WOODS HOLE OCEANOGRAPHIC INSTITUTION

Reference No. 64 - 29

On the Formulas for Correcting Reversing Thermometers

by

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ON THE FORMULAS FOR CORRECTING REVERSING THERMOMETERS

Introduction. The case often arises where a thermometer which has been inserted into a medium of temperature T_w is actually read in a place where the environment is at temperature $t, \neq T_w$. Such a case is the soil thermometer, where the bulb is at T_w and the stem in the air at t; and such a case is the oceanographic reversing thermometer, brought up from a depth-of-reversal (T_w) to the ship laboratory (t). In each case the different cubical expansion coefficients of mercury and glass mean that the stem mercury capillary is taken from the true reading of T_w to a reading of T' by the action of environmental change from T_w to t. Here we assume that the index calibration corrections are already incorporated into T'.

The correction ΔT, where

$$T_w = T' + \Delta T, \qquad \ldots\ldots\ldots(1)$$

is known as the "correction due to the emergence of the stem"[1], and derivations for it in oceanography go back to 1912[2]. Yet the subject is not quite exhausted, and a review of the various forms for ΔT has brought several points to light. This Note attempts to discuss these points. They are

1. The derivation of a correction equation for the unprotected thermometer implicit in a paper by Sverdrup[3], corresponding to Hansen's form[4] for the protected thermometer,

2. An oversight by Sverdrup[3] in attributing a certain form of a correction equation to Feruglio,[2]
3. An error in one of the correction equations as printed in Sverdrup, Johnson and Fleming, "The Oceans",[5]
4. The same error in Defant's "Physical Oceanography".[6]

In order to deal with these points, we must derive ΔT from first principles, following Sverdrup[3].

<u>Derivation of ΔT.</u> Let:

T_w = true water temperature at reversal depth,

T' = actual laboratory reading of reversing thermometer

t = laboratory temperature, as given by the auxiliary thermometer

$\Delta T = T_w - T'$

V = a general volume of the separated mercury column.

V_o = volume of mercury from the 0°C mark to the small-bulb-end of the reversing thermometer

V_w = volume of separated mercury column in reversed thermometer (from small-bulb-end up) as at T_w

V_t = volume of separated mercury column in reversed thermometer (from small-bulb-end up) as at t

γ_{hg} = coefficient of cubical expansion of mercury

γ_g = coefficient of cubical expansion of glass

$K \equiv (\gamma_{hg} - \gamma_g)^{-1}$

V_o, V_w and V_t are expressed in units of degrees Celsius. Hence, by definition,

$$V_w \equiv V_o + T_w \quad \text{(a)}$$
$$V_t \equiv V_o + T' \quad \text{(b)} \qquad \ldots\ldots\ (2)$$

The γ's are increases in volume, per unit volume, per degree Celsius.

From the reversal depth to the laboratory, the temperature change from T_w to t causes a volume change in the separated mercury column. Consider increments of these changes, δT and δV, where δV is so small that the volume V of the column is appreciably constant. Then δV is proportional to both V and δT. The proportionality factor, $\gamma_{hg} - \gamma_g \equiv K^{-1}$, is characteristic of the materials concerned, and, in the small range T_w to t, can be considered absolutely constant. That is,

$$\delta V = \frac{V}{K} \delta T,$$

or, as the increments both tend to zero,

$$dV = \frac{V}{K} dT, \qquad \ldots\ldots\ (3)$$

which may be integrated from T_w to t:

$$\int_{T_w}^{t} \frac{dV}{V} = \frac{1}{K} \int_{T_w}^{t} dT$$

$$\log\left(\frac{V_t}{V_w}\right) = - \frac{(T_w - t)}{K} \,. \qquad \ldots\ldots\ (4)$$

$$\left.\begin{array}{lll} \text{But} & V_t \equiv T' + V_o = T_w - \Delta T + V_o & \text{(a)} \\ \text{and} & V_w \equiv T_w + V_o & \text{(b)} \end{array}\right\} \quad \ldots\ldots (5)$$

so that (4) becomes

$$\log\left(1 - \frac{\Delta T}{T_w + V_o}\right) = - \frac{(T_w - t)}{K} \quad . \qquad \ldots\ldots (6)$$

