}{\tilde{a}^2} \left(\frac{1}{\tilde{a}} \frac{d\tilde{a}}{d\eta} \right)^2 = \frac{\beta M_5}{B (I_3 M_5)^2} \left(\frac{1}{2} D(Y)^2 \left(\frac{dY}{d\eta} \right)^2 + V(Y) \right) \; , \tag{4.12}$$

where η is conformal time ($\tilde{n} = \tilde{a}$). In the above equation it is assumed that $B =$ constant, $C =$ constant. Taking (4.12) to be correct (though there should presumably be $\tilde{a}^{-2}$ multiplying the kinetic term) and changing to general time ($\tilde{n}$ undetermined), we obtain

$$\frac{1}{\tilde{n}^2}\frac{\dot{\tilde{a}}^2}{\tilde{a}^2} = \frac{\beta M_5}{B(I_3 M_5)^2}\left(\frac{1}{2}\frac{\tilde{a}^2}{\tilde{n}^2}D(Y)^2\dot{Y}^2 + V(Y)\right) , \tag{4.13}$$

Recall that $\tilde{n}$ and $\tilde{a}$ are not the physical lapse function and scale factor. Their relation to the components n and a of the physical metric is, according to (4.7) and (4.9),

$$\begin{aligned} n(t,y) &= \tilde{n}(t)\sqrt{\frac{D(t,y)}{BI_3(t)M_5}} \\ a(t,y) &= \tilde{a}(t)\sqrt{\frac{D(t,y)}{BI_3(t)M_5}} , \end{aligned} \tag{4.14}$$

where $D(t,y)$ is given immediately after (4.7) and, as mentioned earlier, $I_3(t) \simeq I_3(0) + \mathcal{O}(\beta/\alpha)$. On the visible brane we have

$$\begin{aligned} n(t,0) &\simeq \tilde{n}(t)\sqrt{\frac{C}{BI_3(0)M_5}} \\ a(t,0) &\simeq \tilde{a}(t)\sqrt{\frac{C}{BI_3(0)M_5}} , \end{aligned} \tag{4.15}$$

where terms of order β/α have been dropped. Combining (4.13) and (4.15) we obtain the derivative of the physical scale factor with respect to the physical proper time measured on the visible brane ($n(t,0) = 1$),

$$\frac{\dot{a}(t,0)^2}{a(t,0)^2} \approx \frac{\beta}{CI_3}\left(\frac{1}{2}a(t,0)^2 D(Y)^2\dot{Y}^2 + V(Y)\right) , \tag{4.16}$$

The Hubble law (4.16) is to be compared with the Hubble law that emerges from the exact five-dimensional equations derived directly from (4.1) (assuming that the brane interaction has only a delta function support at the bulk brane, and taking into account that no energy should flow away across the boundary of spacetime) [17]:

$$\frac{\dot{a}(t,0)^2}{a(t,0)^2} = -\frac{1}{6M_5^3}\alpha e^{-\phi_1}\rho_r - \frac{1}{6M_5^3}\alpha e^{-\phi_0}\rho_d + \frac{1}{36M_5^6}\rho_m^2 + \frac{\mathcal{C}}{a(t,0)^4} , \tag{4.17}$$

where ϕ_0 is the constant value of the breathing modulus at the position of the visible brane, and ϕ_1 and $\mathcal{C}$ are some constants. The tension of the visible brane has to be positive, $-\alpha > 0$, in order to recover a positive gravitational coupling constant, as noted in section 2.3.2; no such restriction is apparent in the effective four-dimensional theory. In the pre-collision era when the brane is empty, the Hubble law (4.17) reduces simply to

$$\frac{\dot{a}(t,0)^2}{a(t,0)^2} = \frac{\mathcal{C}}{a(t,0)^4} \; . \tag{4.18}$$

It is evident that the interbrane distance does not appear as a scalar field, nor does the potential with delta function support enter the equation at all. These are well-known general features of brane cosmology [41], discussed in section 2.3.

Let us also note that matter would presumably be added to the four-dimensional effective theory in the same way as $V(Y)$.[3] Then the gravitational constant to which this matter couples would also not depend on the brane tension but would be simply M_5^{-2}, according to the identification made from (4.2). There would also be no ρ^2-term in the effective theory.

In summary, the Hubble law of the effective four-dimensional theory is not only in quantitative but also in qualitative contradiction with the real Hubble law given by the five-dimensional equations in at least three important respects. In the real Hubble law 1) there is no scalar field corresponding to the interbrane distance, 2) the gravitational coupling has the same sign as the tension of the visible brane, and is not given simply by M_5^{-2} and 3) there is a term proportional to ρ^2.

4.3.5 Integration over the fifth dimension

As the above considerations amply demonstrate, the four-dimensional effective theory might at best be called misleading. However, one may ask whether the problems of the effective theory are *technical*, to be overcome with an improved method of approximation –perhaps including a replacement of the moduli space ansatz (4.7) with a less constrained configuration– or whether they are *conceptual*, underlining a problem in the whole approach of integrating along the fifth dimension to obtain a four-dimensional theory.

Leaving the moduli space approximation aside, let us consider the other ingredient: integration over the fifth dimension[4]. It is clear that such a procedure is not covariant, since it singles out the direction transverse to the branes. Without a metric which has a simple factorisable form[5], the "direction transverse to the branes" is not well-defined in the bulk: one may perform coordinate transformations that mix the t- and y-coordinates in the bulk without affecting them on the branes. So, it is not clear along which path to integrate. One might suggest integrating along a geodesic transverse to the branes, but such a prescription is clearly not unique: a geodesic which starts transverse to the visible brane will not in general be transverse to the hidden brane, and vice versa.

[3] As is done in the cyclic model; see section 4.5.

[4] The discussion in this section draws heavily on [63].

[5] As illustrated in section 2.3.3, brane matter will always cause the metric to be non-factorisable.

At any rate, as a matter of mathematical consistency one should not simply sum (or integrate) tensor quantities (such as the metric) at different points of the manifold but take proper care in transporting them to the same point. It might seem that this is not a problem if one integrates at the level of the action –which is a scalar quantity– as is done in the ekpyrotic scenario, instead of integrating over the equations of motion. However, in order to obtain the four-dimensional equations of motion from the four-dimensional action, one has to vary the action with respect to some four-dimensional quantity. In the ekpyrotic scenario this quantity is $\tilde{n}$, which is in no way covariant as is apparent from (4.14), so that the mathematical status of the resulting equations is somewhat unclear.

