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Problem Definition: By providing an environmentally friendly alternative to traditional vehicles, electric vehicles will transform urban mobility, particularly, in smart cities. In practice, once an electric vehicle is plugged in, the charging station completes charging as soon as possible. Given that the procurement cost of electricity and resulting carbon emissions vary significantly during a day, substantial savings can be achieved by smart charging—delaying charging until the cost is lower. In this paper, we study smart charging as an innovative business model for utility firms.
Practical Relevance: Utility firms are already investing in charging stations and they can achieve significant cost savings through smart charging.
Methodology: We develop a sequential game in which a utility firm first announces pairs of charging price and completion time. Then, each customer selects the pair that maximizes his/her utility. Given the selected completion times, the utility firm solves a dynamic control problem to determine the charging schedule that minimizes the cost of charging under time-varying electricity procurement cost.
Results: We devise an intuitive and easy-to-implement policy for scheduling charging. We prove that this policy is optimal if all customers arrive at the charging station simultaneously. We also characterize properties of the optimal pairs of charging price and completion time. By using real electricity demand and generation data from the largest electricity market in the U.S., we find that cost and emissions savings from smart charging are approximately 20% and 15%, respectively.
Managerial Implications: In contrast to the current practice of charging vehicles without delay, we show that it is economically and environmentally beneficial to delay charging for some vehicles and to set charging prices based on the utility firm's cost of electricity procurement. We also find that most of the savings of smart charging can be achieved by implementing it only a few peak-demand days in a month, highlighting its practical relevance.

Key words: Electric Vehicles, Business Model Innovation, Smart-city Operations, Energy-related Operations, Sustainable Operations.

1. Introduction

Electric vehicles will transform mobility, particularly, in smart cities, by providing an environmentally friendly alternative to traditional vehicles (World Economic Forum 2018). This transformation requires a \$10 billion investment by 2030 for charging stations, which need to offer innovative business models to manage the electricity demand created by vehicle charging (McKinsey & Company 2018). Under the current practice, whenever an electric vehicle is plugged in, its battery is typically charged at the maximum possible speed to complete charging as soon as possible (SEPA 2017). Given that the marginal generation cost of electricity and resulting carbon emissions vary significantly during a day, some experts worry that vehicle charging can create significant cost and emissions (The Washington Post 2015). However, customers do not necessarily require charging as soon as possible, for example, when they park for more than a few hours. Therefore, compared to charging vehicles as soon as possible, substantial cost and emissions savings can be achieved by smart charging—delaying charging until the electricity generation cost is low. In this paper, we study smart charging as an innovative business model for utility firms.

Utility firms expect a significant increase in the number of electric vehicles and some firms, such as Southern California Edison, one of the largest utility firms in California, are already planning to invest in their own charging stations (SEPA 2019, p. 26, Utility Dive 2019, and SCE 2014, p. 49). Given that a utility firm incurs the additional cost of procuring (or generating) electricity for charging electric vehicles and it is responsible for the resulting carbon emissions, the firm is in an ideal position to manage charging (Baker et al. 2019).

Motivated by the need for innovative charging stations, we pose the following research questions. What is the optimal charging policy of a utility firm under given charging completion times? What are the desired charging completion times and how should the firm design a pricing scheme that incentivizes customers to choose them? We consider the perspectives of the two types of utility firms in the U.S.—public and investor-owned. A public utility firm, such as an electric cooperative, is not-for-profit, whereas an investor-owned (or private) utility firm, such as Southern California Edison, is for-profit (DOE 2015, p. 27). For either type of utility firms, a unique challenge of managing electric vehicle charging is that customers have heterogeneous preferences (private information) regarding their sensitivity to the completion time until which their vehicles are charged. This heterogeneity affects the utility firm's decisions on when to charge each electric vehicle, and, in turn, the total charging cost under a time-varying cost for electricity generation. By properly designing the pricing policy, the firm can incentivize customers with different delay-sensitivity to agree to different charging completion times, thus reducing the cost of charging and passing (part of) cost savings to customers.

We model a sequential game between a utility firm and customers with electric vehicles. The firm procures electricity at a time-varying cost to operate charging stations in its service region, and announces pairs of charging price and completion time. Each customer then chooses the pair that maximizes his/her utility which depends on the inconvenience cost that increases in the completion time and the customer's sensitivity to delay. Each customer's sensitivity is his/her private information, but the utility firm knows the distribution of the sensitivity across customer population. We consider both a public utility firm, whose objective is to minimize the sum of charging and inconvenience cost, and an investor-owned (or private) utility firm, whose objective is to maximize the revenue from the charging service minus the charging cost. The key tradeoff is between the charging cost and the inconvenience cost. The utility firm can reduce its charging cost by delaying charging and procuring electricity at a lower cost than charging vehicles as soon as possible, but this leads to a higher inconvenience cost for customers. We also calibrate this analytical model by using real electricity generation and demand data from the PJM Interconnect, the largest electricity market in the U.S., to illustrate our findings.

Our results offer several managerial insights for utility firms that seek innovative solutions to electric vehicle charging. In contrast to the current practice of charging electric vehicles as soon as possible, we show that it is optimal to delay charging for some vehicles and prices should be based on the utility firm's cost of electricity procurement. We do so by first devising an easy-to-implement policy to minimize the cost of charging for given completion times. This policy is an effective solution to a complex, dynamic control problem given the time-varying cost of electricity procurement. The procurement cost-based pricing policy incentivizes customers with low delay-sensitivity to choose later completion times, giving the utility firm the opportunity to charge vehicles when the cost of charging is lower. This results in substantial cost savings, compared to charging vehicles as soon as possible. For example, in the PJM Interconnect, average cost savings across the days in a typical summer month are approximately 20%. We also find that this policy avoids the use of less energy-efficient power plants in the PJM Interconnect (because these plants are usually more costly), resulting in an average emissions savings of approximately 15%. That is, from a policy perspective, allowing utility firms to own and operate charging stations can lead to reductions in both cost and emissions. Finally, most of these reductions can be achieved by implementing smart charging during a few peak-demand days in a month, making customers more receptive to smart charging and highlighting its practical relevance.

2. Literature Review

Our paper contributes to the emerging literature on smart-city operations by studying the electrification of urban mobility (for reviews, see Mak 2018, Hasija et al. 2019, and Qi and Shen 2019). Recent papers have studied various problems in this context. For example, Qi et al. (2018) analyze how a service provider can jointly use its own fleet and excess capacity from shared mobility providers (e.g., Uber drivers) for last-mile delivery. Moreover, electric vehicles have received particular attention in recent years. Mak et al. (2013) and Avci et al. (2014) study battery switching business model where a service provider leases fully charged batteries to vehicle owners by switching them with depleted ones. We complement these papers by studying the charging station business model (where customers own their batteries), which is now more common than battery switching stations. Chocteau et al. (2011) analyze adoption of electric vehicles in commercial fleets. Lim et al. (2015) show that fast charging reduces range anxiety of customers, whereas leasing batteries can increase adoption as well as profitability under high resale anxiety. Instead of focusing on adoption, we focus on a service provider's (i.e., a utility firm's) charging decisions. For a fleet of electric vehicles that are shared by customers, He et al. (2017) determine service regions to maximize coverage while keeping costs low, He et al. (2019) jointly optimize infrastructure planning and repositioning, and Zhang et al. (2019) determine service regions and optimize charging/discharging schedules of these shared vehicles. Sha et al. (2019) investigate how autonomous electric vehicles can provide grid services as well as mobility to customers. Unlike these papers, we consider the interaction between delay-sensitive owners of electric vehicles and charging stations which face time-varying electricity cost. In doing so, we consider the key tradeoff between the charging cost and the inconvenience cost to identify the economic and environmental benefits of smart charging.

Our paper also contributes to the energy related-operations literature, which has investigated several topics, including integration of renewable energy with conventional sources (Wu and Kapuscinski 2013 and Zhou et al. 2019), effects of electricity pricing policies on consumption and investments (Ata et al. 2018 and Kök et al. 2018), supply function equilibrium in wholesale electricity markets (Al-Gwaiz et al. 2016 and Sunar and Birge 2019), and capacity investments in renewable and conventional sources (Hu et al. 2015 and Kök et al. 2019). Our paper is also related to the literature on energy storage operations. Wu et al. (2012) propose a heuristic for managing seasonal energy storage, Zhou et al. (2016) show the importance of negative electricity prices in operating a storage facility, and Kapuscinski et al. (2019) characterize how to operate storage facilities in an electricity network and how much to invest in these facilities so as to minimize the cost of matching supply with demand. Different from the large-scale storage facilities studied in these papers, batteries in electric vehicles are owned by customers and their primary goal is to provide mobility. Therefore, a utility firm needs to offer incentives to customers to manage charging operations. Our paper identifies how the utility firm can incentivize customers with different delay-sensitivity to agree to different charging completion times, thus reducing the cost of charging and passing (part of) cost savings to customers.

To the best of our knowledge, our paper is the first to study smart charging of electric vehicles by a utility firm. In doing so, we propose an easy-to-implement policy for solving the complex, dynamic control problem of determining how much to charge each vehicle at a given time to minimize the charging cost. We also determine how a utility firm should announce charging prices for given completion times to either minimize total cost or maximize its profit. Our numerical analysis demonstrates that smart charging leads to significant cost and emissions savings, providing both economic and environmental benefits.

