Determinants of external imbalances between large economies

by

Julia Niemeläinen

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DETERMINANTS OF EXTERNAL IMBALANCES BETWEEN LARGE ECONOMIES

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Abstract

This dissertation consists of an introduction and three essays on the determination of external imbalances and the real interest rate. In particular, the objective of the dissertation is to analyze the reasons behind the long-lasting current account imbalances observed over the past decades in large open economies, such as the current account deficit of the United States and the current account surplus of China, as well as the concurrent long-term decline in the world real interest rate.

In the first two essays, I focus on demographic change, social security and fiscal policy as potential drivers of households' consumption-saving decisions, the current account and the world real interest rate. The analysis is based on a dynamic general equilibrium model with an embedded life-cycle structure, which allows me to study the role of life-cycle savings and intergenerational transfers in the consumption-saving decisions, and in which fiscal policy is non-Ricardian.

In the first essay I analyze how permanent differences in demographics, social security and fiscal policy affect the external sector and the world real interest rate in the long run, and compare the effects under fixed and variable labour supply, which the literature on demographic change and external imbalances often abstracts from. I find that, relatively high life expectancy and low social security are associated with a positive net foreign asset position in the two-country world. Because the impact of social security is quantitatively large, a country with relatively low life expectancy may become a net creditor if its social security is low enough. This result suggests that analyzing the effects of social security could help us understand the observed low-frequency capital flows from fast-growing emerging economies to industrialized countries. High life expectancy is, in the model, associated with a low international real interest rate, but endogenizing labour supply results in a higher real interest rate for a given level of life expectancy. This indicates that the negative effect of population ageing on the real interest rate might not be as strong as suggested by several earlier studies.

The objective of the second essay is to analyze whether it is possible to replicate the dynamics of China's trade surplus vis-a-vis the United States over the period 1980-2015, by taking into account the differences in demographic factors and social security between the countries. Productivity growth in China has been approximately twice as rapid as in the United States during this period, which, according to traditional neoclassical models, should imply high investment and consumption rates, and result in a trade deficit. In my model, China's relatively rapid population ageing and low social security result in high and increasing savings by Chinese households, so that even when China's high productivity growth is taken into account, the model produces a trade surplus for China during the sample period.

In the third essay, I analyze the impact of the Chinese government's macroeconomic policies on the observed trade balance dynamics and the transmission of these policies to the United States in the 2000s. The analysis is based on a life-cycle model, which includes features that describe the behavior of the Chinese policymakers. In particular, I assume that private agents are excluded from the international financial market, and that the public authorities can directly control the domestic real interest rate and the real exchange rate. I find that macroeconomic policies, especially the undervaluation of China's real exchange rate, and the growth in government expenditures and fiscal deficits run by China and the United States, have had a positive impact on China's trade balance, and raised the international interest rate. However, the overall effect of the policies depends qualitatively on the assumption of the elasticity of intertemporal substitution.

Keywords: external imbalances, real interest rate, demographics, social security, fiscal policy, capital controls, real exchange rate

JEL codes: E21, E22, E43, E62, F21, F32, F41, F42, G28

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Julia Niemeläinen

Contents

1 Introduction

The world economic landscape has transformed greatly over the past few decades. Especially notable is the growth of China, which has replaced Japan as the world's second largest economy (see figure 1.1), and become the world's largest exporter and second largest importer (see figure 1.2), overtaking most of the major advanced economies in this respect.

A feature of the transformation of the international economic landscape is the persistent imbalance of global trade. This is reflected in the persistent current account imbalances run by several countries (see figure 1.3), which have led to an increase in the total value of gross external asset holdings. Concurrently with this observed rise in resource exchange over time, *i.e.* intertemporal trade, the past decades have witnessed a steady decline in the world real interest rate (proxied by the US real interest rate, see figure 1.4). Because the world real interest rate is the price of future consumption in terms of current consumption, *i.e.* the intertemporal terms of trade, it is natural to conjecture that the current account imbalances and the decline in the real interest rate are, at least to some extent, driven by the same factors.

Policymakers have become increasingly concerned over the practical effects of these phenomena. Increased overseas borrowing and financial interlinkages have raised concerns about an increased risk of contagion of financial crises, and fears of so-called sudden stops, *i.e.* situations in which foreign capital is suddenly withdrawn, in countries that are external net borrowers. The decline of the interest rate has made it more difficult for monetary policymakers to stimulate the economy in economic downturns, and given rise to wealth redistributions across nations due to extensive external borrowing and lending. Furthermore, China's active role in international trade and its persistent trade surplus have raised questions regarding to what extent the Chinese government affects the trade surplus with its macroeconomic policies, and whether these policies create an unfair trade advantage in a beggar-thy-neighbor manner.

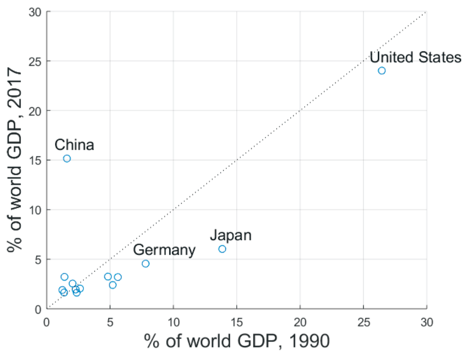

Figure 1.1: The share of the 10 largest economies of the world GDP in 1990 and 2017, current US dollars. Source: World Bank, World Development Indicators 2018.

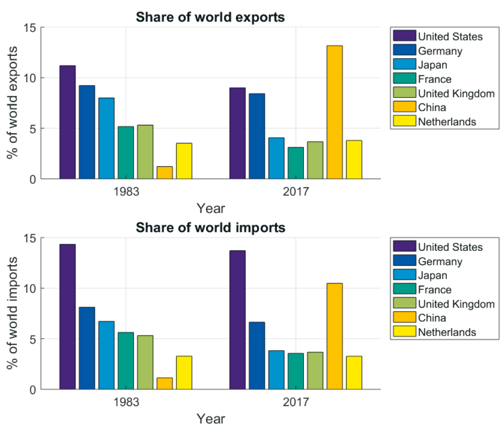

Figure 1.2: The share of the five largest economies of world merchandise exports (upper) and of world merchandise imports (lower) in 1983 and 2017. Source: World Trade Statistical Review 2018.

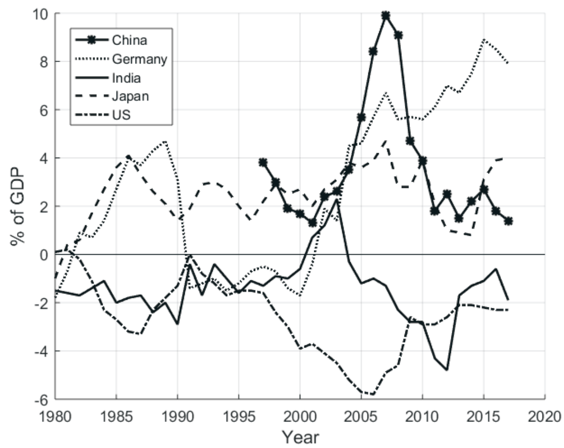

Figure 1.3: Current account as a share of GDP. Source: IMF, World Economic Outlook 2018.

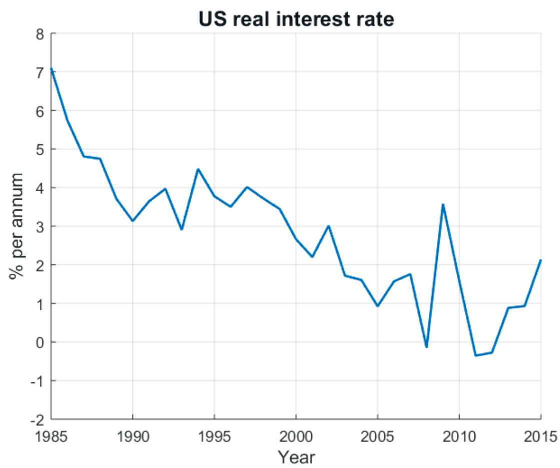

Figure 1.4: Annual interest rate on all US government securities and bonds net of CPI inflation between 1985-2015. Source: IMF, World Economic Outlook 2016.

These global economic trends have also challenged traditional economic theory and policies based on it. A key challenge is to explain capital outflows from fast-growing emerging economies. According to neoclassical economic theory, countries with high productivity growth should experience high consumption and investment rates, and therefore current account deficits. Yet capital outflows from several emerging economies have been observed. Another major challenge is understanding the drivers of the long-term decline in the world real interest rate, and how persistent the decline is, as it has significant implications on the conduct of monetary policy.

Against this background, I analyze the long-run determinants of external imbalances and the real interest rate. Due to the persistent nature of these phenomena, I analyze factors that influence the external relations between countries over the long run, focusing in particular on factors that are i) due to households' consumption-saving decisions over the life-cycle, and ii) due to different government policies, including fiscal policy, social security, exchange rate policy, and interest rate policy. As figures 1.5 and 1.6 show, there are large and persistent differences between countries in household saving rates as well as in demographic characteristics, which, according to previous literature, have a significant effect on households' consumption-saving choices. Furthermore, there have been large differences and long-term changes in government policies, including fiscal policy stances (see figure 1.7), between industrialized and developing countries around the world.

The analysis is done using dynamic general equilibrium models with an embedded life-cycle structure, and the main objective is to test the ability of the theoretical macroeconomic models to explain and replicate the observed dynamics in the external sector and the real interest rate in recent decades.

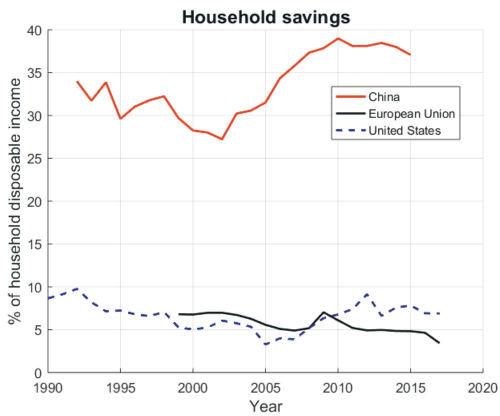

Figure 1.5: Household savings in China, the European Union and the United States. Source: OECD Data.

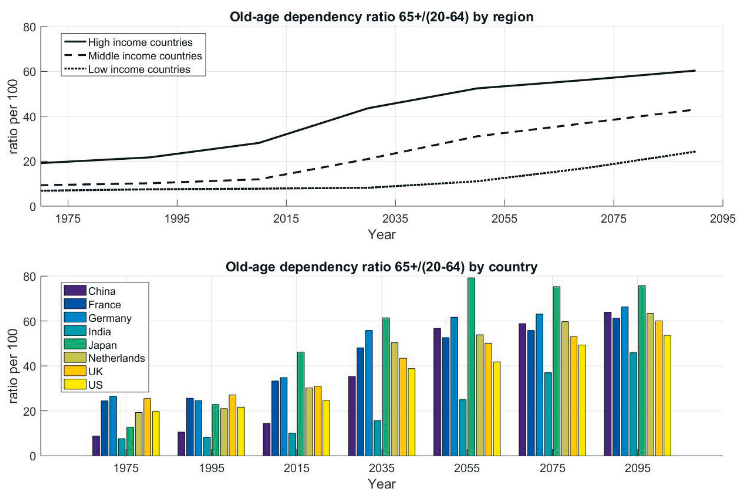

Figure 1.6: Estimates (until 2015) and projections (from 2015 onwards) of the old-age dependency ratio. Source: UN World Population prospects 2017.

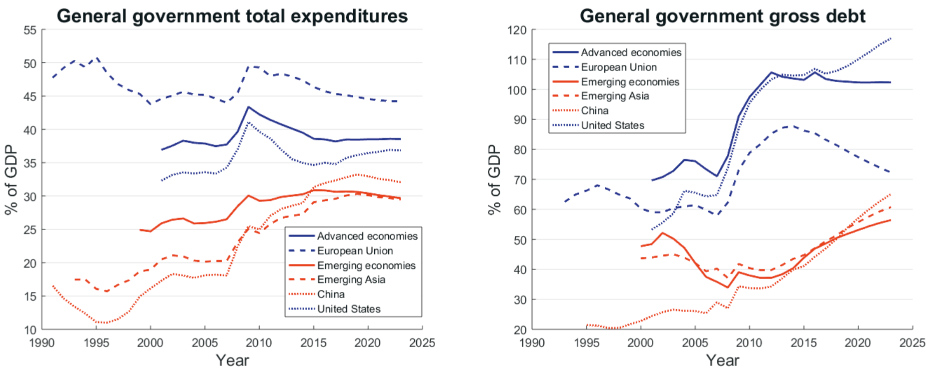

Figure 1.7: General government total expenditures and gross debt. Source: IMF World Economic Outlook 2018.

1.1 Theoretical framework and methodology

The analysis is based on the microfounded macroeconomic general equilibrium theory in a dynamic setting, which I apply to study the determination of the external balance and the real interest rate. I construct a dynamic general equilibrium model consisting of households, a representative firm, and a government, with an embedded life-cycle structure, which allows me to consider life-cycle saving behavior and demographic changes. In the last essay, the model is further extended to take into account features that describe the behavior of Chinese policymakers which include capital controls and control over the real exchange rate. Because the analysis is focused on the drivers of external imbalances between large economies, I use a two-country framework in which both economies are large enough to affect the international real interest rate. This allows the analysis of the simultaneous determination of the real interest rate and the external balance, and the transmission of economic shocks between countries.

Intertemporal trade naturally arises in the two-country framework when differences between the countries (*e.g.* different time preference rates or productivity), induce different optimal time-paths of consumption, savings and investment, so that resources are exchanged between the countries over time. In my analysis, the time preference rate between the countries differs because of differences in their demographic factors and life-cycle characteristics. In addition, households' time-path of consumption and savings is affected by the presence of social security, the level of which differs between the countries, and by other fiscal policies, including fiscal deficits run by the governments, which affect the time-paths of taxation. The key element in the quantitative analysis is that different policies have different effects on different generations and between countries, as they depend on the life-expectancy and fertility rate of the generations.

1.1.1 Life-cycle theory

The analysis of the effects of demographic change and social security on households' savings and the external imbalances relies on the life-cycle hypothesis of Modigliani and Brumberg (1954). According to that hypothesis, households have different consumption-saving profiles over the life-cycle, which implies that aggregate household savings, and therefore the external balance, depend on the demographic structures of the economies. This motivates the choice of departing from the representative agent framework and considering one with heterogenous agents.

To capture life-cycle saving behavior, I use the life-cycle model of Gertler (1999). Its foundations are the two-stage OLG models of Samuelson (1958) and Diamond (1965), and the Yaari (1965) and Blanchard (1985) models with agents with finite horizons.

The two-stage OLG models assume that, at each point in time, individuals of two different generations are alive and maximize their utility over the two periods of their lifetime. Using these models it is thus possible to analyze life-cycle savings and intergenerational transfers (social security). The other building block, the Blanchard (1985)/Yaari (1965) framework, introduces a representative household that faces a constant probability of surviving and therefore a finite expected lifetime, which makes the household discount the future more than in the infinite horizon case.

The Gertler (1999) framework combines the Samuelson (1958)/Diamond (1965) and the Blanchard (1985)/Yaari (1965) models so that one can analyze life-cycle behavior in the presence of uncertainty about lifetime. As in the Diamond (1965) framework, households live through two different stages in life: a stage in which they are young, or workers, and a stage in which they are old, or retirees. Unlike the Diamond (1965) model, households spend a stochastic number of periods in each stage, which is governed by an exogenous survival probability. All workers face the same probability of retiring, and all retirees the same probability of dying. These probabilities can be chosen so as to have realistic average life expectancy, working time, and time spent in retirement faced by the individual agents. At each point in time, different generations are alive. The young in the economy consist of working age population born at different points in time, and the old of retirees, who have been born and retired at different points in time. This feature of the model allows for a realistic, time-varying old age dependency ratio, and therefore the model is well suited to analyzing the effects of population ageing, as observed in most advanced and emerging economies.

In the model, the uncertainty about retirement time causes a risk to workers' wage income, which would lead to unrealistically high household savings. This risk is eliminated by using Epstein-Zin preferences, which allow me to separate income risk aversion and elasticity of intertemporal subsitution, so that one can assume that the agents are risk neutral with respect to income risk. In addition, retirees face a risk because of the uncertainty of the time of death, which means that they face the risk of leaving accidental bequests. This risk is eliminated in the model by assuming a perfect annuities market following Yaari (1965) and Blanchard (1985), which provides perfect insurance against this risk.

Because the young and the old households have different expected lifetimes, their discount factors differ. As the optimal consumption-saving decisions of the households depend on their discount factors, the aggregate effect of policies and shocks depends on the share of each age group in the economy, *i.e.* the demographic composition. Moreover, because the discount factors are endogenous and depend on demographic factors, differences in demographics between economies result in different discount rates between economies, and therefore different dynamic effects across the countries induced by different policies.

I use this framework not only to analyze the effects of time-varying demographic factors, but also of different policies - social security, interest rate policy, exchange rate policy and fiscal policy - under different demographic structures. This is currently a main concern given that, according to the latest demographic forecast by the United Nations, the old age dependency ratio (the number of population aged 65+ to population aged 20-64, for every 100) in the United States is projected to rise from 24.6 in 2015 to 55.0 by 2100, and in China from 14.5 to 64.1.

1.1.2 Social security, fiscal policy, and external imbalances

A key objective of my thesis is to analyze the drivers behind Chinese households' high savings. Given Chinese households' relatively low life expectancy, high household savings are puzzling since, in light of the life-cycle theory, low life expectancy should be associated with low savings. Therefore, features other than life-cycle behavior must drive household savings in China. Accordingly, I analyze the effects of intergenerational transfers on life-cycle savings behavior: in particular, I explore the role of low social security in old age, as it has been suggested by earlier literature (*e.g.* Blanchard et al. (1989)) that intergenerational transfers (a pay-as-you-go social security system) from households with low propensity to consume (the young) to households with high propensity to consume (the old) are likely to lower private savings in a life-cycle setting. The Gertler (1999) life-cycle model makes it possible to study transfers of wealth across generations, taking into account the demographic transition, which implies that the number of people who are contributing to social security changes over time.

In addition to social security, I analyze the effects of government expenditure and budget deficits on households' consumption-saving decisions and the current account. As shown in figure 1.7, government expenditures are higher in developed economies, but over recent decades, they have grown in emerging economies as well, implying a larger tax burden on private agents. Furthermore, fiscal deficits and government debt have grown both in developed economies and in emerging countries, including China, over past decades.

The overlapping generations (OLG) model breaks the Ricardian equivalence, so that the timing of taxes matters. Unlike in the representative agent framework, government deficits induce income redistribution from future generations to current ones, and therefore fiscal deficits affect the equilibrium of the economy and the current account balance.[1] In a similar manner, in the life-cycle economy, higher taxes in the future, which are needed to offset current tax cuts, are discounted by the agents at a higher (endogenous) rate than the risk-free rate used by the government to discount its future expenditures and revenues. Hence, taxes occuring in the future are not fully capitalized by the households and private savings do not fully offset public dissaving, so that a budget deficit has a non-zero net effect on aggregate savings and the current account.

In the first essay I analyze the effects of government expenditures and tax policy on the steady state of the life-cycle economy, assuming that the demo-

[1] See *e.g.* discussion by Obstfeld et al. (1996) in section 3, or by Blanchard et al. (1989).

graphic structures between the economies are identical. As the effects of the timing of taxes depend on whether the taxes are distortionary or lump sum, because labor income taxes affect households' labor supply decisions, I compare the effects of fiscal policy under lump sum and distortionary taxes.

In the third essay, I analyze the effects of fiscal policy on the dynamics of the trade balance of China and the United States in the 2000s. Because of different discount factors between generations and across countries, the implications of fiscal policies are different between economies at different stages of demographic transition. The dynamic simulations of the effects of fiscal policy capture this effect, as the demographic variables are allowed to vary over time.

1.1.3 Intratemporal and intertemporal terms of trade

The first two essays of the thesis focus on factors that endogenously affect the current account and the real interest rate, *i.e.* the intertemporal terms of trade. In the last essay, I also analyze the case in which the government of one country is able to set the level of the domestic interest rate directly. This is motivated by the observation that, as China imposes capital controls on its domestic agents, it has been able to set a domestic interest rate which has differed from the international interest rate. As a result, over the Chinese transition period, the Chinese real interest rate has been lower than the international real interest rate. According to the Mundell-Fleming trilemma, a country can not simultaneously have free capital mobility, a fixed foreign exchange rate, and independent monetary policy, because under fixed exchange rates and free capital mobility, an interest rate spread would lead to capital flows until the uncovered interest parity (UIP) would hold. By preventing capital flows, China can prevent sales and purchases of assets denominated in its currency, and set an interest rate that differs from the international interest rate.

In particular, in the last essay I assume that China can fully prevent capital mobility and control its domestic interest rate, and analyze the quantitative effects of interest rate spreads as observed in the 2000s. The interest rate policy affects the economy both directly and indirectly: as the government can access the world financial market and lend or borrow at a different interest rate than the one faced by domestic private agents, as a result, Ricardian equivalence breaks down.[2]

In addition to analyzing the effects of intertemporal terms of trade manipulation, I analyze the effect of intratemporal terms of trade, *i.e.* the relative

[2] See discussion in section 4.B

price of imports and exports. As pointed out in several studies, including the IMF External Sector Reports 2013-2018, China's real exchange rate was undervalued for a prolonged period in the 2000s. In the model, I assume that the consumers consume domestic and foreign goods, which are imperfect substitutes, and one of the countries is able to directly affect the real exchange rate (the price of the foreign goods at home and the price of domestic goods abroad). This manipulation affects the consumption share of different goods in the households' consumption basket, but also the price of the consumption basket today in comparison to the price of the consumption basket tomorrow. This gives rise to two opposing effects: a negative wealth effect that lowers household savings, and a positive substitution effect, due to which households raise savings in order to substitute consumption into the future. In the third essay, I analyze whether the impact of exogenous undervaluation in Chinese real exchange rate has quantitatively significant effects on its external sector.

Distorting the time paths of terms of trade, both intertemporal and intratemporal, has different effects on the consumption-saving behavior of the different generations. Hence the effects of terms of trade manipulation depends on the demographic structures of the economies, which is one of the topics discussed in the third essay.

1.2 Summary of the essays and directions for future research

This section summarizes the main findings of the essays which constitute the main part of the dissertation, and discusses directions for future research.

1.2.1 Long-term factors behind external imbalances and the real interest rate

In the first essay, I analyze the long-term determinants of the international real interest rate and external imbalances, focusing on the effects of demographic change, social security, and fiscal policy. Because the external imbalances and the decline of the real interest rate, as well as the demographic transition and social security and fiscal policy stances, have been very persistent, in this essay I focus on steady state effects. As opposed to previous literature on demographics and external imbalances, I take into account the labor supply channel by allowing for endogenous labor supply by both the old and the young households. In addition, I present and compare the results under lump sum and distortionary taxation.

In line with earlier literature, I find that a relatively high life expectancy predicts a positive net foreign asset position in the steady state, as it reinforces the saving-for-retirement motive. I therefore find that population ageing is positively associated with aggregrate savings and negatively with the real interest rate. Labor supply responses mitigate the effects of population ageing on aggregate saving, and the effect on the world real interest rate, implying that the link between population ageing and the world interest rate might not be as strong as suggested by previous literature.

In addition, I find that high social security is associated with a negative net foreign asset position. The impact of social security is quantitatively large, and in the model, even a country with a relatively young population may become a net creditor in the steady state if its social security expenditures are sufficiently low. Furthermore, financing social security expenditures with budget deficits reinforces their negative effects on private savings and the external asset position.

The steady state analysis supports the hypothesis that social security and fiscal policy can be important drivers behind the observed external imbalances between emerging and advanced economies, as both social security expenditures and fiscal deficits have typically been low in emerging economies in comparison to advanced countries.

1.2.2 External imbalances between China and the US: a dynamic analysis with a life-cycle model

In the second essay, I use a life-cycle model to analyze the dynamics of the trade balance between China and the United States between 1980 and 2015. The objective of the essay is to analyze the quantitative effects of China's relatively rapid population ageing and low social security expenditures on its households' savings, and to evaluate whether these factors can explain China's persistent trade surplus vis-a-vis the United States, which earlier literature has struggled to explain due to the high productivity growth observed in China over the past decades. Labor supply is assumed to be endogenous, and taxes consist of distortionary labor income taxes.

The key result is that the effects of demographic change and social security on China's external imbalances are strong. Because of China's relatively rapid population ageing and relatively low social security expenditures, its aggregate savings are so high that the trade balance is positive for the majority of the sample period even after taking into account the observed fast productivity growth and its positive impact on investment. The essay complements the recent strand of literature, which explains the capital outflows from China and other emerging economies by focusing on the role of financial markets, by providing an alternative explanation for the high household saving rate and current account surplus, which relies on life-cycle saving behavior.

1.2.3 China's macroeconomic policies and spillover effects

In the third essay, I analyze the effects of the Chinese central government's macroeconomic policies on its external sector, and the transmission of these policies to the United States via the world real interest rate. The essay is motivated by the observations that, during the 2000s, i) the Chinese real interest rate has differed from the international interest rate, ii) the real exchange rate has been undervalued for an extended period, and iii) the Chinese government expenditures and fiscal deficits have grown, as have the fiscal deficits in the United States.

For this purpose, I develop a model embedded with features that describe the behavior of the Chinese policymakers. These features include capital controls, which prevent the private sector from accessing the international financial market, and the existence of a consolidated central bank-government, which is assumed to be able to access the international financial market and to directly control the domestic interest rate and the real exchange rate. As demographic factors are known to affect the external sector and the transmission of macroeconomic policies, and as the demographic structures between China and the United States differ considerably, the analysis is based on the life-cycle framework, which allows the demographic dynamics to be taken into account.

My results support the view that the Chinese government's macroeconomic policies have had a positive effect on its trade balance in the 2000s. Of the three policies analyzed, the undervaluation of the exchange rate improves the trade balance the most. Also, the increase in government expenditures in China, as well as the fiscal deficits run by China and the United States, have had a positive effect on China's trade balance and a negative effect on the trade balance of the United States. However, I also find that the effects of the exchange rate policy on the external balance are qualitatively sensitive to the assumption about elasticity of intertemporal substitution, as this parameter determines whether the real exchange rate undervaluation results in an increase or decrease in private savings.

1.2.4 Directions for future research

The thesis opens several opportunities for future research.

While analyzing the life-cycle savings profiles of the households, one could allow the individuals to care about future generations and leave bequests. Given that, in many countries, household financial wealth is held to an increasing extent by the elderly, this can be an important aspect to consider when analyzing life-cycle saving behavior. Furthermore, it would be possible to introduce an endogenous retirement choice in the spirit of Feldstein (1974) and Munnell (1974).

As regards the assumptions on factor mobility, by allowing for labor mobility between countries, the framework could be used to analyze the migration incentives caused by demographic change and its economic consequences.

In order to better match the dynamics of external imbalances between emerging and industrialized economies, one could allow for different preferences, different labor shares of production, or different capital depreciation rates, as both the labor shares and the types of capital differ significantly between the emerging and the industrialized world.

Finally, given that earlier literature (*e.g.* Fujiwara and Teranishi (2008)) shows that the effects of monetary policy depend on the demographic composition of an economy, it would be of interest to analyze the effects of monetary policy shocks in countries that feature different demographic structures, such as China and the United States. Furthermore, because the life-cycle structure causes the households to discount future financial flows at a higher rate than the government does, it would be of interest to analyze whether the life-cycle framework could provide a solution to the forward guidance puzzle, *i.e.* the observation that, in the standard representative agent New Keynesian model, announcements about the future path of interest rates trigger responses in economic activity that are too large compared to what is observed in the data.

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2 Long-term factors behind external imbalances and the real interest rate

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2.1 Introduction

A large number of the world's economies are characterized by persistent current account imbalances and have therefore accumulated either a large positive net foreign assets position or debt (current accounts of selected economies are shown in figure 2.1). Between 1980 and 2015, 74 of 191 countries experienced a period of at least 20 consecutive years of either current account surpluses or deficits. At the same time, the world real interest rate, proxied by the real interest rate on US government bonds in figure 2.1, has been falling for over three decades.

The drivers of the external imbalances and the fall of the real interest rate have become a key economic question in an environment characterized by integrated financial markets, high cross holdings of foreign assets, and low interest rates. One explanation for the low-frequency capital flows and falling real interest rate is the demographic transition, i.e. falling population growth rate, caused by falling fertility rates and increasing life expectancy in different parts of the world. Recent literature suggests that, by increasing the time spent in retirement and lowering the support ratio, the demographic transition causes excess savings in ageing economies, which results in capital outflows and pushes down the real interest rate. For example, Ferrero (2010) argues that

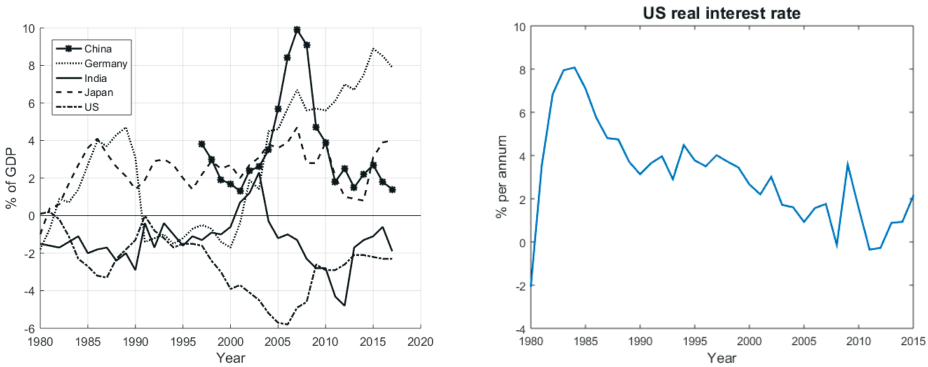

Figure 2.1: Left panel: Current account balances for China, Germany, India, Japan and the United States between 1980-2017 (source: IMF, World Economic Outlook). Right panel: Annual interest rate on all US government securities and bonds (source: IMF, International Financial Statistics) net of CPI inflation between 1980-2015 (source: IMF, World Economic Outlook).

the relatively rapid increase in life expectancy in the G6 countries in comparison with the US can explain a fraction of the dynamics of capital flows between these countries, and Carvalho et al. (2016) argue that demographic changes are a likely explanation for the observed long fall of the real interest rate.

Given the sizeable demographic changes that have taken place in the world's largest and most populous economies (see Appendix A), it seems likely that the demographic transition has had a significant impact on global asset demand, external imbalances and the real interest rate. However, previous literature on the effects of demographics on external imbalances abstracts to a large extent from the effects of ageing through the labor supply channel, nor does it consider the effects of social security on households' saving. Given that observed labor force participation and employment rates as well as the level of social security income differ across regions, especially between emerging and industrialized countries, it seems reasonable to assume that the labor supply channel and social security may have an significant effect on the external imbalances between economies.

The main aim of this paper is to analyze the importance of demographic change, social security and fiscal policy in determining external imbalances and the real interest rate in the long run, and to analyze how the effect of popu-

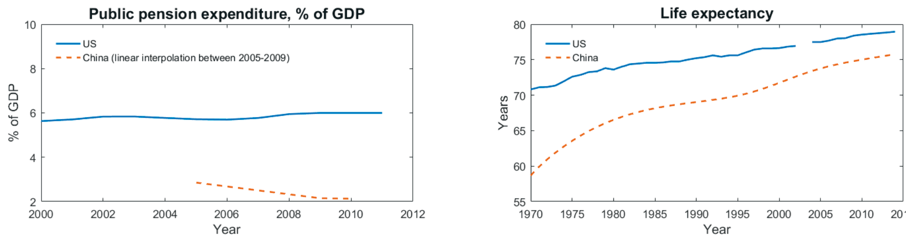

Figure 2.2: Left panel: Public pension spending (% of GDP) for the US and China between 2000-2012 (sources: OECD Data on Social Protection and Asian Development Bank: Social Protection Index Database). Right panel: Life expectancy at birth in the US and China between 1970-2015 (source: World Bank Development Indicators).

lation ageing depends on the level of social security. Furthermore, it aims to analyze how the endogeneity of labor supply and distortionary taxation change the implications of social security and population ageing on the external imbalances. To analyze these questions, I build a dynamic general equilibrium model with the life-cycle structure proposed by Gertler (1999) featuring endogenous labor supply with distortionary taxation and social security. Social security is defined as a pay-as-you-go system consisting of lump-sum transfers to the old, and the model consists of two large economies. The model is calibrated to match an average of China and the United States (US) between 1980-2015.[1] The analysis focuses on the steady state.

The results show that relatively low social security predicts a negative trade balance and a positive stock of foreign assets in the steady state. Because of low pensions, households accumulate more financial wealth for retirement, which increases the capital stock and pushes down the real interest rate. The effect of social security on the external imbalances is quantitatively significant: for the US, the observed difference in pension expenditures (shown in figure 2.2) would predict an average foreign debt of 60 % of GDP between 1980-2015. The model predicts that even if the life expectancy in a country is low, it can become a net external creditor if its social security is low enough. Second, the model predicts that the demographic transition leads to a fall in the real interest rate. However, introducing endogenous labor supply to the model raises

[1]In Appendix 2.D I show the results for the model calibrated to match the US and other OECD countries.

the real interest rate and mutes the steady state effects of population ageing on it. The reason is that agents can prepare for low pensions and higher expected time spent in retirement by increasing their labor supply, and therefore population ageing does not raise household savings to the same extent as with fixed labor supply. The model predicts the country with an older population to have a positive net foreign asset position. For the US, the average differences in life expectancy predict a positive net foreign asset of 20 % of GDP against China. Finally, relatively low levels of government debt and expenditures are also associated with a positive net foreign asset position.

The paper is organized as follows. Section 2 discusses the literature on the life-cycle framework, external imbalances and the real interest rate. Section 3 sets out the model. Section 4 analyzes the effects of social security, population ageing and fiscal policy on external imbalances under different model specifications: with exogenous labor supply, with endogenous labor supply and lump sum taxes, and endogenous labor supply and distortionary taxes. Section 5 concludes.

2.2 Related literature

An extensive literature on international capital flows and the external balance uses life-cycle and overlapping generations models to explain external balances and the interest rate. They provide a fruitful framework for analyzing the effects of demography, social security and fiscal policy in determining the current account.

This paper builds on the life-cycle model by Gertler (1999). The Gertler model is based on the Yaari (1965) and Blanchard (1985) frameworks, which introduce uncertainty of life-time to the macroeconomic models, and on the two-stage overlapping generations model (OLG) by Diamond (1965), in which individuals of two different generations are assumed to be alive at each point in time, and maximize their utility over the two periods of their lifetime. The Gertler (1999) model combines these frameworks by assuming that time spent in both of the stages is finite and determined by an exogenous survival probability. Due to the life-cycle structure, the young households have a motive to save for retirement, and the model facilitates the analysis of demographic factors, social security and fiscal policy. I extend the Gertler (1999) model into a two-country world and provide an extensive analysis of social security, labor supply, taxation and fiscal policy on the external sector in the steady state of the model.

Gertler's model has been applied to study the effects of population ageing by Kilponen et al. (2006), Fujiwara and Teranishi (2008), Ferrero (2010) and Carvalho et al. (2016). Kilponen et al. (2006) analyze the effects of social security and population ageing in a small open economy (Finland), taking the international interest rate as given. Carvalho et al. (2016) analyze the effects of population ageing, and also social security, in a closed economy with the Gertler framework, and assume fixed labor supply. Fujiwara and Teranishi (2008) explore the effects of structural shocks on workers and retirees in the Gertler framework in a closed economy setting, abstracting from social security and assuming lump sum taxes. Ferrero (2010) is, to my knowledge, the only other application of the Gertler framework to study the effects of demographics in a two-country economy, but labor supply is taken as given and the model does not feature social security or distortionary taxation.[2]

This paper is also related to a recent paper by Eugeni (2015), who analyzes the impact of social security on the external imbalances in a two-country, two-stage OLG framework that maintains the structure of the Diamond (1965) model. Eugeni shows that the country with lower social security has a positive stock of net foreign assets and a negative trade balance in the long run in a dynamically efficient steady state, when the countries are equal in other aspects. In comparison to this paper, in Eugeni (2015) the agents only live one period while young and one while old, and therefore she does not study the effects of demographic changes. In addition, her model abstracts from labor market responses and the effects of fiscal policy.

The life-cycle and overlapping generations models provide a theoretical framework for addressing the questions of how population aging, social security and fiscal policy affect the external balance and real interest rate. The large differences observed in labor supply, social security, and fiscal policy, across countries, as well as the challenges that this literature encounters in explaining the drivers of external balances between industrialized and developing countries in particular, motivates the inclusion of labor supply and social security in the analysis and further investigation of the long-term determination of the external imbalances.

[2]For studies of demographic changes and external balances in multi-country, multiperiod overlapping generations models, see Backus et al. (2014), Domeij and Floden (2006) and Saarenheimo (2005).

2.3 The model

The model is a symmetric two-country model which captures the effects of demographic change through a life-cycle structure as in Gertler (1999). The population in a country at time t consists of two groups of individuals: workers, whose total number is N_t^w, and retirees, whose number equals N_t^r. All agents enter the economy as workers at the age of 20, remain workers with probability $\omega_{t,t+1}$ and retire with probability $1-\omega_{t,t+1}$. Every period $(1-\omega_{t,t+1}+n_{t,t+1})\,N_t^w$ new workers are born. Thus the number of workers grows each period at rate $n_{t,t+1}$ and the law of motion for aggregate labor force is

$$N_{t+1}^w = (1-\omega_{t,t+1}+n_{t,t+1})\,N_t^w + \omega_{t,t+1}N_t^w = (1+n_{t,t+1})N_t^w. \tag{2.1}$$

At time t, the probability of a retiree to survive to the next period is $\gamma_{t,t+1}$. The law of motion for the number of retirees is

$$N_{t+1}^r = (1-\omega_{t,t+1})\,N_t^w + \gamma_{t,t+1}N_t^r. \tag{2.2}$$

The ratio of the number of retirees to the number of workers, dependency ratio, is given by $\psi_t = N_t^r/N_t^w$ and can be solved to evolve according to

$$(1+n_{t,t+1})\,\psi_{t+1} = (1-\omega_{t,t+1}) + \gamma_{t,t+1}\psi_t. \tag{2.3}$$

2.3.1 Households

The preferences of households are given by a CES nonexpected utility function of the form

$$V_t^z = \left\{ \left[(C_t^z)^v \,(1-l_t^z)^{1-v} \right]^\rho + \beta_{t,t+1}^z \left[E_t\left(V_{t+1} \mid z\right)^\mu \right]^{\frac{\rho}{\mu}} \right\}^{\frac{1}{\rho}}, \;\; z=\{w\,,\,r\} \tag{2.4}$$

where C_t^z is consumption and l_t^z the fraction of total time allocated to work at time t of a person type z (retiree if $z=r$ and worker if $z=w$). β_t^z is the subjective discount factor, and $E_t\left(V_{t+1} \mid z\right)$ is the expectation of the value function for the next period of the person of type z. The Epstein-Zin preferences allow the separation of income risk aversion from aversion to intertemporal substitution. Parameter ρ captures intertemporal elasticity of substitution, which is given by $\sigma = 1/\left(1-\rho\right)$. Parameter μ captures attitudes towards income risk. Risk neutrality (i.e. $\mu=1$) is assumed, because the only source of income risk is the exogenous probability of retirement and thus the effect of income risk aversion is mitigated.

2.3.1.1 Retirees

Retirees' expectation of the value function is $E_t\left(V_{t+1} \mid r\right) = V^r_{t+1}$ and because the retiree takes into account the survival probability $\gamma_{t,t+1}$, the effective discount factor is $\beta^r_{t,t+1} = \beta\gamma_{t,t+1}$. A retiree born in period j and retired in period i chooses consumption-saving allocation and leisure to maximize

$$V^{jr}_t(i) = \left\{ \left[\left(C^{jr}_t(i)\right)^v \left(1 - l^r_t\right)^{1-v} \right]^\rho + \beta\gamma_{t,t+1} \left(V^{jr}_{t+1}(i)\right)^\rho \right\}^{\frac{1}{\rho}} \tag{2.5}$$

subject to

$$A^{jr}_{t+1}\left(i\right) = \frac{R_t A^{jr}_t\left(i\right)}{\gamma_{t-1,t}} + W_t \xi l^{jr}_t \left(1 - \tau_t\right) + S^{jr}_t\left(i\right) - C^{jr}_t\left(i\right). \tag{2.6}$$

Retirees consume out of their non-human wealth A^r_t, labor income $W_t \xi l^{jr}_t$ net of taxes τ_t and lump sum social security transfer $S^{jr}_t(i)$. The productivity of a unit of labor provided by retirees is only ξ times that of a worker ($\xi \in (0\,,1)$), which leads to a lower labor supply by retirees in the equilibrium.[3] Retirees participate in a perfect annuities market that provides insurance against the uncertainty of the time of death so that each retiree receives a gross return on wealth of $R_{W,t}/\gamma_{t-1,t}$. $R_{W,t}$ is the world interest rate that clears the international capital market. The pension scheme is a public pay-as-you-go (PAYG) pension system in which social security income is financed with transfers from the taxpayers to the retirees.

The first order condition with respect to leisure is

$$l^{jr}_t(i) = 1 - \frac{C^{jr}_t(i)\varsigma}{W_t \xi (1 - \tau_t)}, \tag{2.7}$$

where $\varsigma = \frac{1-v}{v}$. The consumption Euler equation for a retiree is

[3] Evidence from the EU countries suggests that the working decision is relevant for retired population, as 16% of old-age pensioners keep working. The majority do so for financial reasons (see Appendix B: Employed old-age pensioners in the EU-28).

$$C_{t+1}^{jr}(i) = C_t^{jr}(i)\left[\left(\frac{W_t(1-\tau_t)}{W_{t+1}(1-\tau_{t+1})}\right)^{\rho(1-v)}\beta R_{t+1}\right]^{\sigma}. \tag{2.8}$$

The retiree's marginal propensity to consume out of wealth is $\epsilon_t\pi_t$ and decision rule for consumption is given by

$$C_t^{jr}(i) = \epsilon_t\pi_t\left(\frac{R_{W,t}A_t^{jr}(i)}{\gamma_t} + H_t^{jr}(i) + P_t^{jr}(i)\right), \tag{2.9}$$

where $H_t^{jr}(i)$ is the present discounted value of a retiree's lifetime human wealth and $P_t^{jr}(i)$ is the present discounted value of a retiree's lifetime pension benefits, given by

$$H_t^{jr}(i) = W_t(1-\tau_t)\xi l_t^{jr}(i) + \frac{H_{t+1}^{jr}(i)}{R_{t+1}/\gamma_{t+1}}$$

and

$$P_t^r = S_t^r + \frac{P_{t+1}^r}{R_{t+1}/\gamma_{t,t+1}}. \tag{2.10}$$

The marginal propensity to consume evolves according to the nonlinear difference equation

$$\epsilon_t\pi_t = 1 - \frac{\epsilon_t\pi_t}{\epsilon_{t+1}\pi_{t+1}}\gamma_{t,t+1}\left(\frac{W_t(1-\tau_t)}{W_{t+1}(1-\tau_{t+1})}\right)^{\rho\sigma(1-v)}\beta^{\sigma}(R_{t+1})^{\rho\sigma}. \tag{2.11}$$

2.3.1.2 Workers

Workers' expected value function is $E_t\left(V_{t+1} \mid w\right) = \omega_{t,t+1}V_{t+1}^{w} + (1-\omega_{t,t+1})\,V_{t+1}^{r}$ and the effective discount factor is $\beta_{t,t+1}^{w} = \beta$. A worker born in period j chooses consumption-saving allocation and leisure to maximize

$$V_t^{jw} = \left\{\left[\left(C_t^{jw}\right)^{v}\left(1-l_t^{jw}\right)^{1-v}\right]^{\rho} + \beta\left[\omega_{t,t+1}V_{t+1}^{jw} + (1-\omega_{t,t+1})\,V_{t+1}^{jr}\right]^{\rho}\right\}^{\frac{1}{\rho}} \tag{2.12}$$

subject to

$$A_{t+1}^{jw} = R_{W,t}A_t^{jw} + W_t l_t^{jw}(1-\tau_t) - C_t^{jw} - T_t^{jw}. \tag{2.13}$$

Like retirees, workers consume out of nonhuman wealth and wage income net of taxes less a lump sum tax paid by each worker. The proportional income tax (τ_t) is paid both by the workers and the retirees. Workers and retirees in both countries consume a single (numeraire) good that can be traded internationally.

The first order condition with respect to labor is

$$l_t^{wj} = 1 - \frac{C_t^{wj}\varsigma}{W_t(1-\tau_t)}. \tag{2.14}$$

The consumption Euler equation for workers is

$$C_t^{jw}\left[\left(\frac{W_t(1-\tau_t)}{W_{t+1}(1-\tau_{t+1})}\right)^{\rho(1-v)}\beta R_{W,t+1}\Omega_{t+1}\right]^{\sigma} = \omega_{t,t+1}C_{t+1}^{jw} + (1-\omega_{t,t+1})\epsilon_{t+1}^{\frac{\sigma}{1-\sigma}}\chi C_{t+1}^{jr} \tag{2.15}$$

where $\chi = \xi^{-(1-v)}$ and Ω_t is an adjustment term that weights the gross return, *i.e.* an additional discount factor in the workers value function, given by $\Omega_t \equiv \omega_{t-1,t} + (1-\omega_{t-1,t})\epsilon_t^{\frac{1}{1-\sigma}}\chi$.

The worker's marginal propensity to consume out of wealth, π_t, evolves according to

$$\pi_t = 1 - \frac{\pi_t}{\pi_{t+1}}\left(\frac{W_t(1-\tau_t)}{W_{t+1}(1-\tau_{t+1})}\right)^{\rho\sigma(1-v)}\beta^{\sigma}(R_{W,t+1}\Omega_{t+1})^{\sigma-1}. \tag{2.16}$$

The worker's decision rule for consumption is

$$C_t^{jw} = \pi_t(R_{W,t}A_t^{jw} + H_t^{jw} + P_t^{jw}) \tag{2.17}$$

where H_t^{jw} is the present discounted value of a worker's human wealth net of taxation and P_t^{jr} is the present discounted value of a worker's pension benefits once retired. They are given by

$$H_t^{wj} = W_t l_t^{jw} (1 - \tau_t) - T_t^w + \frac{\omega_{t+1} H_{t+1}^{jw}}{\Omega_{t+1} R_{t+1}} + \frac{(1 - \omega_{t+1}) \chi \varepsilon_{t+1}^{1-\frac{1}{\rho}} H_{t+1}^r}{\Omega_{t+1} R_{t+1}}$$

and

$$P_t^{wj} = \frac{\omega_{t+1} P_{t+1}^{jw}}{\Omega_{t+1} R_{t+1}} + \frac{(1 - \omega_{t+1}) \chi \varepsilon_{t+1}^{1-\frac{1}{\rho}} P_{t+1}^r}{\Omega_{t+1} R_{t+1}}.$$

The effective discount rate of the workers is therefore $\Omega_{t+1} R_{t+1} / \omega_{t+1}$, where Ω_t is an additional discount factor which captures the fact that because the workers expect life to be finite, they value the future less than the present and therefore $\Omega_{t+1} > 1$ as discussed in Gertler (1999).