Quite apart from such mathematical concerns, one may ask what is the *physical* justification for integrating over the fifth dimension. In Kaluza-Klein theories, as well as in string theory, the idea is that the observers are not localised in the extra dimensions, and so cannot resolve them. Then it makes sense to integrate over the extra dimensions to obtain an averaged theory. But that is not the situation here: the defining feature of brane cosmology is that the observers *are* localised in the extra dimensions, being confined to a codimension one object – in the simplest case, one point along the extra dimension. Therefore, it does not seem to make sense even from the physical point of view to integrate over the extra dimensions to obtain an effective theory.

The authors of the ekpyrotic scenario have presented an additional justification for using a four-dimensional effective theory. The idea has also been used in the cyclic model, where there is no attempt to derive the effective theory from a fundamental theory. The argument is that there should be some covariant effective four-dimensional theory at low energy, and that the simplest such theory is Einstein gravity plus a scalar field with some potential. There are two counter-arguments to this proposition.

First, it is explicitly known in brane cosmology that one does not recover Einstein gravity plus a scalar field at low energies. As discussed in section 2.3, if the brane does not have a tension that is positive, one does not even approximately recover ordinary gravity. If the brane has positive tension, one has Einstein gravity plus terms quadratic in the brane energy-momentum tensor (the ρ^2-term) plus contributions which cannot be solved from the brane equations, but have to be calculated from the five-dimensional bulk equations. The description of the extra dimension as a scalar field is a feature of Kaluza-Klein theories that does not appear in brane cosmology, and likewise, the ρ^2-behaviour is nowhere to be found in the effective four-dimensional theory or in Kaluza-Klein theories in general.

Second, in the four-dimensional effective theory (as formulated in the ekpyrotic scenario and in the cyclic model), one does not actually even recover Einstein gravity plus a scalar field with some potential at low energy. In the ekpyrotic scenario, the physical Hubble law given by (4.16) is obviously not of that simple form. Redefining the field Y to obtain a canonical kinetic term will result in a potential with a complicated dependence on an integral over $D(Y)a(t(Y))$, certainly not the simplest form one could imagine (though the inclusion of $\tilde{a}^{-2}$ in the kinetic term in (4.12) would make the field redefinition a simple affair). In the cyclic model, the departure from Einstein gravity plus scalar field is even more apparent, as we will see in section 4.5.

To summarise, the correct procedure for obtaining the physical Hubble rate (and other parameters) in brane cosmology is not to integrate over the fifth dimension but to consider the induced equations on the brane. These equations do not in general form a closed system, meaning that one must solve the full five-dimensional equations. This is not surprising. Were one to consider a codimension one domain wall in the visible four-dimensional universe, surely it would not be expected that the dynamics within the wall could be solved entirely without reference to matter in the rest of the universe.

4.4 Problems in five dimensions

In order to place the ekpyrotic scenario on a solid footing it is necessary to study the full five-dimensional equations. The ekpyrotic scenario is still young, and there has not been much work in that direction, so instead of giving an account of results, I will only be able to list some problems that a five-dimensional construction should solve. Apart from [17, 18], the only work on the issue is some criticism and discussion in [2, 5] and some responses in [3, 4].

Unlike in the four-dimensional effective theory, there are no conceptual problems in the five-dimensional approach. The minimal action of five-dimensional heterotic M-theory (4.2) is well-established, and the groundwork for the perturbation analysis in five-dimensional brane cosmologies has also been done [67]. The main problem is that the brane interaction responsible for the breaking of the BPS symmetry and the dynamics is not known in five dimensions, even at the level of an effective description. However, there are some issues that can be discussed without knowing the details of the brane interaction.

4.4.1 Stabilisation of the fifth dimension

Fitting tree-level parameters of (eleven-dimensional) heterotic M-theory to the observed value of the gravitational coupling constant and the inferred value of the grand unified gauge coupling, the size of the eleventh dimension turns out to be a few orders of magnitude larger than the size of the Calabi-Yau dimensions [10, 11]. It was this observation that the universe would look five-dimensional over some energy range that motivated the formulation of heterotic M-theory in five dimensions [32, 33]. The size of the fifth dimension is also important for the phenomenology of the model, as emphasised in the ekpyrotic context in [2, 5].

In the new ekpyrotic scenario the boundary branes collide together and bounce apart, so that the problem of stabilising the brane distance at the desired value is obvious. And even the old ekpyrotic scenario may have problems with stabilisation, quite apart from the results of the four-dimensional effective theory. The stabilisation mechanism of the eleventh dimension is not known in heterotic M-theory. But since the brane interaction that breaks the symmetry of the initial state is supposed to be extremely weak, one may wonder whether a stabilisation mechanism would interfere with the delicate journey of the bulk brane, or vice versa, a point made in [2, 5].

However, it is not clear whether such a stabilisation mechanism is needed. The question is related to an important difference between a brane set-up and a Kaluza-Klein set-up. In the original formulation of heterotic M-theory the gravitational coupling depends on the size of the eleventh dimension as in Kaluza-Klein models generally. However, as we have seen, that is not true when one takes into account the brane nature of the dimensional reduction from eleven to ten dimensions. I have not looked into the issue in detail and do not know how the grand unified gauge coupling behaves in the brane setting. The matter deserves further study, and the need for a stabilisation mechanism in a brane setting should be carefully investigated.

4.4.2 The boundary brane-boundary brane collision

The new ekpyrotic scenario was originally presented because the effective four-dimensional theory indicated that the fifth dimension collapses, so that the eventual collision of the boundary branes rendered the bulk brane superfluous. Even though there is no reason to take the four-dimensional effective theory seriously, a formulation with only the boundary branes has aesthetic appeal, as well as possibly being easier to solve. (There is also the advantage of not having to address the question of the origin of the bulk brane.) However, a collision between two boundary branes is a more violent event than a collision between a boundary brane and a bulk brane, since it involves one dimension vanishing for an instant. One might expect the curvature to become singular at such a collapse, as happens with the moduli metric, signalling the breakdown of the five-dimensional theory.

A proposal for resolving the issue has been made in [6, 7, 135, 136]. The matter has been investigated in some detail in [18]. It turns out, perhaps surprisingly, that it is possible for the transverse direction to collapse to a point and re-expand without the curvature or the energy density becoming singular. This is to be contrasted with a spatially flat or open Friedmann-Robertson-Walker universe, where the simultaneous collapse of the three spatial dimensions necessarily involves a curvature singularity[6]. However, the non-singular

[6]Some spatially closed FRW universes can remain non-singular at the collapse, most notably the 3+1-dimensional Milne universe which contains no matter. However, the reason for avoiding a singularity is different from the ekpyrotic case.

configurations are very constrained. It is possible to construct all non-singular metrics near the collision and use the Israel junction conditions (2.16) to look for constraints on the matter created in the collision.