Now

$$\log \chi = (\chi - 1) - 1/2(\chi - 1)^2 + 1/3(\chi - 1)^3 - \ldots (0 < \chi \leq 2),$$

hence from (6),

$$\frac{\Delta T}{T_w + V_o} + \frac{1/2(\Delta T)^2}{(T_w + V_o)^2} + \frac{1/3(\Delta T)^3}{(T_w + V_o)^3} + \ldots = \frac{T_w - t}{K} \quad , \ldots\ldots (7)$$

from which an expression for ΔT is to be extracted. (This equation shows what is clear from first principles, viz., that ΔT is zero if either $T_w = t$, or $\gamma_{hg} = \gamma_g$, or both).

All ΔT correction formulas are derivable from (or at least explainable by) equation (7). However, this source is useless as it stands, since it contains T_w, the very unknown we finally want to find (see eqn (1)). Now for ΔT (protected) it is clear that both $(T_w - t)$ and $(T_w + V_o)$ can be replaced by $(T' - t)$ and $(T' + V_o)$ or, for better accuracy, by $(T' + \Delta T - t)$ and $(T' + \Delta T + V_o)$, where T' is the protected's temperature-as-read. For ΔT (unprotected), we can say that $(T' + V_o)$ or $(T' + \Delta T + V_o)$ is close to the "instrumental" term $(T_w + V_o)$, where T' is now the unprotected's thermometer-as-read. However this T' is so far different to the true temperature that no substitution can be permitted for T_w in $(T_w - t)$. Indeed, for this T_w we use the value of the corrected reading of the adjacent protected thermometer.

Thus for ΔT (protected), T_w in each term, $(T_w - t)$ and $(T_w + V_o)$, ~~are~~ is replaced by T' or $T' + \Delta T$. For ΔT (unprotected) we leave $(T_w - t)$ untouched and substitute only into $(T_w + V_o)$.

We take forms for ΔT (unprotected) and ΔT (protected) in turn. The general procedure will be to take one term of the series of (7), putting first $T_w \simeq T'$, then $T_w = T' + \Delta T$, then returning to take two terms of the series and repeating the substitutions.

I. Unprotected thermometer corrections. Here we leave $(T_w - t)$ and alter only $(T_w + V_o)$.

(a) Using only one term of the series in equation (7), we get:

$$\Delta T = \frac{(T_w - t)\ (T_w + V_o)}{K}, \qquad \ldots\ldots\ (8)$$

and using $T_w \simeq T'$ as a first approximation,

$$\Delta T = \frac{(T_w - t)\ (T' + V_o)}{K}, \qquad \ldots\ldots\ (9)$$

SCHUMACHER[7]

the usual correction for unprotected thermometers (see, e.g. Lafond[14]).

Replacing T_w by $(T' + \Delta T)$ instead of T' alone, (8) becomes

$$\Delta T = (T_w - t)\ (T' + \Delta T + V_o)\ K^{-1}$$

$$= \frac{(T_w - t)\ (T' + V_o)}{K} + \frac{\Delta T\ (T_w - t)}{K} \qquad \ldots\ldots\ (10)$$

In effect, Schumacher now takes the ΔT from (9) and substitutes into the right hand ΔT of (10). Hence

$$\Delta T = \frac{(T_w - t)\ (T' + V_o)}{K} + \left[\frac{(T_w - t)\ (T' + V_o)}{K}\right] \frac{(T_w - t)}{K}$$

$$\text{i.e. } \Delta T = \frac{(T_w - t)\ (T' + V_o)}{K} \left\{ 1 + \frac{(T_w - t)}{K} \right\} \qquad \text{......(11)}$$

<u>SCHUMACHER</u>[8]

This equation is sometimes used for correcting unprotected thermometers (see, e.g., Wüst[9]).