3. Model

We develop a sequential game between a utility firm and a set of customers with electric vehicles. First, the utility firm announces pairs of charging price and completion time. Then, each customer selects the pair that maximizes his/her utility, which decreases in the charging price, completion time, and the customer's sensitivity to delay. We consider the two main types of utility firms—public and investor-owned. The public utility firm minimizes the sum of charging cost and inconvenience cost of customers due to delay. The investor-owned utility firm (referred to as private utility firm hereafter for brevity) maximizes the revenue from customers minus the charging cost. Both types of utility firms satisfy the electricity demand caused by charging, in addition to the electricity demand from other sources (e.g., residential customers). Denote the utility firm's planning horizon as $[0, T]$, where T is typically 12 to 24 hours. We index customers that arrive at a representative charging station in the utility firm's service area during $t \in [0, T]$ by $n = \{1, \cdots, N\}$, where N is the total number of customers. We provide the details of our model below. All proofs are in the Appendix.

3.1 Customers

To focus on the key tradeoff between electric vehicle charging cost and inconvenience cost due to delay, we first consider a model in which all N customers arrive simultaneously at a charging station at time $t = 0$, i.e., no customers arrive during $t \in (0, T]$. We consider non-simultaneous arrivals in Section 6. For simplicity, we assume that each customer needs the same amount of energy, which is normalized to one unit without loss of generality. We let the maximum charging speed be $\overline{a}$.

Thus, the minimum charging time is $\underline{w} = 1/\overline{a}$. For example, if one unit of energy is 20 kWh and the maximum charging speed is $\overline{a} = 6.67$ kW, the minimum charging time is $\underline{w} = 3$ hours.

For a given charging price p and completion time $\tau \geq \underline{w}$, a customer's utility from the charging service is

$$u(\theta, p, \tau) = u_0 - p - \theta\delta(\tau), \tag{1}$$

where u_0 is a baseline utility (when the price is zero and charging is completed without delay), which does not affect optimization and is assumed to be the same for all customers, $\delta(\tau)$ is the disutility, which increases in the completion time τ, and θ is the customer's sensitivity to delay.

Customers are heterogeneous with respect to the delay-sensitivity parameter θ and can be divided into I classes, which can be ordered from low to high delay-sensitivity such that $\theta^{(1)} < \theta^{(2)} < \cdots < \theta^{(I)}$. For a customer n, θ_n takes the value $\theta^{(i)}$ with probability $\beta^{(i)}$ for $i = \{1, \cdots, I\}$ such that $\sum_{i=1}^{I} \beta^{(i)} = 1$. A customer's delay-sensitivity is the customer's private information, which is unobservable to the utility firm.

Before arrival, customers' delay-sensitivity levels $\{\theta_1, \cdots, \theta_N\}$ realize independently and after arrival, each customer observes the pairs of charging price and completion time $\left\{ \left(p^{(i)}, \tau^{(i)}\right) : i = 1, \ldots, I \right\}$ announced by the utility firm. Each customer chooses the pair that maximizes his/her utility given in (1). Formally, customers in class i with delay-sensitivity $\theta^{(i)}$ choose the pair $\left(p^{(j^*)}, \tau^{(j^*)}\right)$ such that

$$j^* = \arg\max_j u(\theta^{(i)}, p^{(j)}, \tau^{(j)}). \tag{2}$$

The utility firm announces the pairs such that $\left(p^{(i)}, \tau^{(i)}\right)$ maximizes the utility of a customer with delay sensitivity $\theta^{(i)}$. Therefore, $j^* = i$ in (2). That is, for $n = 1, \ldots, N$, customer n chooses charging price p_n and completion time τ_n such that

$$\text{if } \theta_n = \theta^{(i)}, \text{ then } p_n = p^{(i)} \text{ and } \tau_n = \tau^{(i)}. \tag{3}$$

3.2 Utility Firm

There are two types of utility firms: public and private. Public utility firms are either owned by the government or by the customers in their service regions (as electric cooperatives), and they account for 25 percent of the electricity sales in the U.S. (Lazar 2016, pp. 11,12). Their objective is to serve their constituents (i.e., customers) at the minimum cost (EIA 2007 and DOE 2015, p. 27). On the other hand, private utility firms are owned by investors and their objective is to maximize their profits, subject to regulations imposed by state and federal governments (DOE 2015, p. 27). These regulations impose constraints on private utility firms in setting retail prices of electricity (Cawley and Kennard 2018, p. 2). However, given the novelty of electric vehicle charging service, private utilities are not constrained in setting charging prices, which they perceive as a new business model to increase their profits (Baker et al. 2019).

To reflect the above practice, for both utility firms, we formulate a two-stage optimization problem. In the first stage, by announcing pairs of charging price and completion time, a public utility firm minimizes the sum of the expected cost of inconvenience to customers (due to delay) and the expected cost of charging, whereas a private utility firm maximizes its expected profit, i.e., the revenue from customers minus the cost of charging. Both types of utility firms choose the pairs $\{(p^{(i)}, \tau^{(i)}) : i = 1, \ldots, I\}$ such that a customer with delay sensitivity $\theta^{(i)}$ finds it optimal to use the charging service and choose the pair $(p^{(i)}, \tau^{(i)})$. That is, both types of firms are subject to individual rationality and incentive compatibility constraints in choosing the pairs.

In the second stage, given the completion times chosen by customers, both public and private utility firms decide the charging amount for each customer over time to minimize the total cost of charging N vehicles. We formulate the second-stage problem next.

Let $a_n(t) \in [0, \overline{a}]$ for $t \in [0, T]$ denote the charging speed for customer n's electric vehicle at time t, where $\overline{a}$ is the maximum possible charging speed. The total electricity demand due to charging electric vehicles at time t is

$$q(t) = \sum_{n=1}^{N} a_n(t). \tag{4}$$

Let $d(t)$ denote the utility firm's electricity demand from other sources (excluding electric vehicle charging) at time t, and let $\widetilde{c}(q)$ be the cost of procuring (or producing) q units of electricity. Consistent with the practice (see Figure 3) and the literature (e.g., Wu and Kapuscinski 2013 and Kök et al. 2018), we assume that $\widetilde{c}(q)$ is convex in q. Given that the cost of meeting demand $d(t)$ without electric vehicles is $\widetilde{c}(d(t))$, the incremental cost due to vehicle charging is

$$c(q(t)|d(t)) = \widetilde{c}\big(d(t) + q(t)\big) - \widetilde{c}\big(d(t)\big), \tag{5}$$

where $q(t)$ is defined in (4). Because $\widetilde{c}(\cdot)$ is convex, $c(q(t)|d(t))$ is also convex in $q(t)$ for a given $d(t)$ so that the firm can reduce its charging cost by smoothing the total demand $d(t) + q(t)$ over time $t \in [0, T]$. The firm minimizes the cost of charging electric vehicles by choosing the charging schedule $\{a_n(t), n = 1, \ldots, N\}$ for given charging completion times $\{\tau_1, \ldots, \tau_N\}$ selected by customers. Let $C(\tau_1, \ldots, \tau_N)$ denote the minimum charging cost, which is given by:

$$C(\tau_1, \ldots, \tau_N) \doteq \min_{\{a_n(t),\, n=1,\ldots,N\}} \int_0^T c\big(q(t)|d(t)\big)\mathrm{d}t \tag{6}$$

$$\text{s.t.} \int_0^{\tau_n} a_n(t)\,\mathrm{d}t = 1, \quad \forall n = 1, \ldots, N, \tag{7}$$

$$q(t) = \sum_{n=1}^{N} a_n(t), \qquad \forall t \in [0, T], \tag{8}$$

$$0 \le a_n(t) \le \overline{a}, \qquad \forall t \in [0, \tau_n], \forall n = 1, \ldots, N, \tag{9}$$

$$a_n(t) = 0, \qquad \forall t \in (\tau_n, T], \forall n = 1, \ldots, N, \tag{10}$$

where (7) ensures that customer n's electric vehicle is charged with one unit of energy by the customer's chosen completion time τ_n, (8) defines the total electricity demand due to charging electric vehicles, (9) and (10) ensure that a customer's vehicle is only charged before the charging completion time of the customer within the charging speed limit. Given the minimum charging cost by the solution of the second-stage problem (6)-(10), we next formulate the utility firm's first-stage problem.

As discussed in Section 3.1, a utility firm cannot observe customer n's delay-sensitivity θ_n, but it is reasonable to assume that the firm knows the distribution of θ across customers. That is, from the firm's perspective, the delay sensitivity of customer n, denoted by θ_n, is a random variable that takes value $\theta^{(i)}$ with probability $\beta^{(i)}$. Accordingly, the public utility firm's problem of minimizing the sum of the expected inconvenience and charging costs can be written as

$$\min_{\{(\tau^{(i)} \in [\underline{w}, T], p^{(i)}),\, i=1,\ldots,I\}} \mathbb{E}_{\{\theta_n,\, n=1,\ldots,N\}} \left[\sum_{n=1}^{N} \theta_n \delta(\tau_n) + C(\tau_1, \ldots, \tau_N) \right], \tag{11}$$

where $\mathbb{E}_{\{\theta_n,\, n=1,\ldots,N\}}[\cdot]$ denotes the expectation over all N customers' delay sensitivities and customer n chooses the charging price and completion time pair (p_n, τ_n) that maximizes his/her utility as given in (2)-(3), which represent incentive compatibility constraints. The utility firm is also subject to individual rationality constraints for each delay-sensitivity class:

$$u_0 - \theta^{(i)}\delta(\tau^{(i)}) - p^{(i)} \ge \underline{u}, \quad \forall i = 1, \ldots, I, \tag{12}$$

where $\underline{u}$ corresponds to the reservation utility.