Since the brane interaction is supposed to vanish as the branes meet, its energy-momentum tensor contributes only to terms in the Einstein equation which remain bounded at the collision. Therefore the energy-momentum tensor of brane interaction is not relevant for the analysis of possible singularities at the collision, and the issue may be studied on quite general grounds.

The basic requirement for a collision to be non-singular is that physical quantities, in particular the Riemann tensor and the energy-momentum tensor remain bounded, both in the bulk and on the branes. In [18] the issue was studied in a particular local orthonormal frame. (Un)boundedness of the components of the Riemann tensor in a particular local orthonormal frame does not guarantee their (un)boundedness in another local orthonormal frame, since singularities may appear in local Lorentz transformations connecting different frames[7]. However, it is straightforward to check that the constraints on brane matter given below are valid in all local orthonormal frames which respect the homogeneity and isotropy with respect to the spatial dimensions parallel to the brane and reproduce the metric (4.5).

Other conditions necessary for consistency are the no-flow condition and the covariant conservation of the energy-momentum tensor.

The boundedness condition constrains the near-collision metric significantly. The implications for brane matter depend on how rapidly the fifth dimension re-expands. More specifically, writing the scale factor of the fifth dimension in (4.5) after the collision as $b(t,y) = b_k(y)t^k + b_{k+1}(y)t^{k+1} + \mathcal{O}(t^{k+2})$, the implications for brane matter depend on the value of k (which has to be an integer).

In the case $k = 1$ in [18] there is both a sign mistake and a term missing in equation (28) for n. The correct equation is

$$\frac{1}{b_1^2}\left(n_1'' - \frac{b_1'}{b_1}n_1'\right) + n_1 \quad = \quad 2\frac{b_2}{b_1}\ . \tag{4.19}$$

The inclusion of $b_2(y)$ is important, since it means that the brane matter created in the collision is related not only to the velocity $\dot{b}$ but also to the acceleration $\ddot{b}$. The acceleration at collision is not constrained by the boundedness requirement, so that neither is the brane matter created.

In the case $k \geq 2$ the acceleration does not enter and the brane matter is severely

[7] I am grateful to Jorma Louko for pointing this out.

constrained; it has to obey the relations

$$\begin{aligned} \rho_1 + \rho_2 &= 0 \\ p_1 + p_2 &= -4M_5^3 \delta_{2k} \int_0^R dy\, b_k(y) \ , \end{aligned} \tag{4.20}$$

where ρ_i and p_i are the energy density and pressure, respectively, of matter on brane i immediately after the collision. So, the energy density on one brane and the pressure on at least one brane have to be negative.

It is worth emphasising that the above results follow from a direct study of the Riemann tensor, and do not utilise the equations of motion apart from the Israel junction conditions that give the embedding of the branes into spacetime. (The no-flow condition of the Einstein tensor does not yield new information.) Therefore, they are unaffected by changes in the action unless they lead to a change in the junction conditions. Even then, some constraints on brane matter (in the case $k \geq 2$) are likely to remain due to the highly constrained form of the near-collision metrics.

In a way it is not surprising that one of the energy densities created on the branes has to be negative (in the case $k \geq 2$). According to (4.17) the gravitational coupling on one brane will necessarily be negative. For the standard four-dimensional Hubble law $H^2 = 8\pi G_N \rho_m/3$ this would imply a non-positive energy density. Since the Hubble law is not the standard one, this is not necessarily true, and the question would have to be addressed in the context of a full solution of the five-dimensional equations. However, it is clear that in the ekpyrotic brane setting one cannot simply put ordinary matter on both branes without worrying about the details, as originally assumed in the new ekpyrotic scenario.

For the minimal action of heterotic M-theory, (4.2), the no-flow condition and the covariant conservation of the energy-momentum tensor (taking into account an arbitrary brane interaction which vanishes at the collision) forbid the possibility $k = 1$, implying a negative energy density. However, this result can possibly be avoided by turning on more fields in the full action of five-dimensional heterotic M-theory.

To summarise, it is not impossible to have non-singular ekpyrotic boundary brane collisions, but they are highly constrained. However, boundary brane collisions which produce radiation on the visible brane are not ruled out.

4.5 The "cyclic model of the universe"

Some of the ideas of the ekpyrotic scenario have been central in the construction of a spin-off called the "cyclic model of the universe" [13-15]. The set-up is a five-dimensional brane model, with matter produced in collisions between boundary branes, as in the new ekpyrotic scenario. There are two main differences between the cyclic model and the new ekpyrotic scenario.

The first important difference is that instead of being a unique event, ekpyrosis is posited to occur at regular intervals. The history of the universe then consists of an infinite sequence of roughly identical cosmological cycles. Late-time inflation serves to empty the branes between collisions, producing the highly symmetric initial state postulated in the ekpyrotic scenario.

The second important difference is that even though the scenario is motivated by heterotic M-theory, it is not based on it, or on any other theory in the sense that the effective four-dimensional theory would be derived from some more fundamental setting. Instead, the four-dimensional theory –which is similar to that of the ekpyrotic scenario– is simply proposed ad hoc, on the principle of Einstein gravity plus scalar field being the simplest possible covariant low energy description. The connection to a more fundamental theory is hoped to eventually emerge. This connection is especially important since the scale factor of the effective theory collapses to a point, so that a higher-dimensional description is considered necessary to resolve the apparent singularity. The issue has been discussed in [136], and proposals in this direction made in the context of the ekpyrotic scenario [6, 7, 135] have also been referred to in the cyclic model.

Since the cyclic model is not based on a definite fundamental theory or a given five-dimensional action, it is impossible to analyse it by starting from the fundamental theory, as was done for the ekpyrotic scenario in section 4.2. However, it is possible to make a few observations based on general results in brane cosmology; the following remarks mostly follow [18].

The Hubble law. First, as in the ekpyrotic scenario, the physical Hubble law derived from the effective theory is completely different from the real Hubble law in a brane cosmology setting. The Hubble law on the negative tension brane in the cyclic model, given by equations (8) and (12) of [14], is[8]

$$\begin{aligned}\frac{\dot{a}_1^2}{a_1^2} &= \frac{8\pi G}{3}\left(\beta^4\rho + V(\tilde{\phi})\right) + 2\dot{\tilde{\phi}}\coth\tilde{\phi}\sqrt{\dot{\tilde{\phi}}^2 + \frac{8\pi G}{3}\left(\beta^4\rho + V(\tilde{\phi})\right)} \\ &\quad + \dot{\tilde{\phi}}^2(1+\coth^2\tilde{\phi})\ ,\end{aligned} \tag{4.21}$$

where a_1 is the scale factor of the negative tension brane, G is a constant, ρ is the energy density of matter on the brane, $\tilde{\phi}$ is a scalar field related to the size of the fifth dimension[9], $V(\tilde{\phi})$ is the potential responsible for brane movement and $\beta(\tilde{\phi}) = -2\sinh\tilde{\phi}$. Note that the gravitational coupling is given by $8\pi G\beta^4$ and depends on the size of the fifth dimension.