Equation (10) can of course be solved directly for ΔT:

$$\Delta T \left\{ 1 - (T_w - t) K^{-1} \right\} = (T_w - t)\ (T' + V_o) K^{-1}$$

$$\Delta T = \frac{(T_w - t)\ (T' + V_o)}{K - (T_w - t)} \qquad \text{......(12)}$$

<u>SVERDRUP</u>[10]

(b) Using <u>two</u> terms of the series in (7), we might expect to get a slightly more accurate formula for ΔT. Carrying out the algebra,

$$\Delta T \left\{ 1 + \frac{1/2\, \Delta T}{T_w + V_o} \right\} = \frac{(T_w - t)\ (T_w + V_o)}{K}$$

$$\Delta T = \frac{(T_w - t)\ (T_w + V_o)}{K} \left[1 + \frac{1/2\, \Delta T}{T_w + V_o} \right]^{-1},$$

which becomes, using the binomial expansion,

$$\simeq \frac{(T_w - t)\ (T_w + V_o)}{K}\left[1 - \frac{1/2\,\Delta T}{T_w + V_o}\right]$$

$$= \frac{(T_w - t)\ (T_w + V_o)}{K} - \frac{1/2\,\Delta T\ (T_w - t)}{K} \qquad \ldots\ldots\ (13)$$

From here we can either solve directly for ΔT, then use $T_w \simeq T'$:

$$\Delta T \left[1 + \frac{1/2(T_w - t)}{K}\right] = \frac{(T_w - t)\ (T_w + V_o)}{K}$$

$$\Delta T = \frac{(T_w - t)\ (T_w + V_o)}{K + 1/2\ (T_w - t)} \qquad \ldots\ldots\ (14)$$

$$\simeq \frac{(T_w - t)\ (T' + V_o)}{K + 1/2\ (T_w - t)}, \qquad \ldots\ldots\ (15)$$

or we can use $T_w = T' + \Delta T$ first in (13), to get:

$$\Delta T = (T_w - t)\ (T' + \Delta T + V_o)K^{-1} - 1/2\ \Delta T(T_w - t)K^{-1}$$

$$= \frac{(T_w - t)\ (T' + V_o)}{K} + \frac{1/2\ \Delta T(T_w - t)}{K} \qquad \ldots\ldots\ (16)$$

This itself yields two forms, according to whether we follow the "Schumacher step" of putting (9) into the ΔT on the right side of (16):

$$\Delta T = \frac{(T_w - t)(T' + V_o)}{K} + 1/2\left[\frac{(T_w - t)(T' + V_o)}{K}\right]\frac{T_w - t}{K}$$

$$\text{i.e. } \Delta T = \frac{(T_w - t)(T' + V_o)}{K}\left\{1 + \frac{1/2(T_w - t)}{K}\right\}, \qquad \ldots\ldots\ (17)$$

<u>HANSEN</u>[11]

or whether we solve (16) exactly for ΔT:

$$\boxed{\Delta T = \frac{(T_w - t)\ (T' + V_o)}{K - 1/2(T_w - t)}} \quad \ldots\ldots\ (18)$$

This equation is the most exact form so far. Though apparently nowhere quoted in the literature, it is easily derivable from Sverdrup's analysis[3].

II. Protected Thermometer Corrections.

Here we may replace both T_w's, in $(T_w - t)$ and $(T_w + V_o)$, by either T' or $T' + \Delta T$.