In contrast, a private utility firm maximizes its expected profit

$$\max_{\{(\tau^{(i)} \in [\underline{w}, T], p^{(i)}),\, i=1,\ldots,I\}} \mathbb{E}_{\{\theta_n,\, n=1,\ldots,N\}} \left[\sum_{n=1}^{N} p_n - C(\tau_1, \ldots, \tau_N) \right], \tag{13}$$

subject to (12), where (p_n, τ_n) are defined in (2)-(3).

4. Optimal Charging Schedule Given Completion Times

We begin our analysis by considering the second-stage problem of finding the optimal charging schedule that minimizes the cost of charging electric vehicles for given completion times. Recall that this problem is given in (6)-(10) and it is common for both public and private utility firms. This is a continuous-time, optimal control problem, and it can be expressed in the standard form (see the proof of Proposition 1) with state variables $x_n(t) = \int_0^t a_n(s)\,\mathrm{d}s$, where $x_n(t)$ corresponds to the level of electricity charged to vehicle n by time t. An interesting feature of this problem is that constraint (7) imposes terminal conditions $x_n(\tau_n) = 1$, for $n = 1, \ldots, N$, which imply that the number of controls decreases by one as time t exceeds τ_n. Moreover, there are N control variables and N state variables, where N can be large, requiring an easy-to-implement and effective policy to compute the solution.

We devise such a policy by taking advantage of the charging problem's special structure, i.e., the convexity of the electricity procurement cost function. This policy yields one optimal solution for a given sequence of completion times. In this policy, we denote the total demand by $L(t)$ and initially set it to be $d(t)$, the external electricity demand (i.e., excluding charging electric vehicles). In each iteration, we determine one electric vehicle's charging schedule, and update $L(t)$ to include the demand from this vehicle's schedule. When the procedure finishes, $L(t)$ becomes the total electricity demand, including the demand from charging N vehicles. We present this policy below, where we sort customers such that $\tau_N \leq \tau_{N-1} \leq \cdots \leq \tau_1$ without loss of generality.

Juice-filling policy

Step 1. Initialize $n = N$ and $L(t) = d(t)$ over $[0, T]$.

Step 2. Find z_n such that

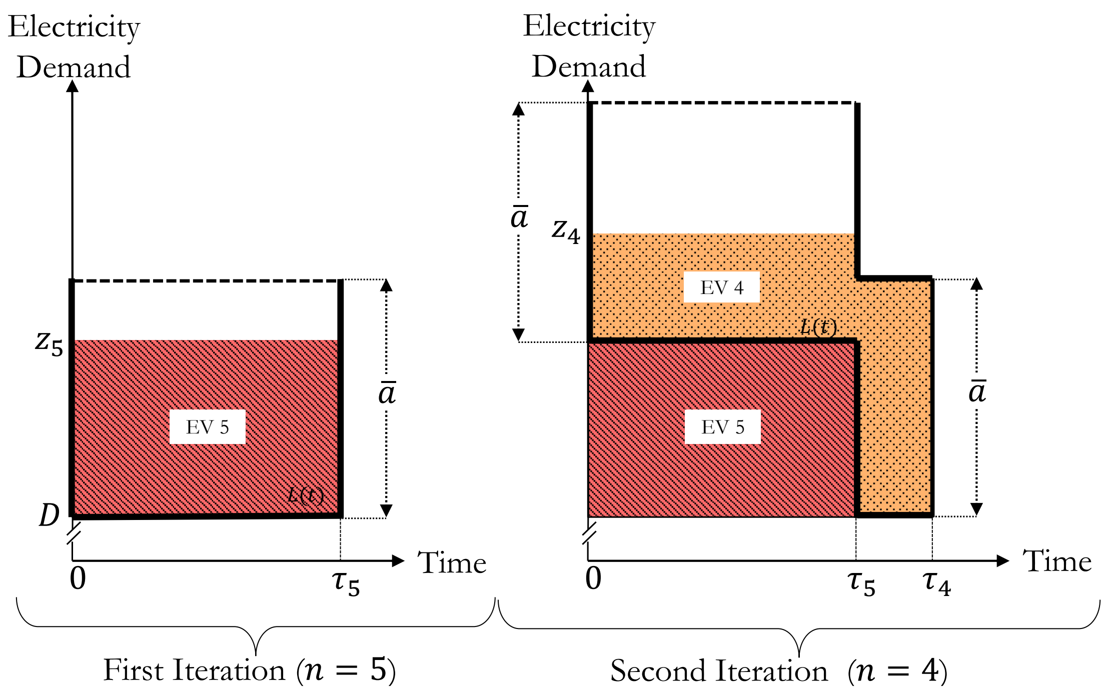
$$\int_0^{\tau_n} \min\left\{\left(z_n - L(t)\right)^+, \overline{a}\right\} \mathrm{d}t = 1. \tag{14}$$

Step 3. Set $a_n(t) = \min\left\{\left(z_n - L(t)\right)^+, \overline{a}\right\}$ for $t \in [0, \tau_n]$.

Step 4. Update $L(t)$ to be $L(t) + a_n(t)$ for $t \in [0, \tau_n]$. If $n = 1$, then stop. Otherwise, decrease n by 1 and go to step 2.

We refer to the above policy as "juice-filling policy" because it resembles the procedure of filling N containers with juice, where each container represents the battery of an electric vehicle and the juice represents the electricity that is charged to the vehicle's battery. The fill-up-to level z_n can be interpreted as the "juice surface level" after filling a container, as illustrated in Figure 1, where we show the first two iterations of the policy for the special case in which the electricity demand $d(t)$ from other sources is constant over time $t \in [0, T]$ and is given as $d(t) = D$.[1] The vertical axis plots the total electricity demand and the horizontal axis plots the time. We consider five electric vehicles (i.e., $N = 5$) with given completion times $\tau_5 \le \tau_4 \le \cdots \le \tau_1$.

Figure 1: (Color online) Juice-filling policy, first two iterations



Notes. We illustrate the first two iterations of the juice-filling policy for the special case when the demand from other sources, $d(t)$, is constant over time.

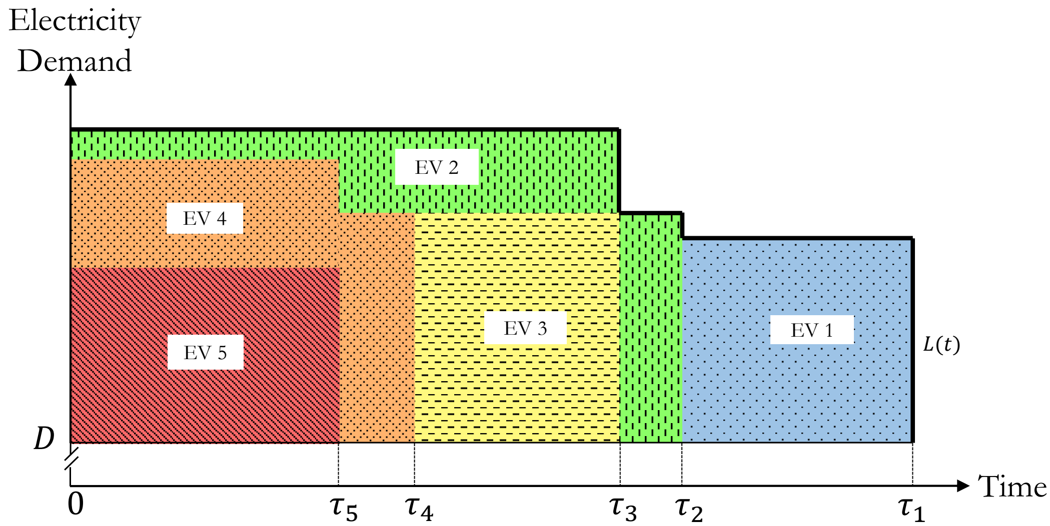
The left panel in Figure 1 illustrates the first iteration: we construct a container (highlighted in bold lines) from time $t = 0$ to $t = \tau_5$. The container's base $L(t)$ is at the demand level D and its height is $\bar{a}$, the maximum charging speed. We fill this container with one unit of juice, which fills the container up to z_5. This juice surface level can be computed by (14). The charging policy is then to set $a_5(t) = z_5 - D$ for $t \in [0, \tau_5]$. Before proceeding to the next electric vehicle, we update $L(t)$ to $L(t) + a_5(t)$.

