12-2019

Platform pricing with strategic buyers: The impact of future production cost

Mei LIN

Singapore Management University, mlin@smu.edu.sg

Xiajun Amy PAN

Quan ZHENG

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Mei Lin
School of Information Systems, Singapore Management University, Singapore, mlin@smu.edu.sg

Xiajun Amy Pan
Warrington College of Business Administration, University of Florida, Gainesville, Florida 32611, amypan@ufl.edu

Quan Zheng*
International Institute of Finance, School of Management, University of Science and Technology of China, Hefei, Anhui 230026, China, benzheng@ustc.edu.cn

Two-sided platforms are often coupled with exclusive hardware products that connect two sides of users, the consumers of the hardware product (i.e., buyers) and the application developers (i.e., sellers). The hardware product in the platform business model introduces three important issues that are not yet well understood in the literature of platform pricing: potentially downward-trending production cost, product quality improvements, and consumers' strategic behaviors. Using analytical modeling, our study explicitly factors in these issues in analyzing a monopoly platform owner's two-sided pricing problem. The platform sequentially introduces and prices quality-improving hardware products, for which the costliness of quality may decrease. Strategic buyers make purchasing and upgrading decisions, which dynamically determines the buyer-side network size. Meanwhile, the seller-side network size is determined endogenously. We find that, an increase in the likelihood or magnitude of future costliness reduction raises the initial buyer-side price of the low-quality product and lowers the seller-side fee. This strategy, in turn, creates an indirect intertemporal effect that allows the platform to also raise the buyer-side price(s) of the product(s) sold later. These findings contrast with conventional wisdom and provide an economic explanation for premium introductory pricing of many platform products. Moreover, we find that strengthening the network effect can result in more pronounced increases in the buyer-side prices.

Key words: Dynamic pricing; two-sided platforms; sequential innovation; network effects; strategic consumers

History: Received: November 2018; Accepted: December 2019 by Subodha Kumar after two revisions.

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* Corresponding Author

1. Introduction

For two-sided platforms, hardware product is often an integral part of the business model. Many well-known platforms connect different groups of users through an exclusive hardware product. Apple products, such as the iPads, iPhones, and Macbooks, bring together the two sides of the platform – users and application developers. The platform hardware is essential such that the users need to purchase the hardware in order to access the third-party applications. Similarly, Google's own smartphone Pixel serves as the platform for users and application developers. Amazon Kindle allows readers to use the device for ebooks from different publishers. More recently, Amazon Echo, the voice-enabled virtual assistant, is gaining popularity. Through Echo, users can access a myriad of "Alexa Skills", which are third-party applications created for Echo. By using voice commands on Echo, users can also make purchases on Amazon.com from the third-party sellers.

When hardware plays a pivotal role in the business model, the platform owner's two-sided strategies are closely tied to the pricing of the hardware device. The iPhone, arguably the most influential and revolutionary consumer electronic in the recent decade, leads Apple's continuing success as a platform owner. In a Bloomberg interview (Grobart 2013), the CEO of Apple Tim Cook spoke on their product strategy: "We never had an objective to sell a low-cost phone. Our primary objective is to sell a great phone and provide a great experience, and we figured out a way to do it at a lower cost." His seemingly paradoxical statement suggests that Apple aims to sell a high-end phone while also focusing on reducing costs. The emphasis on cost consideration leads to a series of interesting questions: How does cost reduction affect platform pricing with a hardware device? Does cost reduction facilitate or discourage high-end positioning of the platform product? How do some other platform-specific characteristics such as network effects come into play? To answer these questions, we need to consider the factors particular to hardware in the context of platform pricing. In this paper, we construct a dynamic two-sided framework to address these questions, accounting for the following factors: production cost, multi-period selling, and strategic consumers.

The production cost of hardware is clearly an important aspect of platform pricing, and its dynamics should be factored into the pricing strategies. Studies on information products, including those related to two-sided platforms, often consider the marginal cost to be zero, which may be limiting for analyzing hardware-based platform business models. For this type of platforms, not only does production cost affect pricing, it also tends to change over time, often with a downward trend. The decreasing cost in part owes to the decreasing manufacturing cost. Platform owners can also play a part in strategically reducing the cost through interactions with their suppliers.

The manufacturing cost, especially that for technology, can decline substantially over time. Decades of academic and industry research has identified learning by doing as an important driver in cost reduction. Starting with Arrow's seminal work (Arrow 1962) that provides a theoretical

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basis for organization-level learning by doing, numerous studies ensued to understand the economic impacts of learning by doing. In a recent discussion on this topic, Thompson (2012) draws on the literature, "organizational learning was shown to affect dynamic pricing strategy because production costs are expected to fall as cumulative production increases" (p. 205), emphasizing the implications for pricing. The cost reduction is also evident in practice, such as the drastic 99 percent reduction in the cost of solar power in the recent years (Chandler 2018). Kavlak et al. (2018) analyze the causes of such cost reduction and find policies that encourage innovation, scale economies, and learning by doing to be especially important.

Aside from the cost reductions on the manufacturing side, the platform owner's strategies in optimizing relationships and contracts with suppliers can be effective in further reducing cost. Apple is known for its success in managing supplier relationships. Its market position offers great bargaining power against its suppliers, allowing Apple to secure a lower production cost for future products (Goldman 2011). Apple also adjusted its operating model by taking the responsibility of procuring parts and materials away from the manufacturers to eliminate the markup (Parker 2013), and to allow the manufacturers to focus solely on assembling and production. This strategy not only directly reduces costs, but also further shifts the bargain power away from the manufacturers to Apple. The effectiveness of these cost cutting strategies is evident. The recent cost saving on the iPhone XS Max is substantial compared to the cost of iPhone X. The upgrades of the core components in the iPhone XS Max only added $20 to the bill of materials, whereas the retail price increased by $100, compared to those of the iPhone X (Lam and Hong 2018). Furthermore, the cost saving is not only reflected by the total component costs. With iPhone XS Max, the bill of materials only made up 35.48 percent of the total cost, a reduction from the 37 percent for the iPhone X (Bluesea Research 2018).

Because production cost is dynamic, it is important to study platform strategies in a multi-period setting. The platform industry is highly innovative – while production cost changes over time, hardware products also advance in quality. Since the release of the original Amazon Echo in 2015, Amazon has integrated a smart home hub in the Echo Plus introduced in 2017. As the platform rolls out different versions of the hardware product sequentially, its pricing strategy is dynamic, to take into account the potential consumer market at the time of releasing a new version and the changes in production cost. Moreover, if the platform chooses to continue offering the older version, the co-existence of different versions requires more sophisticated dynamic pricing considerations.

Faced with the ever-changing platform market, the consumers are often strategic. With the platform sequentially introducing quality-improving products, many consumers become savvy and forward-looking. Their anticipation of future products often sparks wide discussions and generates abundant information on the Internet, which allows consumers to strategically plan their purchases.

Some consumers may choose to wait for the later version of the product, while others may purchase the version that is currently available. Among the latter type of consumers, some may upgrade to the later version when it is released. The platform needs to account for these different preferences and price its products accordingly to optimally segment the consumer market.

With these three factors, we construct a framework that analyzes a monopoly platform's dynamic two-sided pricing strategy, when offering sequentially improving hardware products to consumers. Our paper aims to answer the following questions: When facing strategic consumers and uncertain cost reduction in the future, how does a monopoly platform set the prices for its quality-improving hardware products and the seller-side fee? Is it optimal for the platform to continue offering the low-quality product when the high-quality version is released? If the likelihood or magnitude of costliness reduction increases, how would the platform adjust its two-sided pricing decision in each period? Furthermore, we study the role of network effects in the impacts of future costliness reduction on prices.

Our theoretical model captures sequential introductions of quality-improving products and dynamic pricing decisions with uncertainty in cost reduction. The platform sets the buyer-side price(s) for the hardware product(s) offered and the seller-side fee for joining the platform. The buyers make strategic purchasing decisions. In a two-period time horizon, the low-quality product is introduced in Period 1 and the high-quality product in Period 2. The platform also decides whether to continue offering the low-quality product in Period 2. The hardware product is considered as a durable good, but the buyers have the option of upgrading an existing purchase when a higher quality product is introduced.

We find that the platform always continues to offer the low-quality product in Period 2. Thus, the low and high qualities co-exist in Period 2. The platform's pricing problem is then to optimally segment the buyer-side demand across the two periods and between the two qualities, while balancing the seller-side profit. We analytically derive the platform's optimal buyer-side prices and seller-side fee and further study the impacts of the likelihood and magnitude of costliness reduction in Period 2 on the optimal decisions.

When the likelihood or magnitude of costliness reduction in Period 2 increases, the platform's optimal strategy in Period 1 is to capture less buyer-side demand and raise the buyer-side price. The extent of the buyer-side demand captured initially is critical because it sets up the market demand for the future. When the platform captures fewer buyers in Period 1, more potential buyers are left for Period 2, where the expected market profitability increases because of the increased likelihood or magnitude of costliness reduction. And the platform does so by raising the optimal buyer-side price in Period 1, which also further exploits the higher-valuation buyers. This allows the platform to charge a premium on its introductory product. Meanwhile, the seller-side fee is reduced because the shrunken buyer-side demand makes the platform less attractive to sellers.

Within Period 2, the likelihood and magnitude of costliness reduction have different impacts on the buyer-side prices, depending on the direct and indirect effects created. The likelihood of cost reduction has no direct effect on the buyer-side prices in Period 2 because the uncertainty on cost reduction is realized at that time, rendering the likelihood irrelevant. Similarly, in the scenario without cost reduction, the magnitude of costliness reduction exerts no direct effect on price. Thus, the direct effect only exists for the magnitude of costliness reduction, in the scenario where cost reduction does occur. And the direct effect simply drives down the buyer-side prices, consistent with conventional wisdom. However, the platform's strategy from Period 1 leaves more potential buyers to Period 2, which results in an indirect effect in the opposite direction. Therefore, an increase in the likelihood of cost reduction leads to higher buyer-side prices in Period 2, so does an increase in the magnitude of reduction in the scenario without cost reduction. For the scenario with cost reduction, if the indirect effect is dominant, a greater magnitude of reduction results in higher buyer-side prices for both qualities of products in Period 2; and vice versa. Interestingly, the high-quality product demand always increases while the low-quality product demand always decreases with an increase in either the likelihood or magnitude of costliness reduction, regardless of the platform's pricing strategy.

The network effects experienced by both the high- and low-type of sellers further enhance the impacts of the likelihood and magnitude of costliness reduction on the buyer-side prices and demand. Regardless of the seller type, the network effect has a qualitatively consistent impact. Specifically, when the network effect is stronger, the platform raises the buyer-side price in Period 1 more aggressively in response to the increase in likelihood or magnitude of costliness reduction; as a result, the buyer-side demand in Period 1 also shrinks to a greater extent. The network effect has the same impact on the buyer-side prices in Period 2 and the total buyer-side demand across two periods when the buyer-side prices are raised with more costliness reduction. For the case in which the platform lowers the buyer-side price in Period 2, a stronger network effect induces the platform to do so less aggressively, and the buyer-side demand across both periods increases to a lesser extent. The overall intuition is that strengthened network effect allows the platform to generate the seller-side profit more effectively, so less buyer-side demand is needed.

The remaining of the paper is organized as follows. We discuss the related literature in Section 2. Section 3 introduces the model. In Section 4, we solve the model to derive the optimal buyer-side prices and seller-side fee, and discuss the findings. Section 5 presents the numerical studies based on the model with network effects in both directions (buyer- to seller-side and seller- to buyer-side). Finally, Section 6 concludes the paper.

2. Related Literature

Our work is closely related to four streams of literature: two-sided platforms, sequential innovation, strategic customers and dynamic pricing. To the best of our knowledge, our paper is among the first to consider the dynamic pricing problem of a two-sided platform that sequentially introduces innovative hardware devices in the presence of strategic consumers. We connect the insights from these domains of knowledge to gain a deeper understanding of the dynamic platform problem.

The literature on two-sided platforms explores the platform's pricing problem taking into consideration network effects, user multi-homing, platform governance, and innovation. The earlier works include Rochet and Tirole (2002), Rochet and Tirole (2003), Caillaud and Jullien (2003), Parker and Alstyne (2005), and Armstrong (2006). Continuing from this line of literature, more recent studies examine the innovation issues on platforms. Lin et al. (2011) study the innovation race among sellers of a two-sided market. By analyzing innovation incentives and price competition among sellers, they find the platform's optimal two-sided pricing strategy. They show that the seller-side fee may have a positive impact on sellers' innovation incentives, while the buyer-side fee slows down the innovation race. Boudreau (2012) conducts an empirical study on the effect of the number of applications on software variety. He finds that an increase in the number of application producers leads to an overall reduction in innovation incentives, which creates a tension with the positive network effects assumed by many studies of two-sided markets. Hagiu (2009) accounts for the effect of consumers' preference for variety. He examines the effect of such variety on the platform's pricing strategies and discusses how the seller-side pricing structure influences the sellers' innovation incentives. These studies focus on innovations that drive the products offered by sellers to buyers, whereas we devote our attention to the the platform's strategies in managing the innovative hardware market through two-sided pricing.

Although the studies on two-sided pricing models have been commonly based on static settings to derive crisp insights and maintain analytical tractability, a growing body of research work has begun to explore dynamic strategies in the platform context. Hagiu (2006) investigates price commitment by a platform, where one side of the platform arrives before the other side. He finds that the platform can attract the early-arrival side without committing to a low price for the late-arrival side. Also allowing the consumer side to arrive first, Bhargava et al. (2013) examine the platform's product line expansion strategy with uncertainty on developer-side participation. They find the dependencies of the expansion strategy on the fixed cost for expansion and uncertainty on developer participation. Lin et al. (2011) study sellers' dynamic innovation race to create products for the platform market and find implications on the platform's pricing decisions. Chou et al. (2012) point out that supply chain operational costs may alter the conventional understanding on platform subsidization strategies. Zhu and Lansiti (2012) consider forward-looking consumers

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and focus on a platform's entry problem in competition with an incumbent, with constant quality. Through both analytical modeling and empirical validation, they find that, when both the network effect and consumers' valuation for future applications are sufficiently low, a platform entrant may capture its market with quality advantage. Whereas Zhu and Lansiti (2012) do not model the platform owner's prices, Dou et al. (2012) analytically study a platform's pricing decision on the buyer side. By comparing strategic buyers with myopic buyers, Dou et al. (2012) identify that the two types of buyers exhibit different behaviors only when the platform owner operates a license model or a limited-time freemium model with a positive switching cost.

Compared to the studies on dynamic platform pricing, we are more interested in the platform's dynamic acquisition of the buyer side market through quality-improving products while balancing its profits on the other side. Our findings echo those in the related papers by also illustrating the importance of product quality as well as the platform owner's and buyers' strategic considerations. Furthermore, we emphasize other factors such as multi-product offering, quality improvements, and the decreasing production cost. More recently, costs in the context of two-sided pricing is receiving increasing attention. In our work, the production cost of platform products enters the context of quality innovation. We show that its variability impacts the platform's dynamic prices and strategies in intertemporal market segmentation.