(a) Again referring to (7), we take one term of the series, which leads to (8), and use $T_w \simeq T'$. Then

$$\Delta T = \frac{(T' - t)\ (T' + V_o)}{K} \quad \text{FERUGLIO}^{12} \quad \ldots\ldots\ (19)$$

Sverdrup actually calls equation (8) "Feruglio's formula", which is both incorrect and misleading. The difference is not trivial. Equation (19) is a working form, (8) is not. At any rate, we proceed by putting $T_w = T' + \Delta T$ now in both $(T_w - t)$ and $(T_w + V_o)$ of (8):

$$\Delta T = (T' + \Delta T - t)\ (T' + \Delta t + V_o)K^{-1}$$

$$= (T'^2 + 2T\Delta T + T'V_o + (\Delta T)^2 + V_o\Delta T - tT' - t\Delta T - tV_o)K^{-1}$$

The $(\Delta T)^2$ can be safely neglected, so that the above becomes

$$\Delta T = \frac{(T' - t)\ (T' + V_o)}{K} + \Delta T\left\{\frac{(T' - t) + (T' + V_o)}{K}\right\} \ldots\ldots\ (20)$$

The Schumacher step, putting (19) into the right ΔT of (20), gives

$$\Delta T = \frac{(T' - t)\ (T' + V_o)}{K} + \left[\frac{(T' - t)(T' + V_o)}{K}\right] \frac{(T' - t) + (T' + V_o)}{K}$$

$$\Delta T = \frac{(T' - t)\ (T' + V_o)}{K} \left\{1 + \frac{(T' - t) + (T' + V_o)}{K}\right\} \quad \ldots\ldots\ldots (21)$$

SCHUMACHER[13]

This is the correction formula most commonly used for protected thermometers (see, e.g., Lafond[14]).

If we imagine that $T' = t$, but in the braces only, we get, from (21)

$$\Delta T = \frac{(T' - t)\ (T' + V_o)}{K} \left\{1 + \frac{(T' + V_o)}{K}\right\} \quad \ldots (22)$$

SHOULEJKIN[15]
SUBOW, BOUJEWICZ

Hidaka's formula can be thought of as coming from (21) in the following way: rewrite it as

$$\Delta T = \frac{(T' - t)\ (T' + V_o)}{K \left[1 + \frac{(T' - t) + (T' + V_o)}{K}\right]^{-1}}$$

$$\simeq \frac{(T' - t)\ (T' + V_o)}{K \left[1 - \frac{(T' - t) + (T' + V_o)}{K}\right]}$$

$$\simeq \frac{(T' - t)\ (T' + V_o)}{K - (T' - t + V_o)}, \quad \ldots\ldots\ldots (23)$$

HIDAKA[16]

where we have used the binomial expansion, and dropped one of the T''s in the denominator only.

It is clear that (22) and (23) are being explained, not derived as such.

If (20) is solved exactly for ΔT, then we get:

$$\Delta T \left[1 - \left\{ \frac{(T' - t) + (T' + V_o)}{K} \right\} \right] = \frac{(T' - t)\ (T' + V_o)}{K}$$

$$\Delta T = \frac{(T' - t)\ (T' + V_o)}{K - (T' - t) - (T' + V_o)} \qquad \underline{\text{SVERDRUP}}\text{[17]} \qquad \ldots\ldots.(24)$$

(b) We now return to (7) to take <u>two</u> terms of the series. This leads to (14). Then, if $T_w \simeq T'$ everywhere,

$$\Delta T = \frac{(T' - t)\ (T' + V_o)}{K + 1/2(T' - t)} \qquad \ldots\ldots.(25)$$

Another variation is to put $T_w = T'$ into (18), so that

$$\Delta T = \frac{(T' - t)\ (T' + V_o)}{K - 1/2(T' - t)} \qquad \ldots\ldots.(26)$$

However, for best accuracy try $T_w = T' + \Delta T$ everywhere in (13):

$$\Delta T = (T' + \Delta T - t)\ (T' + \Delta T + V_o)K^{-1} - 1/2\Delta T(T' + \Delta T - t)K^{-1}$$

Multiplying out and collecting,

$$\Delta T = (T'^2 + T'V_o - tT' - tV_o)K^{-1} + \Delta T(\tfrac{3}{2}T' - \tfrac{1}{2}t + V_o)K^{-1}$$

$$= \frac{(T' - t)\ (T' + V_o)}{K} + \Delta T \left\{ \frac{1/2(T' - t) + (T' + V_o)}{K} \right\} \ldots(27)$$