The right panel in Figure 1 illustrates the second iteration: we construct a new container (highlighted in bold) from time $t = 0$ to $t = \tau_4$, such that its base decreases from $L(t) = z_5$ for $t \in [0, \tau_5]$ to $L(t) = D$ for $t \in (\tau_5, \tau_4]$. The top of the container is $\bar{a}$ units above its base. One unit of juice brings the juice surface level to z_4, which can be computed by (14). The charging policy is then to set $a_4(t) = z_4 - L(t)$ for $t \in [0, \tau_5]$ and $a_4(t) = \bar{a}$ for $t \in (\tau_5, \tau_4]$. We update $L(t)$ to $L(t) + a_4(t)$ and proceed to the next container for the next vehicle. The resulting charging policy

[1]Note that the policy is optimal even if $d(t)$ is nonstationary. See Proposition 1.

for the five electric vehicles is given in Figure 2.

Figure 2: (Color online) Juice-filling policy



Proposition 1 *Given external demand $d(t)$, the control policy $\{a_1(t), \ldots, a_N(t)\}$ found by the juice-filling policy is an optimal solution to the charging cost minimization problem in* (6)-(10).

Proposition 1 shows that juice-filling policy is optimal even if the electricity demand from other sources, $d(t)$, is nonstationary. Intuitively, this policy smooths the total demand $d(t)+q(t)$ in every iteration in a greedy fashion. This leads to a total demand that is as smooth as possible over time, as indicated by the total demand (highlighted in bold) in Figure 2. Note that minimizing the charging cost is equivalent to minimizing the utility firm's cost of serving total demand given external demand $d(t)$. This is because the objective function given in (6) is equal to the cost of serving total demand $\int_0^T \widetilde{c}(d(t) + q(t))\mathrm{d}t$ minus the cost of satisfying the external demand $\int_0^T \widetilde{c}(d(t))\mathrm{d}t$, where the latter is a constant that does not affect the optimization. Accordingly, the objective function can be replaced by the cost of serving the total demand. Given that $\widetilde{c}(\cdot)$ is a convex function, the total cost is minimized by smoothing the total demand $d(t) + q(t)$ as much as possible, as done by the juice-filling policy.

The juice-filling policy identifies only one of the optimal solutions to the charging problem given in (6)-(10). There clearly exists (infinitely many) other charging schedules that also achieve the same optimal total demand over time thus attaining the minimum charging cost. Compared to other optimal schedules, the juice-filling policy greatly simplifies the computation due to its sequential nature of determining the charging schedule. Juice-filling is an easy-to-implement, intuitive, and effective policy for finding the solution of a complicated problem. This makes the policy particularly suitable for practical use. Note that the optimality of the juice-filling policy to the charging problem relies on the assumption that all customers arrive simultaneously. This policy can be generalized for non-simultaneous arrivals, as we shall show in Section 6.

We close this section by investigating structural properties of the optimal charging cost function $C(\tau_1, \dots, \tau_N)$ under given completion times.

Lemma 1 *(i) If $\{b_1, \dots, b_N\}$ is any permutation of $\{1, \dots, N\}$, then $C(\tau_{b_1}, \dots, \tau_{b_N}) = C(\tau_1, \dots, \tau_N)$. (ii) The optimal charging cost $C(\tau_1, \dots, \tau_N)$ decreases in the completion time τ_n for any given n.*

Lemma 1 first shows that the optimal charging cost remains the same for any permutation of completion times. This is because each electric vehicle is assumed to have the same demand for electricity. The second part of the lemma indicates that earlier completion times increase the cost of charging electric vehicles. This is because if the utility firm needs to complete charging earlier, it may need to incur a higher cost for procuring electricity. Given that earlier completion times decrease customers' inconvenience cost, this result formalizes the tradeoff between the charging cost and the inconvenience cost. We investigate this tradeoff in the next section to characterize the optimal charging prices for given completion times. Finally, we remark that the charging cost function $C(\tau_1, \tau_2, \dots, \tau_N)$ is generally not convex in τ_n for any n.

5. Optimal Charging Prices and Completion Times

Given the optimal charging schedule determined by the juice-filling policy, we next consider the utility firm's first-stage problem of announcing pairs of charging price and completion time to customers. We characterize the properties of the optimal completion times and charging prices.

5.1 Public Utility Firm

Consider a public utility firm's problem given in (11)-(12). This is a complicated problem due to the nonconvexity of the objective function (11) because the charging cost $C(\tau_1, \dots, \tau_N)$ characterized in Section 4 is not convex and it cannot be expressed in closed-form. Nevertheless, we can reduce the search space for optimal charging prices and completion times significantly by Lemma 2 and Proposition 2 below, where we denote the optimal completion time for customer class i by $\tau^{(i)^*}$.

Lemma 2 *There exists a set of optimal completion times that satisfy $\tau^{(1)*} \geq \tau^{(2)*} \geq \cdots \geq \tau^{(I)*}$.*

Recall that customer classes are ordered by delay sensitivity from low to high such that $\theta^{(1)} < \theta^{(2)} < \cdots < \theta^{(I)}$ (see Section 3.1). Accordingly, Lemma 2 implies that the utility firm should offer the optimal pairs of charging price and completion time such that customers that are more sensitive to delay choose earlier completion times. This result shows that to find the optimal pairs, it is sufficient to only consider charging completion times that are decreasing in the delay sensitivity of customers. We next construct the optimal pricing scheme $\{p^{(1)}, \ldots, p^{(I)}\}$ for given completion times $\tau^{(1)*} \geq \tau^{(2)*} \geq \cdots \geq \tau^{(I)*}$. This pricing scheme in fact implements any decreasing completion times $\tau^{(1)} \geq \tau^{(2)} \geq \cdots \geq \tau^{(I)}$.

Proposition 2 *Given a set of completion times such that $\tau^{(1)} \geq \tau^{(2)} \geq \cdots \geq \tau^{(I)}$, for a public utility firm, the following procedure constructs a pricing scheme such that a customer with delay sensitivity $\theta^{(i)}$ chooses charging price $p^{(i)}$ and completion time $\tau^{(i)}$:*

Set $p^{(I)} \leq u_0 - \theta^{(I)}\delta(\tau^{(I)}) - \underline{u}$ and for $i = I-1, I-2, \ldots, 1$, choose $p^{(i)}$ such that

$$p^{(i)} \in \left[p^{(i+1)} - \theta^{(i+1)}(\delta(\tau^{(i)}) - \delta(\tau^{(i+1)})),\ p^{(i+1)} - \theta^{(i)}(\delta(\tau^{(i)}) - \delta(\tau^{(i+1)}))\right]. \tag{15}$$

Proposition 2 reveals an important managerial insight that a utility firm should set prices for charging electric vehicles in accordance with the cost of inconvenience. This inconvenience-cost based pricing policy incentivizes customers to agree to smart charging (i.e., delaying completion times) in contrast to the current practice of charging vehicles as soon as possible (SEPA 2017). In particular, when a customer in delay-sensitivity class i agrees to a later completion time (compared to the more delay-sensitive class $i+1$), then the customer's price discount should be based on the increased inconvenience cost. Formally, if $\tau^{(i)} > \tau^{(i+1)}$, then

$$p^{(i+1)} - p^{(i)} \in \left[\theta^{(i)}(\delta(\tau^{(i)}) - \delta(\tau^{(i+1)})), \theta^{(i+1)}(\delta(\tau^{(i)}) - \delta(\tau^{(i+1)}))\right]. \tag{16}$$

We next show that this insight holds for a private utility firm as well.

5.2 Private Utility Firm

Consider a private utility firm's problem in Section 3.2 for which Lemma 2 continues to hold, i.e., there exists a set of optimal completion times, where $\tau^{(1)*} \geq \tau^{(2)*} \geq \cdots \geq \tau^{(I)*}$. We next construct a pricing scheme that achieves any decreasing completion times for the private utility firm.

Proposition 3 *Given a set of completion times such that $\tau^{(1)} \geq \tau^{(2)} \geq \cdots \geq \tau^{(I)}$, for a private utility firm, the following procedure constructs a pricing scheme that maximizes its profit and a customer with delay sensitivity $\theta^{(i)}$ chooses charging price $p^{(i)}$ and completion time $\tau^{(i)}$:*

Set $p^{(I)} = u_0 - \theta^{(I)}\delta(\tau^{(I)}) - \underline{u}$ and for $i = I-1, I-2, \ldots, 1$, choose $p^{(i)}$ such that

$$p^{(i)} = p^{(i+1)} - \theta^{(i)}(\delta(\tau^{(i)}) - \delta(\tau^{(i+1)})). \tag{17}$$

Compared to the prices of the public utility firm in Proposition 2, Proposition 3 reveals that the private firm should set the prices at their upper bounds so as to maximize its profit. Moreover, the firm should set prices such that each customer should be indifferent between choosing the completion time designed for his/her class and the completion time for the next class with higher delay-sensitivity. Formally, (17) implies

$$\theta^{(i)}\delta(\tau^{(i)}) + p^{(i)} = \theta^{(i)}\delta(\tau^{(i+1)}) + p^{(i+1)}, \tag{18}$$

so that a customer with delay sensitivity $\theta^{(i)}$ is indifferent between choosing the pair $\left(p^{(i)}, \tau^{(i)}\right)$ designed for his/her class and the pair $\left(p^{(i+1)}, \tau^{(i+1)}\right)$ designed for those with delay sensitivity $\theta^{(i+1)}$.