1.2 Aggregation

Because marginal propensities to consume, both for retirees and workers, do not depend on individual characteristics, aggregate consumption can then be expressed as

$$C_t = \pi_t A_t R_{W,t} (\epsilon_t \lambda_t + 1 - \lambda_t) + \pi_t (H_t^w + P_t^w) + \epsilon_t \pi_t (H_t^r + P_t^r) \tag{2.18}$$

where λ_t is the share of assets held by retirees such that $\lambda_t = \frac{A_t^r}{A_t}$. The distribution of aggregate assets between retirees and workers evolves according to

$$\lambda_{t+1} = \omega_{t,t+1} \left(R_{W,t} \lambda_t \frac{A_t (1 - \epsilon_t \pi_t)}{A_{t+1}} + \frac{W(1 - \tau_t) L_t^r \xi + S_t^r - \epsilon_t \pi_t (H_t^r + P_t^r)}{A_{t+1}} \right) + (1 - \omega_{t,t+1}). \tag{2.19}$$

Aggregate labor supply by workers is

$$L_t^w = N_t^w - \frac{\varsigma}{W_t (1 - \tau_t)} C_t^w \tag{2.20}$$

and that of retirees

$$L_t^r = N_t^r + \frac{\varsigma}{W_t(1-\tau_t)} C_t^r. \tag{2.21}$$

The present discounted value of retirees' aggregate human wealth is

$$H_t^r = \xi L_t^r W_t(1-\tau_t) + \gamma_{t,t+1} \frac{\psi_t}{\psi_{t+1}} \frac{H_{t+1}^r}{(1+n_{t,t+1})R_{t+1}} \tag{2.22}$$

where ψ_t is the old age dependency ratio, *i.e.* $\psi_t = \frac{N_t^r}{N_t^w}$, and $n_{t,t+1}$ is the growth rate of the number of workers between periods t and $t+1$. The discount rate of the present value of total human wealth for current retirees is augmented by the growth rate of the retired labor force $\left(\frac{\psi_t}{\psi_{t+1}}\right)$. Similarly, the present discounted value of workers' aggregate human wealth is

$$\begin{aligned} H_t^w &= L_t^w W_t(1-\tau_t) - t_t^w Y_t \\ &+ \omega_{t,t+1} \frac{H_{t+1}^w}{(1+n_{t,t+1})R_{W,t+1}\Omega_{t+1}} \\ &+ (1-\omega_{t,t+1}) \frac{H_{t+1}^r \epsilon_{t+1}^{\frac{1}{1-\sigma}} \chi}{\psi_{t+1}(1+n_{t,t+1})R_{W,t+1}\Omega_{t+1}}. \end{aligned} \tag{2.23}$$

Present discounted value of retirees' aggregate pension benefits at time t is

$$P_t^r = S_t + \gamma_{t,t+1} \frac{\psi_t}{\psi_{t+1}} \frac{P_{t+1}^r}{R_{t+1}} \tag{2.24}$$

and the present discounted value of social security for workers

$$\begin{aligned} P_t^w = \omega_{t,t+1} & \frac{P_{t+1}^w}{(1+n_{t,t+1})R_{W,t+1}\Omega_{t+1}} \\ & + (1-\omega_{t,t+1}) \frac{P_{t+1}^r \epsilon_{t+1}^{\frac{1}{1-\sigma}} \chi}{\psi_{t+1}(1+n_{t,t+1})R_{W,t+1}\Omega_{t+1}}. \end{aligned} \tag{2.25}$$

2.3.2 Firms

The goods market is competitive and the representative firm produces the consumption good with constant returns to scale under Cobb-Douglas production technology. Aggregate output is given by $Y_t = (X_t L_t)^{\alpha} K_t^{1-\alpha}$, where X_t is the level of exogenous labor augmenting productivity at time t, L_t is the aggregate effective labor force, K_t is physical capital, and $\alpha \in (0, 1)$ is the labor share. The firm chooses labor, capital and investment to maximize the present discounted value of profits given by

$$V(I_{t-1}, K_t) = \left[(X_t L_t)^{\alpha} K_t^{1-\alpha} - W_t L_t - I_t + \frac{V(I_t, K_{t+1})}{R_{t+1}} \right] \tag{2.26}$$

subject to the law of motion of capital accumulation

$$K_{t+1} = (1-\delta) K_t + \left[1 - \frac{\phi}{2} \left(\frac{I_t}{I_{t-1}} - \mu_t \right)^2 \right] I_t. \tag{2.27}$$

The aggregate effective labor force consists of the effective labor input by the two agent types such that $L_t = L_t^w + \xi L_t^r$. Capital depreciates at rate $\delta \in (0, 1)$ and quadratic adjustment cost makes investing new capital costly. The size of the adjustment cost is determined by $\phi > 0$. The term μ_t ensures that investment adjustment costs equal zero along the balanced growth path. Productivity X_t grows at rate x_t, which follows an AR(1) process given by $x_t = (1-\theta)x_{ss} + \theta x_{t-1} + u_t^x$.

2.3.3 Government

The government consumes G_t in each period and pays retirees a total amount of P_t of social security benefits. The expenditures are financed with tax revenues T_t and by issuing debt B_{t+1}. Government spending, social security and fiscal policy are assumed to be an exogenously determined fraction of output: $G_t = \bar{g}_t Y_t$, $P_t = \bar{p}_t Y_t$ and $B_t = \bar{b}_t Y_{t-1}$, and the government adjusts taxes to satisfy its intertemporal budget constraint. The government's per period budget constraint is $B_{t+1} = R_{W,t} B_t + G_t + P_t - T_t$ and the intertemporal budget constraint is

$$R_{W,t} B_t = \sum_{v=0}^{\infty} \frac{T_{t+v}}{\prod_{z=1}^{v} R_{W,t+z}} - \sum_{v=0}^{\infty} \frac{G_{t+v}}{\prod_{z=1}^{v} R_{W,t+z}} - \sum_{v=0}^{\infty} \frac{P_{t+v}}{\prod_{z=1}^{v} R_{W,t+z}} \;. \tag{2.28}$$

The government's per period discount rate equals the world interest rate $R_{W,t}$ and is lower than the households' per period discount rates. With elastic labor supply, total tax revenue is

$$T_t = \tau_t W_t L_t + N_t^w T_t^w \ . \tag{2.29}$$

Labor tax τ_t is endogenously adjusted according to the fiscal policy rule. The rule ties the tax rate to the level of government debt as in Leeper (1991) so that the tax rate follows

$$\tau_t = \tau_{t-1} + \theta \left[b_t - \bar{b}_t \right] . \tag{2.30}$$

In the steady state, government debt is an exogenously given fraction $\bar{b}$ of the output and taxes are adjusted to satisfy the governments budget constraint.

2.3.4 A competitive world equilibrium and the external sector

A competitive world equilibrium is a sequence of quantities and prices such that in each country (i) households maximize utility subject to their budget constraints, (ii) firms maximize profits subject to their technology constraints, (iii) the government chooses a path for taxes and debt, compatible with intertemporal solvency, to finance an exogenous level of total spending, and (iv) all markets clear.

In each economy, households' aggregate nonhuman wealth equals the aggregate capital stock, government bonds and net foreign assets F_t,

$$A_t = K_t + B_t + F_t. \tag{2.31}$$

Net foreign asset position evolves according to

$$F_{t+1} = R_{W,t} F_t + NX_t \tag{2.32}$$

and the trade balance NX_t is determined by the aggregate resource constraint

$$NX_t = Y_t - (C_t + I_t + G_t) \, . \tag{2.33}$$

The current account, the change in the net foreign asset position, consists of net interest payments on foreign assets and the trade balance

$$CA_t = (R_{W,t} - 1) \, F_t + NX_t. \tag{2.34}$$

Return $R_{W,t}$ is equalized across the two countries and in equilibrium, foreign asset positions in the two countries sum up to 0, *i.e.* $F_t + F_t^* = 0$. The law of one price is assumed to hold.

I allow population and productivity to grow in the steady state and therefore the model needs to be stationarized. In the steady-state, quantity variables grow at the constant rate $(1+x)(1+n) \simeq 1+x+n$. The stationarized model is solved for variables in efficiency units, for example, $c_t = C_t / (X_t N_t^w)$ where X_t is the level of productivity and N_t^w the growth rate of population. The equations characterizing the steady state are presented in Appendix 2.G.

2.4 Asymmetries and determination of the external balance

This section analyzes the effects of social security and life expectancy on the steady state trade balance, net foreign asset position and the real interest rate under three model specification: with fixed labor supply and lump sum taxes, with variable labor supply and lump sum taxes only ($\tau_t^w = \tau^w = 0$) and variable labor supply and distortionary taxes only ($T_t^w = T^w = 0$).[4] In addition, the section discusses the role of government debt and expenditures in driving external imbalances.

The model is calibrated such that the steady state values of the key demographic parameters, labor market variables, great ratios and the world interest rate match the average between China and the Unites States between 1980 and 2015.[5] Table 2.1 shows the targeted moments and parameter values.

Following Ferrero (2010) and Carvalho et al. (2016), the discount factor, β, is set at 0.98, the labor share of income, α, at 2/3, and the elasticity of intertemporal substitution, σ, at 0.5. The depreciation rate is 20 %. Consumption share of periodic utility, v, is 0.8 as in Kilponen et al. (2006). The productivity of one unit of labor supplied by a retiree relative to a worker, ξ, is 0.58.

The average of total factor productivity of China and the United States between 1980-2012 is used as proxy for growth in labor augmenting productivity. The annual growth rate of the working age population, $n_{t,t+1}^w = n^w$, matches the average annual population growth rate 1980-2015 of approximately 2 % between China and the US.

The probability of remaining a worker, $\omega_{t,t+1} = \omega$, is calibrated to match the average effective age of retirement of 64 years in China and the United

[4]Labor supply is fixed when $v = 1$ and $\xi = 0$.

[5]See Appendix G for data sources.

Parameter / exogenous variable		**Value**
β	discount factor	0.98
α	labor share	$\frac{2}{3}$
δ	discount rate	0.2
σ	elasticity of intertemporal substitution	0.5
ξ	productivity of a unit of labor, retiree to worker	0.58
v	elasticity of period utility with respect to consumption	0.8
x	growth rate of technology	0.014
n^w	population growth rate, population aged 20-64 years	0.02
γ	probability to survive (if retired)	0.9
ω	probability to stay in the labor force	0.9773
b	government debt, % of gdp	0.3
g	government spending, % of gdp	0.15
s	social security spending, % of gdp	0.04

Table 2.1: Calibration for the open economy. Source: α, σ and δ are from Ferrero (2010). The rest of the parameters are chosen to match data for US and China 1980-2015.

States between 1980-2014.[6] Retirement age is calculated as the average retirement age between the two genders weighted by their share in the labor force. The probability of surviving, $\gamma_{t,t+1} = \gamma$, is calibrated to match the average life expectancy at birth of 74 years between the countries, given the retirement age. Government net debt b is set at 30% of GDP. GDP share of government spending is set at 15 % following the average between China and the US in 1980-2015. The level of social security spending matches the average public pension spending of 4 % of GDP between the countries in 1980-2013.[7]The relative size of the countries, $\frac{X_t^{China} N_t^{w,China}}{X_t^{US} N_t^{w,US}}$, is 1.4, matching the average ratio of the economies' productivity levels $\frac{X_t^{China}}{X_t^{US}}$ times the average ratio of population sizes, $\frac{N_t^{w,China}}{N_t^{w,US}}$, between 1980-2015.

The countries' stationarized net foreign assets evolve according to

$$f_t = R_t f_{t-1}(1 + x_{t-1,t} + n_{t-1,t}) + nx_t \tag{2.35}$$

[6]For China, data is from 1987-2010.

[7]Data on Chinese public pension expenditures is available only for 2004-2013. Therefore it is assumed that China's pension expenditures have remained at the average observed level between these years prior to 2004 as well. Values for 2006-2007 and 2011-2012 have been linearly interpolated.

Therefore in the steady state, the net foreign asset position is given by

$$f = \frac{nx(1+x+n)}{(1+x+n-R)} \,. \tag{2.36}$$

When the economy is dynamically efficient, i.e. $R \geq 1 + x + n$, unless the trade is balanced, one economy must run a positive trade balance while maintaining a negative foreign asset position, and vice versa for the other. When the world interest rate exceeds the growth rate of the economy, the creditor country can finance its imports with the interest payment it receives on its foreign claims. In an inefficient world, a country can have a negative trade balance in addition to a negative foreign asset position in the steady state.

The stationarized current account in the steady state is given by

$$ca = (R-1)\frac{f}{1+x+n} + nx \,. \tag{2.37}$$

The sign of the current account in the steady state is therefore determined by the relative sizes of the trade balance and the net interest payment on foreign assets.

Another feature of the open economy steady state is that because both countries are using the same production technology, the stock of capital as a share of efficient labor force is given by

$$\frac{K}{XL} = \left[\frac{(1-\alpha)}{R-1+\delta}\right]^{\frac{1}{\alpha}} \tag{2.38}$$

which is equalized across the countries. Therefore, output in efficiency units is equalized as well.

2.4.1 Social security

This section analyzes the steady-state effect of differences in social security spending on the external imbalances. Social security spending is increased from 2 to 7 % of GDP in country 2 and kept at 6 % in country 1, which matches the observed differences between China and the US. Between 1980 and 2015, average public pension spending was 6 % of GDP in the US and in China, 2 % between 2005-2015.[8]

[8] Data for Chinese public pension spending is available only from 2005 onwards.

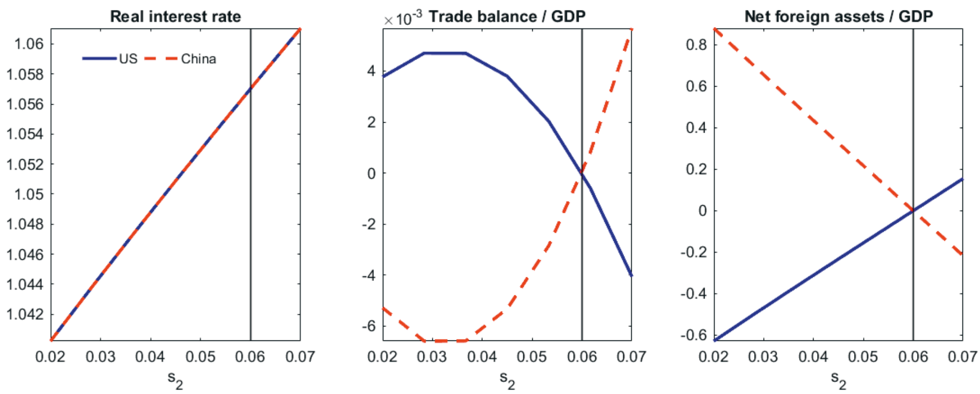

Figure 2.3: **Social security.** Steady state values for $0.02 \leq s_2 \leq 0.07$ and $s_1 = 0.06$ with fixed labor supply. The vertical line shows the steady state when $s_1 = s_2$ and the countries are identical. The point where $s_2 = 0.02$ corresponds to the observed public pension expenditure-to-GDP ratio in China.

Figure 2.3 shows that the country with lower social security spending has a positive net foreign asset position and runs a trade deficit in the steady state (country 2 (China) left of the vertical line; at the vertical line, $s_1 = s_2$).[9] As social security is lower, households insure consumption during retirement by accumulating assets and therefore financial wealth, and aggregate consumption is higher. High aggregate consumption is driven by consumption of the working-age population, since the effect on retirees' consumption is small. Lump sum social security payments constitute a transfer of wealth from the workers to the retirees, and as the old have a higher marginal propensity to consume, high social security increases consumption among retirees as in country 1. However, the intergenerational transfer of wealth increases taxes and, through a negative income effect, lowers consumption among the working-age population. Given the large share of working-age population, the aggregate effect on consumption is negative. As consumption is higher in the country with low social security, it runs a negative trade balance in the steady state. The imports are financed by interest payments on the country's net foreign assets.

The higher the share of social security spending in country 2, the lower the

[9]The prediction of the model - that a country with lower social security has a negative trade balance in a dynamically efficient steady-state - is in line with the results of Eugeni (2015).

incentive to save for retirement and therefore the smaller the amount of world financial wealth. This lowers the capital stock and investments, and raises the real interest rate. A low interest rate benefits the country with an external debt. Less needs to be exported in order to pay back the foreign debt. The bilateral trade balance is therefore smaller when the difference in the countries' social security expenditures is smaller.

When variable labor supply is assumed (figure 2.4), households can also adjust to different levels of social security by adjusting their labor supply, working both during active working life and retirement. This reduces the need to accumulate financial wealth before retirement, as working is also possible in old age, and the households can also maintain higher levels of consumption while working by increasing labor supply. Figure 2.4 shows that with variable labor supply, the interest rate is lower for all levels of social security. The figure also shows that the interest rate is more sensitive to the level of social security when labor supply is fixed, and therefore the impact of social security on the trade balance is also larger. The seemingly large effect of endogenous labor supply on the trade balance is driven by the high real interest rate. By adjusting the value of the discount factor β, it can be seen that the β-adjusted effect of introducing variable labor supply on the trade balance is small.

Figure 2.5 shows the effects of social security with lump sum and distortionary taxes. Distortionary taxes lower the labor supply of both workers and retirees, and propensities to consume are lower and financial wealth higher. Therefore with distortionary taxes, the world financial wealth is lower and the interest rate higher for all levels of social security. Again, the β-adjusted effect of introducing variable labor supply on the trade balance is small.

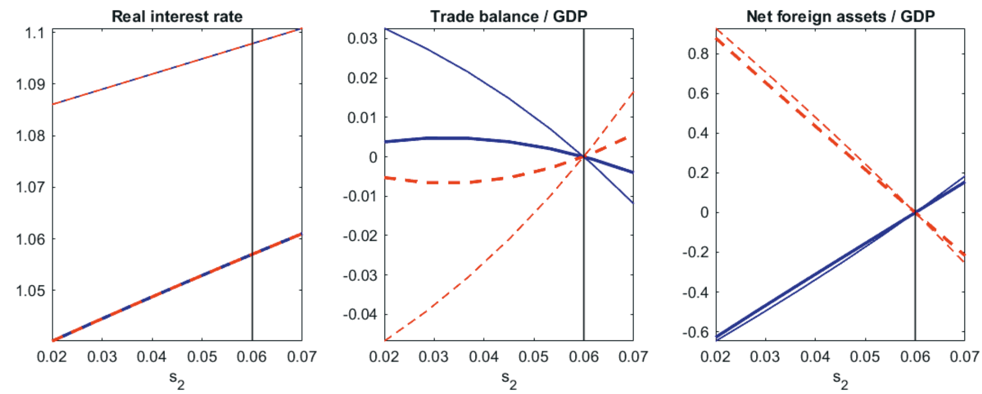

Figure 2.4: **Social security: fixed vs. variable** labor **supply**. Steady state values for $0.02 \leq s_2 \leq 0.07$ and $s_1 = 0.06$ with fixed (thick lines) and variable labor supply (thin lines). The vertical line shows the steady state when $s_1 = s_2$ and the countries are identical. The point where $s_2 = 0.02$ corresponds to the observed public pension expenditure-to-GDP ratio in China.

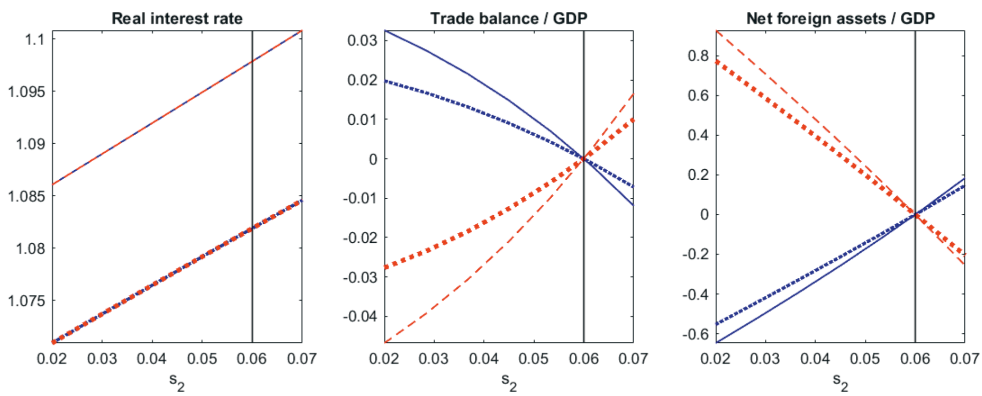

Figure 2.5: **Social security: lump sum vs. distortionary taxes**. Steady state values for $0.02 \leq s_2 \leq 0.07$ and $s_1 = 0.06$ with lump sum taxes (thin lines) and labor income taxes (thick lines). The vertical line shows the steady state when $s_1 = s_2$ and the countries are identical. The point where $s_2 = 0.02$ corresponds to the observed public pension expenditure-to-GDP ratio in China.

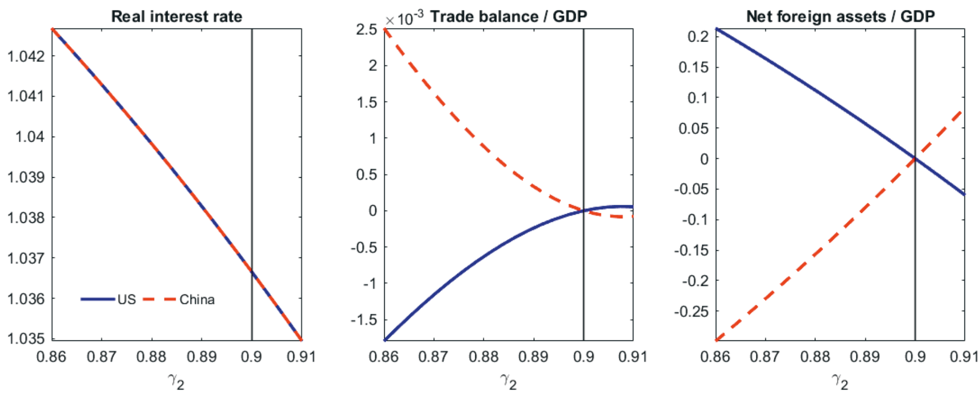

Figure 2.6: **Population ageing.** Steady state values for $0.86 \leq \gamma_2 \leq 0.91$ and $\gamma_1 = 0.9$ with fixed labor supply for US (blue line) and China (red dotted line) . The vertical line shows the steady state when $\gamma_1 = \gamma_2$.

2.4.2 Life expectancy

This section analyses the effects of life expectancy on external imbalances, shown in figure 2.6. Life expectancy varies in country 2 ('China'), while staying constant in country 1 ('US'). The point at which the countries are identical is indicated by the vertical line. When $\gamma_2 < 0.9$ ($\gamma_2 > 0.9$), life expectancy in country 2 is lower (higher) than in country 1. The average life expectancy between the countries between 1980-2015, 74 years, corresponds to $\gamma = 0.9$. The average life expectancy in China in the same period was 72 years, corresponding to $\gamma_2 = 0.87$.

Figure 2.6 shows that the country where life expectancy is higher maintains a negative trade balance and a positive net foreign asset position in the steady state. As can be seen from equations (2.11) and (3.11), a larger γ leads to lower marginal propensities to consume as agents need to spread their consumption over a longer lifetime.[10] Because households need to save for a longer retirement period, its financial wealth and capital stock are higher. For the country with lower life expectancy, the world interest rate is below its autarky rate, and therefore it is willing to borrow from abroad.

Even though the country with higher life expectancy is constantly running a negative trade balance, as the economy is dynamically efficient, the coun-

[10]See Ferrero (2010) for an analytical proof.

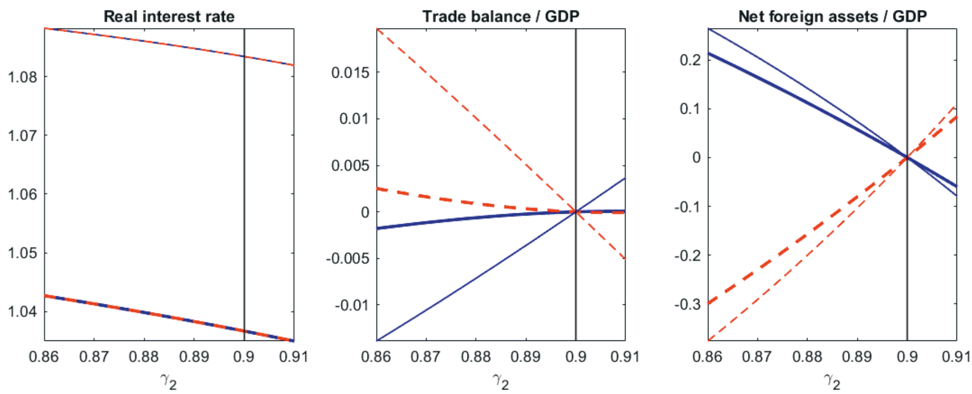

Figure 2.7: **Population ageing: fixed vs. variable labor supply.** Steady state values for $0.86 \leq \gamma_2 \leq 0.91$ and $\gamma_1 = 0.9$ for US (blue line) and China (red dotted line) with fixed labor supply (thick lines) and variable labor supply (thin lines). The vertical line shows the steady state when $\gamma_1 = \gamma_2$.

try earns a high enough interest rate on its foreign assets to be able to hold a positive net foreign asset position. Its negative trade balance also reflects its relatively high aggregate consumption. Even though marginal propensities to consume are higher in the country with a younger population, aggregate consumption in the country with higher life expectancy is higher for three reasons: its residents are wealthier, a higher share of nonhuman wealth is held by retirees, and the dependency ratio is higher.

With variable labor supply (see figure 2.7), the steady state interest rate is higher for all levels of life expectancy. This is higher due to lower financial wealth. A change in life expectancy in country 2 increases its financial wealth, but also leads to higher labor supply, especially from the retirees. Therefore the fall in the real interest rate is muted. Again, the difference in the level of interest rate drives the large differences in trade balance. With β-adjusted calibration, the effect of labor supply on the trade balance is small. With distortionary taxes (see figure 2.8), labor supply among both retirees and workers is lower for all levels of life expectancy and the level of financial wealth is higher, which lowers the interest rate. With lower labor supply and interest rates, marginal propensities to consume are lower and the differences in consumption share of output between the countries smaller. Therefore the external imbalances remain smaller with distortionary taxation.

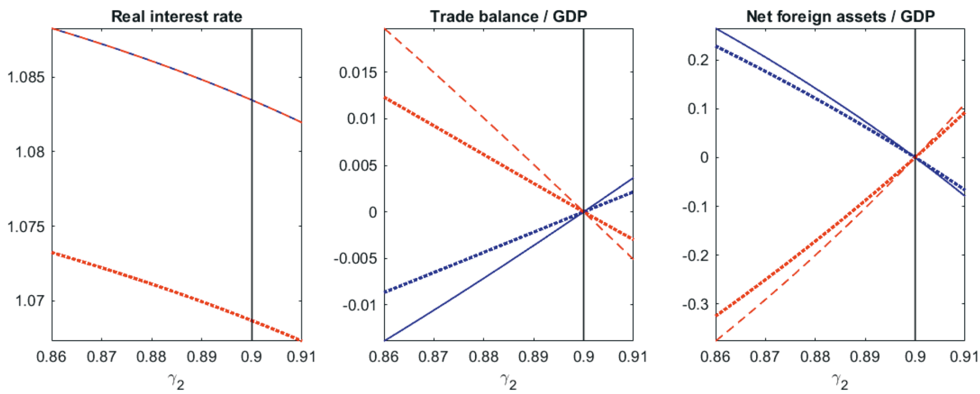

Figure 2.8: **Population ageing: lump sum vs. distortionary taxation.** Steady state values for $0.86 \leq \gamma_2 \leq 0.91$ and $\gamma_1 = 0.9$ for US (blue line) and China (red dotted line) with lump sum taxes (thin lines) and distortionary taxes (thick lines). The vertical line shows the steady state when $\gamma_1 = \gamma_2$.

The steady state effects of life expectancy depend on the level of pensions. Figure 2.9 shows the steady-state implications of life expectancy for different levels of pension expenditures (country 1: 0, 2 and 4 % of GDP; country 2: 4 % of GDP (constant)). As before, life expectancy varies along the x-axis ($0.85 \leq \gamma_1 \leq 0.95$, $\gamma_2 = 0.9$). Even though low life-expectancy lowers the country's financial wealth and predicts a negative net foreign asset position if pension spending is identical in both countries ($s_1 = s_2 = 4\%$; the yellow dotted line), if a country's pension expenditures are small enough relative to the other country, it can nevertheless have a positive net foreign asset position. In addition, with higher social security expenditures, population aging has a stronger impact on the country's external imbalances and the real interest rate. The higher the social security, the larger the tax burden on the working-age population, which lowers the labor supply, encourages savings and pushes down the interest rate. Finally, the impact of the level of social security is greater when life expectancy is low. Because aggregate social security expenditures are by assumption a constant fraction of the GDP, high life expectancy and therefore a high dependency ratio lowers the size of the lump sum transfer per retiree. Therefore, as the world economy grows older, the importance of social security in determining the external balance is mitigated.

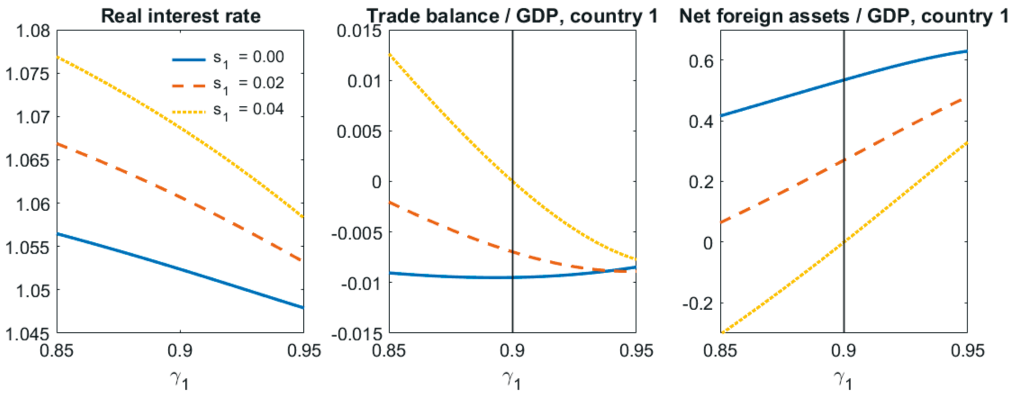

Figure 2.9: **Effect of social security on the steady state impact of population aging.** Life expectancy varies in country 1 such that $0.85 \leq \gamma_1 \leq 0.95$. In country 2, $\gamma_2 = 0.9$. At the vertical line, $\gamma_1 = \gamma_2$. Social security expenditures in country 1 are 0 % of GDP (blue line), 4 % of GDP (red dotted line) or 8 % of GDP (yellow dotted line). Social security expenditures in country 2 are 4 % of GDP.

2.4.3 Government debt

Figure 2.10 shows how a difference in the level of government debt affects the external balance and how the steady state changes when the level of government debt in country 1 changes from 0 to 60 % of GDP while it stays at 30 % of GDP in country 2.

The non-Ricardian effects of government debt in the model are caused by the life-cycle structure and the tax distortions. Due to the life-cycle structure, the expected lifetime of workers is finite and therefore their discount rate exceeds the discount rate of the government. Because the workers less than fully capitalize the higher future tax flows related to higher government debt, government debt increases consumption and lowers savings, which brings down the capital stock and pushes up the interest rate. However, the permanently higher debt level leads to higher taxes in the current period as well, as interest expenses on the public debt are higher. This lowers labor supply and the value of human wealth, lowering consumption and savings. In the aggregate, households are less wealthy in the more indebted country and the country's aggregate consumption is lower. Therefore the country runs a positive trade

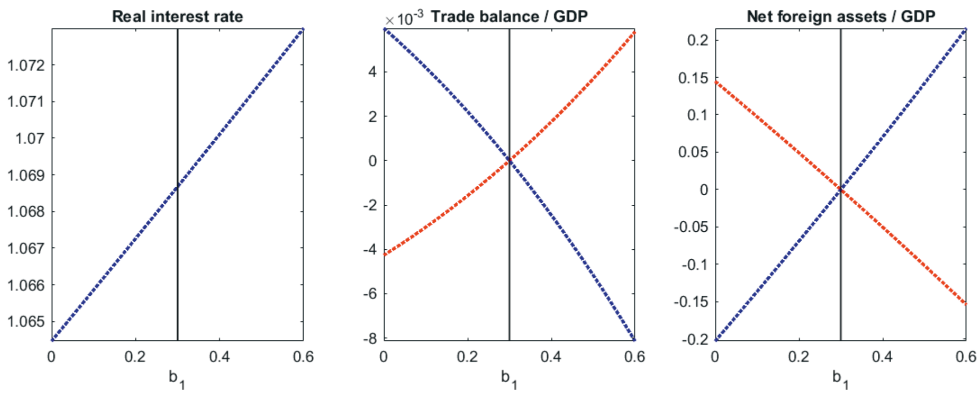

Figure 2.10: **Government debt.** Steady states for $0 \leq b_1 \leq 0.6$ and $b_2 = 0.3$ with variable labor supply and distortionary taxes for country 1 (red line) and country 2 (blue line). At the leftmost vertical line, the countries are identical.

balance and, as its capital supply is lower, holds a negative amount of foreign assets in the steady state.

The level of government debt also has an impact on the steady-state effects of social security. In section 2.4.1 we saw that a country with relatively low social security is a net debtor in a dynamically efficient steady state. However, the result depends on how the social expenditures are financed, since a relatively high level of government debt alone has the opposite influence, as seen above.

Figure 2.11 plots the real interest rates and external balances for different levels of government debt (20-70 % of GDP) and social security (0, 2 or 4 % of GDP) in country 1. The level of government debt and social security expenditures are constant in country 2 (30% and 4% of GDP respectively). With high government debt in country 1, the effect of social security on the trade balance is greater. Therefore the importance of social security in determining the external balance is greater if it is financed with government debt.

2.4.4 Government expenditures

Finally, figure 2.12 shows the steady state effect of government expenditures on the external balance. Government consumption in country 1 ('US') varies between 10 and 20 % of GDP, and stays at 15 % of GDP in country 2 ('China').

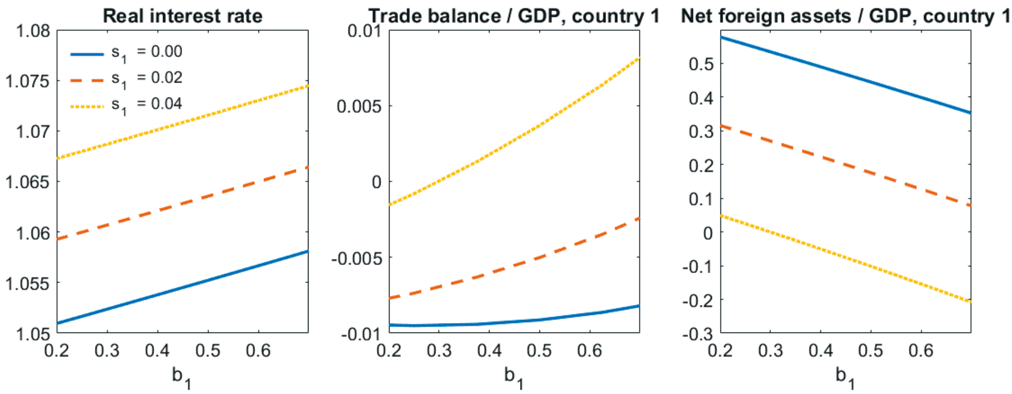

Figure 2.11: **Effect of social security on the impact of government debt.** Government debt varies in both countries such that $0.2 \leq b_1 \leq 0.7$. In country 2, $b_2 = 0.65$. Social security expenditures in country 1 are 0 % of GDP (blue line), 4 % of GDP (red dotted line) or 8 % of GDP (yellow dotted line). Social security expenditures in country 2 are 8 % of GDP.

Between 1980-2015, the average government expenditures between China and US were 15 % of GDP.

The country with lower government consumption has a positive foreign asset position and runs a trade deficit, provided that the economy is dynamically efficient. Tax rate is higher in the country with higher government expenditure, which crowds out private consumption and savings and lowers employment. In the country with less government expenditures consumption as well as employment are higher. Higher private consumption in the steady state leads this country to run a negative trade balance and hold a positive amount of foreign assets in the steady state. The higher the government expenditures in country 1, the more private consumption is crowded out. With the higher real interest rate, capital intensity is lower in both countries. The crowding-out effect is stronger in country 1. As capital intensity is lower, the marginal product of labor falls and output is lower in both countries.

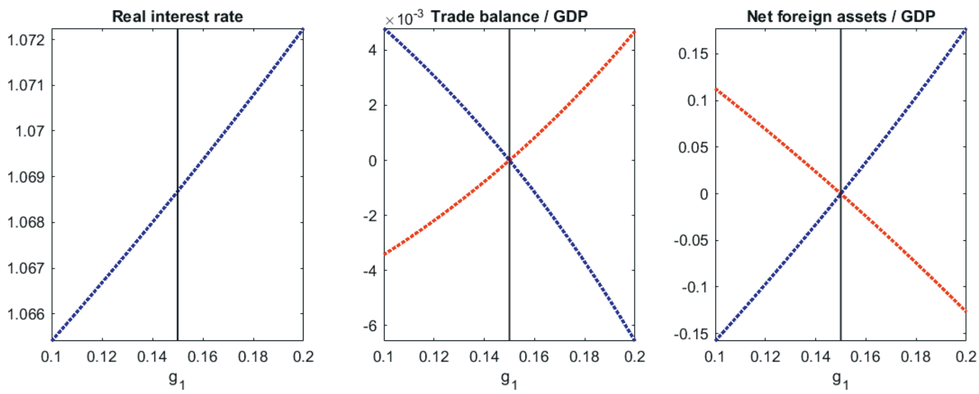

Figure 2.12: **Government spending.** Steady states for $0.1 \leq g_1 \leq 0.2$ and $g_2 = 0.15$ with variable labor supply and distortionary taxesfor country 1 (red line) and country 2 (blue line). At the vertical line, $g_1 = g_2$.

2.5 Conclusions

This paper analyzes the long-term effects of demographic factors, social security and fiscal policy on external imbalances and real interest rate, and explores how these effects are influnced by endogeneity of labor supply and distortionary taxation. The model is based on the Gertler (1999) framework applied to a two-country economy as in Ferrero (2010), extended by social security, variable labor supply and distortionary taxes on labor.

According to my analysis, differences in the level of social security expenditures have a significant impact on external imbalances. The model predicts that countries with low social security expenditures run a negative trade balance and hold a positive stock of foreign assets in a dynamically efficient steady state. The reason is that high social security income during retirement lowers households' savings, leading to low financial wealth and pushing up the world interest rate. Therefore a country with relatively low social security becomes a net external creditor in the long run. For the US, the observed difference in pension expenditures in comparison to China predicts an average foreign debt of 60 % of GDP between 1980-2015. Like earlier literature, the model also predicts a positive net foreign asset position for the relatively aged economy. However, if the aged economy has a high level of social security, it can nevertheless become a net debtor in the long run. High government debt and government expenditures predict a negative stock of foreign assets in the steady state. Therefore high social security, financed by ahigh level of public debt, predicts a weak external balance for a country.

Endogenizing labor supply mutes the effects of the long-term factors on key steady-state variables, including the real interest rate and capital stock. For example, an increase in life-expectancy leads to a smaller increase in aggregate financial wealth than with fixed labor supply, as the negative wealth effect is partly offset by an increase in labor supply. Therefore the negative effect of population ageing on the real interest rate is smaller. This is especially true with lump sum taxes, since the tax distortions caused by labor income taxes discourage the labor supply responses.

The results suggest that the implications of population ageing on external imbalances may change significantly if differences in social security expenditures and labor supply responses are taken into account. For example, social security differentials can be an important factor in explaining the somewhat puzzling observed capital flows from fast-growing emerging economies to the industrialized world, such as from China to the US. Even though the higher life expectancy in the US would by itself predict it to be a net creditor to China in the long run, the fact that China's public social security expenditures and reported government debt have been lower than that of the US between 1980-2015 could lead to the opposite conclusion.

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Appendix

2.A Demographic transition

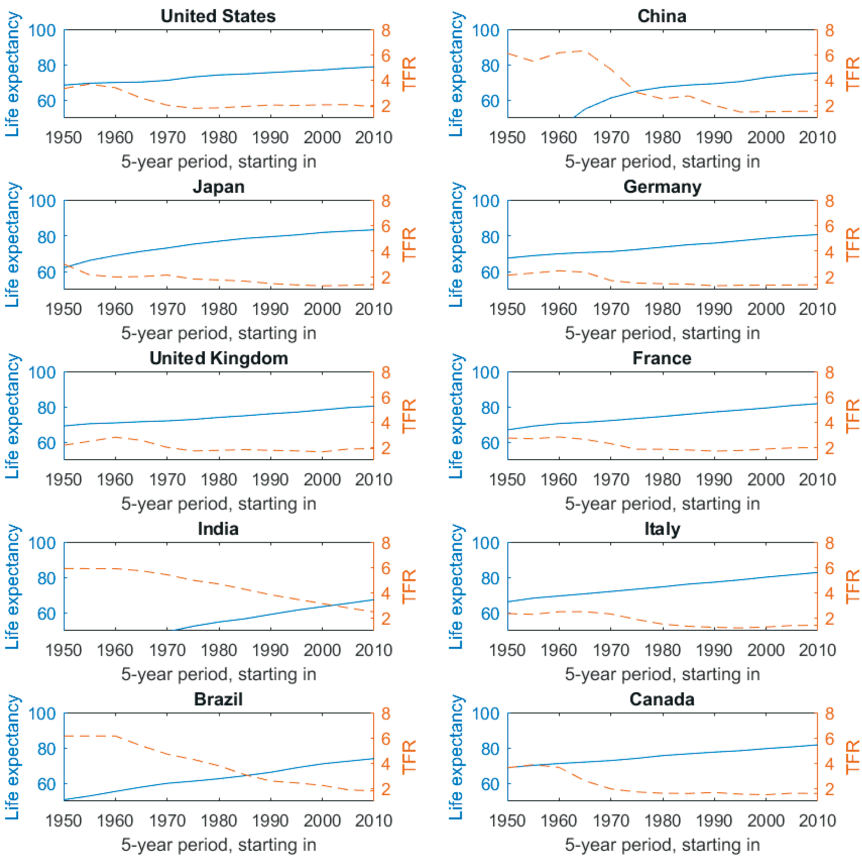

Figure 2.13: Life expectancy at birth and total fertility rate (TFR) in 10 largest economies measured by GDP. Data source: United Nation World Population Prospects 2015 Revision. GDP ranking: World Bank.

2.B Employed old-age pensioners in the EU-28

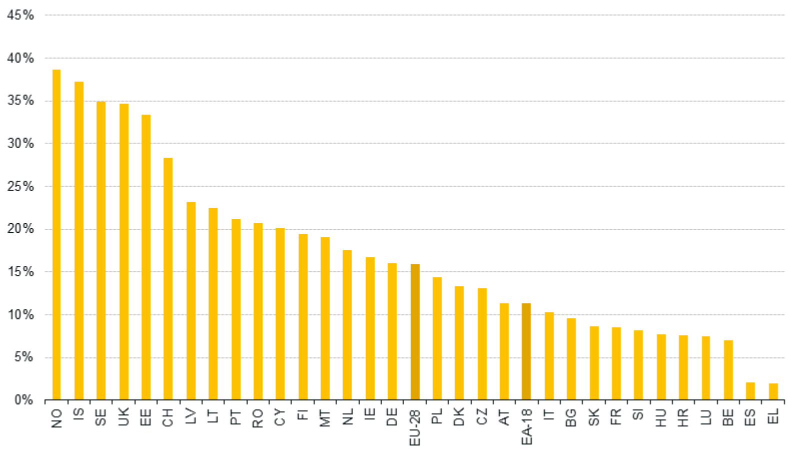

Figure 2.14: The graph shows employed old-age pensioners in the EU-28. A person is counted as employed if (s)he works while receiving an old-age pension, even for a few hours a week. Most old-age pensioners (62.8 %) work for financial reasons. In northern Europe non-financial reasons, such as job satisfaction, are more important, but in southern Europe, financial reasons matter more. Source: Eurostat, Labor Force Survey Statistics (LFS) 2012.

2.C Steady state in a closed economy

This section analyzes the steady-state effects of demographic factors, fiscal policy and social security in a closed economy under three different model specifications: with variable labor supply and distortionary taxes only ($T_t^w = T^w = 0$), with variable labor supply and lump sum taxes only ($\tau_t^w = \tau^w = 0$), and with fixed labor supply and lump sum taxes (as in Ferrero (2010))[11]. When distortionary taxes are ruled out, lump sum taxes adjust endogenously to satisfy the government's intertemporal budget constraint. The purpose of this section is to analyze the effects of endogenous labor supply and distortionary taxation on the steady state, and how the steady-state responses to demographic

[11] See Technical appendix of Ferrero (2010) for the corresponding steady-state equations.

factors, fiscal policy and social security differ under different model specifications. Here the model is calibrated to match the OECD countries in 2016.

2.C.1 The baseline calibration

Table 2.3 presents the steady-state values of the model under the different model specifications. The model is calibrated as in Ferrero (2010) (table 2.2)[12], and the same calibration is used for all model specifications. The additional parameters in the models with variable labor supply, consumption share in periodic utility v and retiree productivity ξ, are chosen so as to match the employment rates of those aged 15-64 and those aged over 65 in the OECD countries in 2014.

Parameter		**Value**	**Source**
β	discount factor	0.98	Ferrero (2010)
α	labor share	$\frac{2}{3}$	Ferrero (2010)
δ	discount rate	0.1	Ferrero (2010)
σ	elasticity of intertemporal substitution	0.5	Ferrero (2010)
ξ	productivity of a unit of labor, retiree / worker	0.58	Chosen to match employment rate in OECD countries
v	elasticity of period utility with respect to consumption	0.66	Chosen to match employment rate in OECD countries
x	growth rate of technology	0.015	Ferrero (2010)
n	population growth rate	0.002	Ferrero (2010)
γ	probability to survive (if retired)	0.93	Ferrero (2010)
ω	probability to stay in the labor force	0.9778	Ferrero (2010)
b	government debt, % of gdp	0.6	Ferrero (2010)
g	government spending, % of gdp	0.2	Ferrero (2010)
s	social security spending, % of gdp	0	-

Table 2.2: Calibration for closed economy. Source: all parameters are from Ferrero (2010) except v and ξ, which are chosen so as to match the employment rate of population aged 15-64 and 65- in OECD countries in 2016.

[12]This is the calibration for the final steady-state for United States in Ferrero (2010).

		Variable labor supply		**Fixed labor supply**
		labor income taxes	*lump sum taxes*	*lump sum taxes*
R	real interest rate	1.046	1.059	1.016
$\epsilon\pi$	marginal propensity to consume, retirees	0.098	0.103	0.087
π	marginal propensity to consume, workers	0.056	0.061	0.046
Ω	adjustment term	1.058	1.055	1.058
ϵ	ratio of marginal propensities to consume	1.729	1.695	1.906
$\frac{K}{XL}$	capital / XL	3.513	3.090	4.844
$\frac{I}{XL}$	investment / XL	0.404	0.356	0.567
$\frac{A}{XL}$	assets / XL	4.420	3.960	5.860
t	tax rate	0.326	0.225	0.200
λ	retirees' share of all financial wealth	0.240	0.270	0.206
$\frac{H}{XL}$	workers' human wealth / XL	7.638	6.740	10.202
$\frac{Hr}{XL}$	retirees' human wealth / XL	0.753	0.929	-
$\frac{Lw}{L}$	workers' labor supply / all labor supply	0.892	0.896	-
$\frac{L}{N_w}$	labor supply / working age population	0.732	0.825	-
$\frac{C}{XL}$	consumption / XL	0.805	0.803	0.787
Y/XL	output / XL	1.512	1.448	1.692
$Net\,saving/XL$	net saving / XL	0.048	0.039	0.082
w	wage / XL	1.008	0.966	-
$\frac{K}{Y}$	capital / GDP	2.324	2.134	2.863
$\frac{I}{Y}$	investment / GDP	0.267	0.246	0.335
$\frac{A}{Y}$	assets / GDP	2.924	2.734	3.463
$\frac{H}{Y}$	workers' human wealth / GDP	5.053	4.653	6.029
$\frac{Hr}{Y}$	retirees' human wealth / GDP	0.498	0.041	-
$\frac{C}{Y}$	consumption / GDP	0.533	0.555	0.465
$Net\,saving/Y$	net saving / GDP	0.035	0.032	0.049

Table 2.3: Steady state values of the model at baseline calibration (table 2.2) with (1) variable labor supply and labor income taxes only, (2) with variable labor supply and lump sum taxes only, and (3) with fixed labor supply and lump sum taxes only.

A comparison of the steady states at the above calibration shows that in the models with endogenous labor supply (first and second column), interest rate is higher and capital stock lower than in the model with fixed labor supply (third column). With fixed labor supply, agents have no labor income during retirement and thus have a motive to save for retirement during working time. With variable labor supply, agents can work during retirement and thus the motive to save during working life is lower. Also, during their working life, agents increase their labor supply, because higher savings and lower consumption in period t would increase the marginal utility of labor if labor supply were kept fixed.[13] The lower aggregate demand for assets increases the interest rate. This reduces the capital stock , and output falls. Both house-

[13]In each period, agents choose labor supply to equate the marginal utility of leisure to wage's consumption value, i.e., $wu_c = u_{1-l}$. As an increase in life expectancy requires the agents to

hold types have higher marginal propensities to consume, and thus aggregate consumption is higher than if labor supply were fixed.

By comparing the models with distortionary and lump sum taxation (first and second column), it can be seen that labor income tax distorts workers' labor supply decision. From equations (2.7) and (2.14) one can see that agents' labor supply decision depends negatively on the tax rate. Therefore households save more for their retirement, and the value of non-human assets, capital, and investment are higher in the steady state than with variable labor supply and lump sum taxation. With a lower real interest rate, agents' marginal propensity to consume and aggregate consumption are lower. The steady-state values of most variables are observed to be closer to the values of the model with fixed labor supply when distortionary taxes are introduced.

2.C.2 Steady-state effects of exogenous changes

The model specification affects the sensitivity of the steady-state values to changes in demographic factors, fiscal policy and social security, which is analyzed in chapters C.2.1 - C.2.3. The results suggest that even though most models with different assumptions on labor supply and taxation produce qualitatively similar predictions of the effects of the factors on the steady state, with variable labor supply the effects are smaller and their relative magnitude differs. Most importantly, the large impact that population aging has on the interest rate in the model with fixed labor supply is muted.