The Schumacher step, (19) into the right side of (27), gives

$$\Delta T = (T' - t)\ (T' + V_o)K^{-1} + \left[(T' - t)\ (T' + V_o)K^{-1} \right] \left\{ 1/2(T' - t) + (T' + V_o) \right\} K^{-1}$$

$$\Delta T = \frac{(T' - t)\ (T' + V_o)}{K} \left\{ 1 + \frac{1/2(T' - t) + (T' + V_o)}{K} \right\} \qquad \ldots\ldots.(28)$$

Finally, the most accurate form is obtained by solving (27) directly for $\triangle T$:

$$\triangle T \left[1 - \frac{1/2(T' - t) + (T' + V_o)}{K} \right] = \frac{(T' - t)\ (T' + V_o)}{K}$$

or

$$\boxed{\triangle T = \frac{(T' - t)\ (T' + V_o)}{K - 1/2(T' - t) - (T' + V_o)}} \qquad \text{.......}(29)$$

<u>HANSEN</u>[18]

This is the equation used, for instance, in the text book by Dietrich.[19]

The various forms for ΔT, unprotected and protected, are written below

	UNPROTECTED	PROTECTED
ONE TERM OF LOGARITHMIC SERIES	$\Delta T = \dfrac{(T_w-t)\ (T'+V_o)}{K}$ (9) SCHUMACHER	$\Delta T = \dfrac{(T'-t)\ (T'+V_o)}{K}$ (19) FERUGLIO
	$\Delta T = \dfrac{(T_w-t)\ (T'+V_o)}{K}\left\{1 + \dfrac{(T_w-t)}{K}\right\}$ (11) SCHUMACHER	$\Delta T = \dfrac{(T'-t)\ (T'+V_o)}{K}\left\{1+ \dfrac{(T'-t)+(T'+V_o)}{K}\right\}$ (21) SCHUMACHER
		$\Delta T = \dfrac{(T'-t)\ (T'+ V_o)}{K}\left\{1 + \dfrac{(T'+ V_o)}{K}\right\}$ (22) SUBOW, BOUJEWICZ, SHOULEJKIN
		$\Delta T = \dfrac{(T'-t)\ (T'+ V_o)}{K-(T'-t+V_o)}$ (23) HIDAKA
	$\Delta T = \dfrac{(T_w-t)\ (T'+V_o)}{K-(T_w-t)}$ (12) SVERDRUP	$\Delta T = \dfrac{(T'-t)\ (T'+V_o)}{K-(T'-t)-(T'+ V_o)}$ (24) SVERDRUP
	$\Delta T = \dfrac{(T_w-t)\ (T'+V_o)}{K+1/2(T_w-t)}$ (15)	$\Delta T = \dfrac{(T'-t)\ (T'+ V_o)}{K+1/2(T'- t)}$ (25)
TWO TERMS OF SERIES		$\Delta T = \dfrac{(T'- t)\ (T'+V_o)}{K-1/2(T'-t)}$ (26)
	$\Delta T = \dfrac{(T_w-t)\ (T'+V_o)}{K}\left\{1+\dfrac{1/2(T_w-t)}{K}\right\}$ (17) HANSEN	$\Delta T = \dfrac{(T'- t)\ (T'+V_o)}{K}\left\{1+ \dfrac{1/2(T'-t)+(T'+V_o)}{K}\right\}$ (28)
	$\boxed{\Delta T = \dfrac{(T_w-t)\ (T'+V_o)}{K-1/2(T_w-t)}}$ (18) SVERDRUP	$\boxed{\Delta T = \dfrac{(T'-t)\ (T'+V_o)}{K-1/2(T'-t)-(T'+V_o)}}$ (29) HANSEN