Another important difference between a public and a private utility firm is in the information rent. Although a public utility firm pays information rent, it considers this as a transfer payment. A private utility firm considers information rent as an additional cost. We highlight this difference by rewriting the private utility firm's objective function (13), using the prices derived in Proposition 3.

Proposition 4 *The private utility firm's objective function in* (13) *can be rewritten as*

$$\min_{\left\{\tau^{(i)}\in[\underline{w},T],\, i=1,\ldots,I\right\}} \mathbb{E}_{\{\theta_n,\, n=1,\ldots,N\}}\left[\sum_{n=1}^{N}\theta_n\delta(\tau_n) + C(\tau_1,\ldots,\tau_N)\right] + N\sum_{i=1}^{I}\beta^{(i)}\sum_{j=i}^{I-1}(\theta^{(j+1)} - \theta^{(j)})\delta(\tau^{(j+1)}),$$

where the expected value term is the public utility firm's cost in (11) *and the second term is the expected value of the information rent incurred by the private utility firm.*

Proposition 4 reveals that the information rent can potentially distort the private firm's decision on charging completion times. That is, compared to the public utility firm, the private utility firm may choose earlier completion times at higher charging prices, or later completion times at lower prices. The direction of the distortion depends on the cost function and customers' sensitivity to delay. The industry regulator or policymakers (see Section 3.2) would be concerned about the magnitude of the distortion, which we shall estimate in Section 7.

We next provide the intuition behind the information rent term by considering the special case where delay sensitivity follows a discrete uniform distribution with $\beta^{(i)} = \frac{1}{I}$ for $i = 1, \ldots, I$. In this case, the information rent term simplifies to $\frac{N}{I}\sum_{i=2}^{I}(i-1)\left(\theta^{(i)} - \theta^{(i-1)}\right)\delta(\tau^{(i)})$ (see the proof of Proposition 4). Because customers in class I have the highest delay sensitivity $\theta^{(I)}$, they receive the reservation utility $\underline{u}$ (see Proposition 3). Accordingly, the private utility firm does not incur an information rent for this class.[2] Consider a customer in class $i < I$ and let the customer's disutility

[2] Customers in class I with the highest delay sensitivity $\theta^{(I)}$ receive the lowest information rent among all classes under pricing schemes of both the public and private utility firms (see Propositions 2 and 3). This is different from the majority of the literature in which the highest class gets the highest information rent. This is because in the literature, a customer's utility typically increases in the customer's private information, whereas in our model a customer's utility decreases in θ.

of inconvenience $\delta(\tau^{(i)})$ increase by one unit and customer's price $p^{(i)}$ decrease by $\theta^{(i)}$ units. Then, the customer in class i maintains the same utility as before these changes. If a customer in class $i-1$ chooses the pair $(p^{(i)}, \tau^{(i)})$ under these changes, then by using (18) it can be seen that the customer's inconvenience cost increases by $\theta^{(i-1)}$ while enjoying a price decrease of $\theta^{(i)}$, resulting in a net gain of $\theta^{(i)} - \theta^{(i-1)}$. To maintain incentive compatibility, one unit increase in $\delta(\tau^{(i)})$ should not only lead to a decrease in $p^{(i-1)}$ by $\theta^{(i)}$, but also decreases in each of the prices $\{p^{(j)}, j = 1, 2 \ldots, i-1\}$ by $\theta^{(i)} - \theta^{(i-1)}$. That is, one unit increase in $\delta(\tau^{(i)})$ should be traded off against $(i-1)(\theta^{(i)} - \theta^{(i-1)})$ units of revenue loss, which is exactly the expression for the information rent.

6. Non-simultaneous Arrivals of Customers

In this section, we extend our model to a more general setting, where customers may arrive at the charging station at different times. In Section 6.1, we show that compared to the main model, finding the optimal charging schedule is more complicated under non-simultaneous arrivals, but the results in Section 5 on optimal pairs of charging price and completion times continue to apply. In Section 6.2, we characterize the optimal schedule as a threshold policy. Given the computational complexity of this optimal policy, we generalize the juice-filling policy for non-simultaneous arrivals.

6.1 Utility Firm's Problem

We first reformulate the utility firm's second-stage problem. Let s_n be the arrival time of customer n. In the main model in Section 3, $s_n = 0$ for all customers. The utility firm's second-stage problem of minimizing the charging cost is given as

$$C(\tau_1, \ldots, \tau_N) \doteq \min_{\{a_n(t),\, n=1,\ldots,N\}} \int_0^T c\big(q(t)|d(t)\big)\mathrm{d}t \tag{19}$$

$$\text{s.t.} \int_{s_n}^{\tau_n} a_n(t)\,\mathrm{d}t = 1, \quad \forall n = 1, \ldots, N, \tag{20}$$

$$q(t) = \sum_{n=1}^{N} a_n(t), \qquad \forall t \in [0, T], \tag{21}$$

$$0 \le a_n(t) \le \bar{a}, \qquad \forall t \in [s_n, \tau_n], \forall n = 1, \ldots, N, \tag{22}$$

$$a_n(t) = 0, \qquad \forall t \notin [s_n, \tau_n], \forall n = 1, \ldots, N. \tag{23}$$

If $s_n = 0$ for $n = 1, 2, \ldots, N$, then the above formulation is equivalent to the second-stage problem of the main model, given in (6)-(10).

As in the main model, given the optimal cost of charging, a public utility firm minimizes the total cost (11) subject to constraints (12) by announcing pairs of charging price and completion time. The only difference from the main model is that a customer's utility is $u(\theta, p, \tau) = u_0 - p - \theta\delta(\tau - s)$, where s is the arrival time of the customer ($s = 0$ in (1)). A private utility firm maximizes its profit (13) by announcing pairs. Each customer selects the pair that maximizes his/her utility, which corresponds to the incentive compatibility constraints. Given that both utility firms' first-stage problems remain the same under non-simultaneous arrivals, the results obtained for simultaneous arrivals in Section 5 continue to apply. That is, the public and private utility firms should set the pairs of charging price and completion times as given in Propositions 2 and 3, respectively. However, the second-stage problem under non-simultaneous arrivals is more involved as we analyze below.

In the following analysis, we assume that the arrival time of each customer s_n is exogenously given and it is public information. That is, we do not consider the case in which customers need to consider a matrix of pairs of charging price and completion time to strategically determine their arrival times. This assumption ensures tractability and isolates the tradeoff between inconvenience cost and charging cost, allowing us to generate insights which can be of value for the case in which arrival times depend on prices.

6.2 Charging Schedule Under Given Completion Times

We next solve the second-stage problem of minimizing the charging cost under given completion times, which is a common problem for both public and private utility firms. To do so, we first pool and sort arrival and completion times of N customers in the ascending order and label them as t_j such that $0 = t_1 = s_1 < t_2 < \cdots < t_{2N} < T$. Thus, the problem horizon consists of $2N$ time intervals: $[t_1, t_2]$, $(t_2, t_3]$, $\cdots$, $(t_{2N}, T]$. Let $\mathcal{I}(t)$ be the set of electric vehicles that are present at the charging station at time t, i.e., $\mathcal{I}(t) \doteq \{k \mid t \in [s_k, \tau_k]\}$.

Unlike the simultaneous-arrival case in Section 4, where there exists an optimal policy that can be implemented by the juice-filling procedure with only one juice surface level (or fill-up-to level) z_n for each vehicle n during the problem horizon $t \in [0, T]$, in the case of non-simultaneous arrivals, we shall show that there exists an optimal policy that can be characterized by one juice-surface-level $z_{n,j}$ for each time interval $(t_j, t_{j+1}]$ for every vehicle n such that there are potentially different $z_{n,j}$'s during the problem horizon $t \in [0, T]$ for vehicle n. That is, these juice surface levels may change across time intervals. They can be determined via the optimization problem given in (A.19)-(A.21) in the Appendix. Because these thresholds are scalars, computing them is simpler than computing the $a_n(t)$ functions in the original problem (19)-(23). For given $z_{n,j}$'s, the charging policy is as follows.

Charging policy for non-simultaneous arrivals (given $z_{n,j}$'s)

Step 1. Initialize $j = 1$ and $L(t) = d(t)$ for $t \in [0, T]$.

Step 2. Set $n = \arg\min_k \{\tau_k | k \in \mathcal{I}(t_j)\}$, which is the electric vehicle with the earliest completion time in the feasible set $\mathcal{I}(t_j)$.

Step 3. Set $a_n(t) = \min\left\{\left(z_{n,j} - L(t)\right)^+, \overline{a}\right\}$ for $t \in [t_j, t_{j+1}]$.

Step 4. Replace $L(t)$ by $L(t)+a_n(t)$ for $t \in [t_j, t_{j+1}]$. If $n = \arg\max_k \{\tau_k | k \in \mathcal{I}(t_j)\}$ and $j = 2N-1$, then stop; if $n = \arg\max_k \{\tau_k | k \in \mathcal{I}(t_j)\}$ but $j < 2N - 1$, increase j by 1 and go to step 2; otherwise, increase n by 1 and go to step 3.