2.C.2.1 Demographic changes

This subsection analyzes the steady-state effects of population growth rate (n), retirement age (ω) and life expectancy (γ), in closed economy under different model specifications. Endogeneity of labor supply affects the implications of population growth, but the effect is small. The implications of retirement age change qualitatively. Introducing variable labor supply significantly affects the implications of population aging.

spread their wealth over a longer lifetime, marginal propensity to consume and consumption fall. The marginal utility of consumption increases, and with endogenous labor supply, agents adjust by increasing their labor supply. Therefore, with higher wage income, agents do not need to reduce consumption as drastically in face of an increased lifetime as they would need to, were labor supply fixed. The higher is the consumption share of periodic utility v, the more the agents work.

Population growth (n) Figure 2.15 shows the steady state of the model for population growth rates between 0.2 % (the baseline calibration) and 2 %. In the steady state, the population growth rate is determined by the growth rate of the labor force n as the dependency ratio, given by $\psi = \frac{1-\omega}{1+n-\gamma}$, is stationary.

Under all specifications, high population growth leads to higher real interest rate and lower capital stock. Population growth affects the steady state slightly more when labor supply is fixed. High population growth decreases the economy's dependency ratio and therefore lowers the share of nonhuman wealth held by retirees. As a younger population has lower marginal propensity to consume out of wealth, high population growth leads to lower consumption. A higher share of agents with less accumulated financial wealth means that capital (in efficiency units) falls, which pushes up the real interest rate.

Retirement age (ω) Figure 2.16 shows the steady state effects of varying ω from 0.975 to 0.978, corresponding to varying the retirement age from 60 to 65 years. An increase in the retirement age, captured by the probability of staying in the labor force ω, increases the horizon of the working-age population and when probability of surviving, γ, is kept constant, it also implies an increase in the life expectancy of the agents.

When labor supply is endogenous, an increase in the probability to stay in the labor force is associated with wealthier households, lower real interest rate and higher consumption. The longer life horizon increases net savings and labor supply in the economy, and real interest rate is lower. The steady-state effects of retirement age with fixed labourlabor supply are the opposite.

Life expectancy (γ) Figure 2.17 shows the steady-state effects of varying γ from 0.8 to 0.96, which corresponds to a life expectancy between 75 and 85 years (when ω is held constant). The level of life expectancy affects workers' and retirees' horizons and changes the economy's demographical composition.

Households' financial wealth and capital stock are higher when life expectancy is high, regardless of whether labor supply is endogenous and independent of taxation. With fixed labor supply, the fall in marginal propensity to consume leads to lower aggregate consumption. Contrarily, with variable labor supply, an increase in retirees' consumption causes the aggregate consumption to increase.

The steady-state effects of life expectancy are smaller in magnitude with variable labor supply than with fixed. With variable labor supply, agents can

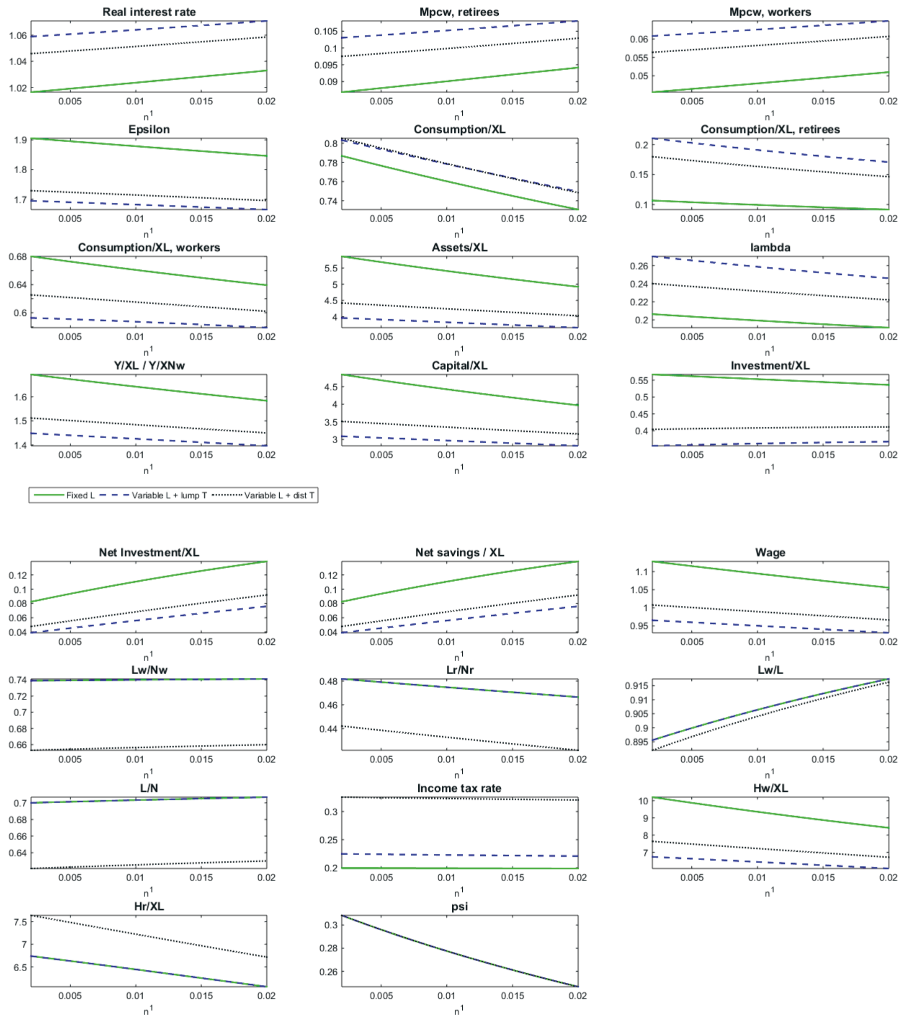

Figure 2.15: Closed economy steady states for population growth levels between 0.2 % (the baseline calibration) and 2 % with variable labor supply and distortionary taxes (black dotted line), variable labor supply and lump sum taxes (blue dotted line) and fixed labor supply (green line). Quantity variables are expressed in efficiency units (X = level of productivity, L = aggregate labor supply).

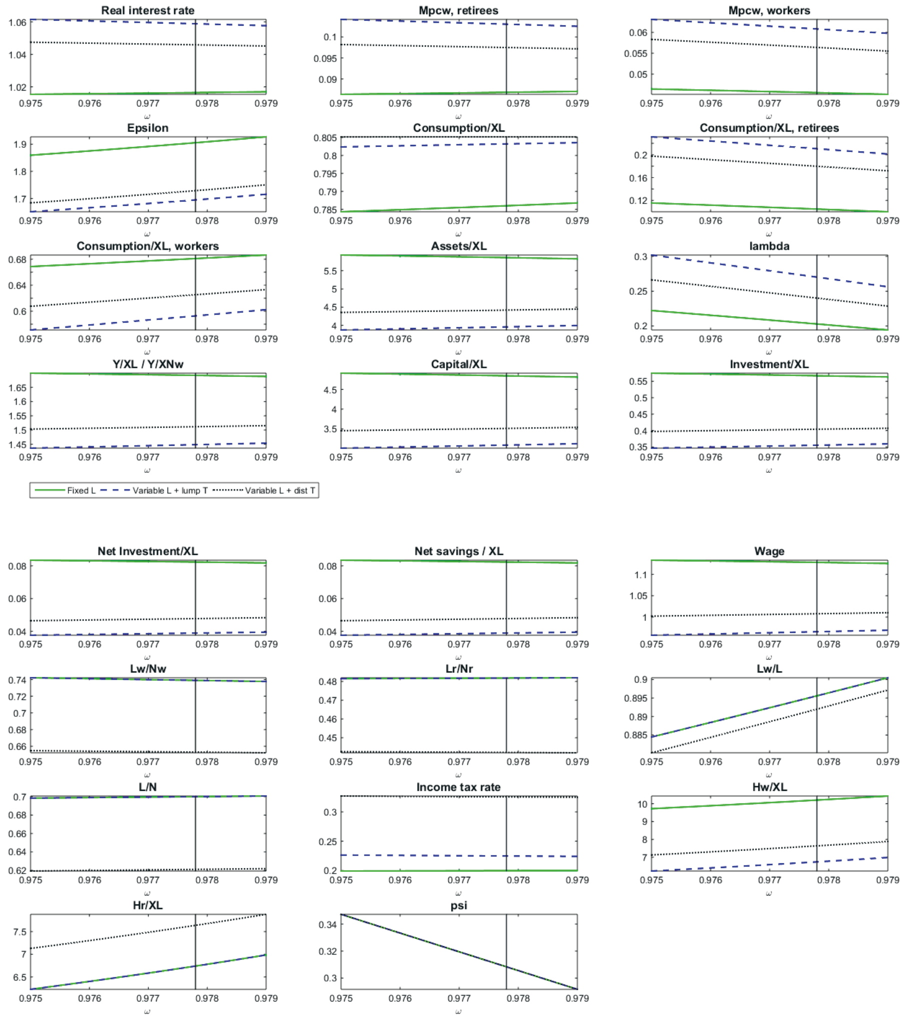

Figure 2.16: Closed economy steady states for $0.975 \leq \omega \leq 0.978$ with variable labor supply and distortionary taxes (black dotted line), variable labor supply and lump sum taxes (blue dotted line) and fixed labor supply (green line). The vertical line indicates the baseline calibration where $\omega \leq 0.9778$. Quantity variables are expressed in efficiency units (X = level of productivity, L = aggregate labor supply).

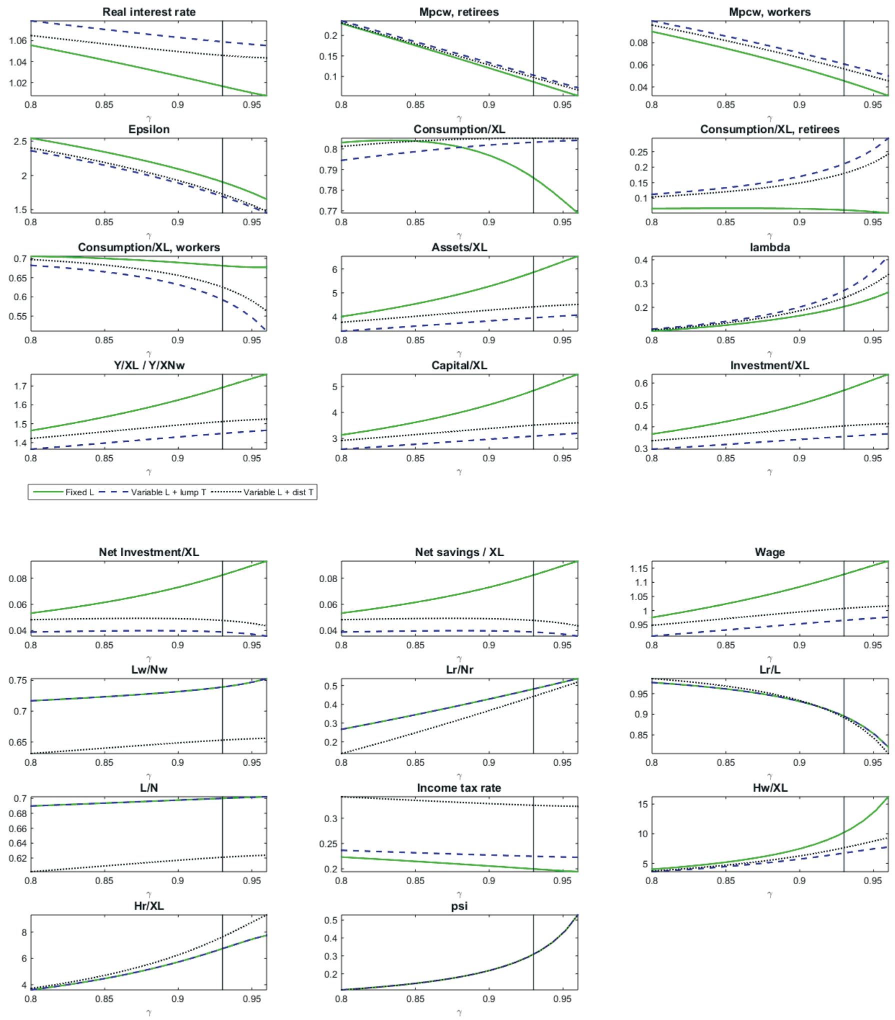

Figure 2.17: Closed economy steady states for $0.8 \leq \gamma \leq 0.96$ with variable labor supply and distortionary taxes (black dotted line), variable labor supply and lump sum taxes (blue dotted line) and fixed labor supply (green line). The vertical line indicates the baseline calibration where $\gamma = 0.93$. Quantity variables are expressed in efficiency units (X = level of productivity, L = aggregate labor supply).

adjust to longer lifetimes by working more, and the observed employment rate of the entire population, especially that of retirees, increases with life expectancy. The motive to save for retirement is smaller, which is reflected in the more modest increase in aggregate household wealth.

2.C.2.2 Fiscal policy

This subsection compares the effects of fiscal policy under different model specifications. Overall, introducing variable labor supply mutes the steady-state effects of fiscal policy and does not change the qualitatively implications documented by Gertler (1999). Labor supply depends negatively on the size of government debt, but the reaction of labor supply to government spending depends on taxation.

Government debt The steady-state effects of government debt are presented in figure 2.18. Government debt varies from 0 to 100 % of GDP. Government expenditures are kept at 20 % of GDP and there are no social security expenditures.

The steady-state effects of changing the size of the government budget deficit are similar under different model specifications. Government debt crowds out capital and investment, which pushes up the real interest rate. As can be seen from the steady-state version of the government budget constraint (see Appendix G, equation (2.53)), a higher level of government debt requires a higher steady state tax rate, because the interest payments on the government debt increase. In each period, households need to save more in order to be able to pay the higher taxes in the future. This leads to an increase in households' financial wealth and to higher aggregate consumption as a share of GDP. With lower capital intensity, real output falls.

With variable labor supply, the steady state is less sensitive to the level of government debt because of the labor supply channel. As government debt increases, increasing the tax rate, labor supply falls. The fall in employment needs to be offset by higher asset accumulation, and the decline in net savings is smaller than with fixed labor supply. As a result, capital intensity falls less, and the increase in real interest rate is smaller.

The choice of taxation in the model mostly affects the levels of the variables. However, with distortionary taxation, workers' labor supply falls more sharply with government debt, due to the tax distortion. This further mutes the increase in the real interest rate.

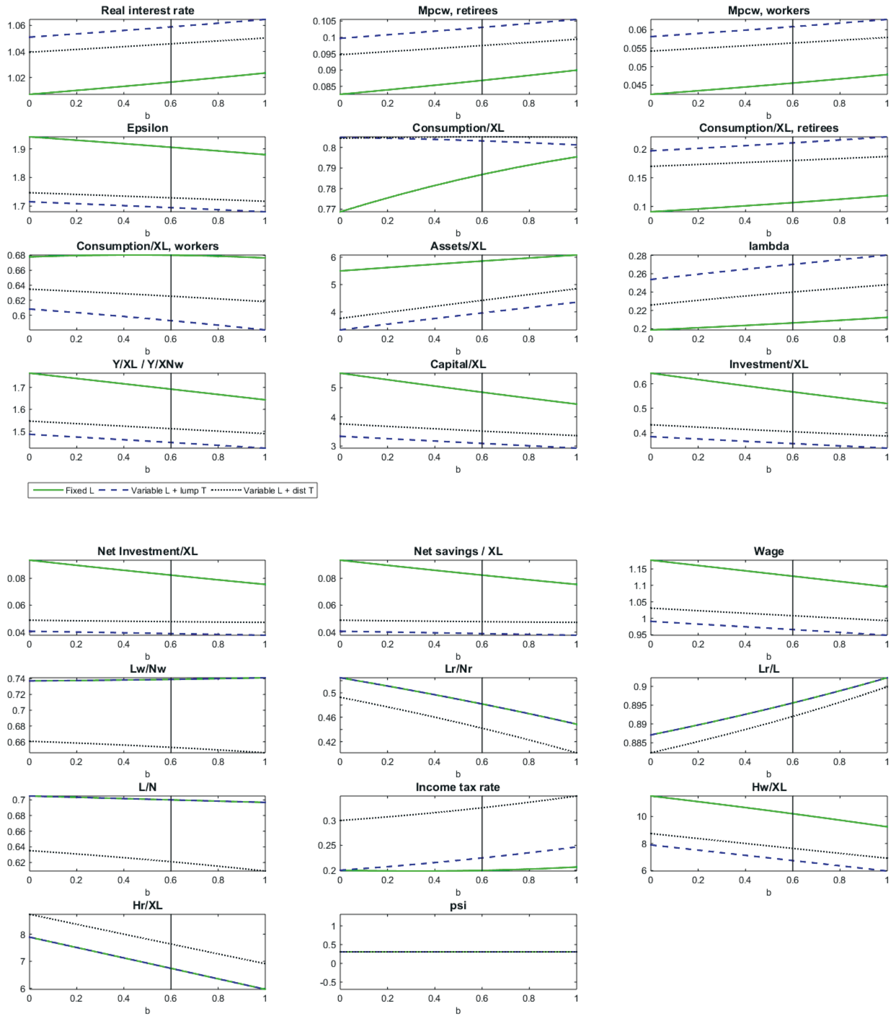

Figure 2.18: Closed economy steady states for $0 \leq b \leq 1$ with variable labor supply and distortionary taxes (black dotted line), variable labor supply and lump sum taxes (blue dotted line) and fixed labor supply (green line). The vertical line indicates the baseline calibration where $b = 0.6$. Quantity variables are expressed in efficiency units (X = level of productivity, L = aggregate labor supply).

Government spending The effect of varying government expenditures from 20 to 25 % of GDP are shown in figure 2.19.

With all model specifications, an increase in government expenditures crowds out private consumption. The increase in taxes implied by the higher government expenditures creates a negative wealth effect on consumption. Households also accumulate less assets which leads to a fall in the capital stock. This increases the real interest rate.

With distortionary taxes, labor supply is lower with high government consumption, whereas with lump sum taxes, labor supply is higher with higher government consumption. As with government debt, the labor supply channel mutes the effect of government expenditures on the steady state.

2.C.2.3 Social security

The steady-state effects of varying the level of social security from 0 to 10 % of GDP with variable labor supply are shown in figure 2.20.[14] Here the government is assumed to run a constant debt/GDP ratio of 60 %, but it should be noted that the effects of social security are sensitive to how they are financed.

Lump sum social security payments to the retirees constitute a transfer of wealth from the young to the old. As the old have a higher marginal propensity to consume, this raises aggregate consumption as a share of GDP. However, as real output falls, consumption in efficiency units falls. As social security income complements the other sources of income in retirement, the incentive to save for retirement is weaker and net saving and household wealth fall. This lowers the capital stock and investment and raises the real interest rate. As the capital stock falls, the marginal product of labor and employment fall. High social security negatively affects retirees' labor supply in particular, and workers provide a larger share of total labor.

When the debt-to-GDP ratio is constant, the higher social security, in the form of a pay-as-you-go system of the model, places a burden on tax payers. With distortionary taxation, the higher tax rate causes labor supply to fall more. This decreases the marginal product of capital, capital as a share of GDP falls less , and therefore the overall effect on the interest rate is muted.

[14] The effects of social security with fixed labor supply in a closed economy are discussed by Gertler (1999)

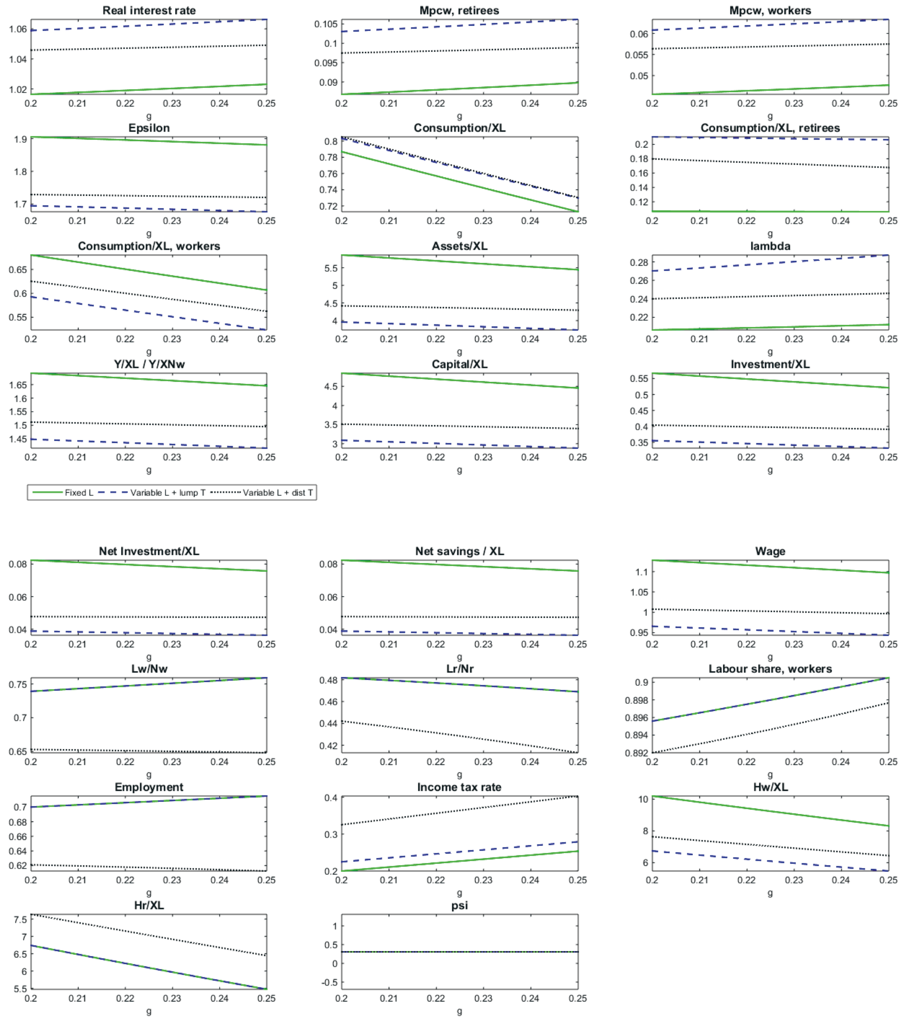

Figure 2.19: Closed economy steady states for $0.2 \leq g \leq 0.25$ with variable labor supply and distortionary taxes (black dotted line), variable labor supply and lump sum taxes (blue dotted line) and fixed labor supply (green line). At the baseline calibration, $g = 0.2$. Quantity variables are expressed in efficiency units (X = level of productivity, L = aggregate labor supply).

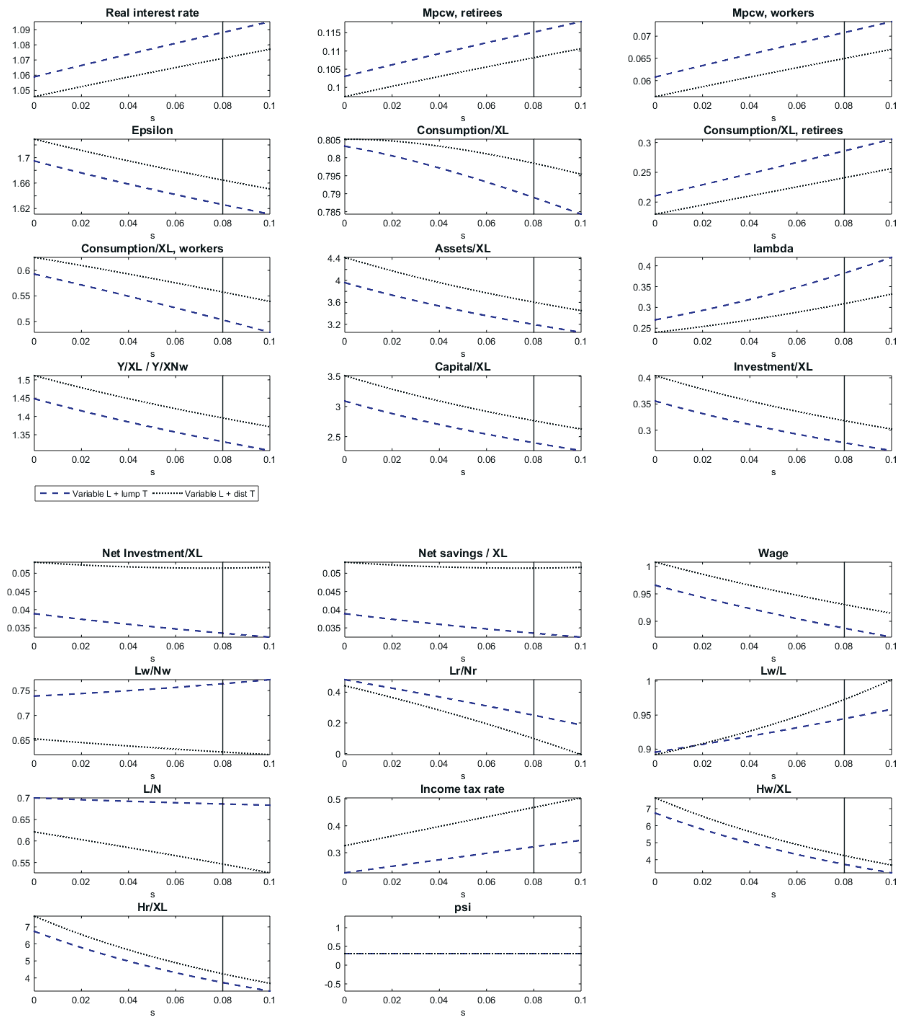

Figure 2.20: Closed economy steady states for $0 \leq s \leq 0.1$ with variable labor supply and distortionary taxes and variable labor supply and lump sum taxes. The vertical line indicates the baseline calibration where $s = 0.08$. Quantity variables are expressed in efficiency units (X = level of productivity, L = aggregate labor supply).

2.D Open economy results for OECD-US

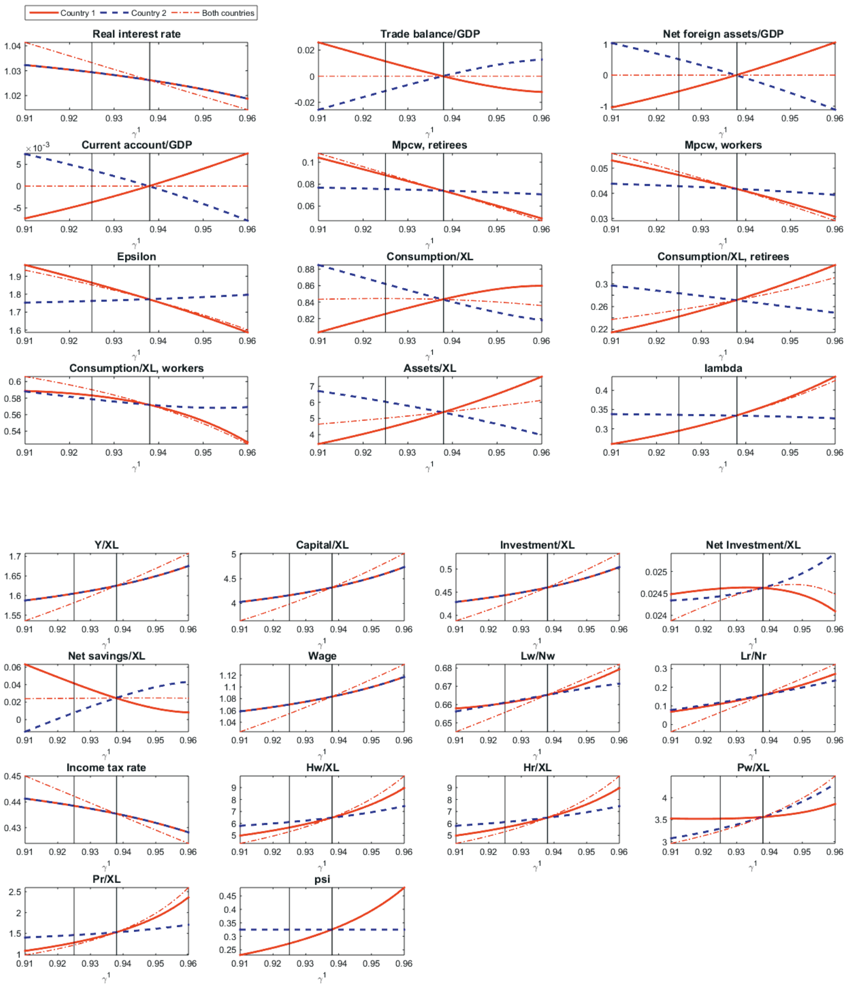

Figure 2.21: Open economy steady states for $0.91 \leq \gamma_1 \leq 0.96$ and $\gamma_2 = 0.9378$ with variable labor supply and distortionary taxes (red line and blue dotted line). The red dotted line shows the steady state when $\gamma_1 = \gamma_2$. At the rightmost vertical line, the countries are identical. At the leftmost vertical line, $\gamma_1 = 0.925$, which corresponds to the observed life expectancy in the US.

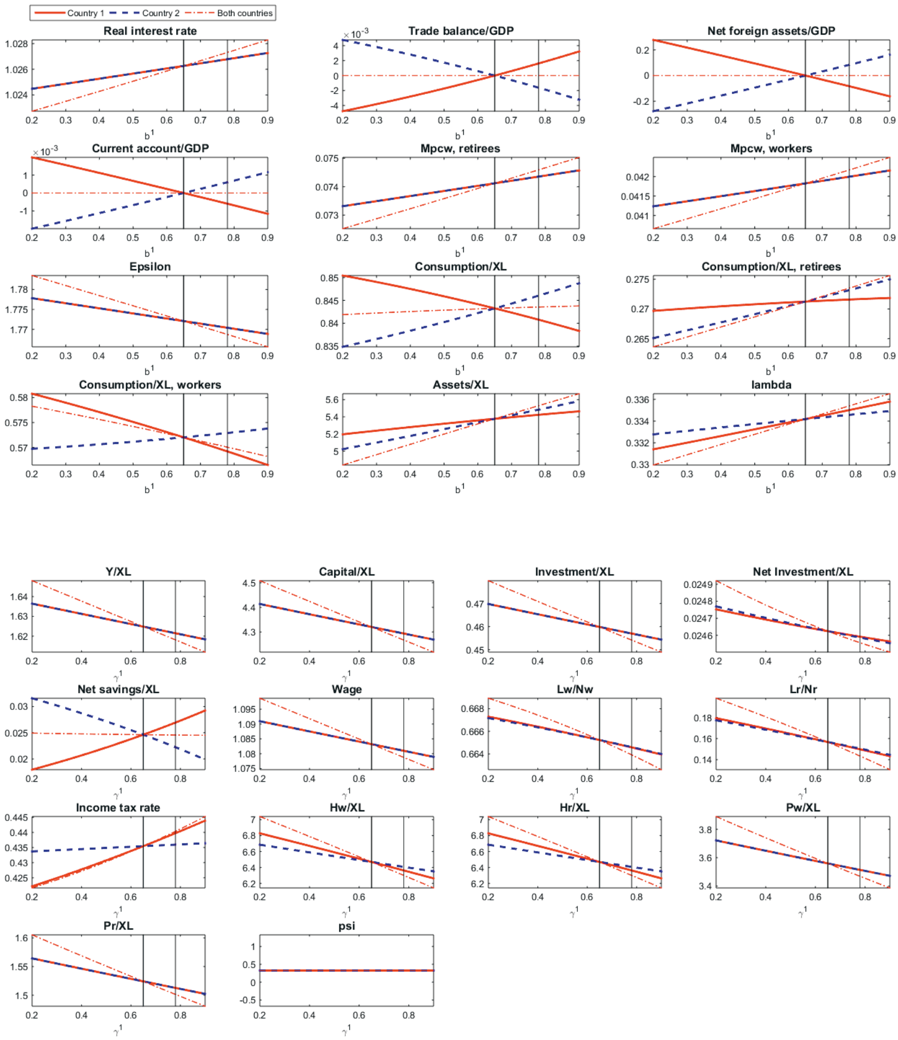

Figure 2.22: Open economy steady states for $0.2 \leq b_1 \leq 0.9$ and $b_2 = 0.65$ with variable labor supply and distortionary taxes (red line and blue dotted line). The red dotted line shows the steady state when $b_1 = b_2$. At the leftmost vertical line, the countries are identical. At the rightmost vertical line, $b_1 = 0.78$, which corresponds to the observed debt-to-GDP ratio in the US.

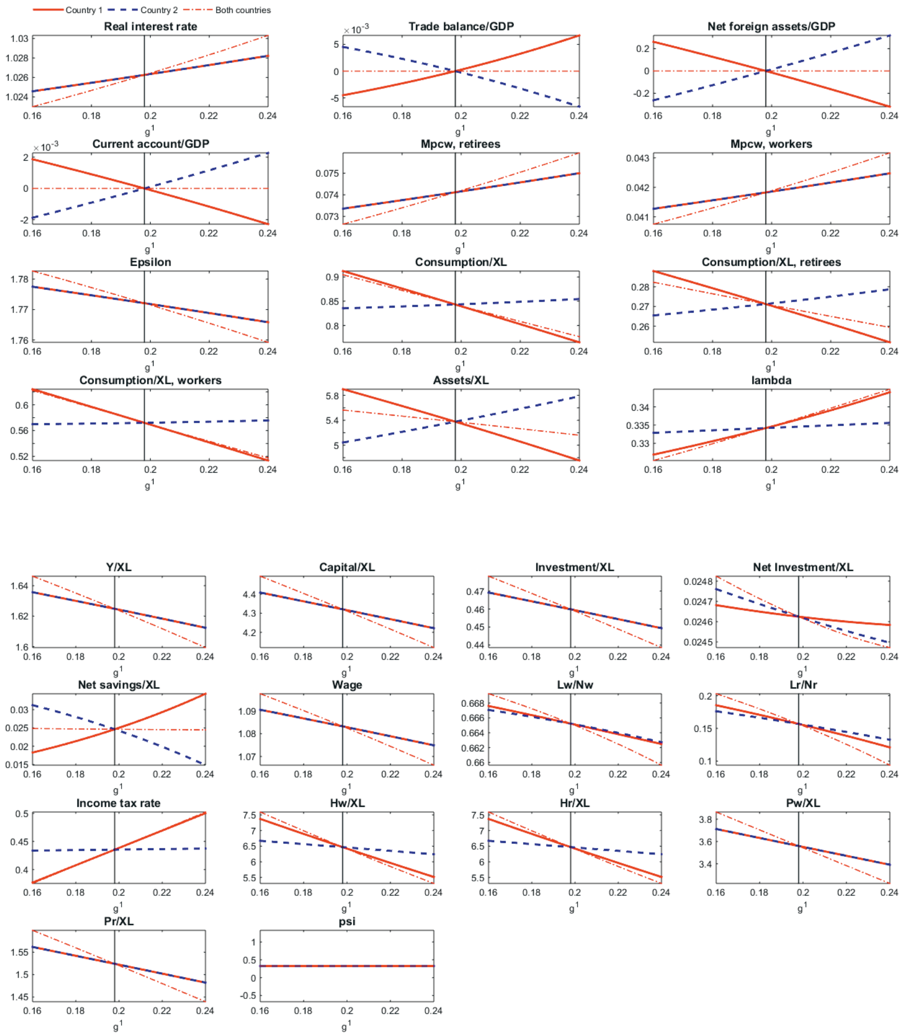

Figure 2.23: Open economy steady states for $0.16 \leq g_1 \leq 0.24$ and $g_2 = 0.198$ with variable labor supply and distortionary taxes (red line and blue dotted line). The red dotted line shows the steady state when $g_1 = g_2$. At the vertical line, the countries are identical. At left $g_1 = 0.16$ corresponds to the observed government expenditure-to-GDP ratio in the US.

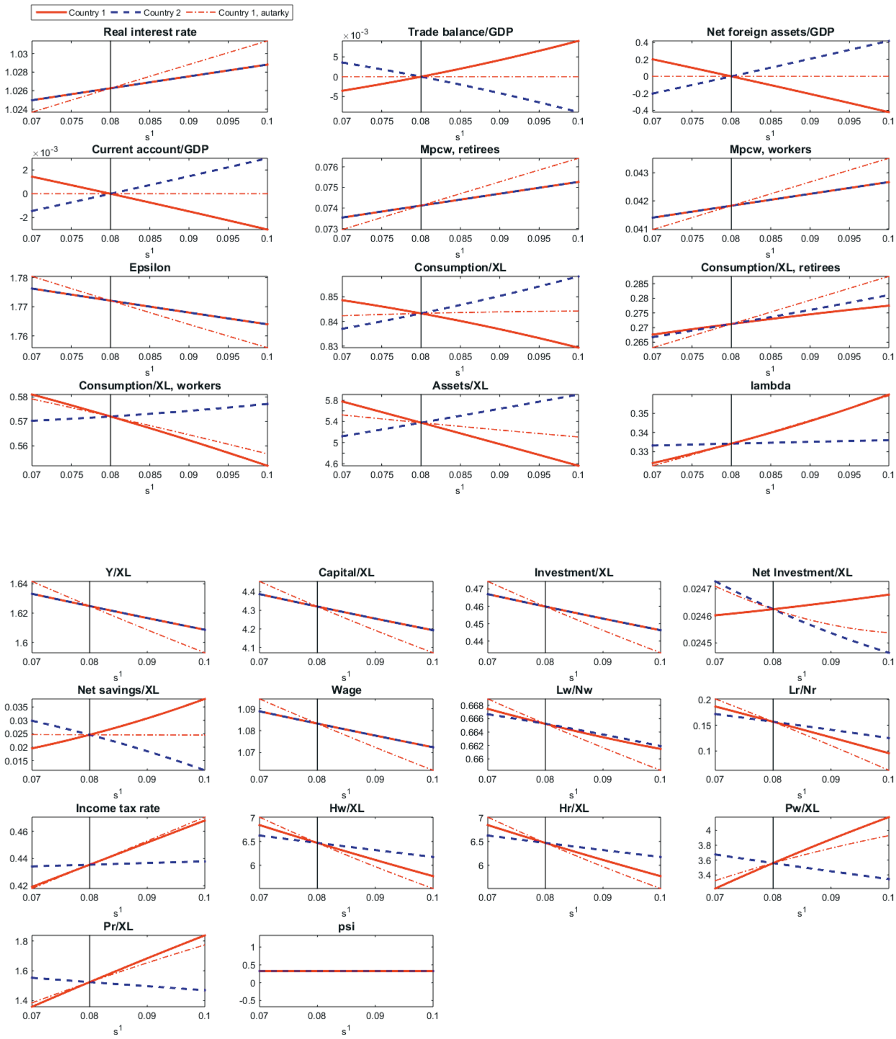

Figure 2.24: Open economy steady states for $0.07 \leq s_1 \leq 0.08$ and $g_2 = 0.08$ with variable labor supply and distortionary taxes (red line and blue dotted line). The red dotted line shows the steady state when $s_1 = s_2$. At the vertical line, the countries are identical. At left, $s_1 = 0.07$ corresponds to the observed public pension expenditure-to-GDP ratio in the US.

2.E Sensitivity of results to parameter values

Productivity of retirees ξ: An increase in the relative productivity of the retirees leads to higher labor supply by retirees[15], lower wages per efficient unit of labor and lower labor supply by workers. The increase in productivity increases the payoff of a unit of labor (see eq. 2.7) and triggers a positive substitution effect on labor supply. As retirees' labor supply increases, their consumption increases due to a positive wealth effect on consumption. An increase in consumption is associated with a fall in savings, an increase in the real interest rates and a lower capital-to-GDP ratio.

The higher the productivity of retirees with respect to workers, the more an increase in γ is accompanied by an increase in retirees' labor supply and the lower is the increase in savings and the fall in the real interest rate.

Consumption elasticity of periodic utility v: A large consumption elasticity of periodic utility leads to higher marginal propensities to consume, higher consumption as a share of GDP, and higher employment for both workers and retirees, as well as lower asset-, and capital-to-GDP ratios. Wages are lower and retirees hold a slightly higher share of total assets.

The effects of population aging are sensitive to consumption elasticity of periodic utility. For low values of elasticity, an increase in γ has nonmonotonic steady-state effects. For high values of elasticity, changes are always monotonic. For example, if consumption share in utility and life expectancy are low, an increase in life expectancy increases the level of nonhuman wealth in the steady-state and lowers the real interest rate. However, if the increase in life expectancy is large enough, the opposite happens. In other words, when leisure share in periodic utility is high enough, at high enough level of life expectancy, level of financial wealth decreases and interest rate rises.

Elasticity of intertemporal subsitution σ: A high σ leads to lower levels of consumption and marginal propensities to consume, and higher assets-, capital- and investment-to-GDP ratios. Interest rate is lower with higher EIS.

The relation of the workers' and retirees' marginal propensities to consume depends in a non-monotonic way on the value of σ due to the fact that workers' marginal propensity to consume is nonmonotonic in σ. It is highest at low values of σ.

[15] For given parameter values, retirees only supply positive amounts of labor when their productivity is high enough.

2.F Data

Table 2.4: Demographic and labor force data

	OECD	OECD without US	US	China	Data source
Average effective age of retirement (all genders, weighted by female labor share), 2009-2014	64	64	65.3	67.8	OECD: Statistics on average effective age of retirement
Female labor share, average 2009-2014	44.7	44.7	46.0	43.9	World Bank World Development Indicators.
Life expectancy at birth, average 2009-2014	80.08	80.13	78.68	75.30	World Bank World Development Indicators.
Average annual population growth rate 2010-2015	0.62	0.57	0.77	0.52	United Nations World Population Prospects: The 2015 Revision
Employment / population, ages 15-64 yrs, 2014	65.8	64.9	68.1	75.1 (2010)	OECD (2016), Labour statistics
Employment / population, ages 65 yrs + , 2014	13.4	11.7	17.7	20.9 (2010)	OECD (2016), Labour statistics
Labor force participation rate, ages 15-64 yrs, 2014	71.2	73.1	72.7	77.4	OECD (2016), Labour statistics
Labor force participation rate, ages 65 yrs +, 2014	13.8	11.9	18.6	21.1 (2010)	OECD (2016), Labour statistics

Table 2.5: Great ratios

	OECD	OECD without US	US	China	Data source
Government spending, % of GDP (g), average 2009-2014	19.8	19.9	15.9	13.3	World Bank, World Development Indicators
General government net debt, % of GDP (b), average 2010-2015	65.3	58.9	77.6	-	International Monetary Fund, World Economic Outlook Database, October 2015
Public pension spending, % of GDP, average 2009-2011	0.08	0.08	0.07	0.02	OECD Data on Social Protection (19.4.2016); China: Asian Development Bank, Social Protection Index Database (25.4.2016)
Private consumption, % of GDP (c/y), average 2009-2014	55.5	55.1	68.5	36.4	World Bank, World Development Indicators: World Bank national accounts data, and OECD National Accounts data files.
Total investment, % of GDP (i/y), average 2010-2015	21.4	21.4	19.3	46.3	World Bank, World Development Indicators
World real interest rate (r), proxied by US annual government bond rate in 2015 net of CPI inflation	2.14 %	-	-		IMF, International Financial Statistics and World Economic Outlook Database (CPI inflation, avegare consumer prices), 7/2016
The average of a GDP-weighted average of OECD countries' TFP growth 2001-2011	0.001				Penn World Table 8.1, data retrieved 20.4.2016, GDP: OECD Data

2.G Steady-state equations

The following equations are stationarized and timing convention is such that the stock variables (A_t, K_t, b_t and F_t) are predetermined. Thus, for instance, equation (3.20) is written as $K_t = (1-\delta)K_{t-1} + \left[1 - \frac{\phi}{2}\left(\frac{I_t}{I_{t-1}} - \mu_t\right)^2\right] I_t$ and then stationarized, resulting in expression (2.44).

1. Retiree's mpcw:

$$\epsilon\pi = 1 - \gamma \left(\frac{1}{1+x}\right)^{\rho\sigma(1-v)} \beta^{\sigma} R^{\sigma-1} \tag{2.39}$$

2. Worker's mpcw:

$$\pi = 1 - \beta^{\sigma} \left(\frac{1}{1+x}\right)^{\rho\sigma(1-v)} (\Omega R)^{\sigma-1} \tag{2.40}$$

3. Adjustment term (where $\chi = \xi^{-(1-v)}$):

$$\Omega = \omega + (1-\omega)\epsilon^{\frac{1}{1-\sigma}}\chi \tag{2.41}$$

4. Production:

$$y = \left(\frac{L}{N^w}\right)^{\alpha} \left(\frac{k}{1+x+n}\right)^{1-\alpha} \tag{2.42}$$

5. Capital:

$$k = \left[\frac{(1-\alpha)}{R-1+\delta}\right]^{\frac{1}{\alpha}} \left(\frac{L}{N^w}\right)(1+x+n) \tag{2.43}$$

7. Investment:

$$i = k\frac{(x+n+\delta)}{(1+x+n)} \tag{2.44}$$

8. Workers' consumption:

$$c^w = \pi\left(\frac{(1-\lambda)aR}{1+x+n} + h^w + p^w\right) \tag{2.45}$$

9. Total consumption:

$$c = \pi \frac{a}{1+x+n} R(\epsilon\lambda + 1 - \lambda) + \pi(h^w + p^w) + \epsilon\pi(h^r + p^r) \tag{2.46}$$

10. Distribution of wealth between generations:

$$\lambda = \left[\psi + \omega \left(\frac{\frac{L-L^w}{L}\alpha y(1-\tau) + s^r y - \epsilon\pi(h^r + p^r)}{a(1+n-\gamma)} \right) \right] * \frac{(1+n-\gamma)}{\left(1 - \frac{\omega\gamma(\beta R_W)^\sigma \left(\frac{1}{1+x}\right)^{\rho\sigma(1-v)}}{(1+x+n)}\right)} \tag{2.47}$$

11. Total assets:

$$a = k + by + f \tag{2.48}$$

12. Workers' human wealth:

$$h^w = \left[\frac{L^w}{L}\alpha y(1-\tau) - t^w y + (1-\omega) \frac{h^r(1+x)\epsilon^{\frac{1}{1-\sigma}}\chi}{\psi R\Omega} \right] \left(1 - \omega \frac{(1+x)}{R\Omega}\right)^{-1} \tag{2.49}$$

13. Retirees' human wealth:

$$h^r = \left[\left(1 - \frac{L_t^w}{L_t}\right) \alpha y(1-\tau) \right] \left(1 - \frac{(1+x)}{R_W/\gamma}\right)^{-1} \tag{2.50}$$

14. Retirees' social security wealth:

$$p^r = sy \left(1 - \frac{(1+x)}{R_W/\gamma}\right)^{-1} \tag{2.51}$$

15. Workers' social security wealth:

$$p^w = \frac{(1-\omega)\epsilon^{\frac{1}{1-\sigma}}\chi p^r(1+x)}{\psi R\Omega} * \frac{1}{1-\omega(1+x)/R\Omega} \tag{2.52}$$

16. Labor tax rate:

$$\tau = \frac{\frac{[R-(1+x+n)]b}{1+x+n} + g + s - t^w}{\alpha} \tag{2.53}$$

17. Aggregate resource constraint:

$$(1-g)y = c + i + nx \tag{2.54}$$

18. Workers' labor supply:

$$\frac{L^w}{L} = \frac{N^w}{L} - \frac{\varsigma}{\alpha y(1-\tau)} c^w \tag{2.55}$$

19. Total labor supply, % of efficient population:

$$\frac{L}{N^w(1+\psi\xi)} = \left(1 + \frac{\varsigma}{\alpha y(1-\tau)}(c^r + c^w)\right)^{-1} \tag{2.56}$$

20. Net foreign assets:

$$f\left(\frac{1+x+n-R}{1+x+n}\right) = nx \tag{2.57}$$

21. Global asset market clearing condition:

$$f_t + \frac{F_t^*}{X_t N_t^w} = f_t + f_t^* \frac{X_t^* N_t^{w*}}{X_t N_t^w} = 0 \tag{2.58}$$

22. Wage in effiency units (detrended by level of technology X_t only; $w_t = \frac{W_t}{X_t}$):

$$w = \alpha y \frac{N^w}{L} \ . \tag{2.59}$$

3 External imbalances between China and the United States: a dynamic analysis with a life-cycle model

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3.1 Introduction

China and the United States, the world's largest economies, have both experienced large and persistent external imbalances over the past decades. China's current account has been consistently positive since 1994, and by 2015 it had accumulated a stock of net foreign assets which amounted to 15 % of its GDP (red lines in figure 3.1). The United States (henceforth, the US) has, in contrast, run a nearly consistent current account deficit since 1982, and accumulated a net foreign debt equal to 40 % of GDP by 2015 (blue lines in figure 3.1). The external imbalances also show up on the countries' bilateral trade balance: since the mid-80s, US imports from China have exceeded its exports to the country, and the US bilateral trade deficit has increased at an accelerating rate until recent years.[1]

[1]Trade balances are pictured in Appendix A (figure 3.20). United States' multilateral trade balance has been negative since 1976. China's multilateral trade balance has been positive since

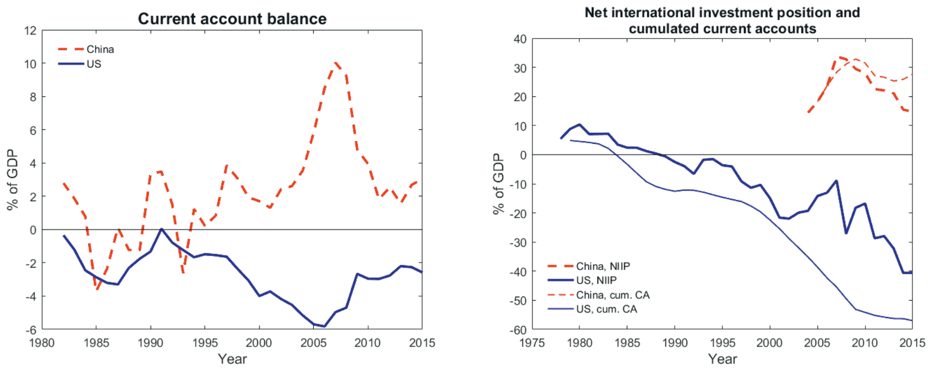

Figure 3.1: Left panel: Current account balance. Right panel: Net international investment position and cumulative current account balance. Source: IMF Balance of Payment Statistics and author's calculations.