Network effects and other key elements of two-sided platforms have been considered in a variety of contexts. Gilbert and Jonnalagedda (2011) anchor on the concept of "contingent product," which is the product that is required to consume a durable good (e.g., ink is the contingent product of printer). They evaluate the lock-in strategy with consideration for strategic consumers and find that the firm's ability to commit to shutting down the production of the durable good plays an important role. Bhargava and Choudhary (2004) study versioning strategies of a platform ("infomediary" in their paper) that provides matching services for the two sides with the option of value-added services. They find that it is optimal for the platform to offer two versions of matching services, those with and without the value-added services, and that the versioning incentives are stronger compared to a traditional seller as a result of network effects. Cheng et al. (2011) evaluate net neutrality policies by studying the the broadband service provider as a platform, which charges a fee to consumers and possibly also a price to the content provider side. By modeling two-sided pricing, they find that abolishing net neutrality benefits the broadband service provider while taxing the content providers; the change to consumer surplus further depends on the relative capabilities of the content providers in generating revenues. Guo et al. (2013) further examine the net neutrality problem by considering the broadband service provider's options to also discriminate the consumer side. Their findings emphasize the importance of the platform making strategic decisions on both the content provider side and the consumer side simultaneously. Hao et al. (2017) focus on mobile advertising platforms and examine different strategies of the platform owner in pricing ads and those of application developers in publishing ads. Chou et al. (2012) incorporate a new element of supply chain operational costs into a two-sided pricing problem. Whereas the conventional theory on platform subsidies may hold, in some cases the platform extracts surplus on both sides to offset the supply chain costs.

Our paper is also related to the literature on sequential innovation. Our model builds on those in Dhebar (1994) and Kornish (2001), which examine the problem of a durable-goods monopolist selling low-quality and high-quality products in the first and second period, respectively. They examine whether there exists an equilibrium pricing strategy when the pace of quality improvement varies. Dhebar (1994) concludes that rapid quality improvement is not desirable even with the option of upgrading the low-quality products, whereas Kornish (2001) shows that any large quality improvement could be optimal under different parameter settings without offering the special upgrading pricing in the second period. Bhattacharya et al. (2003) investigate how to optimally introduce high technology products with an option of holding the low quality products until the high quality products launch. They show that introducing low quality products before high quality products may be still preferred. For topics on sequential innovation, Ramachandran and Krishnan (2008) provide a detailed review.

A key component in most dynamic pricing models is strategic consumer behavior, that is, consumers are forward-looking and may delay their purchases to maximize their utilities over time. Researchers are often interested in how a monopolist optimally prices a single product over time. Stokey (1979) and Bulow (1982) show that a monopolist is forced to price at the marginal cost; Besanko and Winston (1990) prove that the optimal price decreases over time due to consumers' strategic behavior. Levin et al. (2010) analytically illustrate that, for a monopolist offering a perishable product, accounting for consumers' strategic behaviors is critical for obtaining maximum revenues. Our work emphasizes the role of production cost in the firm's and consumers' decisions. We show that, with more reduction in future costliness, consumers' strategic behaviors make possible for the firm to raise the price(s) of the product(s) both initially and in the future.

Dynamic pricing strategies, including skimming or penetration pricing, have been extensively discussed in literature (Liu 2010, Spann et al. 2014). Textbook theories (Kotler and Armstrong 2012) recommend the skimming strategy for differentiated products with sufficient consumer heterogeneity and the penetration strategy for price-sensitive markets with strong competition and network effects. Essentially, they focus on the price trend over time, that is, how firms price their products dynamically under different market conditions. In contrast, we are interested in the dynamic impacts of production cost on prices rather than the price comparison itself across different periods. Specifically, in our work, an increase in the initial price can result from a greater likelihood or magnitude of future cost reduction, different from the economic mechanisms of price skimming.

3. The Model

In this section, we will lay out the model setup for the platform, the buyer side and the seller side. All agents have rational expectations and maximize their own payoff.

3.1. The Platform

Consider a monopoly two-sided platform owner that facilitates transactions between two groups of users through a hardware product exclusively offered by the platform owner. In practice, a platform always faces competition to some extent. We consider the monopoly case to focus on the platform's loyal consumers with limited competition. For example, most Apple users are reluctant to switch to an Android platform. More importantly, this enables us to isolate the effect of production cost, multi-period selling, and strategic consumers on the platform's pricing decisions. The group of users that are buyers join the platform by purchasing the hardware product; the other group of users, sellers, provide the buyer side with applications that run on the hardware device. The platform owner improves the quality of this hardware product sequentially: A low-quality version L is released in Period 1, followed by a high-quality version H in Period 2.

Let q_i denote the quality of product $i = L, H$. As in Liu and Zhang (2013), we take quality as exogenously given to focus on the platform's pricing problem. In our research context, the quality of the platform hardware may be largely determined by the state-of-the-art technology.[1] Following the common assumption (Netessine and Taylor 2007), the production cost of the hardware device is a convex function of quality with $0 < \beta_1 < 1$ denoting the costliness of quality in Period 1. Since technology tends to become less costly over time, the unit costs in Period 1 and 2 are $\beta_1 q_i^2$ and $(\beta_1 - \delta)q_i^2$, respectively, where δ is the costliness reduction from Period 1 to Period 2. To take into consideration the uncertainty of future costliness reduction in Period 2, the costliness is either <u>r</u>educed or <u>c</u>onstant compared to that in Period 1. Specifically, δ is a random variable with values denoted by δ^k, where $k \in \{r, c\}$ denotes the state of the costliness reduction outcome. With probability γ, δ takes on the value of $\delta^r \in (0, \beta_1)$, and with probability $1 - \gamma$ it is $\delta^c = 0$, where $0 \leq \gamma \leq 1$. Furthermore, we follow the assumption adopted in the literature that innovation is not "too rapid" such that quality only improves in absolute terms but not in present-value terms (Liu and Zhang 2013). Mathematically, this implies that $q_L > \alpha q_H$, where $\alpha \in (0, 1)$ is the common per-period discount factor for all players. Violation of this condition would rule out the subgame-perfect equilibrium for sequential product introduction (Dhebar 1994), implying that the optimal pricing strategy may lead to consumer regret.

[1] In some scenarios, quality may also be a result of innovation. While this is an interesting research agenda, an in-depth analysis of the innovation problem warrants a separate research study. Thus, we assume quality is exogenous in our work to focus on the two-sided pricing problem.

The timeline of the events is as follows. In Period 1, only the low-quality product is available. The platform sets the prices charged to both sides, namely, the selling price of product L, p_{L1}, for the buyer side and the entry fee s for the seller side. Then both the buyer- and seller-side demands are realized. In Period 2, the outcome of costliness reduction is realized, and the high-quality product is released. The platform decides whether to continue offering the low-quality product, and sets p_H for the new buyers of product H who have not made a purchase in Period 1, p_U for the upgraders who have purchased the low-quality product in Period 1 and will trade it in for the high-quality version, and p_{L2} for product L if still offered. And again, the demands on both sides are realized. As in the canonical model of vertical differentiation (e.g., Pan and Honhon 2012), the platform can set the selling prices of the low-quality product so high that no consumers purchase it, which is equivalent to not offering the low-quality product. Therefore, we can regard p_H, p_{L2}, and p_U as the platform's only decision variables in Period 2, as these pricing decisions also effectively determine whether the low-quality product is offered.

3.2. The Buyer Side

Consider a unit mass of buyers who are heterogeneous in their valuations (or willingness-to-pay) for quality, θ, which follows a uniform distribution over $[0,1]$. The distribution of buyer valuation is common knowledge to the platform and the buyers. A buyer with valuation θ receives utility $\theta q_i - p_i$ from purchasing the product with quality i for $i = L, H$. Without loss of generality, let a buyer's utility for not joining the platform (by not purchasing the hardware product) be zero. For the hardware-based platform, buyers tend to base purchasing decisions primarily on their valuation for the product quality; therefore, in the main model, we ignore the impact of the seller-side demand on the buyers' purchasing decisions. This simplification allows the model to remain tractable as we examine the platform's dynamic, two-sided pricing strategy. We relax this assumption in Section 5 and show that the results on the pricing strategies from the main model qualitatively hold.

Because the costliness reduction is uncertain, the platform sets the price of low-quality product p_{L1} in Period 1 anticipating such uncertainty and sets the buyer-side prices in Period 2, p_{L2}^k, p_H^k, and p_U^k, where $k \in \{r, c\}$, based on the realized cost in Period 2.

Under the condition that innovation is not too rapid, the buyers with higher valuations purchase the low-quality product in Period 1 at p_{L1} and have an opportunity to upgrade to the high-quality product in Period 2. In Period 2, the remaining buyers who have not joined the platform may purchase the high-quality product at p_H^k or the low-quality product, if offered, at p_{L2}^k. Again, k denotes the outcome of the costliness reduction, as defined previously. The buyers who choose to upgrade to the high quality receive credit for trading in the old version and are charged a discounted price $p_U^k < p_H^k$. Accordingly, let θ_{L1} denote the valuation of the buyer who is indifferent

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between buying the low-quality product in Period 1 and the high-quality product in Period 2; Notice that here the buyer's utility from buying the high-quality product in Period 2 accounts for the uncertainty of the high-quality product price depending on whether the costliness reduction occurs. $\theta_U^k(\geq \theta_{L1})$ denotes the valuation of the buyer who has purchased the low-quality product and is indifferent about whether to upgrade to the high-quality version in Period 2; $\theta_H^k(< \theta_{L1})$ denotes the buyer who is indifferent between purchasing the high- and low-quality products in Period 2; and $\theta_{L2}^k(\leq \theta_H^k)$ denotes the buyer who is indifferent between purchasing the low-quality product and nothing in Period 2 (Figure 1). As suggested in the literature on sequential innovation (Dhebar 1994, Kornish 2001), the indifferent buyers θ_{L1}, θ_U^k, θ_H^k, and θ_{L2}^k must satisfy:

$$\theta_{L1} q_L - p_{L1} = \alpha(\theta_{L1} q_H - \gamma * p_H^r - (1-\gamma) * p_H^c), \tag{1}$$

$$\theta_U^k q_L - p_{L1} = \theta_U^k((1-\alpha)q_L + \alpha q_H) - p_{L1} - \alpha p_U^k, \tag{2}$$

$$\alpha(\theta_H^k q_H - p_H^k) = \alpha(\theta_H^k q_L - p_{L2}^k), \tag{3}$$

$$\alpha(\theta_{L2}^k q_L - p_{L2}^k) = 0. \tag{4}$$

We can further rewrite Eq. (1) to Eq. (4) as:

$$\theta_{L1} = \frac{p_{L1} - \alpha(\gamma p_H^r + (1-\gamma)p_H^c)}{q_L - \alpha q_H}, \tag{5}$$

$$\theta_U^k = \frac{p_U^k}{q_H - q_L}, \tag{6}$$

$$\theta_H^k = \frac{p_H^k - p_{L2}^k}{q_H - q_L}, \tag{7}$$

$$\theta_{L2}^k = \frac{p_{L2}^k}{q_L}. \tag{8}$$

Given the characterization of the indifferent buyers, we can determine the demand for different products as shown in Figure 1. Specifically, the buyers with valuation in $[\theta_{L1}, 1]$ purchase the low-quality product in Period 1, but only a proportion of these buyers, $[\theta_U^k, 1]$, upgrade to the high-quality product in Period 2. Among the remaining buyers, those with valuation in $[\theta_{L2}^k, \theta_H^k)$ and $[\theta_H^k, \theta_{L1})$ purchase the low- and high-quality product, respectively, in Period 2. Note that the platform decides whether to continue offering the low-quality product in Period 2. If the platform chooses to discontinue the low-quality product, the corresponding pricing decisions will imply $\theta_{L2}^k = \theta_H^k$.

3.3. The Seller Side

In addition to the buyer-side prices, the platform charges a seller-side fee s in each period. For analytical tractability, we assume the same entry fee in both periods, which coincides with the

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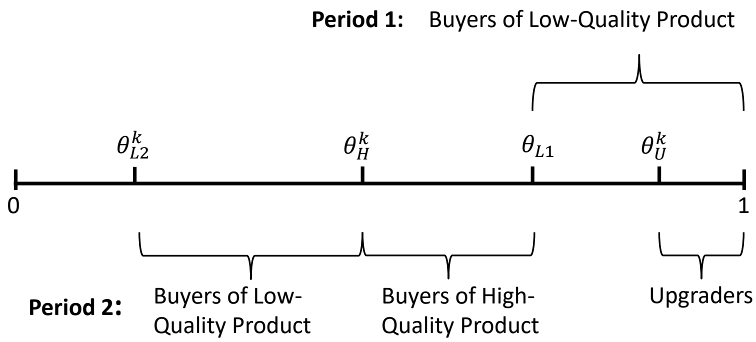

Figure 1 **Indifferent Buyers and Market Segmentation**

observation that the developers' annual fees on the major platforms do not fluctuate with the changing device quality.

Let us consider a potential seller-side market that has proportions z and $1-z$ of sellers that derive high and low cross-side network effects, v_h and v_l, from the buyer-side demand, respectively. In other words, the high-type sellers have a higher valuation for the buyer-side network size than the low-type sellers, when participating on the platform. For the sellers of type $j \in \{l, h\}$, the surplus for joining the platform is the following, based on the standard utility function from Armstrong and Wright (2007):

$$a_j + v_j\Theta - w_j n - s, \tag{9}$$

where a_j is the seller's intrinsic benefit from joining the platform, $v_j\Theta$ is the utility from transacting with the buyer side that has a network size of Θ, and $w_j n$ is the disutility from competition with the other sellers. w_j is the competition effect parameter and n denotes the seller-side network size; thus, with more sellers joining the platform, the negative utility from competition amplifies. Although the term on the competition effect is not present in the utility form in Armstrong and Wright (2007), we introduced it in our model to also capture the competitive intensity.

The sellers' intrinsic benefit of joining the platform describes their valuation *aside* from that related to transacting with the buyer side (Armstrong and Wright 2007, Gold and Hogendorn 2016). In the context of app/game development, these benefits may include learning associated with the technological environment provided by the platform and identifying with the developer community of the platform. The sellers may exhibit different degrees of valuation for such benefits. We let each type of sellers be heterogeneous in their intrinsic benefit a_j, which is uniformly distributed between $[0, 1]$. Thus, the sellers of type j joins the platform if

$$a_j > w_j n + s - v_j\Theta, \tag{10}$$

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which yields the proportion of participating sellers under type j: $1-(w_j n + s - v_j\Theta)$. The total seller-side network size then consists of both types of sellers that join the platform, with the potential seller market normalized to 1:

$$n = z\left[1-(w_h n + s - v_h\Theta)\right] + (1-z)\left[1-(w_l n + s - v_l\Theta)\right]. \tag{11}$$

By solving for n, we obtain:

$$n = \frac{(1-s) + (zv_h + (1-z)v_l)\,\Theta}{1 + (zw_h + (1-z)w_l)} = \frac{(1-s) + \bar{v}\Theta}{1+\bar{w}}, \tag{12}$$

where $\bar{v} = zv_h + (1-z)v_l$ and $\bar{w} = zw_h + (1-z)w_l$.

The platform's profit derived from the seller side in each period is given by sn_t, $t = 1, 2$. In each period, the buyer-side network size Θ may vary. Whereas in Period 1 only the buyers of the low-quality product exert network effect onto the seller side, in Period 2 the buyers from both periods may exert network effect because the sellers in Period 2 can interact with all buyers who have purchased a hardware device.