[8] I have corrected a typo regarding $8\pi G$; it has no effect on the present argument.

[9] In the notation of [14], $\tilde{\phi} = (\phi - \phi_\infty)/\sqrt{6}$.

As an aside, the parameter with respect to which the time derivatives are taken in (4.21) is called the "FRW proper time" in [14]. It is, however, apparently not the physical time; for small brane separation the physical time is given in [14] as $t_5 = \int dt\, e^{-\tilde{\phi}}$. This detail makes no difference to the present argument.

Since the higher-dimensional origin of the potential V is not known, let us consider the case $V = 0$ to allow for comparison with the Einstein equation which arises in the five-dimensional brane setting with an empty bulk. The induced Einstein equation is given by (2.14) with $\Lambda = 0$ and $K = 0$,

$$\frac{\dot{a}^2}{a^2} = -\frac{|\Lambda_1|}{18M_5^6}\rho + \frac{\Lambda_1^2}{36M_5^6} + \frac{1}{36M_5^6}\rho^2 + \frac{\mathcal{C}}{a^4}\,, \tag{4.22}$$

where $\Lambda_1 = -|\Lambda_1|$ is the brane tension.

A comparison of (4.21) and (4.22) shows that the Hubble law of the cyclic model differs from the Hubble law given by the five-dimensional equations in significant respects, as was the case for the Hubble law of the ekpyrotic scenario. In the real Hubble law 1) there is no scalar field corresponding to the interbrane distance, 2) the gravitational coupling has the same sign as the tension of the visible brane, and does not depend on the size of the fifth dimension, 3) there is a term proportional to ρ^2 but no term involving ρ under a square root[10] and 4) there is a term involving the square of the brane tension[11].

In brief, the Hubble law of the cyclic model is completely different from the Hubble law of the brane setting that it is supposed to describe.

The collision. Second, in [14] the spacetime is assumed to be flat immediately before and after the collision, as argued in [6, 7, 135]. However, brane tension (and brane matter) necessarily implies that curvature cannot be neglected [18].

As noted in section 4.4.2, non-singular boundary brane collisions that produce radiation on the positive tension brane are not ruled out. However, one cannot simply produce radiation on the negative tension brane without additional complications, because of the negative gravitational coupling.

The low energy effective description. Third, as is evident from (4.21), the aim of having the simplest possible low energy description in terms of Einstein gravity plus a scalar field is not realised, just like in the ekpyrotic scenario.

[10] As (2.10) shows, it is impossible to obtain such a term in the five-dimensional brane setting without invoking an explicit ρ-dependence in the bulk energy-momentum tensor.

[11] In the Randall-Sundrum model and in the ekpyrotic scenario this term is cancelled by a bulk contribution; no such contribution has been specified in the cyclic model.

Problems that the cyclic model encounters even if one takes the four-dimensional effective theory for granted have been discussed in [87, 138]. It is also pointed out in [87, 138] that, unlike the ekpyrotic scenario, the cyclic model is not an alternative to inflation: the initial state which solves the problems of homogeneity, isotropy and flatness is provided by inflation instead of supersymmetry.

Apart from the above problems, the cyclic model is considerably less attractive than the ekpyrotic scenario simply because it is not founded on a fundamental theory. The effective theory is taken ad hoc, and the necessary higher-dimensional description of the brane collision is adopted from a study in flat spacetime [6, 7, 135, 136] which is not applicable to the cyclic model because of the presence of brane tension and matter [18].

4.6 Summary

The ekpyrotic scenario is a promising concept. Starting, like the pre-big bang scenario, from a set-up as fundamental (and therefore speculative) as string/M-theory and descending down to phenomenology is quite an attractive alternative to the "bottom-up" approach of most inflationary models. The possibility of obtaining a non-singular cosmology is promising, and the conceptual simplicity of the ekpyrotic scenario has definite appeal.

In brief, the ekpyrotic scenario is a welcome new idea.

However, the techniques employed so far in the study of the scenario are mostly not solid, and more careful work is needed to promote the scenario from a promising idea to a concrete model with testable predictions free from technical problems. In particular, the four-dimensional effective theory does not give a correct description, so the analyses within the framework of this theory, including the work on perturbations [1, 2, 7, 113, 114, 120-134] are irrelevant as regards the ekpyrotic scenario.

Nevertheless, work on the four-dimensional effective theory has led to renewed interest in collapsing cosmological backgrounds [113, 125-128, 130, 132-134, 138, 139] which may provide important insights, for example for the pre-big bang scenario. Given the status of the effective four-dimensional theory, it is ironic that the collapse problem seems to also have rekindled serious interest in studying string theory in time-dependent backgrounds, which may be a first step towards a resolution of real cosmological singularities [135, 140-142].

Ultimately, the form of the brane interaction, the absorption of the bulk brane by the visible brane and the small instanton phase transition will have to be addressed in the M-theory context if the ekpyrotic scenario is to be a fundamental description of the early universe. However, it might be possible to do some useful analysis within the context of the five-dimensional effective theory. A promising route might be to consider the bulk brane case, construct a reasonable ansatz for the brane interaction and solve the equations of motion for the background. Then one should do the perturbation analysis, building on existing techniques in brane cosmology [67] and construct an effective description of transferring the brane ripples to perturbations of brane energy density.

The ekpyrotic scenario is only one year old, and the next few years will show whether it leads to a working model that solves cosmological problems or whether its main impact will be as a springboard for new ideas.

Bibliography

[1] Justin Khoury, Burt A. Ovrut, Paul J. Steinhardt and Neil Turok. *The ekpyrotic universe: colliding branes and the origin of the hot big bang*. Phys. Rev. **D64**: 123522, 2001. **hep-th/0103239**.

[2] Renata Kallosh, Lev Kofman and Andrei Linde. *Pyrotechnic universe*. Phys. Rev. **D64**: 123523, 2001. **hep-th/0104073**.

[3] Ron Y. Donagi, Justin Khoury, Burt A. Ovrut, Paul J. Steinhardt and Neil Turok. *Visible branes with negative tension in heterotic M-theory*. JHEP **0111**: 041, 2001. **hep-th/0105199**.

[4] Justin Khoury, Burt A. Ovrut, Paul J. Steinhardt and Neil Turok. *A brief comment on "The pyrotechnic universe"*. **hep-th/0105212**.

[5] Renata Kallosh, Lev Kofman, Andrei Linde, and Arkady Tseytlin. *BPS branes in cosmology*. Phys. Rev. **D64**: 123524, 2001. **hep-th/0106241**.