This paper suggests that demographic developments and differences in the level of social security between China and the US have had a great impact on households' consumption and saving decisions and, therefore, on the observed external imbalances. During the period from 1980 to 2015, China experienced a rapid increase in life expectancy, which converged towards that of the US, but at the same time, its old-age social security expenditures were significantly lower. When analyzed with a two-country life-cycle model, these developments have caused the Chinese population to prepare for the lengthening of the retirement period by increasing their savings, which has resulted in an improvement in China's current account balance and contributed to the build-up of its foreign asset position. The impact of this channel appears quantitatively large, and the model predicts a positive trade balance for China for the majority of the simulation period.

The main contribution of this paper is to provide a complementary explanation for the persistent external imbalances between China and the US. Early research on external imbalances focuses on the industrialized world and struggles to explain the capital flows from fast-growing emerging economies to industrialized countries, because neoclassical theory predicts that capital flows into countries with rapid productivity growth (see *e.g.* the discussion on the "capital allocation puzzle" in Gourinchas and Rey (2014)). A more re-

1990 apart from a brief plunge in 1993.

cent strand of the literature, to which this paper is closely related, focuses on identifying drivers behind the external imbalances between emerging and industrialized economies, including Caballero et al. (2008), Mendoza et al. (2009), Song et al. (2011) and Coeurdacier et al. (2015). This strand of literature emphasizes the importance of financial markets in driving the capital outflows from the emerging world, suggesting for instance that the flight of capital from the emerging markets is due to their underdevelopment and their inability to supply safe assets. However, apart from Coeurdacier et al. (2015), this strand puts less emphasis on explaining the observed high household savings and low propensities to consume, which are the key elements driving the capital flows in this paper. Coeurdacier et al. (2015) suggest that tighter credit constraints in fast-growing emerging economies can lead to higher household savings, and provide an explanation for the current account surpluses. The role of household savings and the current account has also been studied by Eugeni (2015), who suggests that the low level of social security may play an important role in driving the capital outflows from the emerging world.

In this paper, I use a two-country life-cycle model *á la* Gertler (1999) to quantify the importance of social security and demographic transition, as well as productivity growth and government expenditures, in shaping the dynamics of the bilateral trade balance and the net foreign asset position between the US and China between 1980 and 2015. The model is close to Ferrero's (2010), which is a two-country model featuring a Gertler life-cycle structure, and is extended to include a pay-as-you-go social security system, endogenous labor supply and distortionary taxation. The model is calibrated for China and the United States, and demographic change, productivity growth, social security and government expenditures are taken into account as exogenous variables in simulating the transition paths.

The results suggest that demographic transition and the level of social security expenditures have a quantitatively significant impact on households' consumption and saving decisions and, therefore, on the current account, trade balance and net foreign asset positions between China and the US. In the model, population aging leads to high savings as households prepare for the long retirement period by accumulating financial wealth, and because the demographic transition has been faster in China, the impact on its current account is positive. The relatively low social security expenditures in China relative to the US (approximately 2 % of output in China and 6 % of output in the US between 1980 and 2015) have a quantitatively large impact on the household savings, and when both social security and demographic transition are accounted for, the model predicts a trade surplus for China for the majority of the simulation period. Furthermore, even after controlling for high productivity growth, which has a strong negative impact on the trade balance, the model still predicts a positive net foreign asset position and trade balance for China for most years in the simulation period.

The rest of the paper is organized as follows: Section 2 discusses related literature. Section 3 presents the model. Section 4 presents the results of the static and dynamic analysis. Section 5 concludes.

3.2 Related literature

The paper is related to two strands of research: the literature which uses life-cycle and OLG models to analyze questions related to population aging, social security and external imbalances, and, more generally, to the recent literature on the external imbalances between emerging and industrialized economies.

In this paper, I use the life-cycle model by Gertler (1999), which allows me to model an economy with realistic average lengths of life, working time and time spent in retirement, and is therefore well suited to studying the economic implications of the demographic transition and social security. In comparison to large-scale multi-period OLG models, the advantage of the Gertler framework is its parsimony, which allows to obtain tractable analytical solutions for aggregate consumption and labor supply and to track the mechanisms behind the aggregate savings and current account dynamics.

The Gertler (1999) model has previously been used by Kilponen et al. (2006), Fujiwara and Teranishi (2008), Ferrero (2010) and Carvalho et al. (2016), among others. Kilponen et al. (2006) analyze the implications of population aging and social security in a small open economy (Finland), and relax Gertler's assumption of a stationary demographic structure. As opposed to Kilponen et al. (2006), I focus on external imbalances between two large economies, and the relative importance of aging and social security in comparison to total factor productivity (henceforth, TFP) growth and fiscal policy. Fujiwara and Teranishi (2008) analyze the welfare implications of monetary and fiscal policy on different age groups in a closed economy. In comparison, my focus is on the external sector and therefore the analysis is performed in a two-country framework. As I focus on the low-frequency dynamics of the trade balance, I also abstract from nominal frictions. Carvalho et al. (2016) analyze the impact of population aging on the real interest rate in a closed economy, considering also the role of social security. In contrast, my focus is on the external sector, and furthermore, my model features endogenous labor supply together with distortionary taxes, so that the quantitative findings are robust to the effects of social security through the labor supply channel. Ferrero (2010) is, to my knowledge, the only paper applying the Gertler (1999) model to a two-country world to analyze the dynamics of the external balance and the real interest rate. Ferrero (2010) analyzes the effects of demographics, fiscal policy and productivity on the external balance between the US and six other industrialized countries. The above-mentioned difficulty of explaining capital flows between emerging and industrialized countries is present in Ferrero (2010). In fact, even though productivity differentials, together with demographic developments, can explain the majority of the dynamics of the bilateral trade balance between the analyzed industrialized countries during the simulation period (1970-2005), Ferrero's predictions for China are counterfactual. In this paper I extend the Ferrero (2010) model by introducing a pay-as-you-go social security system together with variable labor supply and distortionary labor taxes and restrict the analysis to China and the United States.

Saarenheimo (2005), Domeij and Floden (2006) and Attanasio et al. (2016) use multi-period OLG models to analyze questions related to external imbalances, population aging and social security in multi-country settings. Saarenheimo (2005) studies the effects of population aging on global variables, including the real interest rate, asset prices and external imbalances in a 73-cohort OLG model with five countries, including China, over the period 2000-2050. In addition to the different approach to modeling the life-cycle and the multi-country setting, labor supply is exogenous in the paper, and it focuses on projecting the future rather than explaining the observed dynamics. The model predicts a current account deficit for China for the entire simulation period because of high projected TFP growth, which has proven counterfactual in the first decades of the 2000s. Domeij and Floden (2006) use an OLG model to analyze the effects of demographic transition on the international capital flows in a model economy consisting of 18 OECD countries and the rest of the world, also assuming fixed labor supply. Attanasio et al. (2016) study the role of demographics in shaping the factor prices and capital movements in a 4-region setting, one of the countries being China, with a rich OLG structure. The period of the analysis is between 2010 and 2100, and for the first decades of the millennium the model predicts a negative foreign asset position for China. In addition to using the Gertler (1999) life-cycle framework, this paper differs from the above-mentioned papers as it focuses on the observed dynamics of external imbalances in the past decades and on the bilateral relation between China and the US instead of a multi-country world. In addition, I allow for endogenous labor supply with distortionary taxation, which are not featured in the above-mentioned papers but which allows me to take into account the impact of population aging through the labor supply channel, and ensures that population aging does not result in counterfactual excessive savings by the young.

Two-country OLG models motivated by the Chinese experience include Eugeni (2015) and Coeurdacier et al. (2015). Eugeni (2015) analyzes the role of social security in the formation of external imbalances and the fall of the real interest rate. Eugeni (2015) finds that the absence of a social security system (a pay-as-you-go pension system) leads to higher savings and capital exports in a two-country OLG model in which only one of the two countries has a social security system, suggesting that social security plays a role in explaining the capital outflows from countries such as China. In contrast to Eugeni (2015), my model also features a rich demographic structure so that I am able to analyze the effects of social security together with demographic changes. In addition, I analyze the dynamic effects of a richer set of exogenous variables including TFP growth and fiscal policy, and take into account the impact of social security through the labor supply channel. Coeurdacier et al. (2015) study external imbalances with a 3-period OLG model, in which the external imbalances are driven by the growth differentials between countries and the differences in the credit constraints that the households in different economies are assumed to face. In contrast to Coeurdacier et al. (2015), I abstract from financial frictions to focus on the impact of demographics and social security arising in a frictionless environment.

İmrohoroğlu and Zhao (2018a) and İmrohoroğlu and Zhao (2018b) study Chinese savings in detail with dynastic models. The use of a dynastic model somewhat changes the impact of demographic change on household savings in contrast to a life-cycle model. In dynastic models, individuals derive utility from the well-being of their ancestors and predecessors in addition to their own well-being. This means that the young save to provide for the retired elderly, but also the old save to leave bequests to the young, and the effect of demographic change on aggregate savings depends on which of the different savings motives dominates. In İmrohoroğlu and Zhao (2018a), population ageing results in higher savings by the old and therefore higher aggregate savings: even though there are fewer young to care for, which lowers the savings of the old, and fewer young to provide for the elderly, which lowers the savings of the young, lowering fertility rates lead to a deterioration of the family pension support system. Because of of this, the old are required to save more, resulting in a positive net effect on aggregate savings. Furthermore, in İmrohoroğlu and Zhao (2018a), there are also precautionary savings due to health risks faced by elderly people and labor income risks faced by the young. Precautionary savings by the old, caused by an increase in health costs, are found to be the main explanation for the recent rise in savings in China. İmrohoroğlu and Zhao (2018a) study a closed economy, and analyzing the impact of precautionary saving and family support system on the current account would require a comparison of the relative importance of these factors in the counterpart countries.

İmrohoroğlu and Zhao (2018b) also study the effects of social security reforms in China on saving, capital and output. Social security is found to be negatively associated with household saving also in the dynastic model as in the life-cycle model in this paper. Social security reforms are also studied by Song et al. (2015), who conduct a comprehensive and detailed analysis of the welfare effects of different pension reforms in China during its projected economic and demographic transition in the 21st century. The analysis takes into account features of the Chinese economy such as rural-urban migration and heterogeneous education in different age groups, which I have assumed away for simplicity. To capture the heterogeneity in population, Song et al. (2015) use a rich multi-period OLG structure as opposed to a Gertler type stochastic two-period structure. In Song et al. (2015), the economic transformation and resulting wage growth, which give rise to the welfare redistribution, are due to firm heterogeneity and the financial frictions as in Song et al. (2011), and the impact of demographics and social security on private savings is not discussed. As opposed to Song et al. (2015), I make no model-based assumptions of future economic transition or wage growth, but only analyze the relative effects on savings implied by demographic transition and social security in the two countries.

Finally, several recent papers, including Caballero et al. (2008), Mendoza et al. (2009) and Song et al. (2011), analyze the role of financial markets in the development of external imbalances between emerging and industrialized countries. Caballero et al. (2008) develop a model which rationalizes the international capital flows as a global equilibrium outcome driven by differences in the abilities of different regions to supply financial assets. Mendoza et al. (2009) argue that when countries with less developed financial markets, which result in low enforceability of financial contracts, integrate into the international financial system, capital flows towards more developed financial markets occur. Song et al. (2011) propose a model for the Chinese economy in which the firms are heterogeneous in productivity and in their ability to access financial markets. The capital outflows occur because the productive firms have limited access to financial markets, and therefore the domestic financial markets are unable to provide sufficient investment opportunities to the households despite high productivity growth. In this paper, I assume no frictions in financial markets in order to focus on the potential explanatory power of the life-cycle aspects and social security systems.

3.3 The model

The model is a symmetric two-country model with a life-cycle structure as in Gertler (1999).[2] The population in each country at time t consists of two groups of individuals: workers, whose total number is N_t^w, and retirees, whose number equals N_t^r. All agents enter the economy as workers at the age of 20, and remain workers with probability $\omega_{t,t+1}$ or retire with probability $1-\omega_{t,t+1}$. Retirees face the probability $\gamma_{t,t+1}$ of surviving to the next period. These parameters are calibrated to match the observed average lengths of life, time spent at work and time spent in retirement.

3.3.1 Households

The preferences of households are given by a CES nonexpected utility function of the form

$$V_t^z = \left\{ \left[(C_t^z)^v \left(1 - l_t^z\right)^{1-v} \right]^\rho + \beta_{t,t+1}^z \left[E_t \left(V_{t+1} \mid z \right)^\mu \right]^{\frac{\rho}{\mu}} \right\}^{\frac{1}{\rho}}, \ \ z = \{w, r\}, \quad (3.1)$$

where C_t^z is consumption and l_t^z the fraction of time allocated to work at time t by a person of type z (retiree if $z = r$ and worker if $z = w$). Each individual has one unit of time per period to use for either work or leisure. Parameter β_t^z is the subjective discount factor, and $E_t \left(V_{t+1} \mid z \right)$ is the expectation of the value function in the next period of the person of type z. The Epstein-Zin preferences allow the separation of income risk aversion from aversion to intertemporal substitution. Parameter ρ captures the intertemporal elasticity of substitution, which is given by $\sigma = 1/\left(1-\rho\right)$. Parameter μ captures attitudes towards income risk. Risk neutrality (i.e. $\mu = 1$) is assumed to yield consumption decisions that are linear in wealth, which facilitates the aggregation of the model. The only source of income risk is the exogenous probability to retire and thus the effect of income risk aversion is reasonable.

[2]Appendix B describes the life-cycle structure.

3.3.1.1 Retirees

A retiree's expectation of the value function is $E_t\left(V_{t+1} \mid r\right) = V^r_{t+1}$, and because (s)he takes into account the probability of dying before the next period, the effective discount factor in retirees' case is $\beta^r_{t,t+1} = \beta\gamma_{t,t+1}$. A retiree, born in period j and retired in period i, chooses consumption-saving allocation and leisure to maximize

$$V^{jr}_t(i) = \left\{\left[\left(C^{jr}_t(i)\right)^v (1-l^r_t)^{1-v}\right]^\rho + \beta\gamma_{t,t+1}\left(V^{jr}_{t+1}(i)\right)^\rho\right\}^{\frac{1}{\rho}}, \tag{3.2}$$

subject to

$$A^{jr}_{t+1}(i) = \frac{R_t A^{jr}_t(i)}{\gamma_{t-1,t}} + W_t \xi l^{jr}_t (1-\tau_t) + S^r_t - C^{jr}_t(i). \tag{3.3}$$

Retirees consume out of their non-human wealth A^r_t, labor income $W_t\xi l^{jr}_t$, net of distortionary labor income taxes τ_t, and lump sum social security transfer S^r_t. The productivity of a unit of labor provided by retirees is ξ times that of a worker ($\xi \in (0,1)$), which leads to a lower labor supply by retirees in the equilibrium. Retirees participate in a perfect annuities market which provides insurance against the uncertainty of the time of death, so that each retiree receives a gross return on wealth of $R_t/\gamma_{t-1,t}$. R_t is the real interest rate. The pension scheme is a public PAYG (pay-as-you-go) pension system in which the pension income is a transfer from the tax payers to the retirees.

The first order condition with respect to leisure is

$$l^{jr}_t(i) = 1 - \frac{C^{jr}_t(i)\varsigma}{W_t\xi(1-\tau_t)}, \tag{3.4}$$

where $\varsigma = \frac{1-v}{v}$. The retiree's decision rule for consumption is given by

$$C^{jr}_t(i) = \epsilon_t\pi_t\left(\frac{R_t A^{jr}_t(i)}{\gamma_{t-1,t}} + H^{jr}_t(i) + P^r_t\right), \tag{3.5}$$

where $\epsilon_t\pi_t$ is the marginal propensity to consume (mpc) out of wealth and $H^{jr}_t(i)$ and $P^{jr}_t(i)$ are the present discounted values of a retiree's lifetime human wealth and pension benefits. The marginal propensity to consume evolves according to the nonlinear difference equation

$$\epsilon_t \pi_t = 1 - \frac{\epsilon_t \pi_t}{\epsilon_{t+1}\pi_{t+1}} \gamma_{t,t+1} \left(\frac{W_t(1-\tau_t)}{W_{t+1}(1-\tau_{t+1})} \right)^{\rho\sigma(1-v)} \beta^{\sigma} (R_{t+1})^{\rho\sigma}. \tag{3.6}$$

3.3.1.2 Workers

A worker's expectation of the value function in the next period is $E_t\left(V_{t+1} \mid w\right) = \omega_{t,t+1} V_{t+1}^{w} + \left(1-\omega_{t,t+1}\right) V_{t+1}^{r}$ and the effective discount factor is $\beta_{t,t+1}^{w} = \beta$. A worker born in period j chooses consumption-saving allocation and leisure to maximize

$$V_t^{jw} = \left\{ \left[\left(C_t^{jw} \right)^{v} \left(1 - l_t^{jw} \right)^{1-v} \right]^{\rho} + \beta \left[\omega_{t,t+1} V_{t+1}^{jw} + \left(1-\omega_{t,t+1}\right) V_{t+1}^{jr} \right]^{\rho} \right\}^{\frac{1}{\rho}} \tag{3.7}$$

subject to

$$A_{t+1}^{jw} = R_t A_t^{jw} + W_t l_t^{jw} \left(1-\tau_t\right) - C_t^{jw}. \tag{3.8}$$

As retirees, workers consume out of non-human wealth and wage income net of labor income taxes. Workers and retirees in both countries consume a single (numeraire) good which can be traded internationally.

The first order condition with respect to labor is

$$l_t^{wj} = 1 - \frac{C_t^{wj} \varsigma}{W_t(1-\tau_t)}. \tag{3.9}$$

The consumption Euler equation for workers is

$$\begin{aligned} C_t^{jw} \left[\left(\frac{W_t(1-\tau_t)}{W_{t+1}(1-\tau_{t+1})} \right)^{\rho(1-v)} \beta R_{t+1} \Omega_{t+1} \right]^{\sigma} \\ = \omega_{t,t+1} C_{t+1}^{jw} + (1-\omega_{t,t+1}) \epsilon_{t+1}^{\frac{\sigma}{1-\sigma}} \chi C_{t+1}^{jr}, \end{aligned} \tag{3.10}$$

where $\chi = \xi^{-(1-v)}$ and $\Omega_t \equiv \omega_{t-1,t} + (1-\omega_{t-1,t}) \epsilon_t^{\frac{1}{1-\sigma}} \chi$ is an additional discount factor in the workers' value function. The worker's marginal propensity to consume out of wealth, π_t, evolves according to

$$\pi_t = 1 - \frac{\pi_t}{\pi_{t+1}} \left(\frac{W_t(1-\tau_t)}{W_{t+1}(1-\tau_{t+1})} \right)^{\rho\sigma(1-v)} \beta^{\sigma} (R_{t+1}\Omega_{t+1})^{\sigma-1}. \tag{3.11}$$

The worker's decision rule for consumption is

$$C_t^{jw} = \pi_t(R_t A_t^{jw} + H_t^{jw} + P_t^{jw}), \tag{3.12}$$

where H_t^{jw} is the present discounted value of a worker's human wealth net of taxation and P_t^{jw} is the present discounted value of a worker's pension benefits once retired.

3.3.1.3 Aggregation

Because marginal propensities to consume, both for retirees and workers, do not depend on individual characteristics, aggregate consumption can be expressed as

$$C_t = \pi_t A_t R_t(\epsilon_t \lambda_t + 1 - \lambda_t) + \pi_t(H_t^w + P_t^w) + \epsilon_t \pi_t(H_t^r + P_t^r) \tag{3.13}$$

where λ_t denotes the share of financial assets held by retirees such that $\lambda_t = \frac{A_t^r}{A_t}$. The asset distribution evolves according to

$$\begin{aligned} &\lambda_{t+1} \\ &= \omega_{t,t+1} \left(R_t \lambda_t \frac{A_t}{A_{t+1}} (1 - \epsilon_t \pi_t) + \frac{W(1-\tau_t) L_t^r \xi + S_t^r - \epsilon_t \pi_t (H_t^r + P_t^r)}{A_{t+1}} \right) \\ &+ (1 - \omega_{t,t+1}). \end{aligned} \tag{3.14}$$

The present discounted value of retirees' aggregate human wealth is

$$H_t^r = \xi L_t^r W_t(1-\tau_t) + \gamma_{t,t+1} \frac{\psi_t}{\psi_{t+1}} \frac{H_{t+1}^r}{(1+n_{t,t+1})R_{t+1}} \tag{3.15}$$

where ψ_t is the old age dependency ratio, *i.e.* $\psi_t = \frac{N_t^r}{N_t^w}$, and $n_{t,t+1}$ is the growth rate of the number of workers between periods t and $t+1$. The discount rate of the present value of total human wealth for current retirees is augmented by the growth rate of the retired labor force $\left(\frac{\psi_t}{\psi_{t+1}}\right)$. Similarly, the present discounted value of workers' aggregate human wealth is

$$\begin{aligned} H_t^w = L_t^w W_t(1-\tau_t) + \omega_{t,t+1} \frac{H_{t+1}^w}{(1+n_{t,t+1})R_{t+1}\Omega_{t+1}} \\ + (1-\omega_{t,t+1}) \frac{H_{t+1}^r \epsilon_{t+1}^{\frac{1}{1-\sigma}} \chi}{\psi_{t+1}(1+n_{t,t+1})R_{t+1}\Omega_{t+1}}. \end{aligned} \tag{3.16}$$

Present discounted value of retirees' aggregate pension benefits at time t is

$$P_t^r = S_t + \gamma_{t,t+1} \frac{\psi_t}{\psi_{t+1}} \frac{P_{t+1}^r}{R_{t+1}} \tag{3.17}$$

and the present discounted value of social security for workers is

$$\begin{aligned} P_t^w = \omega_{t,t+1} \frac{P_{t+1}^w}{(1+n_{t,t+1})R_{t+1}\Omega_{t+1}} \\ + (1-\omega_{t,t+1}) \frac{P_{t+1}^r \epsilon_{t+1}^{\frac{1}{1-\sigma}} \chi}{\psi_{t+1}(1+n_{t,t+1})R_{t+1}\Omega_{t+1}}. \end{aligned} \tag{3.18}$$

3.3.2 Firms

The goods market is competitive and the representative firm produces consumption goods with constant returns to scale Cobb-Douglas production technology. Aggregate output is given by $Y_t = (X_t L_t)^\alpha K_t^{1-\alpha}$, where X_t is the level of exogenous labor augmenting productivity at time t, L_t is the aggregate effective labor force, K_t is physical capital, and $\alpha \in (0, 1)$ is the labor share. The firm chooses labor, capital and investment to maximize the present discounted value of profits given by

$$V(I_{t-1}, K_t) = \left[(X_t L_t)^\alpha K_t^{1-\alpha} - W_t L_t - I_t + \frac{V(I_t, K_{t+1})}{R_{t+1}} \right] \tag{3.19}$$

subject to the law of motion of capital accumulation

$$K_{t+1} = (1-\delta) K_t + \left[1 - \frac{\phi}{2} \left(\frac{I_t}{I_{t-1}} - \mu_t \right)^2 \right] I_t. \tag{3.20}$$

The aggregate effective labor force consists of the effective labor input by the two agent types such that $L_t = L_t^w + \xi L_t^r$. Capital depreciates at rate $\delta \in (0, 1)$ and quadratic adjustment cost makes investing new capital costly. The size of the adjustment cost is determined by $\phi > 0$. The term μ_t ensures that along the balanced growth path, investment adjustment costs equal zero. Productivity X_t grows at rate x_t, which follows an AR(1) process given by $x_t = (1-\theta)x_{ss} + \theta x_{t-1} + u_t^x$.

3.3.3 Government

Government consumes G_t in each period and pays retirees a total amount of P_t of social security benefits. The expenditures are financed with tax revenues T_t and by issuing one-period debt B_{t+1}. Government spending, social security and fiscal policy are assumed to be exogenously determined fractions of output: $G_t = \bar{g}_t Y_t$, $P_t = \bar{p}_t Y_t$ and $B_t = \bar{b}_t Y_{t-1}$. The government's per period budget constraint is $B_{t+1} = R_t B_t + G_t + P_t - T_t$ and the intertemporal budget constraint is

$$R_t B_t = \sum_{v=0}^{\infty} \frac{T_{t+v}}{\prod_{z=1}^{v} R_{t+z}} - \sum_{v=0}^{\infty} \frac{G_{t+v}}{\prod_{z=1}^{v} R_{t+z}} - \sum_{v=0}^{\infty} \frac{P_{t+v}}{\prod_{z=1}^{v} R_{t+z}} \ . \tag{3.21}$$

The government's per period discount rate equals the world interest rate R_t and is lower than the households' per period discount rates. With elastic labor supply, total tax revenue is

$$T_t = \tau_t W_t L_t. \tag{3.22}$$

Labor tax τ_t is adjusted according to the fiscal policy rule

$$\tau_t = \tau_{t-1} + \theta \left[b_t - \bar{b}_t \right] \tag{3.23}$$

as in Leeper (1991).

3.3.4 A competitive world equilibrium and the external sector

A competitive world equilibrium is a sequence of quantities and prices such that in each country (i) households maximize utility subject to their budget constraints, (ii) firms maximize profits subject to their technology constraints, (iii) the government chooses a path for taxes and debt, compatible with intertemporal solvency, to finance an exogenous level of total spending, and (iv) all markets clear.

In each economy, households' aggregate non-human wealth equals the aggregate capital stock, government bonds and net foreign assets F_t such that

$$A_t = K_t + B_t + F_t. \tag{3.24}$$

Net foreign asset position evolves according to

$$F_{t+1} = R_t F_t + NX_t \tag{3.25}$$

and the trade balance NX_t is determined by the aggregate resource constraint

$$NX_t = Y_t - C_t - I_t - G_t. \tag{3.26}$$

The current account CA_t consists of the trade balance and net interest payments on foreign assets

$$CA_t = NX_t + (R_t - 1) F_t. \tag{3.27}$$

Return R_t is equalized across the two countries and in equilibrium, foreign asset positions in the two countries sum up to 0, *i.e.* $F_t + F_t^* = 0$ so that the internationally traded asset is in zero net supply. The law of one price is assumed to hold.

3.4 Quantitative analysis

In this section I analyze the drivers of the bilateral external imbalances between US and China during 1980-2015. During this period, China and the US experienced persistent external imbalances both multilaterally and vis-a-vis each other, and accumulated large foreign asset positions of opposite signs. The period coincides with China's transition process towards a market economy, and the features which shaped the Chinese planned economy, especially in the beginning of the period, are not explicitly modeled. China engaged actively in international trade from the beginning of its transition process in 1978, so that the share of exports and imports of GDP grew from approximately 10 % of GDP in 1978 to over 40 % by 1993 (see figure 3.2), and by 2015, it had become the world's largest merchandise trader. Given China's external trade openness during the entire period, it is of interest to study the dynamics of the trade balance since the early 1980s, and a more detailed analysis of the effects of the political system is left for future research.

In what follows, I introduce potential drivers of the external imbalances as exogenous variables into the two-country life-cycle model presented in section 3, and simulate the transition dynamics of the model during 1980-2015. The potential drivers include demographic changes, differences in pension systems, fiscal policy and productivity growth differentials. The dynamic transition of the model economy is simulated in two different ways, first with a standard deterministic simulation and then with a deterministic simulation with updates, in which the paths of exogenous variables are updated in every period. In this case, the agents revise their expectations according to the latest available information. The method is also known as extended path and was originally

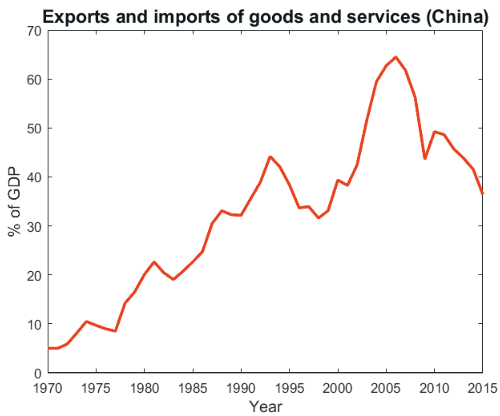

Figure 3.2: Merchandise trade (the sum of merchandise exports and imports) in China as a share of GDP. Source: World Bank World Development Indicators.

proposed by Fair and Taylor (1983).

Section 3.4.1 discusses the evolution of the exogenous variables in US and China between 1980 and 2015. Section 3.4.2 presents the calibration of the model and a comparative static analysis. Section 3.4.3 presents the results of a deterministic simulation and section 3.4.4 a deterministic simulation with updates. Section 3.4.5 discusses the role of the labor supply in the model.

3.4.1 The US and China from 1980 to 2015: demographics, social security, fiscal policy and technological progress

The change in the countries' demographic structures is the largest long-term trend included in the simulation. During the simulation period, both countries experienced a fall in population growth rates and an increase in life expectancy, which has led to an increase of the old-age dependency ratio (the ratio of population aged 65+ to population aged 20-64 years). This trend is projected to continue for several more decades (see figures 3.3 and 3.4). However, both the demographic structures and the pace of change have differed between the countries. In 1980, there were almost twice as many elderly in China for every working-age individual as in the US, but towards the end of the 20th century, the difference in old age dependency ratios declined. This was mainly due to the relatively rapid growth in life expectancy in China, which grew by 8 years (from 67.3 to 75.4 years) between 1980 and 2015, in comparison to 6 years (from 72.9 to 78.9 years) in the US. The convergence of the old-age dependency ratios was dampened because the Chinese population growth rate remained high in relation to the US over the entire period, despite its rapid decline amplified by the introduction of the one-child policy in 1979. In the simulations, the population growth rate and retirement and survival probabilities are adjusted so that the average lengths of time spent as a worker and as a retiree, the population growth rate and the old age dependency ratio match the average values observed in the data.

Based on old United Nations population forecast revisions, the demographic transition has not been fully anticipated. Estimation and forecasting methodology have evolved over time and new census data has become available, resulting in changes in the forecasts. The forecasts of life expectancy at birth both in China and the US have been revised upwards in nearly every revision round (see figure 3.5), and the projections of population growth rates have been revised upwards for the US and downwards for China several times each (see figure 3.6). The time horizon of the demographic projections has also increased over time. Until 1994, the projections were made until 2025, in the years between 1994 and 2008 until 2050, and since 2010 they have been made until 2100. In the simulation with updates, the paths of exogenous demographic variables are adjusted in each period so that the demographic variables in the model match the values of the most recent United Nations World Population Prospect data.

Differences in the level of social security expenditures are also included in the simulation as potential drivers of the external imbalances. In recent years,

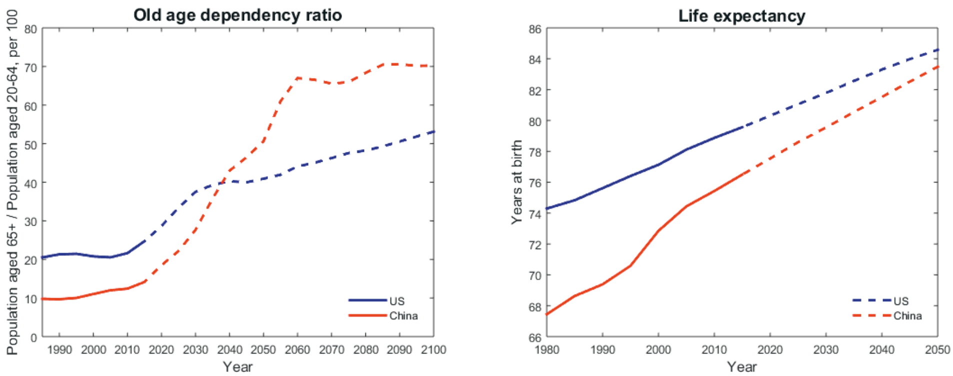

Figure 3.3: Left panel: Old age dependency ratio in the US and China (data frequency: 5 years) between 1985-2015 and projections until 2100. Right panel: Life expectancy at birth in the US and China in 1980-2015 and projections (medium variant) until 2050. Source: United Nations World Population Prospects: The 2015 Revision.

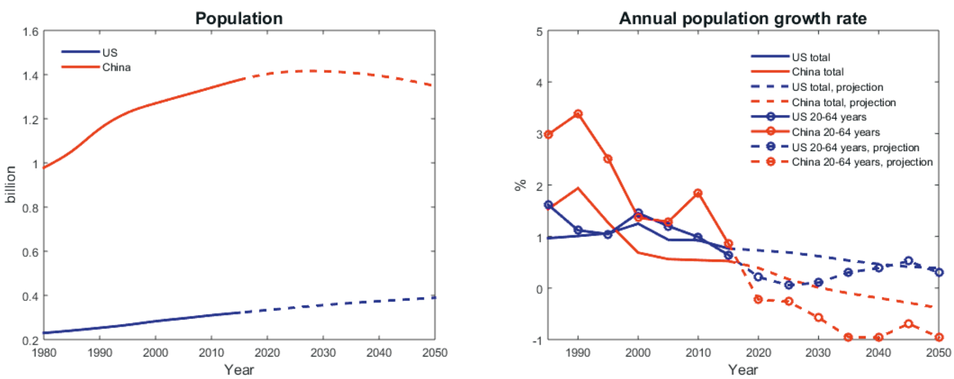

Figure 3.4: Left panel: Total population in the US and China in 1980-2015 and projections until 2050. Right panel: The annual population growth rate in the given year and 5 preceding years 1985-2015 and projections until 2050. Source: United Nations World Population Prospects: The 2015 Revision.

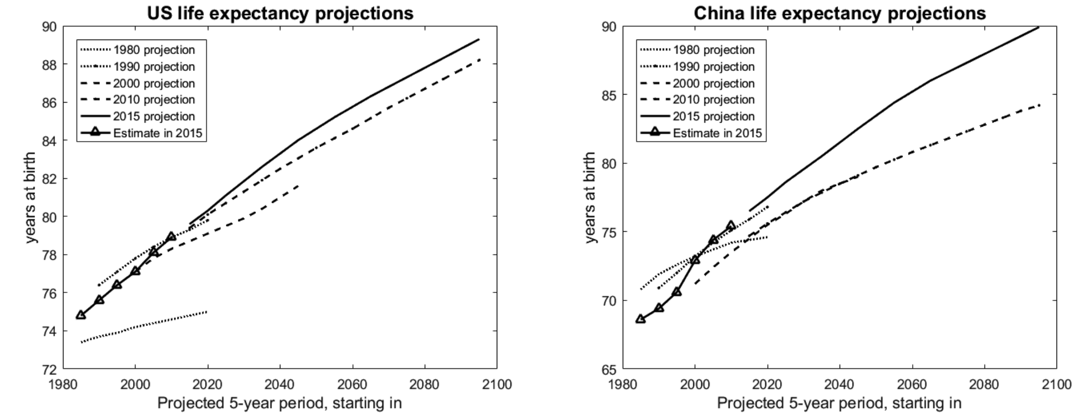

Figure 3.5: Left panel: Life expectancy at birth, US. Right panel: Life expectancy at birth, China. Source: United Nations World Population Prospects: 1980-2015 Revisions.

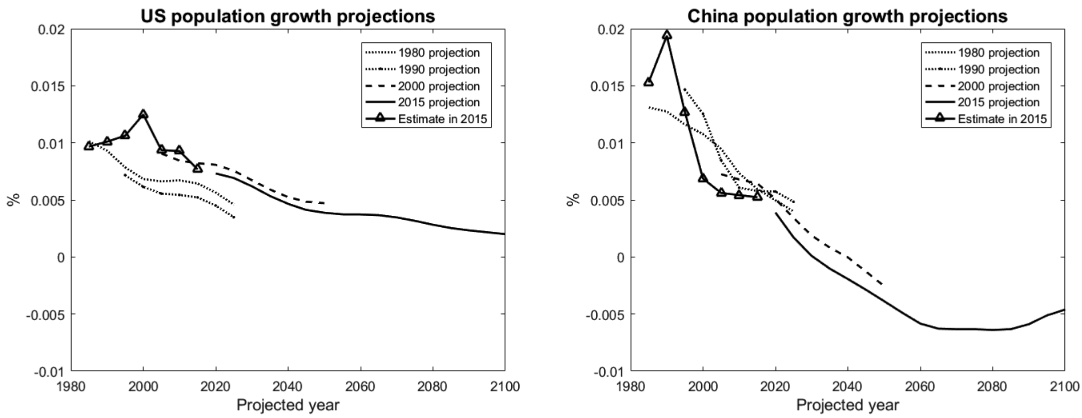

Figure 3.6: Left panel: Population growth rate, US. Right panel: Population growth rate, China. Source: United Nations World Population Prospects: 1980-2015 Revisions.

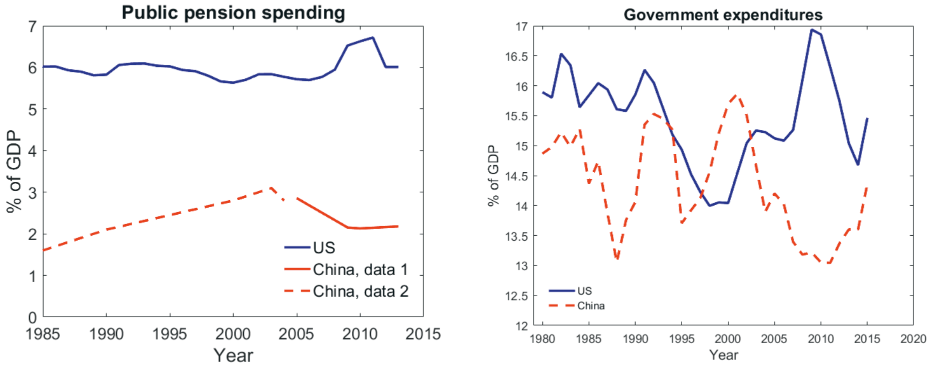

Figure 3.7: Left panel: Public pension expenditures, % of GDP in the US and China. Source: OECD Data on Social Protection and Asian Development Bank: Social Protection Index Database (data 1); Naughton (2007) (data 2). Right panel: General government final consumption expenditures. Source: World Bank, World Development Indicators 2016.

public pension spending in China as a share of GDP has been only one-third that of the US (2.1 % versus 6.6 %; see figure 3.7). The GDP share of public social security expenditures has not changed substantially in either of the countries during the time for which the data is available. China's low GDP share of pension spending is not explained by its younger population structure alone. Had the pension spending been proportional to the old-age dependency ratio, the social security expenditures in China would have needed to be 3.3-4.0 % of GDP to match the US level (see figure 3.3). In addition, the coverage of the pension scheme, defined as the share of population aged 15-65 covered by mandatory pension schemes, is only approximately 27.7 % in China (2010), compared to 71.4 % in the US (2005) (OECD, 2013). In the simulations I run, the pension system is assumed to have full coverage, which is likely to yield conservative results with regard to the impact of social security on households' savings. Assuming heterogeneity in the coverage rate would imply that in China, the vast majority of the population would have no pension income at all. Despite several differences between the Chinese and American public pension systems, they can both be classified as pay-as-you-go based on the 3-pillar classification of pension systems by Pallares-Miralles et al. (2012).[3]

[3]See Appendix C for a description of the old-age pension systems in China and the US and the

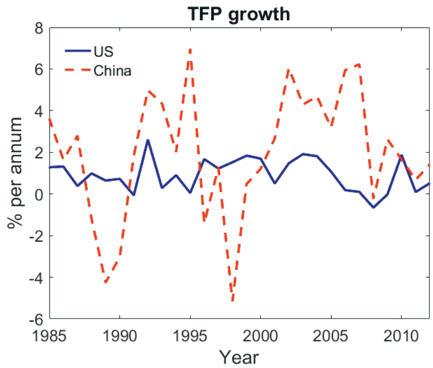

Figure 3.8: Annual TFP growth in the US and China 1980-2012. Source: Penn World Table.

The simulation also takes into account the differences in government expenditures and their impact on the external imbalances. Government expenditures have been higher in the US (on average 15.5 % of GDP) than in China (on average 14.4% of GDP) for most of the simulation period, as shown in figure 3.7. Data for net government debt is not available for China, and therefore government debt is assumed to evolve according to the same fiscal rule in both countries, and the permanent level of government debt is assumed to be 20 % of GDP in both countries.

Finally, productivity growth is included in the simulations as a potential driver of the external imbalances. The counterfactual implications about trade surplus and net foreign debt that several neoclassical models imply for China in the earlier literature (including Ferrero, 2010) are driven by its high productivity growth during most of the simulation period. Figure 3.8 shows the TFP growth rate for China and the US. On average, TFP grew by 1.9 % in China and by 0.9 % in the US every year between 1980 and 2012.

3.4.2 Calibration and the steady state

To analyze the dynamics of the external imbalances between China and the US between 1980 and 2015, the time-invariant model parameters are calibrated to match the means of the GDP shares of the trade balance, consumption and

classification of the pension systems by Pallares-Miralles et al. (2012).

investment, and the mean of the world real interest rate, during the simulation period. The model's time-varying exogenous variables, which include life expectancy, population growth rate, pension expenditures, other government expenditures, and TFP, are set to match the values directly observable in the data.

Table 3.1 presents the values of time-invariant parameters, which are the same in both simulations and for both countries. The labor share of output, α, equals 2/3 and follows the convention in the literature (including Cooley (1995), Gertler (1999), Fujiwara and Teranishi (2008) and Ferrero (2010)).[4] The discount factor, β, equals 0.994 so that the net real interest rate matches the mean value of 3 % over the sample period in the data. Consumption share in utility, v, and the relative productivity of retirees, ξ, are chosen to match employment rates among population aged 15-64 and 65 and over. The value for the consumption share in utility falls between the values used in related papers including Gertler (1999) ($\nu = 0.4$) and Kilponen et al. (2006) ($\nu = 0.8$). There is no consensus in the literature about the value of the elasticity of intertemporal substitution, σ, the values ranging from 0.1 (Hall (1988)) to 2 (Gruber (2013)). A recent meta-study by Havranek et al. (2015) reports that, in a sample of 169 studies, the mean estimate for the elasticity of intertemporal substitution is 0.594 for the US and 0.530 for China; the value chosen here lies in this range. Depreciation rate, δ, and the investment adjustment cost factor, ϕ, are chosen so as to match the mean of the investment-to-output ratio over the sample period.

[4] According to the International Labour Organization (2015), the labor share of output declined both in the US and in China over the past decades. Bai et al. (2006) estimate the labor share of output to be 50-54 % in China. Analyzing the impact of different production technologies is out of the scope of this paper.

Parameter		Value
α	labor share of output	$\frac{2}{3}$
β	discount factor	0.994
δ	depreciation rate	0.2
ϕ	investment adjustment cost factor	3
σ	elasticity of intertemporal substitution	0.55
ξ	productivity of a unit of labor, retiree to worker	0.5
v	elasticity of period utility with respect to consumption	0.7

Table 3.1: Values of the time-invariant parameters in the simulation. The parameters are uniform across the countries.

Table 3.2 presents the values of exogenously determined variables related to demographics, fiscal policy, pensions and TFP in the initial and final states. The dynamics are solved with two different simulation methods (with and without updates on the exogenous variables), and the table presents the values of exogenous variables for both simulations.

In the first simulation (I), the model is calibrated so that in the initial state, the economies match the data in 1980, and converge to a new steady state in which they match the data in 2015. In the initial and final states, productivity and population growth rates (x and n^w) are the same across the countries to prevent one of the economies from outgrowing the other. The annual productivity growth rate is 1 % in both states and economies, which is the average of the observed growth rates between the countries between 1980 and 2015. The annual growth rate of working-age population decreases gradually from 2.15 % in 1980 to 0.5 % in 2015 and thereafter. The probability of staying in the labor force, ω, is the same in the initial and final state, and it is slightly higher in the US than in China ($\omega_{US} = 0.9777$ and $\omega_{China} = 0.9769$), corresponding to an (effective) age of retirement of 63 years in China and and 65 years in the US. Life expectancy, captured by the survival probability γ, differs between the countries in both states. It corresponds to the observed values of 72.9 years (US) and 67.3 years (China) in 1980 and 78.9 years (US) and 75.4 years (China) 2015, respectively. Government spending and public pension spending are permanently at different levels, matching the average in the data between 1980-2015. Government debt in both countries is 20 % of GDP (recall that no data on Chinese government net debt is available).

In the second simulation (II), the model is calibrated so that in the initial state, the economies match the 1980 data and converge to a new steady state. However, the agents' information set regarding the values of exogenous variables during the transition and at the terminal point, as well as the date when the terminal point is reached, is updated in each period. In particular, the agents' information set regarding demographic variables consists of the most recent UN population forecast, so that as each new UN population forecast becomes available, the agents' information set is updated. In the final step of the simulation, the final steady state values match the predicted values for 2100 according to the 2015 UN forecast. The annual steady state growth rate of working-age population decreases gradually from 2.15 % in 1980 to 0 % in the final steady state, which is the average population growth rate projected for the countries for 2100. Life expectancy increases from 72.9 years (US) and 67.3 years (China) in 1980 to 88.7 years (US) and 85.2 years (China) in 2100. Other exogenous variables are as in the first simulation.

		Data	Model			
			I		II	
Exogenous variable			**Initial state 1980**	**Final steady state 2015**	**Initial state 1980**	**Final steady state**
x_1, x_2	technology growth rate (US, China)	0.01 (mean between 1980 and 2015 China and the US)	0.01	0.01	0.01	0.01
n_1, n_2	population growth rate (US, China)	1980: 0.0215 (mean between China and the US) 2050: 0.005 (projected mean in 2015) 2100: 0.00 (projected mean in 2015)	0.0215	0.005	0.0215	0.00
γ_1	probability to survive (if retired) / life expectancy at birth (US)	1980: 72.9 years (estimate in 1980) 2015: 78.9 years (estimate in 2015) 2100: 88.7 years in 2100 (projection in 2015)	0.8758 / 72.9	0.9289 / 78.9	0.8758 / 72.9	0.9580 / 88.7
γ_2	probability to survive (if retired) / life expectancy at birth (China)	1980: 67.3 years (estimate in 1980) 2015: 75.4 years (estimate in 2015) 2100: 85.2 years (projection in 2015)	0.7506/ 67.3	0.9174/ 75.4	0.7506 / 67.3	0.9543 / 85.2
ω_1	probability to stay in the labor force / effective retirement age (US)	65 years	0.9777 / 65	0.9777 / 65	0.9777 / 65	0.9777 / 65
ω_2	probability to stay in the labor force / effective retirement age (China)	63 years	0.9769 / 63	0.9769 / 63	0.9769 / 63	0.9769 / 63
b_1	government debt, % of GDP (US)		0.2	0.2	0.2	0.2
b_2	government debt, % of GDP (China)		0.2	0.2	0.2	0.2
g_1	government spending, % of GDP (US)	0.155	0.155	0.155	0.155	0.155
g_2	government spending, % of GDP (China)	0.144	0.144	0.144	0.144	0.144
s_1	social security spending, % of GDP (US)	0.066	0.060	0.070	0.060	0.060
s_2	social security spending, % of GDP (China)	0.021	0.021	0.021	0.020	0.020

Table 3.2: Values of exogenous variables (technological growth rate, demographic variables, government debt and expenditures, and social security expenditures) in the initial state (*i.e.* in 1980), the final state (*i.e.* in 2015 in the deterministic simulation and in 2100 in the simulation with updates, which is the last year of the United Nation's population forecast) and over time (mean in the period 1980-2015) in the data and the model. (I) Deterministic simulation (II) Simulation with updates.