4. Model Analysis and Results

We solve for the subgame-perfect Nash equilibrium in this dynamic game, such that p_H^k, p_U^k, p_{L2}^k, and the buyers' purchasing decisions are all best responses at the start of Period 2 when p_{L1}, s, and θ_{L1} are given. We analyze this subgame in Section 4.1. In Period 1, all players make forward-looking decisions anticipating such subgame-perfect future strategies. This contrasts with the models of committed pricing, in which the platform makes a static decision for both periods upfront without further optimizing at the start of Period 2. The analysis and results of the optimal strategies are presented in Section 4.2.

4.1. Period 2: Subgame Analysis

We first solve the Period 2 subgame taking the following as given: the low-quality product price in Period 1 (p_{L1}), the seller-side entry fee (s), and the valuation of the indifferent buyer between purchasing the low-quality product in Period 1 and purchasing the high-quality product in Period 2 (θ_{L1}). In Period 2, for $k \in \{r, c\}$, the platform earns profits on (1) the high-quality product from the first-time buyers[2] with valuation $[\theta_H^k, \theta_{L1}]$ and the upgraders with valuation $[\theta_U^k, 1]$, (2) the low-quality product from the buyers with valuation $[\theta_{L2}^k, \theta_H^k]$, and (3) the sellers. Thus, for $k \in \{r, c\}$, the Period 2 profit function is:

$$\Pi_2^k(p_H^k, p_{L2}^k, p_U^k; \theta_{L1}) = \left(\theta_{L1} - \theta_H^k\right)\left[p_H^k - (\beta_1 - \delta^k)q_H^2\right] + \left(1-\theta_U^k\right)\left[p_U^k - (\beta_1 - \delta^k)q_H^2\right]$$

[2] The first-time buyers refer to the buyers who have not made any purchase in Period 1. Mathematically, the valuation of a first-time buyer falls within the range $[0, \theta_{L1}]$.

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$$
\begin{aligned}
&+(\theta_H^k-\theta_{L2}^k)\left[p_{L2}^k-(\beta_1-\delta^k)q_L^2\right]+sn_2 \\
=\;&\underbrace{\left(\theta_{L1}-\frac{p_H^k-p_{L2}^k}{q_H-q_L}\right)\left[p_H^k-(\beta_1-\delta^k)q_H^2\right]}_{\text{Profit from first-time buyers of high-quality product}}+\underbrace{\left(1-\frac{p_U^k}{q_H-q_L}\right)\left[p_U^k-(\beta_1-\delta^k)q_H^2\right]}_{\text{Profit from upgraders}} \\
&+\underbrace{\left(\frac{p_H^k-p_{L2}^k}{q_H-q_L}-\frac{p_{L2}^k}{q_L}\right)\left[p_{L2}^k-(\beta_1-\delta^k)q_L^2\right]}_{\text{Profit from buyers of low-quality product}} \\
&+\underbrace{s\frac{(1-s)+\bar{v}\left(1-\frac{p_{L2}^k}{q_L}\right)}{1+\bar{w}}}_{\text{Profit from seller side}}.
\end{aligned}
\tag{13}
$$

The corresponding profit maximization problem is

$$\max_{p_H^k,p_{L2}^k,p_U^k} \quad \Pi_2^k(p_H^k,p_{L2}^k,p_U^k;\theta_{L1}) \tag{14}$$

$$s.t. \quad \frac{p_{L2}^k}{q_L}\le\frac{p_H^k-p_{L2}^k}{q_H-q_L}\le\theta_{L1}\le\frac{p_U^k}{q_H-q_L}\le 1 \tag{15}$$

$$p_U^k\le p_H^k \tag{16}$$

The constraint $\frac{p_{L2}^k}{q_L}\le\frac{p_H^k-p_{L2}^k}{q_H-q_L}\le\theta_{L1}\le\frac{p_U^k}{q_H-q_L}\le 1$ is to ensure well-defined consumer segments without loss of generality (Pan and Honhon 2012). Moreover, the price to upgrade to the high-quality product needs to be lower than the price for the first-time buyers of the high-quality product. The necessary mathematical assumptions are presented in our analysis in Eq. (40) of Appendix A.

We first examine the product offerings in Period 2 and obtain the following result.

PROPOSITION 1. *The platform always offers both the high- and low-quality products in Period 2.*

Proposition 1 shows that it is not optimal to discontinue the low-quality product after introducing the high-quality product; mathematically, this implies that $\theta_{L2}^{k*}=\frac{p_{L2}^{k*}}{q_L}<\theta_H^{k*}=\frac{p_H^{k*}-p_{L2}^{k*}}{q_H-q_L}$, $k\in\{r,c\}$ always holds for any p_{L1},s and θ_{L1}. Even though offering the low-quality product may cannibalize the demand for the high-quality product, the increased market size from the lower-valuation buyers not only generates more buyer-side revenues, but also leads to additional revenues from the seller side through the network effect. The overall revenue gains dominate the cannibalization effect; thus, the platform always offers both products in Period 2. This offering strategy is often observed in practice. For instance, the previous version of iPhone usually stays on the shelf when the new iPhone is introduced. Also, Echo is still sold along with Echo Plus on Amazon.com.

We then derive the optimal prices:

$$p_H^{k*}(s,\theta_{L1})=\frac{q_H((\beta_1-\delta^k)q_H+\theta_{L1})-s\frac{\bar{v}}{1+\bar{w}}}{2}, \tag{17}$$

$$p_{L2}^{k*}(s,\theta_{L1}) = \frac{q_L((\beta_1-\delta^k)q_L+\theta_{L1}) - s\frac{\bar{v}}{1+\bar{w}}}{2}, \tag{18}$$

$$p_U^{k*}(s,\theta_{L1}) = \frac{(\beta_1-\delta^k)q_H^2+q_H-q_L}{2}. \tag{19}$$

Examining these prices leads to two interesting observations. First, the platform prices each product independently. In other words, each optimal price (Eq. (17) and (18)) is only dependent on the product's own quality level, cost, and the seller-side fee. Second, in contrast, the optimal price for the upgraders does depend on both quality levels, but does not factor in the seller-side fee or the network effect. This is because these buyers already own the low-quality product from Period 1 and the quality improvement is essential in their purchasing decisions. Furthermore, regardless of whether they upgrade to the high-quality product, they already exert network effect onto the seller side; thus, their demand has no additional impact on the seller side.

We further analyze the prices in the following lemmas.

LEMMA 1. *When the highest buyer valuation in Period 2 (θ_{L1}) increases, the optimal price in Period 2 for both the high- and low-quality products increases (i.e., $\frac{\partial p_H^{k*}|_{s,\theta_{L1}}}{\partial\theta_{L1}} > 0, \frac{\partial p_{L2}^{k*}|_{s,\theta_{L1}}}{\partial\theta_{L1}} > 0$) .*

Lemma 1 examines how the buyer-side prices in Period 2 depend on intertemporal market segmentation. The highest buyer valuation in Period 2 is determined by the platform's pricing decisions in Period 1, given buyers' rational expectations on the future prices. In effect, it is the platform's key decision in segmenting the buyer-side market between the two periods. If fewer buyers make purchases in Period 1, not only does the potential demand in Period 2 increase, the highest buyer valuation in Period 2 also increases. The latter incentivizes the platform to exploit the high-valuation buyers through increased prices. We refer to this strategy of raising price to further extract rent from the buyers with higher valuation as *value-driven pricing*, which may actually exclude some lower-valuation buyers and reduce demand. This strategy plays a central role in the subgame-perfect equilibrium results presented in Section 4.2.

LEMMA 2. *In Period 2, if the costliness of production is reduced, the direct effect of costliness reduction drives down the buyer-side prices (i.e., $\frac{\partial p_H^{r*}|_{s,\theta_{L1}}}{\partial\delta^r} < 0, \frac{\partial p_U^{r*}|_{s,\theta_{L1}}}{\partial\delta^r} < 0, \frac{\partial p_{L2}^{r*}|_{s,\theta_{L1}}}{\partial\delta^r} < 0$) and results in more demand for the high-quality product in terms of both the first-time buyers and the upgraders (i.e., $\frac{\partial(\theta_{L1}-\theta_H^{r*})|_{s,\theta_{L1}}}{\partial\delta^r} > 0, \frac{\partial(1-\theta_U^{r*})|_{s,\theta_{L1}}}{\partial\delta^r} > 0$). However, the demand for the low-quality product in Period 2 decreases (i.e., $\frac{\partial(\theta_H^{r*}-\theta_{L2}^{r*})|_{s,\theta_{L1}}}{\partial\delta^r} < 0$).*

The direct effect of the costliness reduction on price in Period 2 follows the conventional wisdom – lower costliness reduces the buyer-side prices for both products. As a result, the demand generally increases. The exception is the demand for the low-quality product in Period 2, which *decreases* even though the price drops with the reduced costliness. The reason lies in the relative price changes at the two quality levels. Because costliness measures the cost for each unit of quality, the costliness reduction leads to more cost savings for the high-quality product ($\delta^r q_H^2$) than for the low-quality product ($\delta^r q_L^2$). Therefore, the optimal pricing strategy allocates some buyers who would otherwise purchase the low-quality product to the high-quality product, resulting in the shrunken low-quality demand.

It is important to note that lower prices with reduced costliness is particular to only Period 2. Given that both the buyers and the platform are forward-looking and have rational expectations, anticipating the possibility of costliness reduction in Period 2 could also impact the platform's strategies in Period 1 (p_{L1} and s), which in turn alters the indifferent buyer (θ_{L1}) that is taken as given here. As the indifferent buyer (θ_{L1}) changes, the buyer-side prices and demand in Period 2 would respond to the costliness reduction differently. We further explore these effects in Section 4.2 by analyzing the subgame-perfect equilibrium results.

4.2. Period 1: Subgame-Perfect Equilibrium

In Period 1, anticipating the subgame-perfect strategy, the platform sets the selling prices to maximize the total profit over the two periods, including the profits from the buyer and seller sides in Period 1 and the discounted expected profit from Period 2. Note that, according to Eq. (12), the seller-side demand in Period 1 is given by

$$n_1 = \frac{(1-s) + \bar{v}(1-\theta_{L1})}{1+\bar{w}}. \tag{20}$$

The platform's profit function can be expressed as:

$$\begin{aligned}
\Pi_1(s, p_{L1}) = \ & (1-\theta_{L1})(p_{L1} - \beta_1 q_L^2) + s n_1 + \gamma[\alpha {\Pi^r}_2(p_H^{r*}, p_{L2}^{r*}, p_U^{r*})] + (1-\gamma)[\alpha {\Pi^c}_2(p_H^{c*}, p_{L2}^{c*}, p_U^{c*})] \\
= \ & \underbrace{\left(1 - \frac{p_{L1} - \alpha(\gamma p_H^{r*} + (1-\gamma)p_H^{c*})}{q_L - \alpha q_H}\right)(p_{L1} - \beta_1 q_L^2)}_{\text{Profit from the buyer-side in Period 1}} + \underbrace{s\frac{(1-s) + \bar{v}(1-\theta_{L1})}{1+\bar{w}}}_{\text{Profit from the seller-side in Period 1}} \\
& + \underbrace{\alpha[\gamma {\Pi^r}_2(p_H^{r*}, p_{L2}^{r*}, p_U^{r*}) + (1-\gamma){\Pi^c}_2(p_H^{c*}, p_{L2}^{c*}, p_U^{c*})]}_{\text{Discounted expected profit from Period 2}}
\end{aligned} \tag{21}$$

The first period profit maximization problem is

$$\max_{s, p_{L1}} \Pi_1(s, p_{L1}) \tag{22}$$

$$s.t. \quad \frac{p_{L1} - \alpha(\gamma p_H^{r*} + (1-\gamma)p_H^{c*})}{q_L - \alpha q_H} \leq 1. \tag{23}$$

Substituting Eq. (17) to (19) into the Period 2 profit function (13) and solving the above total profit maximization problem give the optimal buyer-side price and seller-side fee in Period 1. Due to the complexity of the derivation, we relegate the analytical details to the Appendix A.2..

In the following, we study the comparative statics of the optimal prices with respect to the likelihood and magnitude of costliness reduction (i.e., γ and δ^r respectively) in Period 2. Note that γ captures the degree of uncertainty for cost reduction, while δ^r quantifies the reduction.

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Proposition 2. *When expecting a higher likelihood or a higher magnitude of costliness reduction in Period 2, the platform raises the buyer-side price in Period 1 (i.e.,* $\frac{dp^*_{L1}}{d\gamma}>0$, $\frac{dp^*_{L1}}{d\delta^r}>0$*). This in turn reduces the buyer-side demand in Period 1, which raises the highest buyer valuation in Period 2 (i.e.,* $\frac{d\theta^*_{L1}}{d\gamma}>0$, $\frac{d\theta^*_{L1}}{d\delta^r}>0$*).*

When the platform anticipates a greater likelihood of costliness reduction or more costliness reduction in Period 2, it is optimal to start adjusting the buyer-side price in Period 1. Essentially, an increase in either parameter leads to a higher expected profitability in Period 2. A greater likelihood of costliness reduction shifts the weight from the outcome of no cost change to that of cost reduction, which results in increased profitability; more costliness reduction further raises the the increase in profitability in case of cost reduction. Thus, both create an incentive to allocate more potential buyers to Period 2, which is achieved by raising the optimal buyer-side price in Period 1. Facing an increased price in Period 1, only the buyers with sufficiently high valuation purchase at this early stage. Even though the buyer-side demand is then reduced, the platform is able to extract more surplus from these buyers. In sum, anticipating a greater likelihood of cost reduction or more costliness reduction in Period 2 leads the platform to pursue the value-driven pricing strategy in Period 1, which then increases the highest valuation of the potential buyers remaining for Period 2.

In practice, platforms tend to set a high introductory price to market their initial hardware product as a premium device. Both the first iPhone and the first iPad had a steep price tag of US$499 (for the 4GB iPhone model and the 16GB iPad model), despite the limited features compared to the later versions (Wikipedia 2019b, Smith and Evans 2010). Amazon also took the similar strategy of pricing the first Kindle at US$399 (Wikipedia 2019a). Setting an initial high price point helps to position the product in the market of high-valuation buyers. As these companies usually expect production to be more effective for the following versions of the products, they would have more long-term gain by leaving more consumers for the later version. Our finding suggests that the more the platforms expect to have cheaper production or to lower production cost later on, the higher they may price the initial product. In this regard, our finding is seemingly related to the concept of skimming pricing that a high initial price captures the high-valuation consumers first. However, an importance difference is that, while skimming pricing describes a strategy of market segmentation with sequentially introduced products, our finding illustrates the platform's strategic response to the anticipated changes in the future production cost. In other words, skimming pricing focuses on the decreasing price trend over time, whereas we investigate how the platform's pricing strategy in each stage responds to potential production cost reductions. The economic mechanism in our results is, in fact, drastically different compared to that for skimming pricing.

Proposition 3. *When expecting a higher likelihood or a higher magnitude of costliness reduction in Period 2, the platform lowers the seller-side fee (i.e.,* $\frac{ds^*}{d\gamma} < 0$ *and* $\frac{ds^*}{d\delta^r} < 0$*).*

Proposition 3 shows that the platform's pricing strategy on the seller side is the reverse of that on the buyer side, when either of the cost reduction parameter increases; that is, while the platform raises the optimal buyer-side price ($\frac{dp^*_{L1}}{d\delta^r}>0$ in Proposition 2), it lowers the optimal seller-side fee. As the increased buyer-side price reduces the buyer-side demand in Period 1, the network size for attracting the sellers is smaller. Therefore, the platform needs to lower the seller-side fee in compensation. This is related to the "seesaw principle" discussed in Rochet and Tirole (2006), which suggests that a factor that raises the price on one side tends to reduce the price on the other side as a result of the linkage between the two sides.