[6] Justin Khoury, Burt A. Ovrut, Nathan Seiberg, Paul J. Steinhardt and Neil Turok. *From big crunch to big bang*. Phys. Rev. **D65**: 086007, 2002. **hep-th/0108187**.

[7] Justin Khoury, Burt A. Ovrut, Paul J. Steinhardt and Neil Turok. *Density perturbations in the ekpyrotic scenario*. Phys. Rev. **D66**: 046005, 2002. **hep-th/0109050**.

[8] Lisa Randall and Raman Sundrum. *A large mass hierarchy from a small extra dimension*. Phys. Rev. Lett. **83**: 3370, 1999. **hep-ph/9905221**.

[9] Lisa Randall and Raman Sundrum. *An alternative to compactification*. Phys. Rev. Lett. **83**: 4690, 1999. **hep-th/9906064**.

[10] Petr Hořava and Edward Witten. *Heterotic and type I string dynamics from eleven dimensions*. Nucl. Phys. **B460**: 506, 1996. **hep-th/9510209**.

[11] Petr Hořava and Edward Witten. *Eleven-dimensional supergravity on a manifold with boundary*. Nucl. Phys. **B475**: 94, 1996. **hep-th/9603142**.

[12] Edward Witten. *Strong coupling expansion of Calabi-Yau compactification.* Nucl. Phys. **B471**: 135, 1996. **hep-th/9602070**.

[13] Paul J. Steinhardt and Neil Turok. *A cyclic model of the universe.* **hep-th/0111030**.

[14] Paul J. Steinhardt and Neil Turok. *Cosmic evolution in a cyclic universe.* Phys. Rev. **D65**: 126003, 2002. **hep-th/0111098**.

[15] Paul J. Steinhardt and Neil Turok. *The cyclic universe: an informal introduction.* **astro-ph/0204479**.

[16] Kari Enqvist, Esko Keski-Vakkuri and Syksy Räsänen. *Constraints on brane and bulk ideal fluid in Randall-Sundrum cosmologies.* Phys. Rev. **D64**: 044017, 2001. **hep-th/0007254**.

[17] Kari Enqvist, Esko Keski-Vakkuri and Syksy Räsänen. *Hubble law and brane matter after ekpyrosis.* Nucl. Phys. **B614**: 388, 2001. **hep-th/0106282**.

[18] Syksy Räsänen. *On ekpyrotic brane collisions.* Nucl. Phys. **B626**: 183, 2002. **hep-th/0111279**.

[19] Gunnar Nordström. *On the possibility of a unification of the electromagnetic and gravitation fields.* Phys. Zeitsch. **15**: 504, 1914. Reprinted in Thomas Appelquist et al. (eds): Modern Kaluza-Klein theories, Addison-Wesley Publishing Company, Inc., United States of America, 1987.

[20] Th. Kaluza. *On the unity problem of physics.* Sitzungsber. Preuss. Akad. Wiss. Math. Phys. Klasse 966, 1921. Reprinted in Thomas Appelquist et al. (eds): Modern Kaluza-Klein theories, Addison-Wesley Publishing Company, Inc., United States of America, 1987.

[21] Oskar Klein. *Quantum theory and the five dimensional theory of relativity.* Z. Phys. **37**: 895, 1926. Reprinted in Thomas Appelquist et al. (eds): Modern Kaluza-Klein theories, Addison-Wesley Publishing Company, Inc., United States of America, 1987.

[22] O. Klein. *The atomicity of electricity as a quantum theory law.* Nature **118**: 516, 1926. Reprinted in Thomas Appelquist et al. (eds): Modern Kaluza-Klein theories, Addison-Wesley Publishing Company, Inc., United States of America, 1987.

[23] Daniel Z. Freedman, P. van Nieuwenhuizen and S. Ferrara. *Progress toward a theory of supergravity.* Phys. Rev. **D13**: 3214, 1976.

[24] S. Deser and B. Zumino. *Consistent supergravity.* Phys. Lett. **B62**: 335, 1976.

[25] M.J. Duff, B.E.W. Nilsson and C.N. Pope. *Kaluza-Klein supergravity*. Phys. Rept. **130**: 1, 1986.

[26] S.M. Christensen and M.J. Duff. *New gravitational index theorems and super theorems*. Nucl. Phys. **B154**: 301, 1979.

[27] F.A. Berends, J.W. van Holten, P. van Nieuwenhuizen and B. de Wit. *On field theory for massive and massless spin-5/2 particles*. Nucl. Phys. **B154**: 261, 1979.

[28] C. Aragone and S. Deser. *Consistency problems in hypergravity*. Phys. Lett. **B86**: 161, 1979.

[29] J. Scherk and John H. Schwarz. *Dual models for nonhadrons*. Nucl. Phys. **B81**: 118, 1974.

[30] D. Lüst and S. Theisen. *Lectures on string theory*. Springer-Verlag, Berlin and Heidelberg, 1989.

[31] Joseph Polchinski. *String theory, volume II*. Cambridge University Press, Cambridge, 1998.

[32] André Lukas, Burt A. Ovrut, K.S. Stelle and Daniel Waldram. *The universe as a domain wall*. Phys. Rev. **D59**: 086001, 1999. **hep-th/9803235**.

[33] André Lukas, Burt A. Ovrut, K.S. Stelle and Daniel Waldram. *Heterotic M-theory in five dimensions*. Nucl. Phys. **B552**: 246, 1999. **hep-th/9806051**.

[34] E.G. Adelberger (for the Eöt-Wash group). *Sub-millimeter tests of the gravitational inverse-square law*. **hep-ex/0202008**.

[35] D.E. Groom et al. *The review of particle physics*. The European Physical Journal **C15**: 1, 2000, and 2001 off-year partial update for the 2002 edition available on the PDG WWW pages (URL: http://pdg.lbl.gov/).

[36] Ignatios Antoniadis. *A possible new dimension at a few TeV*. Phys. Lett. **B246**: 377, 1990.

[37] André Lukas, Burt A. Ovrut and Daniel Waldram. *Boundary inflation*. Phys. Rev. **D61**: 023506, 2000. **hep-th/9902071**.

[38] H.A. Chamblin and H.S. Reall. *Dynamic dilatonic domain walls*. Nucl. Phys. **B562**: 133, 1999. **hep-th/9903225**.

[39] Pierre Binétruy, Cédric Deffayet and David Langlois. *Non-conventional cosmology from a brane-universe*. Nucl. Phys. **B565**: 269, 2000. **hep-th/9905012**.

[40] Nemanja Kaloper. *Bent domain walls as braneworlds*. Phys. Rev. **D60**: 123506, 1999. **hep-th/9905210**.