The above charging policy is similar to the juice-filling policy within each time interval because by construction, neither an arrival nor a charging completion occurs during the open time-interval (t_j, t_{j+1}). Accordingly, the charging amount in Step 3 above is equivalent to that in Step 3 of the juice-filling policy. We next prove that there exits $z_{n,j}$'s such that this policy is optimal.

Proposition 5 *Given external demand $d(t)$, there exists $\{z_{n,j} : n = 1, \ldots, N,\ j = 1, \ldots, 2N-1\}$ such that the control policy $\{a_1(t), \ldots, a_N(t)\}$ found by the above charging policy for non-simultaneous arrivals is an optimal solution to the charging cost minimization problem in* (19)-(23).

Although the optimal charging policy can be characterized as described above, threshold $z_{n,j}$'s still must be optimized jointly by solving (A.19)-(A.21) in the Appendix. As the number of electric vehicles increases, such an optimization problem becomes challenging to solve, making it practically intractable to solve the first-stage problem of evaluating the cost of charging many electric vehicles and announcing optimal charging prices and completion times. We next design a computationally efficient and intuitive charging policy for the utility firm by generalizing the juice-filling policy introduced in Section 4 to accommodate non-simultaneous arrivals. We use $\{a_n^G(t) : t \in [s_n, \tau_n], n = 1, \ldots, N\}$ to denote the charging schedule under this policy.

Generalized juice-filling policy

Step 1. Initialize $n = 1$ and $L(t) = d(t)$ for $t \in [0, T]$.

Step 2. Find z_n such that

$$\int_{s_n}^{\tau_n} \min\left\{\left(z_n - L(t)\right)^+, \overline{a}\right\} \mathrm{d}t = 1. \tag{24}$$

Step 3. Set $a_n^G(t) = \min\left\{\left(z_n - L(t)\right)^+, \overline{a}\right\}$ for $t \in [s_n, \tau_n]$.

Step 4. Replace $L(t)$ by $L(t) + a_n^G(t)$ for $t \in [s_n, \tau_n]$. If $n = N$, then stop. Otherwise, increase n by 1 and go to step 2.

The above policy is equivalent to the juice-filling policy if $s_n = 0$ for each customer n. As in the juice-filling policy, the thresholds above are defined for each charging time interval and can be computed sequentially in closed-form (see Step 2). Therefore, the generalized juice-filling policy is much more efficient to compute compared to the optimal policy which relies on threshold $z_{n,j}$'s that are difficult to compute.

The generalized juice filling policy is likely to perform well in determining the charging schedule for a large number of vehicles. Recall that this policy smooths the demand in a greedy fashion in each iteration. Intuitively, as the number of electric vehicles increases, the policy is run for more iterations, so the demand is likely to be smoothed as much as possible. As we illustrate in the next section, using generalized juice-filling policy already enables a utility firm to achieve significant cost savings through smart charging, compared to the current practice of charging vehicles as soon as possible.

7. Numerical Analysis

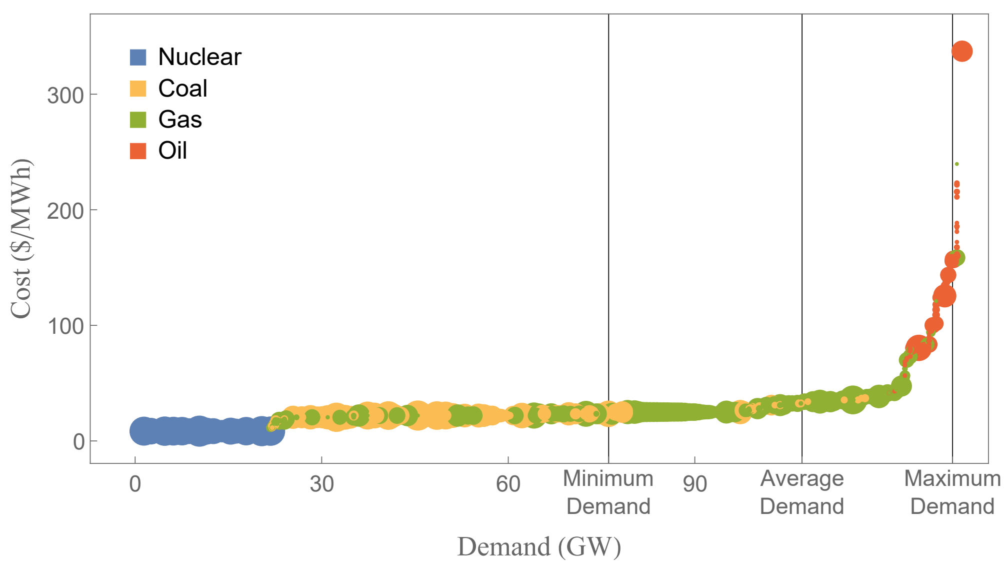
In this section, we illustrate our findings by using real electricity demand and supply data. We quantify the economic and environmental implications of smart charging for a private and public utility firm, considering both cases of simultaneous and non-simultaneous arrivals of customers.

We focus on the PJM Interconnect, the largest wholesale electricity market in the U.S., which serves 65 million customers across thirteen states (PJM 2018, p. 24). We estimate the cost of procuring q units of electricity, i.e., $\tilde{c}(q)$ in (5), by constructing the supply curve for PJM (see Figure 3). We use the hourly electricity demand data for each day of August 2016. We subtract the hourly generation of renewable sources from the electricity demand to obtain the net electricity demand, which is satisfied by conventional sources. We then estimate the generation cost of these conventional sources by using the Emissions and Generation Resource Integrated Database (eGRID) of The Environmental Protection Agency (EPA 2019). eGRID provides detailed information for each of the 448 conventional power plants in the PJM Interconnect, including the heat rate (the amount of thermal energy needed to generate one unit of electrical energy, given in MMBTU/kWh) and nameplate capacity (the theoretical maximum power output of a generator, given in MW).

By multiplying the heat rate with the cost of the fuel source of a power plant (given by EIA 2017 as \$/MMBTU), we calculate the plant's cost of generating electricity (given in \$/kWh). We convert the nameplate capacity to effective capacity for each plant by multiplying the plant's nameplate capacity with the equivalent availability factor (provided by NERC 2019), which accounts for the unavailability due to planned outages for maintenance. We then sort the power plants in the increasing order of their generation costs, which gives a supply curve but does not take into account unplanned outages (due to reliability problems). To account for these, we calibrate the supply curve by multiplying the capacity with a daily scaling factor $\alpha_m \leq 1$ for each day m. We find the factor α_m by minimizing the sum of squared differences between the realized electricity price and the estimated electricity price (based on the adjusted supply curve and the net demand). Figure 3 illustrates the calibrated supply curve for August 25, 2016.

In addition to the cost data, eGRID provides emissions intensity of each conventional power plant, which we use to estimate environmental implications of smart charging. We finally note that, in our numerical analysis, we consider 2,500 electrical vehicles with a daily energy demand of 50 MWh (see Section 7.2). To better illustrate our results, we scale down both the supply curve and the external demand by a factor of 1,000, so that the demand from electric vehicle charging is approximately 2% of the total electricity demand.

Figure 3: (Color online) Supply curve for August 25, 2016



Notes. The above figure is based on The PJM Interconnect. Each circle represents a power plant with its generation cost on the vertical axis and the cumulative capacity up to that plant on the horizontal axis. The size of a circle is proportional to the plant's capacity.

We next estimate customers' waiting cost. Incentivizing customers to wait longer to reduce the cost of charging electric vehicles is a novel business idea and, therefore, there is no prior study that has directly estimated the waiting cost in such a context. It is reasonable to assume that the waiting cost varies significantly across customers. For example, Akşin et al. (2013) estimate that for a call center, the customers' waiting cost varies from a negligible level to approximately $1 per minute and they consider three classes of customers. We also consider a wide range of waiting cost, reflected by the heterogeneity in customers' sensitivity to delays, and we assume that there are five classes of delay-sensitivity. Furthermore, it is reasonable to assume that the waiting cost is convex in the waiting time, i.e., a longer wait leads to a higher marginal waiting cost, which is supported by the psychology literature (Osuna 1985).

For our numerical analysis, we assume that each customer needs to charge 20 kWh for their electric vehicle and each charging station can provide a maximum charging speed of 6.67 kW. Thus, the minimum time required for charging is 3 hours. If the charging completion time τ is longer than 3 hours, an inconvenience cost is incurred. We assume the utility function (1) is

$$u(\theta, \tau, p) \;=\; 50 - \theta(\tau - 3)^2 - p, \quad \tau \geq 3. \tag{25}$$

That is, a charging service that is longer than the 3-hour minimum time leads to an inconvenience cost that quadratically increases in the delay. We also assume that the reservation utility is $\underline{u} = \$40$. In fact, our numerical results depend only on $u_0 - \underline{u} = \$10$, which is the customers' willingness to pay. This translates to 50 cents per kWh, a typical price at charging stations (Blink 2019).