The results from Propositions 2 and 3 discuss the cross-period cost-price effect – it is important for the platform to consider its pricing strategy at the current time while anticipating changes in cost in the future. Such consideration is complex especially when consumers are strategic, as their purchasing timing responds to the price changes. Therefore, the platform must carefully project its market segmentation across the time horizon based on the profitability levels in different periods. Our model illustrates the counterintuitive result that *more* future costliness reduction incentivizes the platform to *raise* the current price, which allows the platform to execute the strategy of value-driven pricing.

We now examine the impacts of the likelihood and the magnitude of costliness reduction on the optimal prices in Period 2, while taking into account the platform's strategies in Period 1. First, let us examine the comparative statics of the prices with respect to the two parameters. The comparative statics with respect to the magnitude of costliness reduction δ^r depend on whether the cost reduction occurs:

$$\begin{aligned}\frac{dp^{r*}_H}{d\delta^r} &= \frac{\partial p^{r*}_H}{\partial\delta^r} + \frac{\partial p^{r*}_H}{\partial s}\frac{ds^*}{d\delta^r} + \frac{\partial p^{r*}_H}{\partial\theta_{L1}}\frac{d\theta^*_{L1}}{d\delta^r} \\ &= \underbrace{\frac{\partial p^{r*}_H}{\partial\delta^r}}_{\text{Direct effect (-)}} \quad \underbrace{-\frac{\bar{v}}{2(1+\bar{w})}\frac{ds^*}{d\delta^r} + \frac{q_H}{2}\frac{d\theta^*_{L1}}{d\delta^r}}_{\text{Indirect effect (+)}}\end{aligned} \tag{24}$$

$$\begin{aligned}\frac{dp^{r*}_{L2}}{d\delta^r} &= \frac{\partial p^{r*}_{L2}}{\partial\delta^r} + \frac{\partial p^{r*}_{L2}}{\partial s}\frac{ds^*}{d\delta^r} + \frac{\partial p^{r*}_{L2}}{\partial\theta_{L1}}\frac{d\theta^*_{L1}}{d\delta^r} \\ &= \underbrace{\frac{\partial p^{r*}_{L2}}{\partial\delta^r}}_{\text{Direct effect (-)}} \quad \underbrace{-\frac{\bar{v}}{2(1+\bar{w})}\frac{ds^*}{d\delta^r} + \frac{q_L}{2}\frac{d\theta^*_{L1}}{d\delta^r}}_{\text{Indirect effect (+)}}\end{aligned} \tag{25}$$

$$\begin{aligned}\frac{dp^{c*}_H}{d\delta^r} &= \frac{\partial p^{c*}_H}{\partial\delta^r} + \frac{\partial p^{c*}_H}{\partial s}\frac{ds^*}{d\delta^r} + \frac{\partial p^{c*}_H}{\partial\theta_{L1}}\frac{d\theta^*_{L1}}{d\delta^r} \\ &= \underbrace{\frac{\partial p^{c*}_H}{\partial\delta^r}}_{\text{Direct effect (0)}} \quad \underbrace{-\frac{\bar{v}}{2(1+\bar{w})}\frac{ds^*}{d\delta^r} + \frac{q_H}{2}\frac{d\theta^*_{L1}}{d\delta^r}}_{\text{Indirect effect (+)}} > 0\end{aligned} \tag{26}$$

$$\begin{aligned}\frac{dp_{L2}^{c*}}{d\delta^r} &= \frac{\partial p_{L2}^{c*}}{\partial \delta^r} + \frac{\partial p_{L2}^{c*}}{\partial s}\frac{ds^*}{d\delta^r} + \frac{\partial p_{L2}^{c*}}{\partial \theta_{L1}}\frac{d\theta_{L1}^*}{d\delta^r} \\ &= \underbrace{\frac{\partial p_{L2}^{c*}}{\partial \delta^r}}_{\text{Direct effect (0)}} \underbrace{-\frac{\bar{v}}{2(1+\bar{w})}\frac{ds^*}{d\delta^r} + \frac{q_L}{2}\frac{d\theta_{L1}^*}{d\delta^r}}_{\text{Indirect effect (+)}} > 0 \end{aligned} \tag{27}$$

The comparative statics with respect to the likelihood of costliness reduction γ are the same for $k \in \{r, c\}$:

$$\begin{aligned}\frac{dp_{H}^{k*}}{d\gamma} &= \frac{\partial p_{H}^{k*}}{\partial \gamma} + \frac{\partial p_{H}^{k*}}{\partial s}\frac{ds^*}{d\gamma} + \frac{\partial p_{H}^{k*}}{\partial \theta_{L1}}\frac{d\theta_{L1}^*}{d\gamma} \\ &= \underbrace{\frac{\partial p_{H}^{k*}}{\partial \gamma}}_{\text{Direct effect (0)}} \underbrace{-\frac{\bar{v}}{2(1+\bar{w})}\frac{ds^*}{d\gamma} + \frac{q_H}{2}\frac{d\theta_{L1}^*}{d\gamma}}_{\text{Indirect effect (+)}} > 0 \end{aligned} \tag{28}$$

$$\begin{aligned}\frac{dp_{L2}^{k*}}{d\gamma} &= \frac{\partial p_{L2}^{k*}}{\partial \gamma} + \frac{\partial p_{L2}^{k*}}{\partial s}\frac{ds^*}{d\gamma} + \frac{\partial p_{L2}^{k*}}{\partial \theta_{L1}}\frac{d\theta_{L1}^*}{d\gamma} \\ &= \underbrace{\frac{\partial p_{L2}^{k*}}{\partial \gamma}}_{\text{Direct effect (0)}} \underbrace{-\frac{\bar{v}}{2(1+\bar{w})}\frac{ds^*}{d\gamma} + \frac{q_L}{2}\frac{d\theta_{L1}^*}{d\gamma}}_{\text{Indirect effect (+)}} > 0 \end{aligned} \tag{29}$$

In the scenario where cost reduction occurs, Eq. (24) and (25) show that the magnitude of costliness reduction affects the buyer-side prices both directly and indirectly. Lemma 2 states that the direct effect is to lower the buyer-side prices in Period 2. However, the indirect effect is positive and stems from the platform's value-driven pricing in Period 1. Intuitively, two forces are in play: First, when the platform implements the value-driven pricing strategy in Period 1, the highest buyer valuation in Period 2 increases ($\frac{d\theta_{L1}^*}{d\delta^r} > 0$ in Proposition 2), which allows the platform to raise the prices ($\frac{\partial p_H^{k*}|_{s,\theta_{L1}}}{\partial \theta_{L1}} > 0$ and $\frac{\partial p_{L2}^{k*}|_{s,\theta_{L1}}}{\partial \theta_{L1}} > 0$ in Lemma 1); second, the value-driven pricing strategy shifts emphasis away from the seller side (Proposition 3), which also offers an opportunity to raise the buyer-side prices. Overall, the net effect from the direct and indirect effects further depends on certain conditions, which are analyzed in the following propositions. For the scenario without any cost reduction (i.e., $\delta = 0$) in Period 2, the direct effect simply vanishes, but the magnitude of costliness reduction still exerts an indirect effect on the prices (Eq. (26) and (27)).

As illustrated by Eq. (28) and (29), the likelihood of costliness reduction affects the buyer-side prices in Period 2 differently compared to the magnitude of costliness reduction, in that the direct effects are absent regardless of whether cost reduction occurs. At the start of Period 2 the outcome of cost reduction is already realized, so the likelihood becomes irrelevant, eliminating the direct effect. However, the positive indirect effects still persist and follow the similar mechanisms as those for the magnitude of costliness reduction described above.

Proposition 4. *When the likelihood of cost reduction γ increases, the platform sets a higher optimal buyer-side price for both products in Period 2 (i.e., $\frac{dp_H^{k*}}{d\gamma}>0$ and $\frac{dp_{L2}^{k*}}{d\gamma}>0$ for $k\in\{r,c\}$). However, the optimal upgrade price is not affected (i.e., $\frac{dp_U^{k*}}{d\gamma}=0$ for $k\in\{r,c\}$).*

When the magnitude of costliness reduction δ^r increases, the optimal pricing strategies for the two scenarios of cost reduction are the following:

- *For the scenario without cost reduction, the platform's pricing strategies follow those under the likelihood of cost reduction: the optimal prices of the two products are higher (i.e., $\frac{dp_H^{c*}}{d\delta^r}>0$ and $\frac{dp_{L2}^{c*}}{d\delta^r}>0$), and the optimal upgrade price is not affected (i.e., $\frac{dp_U^{c*}}{d\delta^r}=0$).*
- *For the scenario with cost reduction, the platform sets a higher buyer-side price for both products (i.e., $\frac{dp_H^{r*}}{d\delta^r}>0$ and $\frac{dp_{L2}^{r*}}{d\delta^r}>0$), when the network effect is sufficiently strong or the quality gap is sufficiently wide; otherwise, the platform lowers the price for both products. Moreover, the platform sets a lower upgrade price in Period 2 (i.e., $\frac{dp_U^{r*}}{d\delta^r}<0$).*

In Period 2, the platform can also employ the strategy of value-driven pricing and raise prices, driven by to the indirect effects coming from Period 1 illustrated by Eq. (24) to (29). An increase in either the likelihood or the magnitude of costliness reduction creates a positive indirect effect by raising the highest buyer valuation for Period 2 (through raised buyer-side price in Period 1, Proposition 2 and Lemma 1) and by reducing the seller-side fee in Period 1 (Proposition 3). Thus, the impacts of the likelihood or the magnitude of costliness reduction on the buyer-side prices in Period 2 depend on how the indirect effect weighs relative to the other effects, if present.

The likelihood of cost reduction does not directly affects the platform's strategies in Period 2 because once the outcome of cost reduction is realized in Period 2, the likelihood is irrelevant. As such, only the indirect effect from Period 1 plays a role in the pricing strategies in Period 2. Specifically, through the indirect effects, an increase in the likelihood of cost reduction enhances the platform's incentive to execute the value-driven pricing strategy in Period 1 (Proposition 2), and consequently in Period 2 (as shown by Eq. (28) and (29)), leading to a higher buyer-side price for both qualities of products in Period 2.

If cost reduction does not occur in Period 2, the magnitude of reduction is also irrelevant for the direct effect. Thus, in this scenario, the impacts of the magnitude of costliness reduction on the prices of two qualities of products and the upgraders follow the same intuition discussed for the likelihood of reduction.

On the other hand, for the scenario where the cost reduction does occur, the magnitude of reduction exerts an additional negative direct effect on some of the prices (Eq. (24) and (25)). The net effect of the direct and indirect effects depends on other conditions. When the network effect is sufficiently strong or the quality gap is sufficiently wide, the platform sets a higher optimal buyer-side price for both products. Given a strong network effect, the platform could rely less on generating a high buyer-side demand; and the value-driven pricing strategy in Period 1, which narrows the buyer-side demand to the higher-valuation buyers, would be executed more aggressively. A larger quality gap implies greater value offering of the high-quality product in Period 2, which encourages the allocation of potential buyers from Period 1 to Period 2. Therefore, both a strong network effect and a large quality gap allow the platform to more aggressively pursue the value-driven pricing strategy in Period 1, which then strengthens the indirect effect in Eq. (24) and (25) and facilitates the value-driven pricing strategy in Period 2 as well.

The value-driven pricing strategy only applies to the first-time buyers, *not* the upgraders, which differ from the first-time buyers in two ways. First, the demand for upgrades does not impact the seller side because these buyers are already part of the network regardless of their upgrading decisions. Second, the valuation of these buyers is less sensitive to the buyer allocation between the two periods; thus, the upgrade pricing is not contingent on the value-driven pricing strategy from Period 1, which then removes the indirect effect discussed above. As a result, the upgrade pricing has little dependency on the other pricing strategies and simply maximizes the profit that can be extracted from the buyer segment that already owns the low-quality product with purchases from Period 1. In the absence of the indirect effect, the likelihood of cost reduction has no effect on the upgrading price in Period 2; similarly, for the scenario without cost reduction, the magnitude of reduction also does not affect the upgrading price. However, in the scenario where the cost reduction does occur, the direct effect of the magnitude of reduction leads to a lower upgrading price in Period 2.

The finding that more costliness reduction may lead to higher prices is in sharp contrast with conventional wisdom. The conventional relationship of lower cost leads to lower price holds in a static setting, where a firm is committed to its pricing strategy. When the cost drops, the firm is better off lowering the price to gain a larger market share. Dynamic pricing alters this economic mechanism, because the platform can optimize its pricing strategy again in the future. The value-driven pricing strategy allows the platform to repeatedly leverage on the high-valuation buyers to raise prices. Further exploiting those buyers in Period 1 sets up more high-valuation buyers for Period 2, which may again lead to increased prices under appropriate conditions.

Proposition 4 underscores that, in a dynamic context, the within-period cost-price effect is not straightforward, as we need to account for the indirect effect that results from the anticipatory strategic decisions in an earlier period. To understand the overall within-period cost effect on price, it is important to consider the factors that affect the platform's anticipatory strategies. We show that, in our model, both the network effect and the product quality improvement can play a role in determining the platform's decisions.

PROPOSITION 5. *An increase in the likelihood of cost reduction γ leads to more demand for the high-quality product in terms of the first-time buyers (i.e., $\frac{d(\theta^{*}_{L1}-\theta^{k*}_{H})}{d\gamma} > 0$). The demand for the upgraders is unaffected (i.e., $\frac{d(1-\theta^{k*}_{U})}{d\gamma} = 0$). The demand for the low-quality product in Period 2 and the total demand across two periods are reduced (i.e., $\frac{d(\theta^{k*}_{H}-\theta^{k*}_{L2})}{d\gamma} < 0$ and $\frac{d(1-\theta^{k*}_{L2})}{d\gamma} < 0$).*

An increase in the magnitude of costliness reduction δ^r also leads to more demand for the high-quality product in terms of the first-time buyers (i.e., $\frac{d(\theta^{*}_{L1}-\theta^{k*}_{H})}{d\delta^r} > 0$) and reduced demand for the low-quality product in Period 2 (i.e., $\frac{d(\theta^{k*}_{H}-\theta^{k*}_{L2})}{d\delta^r} < 0$). Moreover,

- *in the scenario without cost reduction, the demand for the upgraders is unaffected (i.e., $\frac{d(1-\theta^{c*}_{U})}{d\delta^r} = 0$). And the total demand across two periods is always reduced (i.e., $\frac{d(1-\theta^{c*}_{L2})}{d\delta^r} < 0$);*
- *in the scenario with cost reduction, the demand for the upgraders is increased (i.e., $\frac{d(1-\theta^{r*}_{U})}{d\delta^r} > 0$). And the total demand across two periods is reduced (i.e., $\frac{d(1-\theta^{r*}_{L2})}{d\delta^r} < 0$) when the network effect is sufficiently strong or the quality gap is sufficiently wide; and reverse holds.*

As either the likelihood or the magnitude of costliness reduction increases, the platform's pricing strategies described in Proposition 4 shift the buyer demand towards the high-quality product, because these cost parameters lead to more profitability increase for the higher quality product. Specifically, recall that costliness reduction impacts the cost of producing *each unit of quality,* resulting in more total cost reduction for the high-quality product. Thus, the demand for first-time buyers of the high-quality product increases. Meanwhile, the buyer-side demand for the low-quality product is reduced due to the relative prices of the two qualities.