It is interesting to see how the numerical values for ΔT vary according to which formula is used. To see this, we first obtain a figure for K. From the Handbook of Chemistry & Physics[20], $\gamma_{hg} = 0.18186 \times 10^{-3}$ at 20°C, $\gamma_g(\text{Jena}) = 0.2533 \times 10^{-4}$, hence $K^{-1} = 1.5653 \times 10^{-4}$ and K=6388. Depending on the exact type of Jena glass used, K varies somewhat, but let us take 6300 (Jena Glass 16^{III}). The higher it is, the better, of course. Also let $T_w = 5°C$, $T' = 15°C$, $t = 20°C$, $V_o = 100$ in the unprotected case; and $T' = 5°C$, $t = 20°C$ and $V_o = 100$ in the protected case. Then:

Equation	ΔT, Unprotected		Relative Rating for Accuracy	Equation	ΔT, Protected		Relative Rating for Accuracy
	Fraction °C	Decimal °C			Fraction °C	Decimal °C	
(9) Schumacher	-1725/6300	-0.274	3	(19) Feruglio	-1575/6300	-0.250	8
(11) Schumacher	-1725/6315.036	-0.273	4	(21) Schumacher	-1575/6211.268	-0.254	5
				(22) S-B-S	-1575/6196.721	-0.254	3
				(23) Hidaka	-1575/6215	-0.253	6
(12) Sverdrup	-1725/6315	-0.273	3	(24) Sverdrup	-1575/6210	-0.254	4
(15)	-1725/6292.5	-0.274	5	(25)	-1575/6292.5	-0.250	7
				(26)	-1575/6307.5	-0.250	9
(17) Hansen	-1725/6307.509	-0.273	2	(28)	-1575/6203.986	-0.254	2
(18) Sverdrup	-1725/6307.5	-0.273	1	(29) Hansen	-1575/6202.5	-0.254	1

The relative accuracy rating figure is based on the differences of the respective denominators compared to the "true" values in (18) and (29). There is certainly very little variation among them, except perhaps (26), (19), and (25) from the rest of the protected ΔT's. However, it is interesting to note that for the unprotected thermometer the denominators show that (11), supposedly refined over (9), actually overcompensates it, though the decimals do not show this. In the protected case Sverdrup's (24) is indeed better than Schumacher's (21) and Hidaka's (23), but not as good as the Russian (22); again better shown by the demoninators.

It must be stressed that no attempt is being made here to justify equations (15), (25) and (26). They are included only for interest. On the other hand, equation (28) is the natural counterpart of Hansen's (17), and forms a logical step in the sequence of derivations.

<u>Error in Text Books</u>. It may be useful here to point out that in quoting Schumacher's equation for the protected thermometer, viz eqn. (21),

$$\Delta T = \frac{(T' - t)\ (T' + V_0)}{K} \left\{ 1 + \frac{(T' - t)\ +\ (T' + V_0)}{K} \right\},$$

both Sverdrup - Johnson - Fleming's "The Oceans"[5] and Defant's "Physical Oceanography", Vol. I[6], have printed:

$$\Delta T = \frac{(T' - t)\ (T' + V_0)}{K} \left\{ 1 + \frac{(T' - t)\ (T' + V_0)}{K} \right\}.$$

The dropping of the second plus in the braces leads to a considerable error.

For the same values of T', V_o, t and K used previously (protected case),

$$"\Delta T" \text{ (S-J-F,D.)} = -1575/8400 = -0.1875°C,$$

which means a 24% error compared to (29).

Conclusion.

For the most precise work, correction formulas (18) and (29),

$$\left.\begin{aligned} \Delta T(\text{Unprotected}) &= \frac{(T_w - t)\ (T' + V_o)}{K - 1/2(T_w - t)} \\[2ex] \Delta T(\text{protected}) &= \frac{(T' - t)\ (T' + V_o)}{K - 1/2(T' - t) - (T' + V_o)} \end{aligned}\right\}$$

are to be preferred over all others mentioned in the table on p. 11. For this kind of work account must be made of the magnitude of the effect of external hydrostatic pressure on the air bubble in the protected thermometer.[21,22]. Apart from this, however, the above formulas are among the easiest to use on a calculator, and certainly might as well be programmed into a computer rather than the less accurate equations.

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