[41] Tetsuya Shiromizu, Kei-ichi Maeda and Misao Sasaki. *The Einstein equations on the 3-brane world*. Phys. Rev. **D62**: 024012, 2000. **gr-qc/9910076**.

[42] Csaba Csáki, Michael Graesser, Christopher Kolda and John Terning. *Cosmology of one extra dimension with localized gravity*. Phys. Lett. **B462**: 34, 1999. **hep-ph/9906513**.

[43] James M. Cline, Christophe Grojean and Géraldine Servant. *Cosmological expansion in the presence of an extra dimension*. Phys. Rev. Lett. **83**: 4245, 1999. **hep-ph/9906523**.

[44] Walter D. Goldberger and Mark B. Wise. Phys. Rev. **D60**: 107505, 1999. *Bulk fields in the Randall-Sundrum compactification scenario*. **hep-ph/9907218**.

[45] Walter D. Goldberger and Mark B. Wise. *Modulus stabilization with bulk fields*. Phys. Rev. Lett. **83**: 4922, 1999. **hep-ph/9907447**.

[46] O. DeWolfe, D.Z. Freedman, S.S. Gubser and A. Karch. *Modeling the fifth dimension with scalars and gravity*. Phys. Rev. **D62**: 046008, 2000. **hep-th/9909134**.

[47] Panagiota Kanti, Ian I. Kogan, Keith A. Olive and Maxim Pospelov. *Cosmological 3-brane solutions*. Phys. Lett. **B468**: 31, 1999. **hep-ph/9909481**.

[48] Per Kraus. *Dynamics of anti-de Sitter domain walls*. JHEP **9912**: 011, 1999. **hep-th/9910149**.

[49] Markus A. Luty and Raman Sundrum. *Radius stabilization and anomaly-mediated supersymmetry breaking*. Phys. Rev. **D62**: 035008, 2000. **hep-th/9910202**.

[50] Pierre Binétruy, Cédric Deffayet, Ulrich Ellwanger and David Langlois. *Brane cosmological evolution in a bulk with cosmological constant*. Phys. Lett. **B477**: 285, 2000. **hep-th/9910219**.

[51] Csaba Csáki, Michael Graesser, Lisa Randall and John Terning. *Cosmology of brane models with radion stabilization*. Phys. Rev. **D62**: 045015, 1999. **hep-ph/9911406**.

[52] Daisuke Ida. *Brane-world cosmology*. JHEP **0009**: 014, 2000. **gr-qc/9912002**.

[53] Panagiota Kanti, Ian I. Kogan, Keith A. Olive and Maxim Pospelov. *Single-brane cosmological solutions with a stable compact extra dimension*. Phys. Rev. **D61**: 106004, 2000. **hep-ph/9912266**.

[54] Shinji Mukohyama, Tetsuya Shiromizu and Kei-ichi Maeda. *Global structure of exact cosmological solutions in the brane world*. Phys. Rev. **D62**: 024028, 2000. Erratum Phys Rev. **D63**: 029901, 2001. **hep-th/9912287**.

[55] Martin Gremm. *Thick domain walls and singular spaces*. Phys. Rev. **D62**: 044017, 2000. **hep-th/0002040**.

[56] Panagiota Kanti, Keith A. Olive and Maxim Pospelov. *Static solutions for brane models with a bulk scalar field*. Phys. Lett. **B481**: 386, 2000. **hep-ph/0002229**.

[57] R.N. Mohapatra, A. Pérez-Lorenzana and C.S. de S. Pires. *Cosmology of brane-bulk models in five dimensions*. Int. J. Mod. Phys. **A16**: 1431, 2001. **hep-ph/0003328**.

[58] Julien Lesgourgues, Sergio Pastor, Marco Peloso and Lorenzo Sorbo. *Cosmology of the Randall-Sundrum model after dilaton stabilization*. Phys. Lett. **B489**: 411, 2000. **hep-ph/0004086**.

[59] Benjamin Grínstein, Detlef R. Nolte and Witold Skiba. *Adding matter to Poincare invariant branes*. Phys. Rev. **D62**: 086006, 2000. **hep-th/0005001**.

[60] Panagiota Kanti, Keith A. Olive and Maxim Pospelov. *Solving the hierarchy problem in two-brane cosmological models*. Phys. Rev. **D62**: 126004, 2000. **hep-ph/0005146**.

[61] James M. Cline and Hassan Firouzjahi. *Brane-world cosmology of modulus stabilization with a bulk scalar field*. Phys. Rev. **D64**: 023505, 2001. **hep-ph/0005235**.

[62] Kei-ichi Maeda and David Wands. *Dilaton-gravity on the brane*. Phys. Rev. **D62**: 124009, 2000. **hep-th/0008188**.

[63] Andrew Mennim and Richard A. Battye. *Cosmological expansion on a dilatonic brane-world*. Class. Quant. Grav. **18**: 2171, 2001. **hep-th/0008192**.

[64] C. van de Bruck, M. Dorca, C.J.A.P. Martins and M. Parry. *Cosmological consequences of the brane/bulk interaction*. Phys. Lett. **B495**: 183, 2000. **hep-th/0009056**.

[65] Pierre Binétruy, Cédric Deffayet and David Langlois. *The radion in brane cosmology*. Nucl. Phys. **B615**: 219, 2001. **hep-th/0101234**.

[66] Ph. Brax and A.C. Davis. *Cosmological evolution on self-tuned branes and the cosmological constant*. JHEP **0105**: 007, 2001. **hep-th/0104023**.

[67] Hideo Kodama, Akihiro Ishibashi and Osamu Seto. *Brane world cosmology - gauge-invariant formalism for perturbation.* Phys. Rev. **D62**: 064022, 2000. **hep-th/0004160**. Roy Maartens. *Cosmological dynamics on the brane.* Phys. Rev. **D62**: 084023, 2000. **hep-th/0004166**. David Langlois. *Brane cosmological perturbations.* Phys. Rev. **D62**: 126012, 2000. **hep-th/0005025**. Carsten van de Bruck, Miquel Dorca, Robert H. Brandenberger and André Lukas. *Cosmological perturbations in brane-world theories: formalism.* Phys. Rev. **D62**: 123515, 2000. **hep-th/0005032**. Shinji Mukohyama. *Perturbation of junction condition and doubly gauge-invariant variables.* Class. Quant. Grav. **17**: 4777, 2000. **hep-th/0006146**. David Langlois. *Evolution of cosmological perturbations in a brane-universe.* Phys. Rev. Lett **86**: 2212, 2001. **hep-th/0010063**. David Langlois, Roy Maartens, Misao Sasaki and David Wands. *Large-scale cosmological perturbations on the brane.* Phys. Rev. **D63**: 084009, 2001. **hep-th/0012044**. Helen A. Bridgman, Karim A. Malik and David Wands. *Cosmological perturbations in the bulk and on the brane.* Phys. Rev. **D65**: 043502, 2002. **astro-ph/0107245**. Philippe Brax, Carsten van de Bruck and Anne C. Davis. *Brane-world cosmology, bulk scalars and perturbations.* JHEP **0110**: 026, 2001. **hep-th/0108215**. Bernard Leong, Peter Dunsby, Anthony Challinor and Anthony Lasenby. *(1+3) Covariant dynamics of scalar perturbations in braneworlds.* Phys. Rev. **D65**: 104012, 2002. **gr-qc/0111033**.