For the impact of the magnitude of costliness reduction under the scenario with cost reduction, while the changes in the buyer-side demand for the two products in Period 2 do not depend on any conditions (such as those for the pricing strategies in Proposition 4), the changes in the total buyer-side demand across the two periods are contingent on the network effect or quality gap. In particular, a higher magnitude of reduction reduces the total buyer-side demand given sufficiently strong network effect or sufficiently wide quality gap. Following the intuition for Proposition 4, sufficiently strong network effect or wide quality gap allows the platform to rely less on a large buyer-side market and pursue value-driven pricing in both periods. As the platform focuses more on exploiting the high-valuation buyers rather than attracting more buyers, the total demand tends to decrease with costliness reduction. When neither condition holds, the platform would set prices such that the total buyer-side demand increases with more costliness reduction.

In the scenario without cost reduction, the increased magnitude of reduction always reduces the total demand; the same applies when the likelihood of cost reduction increases. As explained for Proposition 4, for these cases, the indirect effects are not offset by the direct effect; thus, the platform's incentive to raise price and pursue the value-driven strategy is sufficiently strong without the conditions of network effect or quality gap.

The demand for upgrades follows from the pricing strategies described in Proposition 4. As the likelihood of cost reduction increases, the upgrading price is not affected; thus, the demand is unchanged. The same result applies in the scenario without cost reduction as the magnitude of reduction increases. In the scenario where cost reduction does occur, a greater magnitude of reduction leads to more upgrade demand because the optimal upgrading price is reduced.

PROPOSITION 6. *The network effects (experienced by both the low- and high-type sellers, i.e., v_l or v_h) play a role in the impacts of the likelihood and the magnitude of costliness reduction on the platform's strategies. As the network effect on either the low- or high-type seller, v_l or v_h, strengthens, the following applies:*

i. The platform more aggressively raises the buyer-side price (i.e., $\frac{\partial^2 p_{L1}^*}{\partial\delta^r\partial v_j} > 0$ and $\frac{\partial^2 p_{L1}^*}{\partial\gamma\partial v_j} > 0$ for $j \in \{l,h\}$) and lowers the seller-side fee in Period 1 (i.e., $\frac{\partial^2 s^*}{\partial\delta^r\partial v_j} < 0$ and $\frac{\partial^2 s^*}{\partial\gamma\partial v_j} < 0$ for $j \in \{l,h\}$) when the likelihood or the magnitude of costliness reduction increases. The shift of the buyer-side demand from Period 1 to Period 2 is also more pronounced (i.e., $\frac{\partial^2(1-\theta_{L1}^*)}{\partial\delta^r\partial v_j} < 0$ and $\frac{\partial^2(1-\theta_{L1}^*)}{\partial\gamma\partial v_j} < 0$ for $j \in \{l,h\}$).

ii. The platform more (less) aggressively raises (reduces) the buyer-side price for both qualities of product in Period 2 (i.e., $\frac{\partial p_H^{r*}}{\partial\delta^r\partial v_j} > 0$, $\frac{\partial^2 p_{L2}^{k*}}{\partial\delta^r\partial v_j} > 0$, $\frac{\partial p_H^{r*}}{\partial\delta^r\partial v_j} > 0$, and $\frac{\partial^2 p_{L2}^{k*}}{\partial\delta^r\partial v_j} > 0$ for $j \in \{l,h\}$ and $k \in \{r,c\}$), when the likelihood or the magnitude of costliness reduction increases. The reduction (increase) of the total market size across both periods is also more pronounced (i.e., $\frac{\partial^2(1-\theta_{L2}^{k*})}{\partial\delta^r\partial v_j} < 0$ and $\frac{\partial^2(1-\theta_{L2}^{k*})}{\partial\gamma\partial v_j} < 0$ for $j \in \{l,h\}$ and $k \in \{r,c\}$).

iii. The increase in the market size of the first-time buyers for the high-quality product is more pronounced (i.e., $\frac{\partial^2(\theta_{L1}^*-\theta_H^{k*})}{\partial\delta^r\partial v_j} > 0$ and $\frac{\partial^2(\theta_{L1}^*-\theta_H^{k*})}{\partial\gamma\partial v_j} > 0$ for $j \in \{l,h\}$ and $k \in \{r,c\}$).

The network effect enables the platform to take on the value-driven pricing strategy more aggressively when the likelihood or the magnitude of costliness reduction increases. As discussed previously, a stronger network effect implies that the platform can more effectively generate revenues on the seller side given the same buyer-side demand. This encourages the value-driven pricing, therefore, the increases of the buyer-side prices and the reduction in the total buyer-side demand across both periods are more pronounced. Consequently, the platform also simultaneously reduces the seller-side fee to a greater extent.

Recall that Proposition 4 also presents the case in which the increase in the costliness reduction induces the platform to lower the buyer-side prices in Period 2 for the scenario with cost reduction, given a weak network effect and narrow quality gap. Strengthened network effect within the range specified for this case has a consistent effect: The platform would reduce the buyer-side prices *less* aggressively. In other words, as the network effect strengthens, the platform would gradually move away from the price-cutting strategy and towards the value-driven pricing strategy where the prices are raised.

The platform's pursuit of value-driven pricing also leads to more emphasis on the high-quality product that attracts the high-valuation buyers. It then follows that a stronger network effect results in shifting more buyers from the low-quality to the high-quality product both intertemporally and within Period 2. In effect, the demand of the first-time buyers for the high-quality product increases.

While the network effects experienced by the two types of sellers generate qualitatively consistent results, the results differ in magnitude. The additional effect that the network effect on the high-type sellers has on the various impacts is weighted by z, the proportion of the high-type sellers; and that related to the low-type sellers is weighted by $1-z$, the proportion of the low-type sellers. In sum, when the sellers value the buyer-side demand differently, the effect generated by a certain type of sellers' valuation for the buyer side is proportional to the number of such type of sellers.

Overall, the network effect facilitates the strategies of collecting the introductory premium on the initial product and expanding the market of the high-quality product when more cost reduction is expected. Our findings offer the economic explanation for such strategies taken by platforms such as Apple and Amazon. We show that it may *not* be that the first iPhone/iPad or the first Kindle were so feature-rich that they justified the high introductory prices, or that the later versions were so under-priced that they attracted large demand. Rather, Apple or Amazon strategically chose to forgo some of the consumer demand initially and were able to gain a large consumer base for the high-quality product, because they may be confident about reducing the production cost later on and had a seller side (i.e., app developer for Apple and publisher for Amazon Kindle) that was strongly linked to the consumer side. Their pricing strategies were enabled by the two-sided platform business model.

The role of the network effect also suggests that, for the platform owners, it is worthwhile to optimize the interactions between the two sides to strengthen the network effect. For App Store or ebook markets, it may be helpful to match the consumers' preferences to the products or even regulate seller competition. For voice-enabled assistant like Echo, the technology of voice processing is key to smoothly connect the consumers to the developers/sellers. Increasing the value each consumer generates for the other side effectively strengthens the network effect, which in turn allows the platform to extract surplus from the high-value consumers of the hardware product more aggressively.

5. Extension: Bi-Directional Network Effects

In this section, we extend our model by considering bi-directional network effects such that both sides benefit from the demand on the opposite side. In other words, compared to the main model with the uni-directional network effect, we incorporate an additional network effect exerted by the seller side onto the buyer side.

Let v_b denote each buyer's marginal utility for an additional seller on the other side of the platform; that is, v_b measures the strength of the seller-to-buyer network effect in each period. Based on Eq. (12), the seller-side demands in Period 1 and 2 are

$$n_1 = \frac{(1-s)+\bar{v}(1-\theta_{L1})}{1+\bar{w}}, \tag{30}$$

$$n_2^k = \frac{(1-s)+\bar{v}(1-\theta_{L2}^k)}{1+\bar{w}}, \tag{31}$$

respectively, where $k \in \{r, c\}$. Consequently, the buyers who purchase in Period 1 obtain additional utility due to the network effect $v_b\,[n_1 + \alpha(\gamma n_2^r + (1-\gamma)n_2^c)]$; for the buyers who purchase in Period 2, the additional utility is $\alpha v_b n_2^k$. Hence, Eq. (2) and (3) remain the same as the network effect utility term on both sides is the same. However, Eq. (1) and (4) are revised as follows:

$$\theta_{L1}q_L + v_b(n_1+\alpha(\gamma n_2^r+(1-\gamma)n_2^c)) - p_{L1} = \alpha(\theta_{L1}q_H + v_b(\gamma n_2^r + (1-\gamma)n_2^c) - \gamma * p_H^r - (1-\gamma)*p_H^c), \tag{32}$$

$$\alpha(\theta_{L2}^k q_L + v_b n_2^k - p_{L2}^k) = 0. \tag{33}$$

The valuations of the indifferent buyers are as follows:

$$\begin{aligned}
\theta_{L1} &= \frac{(p_{L1}-\alpha(\gamma p_H^r + (1-\gamma)p_H^c)) * (1+\bar{w}) - (1-s+\bar{v})v_b}{(q_L - \alpha q_H)(1+\bar{w}) - \bar{v}v_b}, \\
\theta_U^k &= \frac{p_U^k}{q_H - q_L}, \\
\theta_H^k &= \frac{p_H^k - p_{L2}^k}{q_H - q_L}, \\
\theta_{L2}^k &= \frac{p_{L2}^k(1+\bar{w}) - (1-s+\bar{v})v_b}{q_L(1+\bar{w}) - \bar{v}v_b}.
\end{aligned}$$

By substituting θ_{L2}^k into Eq. (31), we obtain

$$n_2^k = \frac{q_L(1-s+\bar{v}) - p_{L2}^k\bar{v}}{q_L(1+\bar{w}) - \bar{v}v_b}. \tag{34}$$

The profit function in Period 2 is:

$$\begin{aligned}
\Pi_2^k(p_H^k, p_{L2}^k, p_U^k; \theta_{L1}) &= \left(\theta_{L1} - \theta_H^k\right)\left[p_H^k - (\beta_1 - \delta^k)q_H^2\right] + (1-\theta_U^k)\left[p_U^k - (\beta_1-\delta^k)q_H^2\right] \\
&\quad + (\theta_H^k - \theta_{L2}^k)\left[p_{L2}^k - (\beta_1 - \delta^k)q_L^2\right] + sn_2 \\
&= \underbrace{\left(\theta_{L1} - \frac{p_H^k - p_{L2}^k}{q_H - q_L}\right)\left[p_H^k - (\beta_1-\delta^k)q_H^2\right]}_{\text{Profit from first-time buyers of high-quality product}} + \underbrace{\left(1 - \frac{p_U^k}{q_H-q_L}\right)\left[p_U^k - (\beta_1 - \delta^k)q_H^2\right]}_{\text{Profit from upgraders}} \\
&\quad + \underbrace{\left(\frac{p_H^k - p_{L2}^k}{q_H - q_L} - \frac{p_{L2}^k(1+\bar{w}) - (1-s+\bar{v})v_b}{q_L(1+\bar{w}) - \bar{v}v_b}\right)\left[p_{L2}^k - (\beta_1 - \delta^k)q_L^2\right]}_{\text{Profit from buyers of low-quality product}} \\
&\quad + \underbrace{s\frac{q_L(1-s+\bar{v}) - p_{L2}^k\bar{v}}{q_L(1+\bar{w}) - \bar{v}v_b}}_{\text{Profit from seller side}},
\end{aligned} \tag{35}$$

Therefore, depending on the costliness realization, the corresponding profit maximization problem is

$$\max_{p_H^k, p_{L2}^k, p_U^k} \quad \Pi_2(p_H^k, p_{L2}^k, p_U^k; s, \theta_{L1}) \tag{36}$$

$$s.t. \quad \frac{p_{L2}^k(1+\bar{w}) - (1-s+\bar{v})v_b}{q_L(1+\bar{w}) - \bar{v}v_b} \leq \frac{p_H^k - p_{L2}^k}{q_H - q_L} \leq \theta_{L1} \leq \frac{p_U^k}{q_H - q_L} \leq 1 \tag{37}$$

$$p_U^k \leq p_H^k \tag{38}$$

Given that the buyers with valuation $(\theta_{L1}, 1]$ purchase the low-quality product in Period 1, the profit function (35) in the bi-directional model differs from (13) in the uni-directional model in the third and fourth terms: the profit from the buyers of the low-quality product and the profit from the seller side due to the seller-to-buyer network effect embodied in the expression of θ_{L2}^k. Anticipating the subgame-perfect strategy, the platform sets the buyer-side price in Period 1 to maximize the total profit over the two periods:

$$\begin{aligned}
\Pi_1(s,p_{L1}) = \; & (1-\theta_{L1})(p_{L1} - \beta_1 q_L^2) + sn_1 + \gamma[\alpha \Pi^r{}_2(p_H^{r*}, p_{L2}^{r*}, p_U^{r*})] + (1-\gamma)[\alpha \Pi^c{}_2(p_H^{c*}, p_{L2}^{c*}, p_U^{c*})] \\
= \; & \underbrace{\left(1 - \frac{(p_{L1} - \alpha(\gamma p_H^r + (1-\gamma)p_H^c)) * (1+\bar{w}) - (1-s+\bar{v})v_b}{(q_L - \alpha q_H)(1+\bar{w}) - \bar{v}v_b}\right)(p_{L1} - \beta_1 q_L^2)}_{\text{Profit from the buyer side in Period 1}} \\
& + \underbrace{s\frac{(1-s) + \bar{v}(1 - \frac{(p_{L1} - \alpha(\gamma p_H^r + (1-\gamma)p_H^c)) * (1+\bar{w}) - (1-s+\bar{v})v_b}{(q_L - \alpha q_H)(1+\bar{w}) - \bar{v}v_b})}{1+\bar{w}}}_{\text{Profit from the seller side in Period 1}} \\
& + \underbrace{\alpha[\gamma \Pi^r{}_2(p_H^{r*}, p_{L2}^{r*}, p_U^{r*}) + (1-\gamma)\Pi^c{}_2(p_H^{c*}, p_{L2}^{c*}, p_U^{c*})]}_{\text{Discounted expected profit from Period 2}}
\end{aligned} \tag{39}$$