Variable		Value in the model (I) *mean* *min / max* *st. dev.*	Value in the model (II) *mean* *min / max* *st. dev.*	Value in data *mean* *min / max* *st. dev.*	Data source
R	World real interest rate	1.03 0.99/1.07 0.02	1.02 1.01/1.04 0.01	1.03 1.00/1.08 0.02	Nominal interest rate: IMF, International Financial Statistics. CPI inflation: IMF, World Economic Outlook 2015.
$\frac{NX_1}{Y_1}$	Bilateral trade balance / output (US)	-0.01 -0.07/0.05 0.03	-0.02 -0.05/0.01 0.01	-0.01 -0.02/-0.01 0.00	Bureau of Economic Analysis (BEA), U.S. Department of Commerce, author's calculation. Period: 1999-2015.
$\frac{NX_2}{Y_2}$	Bilateral trade balance / output (China)	0.00 -0.03/0.03 0.02	0.02 -0.01/0.03 0.01	0.06 0.03/0.09 0.02	Bureau of Economic Analysis (BEA), U.S. Department of Commerce, author's calculation. Period: 1999-2015.
$\frac{NX_1}{Y_1}$	Multilateral trade balance / output (US)	-	-	-0.03 -0.06/-0.00 0.02	World Bank, World Development indicators.
$\frac{NX_2}{Y_2}$	Multilateral trade balance / output (China)	-	-	0.02 0.02 0.03	World Bank, World Development indicators.
$\frac{F_1}{Y_1}$	Net foreign assets / output (US)	-0.16 -0.37/0.00 0.09	-0.39 -0.83/-0.01 0.26	-0.38 -1.72/1.57 0.79	IMF Balance of Payment Statistics and World Economic Outlook
$\frac{F_2}{Y_2}$	Net foreign assets / output (China)	0.11 -0.00/0.18 0.06	0.23 0.01/0.41 0.13	0.09 0.02/0.12 0.03	IMF Balance of Payment Statistics and World Economic Outlook. Period: 2003-2015.
$\frac{C_1}{Y_1}$	Consumption / output (US)	0.53 0.50/0.57 0.02	0.53 0.51/0.56 0.01	0.65 0.60/0.69 0.02	World Bank, World Development indicators.
$\frac{C_2}{Y_2}$	Consumption / output (China)	0.50 0.47/0.53 0.02	0.48 0.47/0.50 0.01	0.44 0.34/0.53 0.06	World Bank, World Development indicators.
$\frac{I_1}{Y_1}$	Investment / output (US)	0.33 0.30/0.34 0.01	0.34 0.31/0.35 0.01	0.22 0.18/0.25 0.02	IMF, World Economic Outlook 2015.
$\frac{I_2}{Y_2}$	Investment / output (China)	0.35 0.33/0.37 0.01	0.36 0.34/0.38 0.01	0.40 0.32/0.48 0.05	IMF, World Economic Outlook 2015.

Table 3.3: Comparison of the key moments (mean, minimum and maximum values and standard deviation) of variables in the data and in the model during the simulation period (1980-2015). (I) Deterministic simulation (II) Simulation with updates.

3.4.2.1 Steady state implications of the model

Throughout the simulation period, China has had a younger population and lower level of social security expenditures than the US. This section presents a comparative static analysis around the steady state to illustrate the effects of life expectancy and social security in the model.

Figure 3.9 shows the steady state effects of life expectancy. The economies are assumed to be in a steady state in which the values of exogenous variables match the average between the countries in the data in 2015, apart from γ, the parameter which determines life expectancy, which is allowed to differ. In the US, γ_1 is fixed at 0.9319, which corresponds to a life expectancy of 79.5 years, and in China, γ_2 varies between 0.92 and 0.9319. At the dotted vertical line the life expectancy in China matches the projected value in 2020 ($\gamma_2 = 0.9247$ corresponding to a life expectancy of 76.5 years), when the difference in life expectancies between the countries is 3 years.

The model predicts that the country with higher life expectancy (the aged country) accumulates a positive net foreign asset position and satisfies its higher consumption demand by importing goods from abroad, running a permanent trade deficit. High life expectancy increases the households' motive to save for retirement, and therefore the households hold more financial wealth in the aged country. Because the world interest rate is below the autarky rate of the country in which life expectancy is lower (the young country), it is willing to borrow from abroad, and so the household wealth of the aged country is invested partly in foreign assets. Even though the aged country is running a permanent trade deficit, it earns a high enough interest rate on its foreign assets to be able to hold a permanent positive net foreign asset position. The marginal propensities to consume (henceforth, mpc's) are lower in the aged country, but its consumption is high because its residents are wealthier and a higher share of non-human wealth is held by retirees, whose mpc is higher than the workers'.

The observed difference in life expectancies between China and the US in 2020, 3 years, implies a positive trade balance of 0.002 % of GDP and a negative net foreign assets position of 4 % of GDP for China in the steady state. The higher the life expectancy in China (the larger γ_2), the smaller its trade surplus and net foreign debt. Also, the higher the life expectancy in China, the lower the world real interest rate, as an increase in life expectancy raises household savings and the aggregate level of financial wealth in the world. The effect is qualitatively similar to that documented by Ferrero (2010).

Figure 3.10 presents the comparative statics with public pension spending. As before, the economies are assumed to be identical so that the values of exogenous variables correspond to the average of observed levels in 2015 apart from social security, which is allowed to differ between the countries. Public pensions spending in the US is set at 6 % of GDP, and varies between 2 and 6 % in China.

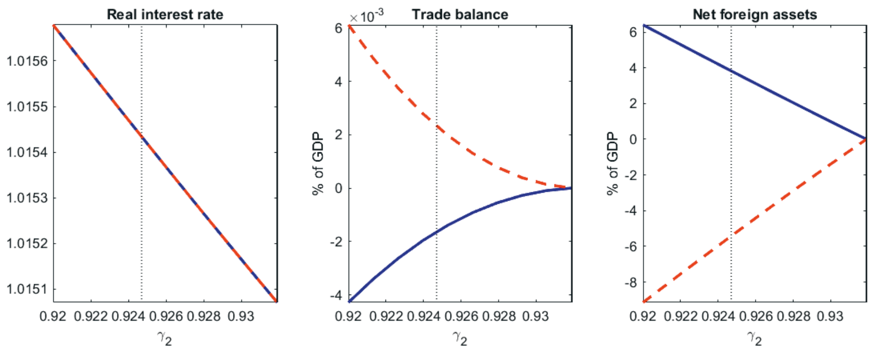

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Figure 3.9: Steady states for $0.92 \leq \gamma_2 \leq 0.9319$ and $\gamma_1 = 0.9319$ (blue line = US and red dotted line = China). At the leftmost vertical line, $\gamma_2 = 0.9247$, which corresponds to the projected life expectancy in China in 2020.

The model predicts that, in the steady state, the country with lower social security expenditures holds a positive net foreign asset position and runs a trade deficit. Because the retirees in that country have lower pension income, the households accumulate more assets before retirement, which pushes down the world real interest rate. The world interest rate is below the autarky rate of the country with higher social security expenditures, which causes capital to flow from the country with low social security (China) to the country with high social security (US). Due to the lower pension expenditures, the country with low pensions can also maintain a lower tax rate, which leads to higher labor supply and level of human wealth, and because of its relatively high human and financial wealth, its aggregate consumption is higher. High consumption demand is partly satisfied with foreign imports, which are financed with interest payments on the net foreign assets held by the country. The effect of social security on external imbalances is quantitatively significant, and it seems likely that social security can be an important factor in understanding the high level of savings in China. The observed 4 % difference in the share of GDP that is spent on public pensions implies a trade deficit of 1.5 % of GDP and a positive net foreign assets position of 70 % of GDP for China in the steady state. The higher the social security spending in China, the smaller is its net foreign asset position and the larger its trade surplus in the steady state.

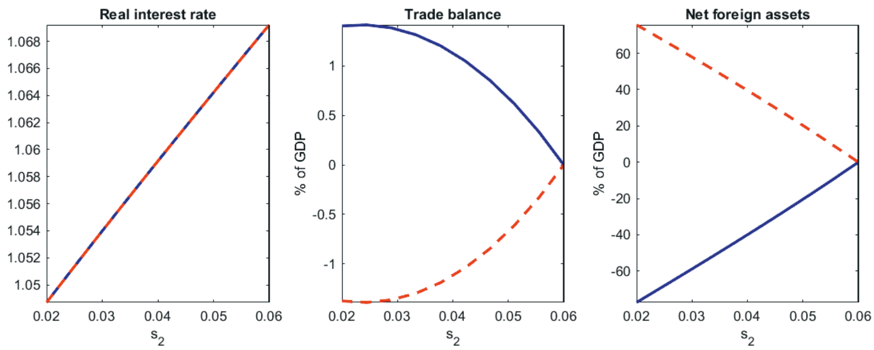

Figure 3.10: Steady states for $0.02 \leq s_2 \leq 0.06$ and $s_1 = 0.06$ (blue line = US and red dotted line = China).

Other features that distinguish the economies are the relatively low effective retirement age and low level of government expenditure in China. Low retirement age predicts a negative net foreign asset position for the US in the steady state. Low government expenditures imply the opposite, namely a positive net foreign asset position in the steady state. In comparison to the effects of life expectancy and social security, however, the effects of retirement age and government expenditure on the steady state variables are relatively small.

3.4.3 Deterministic simulation

The steady state analysis suggests that the low level of social security expenditures can help to explain China's trade surplus vis-a-vis the US. Given that life expectancy has increased more quickly in China than in the US, the steady state analysis also suggests that China's net foreign asset position has improved, implying that its trade balance has been elevated during the period of the demographic transition.

In this subsection, we run a deterministic simulation to analyze the dynamic effects of the demographic transition, social security, government expenditures and TFP fluctuations on the trade balance and net foreign asset position of the two countries. The economies are assumed to initially be in a state in which the exogenous variables match the data in 1980, and then converge to a new steady state, in which the the exogenous variables match the data in 2015. During the transition, the exogenous variables follow the path observed in the data.[5] The approach we follow is sequential: first we introduce demographic transition, while keeping all other factors constant; then fiscal policy shocks and differences in public social security spending; and finally, temporary fluctuations in TFP growth rates.

3.4.3.1 The dynamic effects of demographic transition

The growth rate of population aged 15 to 64 fell from 2.9 to 0.2 % in China, and from 1.4 to 0.5 % in the US, between 1980 and 2015. Life expectancy increased from 67.4 to 75.4 years in China, and from 74.3 to 78.9 years in the US over the same period (United Nations (2015)). In the model, we are able to match the population growth rates during the period 1980-2015 exactly (see the data in figure 3.4), but need to assume a faster increase in life expectancy than in the data to avoid numerical problems that would otherwise arise in the model solution algorithm. In the initial and final states, population growth rate is assumed to be the average of the observed growth rates (2.15 % in 1980 and 0.5 % 2015) between the countries.

The results of the dynamic simulation with demographic transition are shown in figures 3.11 and 3.12. Consistent with the steady state results, higher retirement age and life expectancy together imply a positive net foreign asset position and negative trade balance for the US in the initial state. The size of the net foreign asset position is approximately 63 % of GDP and the trade deficit is 1 % of GDP for the US. In the initial state, the US households hold more financial wealth than the Chinese households because of higher life expectancy, and the US net foreign asset position is therefore positive. China holds a negative foreign asset position and runs a trade surplus.

After the initial state, the economy faces transitory and permanent changes to the demographic variables as described above. As life expectancy increases permanently in both countries, the mpc's of and consumption by both retirees and workers initially fall, as the households increase their savings because of the longer time they expect to spend in retirement. The mpc's decline more in China because the increase in Chinese households' life expectancy is larger. As savings increase, the real interest rate falls, and investment rises as the economies start to adjust to a new steady state with higher capital stock. The net effect of the demographic shock on the trade balance is positive in China, because the increase in savings exceeds the rise in investment, and negative in

[5] The values of g_1, g_2, s_1, s_2, nw_1, nw_2, x_1 and x_2 match exactly the values in the data, plotted in figures 3.4, 3.7 and 3.8. The life expectancy parameters, γ_1 and γ_2, converge faster than in the data to obtain model convergence.

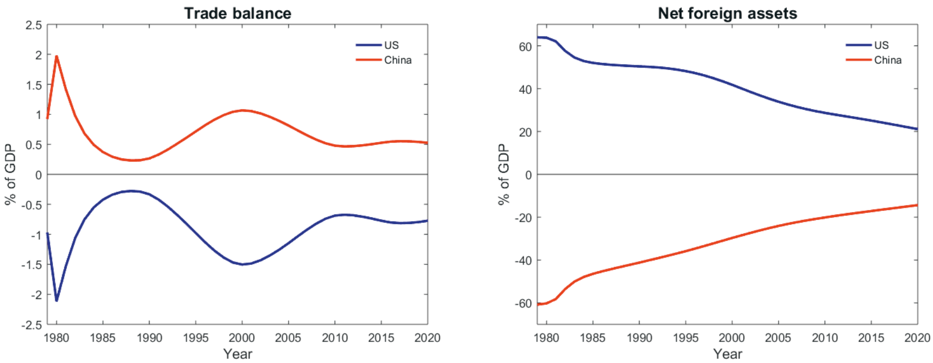

Figure 3.11: Left panel: Effects of a permanent demographic change on the trade balance. Right panel: Effects of a permanent demographic change on the net foreign asset position.

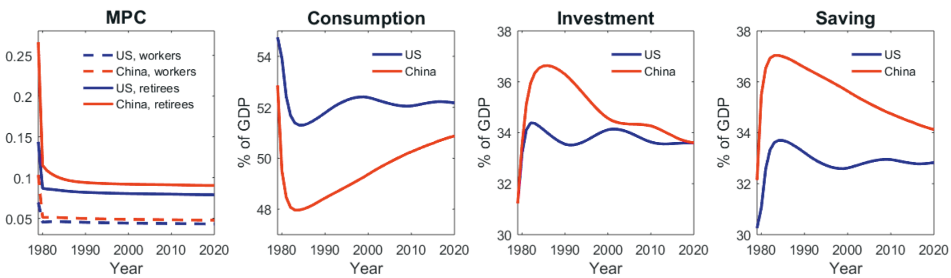

Figure 3.12: Effects of demographic changes on the marginal propensities to consume and the GDP shares of consumption, investment and saving.

the US, where the effect on investments is larger. The reason is that the increase in Chinese savings pushes the real interest rate below the autarky level of the US interest rate, and therefore the change in investments exceeds the change in domestic savings caused by the aging of its domestic population. As the US trade balance weakens, and as the fall in the interest rate causes the returns on its foreign assets to fall, its stock of foreign assets starts to decline, and China's stock of foreign assets begins to grow.

As the age structures between the countries become more similar, the external imbalances gradually become smaller. As the capital stock approaches the new steady state level, investments decline, with a positive impact on the trade balance both in the US and in China. However, the trade surplus of China becomes smaller because of the simultaneous rise in the aggregate consumption share. The rise in consumption share is driven by rising consumption among the retirees, which is in turn caused by the increase in aggregate wealth held by the retirees, and their rising share in the population, given that their marginal propensities to consume are higher than the workers'.

The fluctuations in the simulated trade balance are caused by population growth fluctuations during the period. In periods of relatively high population growth in China, investments need to increase to maintain an equal rate of return across countries, which temporarily weakens the trade balance. Overall, the relatively rapid population ageing in China causes a slow and continuous decline in the net foreign asset position of the US, which resembles the downward trend observed in the data.

3.4.3.2 The effects of social security and fiscal policy

Between 1980 and 2015, the output shares of social security expenditures and general government expenditures were higher in the US than in China over nearly every period (see figure 3.7). In this section, we analyze the dynamics of the trade balance, taking into account government expenditures and social security spending, in addition to the demographic transition described in the previous section. Public pension spending is assumed to grow slowly from 6 % to 7 % of GDP in the US, and to remain at 2.1 % of GDP in China over the simulation period. The GDP share of general government expenditures exactly matches the data, and we assume it to be 15.5 % in the US and 14.4 % in China in the initial and final states. Government debt as a share of GDP is endogenously determined by the fiscal rule (equation (3.23)), and we assume it to be 20 % in the initial and final states in both countries, as no data on Chinese government net debt is available.

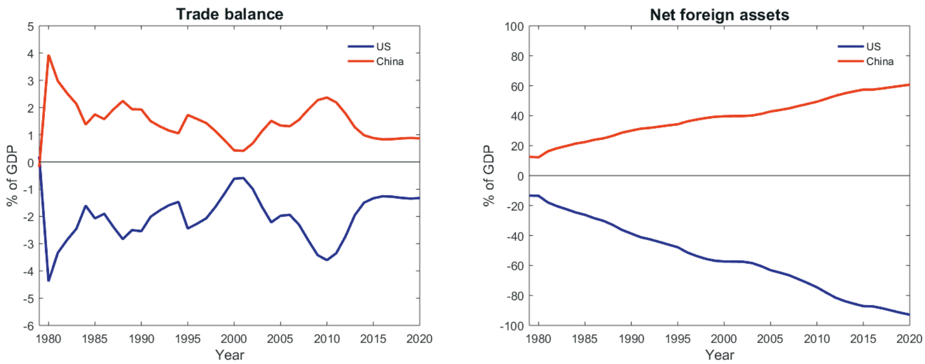

Figure 3.13: Effects of a permanent demographic changes with social security and fiscal policy on the trade balance (left panel) and on the net foreign asset position (right panel).

The results of the dynamic simulation are shown in figure 3.13. The effect of public pension expenditures on the external imbalances is noticeable. Consistent with the steady state results, high public pension expenditures in the US have a negative effect on its initial net foreign asset position: whereas the demographic factors would predict the US to hold a positive net foreign asset position of approximately 60 % of GDP (see figure 3.12), when differences in social security expenditures are accounted for, the initial net foreign asset position is negative (figure 3.13). Despite life expectancy being lower in China, low pensions raise the aggregate level of non-human wealth in the economy, resulting in a positive net foreign asset position in the initial state.

Relatively high government expenditures have a small negative effect on the US' net foreign asset position in the steady state. High government expenditures increase the tax rate, lowering the labor income of employees, which crowds out private consumption and savings. The impact on steady state trade balance is positive because of the negative wealth effect on consumption, but the net foreign asset position is weaker because of lower savings.

The dynamics of the trade balance and the net foreign assets are mainly driven by the demographic changes, which cause a large increase in Chinese savings and result in an increase in its net foreign asset position vis-a-vis the US. When the differences in social security and government expenditures are also accounted for, the model dynamics qualitatively match the data well.

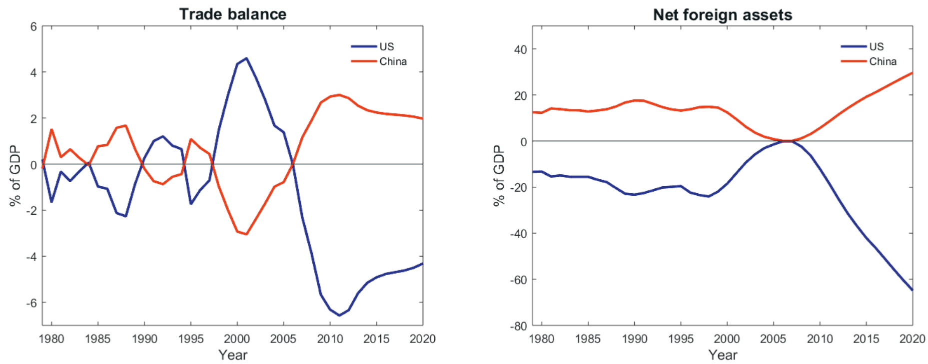

Figure 3.14: Effects of a permanent demographic changes, social security, government expenditures and TFP growth on the trade balance (left panel) and on the net foreign asset position (right panel).

3.4.3.3 The effects of productivity growth fluctuations

During the period 1980-2015, the average annual TFP growth rate was 1.9 % in China and 0.9 % in the US. In this section, we introduce temporary TFP fluctuations, which match the data exactly (see figure 3.8), in addition to demographic, social security and government expenditure shocks. In the initial and final states, productivity growth rate is assumed to be constant at 1 %.

Figure 3.14 reports the resulting dynamics. As expected, TFP shocks have a strong impact on the trade balance. Periods of relatively high productivity growth in China raise investment and consumption, which weaken its trade balance and the net foreign asset position. However, due to the underlying demographic trends and differences in social security and government expenditures between China and the US, the trend in the Chinese net foreign asset position is increasing. Indeed, only during the period of high TFP growth in China in the mid-2000s does the model counterfactually predict the net foreign asset position to decline to zero. For most of the periods in the sample years, the model predicts a positive trade balance for China.

Figure 3.15 shows the simulation decomposed into the effects of demographic transition and the marginal effects of pensions, fiscal policy and TFP. Even though the negative impact of TFP shocks weakens the US trade balance, especially in the early 2000s, the opposite effects of demographic factors, social security and fiscal policy predict a trade deficit for the US, especially at the

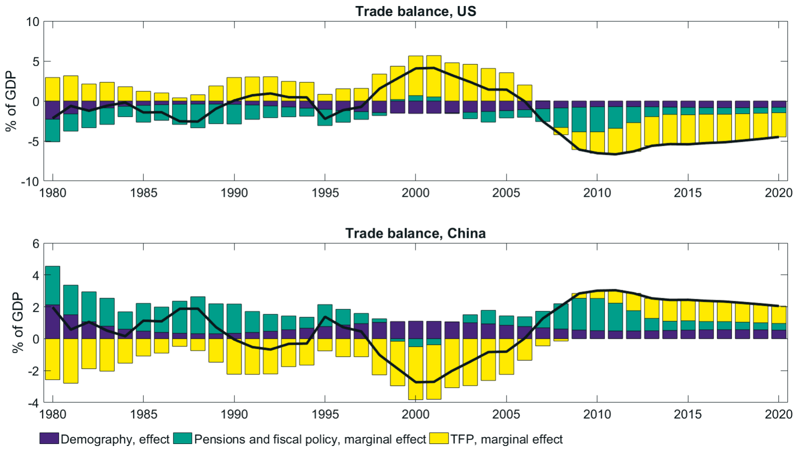

Figure 3.15: Decomposition of the effects of demographic factors and marginal effect of pensions and fiscal policy and TFP on the dynamics of the trade balance.

beginning of the simulation period. Social security is a key element in driving the results: without it, the model would indeed predict a counterfactual trade deficit and foreign debt for China for almost the entire simulation period.

3.4.4 Deterministic simulation with updates

The deterministic simulation used in the previous section assumes that the paths of the exogenous variables are known to the agents at the beginning of the simulation period. This section presents an alternative solution to the model, in which a deterministic simulation is performed for every period of the simulation, but the shocks and paths of exogenous variables are allowed to change. In particular, for each period, the initial values of the exogenous and endogenous variables are given by the simulation for the previous period, and new transition paths are solved based on the updated information. Innovations in exogenous variables are unanticipated, unlike in the previous simulation. This approach allows the agents to update their knowledge on the future paths of exogenous variablesas they become available in the data. The transition paths are constructed as a compilation of values of different rounds of simulation.

The assumptions regarding the future paths of exogenous variables are as follows. In each period, the agents are assumed to know the paths of life expectancy and population growth as given in the respective vintage of the UN population forecast (plotted for selected years in figures 3.5 and 3.6). TFP growth rate and government expenditures are assumed to follow AR(1) processes known by the agents, who estimate the autoregressive parameter given past data in each period. The agents observe the current exogenous realization of TFP and government expenditures, and forecast the future values given the AR(1) process. The agents thus assume that, after the temporary shock has occurred, these variables slowly converge to their steady state values. In each period there is a shock to TFP and government expenditures so that the compiled series of these variables match the data exactly (plotted in figures 3.7 and 3.8). Therefore, unlike in the deterministic simulation, the agents are not assumed to know the entire path of e.g. productivity growth at the beginning of the simulation. Pension expenditures are assumed to be constant over time. With respect to the endogenous variables, we assume that the economies are otherwise in a steady state implied by the exogenous variables apart from net foreign assets and trade balance, which are assumed to be zero at the beginning of the period.

The results of the simulation are presented in figure 3.16. The simulated paths of the trade balance and the net foreign asset position match the data better than those obtained with the deterministic simulation. The Chinese trade balance is positive for almost the entire simulation period, apart from a sharp deterioration caused by a sharp unexpected downward revision in life expectancy in the 1982 population forecast. Periods such as the mid-2000s, when the TFP grew at record rates reaching up to 6 %, cause the trade balance to decline, but it remains positive for the entire simulation period. In contrast, with the deterministic simulation Chinese trade balance is negative between 1990-1994 (-0.4 % of GDP on average) and 1998-2005 (-1.7 % of GDP on average). With the simulation with updates, China is predicted to have a positive net foreign asset position which averages 23 % of its GDP during the simulation period. For the US as well, the simulated trade balance and the negative and deteriorating foreign asset position match the data well.

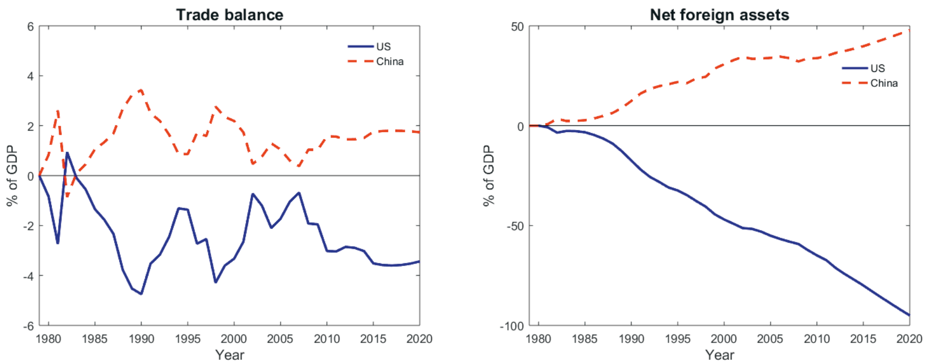

Figure 3.16: Effects of a permanent demographic changes, social security, government expenditures and TFP growth on the trade balance, simulation with updates (left panel) and on the net foreign asset position (right panel).

3.4.5 Life expectancy and the labor supply channel

The dynamics of the simulations in sections 3.4.3 and 3.4.4, and especially the persistent trade surplus in China, are driven by the demographic transition. A demographic shock, which induces an increase in life expectancy, lowers aggregate consumption and increases savings in the model. However, if labor supply is assumed to be constant, the impact of the increase in life expectancy can be unrealistically large in the sense that it could lead to counterfactual excessive savings by the young and a dynamically inefficient world interest rate. In the US data, the share of wealth held by the elderly has in fact increased over the past 30 years (see left panel in figure 3.17). The endogeneity of labor supply helps to address this problem, as an increase in life expectancy then results in a more modest increase in savings by the working-age population as the households also adjust their labor supply in response to a demographic shock. The resulting labor supply dynamics match the data qualitatively well.

In this section, in order to illustrate the impact of the labor supply channel in the model, we compare the dynamic effects of an increase in life expectancy with variable labor supply and fixed labor supply (in the latter case, we assume that $v = 1, \xi = 0$). In the initial state, life expectancy is assumed to be 74.3 years in the US and 67.5 years in China, and to slowly converge to 79.5 years in the US and 76.6 years in China, corresponding to estimates of life expectancies in 1985 and projected values in 2020 according to the data from United Nations

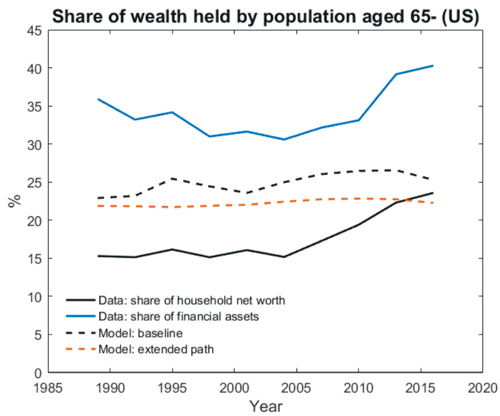

Figure 3.17: Share of wealth held by population aged 65-. Source: Federal Reserve, Survey of Consumer Finances (SFC) 1989-2016.

(2015).[6]

As previously discussed, an increase in life expectancy lowers the marginal propensities to consume (equations (3.6) and (3.11)), which leads to a decline in consumption (equations (3.5) and (3.12)), unless offset by an increase in the present value of life-time wealth. With variable labor supply, the decline in consumption and the increase in the marginal utility of consumption require an increase in labor supply l^r and l^w for a given wage and tax rate for the intratemporal labor supply optimality conditions (equations (3.4) and (3.9)) to hold. However, an increase in labor supply in retirement raises the present discounted value of workers' human wealth H_t^{jw}. As workers anticipate higher labor income during their retirement, the negative wealth effect on their consumption is smaller.[7] If the retirees' labor supply increases sufficiently, the workers react by reducing their labor supply and increasing consumption, and thus the increase of life expectancy may result in a decline in the world aggre-

[6] The preference parameters are calibrated as in table 3.1. Social security expenditures, the level of government expenditures and the steady state value of government debt are assumed to be at the same level in both countries (4%, 15 % and 20 % of GDP, respectively). Growth rate of working rate population is fixed (2.15%). The simulation abstracts from temporary TFP and government expenditure shocks.

[7] See also discussion by Fujiwara and Teranishi (2008) on the non-monotonic effects of life expectancy in the Gertler (1999) framework.

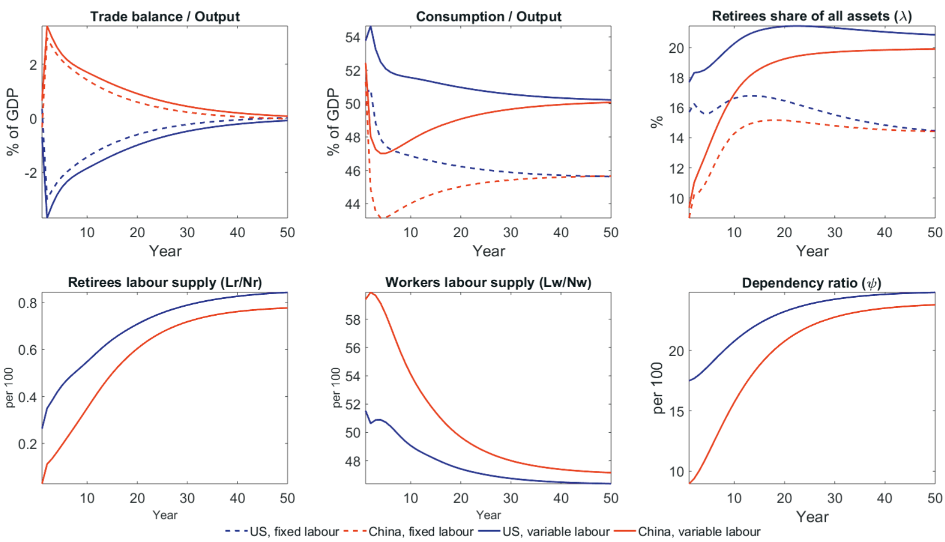

Figure 3.18: Impact of population aging with variable and fixed labor supply.

gate financial wealth, and an increase in real interest rate.

In the current simulation (see figure 3.18), retirees' labor supply is in the initial state higher in the country with higher life expectancy (US) because of the negative wealth effect discussed above. In the US, high labor supply in retirement increases the present value of workers' human wealth so that labor supply during working life is also lower in the US than in China. An increase in longevity increases retirees' labor supply so much that after the initial impact (which lasts for 2 to 4 periods), workers' labor supply declines in both countries. The increase in expected labor income dampens the negative wealth effect, and savings by workers increase less than in the model with fixed labor supply, and therefore the share of financial wealth held by the retirees does not fall in either economy. Furthermore, in the model with variable labor supply, population aging results in a smaller increase in the level of financial wealth and a smaller decline in the world real interest rate, resulting in lower investment and higher consumption than the fixed labor supply model. The impact on the external balance is similar, because the higher consumption in the variable labor supply model is coupled with a low investment-to-output ratio.

The labor supply dynamics of the model match the data qualitatively. Labor force statistics display consistently higher hours worked per worker for China than the United States (see left panel in figure 3.19).[8] Second, there has been a persistent decline in hours worked both in the US and China, which in the model arises as a result of population aging. Third, according to the 2010 population census in China, labor income was a primary source of support for 28% of the rural population aged 65 and over, and for 4 % of the urban population in China[9], and in the US, employment to population ratio among the population over 65 has displayed an upward trend since the early 1980s (see right panel in figure 3.19), suggesting that retirees' role in income provision is important.

[8] Also, on the extensive margin, the employment to population ratio, as well as the labor force participation rate, has been historically higher in China than in the US both among population aged 15-64 and population aged 65 and over (the right panel in figure 3.19).

[9] Original source: National Bureau of Statistics of China (2012), cited by Jiang et al. (2016). In 2004, working was the primary source of income 8% of urban men and 4% of urban women, and 43% of rural men and 23% of rural women. Original source: National Bureau of Statistics of China (2005), cited by Naughton (2007).

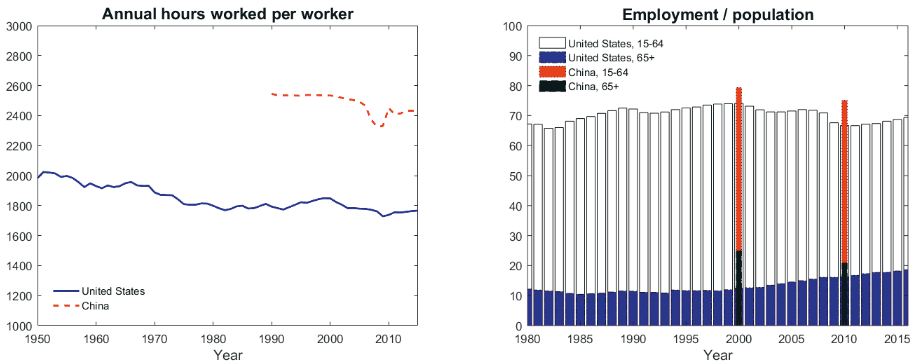

Figure 3.19: Left panel: Annual hours worked per worker. Source: The Conference Board Total Economy Database 2015. Right panel: Employment to population ratio on age groups 15-64 and 65 and over. Source: OECD Labor Force Survey 2018.

3.5 Conclusions

This paper examines whether the relatively rapid demographic transition in China combined with its low level of social security can help to explain why China has accumulated a large positive net foreign asset position over the past decades and why the country has persistently run a positive trade balance vis-a-vis the US. The analysis is performed with a model which features a life-cycle structure and a pay-as-you-go pension system along with endogenous labor supply and distortionary taxation.

The results show that demographic transition, together with the low level of social security expenditures, helps to explain the observed long-lasting trade and current account surpluses of China vis-a-vis the US. The rapid increase in life expectancy increases Chinese savings and generates current account and trade surpluses. The model predicts a long-lasting improvement in China's net foreign asset position and a positive trade balance for most of the simulation period. The role of low social security income in explaining the observed pattern is crucial. Even though life expectancy grew more quickly in China, it still remained lower than in the US for the entire simulation period, and therefore without social security, the model would counterfactually predict a negative trade balance and net foreign asset position for China.

Temporary TFP fluctuations remain important drivers of the trade balance, but even after controlling for the effects of productivity growth, the model predicts a positive trade balance and net foreign asset position for China for most of the simulation period. TFP growth rate fluctuations cause more volatility in the trade balance when the model is solved deterministically than when the alternative method, in which the temporary shocks are not known in advance and agents learn gradually, is used. Therefore the impact of the TFP fluctuations is smaller with the latter method, and the results better aligned with data.

As the features in the model are not specific to China, the model could also be used to analyze the relationship between other emerging and industrialized economies. Given the similarities in demographic trends in other emerging economies and the relatively low level of social security commonly observed, the model is likely to predict a similar pattern for several other emerging economies.

Finally, the analysis omits the effects of the Chinese central bank policies, including foreign exchange policies, capital controls and trade policies, as well as financial market imperfections. I leave this for future research.

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Appendix

3.A Trade balance

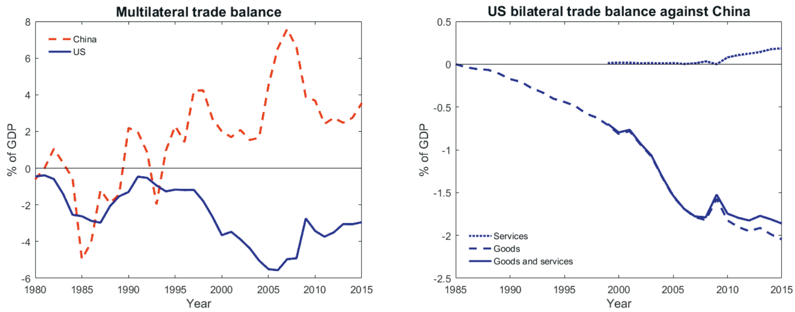

Figure 3.20: Left: External balance on goods and services 1980-2015 for China and the US. Source: World Bank, World Development Indicators. Right: US bilateral trade balance against China 1985-2015. Sources: US Census Bureau (goods) and Bureau of Economic Analysis (goods and services since 1999).

3.B Data fit

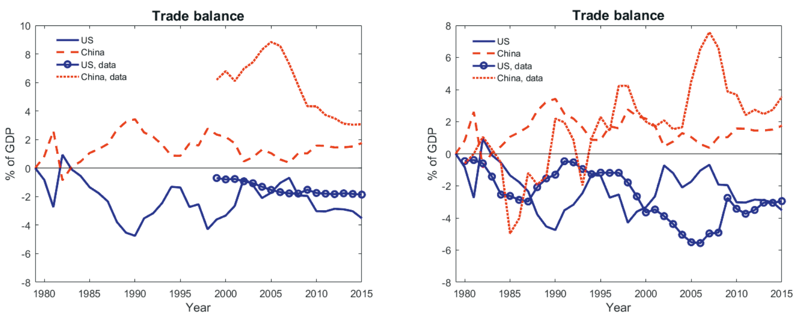

Figure 3.21: Simulation results (simulation with updates) with bilateral trade balance data (left) and multilateral trade balance data (right).

3.C Life-cycle structure

Population in a country at time t consists of two groups of individuals: workers, whose total number is N_t^w, and retirees, whose number equals N_t^r. All agents enter the economy as workers at the age of 20 and remain workers with probability $\omega_{t,t+1}$ and retire with probability $1 - \omega_{t,t+1}$. Every period $(1 - \omega_{t,t+1} + n_{t,t+1}) N_t^w$ new workers are born. Thus the number of workers grows each period at rate $n_{t,t+1}$ and the law of motion for aggregate labor force is

$$N_{t+1}^w = (1 - \omega_{t,t+1} + n_{t,t+1}) N_t^w + \omega_{t,t+1} N_t^w = (1 + n_{t,t+1}) N_t^w \ . \tag{3.28}$$

At time t, probability of a retiree to survive to the next period is $\gamma_{t,t+1}$. The law of motion for the number of retirees is

$$N_{t+1}^r = (1 - \omega_{t,t+1}) N_t^w + \gamma_{t,t+1} N_t^r \ . \tag{3.29}$$

The ratio of number of retirees to the number of workers, dependency ratio, is given by $\psi_t = N_t^r / N_t^w$ and can be solved to evolve according to

$$(1 + n_{t,t+1}) \psi_{t+1} = (1 - \omega_{t,t+1}) + \gamma_{t,t+1} \psi_t \ . \tag{3.30}$$

3.D Pension systems

China In the early 1980s, the Chinese pension system covered mostly urban workers in the public sector and state-owned enterprises, and until the early 1990s, all China's pension liabilities were unfunded (Naughton, 2007). After the beginning of the 1980s, the Chinese economy and demographic structure underwent changes that necessitated several reforms in the pension system. The targets of the reforms included increasing the pension coverage to urban employees of private enterprises and, eventually, to rural residents, and tackling the challenge posed by the aging population by setting up a partly funded system.

The current two-tier pension system was introduced in 1998 and revised in 2006 (OECD, 2015). It consists of a public basic pension, which is funded on a pay-as-you-go basis, and a mandatory employee contribution to a second-tier plan. The coverage of the pension system remains mostly limited to urban workers. The second-tier plan, which is a funded individual-account system, was de facto functional in only 11 out of the 33 Chinese provinces. The pension scheme covers 27.7 % of the population aged 15-65 and 33.5 % of the labor force. Depending on individual earnings, the gross replacement rate of the basic pensions system, defined as the pension benefits as a share of individual lifetime average earnings, varies between 30 and 50 % (OECD, 2015). With the defined contribution pillar, the gross replacement rate is approximately 75%.

The US The public pension system in the United States is called Social Security and originates in the Social Security Act of 1935. It is a defined benefit, earnings-based public pension scheme. A means-tested old-aged pension benefit (Supplemental Security Income) provides additional income for low-income pensioners. The social security covered 71.4 % of the population aged 15 to 65 and 92.2 % of the labor force in 2010 (OECD, 2015). The gross replacement rate of the social security varies between 25 % and 45 %, which is low in comparison to the OECD average (OECD, 2015). Private pension funds are highly important in the United States. An approximation by the OECD (2015) shows that if all workers are assumed to contribute annually at the rate of 9% into a voluntary private pension fund throughout their life, the replacement rate of an average earner is 82%.

The social security system is partly funded. Almost all wage income is subject to FICA taxes (FICA = Federal Insurance Contributions Act tax), which are administered by the Social Security Trust Funds. The Federal Old-Age and Survivors Insurance (OASI) fund holds the accumulated old-age pension fund assets and pays the benefits (The Board of Trustees, Federal Old-Age and Survivors Insurance and Federal Disability Insurance Trust Funds, 2016). Any excess income is deposited into the fund and invested in special US government bonds (Special Issue Securities; www.ssa.gov). The OASI funds reserves at the end of 2015 were 2780 billion USD. Income in 2015 was 798 billion USD (of which 679 billion USD consisted of payroll taxes) and costs 776 billion USD (of which benefit payments amounted to 769 billion) (The Board of Trustees, Federal Old-Age and Survivors Insurance and Federal Disability Insurance Trust Funds, 2016).

Classification of the pension systems by Pallares-Miralles et al. (2012) The classification is based on three principles: i) what the basic form of benefit promise is, ii) how the benefits are financed and iii) whether the system is privately or publicly managed. The classification is a simplified version of the World Bank's 5-pillar classification. The majority of pension systems under pillars 0 and 1 are PAYG, which supports the classification of the Chinese and American systems as PAYG systems. The Chinese pension system consists of pillars 0 (mandatory, publicly managed pension schemes aiming at adequacy) and 1 (mandatory, publicly managed systems aiming at income replacement), both consisting of defined benefits rather than defined contributions. The American pension system, managed under "Social Security", consists of targeted programs under pillar 0 and defined benefit pension schemes under pillar 1. Neither the Chinese nor the American system have elements of pillar 2, which is a scheme that also aims at income replacement, but is privately managed.

3.E Sensitivity to TFP growth assumptions

News shocks on TFP growth are known to cause aggregate fluctuations especially in low-frequency analysis (see Fujiwara (2010)). In order to capture the impact of different assumptions about TFP growth on the aggregate fluctuations the model has been simulated under three alternative TFP growth scenarios.

Choosing the assumptions about future long term TFP growth over the simulation period is not trivial. Even though TFP gains can still be expected due to resource reallocation and policy reforms, as discussed by Adler et al. (2017), China's integration to the world trade has matured and the effects of structural transformations may be fading. In all scenarios, I assume that current TFP growth exactly matches the data (shown in figure 3.8), but future TFP growth rate follows a linear trend, in which the Chinese TFP growth exceeds the US growth rate for varying periods of time.

In the first scenario, I assume that the Chinese TFP growth rate is 2 % in 1980 and slows down at a constant rate over the next 35 years, so that in 2015, the growth rates in China and the US are equalized. The second scenario is similar but the TFP growth rates are equalized only in 2030, meaning that TFP growth rate remains higher in China for 50 years. In the last scenario, China experiences a 35-year period of relatively high TFP growth between 1980 and 2015, after which its TFP growth slows down to 0.75 % between 2016 and 2025, and then grows again and becomes equal to the US growth rate of 1 % in 2030. In all scenarios, TFP growth rates are equal in the steady state to gain stationarity.

In these scenarios, the results are not qualitatively affected. However, the larger and the more persistent the TFP growth differential, the smaller the trade balance, especially early in the simulation period.

3.F Parameter sensitivity

		NX_t/Y_t	NX_t^*/Y_t^*	F_t/Y_t	F_t^*/Y_t^*	R_t
Baseline simulation						
	mean	-0.02	0.02	-0.39	0.23	1.02
	min / max	-0.05/0.01	-0.01/0.03	-0.83/-0.01	0.01/0.41	1.01/1.04
	standard deviation	0.01	0.01	0.26	0.13	0.01
β discount factor						
$\beta = 0.990$	mean	-0.02	0.01	-0.38	0.23	1.03
	min / max	-0.05/0.01	-0.01/0.03	-0.81/-0.01	-0.01/0.40	1.01/1.04
	standard deviation	0.01	0.01	0.25	0.13	0.01
$\beta = 0.998$	mean	-0.03	0.02	-0.41	0.24	1.02
	min / max	-0.05/0.01	-0.01/0.04	-0.85/-0.01	0.01/0.42	1.00/1.03
	standard deviation	0.01	0.01	0.26	0.13	0.01
σ elasticity of intertemporal substitution						
$\sigma = 0.45$	mean	-0.02	0.01	-0.29	0.17	1.03
	min / max	-0.04/0.02	-0.02/0.03	-0.68/0.02	-0.02/0.34	1.02/1.05
	standard deviation	0.01	0.01	0.22	0.12	0.01
δ depreciation rate						
$\delta = 0.15$	mean	-0.02	0.01	-0.28	0.16	1.04
	min / max	-0.04/0.02	-0.02/0.03	-0.67/0.01	-0.01/0.33	1.02/1.06
	standard deviation	0.01	0.01	0.21	0.11	0.01
ϕ investment adjustment cost factor						
$\phi = 0.2$	mean	-0.03	0.02	-0.35	0.20	1.02
	min / max	-0.07/0.03	-0.03/0.05	-0.81/0.08	-0.07/0.40	1.01/1.05
	standard deviation	0.02	0.02	0.27	0.15	0.01
$\phi = 10$	mean	-0.02	0.02	-0.43	0.26	1.02
	min / max	-0.04/-0.00	0.00/0.04	-0.55/-0.02	0.02/0.43	1.01/1.04
	standard deviation	0.01	0.01	0.25	0.12	0.01
ξ productivity of a unit of labor, retiree to worker						
$\xi = 0.6$	mean	-0.02	0.01	-0.33	0.20	1.03
	min / max	-0.04/0.01	-0.01/0.03	-0.73/-0.00	0.05/0.36	1.01/1.04
	standard deviation	0.01	0.01	0.23	0.12	0.01
v elasticity of period utility with respect to consumption						
$v = 0.75$	mean	-0.02	0.01	-0.38	0.23	1.03
	min / max	-0.04/0.01	-0.01/0.03	-0.80/-0.01	0.01/0.40	1.01/1.04
	standard deviation	0.01	0.01	0.25	0.13	0.01

Table 3.4: The table shows the sensitivity of the mean, minimum and maximum values and standard deviation of the simulated variables to changes in single parameter values during the sample period 1980-2015 in the simulation with updates. The rest of the parameters are calibrated as in the baseline simulation (table 3.1).

3.G Technical appendix

Derivation of the model

The appendix presents derivation of retiree's and worker's problems, the aggregation result, the problem of the firm, dynamic stationarized equations and a characterization of the steady state of the stationarized model.

1.1 Households

Retirees

A retiree born in period j and retired in period i chooses consumption-saving allocation and labor input to maximize

$$V_t^{jr}(i) = max \left\{ \left[\left(C_t^{jr}(i) \right)^v (1 - l_t^r)^{1-v} \right]^\rho + \beta \gamma_{t,t+1} \left(V_{t+1}^{jr}(i) \right)^\rho \right\}^{\frac{1}{\rho}} \tag{3.31}$$

subject to

$$A_{t+1}^{jr}(i) = \frac{R_t A_t^{jr}(i)}{\gamma_{t-1,t}} + W_t \xi l_t^{jr} (1 - \tau_t) + S_t^r - C_t^{jr}(i). \tag{3.32}$$

R_t is the world interest rate that clears the international capital market and $\gamma_{t,t+1}$ (henceworth, $\gamma_{t,t+1} \equiv \gamma_{t+1}$) is the retirees' probability to survive from one period to the next. $W\xi l_t^r(1 - \tau_t)$ is the net wage income and S_t^r is the pension income from the government. The proportional income tax τ_t is paid both by the workers and the retirees.