[68] S. Perlmutter et al. (The Supernova Cosmology Project) *Discovery of a supernova explosion at half the age of the universe and its cosmological implications.* Nature **391**: 51, 1998. **astro-ph/9712212**.

[69] Adam G. Riess et al. *Observational evidence from supernovae for an accelerating universe and a cosmological constant.* Astron. J. **116**: 1009, 1998. **astro-ph/9805201**.

[70] G. Efstathiou et al. *Evidence for a non-zero Lambda and a low matter density from a combined analysis of the 2dF Galaxy Redshift Survey and cosmic microwave background anisotropies.* **astro-ph/0109152**.

[71] Alessandro Melchiorri. *CMB and cosmological parameters: current status and prospects.* **astro-ph/0204017**.

[72] Klaus Behrndt and Mirjam Cvetič. *Supersymmetric domain-wall world from D=5 simple gauged supergravity.* Phys. Lett. **B475**: 253, 2000. **hep-th/9909058**.

[73] Francisco A. Brito, Mirjam Cvetič and SangChul Yoon. *From a thick to a thin supergravity domain wall.* Phys. Rev. **D64**: 064021, 2001. **hep-ph/0105010**.

[74] Keith A. Olive. *Big bang nucleosynthesis, implications of recent CMB data and supersymmetric dark matter.* **astro-ph/0202486**.

[75] C.B. Netterfield et al. *A measurement by BOOMERANG of multiple peaks in the angular power spectrum of the cosmic microwave background*. Astrophys. J. **571**: 604, 2002. **astro-ph/0104460**.

[76] C. Pryke et al. *Cosmological parameter extraction from the first season of observations with DASI*. Astrophys. J. **568**: 46, 2002. **astro-ph/0104490**.

[77] R. Stompor et al. *Cosmological implications of the MAXIMA-I high resolution cosmic microwave background anisotropy measurement*. Astrophys. J. **561**: L7, 2001. **astro-ph/0105062**.

[78] P. de Bernardis et al. *Multiple peaks in the angular power spectrum of the cosmic microwave background: significance and consequences for cosmology*. Astrophys. J. **564**: 559, 2002. **astro-ph/0105296**.

[79] G. Efstathiou and S.J. Moody. *Maximum likelihood estimates of the two- and three-dimensional power spectra of the APM galaxy survey*. **astro-ph/0010478**.

[80] Scott Dodelson et al. for the SDSS Collaboration. *The 3D power spectrum from angular clustering of galaxies in early SDSS data*. Astrophys. J. **572**: 140, 2001. **astro-ph/0107421**.

[81] Michael S. Turner. *Dark energy and the new cosmology*. **astro-ph/0108103**.

[82] V.A. Kuzmin, V.A. Rubakov and M.E. Shaposhnikov. *On the anomalous electroweak baryon number nonconservation in the early universe*. Phys. Lett. **B155**: 36, 1985.

[83] Antonio Riotto and Mark Trodden. *Recent progress in baryogenesis*. Ann. Rev. Nucl. Part. Sci. **49**: 35, 1999. **hep-ph/9901362**.

[84] Andrei Linde. *Particle physics and inflationary cosmology*. Harwood Academic Publishers GmbH, Chur, Switzerland, 1990.

[85] Edward W. Kolb and Michael S. Turner. *The early universe*. Addison-Wesley Publishing Company, United States of America, 1994.

[86] Robert H. Brandenberger. *Inflationary cosmology: progress and problems*. **hep-ph/9910410**.

[87] Andrei Linde. *Inflationary theory versus ekpyrotic/cyclic scenario*. **hep-th/0205259**.

[88] N.C. Tsamis and R.P. Woodard. *The quantum gravitational back-reaction on inflation*. Annals Phys. **253**: 1, 1997. **hep-ph/9602316**.

[89] L.R. Abramo, N.C. Tsamis and R.P. Woodard. *Cosmological density perturbations from a quantum gravitational model of inflation.* Fortsch. Phys. **47**: 389, 1999. **astro-ph/9803172**.

[90] A.A. Starobinsky. *A new type of isotropic cosmological models without singularity.* Phys. Lett. **B91**: 99, 1980.

[91] A.A. Starobinsky. *The perturbation spectrum evolving from a nonsingular, initially de Sitter cosmology, and the microwave background anisotropy.* Pis'ma Astron. Zh. **9**: 579, 1983. Reprinted in Sov. Astron. Lett. **9**: 5, 1983.

[92] A.D. Linde. *Chaotic inflation.* Phys. Lett. **B129**: 177, 1983.

[93] Andrei Linde. *Hybrid inflation.* Phys. Rev. **D49**: 748, 1993. **astro-ph/9307002**.

[94] Jennie H. Traschen and Robert H. Brandenberger. *Particle production during out-of-equilibrium phase transitions.* Phys. Rev. **D42**: 2491, 1990.

[95] Lev Kofman, Andrei Linde and Alexei Starobinsky. *Towards the theory of reheating after inflation.* Phys. Rev. **D56**: 3258, 1997. **hep-ph/9704452**.

[96] L.H. Ford. *Gravitational particle production and inflation.* Phys. Rev. **D35**: 2955, 1987.

[97] Gary Felder, Lev Kofman and Andrei Linde. *Gravitational particle production and the moduli problem. JHEP* **0002**: 027, 2000. **hep-ph/9909508**.

[98] Mark Trodden and Tanmay Vachaspati. *Causality and cosmic inflation.* Phys. Rev. **D61**: 023502, 2000. **gr-qc/9811037**.

[99] Fred C. Adams, Katherine Freese and Alan H. Guth. *Constraints on the scalar field potential in inflationary models.* Phys. Rev. **D43**: 965, 1991.

[100] Arvind Borde, Alan H. Guth and Alexander Vilenkin. *Inflation is not past-eternal.* **gr-qc/0110012**.

[101] A. Vilenkin. *Quantum origin of the universe.* Nucl. Phys. **B252**: 141, 1985.

[102] N.D. Birrell and P.C.W. Davies. *Quantum fields in curved space.* Cambridge University Press, Cambridge, 1983.