The Period 1 profit maximization problem is

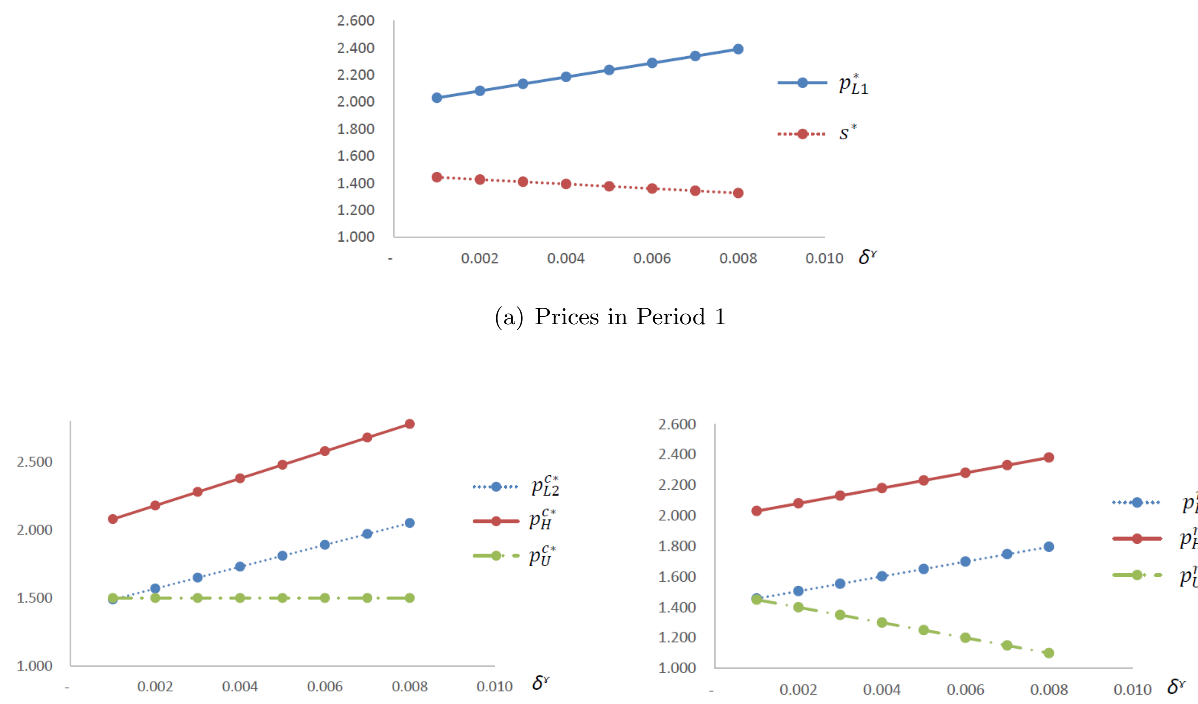
$$\max_{s, p_{L1}} \Pi_1(s, p_{L1})$$

$$s.t. \quad \frac{(p_{L1} - \alpha(\gamma p_H^r + (1-\gamma)p_H^c)) * (1+\bar{w}) - (1-s+\bar{v})v_b}{(q_L - \alpha q_H)(1+\bar{w}) - \bar{v}v_b} \leq 1.$$

The overall profit maximization problem (39) departs from (21) in the expression of θ_{L1} due to the network effect in the buyers' utility. We solve this profit-maximizing problem and obtain the closed-form optimal prices. Due to the complexity of the expressions, we perform numerical analysis to derive the comparative statics with respect to the likelihood and magnitude of costliness reduction.

We test a wide range of parameter values that satisfy the second order conditions, valuation bounds relationship among the indifferent buyers ($0 < \theta_{L2}^{k*} < \theta_H^{k*} < \theta_{L1}^* < \theta_U^{k*} < 1$), and $p_U^{k*} \leq p_H^{k*}, s^* > 0$, consistent with the approach in the uni-directional model. We consider $q_H = 10, q_L \in \{1, 1.5, ..., 9\}$, $\alpha, \beta_1 \in \{0.01, 0.02, ..., 1\}$, $\gamma, v_b \in \{0.1, 0.2, ..., 0.9\}$, and $\bar{v}, \bar{w} \in \{2, 2.05, ..., 8\}$. In each parameter set, we vary δ^r from 0.001 to β_1 at the step size of 0.001 to check how the optimal prices change with δ^r.

Our numerical results show that the analytical findings on the platform's pricing strategies under the uni-directional model hold qualitatively under the bi-directional model. Figures 2 and 3 illustrate how the optimal prices change with δ^r for two different α values ($\alpha = 0.78, 0.58$) but under the same value for the other parameters ($q_H = 10$, $q_L = 8$, $\gamma = 0.5$, $\beta_1 = 0.01$, $v_b = 0.8$, $\bar{v} = 4$, and $\bar{w} = 4.45$). The pricing strategies shown in these figures are representative among all the problem instances. In particular, Figures 2a and 3a indicate that the platform always raises the buyer-side price and lowers the seller-side fee in Period 1 anticipating more costliness reduction in Period 2, consistent with Propositions 2 and 3. When the costliness is not reduced in Period 2, from Figures 2b and 3b, we can see that the platform would always set a higher price for the low- and high-quality products but the upgrade prices remains the same, as previously discussed in Proposition 4. Figures 2c and 3c show that more costliness reduction always leads to a lower upgrade price in Period 2. In contrast, more costliness reduction in Period 2 may lead to higher or lower prices for both low-quality and high-quality products in Period 2.



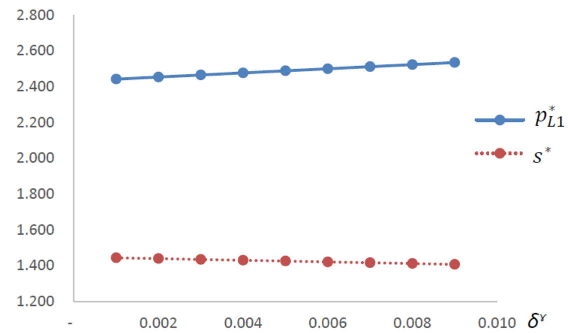
(a) Prices in Period 1

(b) Prices without Costliness Reduction in Period 2

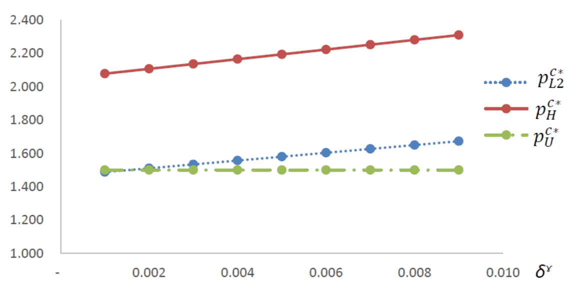
(c) Prices with Costliness Reduction in Period 2

Figure 2 Platform's Pricing Changes with Costliness Reduction Magnitude for
$q_H = 10, q_L = 8, \gamma = 0.5, \alpha = 0.78, v_b = 0.8, \bar{v} = 4, \beta_1 = 0.01, \bar{w} = 4.45$

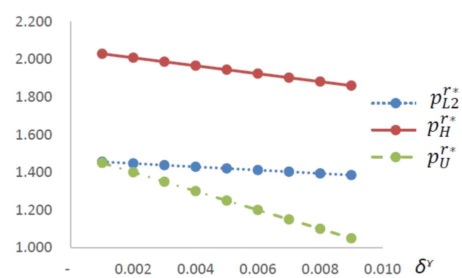
Figure 4 illustrates how the optimal prices change with γ varying from 0.1 to 0.9 at the step size of 0.1 when $q_H = 10$, $q_L = 8$, $\alpha = 0.78$, $\delta^r = 0.003$, $\beta_1 = 0.01$, $v_b = 0.8$, $\bar{v} = 4$, and $\bar{w} = 4.45$. As the likelihood of costliness reduction γ increases, the platform always sets a higher optimal



(a) Prices in Period 1



(b) Prices without Costliness Reduction in Period 2



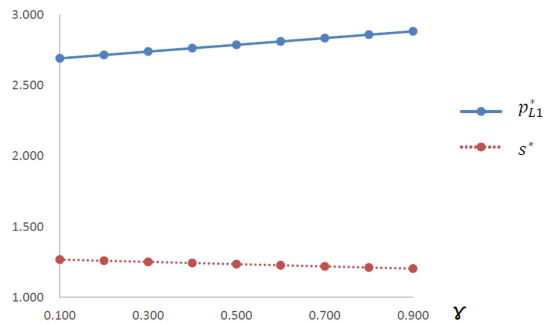
(c) Prices with Costliness Reduction in Period 2

Figure 3 Platform's Pricing Changes with Costliness Reduction Magnitude for $q_H = 10, q_L = 8, \gamma = 0.5, \alpha = 0.58, v_b = 0.8, \bar{v} = 4, \beta_1 = 0.01, \bar{w} = 4.45$

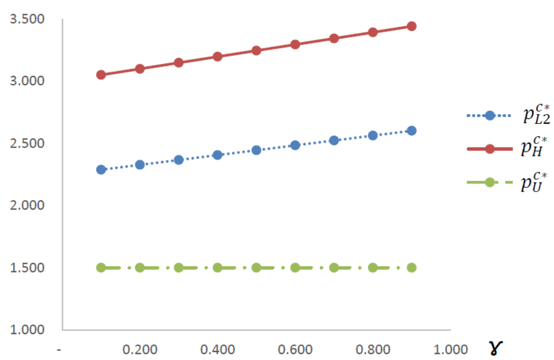
buyer-side price in both periods and lowers the seller-side fee. However, the optimal upgrade price is not affected, consistent with the findings in the model with uni-directional network effect.

6. Conclusion

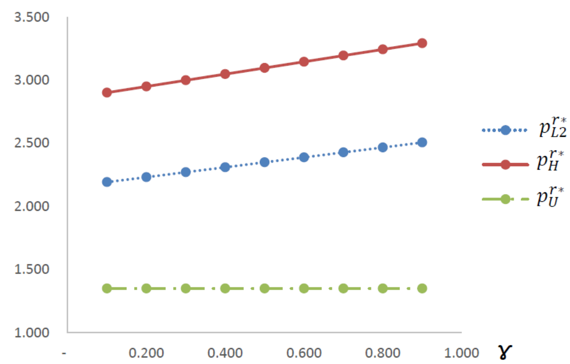
In this paper, we examine the dynamic pricing decisions for a hardware-based platform that offer quality-improving hardware products. The products are introduced sequentially with possibly decreasing production costliness. First, we find that it is always optimal to continue to provide the low-quality product when the high-quality version is released. Second, an increase in the likelihood or magnitude of costliness reduction leads to a higher buyer-side price in Period 1 so that more potential buyers are shifted to Period 2. Meanwhile, the platform lowers the seller-side fee. Furthermore, an increase in the likelihood or magnitude of costliness reduction can also induce the platform to raise the buyer-side prices in Period 2. These findings are in sharp contrast with the conventional wisdom that lower cost leads to lower price in a static setting. Moreover, the impact of costliness reduction on dynamic pricing is also affected by the network effect. A stronger network effect induces the platform to more aggressively raise the buyer-side prices.



(a) Prices in Period 1



(b) Prices without Costliness Reduction in Period 2



(c) Prices with Costliness Reduction in Period 2

Figure 4 Platform's Pricing Changes with Costliness Reduction Likelihood for $q_H = 10, q_L = 8, \delta^r = 0.003, \alpha = 0.78, v_b = 0.8, \bar{v} = 4, \beta_1 = 0.01, \bar{w} = 4.45$

A few limitations exist in the current paper and point to several directions for future research. First, we have taken the qualities of the products introduced in both periods as given for tractability. In practice, firms can strategically design different versions of goods and determine the features to be included in each version dynamically. A future study that focuses more on dynamic quality choices will be relevant for exploring other aspects of the dynamic pricing question in the presence of different cost structures. Another interesting extension is to relax the monopoly assumption. Anticipating more effective production in the future may lead to initial quality differentiation of platforms so as to target different consumer segments and mitigate market competition. It will be worthwhile to examine this topic in more depth.

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Appendix A: Proofs

A.1. Period 2 Subgame Analysis

The Hessian matrix for the profit function in Period 2 is, for $k \in \{r, c\}$,

$$\begin{bmatrix} \frac{\partial^2 \Pi_2^k}{\partial {p_H^k}^2} & \frac{\partial^2 \Pi_2^k}{\partial p_H^k \partial p_{L2}^k} & \frac{\partial^2 \Pi_2^k}{\partial p_H^k \partial p_U^k} \\ \frac{\partial^2 \Pi_2^k}{\partial p_H^k \partial p_{L2}^k} & \frac{\partial^2 \Pi_2^k}{\partial {p_{L2}^k}^2} & \frac{\partial^2 \Pi_2^k}{\partial p_{L2}^k \partial p_U^k} \\ \frac{\partial^2 \Pi_2^k}{\partial p_H^k \partial p_U^k} & \frac{\partial^2 \Pi_2^k}{\partial p_{L2}^k \partial p_U^k} & \frac{\partial^2 \Pi_2^k}{\partial {p_U^k}^2} \end{bmatrix} = \begin{bmatrix} \frac{-2}{q_H - q_L} & \frac{2}{q_H - q_L} & 0 \\ \frac{2}{q_H - q_L} & -2(\frac{1}{q_H - q_L} + \frac{1}{q_L}) & 0 \\ 0 & 0 & \frac{-2}{q_H - q_L} \end{bmatrix}.$$

It can be easily verified that the Hessian matrix is non-positive and the second order condition (SOC) is met. From the first order condition (FOC), we obtain

$$\begin{aligned} p_H^{k*}(s, \theta_{L1}) &= \frac{q_H((\beta_1 - \delta^k) q_H + \theta_{L1}) - s\frac{\bar{v}}{1+\bar{w}}}{2}, \\ p_{L2}^{k*}(s, \theta_{L1}) &= \frac{q_L((\beta_1 - \delta^k) q_L + \theta_{L1}) - s\frac{\bar{v}}{1+\bar{w}}}{2}, \\ p_U^{k*}(s, \theta_{L1}) &= \frac{(\beta_1 - \delta^k) q_H^2 + q_H - q_L}{2}, \\ \theta_H^{k*}(s, \theta_{L1}) &= \frac{(\beta_1 - \delta^k)(q_H + q_L) + \theta_{L1}}{2}, \\ \theta_{L2}^{k*}(s, \theta_{L1}) &= \frac{q_L((\beta_1 - \delta^k) q_L + \theta_{L1}) - s\frac{\bar{v}}{1+\bar{w}}}{2q_L}, \\ \theta_U^{k*}(s, \theta_{L1}) &= \frac{(\beta_1 - \delta^k) q_H^2 + q_H - q_L}{2(q_H - q_L)}. \end{aligned}$$

To satisfy Constraints (15) and (16), we need the following conditions:

$$\max\{(\beta_1 - \delta^k)(q_H + q_L), \frac{s\bar{v} + (q_H - q_L)(1 + \bar{w})}{q_H(1 + \bar{w})}\} < \theta_{L1} \leq \frac{q_H(1 + (\beta_1 - \delta^k) q_H) - q_L}{2(q_H - q_L)} \leq 1 \tag{40}$$

After solving for θ_{L1}^* in Period 1, we can get the conditions on the parameters.

Proof of Proposition 1 From the above expressions for $p_H^{k*}(s, \theta_{L1})$ and $p_{L2}^{k*}(s, \theta_{L1})$, we have

$$\begin{aligned} \frac{p_H^{k*}(s, \theta_{L1}) - p_{L2}^{k*}(s, \theta_{L1})}{q_H - q_L} &= \frac{1}{2}((\beta_1 - \delta^k)(q_H + q_L) + \theta_{L1}), \\ \frac{p_{L2}^{k*}(s, \theta_{L1})}{q_L} &= \frac{1}{2}((\beta_1 - \delta^k) q_L + \theta_{L1} - s\frac{\bar{v}}{q_L(1 + \bar{w})}), \end{aligned}$$

which imply that $\frac{p_{L2}^{k*}(s,\theta_{L1})}{q_L} < \frac{p_H^{k*}(s,\theta_{L1}) - p_{L2}^{k*}(s,\theta_{L1})}{q_H - q_L}$ always holds and both the high- and low-quality products are always offered at optimum.