The first order condition with respect to asset accumulation is

$$v(C_t^{jr}(i))^{v\rho-1}(1 - l_t^{jr})^{\rho(1-v)} = \beta \gamma_{t+1} (V_{t+1}^{jr}(i))^{\rho-1} \frac{\partial V_{t+1}^{jr}(i)}{\partial A_{t+1}^{jr}(i)}. \tag{3.33}$$

The envelope condition is

$$\frac{\partial V_t^{jr}(i)}{\partial A_t^{jr}(i)} = (V_t^{jr})^{1-\rho} v (C_t^{jr}(i))^{v\rho-1} (1 - l_t^{jr})^{(1-v)\rho} \frac{R_t}{\gamma_t}. \tag{3.34}$$

The first order condition with respect to labor is

$$l_t^{jr}(i) = 1 - \frac{C_t^{jr}(i)\varsigma}{W_t\xi(1-\tau_t)} \tag{3.35}$$

where $\varsigma = \frac{1-v}{v}$.

Combining the first order conditions and the lagged envelope condition $\left(\frac{\partial V_{t+1}^{jr}(i)}{\partial A_{t+1}^{jr}(i)}\right)$, and noting that $\sigma = \frac{1}{1-\rho}$ gives the following Euler equation for retirees:

$$C_{t+1}^{jr}(i) = C_t^{jr}(i)\left[\left(\frac{W_t(1-\tau_t)}{W_{t+1}(1-\tau_{t+1})}\right)^{\rho(1-v)}\beta R_{t+1}\right]^{\sigma}. \tag{3.36}$$

Guess that consumption is a fraction of total lifetime wealth:

$$C_t^{jr}(i) = \epsilon_t\pi_t\left(\frac{R_t A_t^{jr}(i)}{\gamma_t} + H_t^{jr}(i) + P_t^r\right) \tag{3.37}$$

where H_t^{jr} is the present discounted value of a retiree's lifetime human wealth and P_t^r is the present discounted value of a retiree's lifetime pension benefits. The present discounted value of a retiree's human wealth can be written as

$$H_t^{jr}(i) = W_t(1-\tau_t)\xi l_t^{jr}(i) + \frac{W_{t+1}(1-\tau_{t+1})\xi l_{t+1}^{jr}(i)}{R_{t+1}/\gamma_{t+1}} + \frac{W_{t+2}(1-\tau_{t+1})\xi l_{t+1}^{jr}(j)}{(R_{t+1}/\gamma_{t+1})(R_{t+2}/\gamma_{t+2})} + ...$$

$$= \sum_{v=0}^{\infty}\frac{W_{t+\mathrm{v}}(1-\tau_{t+v})\xi l_{t+v}^{jr}(i)}{\prod_{s=1}^{v}(R_{t+s}/\gamma_{t+s})}$$

$$= W_t(1-\tau_t)\xi l_t^{jr}(i) + \frac{H_{t+1}^{jr}(i)}{R_{t+1}/\gamma_{t+1}} \quad . \tag{3.38}$$

Similarly, the present discounted value of a retiree's pension benefits can be written as

$$P_t^r = S_t^r + \frac{P_{t+1}^r}{R_{t+1}/\gamma_{t+1}} \quad . \tag{3.39}$$

Combining the Euler equation (3.36) with the guess (3.37) as follows gives a law of motion for the marginal propensity to consume as follows.

First, combine the budget constraint (3.32) with the guess (3.37) to have the following expression for consumption:

$$C_t^{jr}(i) = \epsilon_t \pi_t \left(\frac{R_t}{\gamma_t} * \frac{\gamma_t}{R_t} (A_{t+1}^{jr}(i) - W_t \xi l_t^{jr}(i) (1-\tau_t) - S_t^r - C_t^{jr}(i)) + H_t^{jr}(i) + P_t^r \right) \quad . \tag{3.40}$$

Second, substitute this expression (3.40), a lagged expression of the guess (3.37) and the expressions for present value of human (3.38) and social security (3.39) wealth into the Euler equation (3.36) to obtain the following expression

$$\begin{aligned} \epsilon_{t+1}\pi_{t+1} \left(\frac{R_{t+1} A_{t+1}^{jr}(i)}{\gamma_{t+1}} + H_{t+1}^{jr}(i) + P_{t+1}^r \right) = \\ \left\{ \epsilon_t \pi_t \left(A_{t+1}^{jr}(i) + \frac{H_{t+1}^{jr}(i) + P_{t+1}^r}{R_{t+1}/\gamma_{t+1}} + C_t^{jr}(i) \right) \right\} \\ * \left[\left(\frac{W_t(1-\tau_t)}{W_{t+1}(1-\tau_{t+1})} \right)^{\rho(1-v)} \beta R_{t+1} \right]^{\sigma} . \end{aligned} \tag{3.41}$$

Dividing both sides of the equation by the LHS (left-hand-side) of itself and using the Euler equation and the conjecture again, retiree's marginal propensity to consume can be solved to evolve according to

$$\epsilon_t \pi_t = 1 - \frac{\epsilon_t \pi_t}{\epsilon_{t+1}\pi_{t+1}} \gamma_{t+1} \left(\frac{W_t(1-\tau_t)}{W_{t+1}(1-\tau_{t+1})} \right)^{\rho\sigma(1-v)} \beta^{\sigma} (R_{t+1})^{\rho\sigma} \quad . \tag{3.42}$$

To obtain a solution for the value function, first conjecture that it is linear in consumption according to

$$V_t^{jr}(i) = \triangle_t^r C_t^{jr}(i) \left(\frac{\varsigma}{W_t \xi (1-\tau_t)} \right)^{1-v}, \tag{3.43}$$

substitute the conjecture into the value function and solve for $\triangle_t^r$ to obtain the following expression

$$(\triangle_t^r)^\rho = 1 + \frac{\beta\gamma_{t+1}(\triangle_{t+1}^r)^\rho \left[C_{t+1}^{jr}(i)\left(\frac{\varsigma}{W_{t+1}\xi(1-\tau_{t+1})}\right)^{1-v}\right]^\rho}{\left(C_t^{jr}(i)(\frac{\varsigma}{W_t\xi(1-\tau_t)})^{1-v}\right)^\rho}.$$

Then, substitute $C_{t+1}^{jr}(i)$ from the Euler equation (3.36) and use the equation (3.42) to verify that

$$\triangle_t^r = (\epsilon_t \pi_t)^{-\frac{1}{\rho}}. \tag{3.44}$$

and thus

$$V_t^{jr}(i) = (\epsilon_t \pi_t)^{-\frac{1}{\rho}} C_t^{jr}(i)\left(\frac{\varsigma}{W_t\xi(1-\tau_t)}\right)^{1-v}. \tag{3.45}$$

Workers

A worker born in period j chooses consumption-saving allocation and labor input to solve

$$V_t^{jw} = max\left\{\left[\left(C_t^{jw}\right)^v\left(1-l_t^{jw}\right)^{1-v}\right]^\rho + \beta\left[\omega_{t+1}V_{t+1}^{jw} + (1-\omega_{t+1})V_{t+1}^{jr}\right]^\rho\right\}^{\frac{1}{\rho}} \tag{3.46}$$

where $\omega_{t,t+1}$ (henceworth, $\omega_{t,t+1} \equiv \omega_{t+1}$) is the workers' probability to remain a worker, subject to

$$A_{t+1}^{jw} = R_t A_t^{jw} + W_t l_t^{jw}(1-\tau_t) - C_t^{jw} - T_t^{jw} \ . \tag{3.47}$$

The first order condition with respect to asset accumulation is

$$v(C_t^{jw})^{v\rho-1}(1-l_t^{jw})^{\rho(1-v)} = \\ \beta\left(\omega_{t+1}V_{t+1}^{jw} + (1-\omega_{t+1})V_{t+1}^{jw}\right)^{\rho-1}\left[\omega_{t+1}\frac{\partial V_{t+1}^w}{\partial A_{t+1}^w} + (1-\omega_{t+1})\frac{\partial V_{t+1}^{rj}}{\partial A_{t+1}^{wj}}\right]. \tag{3.48}$$

The envelope conditions are

$$\frac{\partial V_t^{jw}}{\partial A_t^{jw}} = (V_t^w)^{1-\rho} v (C_t^w)^{v\rho-1} (1 - l_t^w)^{(1-v)\rho} R_t \tag{3.49}$$

and

$$\frac{\partial V_t^{jr}(i)}{\partial A_t^{jw}} = \frac{\partial V_t^{jr}(i)}{\partial A_t^{jr}} \frac{\partial A_t^{jr}}{\partial A_t^{jw}} = \frac{\partial V_t^{jr}(i)}{\partial A_t^{jr}} = (V_t^r)^{1-\rho} v (C_t^r)^{v\rho-1} (1 - l_t^r)^{(1-v)\rho} R_t. \tag{3.50}$$

The first order condition with respect to labor is

$$l_t^w(j) = 1 - \frac{C_t^w(i)\varsigma}{W_t(1-\tau_t)}. \tag{3.51}$$

Combining the first order conditions and the envelope conditions gives the Euler equation for the workers. First, substitute into the first order condition for asset accumulation (3.48) the envelope conditions taking into account the assumption of no compensation for the risk of death :

$$\begin{aligned} v(C_t^{jw})^{v\rho-1}(1 - l_t^{jw})^{\rho(1-v)} = & \\ & \beta(\omega_{t+1} V_{t+1}^{jw} + (1-\omega_{t+1}) V_{t+1}^{jw})^{\rho-1} * \\ & (\omega_{t+1} (V_{t+1}^w)^{1-\rho} v (C_{t+1}^w)^{v\rho-1} (1 - l_{t+1}^w)^{(1-v)\rho} R_{t+1} \\ & + (1-\omega_{t+1})(V_{t+1}^r)^{1-\rho} v (C_{t+1}^r)^{v\rho-1} (1 - l_{t+1}^r)^{(1-v)\rho} R_{t+1}). \end{aligned}$$

Then substitute the labor FOC (3.51) to have

$$\begin{aligned} C_t^w \left(\frac{W_{t+1}(1-\tau_{t+1})}{W_t(1-\tau_t)} \right)^{\frac{\rho(1-v)}{\rho-1}} (\beta R_{t+1})^\sigma = & \\ & (\omega_{t+1} V_{t+1}^{jw} + (1-\omega_{t+1}) V_{t+1}^{jw}) * \\ \Big[\omega_{t+1} (V_{t+1}^w)^{1-\rho} (C_{t+1}^w)^{\rho-1} + (1-\omega_{t+1}) & (V_{t+1}^r)^{1-\rho} v (C_{t+1}^r)^{\rho-1} \xi^{(v-1)\rho} \Big]^{-\sigma}. \end{aligned} \tag{3.52}$$

Conjecture that similarly as for retirees, the value function is linear in consumption:

$$V_t^{jw} = \triangle_t^{jw} C_t^{jw} \left(\frac{\varsigma}{W_t(1-\tau_t)} \right)^{1-v}. \tag{3.53}$$

In addition, define the adjustment term to be

$$\Omega_t \equiv \omega_{t-1,t} + (1-\omega_{t-1,t})\epsilon_t^{\frac{1}{1-\sigma}} \chi \tag{3.54}$$

and define $\chi = \xi^{-(1-v)}$.

Substitute the conjectures for the value functions (3.43) and (3.53) into equation (3.52). The resulting Euler equation takes the form

$$C_t^{jw} \left[\left(\frac{W_t(1-\tau_t)}{W_{t+1}(1-\tau_{t+1})} \right)^{\rho(1-v)} \beta R_{,t+1} \Omega_{t+1} \right]^{\sigma} = \omega_{t+1} C_{t+1}^{jw} + (1-\omega_{t+1}) \left(\frac{\triangle_{t+1}^{r}}{\triangle_{t+1}^{w}} \right) \chi C_{t+1}^{jr} \tag{3.55}$$

which equals

$$C_t^{jw} \left[\left(\frac{W_t(1-\tau_t)}{W_{t+1}(1-\tau_{t+1})} \right)^{\rho(1-v)} \beta R_{,t+1} \Omega_{t+1} \right]^{\sigma} = \omega_{t+1} C_{t+1}^{jw} + (1-\omega_{t+1})(\epsilon_{t+1})^{\frac{\sigma}{1-\sigma}} \chi C_{t+1}^{jr}.$$

Guess the consumption to be a fraction of total lifetime wealth

$$C_t^{jw} = \pi_t (R_{W,t} A_t^{jw} + H_t^{jw} + P_t^{jw}) \tag{3.56}$$

where H_t^{jw} is the present discounted value of a worker's human wealth net of taxation and P_t^{w} is the present discounted value of a worker's pension benefits that (s)he can assume to get once retired.

Substitute the per period budget constraint (3.47) to get the following expression:

$$C_t^{jw} = \pi_t (A_{t+1}^{jw} - W_t l_t^{jw} (1-\tau_t) + C_t^{jw} + T_t^{w} + H_t^{jw} + P_t^{w}).$$

For a retiree who just abandoned the labor force, the consumption is guessed to be proportional to the assets they had as workers which they carry to their first period of retirement and human and social security as retirees:

$$C_t^{r|w} = \varepsilon_t \pi_t (R_t A_t^{jw} + H_t^{jr} + P_t^{jr}) \tag{3.57}$$

with the budget constraint being

$$A_{t+1}^{jr|w} = R_t A_t^{jw} + W_t l_t^{jw} (1 - \tau_t) - C_t^{jw} - T_t^w. \tag{3.58}$$

Substitute the budget constraint (3.58) into the conjecture (3.57) to have

$$C_t^{r|w} = \varepsilon_t \pi_t (A_{t+1}^{jr|w} - W_t l_t^{jw} (1 - \tau_t) + C_t^{jw} + T^w + H_t^{jr} + P_t^{jr})$$

Combining the Euler equation (3.55) with the guesses (3.56) and (3.57) and the budget constraints (3.47) and (3.58) as follows gives a law of motion for the marginal propensity to consume as follows. First, subsitute the guesses into the Euler equation to have

$$\pi_t (R_{W,t} A_t^{jw} + H_t^{jw} + P_t^{jw}) \left[\left(\frac{W_t (1 - \tau_t)}{W_{t+1} (1 - \tau_{t+1})} \right)^{\rho(1-v)} \beta R_{t+1} \Omega_{t+1} \right]^{\sigma} = \omega_{t+1} \pi_{t+1} (R_{t+1} A_{t+1}^{jw} + H_{t+1}^{jw} + P_{t+1}^{jw}) + (1 - \omega_{t+1}) \left(\frac{\triangle_{t+1}^r}{\triangle_{t+1}^w} \right) \chi \varepsilon_{t+1} \pi_{t+1} (R_{t+1} A_{t+1}^{jw} + H_{t+1}^{jr} + P_{t+1}^{jr}) \tag{3.59}$$

and substitute the per period budget constraints (3.47) and (3.58) to have

$$\pi_t (R_{W,t} A_t^{jw} + H_t^{jw} + P_t^{jw}) \left[\left(\frac{W_t (1 - \tau_t)}{W_{t+1} (1 - \tau_{t+1})} \right)^{\rho(1-v)} \beta R_{t+1} \Omega_{t+1} \right]^{\sigma} = \omega_{t+1} \pi_{t+1} (R_{W,t,t+1} (R_{W,t} A_t^{jw} + W_t l_t^{jw} (1 - \tau_t) - C_t^{jw} - T_t^w) + H_{t+1}^{jw} + P_{t+1}^{jw}) + (1 - \omega_{t+1}) \left(\frac{\triangle_{t+1}^r}{\triangle_{t+1}^w} \right) \chi \varepsilon_{t+1} \pi_{t+1} (R_{t+1} (R_{W,t} A_t^{jw} + W_t l_t^{jw} (1 - \tau_t) - C_t^{jw} - T_t^w) + H_{t+1}^{jr} + P_{t+1}^{jr}). \tag{3.60}$$

Substituting again the guess for C_t^{wj}, collecting terms and using the definition of the adjustment term, collecting terms with A_t^w on the LHS and dividing the equality by $\Omega_{t+1}\pi_{t+1}R_{t+1}$, this expression becomes

$$R_t A_t^w * \left[1-\pi_t - \frac{\pi_t}{\pi_{t+1}}\left(\frac{W_t(1-\tau_t)}{W_{t+1}(1-\tau_{t+1})}\right)^{\rho(1-v)\sigma}\beta^\sigma (R_{t+1}\Omega_{t+1})^{\sigma-1}\right] =$$
$$\left[\pi + \frac{\pi_t}{\pi_{t+1}}\left(\frac{W_t(1-\tau_t)}{W_{t+1}(1-\tau_{t+1})}\right)^{\rho(1-v)\sigma}\beta^\sigma (R_{t+1}\Omega_{t+1})^{\sigma-1}\right](H_t^w + P_t^w) - W_t l_t^w(1-\tau_t) + T_t$$
$$-\frac{\omega_{t+1}}{R_{t+1}\Omega_{t+1}}(H_{t+1}^w + P_{t+1}^w) - \frac{(1-\omega_{t+1})}{R_{t+1}\Omega_{t+1}}\left(\frac{\triangle_{t+1}^r}{\triangle_{t+1}^w}\right)\chi\varepsilon_{t+1}(H_{t+1}^r + P_{t+1}^r).$$

Conjecturing that $H_t^{wj} = W_t l_t^{jw}(1-\tau_t) - T_t^w + \frac{\omega_{t+1}H_{t+1}^{jw}}{\Omega_{t+1}R_{t+1}} + \frac{(1-\omega_{t+1})\left(\frac{\triangle_{t+1}^r}{\triangle_{t+1}^w}\right)\chi\varepsilon_{t+1}H_{t+1}^r}{\Omega_{t+1}R_{t+1}}$ and
$P_t^{wj} = \frac{\omega_{t+1}P_{t+1}^{jw}}{\Omega_{t+1}R_{t+1}} + \frac{(1-\omega_{t+1})\left(\frac{\triangle_{t+1}^r}{\triangle_{t+1}^w}\right)\chi\varepsilon_{t+1}P_{t+1}^r}{\Omega_{t+1}R_{t+1}}$, this can be written as

$$(R_t A_t^w + H_t^{wj} + P_t^{wj}) * \left[1-\pi_t - \frac{\pi_t}{\pi_{t+1}}\left(\frac{W_t(1-\tau_t)}{W_{t+1}(1-\tau_{t+1})}\right)^{\rho(1-v)\sigma}\beta^\sigma (R_{t+1}\Omega_{t+1})^{\sigma-1}\right] = 0$$

which implies that the workers´ marginal propensity to consume can be solved to evolve according to

$$\pi_t = \left[1 - \frac{\pi_t}{\pi_{t+1}}\left(\frac{W_t(1-\tau_t)}{W_{t+1}(1-\tau_{t+1})}\right)^{\rho\sigma(1-v)}\beta^\sigma (R_{t+1}\Omega_{t+1})^{\sigma-1}\right]. \tag{3.61}$$

To obtain a solution for the value function, subsitute the conjecture of the value function (3.53) into the value function (3.46) and solve for $\triangle_t^r$ to obtain the following expression

$$(\triangle_t^w)^\rho = 1 + \frac{\beta^\sigma\left(\frac{W_t(1-\tau_t)}{W_{t+1}(1-\tau_{t+1})}\right)^{(1-v)\rho}(\triangle_{t+1}^w)^\rho\left[\omega_{t+1}C_{t+1}^w + (1-\omega_{t+1})\left(\frac{\Delta_{t+1}^r}{\Delta_{t+1}^w}\right)\chi C_{t+1}^r\right]^\rho}{(C_t^w)^\rho}.$$

Then, substitute the Euler equation (3.55) into this expression and use the equation (3.61) to verify that

$$\triangle_t^w = (\pi_t)^{-\frac{1}{\rho}} \quad . \tag{3.62}$$

and thus

$$V_t^w = (\pi_t)^{-\frac{1}{\rho}} C_t^r (\frac{\varsigma}{W_t(1-\tau_t)})^{1-v} \quad . \tag{3.63}$$

1.2 Aggregation

Total assets

Because of the perfect annuity market, $R_t A_{t-1}^r$ of the assets of retirees of period $t-1$ is carried to the next period t. Of workers who retire between periods $t-1$ and t, $(1-\omega_{t-1,t})(R_{W,t-1}A_{t-1}^w + W_{t-1}l_{t-1}^w\,(1-\tau_{t-1}) - C_{t-1}^w - T_{t-1}^w)$ adds to the retirees assets in the beginning of period t. Thus retirees' aggregate assets evolve according to

$$A_{t+1}^r = R_t A_t^r + (1-\omega_{t+1})(R_{t+1}A_t^w + W_t l_t^w\,(1-\tau_t)\,N_t^w - C_t^w - T_t^w). \tag{3.64}$$

Workers aggregate assets evolve according to

$$A_{t+1}^w = \omega_{t+1}(R_{t+1}A_t^w + W_t l_t^w\,(1-\tau_t)\,N_t^w - C_t^w - T_t^w). \tag{3.65}$$

Consumption

Retirees' and workers' marginal propensities to consume do not depend on individual characteristics. The aggregate consumption in each group is thus total wealth times the groups' marginal propensities to consume.

Retirees' aggregate consumption is

$$C_t^r = \epsilon_t \pi_t (R_t A_t^r + H_t^r + P_t^r) \tag{3.66}$$

and workers' aggregate consumption is

$$C_t^w = \pi_t (R_t A_t^w + H_t^w + P_t^w). \tag{3.67}$$

Total aggregate consumption is

$$C_t = C_t^r + C_t^w = \epsilon_t \pi_t (R_{W,t}A_t^r + H_t^r + P_t^r) + \pi_t (R_{W,t}A_t^w + H_t^w + P_t^w). \tag{3.68}$$

Denoting the share of assets held by retirees $\lambda_t = \frac{A_t^r}{A_t}$, total aggregate consumption is given by

$$C_t = \pi_t A_t R_{W,t}(\epsilon_t \lambda_t + 1 - \lambda_t) + \pi_t(H_t^w + P_t^w) + \epsilon_t \pi_t (H_t^r + P_t^r). \quad (3.69)$$

Human wealth

The present discounted value of retirees' aggregate human wealth is

$$H_t^r = \left(1 - \frac{L_t^w}{L_t}\right) \alpha Y_t (1 - \tau_t) + \gamma_{t+1} \frac{\psi_t}{\psi_{t+1}} \frac{H_{t+1}^r}{(1 + n_{t,t+1}) R_{t+1}} \quad (3.70)$$

and the present discounted value of workers' aggregate human wealth is

$$\begin{aligned} H_t^w = \frac{L_t^w}{L_t} \alpha Y_t (1 - \tau_t) - t_t^w Y_t & \\ + \omega_{t+1} \frac{H_{t+1}^w}{(1 + n_{t,t+1}) R_{t+1} \Omega_{t+1}} & \\ + (1 - \omega_{t+1}) \frac{H_{t+1}^r \epsilon_{t+1}^{\frac{1}{1-\sigma}} \chi}{\psi_{t+1} (1 + n_{t,t+1}) R_{t+1} \Omega_{t+1}}. & \quad (3.71) \end{aligned}$$

Social security wealth

With $S_t = S_t^{jr} N_t^r$, the present discounted value of retirees´ aggregate pension benefits at time t is

$$P_t^r = S_t + \gamma_{t,t+1} \frac{P_{t+1}^r}{(1 + n_{t,t+1}^r) R_{t+1}} \quad . \quad (3.72)$$

In the current period, workers receive no social security payments (S_t), but expect to receive social security once retired. The total value of social security payments for the working force at time t equals

$$P_t^w = \omega_{t,t+1} \frac{P_{t+1}^w}{(1+n_{t,t+1}^w) R_{t+1} \Omega_{t+1}} + (1-\omega_{t,t+1}) \frac{P_{t+1}^r \epsilon_{t+1}^{\frac{1}{1-\sigma}} \chi}{\psi_{t+1}(1+n_{t,t+1}) R_{t+1} \Omega_{t+1}}. \quad (3.73)$$

Labor supply

Because the individual labor supply for retirees (3.35) is linear in consumption which is linear in wealth (3.37), the aggregate labor supply by retirees is:

$$L_t^r = N_t^r - \frac{\varsigma}{\xi W_t (1-\tau_t)} C_t^r. \quad (3.74)$$

Similarly, aggregate labor supply by workers is

$$L_t^w = N_t^w - \frac{\varsigma}{W_t (1-\tau_t)} C_t^w. \quad (3.75)$$

Distribution of wealth

Substituting expressions (3.65) and (3.67) into (3.64), the distribution of wealth between workers and retirees can be shown to evolve as follows:

$$\lambda_{t+1} = \omega_{t,t+1} (R_{W,t} \lambda_t \frac{A_t}{A_{t+1}} (1-\epsilon_t \pi_t) + \frac{N_t^r W_t (1-\tau_t) l_t^r \xi}{A_{t+1}} + \frac{S_t^r - \epsilon_t \pi_t (H_t^r + P_t^r)}{A_{t+1}}) + (1-\omega_{t+1}). \quad (3.76)$$

1.2 Firms

Firms operate in a competitive market and employ labor, capital and investment to maximize present discounted value of profits which is given by

$$V\left(I_{t-1}, K_t\right) = max\left[\left(X_t\left(L_t^w + \xi L_t^r\right)\right)^\alpha K_t^{1-\alpha} - W_t\left(L_t^w + \xi L_t^r\right) - I_t + \frac{V(I_t, K_{t+1})}{R_{W,t+1}}\right] \tag{3.77}$$

subject to the law of motion of capital

$$K_{t+1} = (1-\delta)K_t + \left[1 - \frac{\phi}{2}\left(\frac{I_t}{I_{t-1}} - \mu_t\right)^2\right] I_t \ . \tag{3.78}$$

The capital is owned by the firm and wage is paid to the effective labor force. Production takes place according to a constant returns to scale Cobb-Douglas production function

$$Y_t = \left(X_t\left(L_t^w + \xi L_t^r\right)\right)^\alpha K_t^{1-\alpha} \tag{3.79}$$

where X_t is the level of exogenous labor augmenting productivity at time t. Effective labor supplied by the retirees is given by ξL_t^r. Productivity grows according to $X_{t+1} = (1 + x_{t,t+1})X_t$. α is the labor share. The Lagrangian is

$$\begin{aligned}\mathcal{L} = &\left(X_t\left(L_t^w + \xi L_t^r\right)\right)^\alpha K_t^{1-\alpha} - W_t\left(L_t^w + \xi L_t^r\right) - I_t \\ &+ \frac{V(I_t, K_{t+1})}{R_{t+1}} - q_t\left[(1-\delta)K_t + \left[1 - \frac{\phi}{2}\left(\frac{I_t}{I_{t-1}} - \mu_t\right)^2\right] I_t - K_{t+1}\right]\end{aligned}$$

and the first order conditions with respect to labor force, capital and investment solve

$$W_t = \alpha \frac{Y_t}{L_t^w + \xi L_t^r} \tag{3.80}$$

$$\frac{\partial \mathcal{L}}{\partial I_t} = -1 + \frac{\partial V(I_t,K_{t+1})}{\partial I_t} - q_t - q_t\left(\frac{I_t}{I_{t-1}} - \mu_t\right)^2 + q_t\phi\frac{I_t}{I_{t-1}}\left(\frac{I_t}{I_{t-1}} - \mu_t\right) = 0$$

and

$$\frac{\partial \mathcal{L}}{\partial K_{t+1}} = \frac{\partial V(I_t,K_{t+1})}{\partial K_{t+1}} - q_t = 0.$$

The envelope conditions are

$$\frac{\partial V(I_{t-1},K_t)}{\partial I_{t-1}} = q_t\phi\left(\frac{I_t}{I_{t-1}}\right)^2\left(\frac{I_t}{I_{t-1}} - \mu_t\right)$$

and

$$\frac{\partial V(I_{t-1},K_t)}{\partial K_t} = (1-\alpha)\frac{Y_t}{K_{t\,t}} + (1-\delta)q_t.$$

Substituting the envelope conditions into the first order conditions gives the following first order conditions for capital and investment:

$$q_t = \frac{1}{R_{t+1}}\left[(1-\alpha)\frac{Y_{t+1}}{K_{t+1}} + (1-\delta)q_{t+1}\right] \tag{3.81}$$

and

$$q_t\left[1 - \frac{\phi}{2}\left(\frac{I_t}{I_{t-1}} - \mu_{t-1,t}\right)^2 - \phi\left(\frac{I_t}{I_{t-1}} - \mu_{t-1,t}\right)\frac{I_t}{I_{t+1}}\right] = 1 - \frac{\phi q_{t+1}}{R_{t+1}}\left(\frac{I_{t+1}}{I_t} - \mu_{t,t+1}\right)\left(\frac{I_{t+1}}{I_t}\right)^2. \tag{3.82}$$

1.3 Government

In addition to wasteful spending G_t, government pays the retirees social security benefits S_t, and finances these with tax revenues and by issuing debt. Government spending and social security are assumed exogenous fraction of output: $\frac{G_t}{Y_t} = g_t$ and $\frac{S_t}{Y_t} = s_t$. Government's per period budget constraint is

$$B_{t+1} = R_t B_t + G_t + S_t - T_t \tag{3.83}$$

and iterating forward, the intertemporal budget constraint is

$$R_t B_t = \sum_{v=0}^{\infty}\frac{T_{t+v}}{\prod_{z=1}^{v} R_{t+z}} - \sum_{v=0}^{\infty}\frac{G_{t+v}}{\prod_{z=1}^{v} R_{t+z}} - \sum_{v=0}^{\infty}\frac{S_{t+v}}{\prod_{z=1}^{v} R_{t+z}}. \tag{3.84}$$

With elastic labor supply, total tax revenue is

$$T_t = \tau_t W_t \left(\xi l_t^r N_t^r + l_t^w N_t^w \right) + T_t^w \quad . \tag{3.85}$$

The lump sum tax $T_t^w = N_t^w T_t^{jw}$ is exogenous. labor tax τ_t endegenously adjusts according to the following fiscal policy rule:

$$\tau_t = \tau_{t-1} + \theta \left[b_t - \bar{b_t} \right], \tag{3.86}$$

where $\bar{b_t}$ is an exogenous steady state debt/output share.

1.4 A competitive world equilibrium and the external sector

A competitive world equilibrium is a sequence of quantities and prices such that in each country (i) households maximize utility subject to their budget constraint, (ii) firms maximize profits subject to their technology constraints, (iii) the government chooses a path for taxes and debt, compatible with intertemporal solvency, to finance exogenous level of total spending, and (iv) all markets clear.

In each economy, total assets are the sum of capital stock, government bonds and net foreign assets F_t:

$$A_t = K_t + B_t + F_t. \tag{3.87}$$

Net foreign asset position evolves according to

$$F_{t+1} = R_{t+1} F_t + NX_t \tag{3.88}$$

where NX_t is the trade balance

$$NX_t = Y_t - (C_t + I_t + G_t). \tag{3.89}$$

Return $R_{W,t}$ is equalized across the two countries because of the clearing of the international asset markets, which implies that internationally traded assets are in zero net supply:

$$F_t + F_t^* = 0. \tag{3.90}$$

Denoting the relative size of the economies by

$$RS_t = \frac{X_t N_t}{X_t^* N_t^*}, \tag{3.91}$$

the relative size of the economies evolves according to

$$\frac{X^*_{t+1}N^*_{t+1}}{X_{t+1}N_{t+1}} = \frac{X^*_t N^*_t(1 + x^*_{t,t+1} + n^*_{t,t+1})}{X_t N_t(1 + x_{t,t+1} + n_{t,t+1})}. \tag{3.92}$$

Given an initial relative size $\bar{RS}$ in period 1, the size in period T is

$$\begin{aligned} RS_T &= \bar{RS}(1 + x_1 + n_1)(1 + x_2 + n_2)...(1 + x_T + n_T) \\ &= \bar{RS}\prod_{s=1}^{T-1}(1 + x_{1+s} + n_{1+s}). \end{aligned} \tag{3.93}$$

Exogenous policy variables, government spending to output ratio g_t and social security to output ratio s_t, follow first order autoregressive processes as follows:

$$g_t = (1 - \theta_t)g^{ss} + \theta_t g_{t-1} + u^g_t$$

and

$$s_t = (1 - \theta_t)s^{ss} + \theta_t s_{t-1} + u^s_t.$$

4 China's macroeconomic policies and spillover effects

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4.1 Introduction

China's rapid economic growth and its rise to become the world's largest merchandise trader[1] has created tensions between China and other large economies. These tensions come from the fact that China's trade has been persistently imbalanced, and the imbalances have been interpreted as being, at least to some extent, a result of government policies which distort both the country's intratemporal terms of trade, *i.e.* the price of foreign goods in the domestic market, and the intertemporal terms of trade, *i.e.* the real interest rate, at the expense of China's trading partners. Throughout the 2000s, China has run trade and current account surpluses against the rest of the world and against its main trading partner, the United States[2].

The objective of this paper is to provide a quantitative analysis of the effects of China's macroeconomic policies on the dynamics of its trade balance and the real interest rate in the 2000s, and the spillover effects of these policies on the United States. The policies analyzed are i) capital controls, which allow the government to set a domestic interest rate that differs from the world interest rate, ii) undervaluation of the real exchange rate, and iii) fiscal policy, especially the strong growth of the Chinese general government's expenditures and the large fiscal deficits observed during the 2000s.

[1]Source: WTO World Trade Statistical Review, 2017.

[2]Current account balance is shown in Appendix 4.D, figure 4.15.

While earlier theoretical literature on capital controls and real exchange rates provides qualitative indications on the effects of these policies on the external sector, the novelty of this paper lies in providing a theory-based, quantitative and dynamic assessment of the importance of government policies on the observed external imbalances between China and the United States in the 2000s. In the analysis, I take into account differences in demographic structures as a factor which affects the transmission of government policies. I also analyze the spillover effects that Chinese macroeconomic policies are likely to have on its trading partners.

In order to quantify the impact of the above-mentioned policies, I build a two-country dynamic general equilibrium model embedded with policy instruments describing the behavior of Chinese policymakers. I assume that a consolidated Chinese government-central bank can impose capital controls, which prevent the private Chinese agents from holding foreign assets, and control directly an exogenously given real exchange rate. The model also features a life-cycle structure in which the households live through two different stages in life and the sizes of cohorts vary over time. In this framework, fiscal policy is non-Ricardian, and its effects depend on the demographic parameters. The life-cycle feature is important for the quantitative analysis since the demographic structures between the economies differ considerably. The life-cycle structure also facilitates the analysis of life-cycle behavior on households' consumption-saving choices, which is important given that the Chinese current account surplus is driven by Chinese households' high saving rate.

The reasons for considering the above-mentioned policies as a potential cause for the trade imbalances are as follow.

First, despite maintaining an open trade stance, China's restrictions on its capital account have been relatively high by international standards (see the left panel in figure 4.1), and free capital mobility, especially in portfolio investments, has been prevented.[3,4] Even though liberalization of the capital account has been set as an official target, and measures to allow higher capital mobility have taken place, the schedule of full capital account liberalization remains unknown. Capital controls have given China the ability to exercise an independent interest rate policy despite its exchange rate regime, in which the yuan has

[3] The historical development of the capital control index is shown in figure 4.13 in Appendix 4.D.

[4] Foreign direct investments (FDI) have been less stringently regulated, which is reflected in a relatively large share of FDI in foreign liabilities (figure 4.14 in Appendix 4.D). For a review of capital controls on different asset classes, see Fernández et al. (2016).

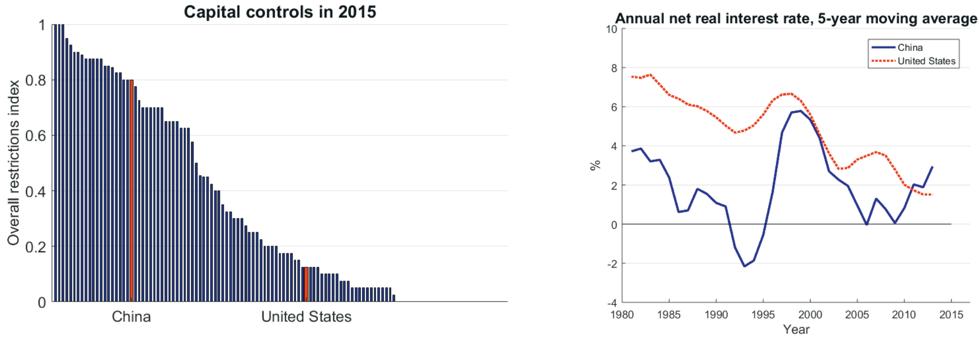

Figure 4.1: Left panel: Capital controls index (overall restrictions index, including all asset classes). Source: Fernández et al. (2016). Right panel: Real interest rates (lending interest rate adjusted for inflation as measured by the GDP deflator). Source: World Bank World Development Indicators 2018 and author's calculations.

effectively been anchored to the U.S. dollar (IMF, 2014)[5]. According to World Bank data (see right panel in figure 4.1), the United States' real interest rate has exceeded the Chinese real interest rate rate considerably from the 1980s until the Asian financial crises in 1997; in the 2000s, the Chinese real interest rate has been below the US real interest rate (5 basis points on average) as well. By controlling the domestic real interest rate, the government has been able to distort the intertemporal terms of trade faced by the private sector and thereby the trade balance. As the private sector is largely prevented from accessing the international financial market, persistent trade surpluses have led to the accumulation of a sizeable foreign reserve position held by the People's Bank of China (PBOC), which constitutes the majority of China's external assets (see figure 4.14 in Appendix 4.D). To capture this, in the model, I assume that the consolidated central bank is the only Chinese agent able to access the international financial market. The domestic interest rate is assumed to be exogenous and directly controlled by the central bank.

Second, the Chinese real exchange rate is estimated to have been undervalued for an extended period of time during the 2000s and 2010s. According to Goldstein and Lardy (2009), the undervaluation was approximately 20 % from 2002 to 2008. Also according to the IMF Series of External Sector Reports (IMF,

[5]Since 2016, the yuan has been classified as de facto anchored to a composite basket instead of the dollar.

2014, 2015, 2016), the REER was slightly undervalued until as late as 2014 after which it reached a level which the IMF describes as being in line with the fundamentals.[6][7] Figure 4.2 presents estimates of the REER undervaluation based on Goldstein and Lardy (2009) and by the IMF (the right panel). From the figure it can also be seen that the REER has closely followed the bilateral real exchange rate of the renmimbi vis-a-vis the dollar (the left panel). The period of renmimbi undervaluation coincides with the substantial increase in the current account surplus in 2006-2007.[8] In the simulations, I assumed that the real exchange rate is exogenous and matches the estimates by Goldstein and Lardy (2009) and the IMF.

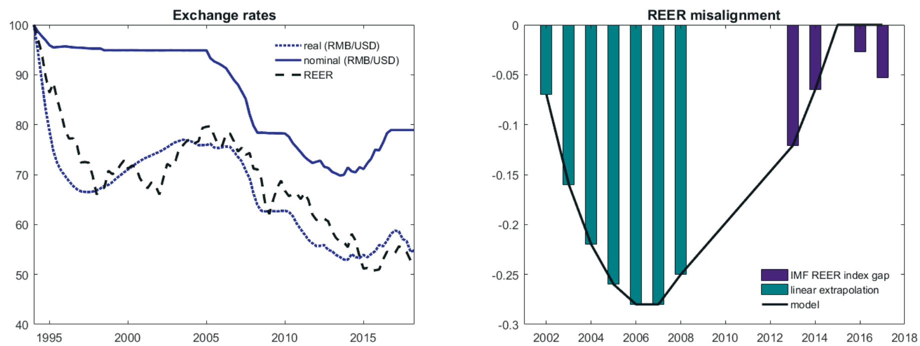

Figure 4.2: Left panel: Nominal China / US exchange rate (solid line), real China / US exchange rate (dashed blue line), China's real effective exchange rate (dotted black line). All data is quarterly. Data sources: Federal Reserve Bank of St. Louis (exchange rates), OECD (inflation).
Right panel: China REER misalignment based on IMF REER index gap and linear extrapolation as in Goldstein and Lardy (2009).

Third, the size of the general government, measured by the government consumption as a share of GDP, has grown markedly in the 2000s in China, and government expenditures have been financed, to a growing extent, with

[6]For the methodology of the undervaluation assessment, see IMF, 2019.

[7]Alternative estimates on the size of the undervaluation are summarized by Goldstein (2008). 17 out of 18 studies from the years 2000-2007 find that the real effective exchange rate (henceforth, REER) of the renmimbi was undervalued in the first part of the 2000s, by 17 % in years 2000-2004 and by 26 % between 2005-2007 on average.

[8]See figure 4.15 in Appendix 4.D.

net borrowing (see figure 4.5). General government total expenditures grew from approximately 17 % to over 30 % of GDP between 2000 and 2015, and the growth in government revenues has not sufficed to close the fiscal deficit, especially in the latter part of the 2010s. The observed fiscal deficits have the potential to explain China's high savings rate to the extent that they have led to an increase in private savings in anticipation of high future tax rates. At the same time, the United States has also been running a persistent government budget deficit, which widened in the aftermath of the financial crisis and has led to a substantial increase in public debt.

Finally, the demographic structures of China and the United States differ significantly from each other, and demographic developments have changed the population age structures both in China and the US in the past decades (see the United Nations' 2017 demographic projections, figure 4.3). Demographic changes have been more evident in China than in the US, and this asymmetry can potentially explain at least to some extent the observed imbalances in the external sector. In addition, the demographic structure affects the transmission of public policies in the economies by altering the share of different age groups in the economy, whose marginal consumption and saving propensities differ from each other, and by affecting the households' discount rate. Therefore it is important to analyze the impacts of the policies in a model that features a life-cycle structure.

According to my analysis, the macroeconomic policies have had a positive effect on the Chinese trade balance in the 2000s, driven by the real exchange rate undervaluation, which has caused households to postpone consumption into the future. Observed fiscal policy has also had a positive impact on the Chinese trade balance. The impact of the interest rate policy on the trade balance has been negative, because the policy of low interest rates has encouraged investment and discouraged savings. The removal of capital controls, which would lead to an equalization of the domestic interest rate with the international interest rate, would have a positive effect on the Chinese trade balance. Furthermore, I find that the effect of the macroeconomic policies on the real interest rate has been positive, as they have resulted in a reduction in world aggregate savings. Demographic change has, on the contrary, put downward pressure on the international interest rate, as it has strengthened the saving-for-retirement motive in both economies. Demographic change has also resulted in a long-term improvement in China's trade surplus because of the relatively large increase in life expectancy, even though the periods of high population growth in the early 2000s have lowered the trade surplus temporarily. The impact of the exchange rate policy is less clear as it is sensitive to the value of elasticity of intertemporal substitution.

4.2 Related literature

My paper is related mainly to the four following strands of literature: first, theoretical literature that applies the theory on capital controls and real exchange rates to study the consequences of capital controls and exchange rate policy in the Chinese context; second, a broader strand of literature on the optimality of capital controls analyzed with two-country models; third, theoretical literature on real exchange rate and terms of trade manipulation; and fourth, literature linking demographics and the dynamics of the external sector.

The first strand of literature on the consequences of capital controls and exchange rate policy in the Chinese context includes papers by Song et al. (2014), Bacchetta et al. (2013) and Chang et al. (2015). Song et al. (2014) analyze the economic consequences of capital controls and domestic interest rate and exchange rate policies on measures of economic performance, including the trade surplus, with a dynamic two-period OLG model that features the endogenous economic transition mechanism by Song et al. (2011). Like Song et al. (2014), Bacchetta et al. (2013) analyze the impact of capital controls and reserve accumulation in a model in which a consolidated government-central bank is by assumption the only agent that can access the international financial market. In Bacchetta et al. (2013) the economy is a simple endowment economy, where the households are assumed to be borrowing-constrained, and exogenous output growth and endowment fluctuations give rise to consumption smoothing motives. Bacchetta et al. (2013) show that in a fast-growing economy like China, the central bank can subsidize or tax households' borrowing and saving by manipulating the interest rate, and influence the intertemporal allocation of resources in a welfare-improving way. My paper is close to Song et al. (2014) and Bacchetta et al. (2013) as it assumes that domestic (Chinese) households cannot access the international financial market. In addition, like Song et al. (2014), I assume that there is a consolidated government-central bank which accumulates foreign reserves and exercises interest rate and exchange rate policies. However, in contrast to Song et al. (2014), I generalize the two-stage OLG structure into a Gertler (1999) life-cycle model, in which the households are assumed to spend realistic average lengths of time as workers and retirees. This allows me to take into account the impact of the demographic transition and rationalizes the observed high saving propensity through population ageing. Unlike Song et al. (2014), my analysis is quantitative, and it is performed in a two-country framework, which allows me to analyze the interest rate dynamics, the spillover effects on the foreign economy through the interest rate, and the feedback effects on the domestic economy through endogenous foreign demand.

Chang et al. (2015) study the impact of China's macroeconomic policies on its welfare and the external balance with a New-Keynesian DSGE model with a focus on the monetary policy trade-off that sterilization of the PBOC interventions involves. In the model, sterilization is costly if the interest rate on domestic government bonds is above the world interest rate, but unsterilized foreign exchange interventions would result in an increase of money supply and higher inflation. My work is related to the paper by Chang et al. (2015) as I analyze the impact of government dissaving through bond issuance, but unlike Chang et al. (2015), who focus on the case in which government reserve accumulation can only be financed by bond issuance (or money creation), my focus is on the case where the government can accumulate reserves with taxation, as the households in my model are heterogeneous and therefore tax policy is non-neutral.

Second, my work is related to the following papers, which study the effects of capital controls in two country models. Costinot et al. (2014) analyze the use of capital controls as a tool to manipulate the world real interest rate and thereby the intertemporal terms of trade. De Paoli and Lipinska (2013), Brunnermeier and Sannikov (2015) and Heathcote and Perri (2016) focus on the role of aggregate uncertainty and incomplete asset markets, Devereux and Yetman (2014) analyze the possibility of capital controls to restore monetary policy independence at the zero lower bound, and Coeurdacier et al. (2015) analyze the impact of growth and credit constraints on external imbalances. Jeanne and Korinek (2010), Bianchi (2011), Korinek (2017) and Benigno et al. (2016) focus on the optimality of capital controls as a measure to correct pecuniary externalities related to foreign borrowing. The main difference between my paper and this strand of literature is that the analysis in the latter is normative whereas mine is positive and aims at explaining the link between observed capital controls and dynamics in the external sector. This strand also abstracts from consideration of the demographic factors.

Third, the analysis of the effects of the real exchange rate on the external sector is related to Harberger (1950), Laursen and Metzler (1950), Obstfeld and Rogoff (1995),Corsetti and Pesenti (2001) and Cashin and McDermott (2003). Harberger (1950) and Laursen and Metzler (1950) analyze the link between currency valuation and the trade balance, and explore the negative income effect of an adverse terms of trade shock on private savings. Obstfeld and

Rogoff (1995) and Corsetti and Pesenti (2001) analyze the transmission of monetary and fiscal policy shocks in a two-country DSGE models, discussing also the substitution effect related to terms of trade shocks. Cashin and McDermott (2003) explore the role of intertemporal and intratemporal substitution in influencing the responses to terms-of-trade shocks in a small open economy. Uribe and Schmitt-Grohé (2017) discuss the empirical evidence on terms-of-trade shock and the external sector.

Finally, the paper is related to literature on the impacts of demographic change on the real interest rate and the trade balance, and on the distributional effects of policy shocks, including papers by Kilponen et al. (2006), Fujiwara and Teranishi (2008), Ferrero (2010), Carvalho et al. (2016) and Niemeläinen (2017), which all build on the Gertler (1999) framework. Fujiwara and Teranishi (2008) show that the impact of monetary policy shocks, in the context of the model, depend on the demographic structure of the economy. Ferrero (2010) and Carvalho et al. (2016) discuss the role of demographic change as a driver behind the decline of the real interest rate. The main difference between my model and the previous literature with the Gertler (1999) framework are the embedded policy instruments. In addition, with the exception of Ferrero (2010) and Niemeläinen (2017), the previous literature studies closed or small open economies.

4.3 The model

The model economy consists of two countries populated by finitely lived agents with the Gertler (1999) life-cycle structure. In country 1 (China), a consolidated government-central bank imposes capital controls on outflows so that the domestic private agents can invest in domestic firms or government bonds but not in foreign government bonds or firms, and on inflows so that foreign investors cannot hold private domestic assets. The government-central bank issues domestically held debt to match the domestic asset demand given the interest rate, which is exogenously set by the government-central bank. It also collects taxes, consumes and intervenes in the foreign exchange markets by acquiring the net income by exporters and using the proceeds to acquire foreign reserves according to its fiscal policy rule. The foreign reserves are composed solely of the government bonds of country 2.

In both economies, a representative firm produces an internationally traded good with Cobb-Douglas production technology with labor-augmenting productivity.

4.3.1 Households

All households are assumed to be born as workers at the age of 20 years and face the probability $1-\omega_{t,t+1}$ to retire between periods t and $t+1$. In retirement, households die with probability $1-\gamma_{t,t+1}$. The growth rate of the number of workers is $n^w_{t,t+1}$ and the old age dependency ratio, ψ_t, evolves according to

$$\psi_{t+1}(1+n^w_{t,t+1}) = (1-\omega_{t,t+1}) + \gamma_{t,t+1}\psi_t \tag{4.1}$$

where ψ_t is defined as $\psi_t = \frac{N^t_t}{N^w_t}$.