[103] Gary T. Horowitz and Robert M. Wald. *Dynamics of Einstein's equation modified by a higher-order derivative term.* Phys. Rev. **D17**: 414, 1978.

[104] Jonathan Z. Simon. *The stability of flat space, semiclassical gravity, and higher derivatives.* Phys. Rev. **D43**: 3308, 1991.

[105] Jonathan Z. Simon. *No Starobinsky inflation from self-consistent semiclassical gravity.* Phys. Rev. **D45**: 1953, 1992.

[106] Leonard Parker and Jonathan Z. Simon. *Einstein equation with quantum corrections reduced to second order.* Phys. Rev. **D47**: 1339, 1993. **gr-qc/9211002**.

[107] S. Carlip. *Quantum gravity: a progress report.* Rept. Prog. Phys. **64**: 885, 2001. **gr-qc/0108040**.

[108] G. Veneziano. *String cosmology: the pre-big bang scenario.* **hep-th/0002094**.

[109] Edmund J. Copeland. *String cosmology.* **hep-th/0202028**.

[110] M. Gasperini and G. Veneziano. *The pre-big bang scenario in string cosmology.* **hep-th/0207130**.

[111] Kari Enqvist and Martin S. Sloth. *Adiabatic CMB perturbations in pre-big bang string cosmology.* Nucl. Phys. **B626**: 395, 2002. **hep-ph/0109214**.

[112] David H. Lyth and David Wands. *Generating the curvature perturbation without an inflaton.* Phys. Lett. **B524**: 5, 2002. **hep-ph/0110002**.

[113] Fabio Finelli and Robert Brandenberger. *On the generation of a scale-invariant spectrum of adiabatic fluctuations in cosmological models with a contracting phase.* Phys.Rev. **D65**: 103522, 2002. **hep-th/0112249**.

[114] Jai-chan Hwang and Hyerim Noh. *Identification of perturbation modes and controversies in ekpyrotic perturbations.* **hep-th/0203193**.

[115] M. Gasperini, M. Maggiore and G. Veneziano. *Towards a non-singular pre-big bang cosmology.* Nucl. Phys. **B494**: 315, 1997. **hep-th/9611039**.

[116] Ram Brustein and Richard Madden. *Graceful exit and energy conditions in string cosmology.* Phys. Lett. **B410**: 110, 1996. **hep-th/9702043**.

[117] Stefano Foffa, Michele Maggiore and Riccardo Sturani. *Loop corrections and graceful exit in string cosmology.* Nucl. Phys. **B552**: 395, 1999. **hep-th/9903008**.

[118] C. Cartier, E.J. Copeland and R. Madden. *The graceful exit in pre-big bang string cosmology.* JHEP **0001**: 035, 2000. **hep-th/9910169**.

[119] Nemanja Kaloper, Andrei Linde and Raphael Bousso. *Pre-big-bang requires the universe to be exponentially large from the very beginning*. Phys. Rev. **D59**: 043508, 1999. **hep-th/9801073**.

[120] David H. Lyth. *The primordial curvature perturbation in the ekpyrotic Universe*. Phys. Lett. **B524**: 1, 2002. **hep-ph/0106153**.

[121] Robert Brandenberger and Fabio Finelli. *On the spectrum of fluctuations in an effective field theory of the ekpyrotic universe*. JHEP **0111**: 056, 2001. **hep-th/0109004**.

[122] Jai-chan Hwang. *Cosmological structure problem of the ekpyrotic scenario*. Phys. Rev. **D65**: 063514, 2002. **astro-ph/0109045**.

[123] David H. Lyth. *The failure of cosmological perturbation theory in the new ekpyrotic scenario*. Phys. Lett. **B526**: 173, 2002. **hep-ph/0110007**.

[124] Ruth Durrer. *Clarifying perturbations in the ekpyrotic universe*. **hep-th/0112026**.

[125] Jai-chan Hwang and Hyerim Noh. *Non-singular big-bounces and evolution of linear fluctuations*. Phys. Rev. **D65**: 124010, 2002. **astro-ph/0112079**.

[126] Jérôme Martin, Patrick Peter, Nelson Pinto-Neto and Dominik J. Schwarz. *Passing through the bounce in the ekpyrotic models*. Phys. Rev. **D65**: 123513, 2002. **hep-th/0112128**.

[127] Patrick Peter and Nelson Pinto-Neto. *Primordial perturbations in a non singular bouncing universe model*. **hep-th/0203013**.

[128] Ruth Durrer and Filippo Vernizzi. *Adiabatic perturbations in pre big bang models: matching conditions and scale invariance*. **hep-ph/0203275**.

[129] Jérôme Martin, Patrick Peter, Nelson Pinto-Neto, Dominik J. Schwarz. *Comment on "Density perturbations in the ekpyrotic scenario"*. **hep-th/0204222**.

[130] Patrick Peter, Jérôme Martin, Nelson Pinto-Neto, Dominik J. Schwarz. *Perturbations in the ekpyrotic scenarios*. **hep-th/0204227**.

[131] A. Notari and A. Riotto. *Isocurvature perturbations in the ekpyrotic universe*. **hep-th/0205019**.

[132] Fabio Finelli. *Assisted contraction*. **hep-th/0206112**.

[133] Christopher Gordon and Neil Turok. *Cosmological perturbations through a general relativistic bounce*. **hep-th/0206138**.

[134] Shinji Tsujikawa, Robert Brandenberger and Fabio Finelli. *On the construction of nonsingular pre-big-bang and ekpyrotic cosmologies and the resulting density perturbations.* **hep-th/0207228**.

[135] Nathan Seiberg. *From big crunch to big bang – is it possible?* **hep-th/0201039**.

[136] Andrew J. Tolley and Neil Turok. *Quantum fields in a big crunch/big bang spacetime.* **hep-th/0204091**.

[137] Neil Turok. *Density perturbations in the ekpyrotic scenario.* Talk at the "Workshop on the ekpyrotic universe", Annecy, 11.12.2001.

[138] Gary Felder, Andrei Frolov, Lev Kofman and Andrei Linde. *Cosmology with negative potentials.* Phys. Rev. **D66**: 023507, 2002. **hep-th/0202017**.

[139] Jnanadeva Maharana. *From big crunch to big bang: a quantum string cosmology perspective.* Phys. Rev. **D65**: 103504, 2002. **gr-qc/0110120**.

[140] Vijay Balasubramanian, S.F. Hassan, Esko Keski-Vakkuri and Asad Naqvi. *A spacetime orbifold: a toy model for a cosmological singularity.* **hep-th/0202187**.

[141] Nikita A. Nekrasov. *Milne universe, tachyons, and quantum group.* **hep-th/0203112**.

[142] L. Cornalba and Miguel S. Costa. *A new cosmological scenario in string theory.* **hep-th/0203031**.