Proof of Lemma 1 From the expressions for $p_H^{k*}(s, \theta_{L1}), p_{L2}^{k*}(s, \theta_{L1})$, we obtain

$$\begin{aligned} \frac{\partial p_H^{k*}(s, \theta_{L1})}{\partial \theta_{L1}} &= \frac{q_H}{2} > 0, \\ \frac{\partial p_{L2}^{k*}(s, \theta_{L1})}{\partial \theta_{L1}} &= \frac{q_L}{2} > 0. \end{aligned}$$

Proof of Lemma 2

$$\begin{aligned} \frac{\partial p_H^{r*}(s, \theta_{L1})}{\partial \delta^r} &= \frac{-q_H^2}{2} < 0, \\ \frac{\partial p_{L2}^{r*}(s, \theta_{L1})}{\partial \delta^r} &= \frac{-q_L^2}{2} < 0, \end{aligned}$$

$$
\begin{aligned}
\frac{\partial p_U^{r*}(s,\theta_{L1})}{\partial \delta^r} &= \frac{-q_H^2}{2} < 0,\\
\frac{\partial(\theta_{L1}^* - \theta_H^{r*})|\theta_{L1}}{\partial \delta^r} &= \frac{q_H + q_L}{2} > 0,\\
\frac{\partial(1-\theta_U^{r*})|\theta_{L1}}{\partial \delta^r} &= \frac{q_H^2}{2(q_H - q_L)} > 0,\\
\frac{\partial(\theta_H^{r*} - \theta_{L2}^{r*})|\theta_{L1}}{\partial \delta^r} &= \frac{-q_H}{2} < 0.
\end{aligned}
$$

A.2. Period 1 Equilibrium Analysis

We write the Period 1 profit as a function of p_{L1} and s by substituting Eq. (17) to (19) into the Period 2 profit function (13). The Hessian matrix for the profit function in Period 1 is

$$
\begin{bmatrix} \frac{\partial^2 \Pi_1}{\partial p_{L1}^2} & \frac{\partial^2 \Pi_1}{\partial p_{L1}\partial p_s} \\ \frac{\partial^2 \Pi_1}{\partial p_{L1}\partial p_s} & \frac{\partial^2 \Pi_1}{\partial s^2} \end{bmatrix} = \begin{bmatrix} \frac{6\alpha q_H - 8q_L}{(\alpha q_H - 2q_L)^2} & \frac{\bar{v}(\alpha q_H(2+3\alpha) - 4q_L(1+\alpha))}{(\alpha q_H - 2q_L)^2(1+\bar{W})} \\ \frac{\bar{v}(\alpha q_H(2+3\alpha) - 4q_L(1+\alpha))}{(\alpha q_H - 2q_L)^2(1+\bar{w})} & \frac{\alpha \bar{v}^2(\alpha^2 q_H(q_H + 3q_L) - 4q_L^2(1+\alpha))}{2q_L(\alpha q_H - 2q_L)^2(1+\bar{w})^2} - \frac{2(1+\alpha)}{1+\bar{w}} \end{bmatrix}
$$

The SOC requires that $\bar{v} < 2\sqrt{\frac{(1+\alpha)(3\alpha q_H - 4q_L)q_L(1+\bar{w})}{3\alpha^2 q_H - 4(1+\alpha)q_L}}$. For notational convenience, let $A = 4(1+\alpha)(4q_L - 3\alpha q_H)q_L(1+\bar{w}) + (3\alpha^2 q_H - 4(1+\alpha)q_L)\bar{v}^2$. By solving the FOCs, we get

$$
\begin{aligned}
p_{L1}^* = {} & \frac{1}{2A} \\
& \{\alpha^3 q_H(\bar{v}(q_H(-1+\beta_1 q_H - \delta^r \gamma q_H)\bar{v} + 3q_L(2+\bar{v} - \beta_1 q_L \bar{v} + \delta^r \gamma q_L \bar{v})) + 4q_H(1-\beta_1 q_H + \delta^r \gamma q_H)q_L(1+\bar{w})) \\
& +8q_L^2(-\bar{v}(1+\bar{v}) + 2q_L(1+\beta_1 q_L)(1+\bar{w})) + 4\alpha q_L(\bar{v}((-\beta_1 + \delta^r \gamma)q_H^2 \bar{v} + q_H(1+\bar{v}) + q_L(-4 + (-3+\beta_1 q_L - \delta^r \gamma q_L)\bar{v})) \\
& +2q_L(2q_L(1+\beta_1 q_L) - q_H(2+\beta_1 q_L))(1+\bar{w})) + 2\alpha^2 q_L(\bar{v}(-2(\beta_1 - \delta^r \gamma)q_H^2 \bar{v} + q_H(5+5\bar{v} + \delta^r \gamma q_L \bar{v}) \\
& +2q_L(-2 + (-1+\beta_1 q_L - \delta^r \gamma q_L)\bar{v})) + 2q_H(q_H + (-\beta_1 + \delta^r \gamma)q_H^2 - 2q_L(2+\beta_1 q_L))(1+\bar{w})))\}, \\
s^* = {} & \frac{1}{A}\{q_L(4q_L(2+\bar{v} - \beta_1 q_L \bar{v}) + 3\alpha^2 q_H(-2 + (-1+\beta_1 q_L - \delta^r \gamma q_L)\bar{v}) \\
& +2\alpha(-3q_H + 4q_L + 2(q_H - q_L)(-1 + (\beta_1 - \delta^r \gamma)(q_H + q_L))\bar{v}))(1+\bar{w})\}.
\end{aligned}
$$

Combining the above expressions for p_{L1}^* and s^* with $\theta_{L1} = \frac{p_{L1}^* - \alpha(\gamma p_H^{r*} + (1-\gamma)p_H^{c*})}{q_L - \alpha q_H}$ and (17) to (19), we obtain the optimal prices $p_H^{r*}, p_{L2}^{r*}, p_U^{r*}, p_H^{c*}, p_{L2}^{c*}, p_U^{c*}$ as follows:

$$
\begin{aligned}
p_H^{r*} = {} & \frac{1}{2A}\{(4q_L^2 \bar{v}(-2(1+\alpha) + (-1+\beta_1 q_L + \alpha(-1+\beta_1 q_L - \delta^r \gamma q_L))\bar{v}) + \alpha(5\beta_1 - \delta^r(3+2\gamma))q_H^3(\alpha \bar{v}^2 - 4(1+\alpha)q_L(1+\bar{w})) \\
& +q_H q_L(-4\bar{v}(1+\bar{v}) + \alpha \bar{v}(2 - 2\delta^r \gamma q_L \bar{v} + 3\alpha(2+\bar{v} - \beta_1 q_L \bar{v} + \delta^r \gamma q_L \bar{v})) + 8(1+\alpha)q_L(1+\beta_1 q_L)(1+\bar{w})) \\
& +q_H^2((\alpha^2 + 4(-(1+2\alpha)\beta_1 + \delta^r + \alpha \delta^r(1+\gamma))q_L)\bar{v}^2 - 4(1+\alpha)q_L(\alpha + 4(-\beta_1 + \delta^r)q_L)(1+\bar{w})))\}, \\
p_H^{c*} = {} & \frac{1}{2A}\{(4q_L^2 \bar{v}(-2(1+\alpha) + (-1+\beta_1 q_L + \alpha(-1+\beta_1 q_L - \delta^r \gamma q_L))\bar{v}) + \alpha(5\beta_1 - 2\delta^r \gamma)q_H^3(\alpha \bar{v}^2 - 4(1+\alpha)q_L(1+\bar{w})) \\
& +q_H^2((\alpha^2 - 4(\beta_1 + 2\alpha\beta_1 - \alpha\delta^r \gamma)q_L)\bar{v}^2 - 4(1+\alpha)q_L(\alpha - 4\beta_1 q_L)(1+\bar{w})) \\
& +q_H q_L(-4\bar{v}(1+\bar{v}) + \alpha \bar{v}(2 - 2\delta^r \gamma q_L \bar{v} + 3\alpha(2+\bar{v} - \beta_1 q_L \bar{v} + \delta^r \gamma q_L \bar{v})) + 8(1+\alpha)q_L(1+\beta_1 q_L)(1+\bar{w})))\}, \\
p_{L2}^{r*} = {} & \frac{q_L}{2A}\{\bar{v}(6(1+\alpha)(\alpha q_H - 2q_L) + (2\alpha q_H(2(1+\alpha) + (-2+\alpha)(\beta_1 - \delta^r \gamma)q_H) + (-8 + \alpha(-8 + 3\alpha\delta^r(-1+\gamma)q_H))q_L \\
& +2\delta^r(2 + \alpha(2-3\gamma))q_L^2)\bar{v}) - 4(1+\alpha)q_L(\alpha q_H(1 + 2\beta_1 q_H - 2\delta^r \gamma q_H + 3\beta_1 q_L - 3\delta^r q_L) - 2q_L(1 + 3\beta_1 q_L - 2\delta^r q_L))(1+\bar{w})\}, \\
p_U^{r*} = {} & \frac{q_H + (\beta_1 - \delta^r)q_H^2 - q_L}{2}, \\
p_U^{c*} = {} & \frac{q_H + \beta_1 q_H^2 - q_L}{2}.
\end{aligned}
$$

Using the expressions for $p_H^{k*}, p_U^{k*}, p_{L1}^{*}, p_{L2}^{k*}$, and s^* and (5) to (8), we obtain expressions for the θ thresholds. Moreover, we restrict our analysis on the parameter constellation such that $\theta_H^{k*} < \theta_{L1}^* < \theta_U^{k*} < 1, p_U^{k*} < p_H^{k*}$ are satisfied as this case is the focus of our paper.

Proof of Proposition 2

$$\begin{aligned}
\frac{dp_{L1}^*}{\partial\delta^r} &= \frac{\alpha\gamma((-\alpha^2 q_H^3 + 4(1+\alpha)q_H^2 q_L + \alpha(2+3\alpha)q_H q_L^2 - 4(1+\alpha)q_L^3)\bar{v}^2 + 4\alpha(1+\alpha)q_H^3 q_L(1+\bar{w}))}{2A} > 0,\\
\frac{dp_{L1}^*}{d\gamma} &= \frac{\alpha\delta^r((-\alpha^2 q_H^3 + 4(1+\alpha)q_H^2 q_L + \alpha(2+3\alpha)q_H q_L^2 - 4(1+\alpha)q_L^3)\bar{v}^2 + 4\alpha(1+\alpha)q_H^3 q_L(1+\bar{w}))}{2A} > 0,\\
\frac{d\theta_{L1}^*}{d\delta^r} &= -\frac{2\alpha\gamma((\alpha q_H^2 + q_L^2)\bar{v}^2 - 4(1+\alpha)q_H^2 q_L(1+\bar{w}))}{A} > 0,\\
\frac{d\theta_{L1}^*}{d\gamma} &= -\frac{2\alpha\delta^r((\alpha q_H^2 + q_L^2)\bar{v}^2 - 4(1+\alpha)q_H^2 q_L(1+\bar{w}))}{A} > 0.
\end{aligned}$$

Proof of Proposition 3

$$\begin{aligned}
\frac{ds^*}{d\delta^r} &= -\frac{\alpha\gamma q_L(4q_H^2 + 3\alpha q_H q_L - 4q_L^2)\bar{v}(1+\bar{w})}{A} < 0.\\
\frac{ds^*}{d\gamma} &= -\frac{\alpha\delta^r q_L(4q_H^2 + 3\alpha q_H q_L - 4q_L^2)\bar{v}(1+\bar{w})}{A} < 0.
\end{aligned}$$

Proof of Proposition 4

$$\begin{aligned}
\frac{dp_H^{r*}}{d\delta^r} &= \frac{1}{2A}\left\{(-\alpha^2(3+2\gamma)q_H^3 + 4(1+\alpha+\alpha\gamma)q_H^2 q_L + \alpha(-2+3\alpha)\gamma q_H q_L^2 - 4\alpha\gamma q_L^3)\bar{v}^2\right.\\
&\quad \left.+4(1+\alpha)q_H^2(\alpha(3+2\gamma)q_H - 4q_L)q_L(1+\bar{w})\right\}\\
\frac{dp_H^{c*}}{d\delta^r} &= \frac{1}{2A}\left\{\alpha\gamma((-2\alpha q_H^3 + 4q_H^2 q_L + (-2+3\alpha)q_H q_L^2 - 4q_L^3)\bar{v}^2 + 8(1+\alpha)q_H^3 q_L(1+\bar{w}))\right\} > 0\\
\frac{dp_H^{r*}}{d\gamma} &= \frac{1}{2A}\left\{\alpha\delta^r((-2\alpha q_H^3 + 4q_H^2 q_L + (-2+3\alpha)q_H q_L^2 - 4q_L^3)\bar{v}^2 + 8(1+\alpha)q_H^3 q_L(1+\bar{w}))\right\} > 0\\
\frac{dp_H^{c*}}{d\gamma} &= \frac{1}{2A}\left\{\alpha\delta^r((-2\alpha q_H^3 + 4q_H^2 q_L + (-2+3\alpha)q_H q_L^2 - 4q_L^3)\bar{v}^2 + 8(1+\alpha)q_H^3 q_L(1+\bar{w}))\right\} > 0\\
\frac{dp_{L2}^{r*}}{d\delta^r} &= \frac{1}{2A}\left\{q_L((-2(-2+\alpha)\alpha\gamma q_H^2 + 3\alpha^2(-1+\gamma)q_H q_L + 2(2+\alpha(2-3\gamma))q_L^2)\bar{v}^2\right.\\
&\quad \left.+4(1+\alpha)q_L(-4q_L^2 + \alpha q_H(2\gamma q_H + 3q_L))(1+\bar{w}))\right\}\\
\frac{dp_{L2}^{c*}}{d\delta^r} &= \frac{1}{2A}\left\{\alpha\gamma q_L((-2(-2+\alpha)q_H^2 + 3\alpha q_H q_L - 6q_L^2)\bar{v}^2 + 8(1+\alpha)q_H^2 q_L(1+\bar{w}))\right\} > 0\\
\frac{dp_{L2}^{r*}}{d\gamma} &= \frac{1}{2A}\left\{\alpha\delta^r q_L((-2(-2+\alpha)q_H^2 + 3\alpha q_H q_L - 6q_L^2)\bar{v}^2 + 8(1+\alpha)q_H^2 q_L(1+\bar{w}))\right\} > 0\\
\frac{dp_{L2}^{c*}}{d\gamma} &= \frac{1}{2A}\left\{\alpha\delta^r q_L((-2(-2+\alpha)q_H^2 + 3\alpha q_H q_L - 6q_L^2)\bar{v}^2 + 8(1+\alpha)q_H^2 q_L(1+\bar{w}))\right\} > 0\\
\frac{dp_U^{r*}}{d\delta^r} &= -\frac{q_H^2}{2} < 0\\
\frac{dp_U^{c*}}{d\delta^r} &= \frac{dp_U^{r*}}{d\gamma} = \frac{dp_U^{c*}}{d\gamma} = 0
\end{aligned}$$

We have $\frac{dp_H^{r*}}{d\delta^r} > 0$ when:

$$\begin{aligned}
\bar{v} &> \tilde{v}^r = 2\sqrt{\frac{((1+\alpha)q_H^2(\alpha(3+2\gamma)q_H - 4q_L)q_L(1+\bar{w}))}{(\alpha^2(3+2\gamma)q_H^3 - 4(1+\alpha+\alpha\gamma)q_H^2 q_L + (2-3\alpha)\alpha\gamma q_H q_L^2 + 4\alpha\gamma q_L^3)}}\\
&\text{or}\\
\alpha &> \frac{4q_L}{5q_H} \text{ and } \left[\alpha < \frac{4q_L}{5q_H} \text{ or } \gamma \geq \frac{2q_L}{\alpha q_H} - \frac{3}{2}\right].
\end{aligned}$$

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The second condition can further be simplified as follows:

$$\alpha > \frac{4q_L}{5q_H} \quad \text{and} \quad \gamma \geq \frac{2q_L}{\alpha q_H} - \frac{3}{2}$$
$$\Rightarrow \frac{q_L}{q_H} < \frac{5\alpha}{4} \quad \text{and} \quad \frac{q_L}{q_H} \leq \frac{\alpha}{2}\gamma + \frac{3\alpha}{4}.$$

To sum up, $\frac{dp_H^{r*}}{d\delta^r} > 0$ when $\bar{v} > \tilde{v}^r$ or $\frac{q_L}{q_H} \leq \frac{\alpha}{2}\gamma + \frac{3\alpha}{4}$ because $\frac{q_L}{q_H} < \frac{5\alpha}{4}$ is weaker.