4.3.1.1 Retirees

A retiree born in period j and retired in period i chooses consumption-saving allocation to maximize her/his expected continuation value given by

$$V^{jr}_t(i) = \max_{C^{jr,1}_t(i), C^{jr,2}_t(i), A^{jr,1}_{t+1}(i)} \left\{ \left(C^{jr}_t(i)\right)^{\rho} + \beta\gamma_{t,t+1}\left(V^{jr}_{t+1}(i)\right)^{\rho} \right\}^{\frac{1}{\rho}}. \tag{4.2}$$

Households consume both domestic goods, $C^{jr,1}_t(i)$, and foreign goods, $C^{jr,2}_t(i)$, and the consumption index of a retiree is given by

$$C^{jr}_t(i) = \left(\left(C^{jr,1}_t(i)\right)^{\frac{\varepsilon-1}{\varepsilon}} + \left(C^{jr,2}_t(i)\right)^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{4.3}$$

where ε is the elasticity of substitution between domestic and foreign goods (Armington elasticity). The budget constraint of the retiree is

$$A^{jr}_{t+1}(i) = \frac{R^d_t A^{jr}_t(i)}{\gamma_{t-1,t}} - C^{jr,1}_t(i) - e_t C^{jr,2}_t(i) \tag{4.4}$$

where R^d_t is the domestic interest rate set by the local authority, e_t is the price of the foreign good in terms of a domestic good (the real exchange rate), $e_t C^{jr,2}_t(i)$ consumption expenditures of the foreign good in local consumption units and $A^{jr}_t(i)$ the value of financial wealth.

The first order condition with respect to asset accumulation is

$$\left(\left(C_t^{jr,1}(i)\right)^{\frac{\varepsilon-1}{\varepsilon}}+\left(C_t^{jr,2}(i)\right)^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon\rho}{\varepsilon-1}-1}\left(C_t^{jr,1}(i)\right)^{\frac{\varepsilon-1}{\varepsilon}-1}$$

$$=\beta\gamma_{t+1}(V_{t+1}^{jr}(i))^{\rho-1}\frac{\partial V_{t+1}^{jr}(i)}{\partial A_{t+1}^{jr}(i)} \quad (4.5)$$

and the first order condition with respect to foreign consumption is

$$C_t^{jr,1}(i)e_t^{-\varepsilon}=C_t^{jr,2}(i). \quad (4.6)$$

Retiree's Euler equation is

$$C_{t+1}^{jr,1}(i)=C_t^{jr,1}(i)\left[\beta R_{t+1}^d\right]^{\sigma}\left(\frac{1+e_t^{1-\varepsilon}}{1+e_{t+1}^{1-\varepsilon}}\right)^{\frac{\varepsilon-\sigma}{\varepsilon-1}}, \quad (4.7)$$

and their marginal propensity to consume evolves according to

$$\epsilon_t\pi_t=1-\frac{\epsilon_t\pi_t}{\epsilon_{t+1}\pi_{t+1}}\gamma_{t,t+1}\beta^{\sigma}(R_{t+1}^d)^{\rho\sigma}\left(\frac{1+e_t^{1-\varepsilon}}{1+e_{t+1}^{1-\varepsilon}}\right)^{\frac{1-\sigma}{\varepsilon-1}}. \quad (4.8)$$

4.3.1.2 Workers

A worker born in period j chooses consumption-saving allocation to maximize her/his expected continuation value given by

$$V_t^{jw}=\max_{C_t^{jw,1},C_t^{jw,2},A_{t+1}^{jw}}\left\{\left(C_t^{jw}\right)^{\rho}+\beta\left[\omega_{t+1}V_{t+1}^{jw}+(1-\omega_{t+1})V_{t+1}^{jr}\right]^{\rho}\right\}^{\frac{1}{\rho}} \quad (4.9)$$

where $C_t^{jw}=\left(\left(C_t^{jw,1}\right)^{\frac{\varepsilon-1}{\varepsilon}}+\left(C_t^{jw,2}\right)^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}}$ is subject to the periodic budget constraint

$$A_{t+1}^{jw}=R_t^dA_t^{jw}+W_t-C_t^{jw,1}-e_tC_t^{jw,2}-T_t^{jw}. \quad (4.10)$$

The first order condition with respect to asset accumulation is

$$\left(\left(C_t^{jw,1}\right)^{\frac{\varepsilon-1}{\varepsilon}}+\left(C_t^{jw,2}\right)^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon\rho-\varepsilon+1}{\varepsilon-1}}\left(C_t^{jw,1}\right)^{-\frac{1}{\varepsilon}}$$

$$=\beta\left(\omega_{t+1}V_{t+1}^{jw}+(1-\omega_{t+1})V_{t+1}^{jr}\right)^{\rho-1}\left[\omega_{t+1}\frac{\partial V_{t+1}^{w}}{\partial A_{t+1}^{w}}+(1-\omega_{t+1})\frac{\partial V_{t+1}^{rj}}{\partial A_{t+1}^{wj}}\right] \quad (4.11)$$

and the first order condition with respect to consumption of foreign goods is

$$C_t^{jw,1}e_t^{-\varepsilon}=C_t^{jw,2}. \quad (4.12)$$

The Euler equation for the worker is

$$\omega_{t+1}C_{t+1}^{jw}+(1-\omega_{t+1})(\epsilon_{t+1})^{\frac{\sigma}{1-\sigma}}C_{t+1}^{jr}=C_t^{jw,1}\left[\beta R_{,t+1}^{d}\Omega_{t+1}\right]^{\sigma}\left(\frac{1+e_t^{1-\varepsilon}}{1+e_{t+1}^{1-\varepsilon}}\right)^{\frac{\varepsilon-\sigma}{\varepsilon-1}} \quad (4.13)$$

and her/his marginal propensity to consume evolves according to

$$\pi_t=1-\frac{\pi_t}{\pi_{t+1}}\beta^{\sigma}(R_{t+1}^{d}\Omega_{t+1})^{\sigma-1}\left(\frac{1+e_t^{1-\varepsilon}}{1+e_{t+1}^{1-\varepsilon}}\right)^{\frac{1-\sigma}{\varepsilon-1}}. \quad (4.14)$$

The workers' human wealth equals

$$H_t^{wj}=W_t-T_t^{jw}+\frac{\omega_{t+1}H_{t+1}^{jw}}{\Omega_{t+1}R_{t+1}^{d}} \quad (4.15)$$

where Ω_t is a variable which augments the interest rate in worker's marginal propensity to consume (eq. 4.14), defined as

$$\Omega_t\equiv\omega_{t-1,t}+(1-\omega_{t-1,t})\epsilon_t^{\frac{1}{1-\sigma}}. \quad (4.16)$$

4.3.2 Aggregation

Because the marginal propensities to consume (equations (4.8) and (4.14)) do not depend on individual characteristics, aggregate consumption expenditures by the retirees can be solved to be a fraction of aggregate financial wealth held by the retirees

$$C_t^{r}=\epsilon_t\pi_t(R_t^{d}\lambda_t A_t) \quad (4.17)$$

where $\lambda_t \equiv \frac{A_t^r}{A_t}$ is the share of non-human wealth held by retirees. Similarly, aggregate consumption expenditures of workers is given by

$$C_t^w = \pi_t (R_t^d (1 - \lambda_t) A_t + H_t^w). \tag{4.18}$$

Retirees and workers consume domestic and foreign goods in the same proportion, the consumption shares of domestic and foreign goods is determined by

$$C_t^1 e_t^{-\varepsilon} = C_t^2. \tag{4.19}$$

Given that aggregate consumption expenditures in the domestic currency are $C_t = C_t^1 + e_t C_t^2 = C_t^1(1 + e_t^{1-\varepsilon}) = \pi_t A_t R_t^d(\epsilon_t \lambda_t + 1 - \lambda_t) + \pi_t(H_t^w)$, aggregate consumption expenditures of domestic good in country 1 are given by

$$C_t^1 = \frac{\left(\pi_t A_t R_t^d(\epsilon_t \lambda_t + 1 - \lambda_t) + \pi_t(H_t^w)\right)}{(1 + e_t^{1-\varepsilon})} \tag{4.20}$$

and aggregate consumption expenditures of the foreign good in country 1 by

$$C_t^2 = \frac{e_t^{-\varepsilon}\left(\pi_t A_t R_t^d(\epsilon_t \lambda_t + 1 - \lambda_t) + \pi_t(H_t^w)\right)}{(1 + e_t^{1-\varepsilon})}. \tag{4.21}$$

Households' aggregate financial wealth is the sum of the domestic capital stock and government bonds, given by

$$A_t = K_t + B_t. \tag{4.22}$$

4.3.3 Consolidated government-central bank

The consolidated government-central bank issues real one-period bonds, consumes, collects taxes and trades international assets. Because of capital controls, it is the only agent that can hold foreign assets. The government's flow budget constraint is given by

$$B_{t+1} - e_t B_{t+1}^* = R_t^d B_t - R_{w,t} e_t B_t^* + G_t - T_t \tag{4.23}$$

where B_t stands for the outstanding amount of domestic government bonds, and B_t^* for the outstanding value of foreign assets. The left-hand-side of equation (4.23) is the net debt of the central-bank government.[9] $R_{w,t}$ stands for the

[9] I follow the convention by Bacchetta et al. (2013) and Chang et al. (2015) where a positive value

interest rate on foreign assets, $R_{d,t}$ the interest rate on domestic government debt, G_t government expenditures, and T_t taxes.

In "normal times" the government-central bank follows a balanced budget rule, given by

$$T_t = (R_t^d - (1 + x_{t,t+1} + n_{t,t+1}))B_t \\ - e_t(R_{w,t} - (1 + x_{t,t+1} + n_{t,t+1}))B_t^* + G_t, \quad (4.24)$$

which keeps net government wealth constant. In the quantitative analysis I allow the government-central bank to follow fiscal policy so that the tax-to-output ratio, $\tau_t \equiv \frac{T_t}{Y_t}$, matches an exogenously given value which is calibrated to directly match the data. Government spending is an exogenously determined fraction of the output $\frac{G_t}{Y_t} = \bar{g}_t$ also calibrated to match the data.

4.3.4 The external sector

The net foreign asset position in country 1 evolves according to

$$e_t B_{t+1}^* = e_t R_{t+1} B_t^* + NX_t \quad (4.25)$$

where NX_t is the trade balance, defined as

$$NX_t \equiv Y_t - (C_t + I_t + G_t) \quad (4.26)$$

where C_t are aggregate domestic consumption expenditures. The interest rate on internationally traded assets,$R_{W,t}$, pinned down by the financial market clearing condition

$$B_t^{1,*} + B_t^{2,*} = 0 \quad (4.27)$$

which states that the assets are in zero net supply. In country 2, by arbitrage, the international interest rate equals the domestic interest rate. However, the central bank-government in country 1 is assumed to be able to control the domestic interest rate in country 1 directly so that it can differ temporarily from the international interest rate ($R_t^d \neq R_{w,t}$). This is possible because the capital controls imposed by country 1 prevent free capital mobility.

The internationally traded foreign assets are denoted in country 2 consumption units. Equations for country 2 are analogous, but they are denoted in local

of B_t^* indicates a net creditor position.

consumption units. The rate of exchange in country 2 is the inverse of country 1's rate of exchange, $e_t^* = \frac{1}{e_t}$.

Definition 1 (a competitive world equilibrium)
A competitive world equilibrium without policy interventions is a sequence of quantities and prices such that in each country (i) households maximize utility subject to their budget constraint, (ii) firms maximize profits subject to their technology constraints, (iii) the government chooses a path for taxes and debt, compatible with intertemporal solvency, to finance exogenous level of total spending, and (iv) all markets clear.

4.3.5 Transmission channels of transitory macroeconomic policies

In the quantitative analysis, I allow country 1 to practice transitory macroeconomic policies which cause the economy to deviate temporarily from the equilibrium conditions laid out in the previous section. In this section I describe the equilibrium conditions under activist macroeconomic policies and describe the policy transmission channels. In describing the transmission channels, I use the following decomposition of central bank - government's periodic budget constraint (eq. (4.28)):

$$\begin{aligned} e_t B_{t+1}^* &= B_{t+1} + e_t R_{w,t} B_t^* - R_t^d B_t + T_t - G_t \qquad (4.28) \\ &= \underbrace{A_{t+1} - K_{t+1}}_{\text{private saving gap}} + \underbrace{e_t R_{w,t} B_t^* - R_t^d B_t + T_t - G_t}_{\text{net government savings}}. \end{aligned}$$

The second equality follows from the fact that the government-central bank issues domestic debt to meet the domestic asset demand at the given domestic interest rate R_t^d, and therefore domestic government debt equals the domestic private saving gap ($B_{t+1} = A_{t+1} - K_{t+1}$). Therefore, the value of foreign assets equals the sum of the private saving gap and net government savings.

4.3.5.1 Exchange rate policy

The exchange rate, e_t, is exogenous, and the government of country 1 is assumed to be able to control it directly. An exchange rate policy shock (a terms of trade shock) is defined as a temporary under-/overvaluation of country 1's currency ($e_t > 1/ \ e_t < 1$), and simultaneous over-/undervaluation of country 2's exchange rate e_t^*. I define the equilibrium under exchange rate policy as follows:

> **Definition 2 (a competitive world equilibrium under active exchange rate policy)**
> A competitive world equilibrium under active exchange rate policy is a sequence of quantities and prices such that given the exogenous level of exchange rate e_t for $t \in [n, m]$, $n, m \in \mathbb{N}$, in each country (i) households maximize utility subject to their budget constraints, (ii) firms maximize profits subject to their technology constraints, (iii) the government chooses a path for taxes and debt, compatible with intertemporal solvency, to finance exogenous level of total spending, and (iv) all markets clear.

The impact of a terms of trade shock on private savings, A_{t+1}, is ambiguous. On one hand, the depreciation of the foreign exchange rate of country 1 ($e_t > 1$) lowers households' income, which impacts negatively consumption and savings (*i.e.* the Harberger-Laursen-Metzler (HLM) effect).[10] However, the negative terms of trade shock also induces a positive substitution effect on current savings, because it increases the relative price of current consumption in comparison to future consumption. With low elasticity of intertemporal substitution, σ, the HLM effect dominates, and the impact of real exchange rate appreciation on savings is negative.

In addition to intertemporal substitution, the households respond to the terms of trade shock by intratemporal substitution. The depreciation of the domestic exchange rate makes foreign goods more expensive in comparison to domestic goods, increasing the optimal consumption share of domestic goods and decreasing the share of foreign goods in the consumption basket, which can be seen from the optimality condition of period t ratio of foreign to domestic goods

$$\boldsymbol{e_t}^{-\varepsilon} = \frac{C_t^{jz,2}(i)}{C_t^{jz,1}(i)}$$

[10] The Harberger-Laursen-Metzler (HLM) effect, after Harberger (1950) and Laursen and Metzler (1950), is also known as the consumption-smoothing effect, as discussed by Cashin and McDermott (2003). Uribe and Schmitt-Grohé (2017) discuss empirical evidence on the HLM effect.

where $z = \{w, r\}$. With high elasticity of substitution between home and foreign goods ε, an adverse exchange rate shock shifts the consumption in the current period from the expensive foreign good to the domestic good.

The relative size of σ and ε determines whether the substitution towards domestic goods or the substitution of consumption into future periods has a larger impact on the consumption of the domestic goods. The retirees' and workers' consumption expenditures of domestic goods are decreasing in e_t as long as $\sigma > \varepsilon$, as then the substitution towards future periods is larger than the increase in the consumption of domestic goods.

To develop intuition for the dynamics of the real interest rate, the workers' savings can be shown in the limiting 2-period version of the model to be a concave function of the real exchange rate (see Appendix section 4.A). Therefore a symmetric terms of trade shock has a larger impact on the savings behavior of the country with an overvalued real exchange rate. If $\sigma > 1$, as in the simulations in section 4, the substitution effect dominates, and the currency undervaluation leads to an increase in domestic savings, and a decline in savings abroad. Because the decline in savings abroad exceeds the increase in domestic savings, the impact on world aggregate financial wealth is negative, and therefore the policy leads to an increase in the world real interest rate.

The terms of trade shock also has a direct impact on the external wealth held by the central bank-government. If country 1 is a net creditor, the undervaluation worsens its foreign asset position in terms of domestic consumption units. If the country follows the balanced budget rule (equation 4.24), fiscal policy must be adjusted to take the valuation effect into account. A worsening of the net external wealth calls for an increase in taxes, to keep the value of government net wealth in domestic consumption units constant.

4.3.5.2 Interest rate policy

The domestic interest is exogenous and the central bank-government in country 1 is assumed to be able to control it directly. When the domestic central bank-government is not practising active interest rate policy, the domestic interest rate equals the international interest rate determined by the asset market clearing condition ($R_t^d = R_t$). Interest rate policy is defined as a transitory deviation of the domestic interest rate from the international interest rate ($R_t^d \neq R_t$). I define the competitive world equilibrium under interest rate policy as follows:

Definition 3 (a competitive world equilibrium under active interest rate policy)
A competitive world equilibrium under active interest rate policy is a sequence of quantities and prices such that, given the exogenous level of the country 1 interest rate R_t^d for $t \in [n, m]$, $n, m \in \mathbb{N}$, in each country, (i) households maximize utility subject to their budget constraint, (ii) firms maximize profits subject to their technology constraints, (iii) the government chooses a path for taxes and debt, compatible with intertemporal solvency, to finance exogenous level of total spending, and (iv) all markets clear.

The impact of the interest rate policy on private savings is ambiguous. An increase in the domestic interest rate ($R_t^d > R_t$) has a positive impact on households' savings, A_{t+1}, because the price of present day consumption in terms of future consumption increases (substitution effect), and because the present value of lifetime wealth falls.[11] On the other hand, the policy has a positive income effect, which increases households' consumption and lowers savings. With high elasticity of intertemporal substitution, the substitution effect dominates, and the savings are increasing in interest rate. The impact of the increase in the interest rate on investments and the capital stock, K_{t+1}, is negative. The policy increases the marginal cost of capital and therefore drags down investment and output. With high elasticity of intertemporal substitution, the policy's impact on the private saving gap is unambiguously positive, as it has a positive impact on households' savings and a negative impact on investment.

To illustrate the impact of the interest rate policy on net government saving, eq. (4.28) can be further decomposed as follows:

$$\begin{aligned} e_t B_{t+1}^* &= A_{t+1} - K_{t+1} - R_t^d B_t + e_t R_{w,t} B_t^* + R_{w,t} B_t - R_{w,t} B_t + T_t - G_t \\ &= \underbrace{A_{t+1} - K_{t+1}}_{private\,saving\,gap} + \underbrace{R_{w,t}\left(e_t B_t^* - B_t\right)}_{interest\,income} - \underbrace{(R_t^d - R_{w,t}) B_t}_{policy\,cost/benefit} + T_t - G_t. \end{aligned} \tag{4.29}$$

Because the interest payments on domestic bonds differs from the interest income on foreign assets, the interest rate policy induces a cost or benefit to the

[11] In the workers' case, the negative wealth effect is reinforced by the presence of the additional discount factor Ω_t, which captures the expected finiteness of life, and the survival probability at work, $\omega_{t,t+1}$, which captures the fact that the worker has a positive probability to lose their future work stream. This can be seen in the workers' decision rule for consumption and the marginal propensity to consume (see eq. (4.18)). In retirees' case, the expected finiteness of life is mitigated by the perfect annuity market and no such reinforcement mechanism exists.

government, depending on the sign of the interest rate spread and whether the government is a net debtor or lender. Net government savings consist of interest income/expenses on net government assets less the cost of running the interest rate policy, and tax income net of government consumption expenditures. An increase in the domestic interest rate lowers net government savings as it induces a positive policy cost, but as the government follows the balanced budget rule, which keeps net government wealth constant, taxes depend positively on the domestic interest rate (see eq. (4.24)). The net effect on government savings is therefore ambiguous.

The policy increases demand for the foreign (internationally traded) bond, if the net effect on the external asset position is positive. The increasing asset demand lowers the interest rate in the international asset market, and therefore the domestic interest rate in country 2, increasing its capital stock and output.

4.3.5.3 Government expenditures and fiscal deficit

In "normal times", the government is assumed to follow the balanced budget rule (eq. (4.24)), which keeps the government's net wealth constant. In analyzing the quantitative impacts of fiscal policy, the government is allowed to deviate temporarily from the balanced budget rule and directly choose an exogenous tax rate τ_t. I also allow for transitory government expenditure shocks. I focus on the case characterized by the tax policy where $\tau_t < T_t$, since both China and the United States have run budget deficits for the most years in the period of interest (2000s). I define the competitive world equilibrium under active fiscal policy as follows:

> **Definition 4 (a competitive world equilibrium under active fiscal policy)**
> A competitive world equilibrium under active fiscal policy is a sequence of quantities and prices such that, given an exogenous level of tax-to-GDP ratio τ_t for $t \in [n, m]$, $n, m \in \mathbb{N}$, in each country (i) households maximize utility subject to their budget constraint, (ii) firms maximize profits subject to their technology constraints, (iii) the government chooses a path for taxes and debt for $t \notin [n, m]$, compatible with intertemporal solvency, to finance exogenous level of total spending, and (iv) all markets clear.

An increase in government expenditures, when financed with a balanced budget, has a negative impact on households' consumption because of a rise in tax liabilities. However, because of the expected finitude of working time and life, the households have a higher discount rate than the government, and therefore they do not fully capitalize the future tax stream associated with higher government expenditures. Therefore the decline in private consumption does not offset the rise in public consumption, and the net effect on national savings is negative.

Running a fiscal deficit has a direct negative impact on external asset holdings (see eq. (4.29)) through a decline in government savings, and an indirect effect through private savings. Lower taxes raise working age households' disposable income with a positive effect on current consumption. On the other hand, the agents anticipate higher taxes in the future, which has a positive effect on private savings. Because the agents do not fully capitalize future tax liabilities, which are postponed further into the future because of the budget deficits, the increase in private savings does not compensate for the public dissaving, and the net effect on the national savings is negative.

The households' discount rate varies with life expectancy and population growth rate, so that the extent to which the government debt increases the households' wealth depends on individuals' retirement age and life expectancy, and the demographic structure of the economy. An economy with lower life expectancy and higher population growth rate has a higher discount factor, and so the impact of government expenditures and debt on private consumption is stronger.

4.4 Quantitative analysis

In this section I evaluate the dynamic effects of the demographic transition and macroeconomic policies (exchange rate policy, interest rate policy and fiscal policy) on the external sector and the real interest rate in the 2000s. The quantification of the exchange rate policy is based on the estimates on the percentage deviation of the renmimbi real exchange rate from its equilibrium level by Goldstein and Lardy (2009) and by the IMF External Sector Reports 2013-2018, shown in figure 4.2. The quantification of the interest rate policy is based on the observed real interest rates in the World Bank data (see figure 4.1), and the quantification of the government deficits on the observations on government expenditures and fiscal deficits as in the IMF World Economic Outlook database (figure 4.5). Subsection 4.4.1 presents the calibration of the model and assumed paths of exogenous variables. In subsection 4.4.2, I analyze the effects of demographic change on the world real interest rate and the trade balance, and the resulting dynamics are used as a benchmark for the analysis of the impacts of the macroeconomic policies in section 4.4.3.

4.4.1 Calibration and exogenous variables

Table 4.1 reports the values of the calibrated parameters in the simulation. The calibration follows the literature so that the labor share of income, α, the discount factor, β, and the capital depreciation rate, δ, equal the values in the Gertler (1999) model.[12] The parameter for the intertemporal elasticity of substitution is set at $\sigma = 1.5$, which is a compromise between the two strands of literature, namely models with the Gertler framework (such as Gertler (1999), Fujiwara and Teranishi (2008), Ferrero (2010), and Kilponen et al. (2006)) in which the elasticity of intertemporal substitution takes values below unity, and the model by Song et al. (2014), a general equilibrium model with similar policy features as the one in this paper, in which the value of the parameter of elasticity of intertemporal substitution is 2. Because the results are sensitive to the value of σ, I report the main results for $\sigma = 0.5$ in section 4.C of the Appendix. In setting the value of elasticity of substitution between domestic and foreign goods, ε, I follow Song et al. (2014).

The values of the exogenous variables are shown in table 4.2. The frequency of the data is annual. The simulation entails a permanent demographic change which takes place over 39 periods as follows. In the initial period, life expectancies in both countries match the values in the data in 2001, and in the final period, the values in 2040. In the initial state, population growth rates equal the average population growth rate (the rate of growth of population between ages 20-64) between the two countries in 2001, and in the final state, an approximation of the value in 2040[13]. The probability of staying in the labor force is constant and calibrated to match an average retirement age of 65 years as in Auerbach and Kotlikoff (1987), Gertler (1999) and Ferrero (2010). Technological growth rate is assumed to be constant and equal between the countries and

[12] Because the analysis focuses on the effects of macroeconomic policies of the government, and demographic factors which affect the households, the calibration of the supply side parameters, α and δ, follows the convention in the literature. In addition, the discount factor β is calibrated as in Gertler (1999) as I am mainly interested in the workers' effective discounted factor, which is augmented by the adjustment term Ω_t and varies over time due to the demographic factors. I abstract from any differences in the values of these parameters across the countries. The analysis of cross-country or time variation of the parameters is left for future research.

[13] The average growth rate in 2040 is negative. I assume that there is no population growth in the final steady state.

matches the average in the data between 2001-2014. Government consumption is assumed to be a constant multiple of the GDP, matching the average in the data between 2001-2017 in each country. The government-central bank in both countries is assumed to have negative net wealth. The net government debt-to-GDP ratio of country 2 (US) is 31.5 % corresponding to the observed net debt in 2001 in the data. China's net debt is assumed to be 50 % smaller in efficiency units, corresponding to 17.5 % of GDP in the initial state (China).[14] In the simulation with fiscal policy, the increase in government net debt is taken into account endogenously. In other simulations, the government is assumed to follow a balanced budget rule.

The deterministic simulations are performed with the extended path algorithm, which allows the introduction ofunanticipated changes to exogenous policy variables.

Parameter		**Value**	**Source**
α	labor share of income	2/3	Gertler (1999)
β	discount factor	0.96	Gertler (1999)
δ	depreciation rate	0.1	Gertler (1999)
σ	elasticity of intertemporal substitution	1.5	Ferrero (2010)
ε	elasticity of substitution between home and foreign goods	2	Song et al. (2014)

Table 4.1: Calibration

Exogenous variable		**Initial state (2001)**	**Final state (2040)**	**Data source / reference**
γ^1	probability to survive, China	0.9062	0.9374	UN Population Prospects 2017
γ^2	probability to survive, US	0.9227	0.9465	UN Population Prospects 2017
$\omega^1 = \omega^2$	probability to stay in labor force	0.9778	0.9778	Auerbach and Kotlikoff (1987), Gertler (1999), Ferrero (2010)
$n^1 = n^2$	population growth rate	1.35 %	0.00 %	UN Population Prospects 2017
$x^1 = x^2$	technology growth rate	1.00 %	1.00 %	Penn World Table 9.0
g^1	government spending to ouput	16.2 %	16.2 %/30.2 %*	IMF World Economic Outlook 2018
g^2	government spending to ouput	30.2 %	30.2 %	IMF World Economic Outlook 2018
$\frac{B^1_{t+1} - e_t B^{1,*}_{t+1}}{Y^1_t}$	government net debt to ouput	17.5 %	(endogenous)	Estimate base on gross debt data from IMF World Economic Outlook 2018
$\frac{B^2_{t+1} - e^*_t B^{2,*}_{t+1}}{Y^2_t}$	government net debt to ouput	35.1 %	(endogenous)	World Bank, WEO

Table 4.2: Exogenous variables. *Final value in the simulation with fiscal policy.

[14]Data on China's general government net debt is not available.

4.4.2 Demographic change

Over the first decades of the 21st century, increase in longevity and decline in population growth rates have led to societal ageing, *i.e.* an increase in the old-age dependency ratio, both in China and the US (see figure 4.3). Because longevity has increased more in China, and the population growth rate has declined substantially more quickly, societal ageing has been faster in China. The old age dependency ratio rose by 37 % in China (from 10.5 to 14.5), and by 14 % in the US (from 21.7 to 24.7), between 2000 and 2015.

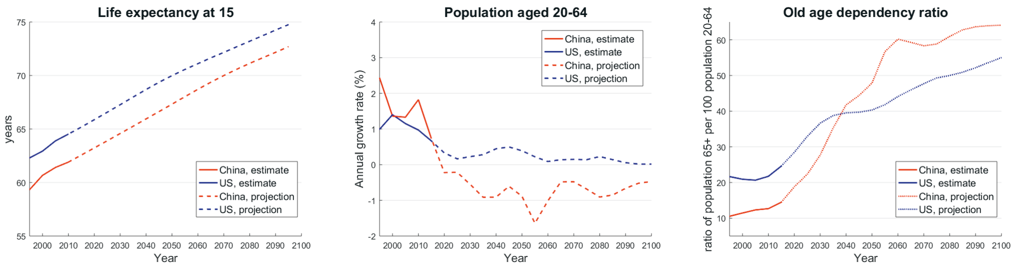

Figure 4.3: Historical estimates and long term projections of demographic trends in China and the US. "Year" on the x-axis corresponds to a five-year period in the data; e.g. "1980" corresponds to an estimated value in 1980-1985. Source: United Natios World Population Prospects 2017.

Societal ageing is projected to continue throughout the century and severely affect the demographic structure of the Chinese population in particular, due to a projected decline in the size of the working-age population.

Figure 4.4 shows the dynamic effects of the demographic changes, which follow the trends in the data closely, on the trade balance and the real interest rate. The increase of life expectancy raises saving in both economies and results in a fall in the real interest rate, because increasing longevity calls for higher savings to sustain the optimal income level in the lengthening retirement period. Because life expectancy increases more in China, the effect on savings of Chinese households is more pronounced, and therefore population ageing has a positive long-term effect on its trade balance. Also, the strong fall in the population growth rate has a positive impact on the trade balance in the long run. Temporary population growth surges, including the sharp increase in working-age population growth rate in China in early 2000s, result in temporary deteriorations of the Chinese trade balance, and a temporary halt in the fall of the real interest rate.

Here, the government-central bank is assumed to be passive and to not practice active macroeconomic policies: the domestic interest is equal to the international interest rate ($R_t^d = R_{w,t}$), the law of one price holds ($e_t = 1$) and the government-central bank follows the balanced budget rule (equation (4.24)) which keeps its net wealth constant, i.e. acts as an intermediator between the domestic private sector and the international financial market.

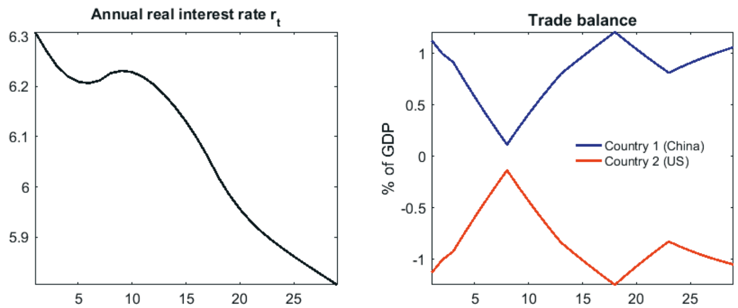

Figure 4.4: Impact of demographic change on the net real interest rate and the trade balance.

4.4.3 Policy interventions

Despite the transition of the Chinese economy over the past decades, the government still plays a central role in economic activity as a feature of a socialist market economy. In this section I analyze to what extent the Chinese central government has affected the observed trade imbalances and the dynamics of the intertemporal terms of trade, i.e. the real interest rate, via three policies: capital controls and interest rate policy, exchange rate policy, and fiscal policy.

In order to analyze the impacts of the macroeconomic policies on the external balance and the real interest rate, I construct a series of policy variables for the real exchange rate e_t, real interest rate R_t^d, tax τ_t and government expenditures g_t. The quantification of the policies is based on the following facts.

First, as discussed in the introduction, the capital controls prevent private international capital flows to a large extent, and therefore the government has been able to control the Chinese real interest rate, which has, despite its convergence, stayed below the US real rate in the 2000s. Accordingly, in the quantitative analysis, the domestic interest rate R_t^d is assumed to be 50 basis points below the world interest rate in 2002, and the spread is assumed to diminish gradually so that the interest rates are equalized after 2015 ($R_t^d = .995 \times R_{w,t}$ for $t = 1$ and follows and AR(1) process). The initial spread of 50 basis points is based on the average interest rate spread between China and the US between 2002-2015, and the decline of the spread is based on the assumption that the capital controls are gradually removed, which leads to equalization of the interest rates across the countries (figure 4.1).

Second, for an extended period of time, between 2001 and 2014, the Chinese real exchange rate was estimated to be undervalued. In the simulation, the central bank-government manipulates the price of the foreign goods in terms of domestic goods so that the real exchange rate is undervalued for 14 periods, which corresponds to the period of renmimbi undervaluation during 2001-2014 according to Goldstein and Lardy (2009) and the IMF ($e_t < 1$ for $t = 1 : 14$). The renmimbi undervaluation begins in 2002, reaches its peak in 2006-2007, and the equilibrium level is restored in 2015. Both the estimates and the policy variable, which is constructed based on the estimates, are shown in figure 4.2.

Third, the quantitative analysis of fiscal policy is based on the observed changes in government expenditures between 2001 and 2015, and the fiscal deficits in the 2000s. China has experienced a large growth in both total government expenditures and revenues between 2001 and 2015 (left panel in figure 4.5). However, for the majority of the period, the general government has run a budget deficit. After the early 2000s, the government deficit was reduced, but has grown sharply again since 2012 with the government trying to boost economic growth with expansionary fiscal policy. In the United States, general government expenditures and revenues have remained stable at approximately 30 % of GDP in the 2000s. The general government has run a budget deficit since 2002, and due to a sharp rise in government expenditures and a decline in revenues during and after the financial crises, the deficit increased to almost 12 % of the GDP and led to a substantial increase in the government's net debt. In the simulation, I use total government revenues as a share of GDP to estimate the total tax revenue of the government, and total government expenditures (which include net interest payments) to obtain an estimate of total government expenditures. In order to capture the effects of the budget deficits, I run as a counterfactual a simulation in which the government expenditures follow the dynamics in the data and the government is assumed to run a balanced budget, and compare the dynamics to the case in which the government's tax revenues follow the government revenues in the data, which results in budget deficits in both countries, and an increase in the total government net debt.

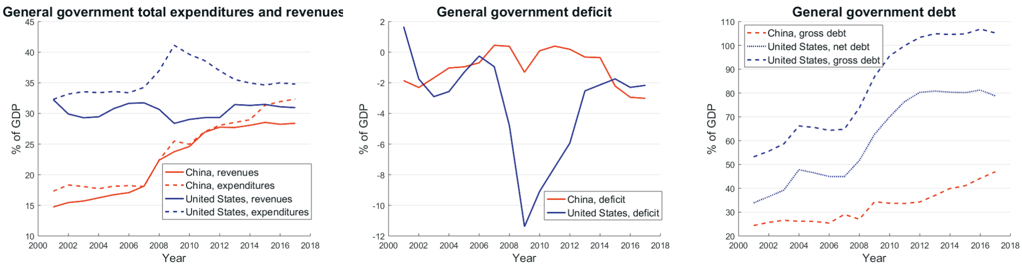

Figure 4.5: General government total expenditures and revenues, deficit and debt in 2001-2015. Data source: IMF World Economic Outlook Database October 2018.

The quantitative results of the simulation with active exchange rate, interest rate and fiscal policies are shown in figure 4.6, together with the results of the benchmark simulation of section 4.4.2. Overall, the policies raise the world real interest rate (black solid line) by approximately 0 to 30 basis points relative to the benchmark (black dashed line) in the short run (while the policies are active), and after a brief initial deterioration in the trade balance caused by a surge in investments, improve the trade balance of country 1 (China). In the long run, they increase the interest rate by approximately 10 basis points relative to the benchmark, and worsen the trade balance of country 1 (after its return to the laissez-faire economy). The results are mainly driven by the undervaluation of the exchange rate. Sections 4.4.3.1-4.4.3.3 analyze the impact of individual policies in more detail.

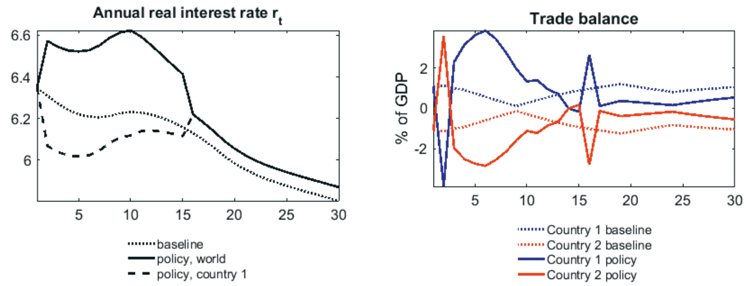

Figure 4.6: Impact of macroeconomic policies and demographics on the real interest rate and trade balance. The "baseline" variables refer to the interest rate and trade balances simulated in section 4.4.2 with demographic change under passive policy (where $R_t^d = R_{w,t}$). The "policy" variables refer to the simulated interest rates and trade balances with demographic change and government policy interventions (where $R_t^d \neq R_{w,t}$).

4.4.3.1 Real exchange rate undervaluation

Figure 4.7 shows the results of exchange rate undervaluation on the model dynamics, plotted together with the benchmark policy in which the real exchange rate is at its equilibrium level.

The undervaluation of the real exchange rate in country 1, e_t, has a positive impact on the trade balance of country 1 (China) in the short run and results in a large increase in the trade balance around 2006-2007 as observed in the data, which is mainly caused by a fall in aggregate consumption expenditures. Because households expect the real exchange rate to appreciate, they substitute consumption into the future and save more, and the private savings gap ($B_t = A_t - K_t$) increases. The government, which acts as an intermediator between the domestic and foreign asset markets, issues domestic debt to meet the asset demand, and invests the proceeds in international asset markets. After the equilibrium level of the real exchange rate is restored, consumption expenditures in country 1 increase, reflecting the higher level of financial wealth held by the households, with a negative impact on the trade balance. Because of the symmetry of the countries, an opposite effect takes place in country 2 as its real exchange rate, e_t^*, appreciates.

The impact of the exchange rate policy on the world interest rate is positive due to the nonlinearity of the savings function with respect to the exchange rate (discussed in section 4.3.5). The reduction of savings in country 2 exceeds the increase in savings in country 1, and the decline in the world financial wealth leads to a positive effect on the world real interest rate.

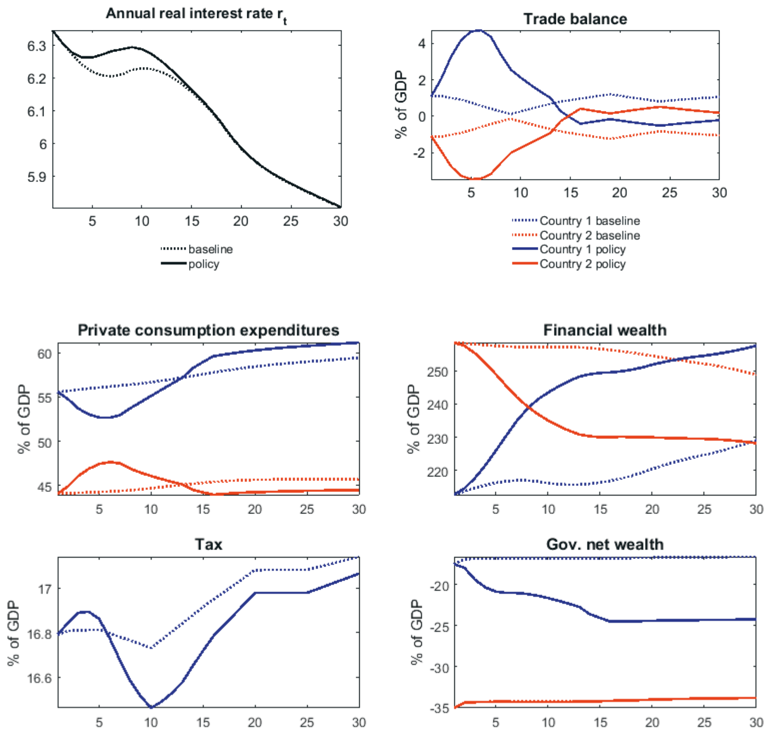

Figure 4.7: Impact of exchange rate policy and demographics on the economy.

4.4.3.2 Capital controls and interest rate policy

Figure 4.8 shows the results of interest rate policy on the model dynamics, together with the benchmark where the domestic and the world interest rate are equal. The interest rate policy affects the trade balance in country 1 negatively in the short run, relative to the benchmark. The negative trade account effect is driven by a fall in investment, caused by the decline in the marginal cost of capital, and by an increase in the consumption of the working age population, which is driven by the rise in wages and the present value of human wealth. The impact on the consumption of the old is negative, because marginal propensities to consume out of lifetime wealth fall, as well as the level of financial wealth. In other words, the substitution effect dominates in the workers' case, and the income effect in the retirees' case. However, because of the large proportion of the working age population, the net effect on consumption is positive. As there are no adjustment costs in the model, there is a large shift in investments in the periods in which the policy begins and ends, as the capital stock is instantly adjusted to the optimal level.

In the long run, the interest rate policy leads to a lower level of financial wealth. After the domestic interest rate returns to the world equilibrium level, the net effect of the interest rate policy on aggregate consumption becomes negative.

As a result of the interest rate policy, the world real interest rate increases relative to benchmark. As the policy lowers the private savings gap, $B_t = A_t - K_t$, in country 1, and therefore its government-central bank's domestic debt, it also has a negative impact on the government-central bank's net foreign asset holdings. In the international financial market, lower asset demand pushes up the interest rate, which increases investment and output and lowers consumption in country 2.

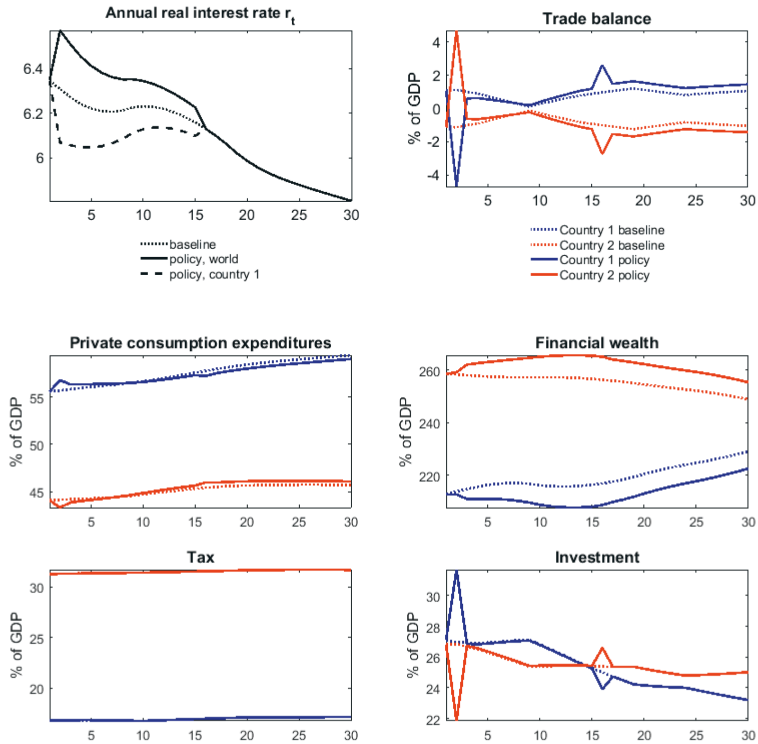

Figure 4.8: Impact of interest rate policy and demographics on the economy.

4.4.3.3 Government spending and budget deficits

Figure 4.9 shows the effects of government expenditure shocks, both assuming that the government runs a balanced budget and taking into account the observed fiscal deficits, together with the benchmark simulation of section 4.4.2, where government expenditures are constant and both governments run a balanced budget.

The observed government expenditure fluctuations, had the government budgets been balanced (the dashed lines in figure 4.9), would have led to an improvement in China's, and a deterioration in the US', trade balance. The increase in public consumption raises taxes and crowds out private consumption. However, the workers' discount rate exceeds the government's discount rate, the riskless interest rate R^w_{t+1}, (see eq. 4.15) because of the anticipated finitude of working life and overall lifetime. Therefore the workers do not fully capitalize the increase in future tax liabilities, and the fall in private consumption does not fully offset the rise in government expenditures (i.e. increase in private saving does not fully compensate for the decline in public saving), resulting in a reduction in domestic savings, a decline in the capital stock and an increase in the world real interest rate. The increase in government expenditures, as it results in an increase in the world interest rate, implies an intergenerational transfer to the old, who live out of savings and interest income only. Because the discount rate of Chinese workers is lower due to lower life expectancy, the reduction in private consumption expenditures is more prounounced than in the United States, resulting in a smaller decline in aggregate savings, and therefore the net effect on its trade balance is positive.

The observed fiscal deficits (solid lines in figure 4.9) lead to an improvement of the Chinese trade balance, and rise in the world interest rate, relative to the counterfactual where government expenditures are financed with a balanced budget. As the households' discount rate is higher than the government's, postponing tax payments further into the future raises the present value of human wealth relative to taxes timed according to a balanced budget rule. Therefore, when financed with debt, government expenditures crowd out less private consumption than with a balanced budget, and the increase of private savings is smaller. As national savings are lower, the world interest rate is higher, resulting in lower investments and capital stock. The increase in life expectancy raises the households' discount rate and the crowding-out effect of government deficit on private consumption becomes muted, and the current account effect stronger. To the contrary, an anticipated decline in the population growth rate raises the present discounted value of tax liabilities, which means that the external sector becomes less sensitive to fiscal policy (government budget deficits). The quantitative effect of observed government expenditures and deficits on the trade surplus is small, approximately 0.1-0.5 % of GDP, and on the interest rate, less than 10 basis points at the most.

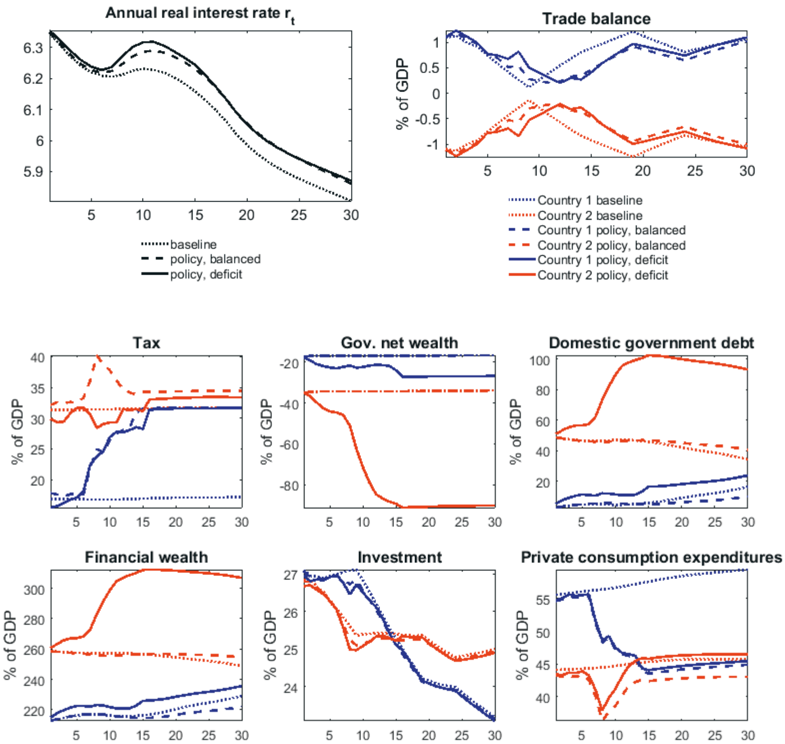

Figure 4.9: Impact of fiscal policy and demographics on the economy.

4.5 Conclusions

This paper analyzes the impact of China's macroeconomic policies, including capital controls and interest rate policy, exchange rate policy and fiscal policy, on China's trade balance vis-a-vis the United States and the world real interest rate in the 2000s. In the analysis, I take into account the direct effects of demographic factors on the external imbalances as well as the indirect effects through the impact that demographics have on the transmission of the macroeconomic policies. The paper is motivated by the observed persistence of the external imbalances in China and the United States, the central role of the public sector in the Chinese economy, and the claims that China has been able to affect both the interest rate dynamics and the trade balance by practising policies that affect both intratemporal and intertemporal terms of trade.

In order to quantify the importance of these factors in explaining the observed trade balance and interest rate dynamics, I construct a dynamic general equilibrium model embedded with features that describe the behavior of the Chinese policy makers. The policies I focus on are capital controls and interest rate policy, exchange rate policy and fiscal policy. I construct a series of policy variables based on the observations that i) the Chinese real exchange rate was undervalued between 2001 and 2014; ii) the US real interest rate has exceeded the Chinese real rate in the 2000s despite their convergence; and, iii) Chinese government expenditures and budget deficits have grown in the 2000s, and simulate with the model to analyze the dynamics of the trade balances and the real interest rate in the 2000s.