We let the delay-sensitivity θ take five possible values: $\theta^{(1)} = 0.1$, $\theta^{(2)} = 2$, $\theta^{(3)} = 4$, $\theta^{(4)} = 6$, and $\theta^{(5)} = 8$ with equal probabilities. These values also represent the inconvenience cost (in dollars) for the first hour of delay. We tried expanding the range of θ to include higher waiting cost, but those highly delay-sensitive customers are served almost without delay in the optimal solution. As a result, their demand can be regarded as non-schedulable and included in the demand $d(t)$ from other sources.

7.1 Simultaneous Arrivals

We first illustrate the results in Sections 4 and 5. We consider $N = 500$ customers who request charging service simultaneously, with $I = 5$ customer classes. We assume that there are 100 customers from each delay-sensitivity level $\theta^{(i)}$ for $i = 1, \ldots, 5$. That is, instead of independently drawing each customer's delay sensitivity from the delay-sensitivity distribution, we assume that the same number of customers from each class request charging. This assumption simplifies the computation and it is reasonable given that the number of customers N is much greater than the number of customer classes I. Each class of 100 customers requires 2,000 kWh or 2 MWh. Thus, the total energy required for charging 500 electric vehicles is 10 MWh.

Figure 4 and Table 1 compare four charging policies for electric vehicles. The utility firm's electricity demand (excluding electric vehicle charging) is taken from 3 p.m. to 9 p.m. on a typical peak day. The marginal cost (the numbers below the external demand curve) during the peak hours is more than twice as high as that during the off-peak hours ($158/MWh versus $72/MWh). All 500 customers request charging service at the beginning of this 6-hour period. We shall analyze the case with non-simultaneous arrivals in Section 7.2.

Figure 4 (a) shows the as soon as possible (ASAP) charging schedule, in which all 500 electric vehicles are charged during the first 3 hours of the 6-hour period. This corresponds to the current practice in charging stations (SEPA 2017). The total charging demand is 3.333 MW, burdening

Figure 4: (Color online) Electric vehicle charging policies under simultaneous arrivals

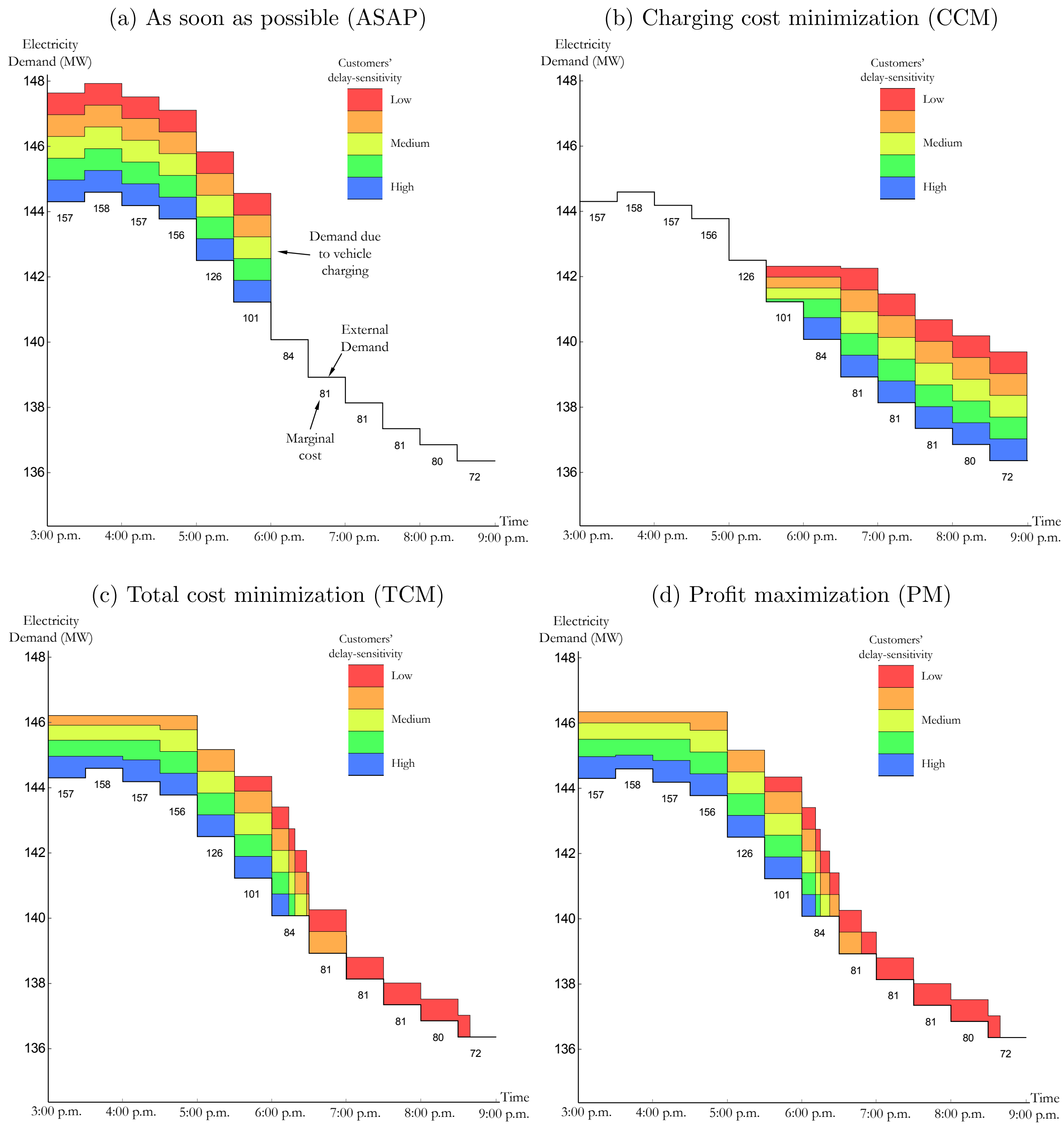


Table 1: Comparison of electric vehicle charging policies

		As soon as possible (ASAP)	Charging cost minimization (CCM)	Total cost minimization (TCM)	Profit maximization (PM)
Completion times (hours)	$\tau^{(1)}$	3	6	5.66	5.66
	$\tau^{(2)}$	3	6	4.01	3.80
	$\tau^{(3)}$	3	6	3.47	3.37
	$\tau^{(4)}$	3	6	3.31	3.25
	$\tau^{(5)}$	3	6	3.23	3.18
Inconvenience cost ($)		0	18090	459.26	318.45
Charging cost ($)		5455.68	913.30	1765.52	2012.31
Charging cost ($ per kWh)		0.546	0.091	0.177	0.201
Total cost ($)		5455.68	19003.3	2224.78	2330.76
Prices ($)	$p^{(1)}$	10	-	6.640	7.608
	$p^{(2)}$	10		7.247	8.252
	$p^{(3)}$	10		8.831	9.252
	$p^{(4)}$	10		9.320	9.562
	$p^{(5)}$	10		9.585	9.730
Total payment ($= 100 \sum_{i=1}^{5} p^{(i)}$)		5000	-	4162.38	4440.33
Profit (= total payment − charging cost)		−455.68	-	2396.85	2428.01
Information rent ($) for customers with delay sensitivity	$\theta^{(1)}$	0	-	2.652	1.684
	$\theta^{(2)}$	0		0.732	0.470
	$\theta^{(3)}$	0		0.296	0.191
	$\theta^{(4)}$	0		0.104	0.068
	$\theta^{(5)}$	0		0	0
Total information rent ($)		0	-	378.36	241.23
CO_2 emissions (kilograms per kWh of EV charging)		7.312	0.904	2.007	2.741
Total CO_2 emissions from charging (metric tons)		73.12	9.04	20.07	27.41

the utility firm because the marginal cost is already very high during the first 3 hours. Under such a policy, each kWh charged costs $0.546 on average, while customers pay $0.5 per kWh ($10 for 20 kWh as discussed earlier), leading to a loss to the charging station, as shown in Table 1. The corresponding CO_2 emissions are also high because the additional electricity used during peak hours is mostly generated by the least efficient peaking power plants with the highest emissions intensity.

Figure 4 (b) illustrates the policy of minimizing the charging cost. All electric vehicles are charged to smooth the total electricity demand as much as possible. The average charging cost is only $0.091 per kWh and the emissions intensity is 0.9 kg per kWh. However, this policy is impractical because it results in hefty inconvenience cost—so high that customers must be paid rather than paying for charging (thus the prices are blank in Table 1). Nevertheless, we consider this policy to understand the cost and emissions reduction potential of smart charging.

The total cost minimization policy minimizes the sum of customers' inconvenience cost and charging cost, as shown in Figure 4 (c). This is the objective of a public utility firm given in (11). From 3:00 to 5:00 p.m., the total charging demand is reduced to about half of that under ASAP policy and the total system demand during these two hours is smoothed. Charging is delayed to later hours when the marginal cost is lower, and the delay is allocated to various customer groups depending on their delay sensitivity. Both the charging cost and the CO_2 emissions are substantially lower compared to the ASAP policy.

Under the total cost minimization policy, the prices shown in Table 1 are incentive compatible: Customers who are least delay-sensitive choose to have the longest delay and pay \$6.64 for charging service as opposed to \$10 under the current practice of ASAP charging. This is an example of how the firm can pass part of the cost savings from smart charging to customers by offering lower prices. Customers who are more sensitive to delay pay more (or save less) in exchange for less delay in charging.