Similarly, $\frac{dp_{L2}^{r*}}{d\delta^r} > 0$ when:

$$\bar{v} \ > \ \hat{v}^r = 2\sqrt{\frac{-(((1+\alpha)q_L(-4q_L^2+\alpha q_H(2\gamma q_H+3q_L))(1+\bar{w}))}{(-2(-2+\alpha)\alpha\gamma q_H^2+3\alpha^2(-1+\gamma)q_Hq_L+2(2+\alpha(2-3\gamma))q_L^2))}}$$

or

$$\alpha \ > \ \frac{4q_L^2}{2q_H^2+3q_Hq_L} \text{ and } \left[\alpha \leq \frac{4q_L^2}{2q_H^2+3q_Hq_L} \text{ or } \gamma \geq \frac{q_L(4q_L-3\alpha q_H)}{2\alpha q_H^2}\right].$$

The second condition can be further simplified to $4q_L^2 \leq 2\alpha\gamma q_H^2 + 3\alpha q_H q_L$. To sum up, $\frac{dp_{L2}^{r*}}{d\delta^r} > 0$ when $\bar{v} > \hat{v}^r$ or $4q_L^2 \leq 2\alpha\gamma q_H^2 + 3\alpha q_H q_L$.

Proof of Proposition 5

$$\begin{aligned}
\frac{d(\theta_{L1}^* - \theta_H^{r*})}{d\delta^r} &= \frac{1}{2A}\{(-(\alpha^2 q_H((-3+2\gamma)q_H-3q_L)+4q_L(q_H+q_L)+2\alpha q_L(2q_H+(2+\gamma)q_L))\bar{v}^2 \\
&\quad +4(1+\alpha)q_L(\alpha q_H((-3+2\gamma)q_H-3q_L)+4q_L(q_H+q_L))(1+\bar{w}))\} > 0 \\
\frac{d(1-\theta_U^{r*})}{d\delta^r} &= \frac{q_H^2}{2q_H-2q_L} > 0 \\
\frac{d(\theta_H^{r*} - \theta_{L2}^{r*})}{d\delta^r} &= \frac{1}{2A}\{(4q_Hq_L - 3\alpha^2 q_H(q_H+\gamma q_L)+4\alpha(q_Hq_L+\gamma(-q_H^2+q_L^2)))\bar{v}^2+4(1+\alpha)q_H(3\alpha q_H-4q_L)q_L(1+\bar{w})\} < 0 \\
\frac{d(1-\theta_{L2}^{r*})}{d\delta^r} &= -\frac{1}{2A}\{((-2(-2+\alpha)\alpha\gamma q_H^2+3\alpha^2(-1+\gamma)q_Hq_L+ \\
&\quad 2(2+\alpha(2-3\gamma))q_L^2)\bar{v}^2+4(1+\alpha)q_L(-4q_L^2+\alpha q_H(2\gamma q_H+3q_L))(1+\bar{w})\}
\end{aligned}$$

$\frac{d(1-\theta_{L2}^*)}{d\delta^r} < 0$ when $\bar{v} > \hat{v}^r$ or $4q_L^2 \leq 2\alpha\gamma q_H^2 + 3\alpha q_H q_L$, which are the same conditions for $\frac{dp_{L2}^{r*}}{d\delta^r} > 0$ as $\theta_{L2}^{r*} = \frac{p_{L2}^{r*}}{q_L}$.

$$\begin{aligned}
\frac{d(\theta_{L1}^* - \theta_H^{r*})}{d\gamma} &= -\frac{\alpha\delta^r((\alpha q_H^2+q_L^2)\bar{v}^2-4(1+\alpha)q_H^2q_L(1+\bar{w}))}{A} > 0 \\
\frac{d(1-\theta_U^{r*})}{d\gamma} &= 0 \\
\frac{d(\theta_H^{r*} - \theta_{L2}^{r*})}{d\gamma} &= \frac{\alpha\delta^r(-4q_H^2-3\alpha q_Hq_L+4q_L^2)\bar{v}^2}{2A} < 0 \\
\frac{d(1-\theta_{L2}^{r*})}{d\gamma} &= \frac{\alpha\delta^r((2(-2+\alpha)q_H^2-3\alpha q_Hq_L+6q_L^2)\bar{v}^2-8(1+\alpha)q_H^2q_L(1+\bar{w}))}{2A} < 0 \\
\frac{d(\theta_{L1}^* - \theta_H^{c*})}{d\delta^r} &= -\frac{\alpha\gamma((\alpha q_H^2+q_L^2)\bar{v}^2-4(1+\alpha)q_H^2q_L(1+\bar{w}))}{A} > 0 \\
\frac{d(1-\theta_U^{c*})}{d\delta^r} &= 0 \\
\frac{d(\theta_H^{c*} - \theta_{L2}^{c*})}{d\delta^r} &= \frac{\alpha\gamma(-4q_H^2-3\alpha q_Hq_L+4q_L^2)\bar{v}^2)}{2A} < 0 \\
\frac{d(1-\theta_{L2}^{c*})}{d\delta^r} &= -\frac{\alpha\gamma((2(-2+\alpha)q_H^2-3\alpha q_Hq_L+6q_L^2)\bar{v}^2-8(1+\alpha)q_H^2q_L(1+\bar{w}))}{A} < 0 \\
\frac{d(\theta_{L1}^* - \theta_H^{c*})}{d\gamma} &= -\frac{\alpha\delta^r((\alpha q_H^2+q_L^2)\bar{v}^2-4(1+\alpha)q_H^2q_L(1+\bar{w}))}{A} > 0 \\
\frac{d(1-\theta_U^{c*})}{d\gamma} &= 0
\end{aligned}$$

$$\frac{d(\theta_H^{c*}-\theta_{L2}^{c*})}{d\gamma} = \frac{\alpha\delta^r(-4q_H^2-3\alpha q_Hq_L+4q_L^2)\bar{v}^2}{2A} < 0$$
$$\frac{d(1-\theta_{L2}^{c*})}{d\gamma} = \frac{\alpha\delta^r((2(-2+\alpha)q_H^2-3\alpha q_Hq_L+6q_L^2)\bar{v}^2-8(1+\alpha)q_H^2q_L(1+\bar{w}))}{2A} < 0$$

Proof of Proposition 6 i.

$$\frac{\partial^2 p_{L1}^*}{\partial\delta^r\partial\bar{v}} = -\frac{(4\alpha(1+\alpha)\gamma q_L^2(\alpha(2+3\alpha)q_H-4(1+\alpha)q_L)(4q_H^2+3\alpha q_Hq_L-4q_L^2)\bar{v}(1+\bar{w})}{A^2} > 0$$
$$\frac{\partial^2 s^*}{\partial\delta^r\partial\bar{v}} = \frac{\alpha\gamma q_L(4q_H^2+3\alpha q_Hq_L-4q_L^2)(1+\bar{w})((3\alpha^2q_H-4(1+\alpha)q_L)\bar{v}^2+4(1+\alpha)(3\alpha q_H-4q_L)q_L(1+\bar{w}))}{A^2} < 0$$
$$\frac{\partial^2(1-\theta_{L1}^*)}{\partial\delta^r\partial\bar{v}} = \frac{12\alpha(1+\alpha)\gamma(\alpha q_H-2q_L)q_L(4q_H^2+3\alpha q_Hq_L-4q_L^2)\bar{v}(1+\bar{w})}{A^2} < 0$$
$$\frac{\partial^2 p_{L1}^*}{\partial\gamma\partial\bar{v}} = -\frac{(4\alpha(1+\alpha)\delta^r q_L^2(\alpha(2+3\alpha)q_H-4(1+\alpha)q_L)(4q_H^2+3\alpha q_Hq_L-4q_L^2)\bar{v}(1+\bar{w})}{A^2} > 0$$
$$\frac{\partial^2 s^*}{\partial\gamma\partial\bar{v}} = \frac{\alpha\delta^r q_L(4q_H^2+3\alpha q_Hq_L-4q_L^2)(1+\bar{w})((3\alpha^2q_H-4(1+\alpha)q_L)\bar{v}^2+4(1+\alpha)(3\alpha q_H-4q_L)q_L(1+\bar{w}))}{A^2} < 0$$
$$\frac{\partial^2(1-\theta_{L1}^*)}{\partial\gamma\partial\bar{v}} = \frac{2\alpha\delta^r((\alpha q_H^2+q_L^2)\bar{v}^2-4(1+\alpha)q_H^2q_L(1+\bar{w}))}{A^2} < 0$$

ii.

$$\frac{\partial p_H^{r*}}{\partial\delta^r\partial\bar{v}} = -\frac{(4\alpha(1+\alpha)\gamma((-2+3\alpha)q_H-4q_L)q_L^2(4q_H^2+3\alpha q_Hq_L-4q_L^2)\bar{v}(1+\bar{w})}{A^2} > 0$$
$$\frac{\partial p_H^{c*}}{\partial\delta^r\partial\bar{v}} = -\frac{(4\alpha(1+\alpha)\gamma((-2+3\alpha)q_H-4q_L)q_L^2(4q_H^2+3\alpha q_Hq_L-4q_L^2)\bar{v}(1+\bar{w})}{A^2} > 0$$
$$\frac{\partial^2 p_{L2}^{r*}}{\partial\delta^r\partial\bar{v}} = -\frac{(12\alpha(1+\alpha)\gamma(\alpha q_H-2q_L)q_L^2(4q_H^2+3\alpha q_Hq_L-4q_L^2)\bar{v}(1+\bar{w})}{A^2} > 0$$
$$\frac{\partial^2 p_{L2}^{c*}}{\partial\delta^r\partial\bar{v}} = -\frac{(12\alpha(1+\alpha)\gamma(\alpha q_H-2q_L)q_L^2(4q_H^2+3\alpha q_Hq_L-4q_L^2)\bar{v}(1+\bar{w})}{A^2} > 0$$
$$\frac{\partial p_H^{r*}}{\partial\gamma\partial\bar{v}} = -\frac{(4\alpha(1+\alpha)\delta^r((-2+3\alpha)q_H-4q_L)q_L^2(4q_H^2+3\alpha q_Hq_L-4q_L^2)\bar{v}(1+\bar{w})}{A^2} > 0$$
$$\frac{\partial p_H^{c*}}{\partial\gamma\partial\bar{v}} = -\frac{(4\alpha(1+\alpha)\delta^r((-2+3\alpha)q_H-4q_L)q_L^2(4q_H^2+3\alpha q_Hq_L-4q_L^2)\bar{v}(1+\bar{w})}{A^2} > 0$$
$$\frac{\partial^2 p_{L2}^{r*}}{\partial\gamma\partial\bar{v}} = -\frac{(12\alpha(1+\alpha)\delta^r(\alpha q_H-2q_L)q_L^2(4q_H^2+3\alpha q_Hq_L-4q_L^2)\bar{v}(1+\bar{w})}{A^2} > 0$$
$$\frac{\partial^2 p_{L2}^{c*}}{\partial\gamma\partial\bar{v}} = -\frac{(12\alpha(1+\alpha)\delta^r(\alpha q_H-2q_L)q_L^2(4q_H^2+3\alpha q_Hq_L-4q_L^2)\bar{v}(1+\bar{w})}{A^2} > 0$$
$$\frac{\partial^2(1-\theta_{L2}^{r*})}{\partial\delta^r\partial\bar{v}} = \frac{12\alpha(1+\alpha)\gamma(\alpha q_H-2q_L)q_L(4q_H^2+3\alpha q_Hq_L-4q_L^2)\bar{v}(1+\bar{w}}{A^2} < 0$$
$$\frac{\partial^2(1-\theta_{L2}^{c*})}{\partial\delta^r\partial\bar{v}} = \frac{12\alpha(1+\alpha)\gamma(\alpha q_H-2q_L)q_L(4q_H^2+3\alpha q_Hq_L-4q_L^2)\bar{v}(1+\bar{w})}{A^2} < 0$$
$$\frac{\partial^2(1-\theta_{L2}^{r*})}{\partial\gamma\partial\bar{v}} = \frac{12\alpha(1+\alpha)\delta^r(\alpha q_H-2q_L)q_L(4q_H^2+3\alpha q_Hq_L-4q_L^2)\bar{v}(1+\bar{w})}{A^2} < 0$$
$$\frac{\partial^2(1-\theta_{L2}^{c*})}{\partial\gamma\partial\bar{v}} = \frac{12\alpha(1+\alpha)\delta^r(\alpha q_H-2q_L)q_L(4q_H^2+3\alpha q_Hq_L-4q_L^2)\bar{v}(1+\bar{w})}{A^2} < 0$$

iii.

$$\frac{\partial^2(\theta_{L1}^*-\theta_H^{r*})}{\partial\delta^r\partial\bar{v}} = \frac{8\alpha(1+\alpha)\gamma q_L^2(4q_H^2+3\alpha q_Hq_L-4q_L^2)\bar{v}(1+\bar{w})}{A^2} > 0$$
$$\frac{\partial^2(\theta_{L1}^*-\theta_H^{c*})}{\partial\delta^r\partial\bar{v}} = \frac{8\alpha(1+\alpha)\gamma q_L^2(4q_H^2+3\alpha q_Hq_L-4q_L^2)\bar{v}(1+\bar{w})}{A^2} > 0$$
$$\frac{\partial^2(\theta_{L1}^*-\theta_H^{r*})}{\partial\gamma\partial\bar{v}} = \frac{8\alpha(1+\alpha)\delta^r q_L^2(4q_H^2+3\alpha q_Hq_L-4q_L^2)\bar{v}(1+\bar{w})}{A^2} > 0$$
$$\frac{\partial^2(\theta_{L1}^*-\theta_H^{c*})}{\partial\gamma\partial\bar{v}} = \frac{8\alpha(1+\alpha)\delta^r q_L^2(4q_H^2+3\alpha q_Hq_L-4q_L^2)\bar{v}(1+\bar{w})}{A^2} > 0$$