According to my analysis, the macroeconomic policies overall have had a strong positive impact on the trade balance, and also a positive impact on the interest rate in the 2000s. The effects are driven by the real exchange rate policy. The demographic developments have contributed to the fall of the interest rate by approximately 50 basis points since 2001. The impact of demographic change on the trade balance has been negative in the first decade, and positive in the second decade of the century. However, the impact of the exchange rate policy is qualitatively sensitive to the assumptions on the elasticity of intertemporal substitution. The impact of the demographic change and the other policies are qualitatively robust to this assumption.

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Appendix

4.A Exchange rate and savings

The life-cycle model of this paper nests a two-period OLG model in the special case when $\omega_{t,t+1} = 0$ and $\gamma_{t,t+1} = 0 \ \forall \, t$. In this case, retirees consume all of their wealth, and the aggregate savings in the economy consist of workers' savings only. Then, as also shown by Song et al. (2014), workers' savings are given by

$$A^w_{t+1} = (W_t - T^w_t) \left[1 - \left(1 + \beta^\sigma R^{\sigma-1}_{t+1} \left(\frac{1 + e^{1-\varepsilon}_{t+1}}{1 + e^{1-\varepsilon}_t} \right)^{\frac{\sigma-1}{\varepsilon-1}} \right)^{-1} \right] \tag{4.30}$$

which is an increasing/decreasing function of the exchange rate e_t if $\sigma > 1/\sigma < 1$ (see figure 4.10). The first derivative of the savings with respect to the exchange rate is

$$\frac{\partial A^w_{t+1}}{\partial e_t} = \frac{\beta^\sigma {R_{t+1}}^{\sigma-1} (\sigma - 1) \left(\frac{1+e^{1-\varepsilon}_{t+1}}{1+e^{1-\varepsilon}_t} \right)^{\frac{\sigma-\varepsilon}{\varepsilon-1}} \frac{(1+e^{1-\varepsilon}_{t+1}) e^{-\varepsilon}_t}{(1+e^{1-\varepsilon}_t)^2}}{\left(1 + \beta^\sigma R^{\sigma-1}_{t+1} \left(\frac{1+e^{1-\varepsilon}_{t+1}}{1+e^{1-\varepsilon}_t} \right)^{\frac{\sigma-1}{\varepsilon-1}} \right)^2} (W_t - T^w_t)$$

and it is positive if $\sigma > 1$. Furthermore, because the savings increase/decrease at a diminishing rate, overvaluation of the exchange rate ($e_t < 1$) has a larger effect on savings than undervaluation of the exchange rate ($e_t > 1$).

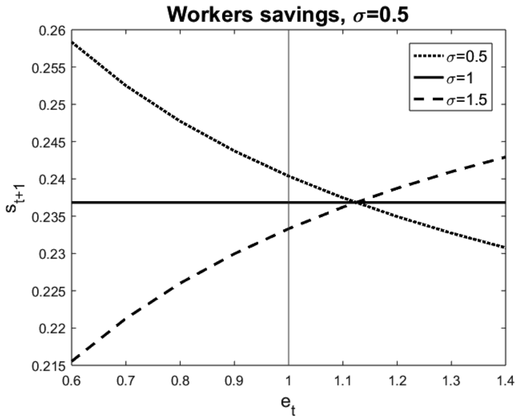


Figure 4.10: Workers' savings as function of the exchange rate e_t in the two-period OLG version of the model (with different elasticities of intertemporal substitution, $\sigma = 0.5$, $\sigma = 1$ and $\sigma = 1.5$).

4.B Capital controls and Ricardian equivalence

To develop the intuition for how the domestic interest rate policy breaks down Ricardian equivalence, consider a two-period small open endowment economy consisting of two agents: a representative household and government, which takes the international interest rate R as given, but can set a domestic interest rate R_d different from the international interest rate. Assume that the government and household have no initial wealth in period 1, and in period 2, all wealth is consumed. Assume that the household can not access external financial markets so that its only savings vehicle is the domestic government bond so that $A_{t+1} = B_{t+1}$ where A_{t+1} = household's financial wealth and B_{t+1} = domestic government bond.

In this economy, the household maximises utility, given by

$$U = C_1 + \beta C_2 \tag{4.31}$$

subject to the budget constraint $Y_1 - T_1 = C_1 + A_2$ in period 1 and $Y_2 - T_2 + R_2 A_2 = C_2$ in period 2. Solving the household's constrained optimization problem gives the standard Euler equation, $C_2 = \beta R C_1$, which, when combined with the budget constraints, gives the optimal consumption expenditures in periods 1 and 2, given by $C_1 = \frac{1}{1+\beta R}\left[Y_1 - T_1 + \frac{Y_2 - T_2}{R}\right]$ and $C_2 = \frac{\beta R}{1+\beta R}\left[Y_1 - T_1 + \frac{Y_2 - T_2}{R}\right]$.

The government's flow budget constraint is given by $B_{t+1} - B^*_{t+1} = R_d B_t - RB^*_t + G_t - T_t$, where B_{t+1} is the government's domestic debt and B^*_{t+1} foreign assets. The government's budget constraint is given by $B_2 - B^*_2 = G_1 - T_1$ period 1 and by $0 = R_d B_2 - RB^*_2 + G_2 - T_2 = R_d(B_2 - B^*_2) + (R_d - R)B^*_2 + G_2 - T_2$ in period 2. The government's interemporal budget constraint then solves as $T_1 + \frac{T_2}{R_d} - B^*_2\left(1 - \frac{R}{R_d}\right) = G_1 + \frac{G_2}{R_d}$.

If the government is a net external creditor, $B^*_2 > 0$, and if the domestic interest rate is below the international interest rate, $R_d < R$, the government can save at a higher rate than the households, and for a given domestic interest rate R_d, external saving by the government relaxes the budget constraint. Then, as the households internalize the government's intertemporal budget constraint, consumption in both periods depends positively on government's net external asset holdings, given by

$$C_1 = \frac{1}{1+\beta R_d}\left[Y_1 + \frac{Y_2}{R_d} - G_1 - \frac{G_2}{R_d} - B^*_2\left(1 - \frac{R}{R_d}\right)\right]$$

in period 1 and by

$$C_2 = \frac{\beta R_d}{1+\beta R_d}\left[Y_1 + \frac{Y_2}{R_d} - G_1 - \frac{G_2}{R_d} - B^*_2\left(1 - \frac{R}{R_d}\right)\right]$$

in period 2. The Ricardian equivalence in this case does not hold.

The current account is defined as $CA_t = Y_t - C_t - G_t + (R_{w,t} - 1)B^*_t = B^*_{t+1} - B^*_t$. Given the assumption of a two-period economy, the current account is given by $CA_1 = Y_1 - C_1 - G_1 = B^*_2$ in period 1 and by $CA_2 = Y_2 - C_2 - G_2 + (R-1)B^*_2 = -B^*_2$ in period 2. By substituting C_1 into period 1's current account condition, one can solve for the period 1 current account, which is given by

$$B^*_2 = \frac{\beta R_d}{1 + \beta R_d - \left(\mathbf{1} - \frac{\mathbf{R}}{\mathbf{R_d}}\right)}(Y_1 - G_1) - \frac{1}{1 + \beta R - \left(\mathbf{1} - \frac{\mathbf{R}}{\mathbf{R_d}}\right)}\left[\frac{Y_2}{R_d} - \frac{G_2}{R_d}\right].$$

If $R_d \neq R$, the current account differs from that under Ricardian equivalence. Specifically, a low domestic interest rate, $R_d < R$, is associated with a smaller external asset position than if $R_d = R$. This is because (even though low interest rate lowers saving) the positive wealth effect associated with public external debt results has a positive effect on consumption, which at the optimum in period 1 equals $C_1 = \left[\frac{1}{1+\beta R_d} - \frac{\beta R_d\left(1-\frac{R}{R_d}\right)}{1+\beta R_d-\left(1-\frac{R}{R_d}\right)}\right]\left[Y_1 - G_1 + \frac{Y_2-G_2}{R_d}\right]$. Therefore, in addition to the normal interest rate channel, the capital control breaks down the Ricardian equivalence and induces a positive wealth effect on domestic households.

4.C Robustness of the quantitative results to EIS

There is a discrepancy in the literature about the value of the elasticity of intertemporal substitution parameter σ. Papers which study the impact of demographic change with life-cycle models, such as Gertler (1999), Fujiwara and Teranishi (2008), Ferrero (2010), and Kilponen et al. (2006), typically set the parameter value below one. Models that study the effect of terms of trade shocks on consumption-saving decisions, including Song et al. (2014), typically have higher elasticities of intertemporal substitution.

Figure 4.11 shows the effect of demographic change with $\sigma = 1.5$ and $\sigma = 0.5$. With a low value of σ, demographic change leads to a larger decline (appr. 150 basis points) of the real interest rate. However, qualitatively the result is not sensitive to this assumption, as the demographic change produces a fall in the interest rate (appr. 50 basis points) even with $\sigma = 1.5$. The simulation with low sigma, which leads to a lower real interest rate, is associated with smaller gross trade balance / GDP ratios.

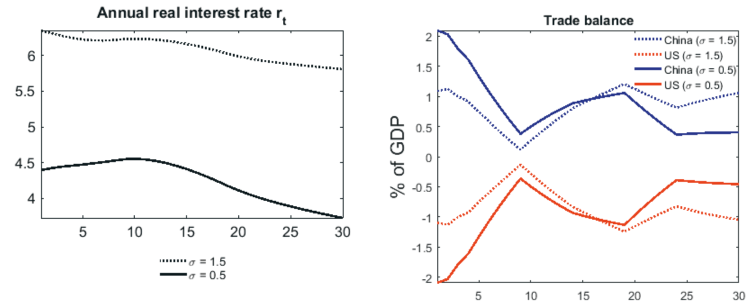
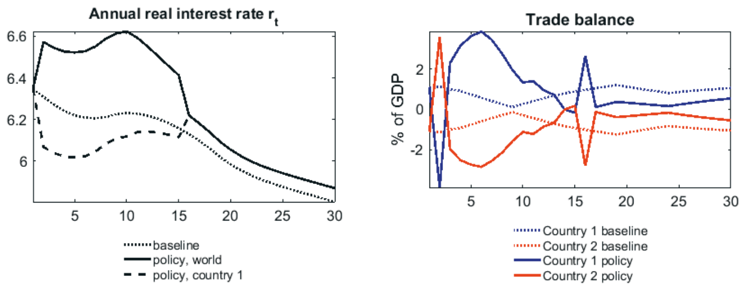


Figure 4.11: Impact of demographics on the real interest rate and trade balance with different elasticities of intertemporal substitution ($\sigma = 1.5$ and $\sigma = 0.5$).

Figure 4.12 shows the effect of macroeconomic policies with $\sigma = 1.5$ and $\sigma = 0.5$. The impact of the macroeconomic policies on the world interest rate is qualitatively robust to the assumption for σ in the short run, but not in the long run. In the long run, the simulation with policies and with $\sigma = 1.5$ results in an interest rate which is appr. 0-10 basis points above the benchmark interest rate, but if $\sigma = 0.5$, the long run interest rate is appr. 30 basis points below the benchmark interest rate. This, as well as the sensitivity of the trade balance dynamics to the value of σ, is mainly due to the fact that the exchange rate policy has opposite effects with different values of σ. As discussed in section 4.3.5, exchange rate policy is dominated by the HLM effect when $\sigma < 1$, and by the substitution effect when $\sigma > 1$. The impact of fiscal policy on the trade balance is also sensitive to the value of σ, as fiscal policy has a stronger impact with $\sigma < 1$. Then the increase of taxes does not have quite as large a negative effect on private savings, and the net effect on government savings is larger. Because of the sensitivity of the results to parameter values, and because of the illustrative nature of the policy shocks, the results must be interpreted with caution.

$\sigma = 1.5$



$\sigma = 0.5$

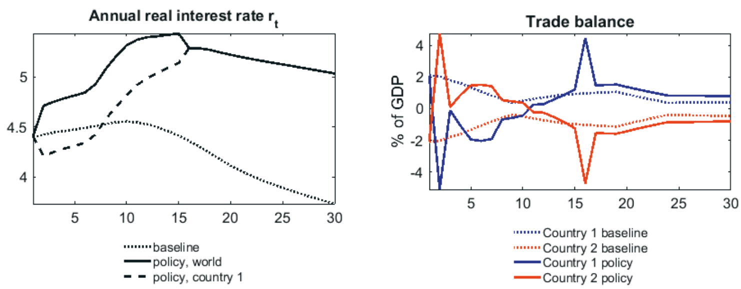


Figure 4.12: Impact of macroeconomic policies and demographics on the real interest rate and trade balance with different elasticities of intertemporal substitution ($\sigma = 1.5$ and $\sigma = 0.5$).

4.D Figures

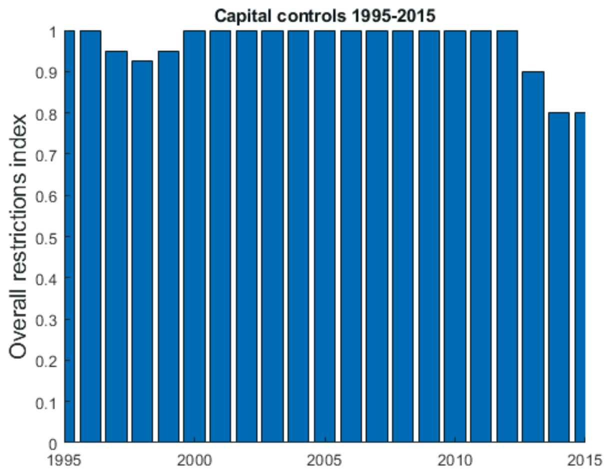


Figure 4.13: Capital controls in China 1995-2015, overall restrictions index. Source: Fernández et al. (2016).

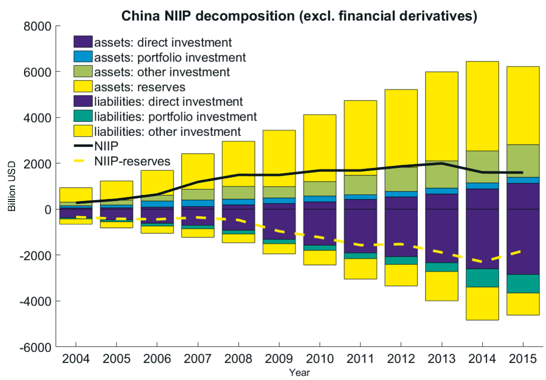


Figure 4.14: China's external assets and liabilities. Source: IMF 2017.

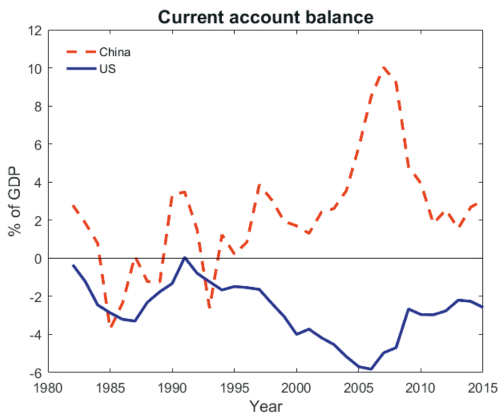


Figure 4.15: Current account balance of China and the United States. Source: IMF 2018.

4.E Technical appendix

This appendix presents the derivation of the retirees' and workers' problems and the aggregation result.

1.1 Households

Retirees

A retiree born in period j and retired in period i chooses consumption-saving allocation and labor input to maximize

$$V_t^{jr}(i) = max\left\{\left[C_t^{jr}(i)\right]^\rho + \beta\gamma_{t,t+1}\left(V_{t+1}^{jr}(i)\right)^\rho\right\}^{\frac{1}{\rho}}. \tag{4.32}$$

Households consume domestic goods $C_t^{jr,1}(i)$ and foreign goods $C_t^{jr,2}(i)$ so that the aggregate consumption index is given by

$$C_t^{jr}(i) = \left(\left(C_t^{jr,1}(i)\right)^{\frac{\varepsilon-1}{\varepsilon}} + \left(C_t^{jr,2}(i)\right)^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}}$$

where ε is the elasticity of substitution between home and foreign goods. R_t^d is the domestic interest rate set by the local authority and $\gamma_{t,t+1}$ is the retirees' probability to survive from one period to the next. Then the value function can be written as

$$V_t^{jr}(i) = \max_{C_t^{jr,1}(i), C_t^{jr,2}(i), A_{t+1}^{jr,1}(i)} \tag{4.33}$$

$$\left\{ \left[\left(\left(C_t^{jr,1}(i) \right)^{\frac{\varepsilon-1}{\varepsilon}} + \left(C_t^{jr,2}(i) \right)^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}} \right]^{\rho} + \beta\gamma_{t,t+1} \left(V_{t+1}^{jr}(i) \right)^{\rho} \right\}^{\frac{1}{\rho}}.$$

The periodic budget constraint is

$$A_{t+1}^{jr}(i) = \frac{R_t^d A_t^{jr}(i)}{\gamma_{t-1,t}} - C_t^{jr,1}(i) - e_t C_t^{jr,2}(i) \tag{4.34}$$

where $A_{t+1}^{jr}(i)$ is financial wealth and e_t is the price of the foreign good in terms of a domestic good *i.e.* the real exchange rate. By solving the periodic budget constraint for domestic consumption and substituting into the value function, one obtains

$$V_t^{jr}(i) = \max_{C_t^{jr,1}(i), C_t^{jr,2}(i), A_{t+1}^{jr,1}(i)} \tag{4.35}$$

$$\left\{ \left(\left(\frac{R_t^d A_t^{jr}(i)}{\gamma_{t-1,t}} - A_{t+1}^{jr}(i) - e_t C_t^{jr,2}(i) \right)^{\frac{\varepsilon-1}{\varepsilon}} + \left(C_t^{jr,2}(i) \right)^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\rho\varepsilon}{\varepsilon-1}} + \beta\gamma_{t,t+1} \left(V_{t+1}^{jr}(i) \right)^{\rho} \right\}^{\frac{1}{\rho}}.$$

The first order condition with respect to asset accumulation is

$$\left(\left(C_t^{jr,1}(i) \right)^{\frac{\varepsilon-1}{\varepsilon}} + \left(C_t^{jr,2}(i) \right)^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon\rho}{\varepsilon-1}-1} \left(C_t^{jr,1}(i) \right)^{\frac{\varepsilon-1}{\varepsilon}-1}$$

$$= \beta\gamma_{t+1} (V_{t+1}^{jr}(i))^{\rho-1} \frac{\partial V_{t+1}^{jr}(i)}{\partial A_{t+1}^{jr}(i)} \tag{4.36}$$

and the first order condition with respect to foreign consumption is

$$C_t^{jr,1}(i)e_t^{-\varepsilon} = C_t^{jr,2}(i). \tag{4.37}$$

Substituting the latter into the former, one obtains

$$\left(C_t^{jr,1}(i)\right)^{\rho-1}\left(1+e_t^{1-\varepsilon}\right)^{\frac{\varepsilon(\rho-1)+1}{\varepsilon-1}} = \beta\gamma_{t+1}(V_{t+1}^{jr}(i))^{\rho-1}\frac{\partial V_{t+1}^{jr}(i)}{\partial A_{t+1}^{jr}(i)}. \tag{4.38}$$

The envelope condition is

$$\frac{\partial V_t^{jr}(i)}{\partial A_t^{jr}(i)} = (V_t^{jr})^{1-\rho}(C_t^{jr,1}(i))^{\rho-1}\left(1+e_t^{1-\varepsilon}\right)^{\frac{\varepsilon(\rho-1)+1}{\varepsilon-1}}\frac{R_t^d}{\gamma_{t-1,t}}. \tag{4.39}$$

Combining the first order conditions and the lagged envelope condition $\left(\frac{\partial V_{t+1}^{jr}(i)}{\partial A_{t+1}^{jr}(i)}\right)$, and noting that $\sigma = \frac{1}{1-\rho}$ gives the following Euler equation for retirees:

$$C_{t+1}^{jr,1}(i) = C_t^{jr,1}(i)\left[\beta R_{t+1}^d\right]^{\sigma}\left(\frac{1+e_t^{1-\varepsilon}}{1+e_{t+1}^{1-\varepsilon}}\right)^{\frac{\varepsilon-\sigma}{\varepsilon-1}}. \tag{4.40}$$

Guess that aggregate consumption is a fraction of total lifetime wealth:

$$C_t^{jr,1}(i) + e_t C_t^{jr,2}(i) = \epsilon_t\pi_t\left(\frac{R_t^d A_t^{jr}(i)}{\gamma_t}\right) = \left(1+e_t^{1-\varepsilon}\right)C_t^{jr,1}(i) \tag{4.41}$$

where the last equality follows from eq. (4.37). Then, combining the Euler equation (4.40) with the guess (4.41) gives a law of motion for the marginal propensity to consume as follows. First, combine the budget constraint (4.34) with the guess (4.41) to have the following expression for consumption:

$$C_t^{jr,1}(i) + e_t C_t^{jr,2}(i) = \epsilon_t\pi_t\left(A_{t+1}^{jr}(i) + C_t^{jr}(i)\right) = \left(1+e_t^{1-\varepsilon}\right)C_t^{jr,1}(i). \tag{4.42}$$

Second, substitute this expression (4.42) and lagged expression of the guess (4.41) into the Euler equation (4.40) to obtain the following expression

$$\frac{\epsilon_{t+1}\pi_{t+1}}{(1+e_{t+1}^{1-\varepsilon})}\left(\frac{R_{t+1}^{d}A_{t+1}^{jr}(i)}{\gamma_{t,t+1}}\right)$$
$$=\left\{\frac{\epsilon_t\pi_t}{(1+e_t^{1-\varepsilon})}\left(A_{t+1}^{jr}(i)+C_t^{jr}(i)\right)\right\}*\left[\beta R_{t+1}^{d}\right]^{\sigma}\left(\frac{1+e_t^{1-\varepsilon}}{1+e_{t+1}^{1-\varepsilon}}\right)^{\frac{\varepsilon-\sigma}{\varepsilon-1}}.$$

Dividing both sides of the equation by the LHS (left-hand-side) of itself gives

$$1=\frac{\left\{\frac{\epsilon_t\pi_t}{(1+e_t^{1-\varepsilon})}\left(A_{t+1}^{jr}(i)+\left(1+e_t^{1-\varepsilon}\right)C_t^{jr,1}(i)\right)\right\}*\left[\beta R_{t+1}^{d}\right]^{\sigma}\left(\frac{1+e_t^{1-\varepsilon}}{1+e_{t+1}^{1-\varepsilon}}\right)^{\frac{\varepsilon-\sigma}{\varepsilon-1}}}{\frac{\epsilon_{t+1}\pi_{t+1}}{(1+e_{t+1}^{1-\varepsilon})}\left(\frac{R_{t+1}^{d}A_{t+1}^{jr}(i)}{\gamma_{t,t+1}}\right)}$$
$$=\frac{\frac{\epsilon_t\pi_t}{(1+e_t^{1-\varepsilon})}A_{t+1}^{jr}(i)\left[\beta R_{t+1}^{d}\right]^{\sigma}\left(\frac{1+e_t^{1-\varepsilon}}{1+e_{t+1}^{1-\varepsilon}}\right)^{\frac{\varepsilon-\sigma}{\varepsilon-1}}}{\frac{\epsilon_{t+1}\pi_{t+1}}{(1+e_{t+1}^{1-\varepsilon})}\left(\frac{R_{t+1}^{d}A_{t+1}^{jr}(i)}{\gamma_{t,t+1}}\right)}$$
$$+\frac{\frac{\epsilon_t\pi_t}{(1+e_t^{1-\varepsilon})}\left(1+e_t^{1-\varepsilon}\right)C_t^{jr,1}(i)\left[\beta R_{t+1}^{d}\right]^{\sigma}\left(\frac{1+e_t^{1-\varepsilon}}{1+e_{t+1}^{1-\varepsilon}}\right)^{\frac{\varepsilon-\sigma}{\varepsilon-1}}}{\frac{\epsilon_{t+1}\pi_{t+1}}{(1+e_{t+1}^{1-\varepsilon})}\left(\frac{R_{t+1}^{d}A_{t+1}^{jr}(i)}{\gamma_{t,t+1}}\right)}$$
$$=\frac{\epsilon_t\pi_t}{\epsilon_{t+1}\pi_{t+1}}\beta^{\sigma}\left(R_{t+1}^{d}\right)^{\sigma-1}\gamma_{t,t+1}\left(\frac{1+e_t^{1-\varepsilon}}{1+e_{t+1}^{1-\varepsilon}}\right)^{\frac{1-\sigma}{\varepsilon-1}}$$
$$+\frac{\epsilon_t\pi_t C_t^{jr,1}(i)\left[\beta R_{t+1}^{d}\right]^{\sigma}\left(\frac{1+e_t^{1-\varepsilon}}{1+e_{t+1}^{1-\varepsilon}}\right)^{\frac{\varepsilon-\sigma}{\varepsilon-1}}}{\frac{\epsilon_{t+1}\pi_{t+1}}{(1+e_{t+1}^{1-\varepsilon})}\left(\frac{R_{t+1}^{d}A_{t+1}^{jr}(i)}{\gamma_{t,t+1}}\right)}$$

By the conjecture (eq. (4.41)), the denominator of the last term equals

$$\frac{\epsilon_{t+1}\pi_{t+1}}{\left(1+e_{t+1}^{1-\varepsilon}\right)}\left(\frac{R_{t+1}^{d}A_{t+1}^{jr}(i)}{\gamma_{t+1}}\right)=C_{t+1}^{jr,1}(i)=C_{t}^{jr,1}(i)\left[\beta R_{t+1}^{d}\right]^{\sigma}\left(\frac{1+e_{t}^{1-\varepsilon}}{1+e_{t+1}^{1-\varepsilon}}\right)^{\frac{\varepsilon-\sigma}{\varepsilon-1}}$$

where the last equality follows from the Euler equation (eq. (4.40). Then the retiree's marginal propensity to consume can be solved to evolve according to

$$\epsilon_t\pi_t=1-\frac{\epsilon_t\pi_t}{\epsilon_{t+1}\pi_{t+1}}\gamma_{t,t+1}\beta^{\sigma}(R_{t+1}^{d})^{\rho\sigma}\left(\frac{1+e_{t}^{1-\varepsilon}}{1+e_{t+1}^{1-\varepsilon}}\right)^{\frac{1-\sigma}{\varepsilon-1}}. \tag{4.43}$$

To obtain a solution for the value function, first conjecture that it is linear on consumption (non-linear on exchange rate) according to

$$\begin{aligned} V_t^{jr}(i) &= \triangle_t^r C_t^{jr}(i)\left(1+e_t^{1-\varepsilon}\right)^{\frac{1}{\varepsilon-1}} \qquad (4.44)\\ &= \triangle_t^r C_t^{jr,1}(i)\left(1+e_t^{1-\varepsilon}\right)^{\frac{\varepsilon}{\varepsilon-1}}. \end{aligned}$$

Substitute the conjecture into the value function and solve for $\triangle_t^r$ to obtain the following expression

$$(\triangle_t^r)^{\rho}=1+\frac{\beta\gamma_{t,t+1}(\triangle_{t+1}^r)^{\rho}\left[C_{t+1}^{jr,1}(i)\left(1+e_{t+1}^{1-\varepsilon}\right)^{\frac{\varepsilon}{\varepsilon-1}}\right]^{\rho}}{\left(C_t^{jr,1}(i)\left(1+e_t^{1-\varepsilon}\right)^{\frac{\varepsilon}{\varepsilon-1}}\right)^{\rho}}.$$

Then, substitute $C_{t+1}^{jr}(i)$ from the Euler equation (4.40) and use the equation (4.43) to verify that

$$\triangle_t^r=(\epsilon_t\pi_t)^{-\frac{1}{\rho}} \tag{4.45}$$

and thus

$$V_t^{jr}(i) = (\epsilon_t \pi_t)^{-\frac{1}{\rho}} C_t^{jr,1}(i) \left(1 + e_t^{1-\varepsilon}\right)^{\frac{\varepsilon}{\varepsilon-1}} \\ = (\epsilon_t \pi_t)^{-\frac{1}{\rho}} C_t^{jr}(i) \left(1 + e_t^{1-\varepsilon}\right)^{\frac{1}{\varepsilon-1}} . \tag{4.46}$$

Workers

As retirees, workers consume domestic and foreign goods. A worker born in period j chooses consumption-saving allocation to solve

$$V_t^{jw} = \max_{C_t^{jw,1}, C_t^{jw,2}, A_{t+1}^{jw}} \\ \left\{ \left[\left(\left(C_t^{jw,1}\right)^{\frac{\varepsilon-1}{\varepsilon}} + \left(C_t^{jw,2}\right)^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}} \right]^{\rho} + \beta \left[\omega_{t+1} V_{t+1}^{jw} + (1 - \omega_{t+1}) V_{t+1}^{jr} \right]^{\rho} \right\}^{\frac{1}{\rho}} \tag{4.47}$$

subject to

$$A_{t+1}^{jw} = R_t^d A_t^{jw} + W_t - C_t^{jw,1} - e_t C_t^{jw,2} - T_t^{jw} \ . \tag{4.48}$$

Solving for consumption of the domestic good, $C_t^{jw,1}$, from the budget constraint and substituting it into the value function gives

$$V_t^{jw} = \max_{C_t^{jw,2}, A_{t+1}^{jw}} \\ (\left[\left(\left(R_t^d A_t^{jw} + W_t - e_t C_t^{jw,2} - T_t^{jw} - A_{t+1}^{jw} \right)^{\frac{\varepsilon-1}{\varepsilon}} + \left(C_t^{jw,2}\right)^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}} \right]^{\rho} \\ + \beta \left[\omega_{t+1} V_{t+1}^{jw} + (1 - \omega_{t+1}) V_{t+1}^{jr} \right]^{\rho})^{\frac{1}{\rho}}. \tag{4.49}$$

The first order condition with respect to asset accumulation is

$$\left(\left(C_t^{jw,1}\right)^{\frac{\varepsilon-1}{\varepsilon}}+\left(C_t^{jw,2}\right)^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon\rho-\varepsilon+1}{\varepsilon-1}}\left(C_t^{jw,1}\right)^{-\frac{1}{\varepsilon}}$$
$$=\beta\left(\omega_{t+1}V_{t+1}^{jw}+(1-\omega_{t+1})V_{t+1}^{jr}\right)^{\rho-1}\left[\omega_{t+1}\frac{\partial V_{t+1}^{w}}{\partial A_{t+1}^{w}}+(1-\omega_{t+1})\frac{\partial V_{t+1}^{rj}}{\partial A_{t+1}^{wj}}\right] \quad (4.50)$$

and the first order condition with respect to consumption of foreign goods gives

$$C_t^{jw,1}e_t^{-\varepsilon}=C_t^{jw,2}. \quad (4.51)$$

Substituting eq. (4.51) into the first order condition eq. (4.52) gives

$$\left(C_t^{jw,1}\right)^{\rho-1}(1+e_t^{1-\varepsilon})^{\frac{\varepsilon\rho-\varepsilon+1}{\varepsilon-1}}$$
$$=\beta\left(\omega_{t+1}V_{t+1}^{jw}+(1-\omega_{t+1})V_{t+1}^{jr}\right)^{\rho-1}\left[\omega_{t+1}\frac{\partial V_{t+1}^{w}}{\partial A_{t+1}^{w}}+(1-\omega_{t+1})\frac{\partial V_{t+1}^{rj}}{\partial A_{t+1}^{wj}}\right] \quad (4.52)$$

The envelope conditions are

$$\frac{\partial V_t^{jw}}{\partial A_t^{jw}}=(V_t^{w})^{1-\rho}(C_t^{jw,1})^{\rho-1}R_t^d\left(1+e_t^{1-\varepsilon}\right)^{\frac{\varepsilon(\rho-1)+1}{\varepsilon-1}} \quad (4.53)$$

and

$$\frac{\partial V_t^{jr}}{\partial A_t^{jw}}=\frac{\partial V_t^{jr}}{\partial A_t^{jr}}\frac{\partial A_t^{jr}}{\partial A_t^{jw}}=\frac{\partial V_t^{jr}}{\partial A_t^{jr}}=(V_t^{jr})^{1-\rho}(C_t^{jr,1})^{\rho-1}\left(1+e_t^{1-\varepsilon}\right)^{\frac{\varepsilon(\rho-1)+1}{\varepsilon-1}}R_t^d \quad (4.54)$$

where it has been taken into account that the agent that has been a worker in period t, and becomes a retiree in period $t+1$, does not take part in the mutual fund.

Combining the first order conditions and the envelope conditions gives the Euler equation for the workers. First, substitute into the first order condition for asset accumulation (4.52) the envelope conditions taking into account the assumption of no compensation for the risk of death :

$$(C_t^{jw,1})^{\rho-1}(1+e_t^{1-\varepsilon})^{\frac{\varepsilon\rho-\varepsilon+1}{\varepsilon-1}} = \beta(\omega_{t+1}V_{t+1}^{jw} + (1-\omega_{t+1})V_{t+1}^{jr})^{\rho-1} * \\ (\omega_{t,t+1}(V_{t+1}^{jw})^{1-\rho}(C_{t+1}^{jw,1})^{\rho-1}R_{t+1}^d\left(1+e_{t+1}^{1-\varepsilon}\right)^{\frac{\varepsilon(\rho-1)+1}{\varepsilon-1}} \\ + (1-\omega_{t+1})(V_{t+1}^{jr})^{1-\rho}(C_{t+1}^{jr,1})^{\rho-1}R_{t+1}^d\left(1+e_{t+1}^{1-\varepsilon}\right)^{\frac{\varepsilon(\rho-1)+1}{\varepsilon-1}}). \quad (4.55)$$

Conjecture that, as for retirees, the value function depends on consumption as follows:

$$V_t^{jw} = \triangle_t^{jw} C_t^{jw,1}(1+e_t^{1-\varepsilon})^{\frac{\varepsilon}{\varepsilon-1}} \quad . \quad (4.56)$$

In addition, define the adjustment term to be

$$\Omega_t \equiv \omega_{t-1,t} + (1-\omega_{t-1,t})\epsilon_t^{\frac{1}{1-\sigma}}. \quad (4.57)$$

Substitute the conjectures for the value functions (4.44) and (4.56) into equation (4.55). This gives

$$(C_t^{jw,1})^{\rho-1}(1+e_t^{1-\varepsilon})^{\frac{\varepsilon\rho-\varepsilon+1}{\varepsilon-1}}(\beta R_{t+1})^{-1} = \\ (\omega_{t+1}\triangle_{t+1}^{jw}C_{t+1}^{jw,1}(1+e_{t+1}^{1-\varepsilon})^{\frac{\varepsilon}{\varepsilon-1}} + (1-\omega_{t+1})\triangle_{t+1}^{r}C_{t+1}^{jr,1}(i)\left(1+e_{t+1}^{1-\varepsilon}\right)^{\frac{\varepsilon}{\varepsilon-1}})^{\rho-1} * \\ \left[\omega_{t,t+1}(\triangle_{t+1}^{jw})^{1-\rho}\left(1+e_{t+1}^{1-\varepsilon}\right)^{\frac{1}{\varepsilon-1}} + (1-\omega_{t+1})(\triangle_{t+1}^{r})^{1-\rho}\left(1+e_{t+1}^{1-\varepsilon}\right)^{\frac{1}{\varepsilon-1}}\right]. \quad (4.58)$$

After some algebra, obtain the following expression for the Euler equation:

$$C_t^{jw,1}\left[\beta R_{,t+1}^d\Omega_{t+1}\right]^{\sigma}\left(\frac{1+e_t^{1-\varepsilon}}{1+e_{t+1}^{1-\varepsilon}}\right)^{\frac{\varepsilon-\sigma}{\varepsilon-1}} = \omega_{t+1}C_{t+1}^{jw,1} + (1-\omega_{t+1})\left(\frac{\triangle_{t+1}^{r}}{\triangle_{t+1}^{w}}\right)C_{t+1}^{jr,1}. \quad (4.59)$$

Guess the consumption to be a fraction of total lifetime wealth

$$C_t^{jw} = \pi_t(R_t^d A_t^{jw} + H_t^{jw}) = C_t^{jw,1}(1+e_t^{\mathbf{1}-\varepsilon}) \quad (4.60)$$

where H_t^{jw} is the present discounted value of a worker's human wealth net of taxation. Then substitute the per period budget constraint (4.48) to get the following expression

$$\begin{aligned} C_t^{jw} &= \pi_t(A_{t+1}^{jw} - W_t + C_t^{jw} + T_t^{jw} - H_t^{jw}) \\ &= \pi_t(A_{t+1}^{jw} - W_t + C_t^{jw,1} + e_t C_t^{jw,2} + T_t^{jw} - H_t^{jw}). \end{aligned}$$

Note that for a retiree who has just abandoned the labor force, the consumption is guessed to be proportional to the assets they had as workers which they carry to their first period of retirement

$$C_t^{r|w} = \varepsilon_t \pi_t (R_t^d A_t^{jw}) \tag{4.61}$$

with the budget constraint being

$$\begin{aligned} A_{t+1}^{jr|w} &= R_t^d A_t^{jw} + W_t - C_t^{jw} - T_t^{jw} \\ &= R_t^d A_t^{jw} + W_t - C_t^{jw,1} - e_t C_t^{jw,2} - T_t^{jw} \\ &= R_t^d A_t^{jw} + W_t - C_t^{jw,1}(1 + e_t^{1-\varepsilon}) - T_t^{jw}. \end{aligned} \tag{4.62}$$

Combining the Euler equation (4.59) with the guesses (4.60) and (4.61) and the budget constraints (4.48) and (4.62) as follows gives a law of motion for the marginal propensity to consume:

First, subsitute the guesses into the Euler equation to have

$$\begin{aligned} &\frac{\pi_t(R_t^d A_t^{jw} + H_t^{jw})}{(1 + e_t^{1-\varepsilon})} \left(\frac{1 + e_t^{1-\varepsilon}}{1 + e_{t+1}^{1-\varepsilon}} \right)^{\frac{\varepsilon-\sigma}{\varepsilon-1}} \left[\beta R_{t+1}^d \Omega_{t+1} \right]^{\sigma} = \\ &\omega_{t+1} \frac{\pi_{t+1}(R_{t+1}^d A_{t+1}^{jw} + H_{t+1}^{jw})}{(1 + e_{t+1}^{1-\varepsilon})} + (1 - \omega_{t+1}) \left(\frac{\triangle_{t+1}^r}{\triangle_{t+1}^w} \right) \frac{\varepsilon_{t+1} \pi_{t+1}(R_{t+1}^d A_{t+1}^{jw})}{(1 + e_{t+1}^{1-\varepsilon})} \end{aligned} \tag{4.63}$$

and substitute the per period budget constraints (4.48) and (4.62) to have

$$\frac{\pi_t(R_t^d A_t^{jw} + H_t^{jw})}{(1+e_t^{1-\varepsilon})}\left(\frac{1+e_t^{1-\varepsilon}}{1+e_{t+1}^{1-\varepsilon}}\right)^{\frac{\varepsilon-\sigma}{\varepsilon-1}}\left[\beta R_{t+1}^d \Omega_{t+1}\right]^{\sigma} =$$
$$\omega_{t+1}\frac{\pi_{t+1}(R_{t,t+1}^d(R_t^d A_t^{jw} + W_t - C_t^{jw,1}(1+e_t^{1-\varepsilon}) - T_t^{jw}) + H_{t+1}^{jw})}{(1+e_{t+1}^{1-\varepsilon})} +$$
$$(1-\omega_{t+1})\left(\frac{\triangle_{t+1}^r}{\triangle_{t+1}^w}\right)\frac{\varepsilon_{t+1}\pi_{t+1}(R_{t+1}^d(R_t^d A_t^{jw} + W_t - C_t^{jw,1}(1+e_t^{1-\varepsilon}) - T_t^{jw}))}{(1+e_{t+1}^{1-\varepsilon})}. \tag{4.64}$$

Substituting again the guess for C_t^{wj}, collecting terms and using the definition of the adjustment term, this equals

$$\Omega_{t+1}\pi_{t+1}R_{t+1}^d\left[R_t^d A_t^w(1-\pi_t) + W_t - \pi_t(H_t^w) - T_t^w\right]$$
$$+\,\omega_{t+1}\pi_{t+1}H_{t+1}^w =$$
$$\pi_t(R_t^d A_t^{jw})\left[\beta R_{t+1}^d\Omega_{t+1}\right]^{\sigma}\left(\frac{1+e_t^{1-\varepsilon}}{1+e_{t+1}^{1-\varepsilon}}\right)^{\frac{1-\sigma}{\varepsilon-1}} + \pi_t(H_t^{jw})\left[\beta R_{t+1}^d\Omega_{t+1}\right]^{\sigma}\left(\frac{1+e_t^{1-\varepsilon}}{1+e_{t+1}^{1-\varepsilon}}\right)^{\frac{1-\sigma}{\varepsilon-1}}.$$

By collecting terms with A_t^{jw} on the LHS, dividing the equality by $\Omega_{t+1}\pi_{t+1}R_{t+1}^d$ and conjecturing that $H_t^{wj} = W_t - T_t^{jw} + \frac{\omega_{t+1}H_{t+1}^{jw}}{\Omega_{t+1}R_{t+1}^d}$ this expression can be written as

$$(R_t^d A_t^{jw} + H_t^{wj}) * \left[1-\pi_t - \frac{\pi_t}{\pi_{t+1}}\beta^{\sigma}(R_{t+1}^d\Omega_{t+1})^{\sigma-1}\left(\frac{1+e_t^{1-\varepsilon}}{1+e_{t+1}^{1-\varepsilon}}\right)^{\frac{1-\sigma}{\varepsilon-1}}\right] = 0\ .$$

This implies that

$$\left[1-\pi_t - \frac{\pi_t}{\pi_{t+1}}\beta^{\sigma}(R_{t+1}^d\Omega_{t+1})^{\sigma-1}\left(\frac{1+e_t^{1-\varepsilon}}{1+e_{t+1}^{1-\varepsilon}}\right)^{\frac{1-\sigma}{\varepsilon-1}}\right] = 0$$

from where workers´ marginal propensity to consume can be solved to evolve according to

$$\pi_t = 1 - \frac{\pi_t}{\pi_{t+1}} \beta^{\sigma} (R^d_{t+1}\Omega_{t+1})^{\sigma-1} \left(\frac{1+e_t^{1-\varepsilon}}{1+e_{t+1}^{1-\varepsilon}} \right)^{\frac{1-\sigma}{\varepsilon-1}} . \tag{4.65}$$

1.2 Aggregation

Total assets

Because of the perfect annuity market, $R^d_t A^r_{t-1}$ of periods' $t-1$ retirees is carried to the next period. Of workers who retire between these periods, $(1-\omega_{t-1,t})(R^d_{t,t-1}A^w_{t-1} + W_{t-1} - C^w_{t-1} - T^w_{t-1})$ adds to the retirees assets in the beginning of period t, where $T^w_t = N^w_t T^{jw}_t$. Thus retirees' aggregate assets evolve according to

$$A^r_{t+1} = R^d_t A^r_t - C^r_t + (1-\omega_{t+1})(R^d_t A^w_t + W_t N^w_t - C^w_t - T^w_t). \tag{4.66}$$

Workers' aggregate assets evolve according to

$$A^w_{t+1} = \omega_{t+1}(R^d_t A^w_t + W_t N^w_t - C^w_t - T^w_t). \tag{4.67}$$

Aggregate resource constraint is given by

$$\begin{aligned} A_{t+1} &= A^w_{t+1} + A^r_{t+1} \\ &= R^d_t A_t + W_t N_t - C_t - T^w_t. \end{aligned} \tag{4.68}$$

Consumption

Marginal propensities to consume, both for retirees and workers, do not depend on individual characteristics. The aggregate consumption in each group is thus total wealth times the groups' marginal propensities to consume.

Aggregate consumption for retirees is

$$C^r_t = \epsilon_t \pi_t (R^d_t A^r_t) \ . \tag{4.69}$$

Retirees aggregate consumption is divided between home and foreign goods to satisfy the first order condition with respect to foreign consumption, $C^{r,1}_t e_t^{-\varepsilon} =$

$C_t^{r,2}$, so that retirees' consumption of domestic goods is

$$C_t^{r,1} = \frac{\epsilon_t \pi_t (R_t^d A_t^r)}{(1 + e_t^{1-\varepsilon})} \tag{4.70}$$

and retirees' consumption of foreign goods

$$C_t^{r,2} = \frac{e_t^{-\varepsilon} \epsilon_t \pi_t (R_t^d A_t^r)}{(1 + e_t^{1-\varepsilon})}. \tag{4.71}$$

Similarly for workers, the aggregate consumption is

$$C_t^w = \pi_t (R_t^d A_t^w + H_t^w). \tag{4.72}$$

Workers' consumption of domestic goods is

$$C_t^{w,1} = \frac{\pi_t (R_t^d A_t^w + H_t^w)}{(1 + e_t^{1-\varepsilon})} \tag{4.73}$$

and workers' consumption of foreign goods is

$$C_t^{w,2} = \frac{e_t^{-\varepsilon} \pi_t (R_t^d A_t^w + H_t^w)}{(1 + e_t^{1-\varepsilon})}. \tag{4.74}$$

Total aggregate consumption is

$$C_t = C_t^r + C_t^w = \epsilon_t \pi_t (R_t^d A_t^r) + \pi_t (R_t^d A_t^w + H_t^w). \tag{4.75}$$

Denoting the share of assets held by retirees $\lambda_t = \frac{A_t^r}{A_t}$, total aggregate consumption can be written as

$$C_t = \pi_t A_t R_t^d (\epsilon_t \lambda_t + 1 - \lambda_t) + \pi_t (H_t^w). \tag{4.76}$$

Consumption for retirees can be written as

$$C_t^r = \epsilon_t \pi_t (R_t^d \lambda_t A_t) \tag{4.77}$$

and consumption for workers

$$C_t^w = \pi_t (R_t^d \, (1 - \lambda_t) \, A_t + H_t^w). \tag{4.78}$$

On the aggregate level, consumption of home and foreign goods is divided as follows:

$$C_t^1 e_t^{-\varepsilon} = C_t^2 \tag{4.79}$$

and given that aggregate consumption is $C_t = C_t^1 + e_t C_t^2 = C_t^1(1 + e_t^{1-\varepsilon}) = \pi_t A_t R_t^d(\epsilon_t \lambda_t + 1 - \lambda_t) + \pi_t(H_t^w)$, aggregate consumption of domestic good in country 1 is

$$C_t^1 = \frac{\Big(\pi_t A_t R_t^d(\epsilon_t \lambda_t + 1 - \lambda_t) + \pi_t(H_t^w)\Big)}{(1 + e_t^{1-\varepsilon})} \tag{4.80}$$

and aggregate consumption of foreign goods in country 1 is

$$C_t^2 = \frac{e_t^{-\varepsilon}\Big(\pi_t A_t R_t^d(\epsilon_t \lambda_t + 1 - \lambda_t) + \pi_t(H_t^w)\Big)}{(1 + e_t^{1-\varepsilon})}. \tag{4.81}$$

Human wealth

The present discounted value of workers' aggregate human wealth is

$$\begin{aligned} H_t^w &= N_t^w W_t - T_t^w + \omega_{t,t+1} N_t^w \frac{\frac{H_{t+1}^w}{N_{t+1}^w}}{R_{t+1}^d \Omega_{t+1}} \\ &= N_t^w W_t - T_t^w + \omega_{t,t+1} N_t^w \frac{\frac{H_{t+1}^w}{N_t^w(1+n_t)}}{R_{t+1}^d \Omega_{t+1}} \\ &= N_t^w W_t - T_t^w + \omega_{t,t+1} \frac{H_{t+1}^w}{(1+n_t) R^d{}_{t+1} \Omega_{t+1}}. \end{aligned} \tag{4.82}$$

Distribution of wealth

Substituting expressions (4.67) and (4.78) into (4.66), the distribution of wealth between workers and retirees can be shown to evolve as follows:

$$
\begin{aligned}
A_{t+1}^{r} &= R_t^d A_t^r - C_t^r + (1-\omega_{t+1})(R_{t+1}^d A_t^w + W_t N_t^w - C_t^w - T_t^w) \\
&= R_t^d A_t^r (1-\epsilon_t \pi_t) + (1-\omega_{t+1}) \frac{A_{t+1}^w}{\omega_{t+1}} \qquad (4.83)
\end{aligned}
$$

Substituting the definition if $\lambda_t = \frac{A_t^r}{A_t}$, this can be rewritten as

$$
A_{t+1}\lambda_{t+1} = R_t^d \lambda_t A_t (1-\epsilon_t \pi_t) + (1-\omega_{t+1}) \frac{\lambda_{t+1} A_{t+1}}{\omega_{t+1}}
$$

$$
\lambda_{t+1} = \omega_{t,t+1}(R_t^d \lambda_t \frac{A_t}{A_{t+1}}(1-\epsilon_t \pi_t)) + (1-\omega_{t,t+1}). \qquad (4.84)
$$