Finally, Figure 4 (d) and the last column in Table 1 reveal the solution that maximizes the profit, which is the objective function of a private utility firm in (13). Compared to the total cost minimization of a public utility firm, the profit maximization of a private utility firm completes charging service slightly earlier for all customers except for the least delay-sensitive ones. The inconvenience cost is thus reduced and the customers are willing to pay higher prices to the private utility firm, compared to a public one. The increase in revenue (\$278 or 6.7%) exceeds the cost increase (\$247 or 14%), resulting in a maximum profit of \$2,428, which is only 1.3% higher than the profit of a public utility firm; the CO_2 emissions increase by 37%, because the petroleum-fired peaking power plants are used more often and they have much higher emissions intensity. Nevertheless, both total cost minimization and profit maximization policies are substantially better than charging electric vehicles as soon as possible: The respective charging cost reduction is 67.6% and 63.1% of the cost under ASAP policy, and the CO_2 emissions reduction is 72.5% and 62.5%, respectively.

In the above example, the assumption that all customers arrive simultaneously at a peak-demand time allows us to illustrate the fundamental tradeoff between inconvenience and charging costs. Next, we examine a more general setting in which customers arrive non-simultaneously over time.

7.2 Non-Simultaneous Arrivals

We now consider a total of $N = 2,500$ customers, arriving at 5 different times over the course of a day: 8:00 a.m., 10:30 a.m., 1:00 p.m, 3:30 p.m., and 6:00 p.m. At each time, 500 customers arrive, with 100 customers being of type $\theta^{(i)}$, $i = 1, \ldots, 5$. As in Section 7.1, each customer demands 20kWh (thus the total energy required for charging is 50 MWh over the course of a day), and the customers' utility function is given by (25). Note that two adjacent arrival times are 2.5 hours apart, shorter than minimum charging time of 3 hours. This approximates the realistic situation where customers request charging service as other vehicles are being charged. We also experimented using a finer division of periods (e.g., 10 or 15 arrival times), and the results are found to be similar. We choose to present the results of the simpler model for the ease of illustration (see Figure 5) as well as for computational efficiency. The computation is demanding: For each of the 31 days in August 2016, we optimize the charging schedule under the objectives of a public and private utility firm and compare the results with ASAP policy.

Because the objective function of a utility firm in the first-stage problem is generally neither convex nor concave, we employ a global search algorithm with the following constraints: (a) The charging completion times decrease in the customers' sensitivity to delay (Lemma 2), and (b) For customers with the same level of delay-sensitivity, later arrivals also have later completion. For the solution of the second-stage problem of the utility firm, we use the generalized juice-filling policy. The computational procedure takes approximately two hours to converge to a set of optimal completion times (with optimal charging schedule) for each day.

Below, we first present the results on the cost and emissions reductions to demonstrate the overall benefits of incentivizing customers to delay charging their electric vehicles, and then we illustrate these benefits by analyzing a typical peak day.

Table 2 details the cost and CO_2 emissions of charging electric vehicles for each day in August 2016 under the following policies: ASAP, total cost minimization (for a public utility firm), and profit maximization (for a private utility firm). The charging cost varies drastically from day to day. For two days (August 11th and 25th), the ASAP charging cost exceeds \$14,000 (or \$0.28 per kWh), and either total cost minimization or profit maximization policy reduces the charging cost by over 50%. These two days also see very high electricity prices in the data: \$175.65 and \$198.40 per MWh, respectively. There are five days (August 10th, 12th, 13th, 18th, 26th) for which the ASAP charging cost ranges from \$3,000 to about \$6,000, the other two policies achieve a cost reduction of 7 to 35%. There are 14 days for which they reduce cost between 2 to 4%, and for

Table 2: Costs and emissions of electric vehicle charging for a typical summer month

Based on PJM demand and power plants data in August 2016
ASAP = Charge as soon as possible, TCM = Total cost minimization of a public utility firm, PM = Profit maximization of a private utility firm

day	Cost of EV charging ($)					CO_2 emissions due to EV charging (metric tons)				
	ASAP	TCM	$\frac{ASAP-TCM}{ASAP}$	PM	$\frac{ASAP-PM}{ASAP}$	ASAP	TCM	$\frac{ASAP-TCM}{ASAP}$	PM	$\frac{ASAP-PM}{ASAP}$
1	2291.5	2222.6	3.01%	2225.4	2.89%	38.7	38.7	0.10%	38.7	0.12%
2	2218.6	2159.5	2.67%	2162.2	2.55%	37.7	37.6	0.37%	37.6	0.27%
3	2366.6	2304.2	2.64%	2306.8	2.53%	38.8	38.5	0.73%	38.5	0.75%
4	2011.7	1967.6	2.19%	1970.8	2.03%	35.7	35.7	-0.06%	35.7	-0.01%
5	1557.7	1557.3	0.03%	1557.3	0.03%	29.2	29.3	-0.03%	29.3	-0.03%
6	1450.9	1449.2	0.12%	1449.3	0.12%	29.1	29.1	0.07%	29.1	0.08%
7	1552.5	1551.8	0.04%	1551.8	0.04%	30.7	30.7	0.03%	30.7	0.03%
8	2891.8	2792.0	3.45%	2801.9	3.11%	40.2	40.0	0.56%	40.1	0.48%
9	2601.6	2507.6	3.61%	2513.3	3.39%	38.6	38.3	0.94%	38.3	0.86%
10	6342.7	4079.4	35.68%	4174.7	34.18%	77.8	47.1	39.45%	47.9	38.44%
11	14587.8	6454.8	55.75%	6986.5	52.11%	185.9	89.0	52.12%	98.1	47.22%
12	5257.0	3836.6	27.02%	3910.4	25.61%	63.7	45.8	28.07%	46.4	27.10%
13	3953.4	3611.4	8.65%	3622.9	8.36%	48.2	43.7	9.30%	43.8	9.10%
14	2186.1	2139.4	2.14%	2142.4	2.00%	36.9	36.6	0.89%	36.6	0.78%
15	1625.6	1625.0	0.03%	1625.1	0.03%	31.4	31.4	0.07%	31.4	0.07%
16	2557.1	2477.5	3.11%	2480.8	2.98%	38.6	38.5	0.34%	38.5	0.29%
17	2900.2	2788.8	3.84%	2798.2	3.52%	39.5	39.2	0.90%	39.2	0.85%
18	3439.8	3144.0	8.60%	3159.7	8.14%	43.8	41.0	6.34%	41.1	6.07%
19	1745.4	1741.2	0.25%	1741.3	0.24%	33.9	33.8	0.09%	33.8	0.09%
20	1619.8	1619.1	0.04%	1619.2	0.04%	31.1	31.1	0.05%	31.1	0.05%
21	1209.6	1209.6	0.00%	1209.6	0.00%	26.5	26.5	-0.01%	26.5	-0.01%
22	1440.4	1438.6	0.13%	1438.7	0.12%	29.2	29.2	-0.04%	29.2	-0.03%
23	1437.2	1435.9	0.09%	1435.9	0.09%	27.9	27.9	0.00%	27.9	0.00%
24	1962.3	1922.5	2.03%	1924.9	1.91%	34.8	34.7	0.47%	34.6	0.49%
25	14268.6	5707.5	60.00%	6230.9	56.33%	174	82.7	52.46%	94.0	45.95%
26	3082.0	2860.2	7.20%	2872.2	6.81%	40.6	39.4	3.08%	39.4	2.96%
27	1440.6	1438.9	0.12%	1439.0	0.12%	29.5	29.5	0.05%	29.5	0.06%
28	2245.3	2182.9	2.78%	2185.8	2.65%	36.2	36.1	0.29%	36.1	0.33%
29	2433.0	2344.5	3.64%	2347.8	3.50%	37.6	37.0	1.41%	37.1	1.27%
30	2239.0	2175.2	2.85%	2178.0	2.72%	36.4	36.1	0.76%	36.2	0.66%
31	2380.8	2302.9	3.27%	2308.4	3.04%	38.6	38.7	-0.08%	38.7	-0.12%
Total	99296.8	77047.8	22.4%	78371.2	21.1%	1460.9	1212.7	17.0%	1235.0	15.5%

the remaining 10 days, the cost reduction is no more than 0.25%. For the entire month, total cost minimization and profit maximization policies reduce the total cost of charging electric vehicles by 22.4% and 21.1%, respectively.

The CO_2 emissions reductions are generally correlated with the cost reductions, but there are two caveats. First, the emissions reduction percentages are typically lower than the cost reduction percentages because the public and private utility firm's objectives do not include emissions. There are five days for which the total cost minimization and profit maximization policies increase emissions slightly (less than 0.12%). Second, the emissions reduction percentages can exceed the cost reduction percentages (notably on August 10th and 12th), primarily because of the increased use of natural gas-fired generators with low emissions. In aggregate, the two policies reduce the monthly CO_2 emissions of electric vehicle charging by 17% and 15.5%, respectively.

These results have important practical implications: First, to achieve most of the benefits, the incentives for customers to delay electric vehicle charging need to be used only during the peak days. In fact, a public or a private utility firm, by employing smart charging in only 4 peak days (August 10th, 11th, 12th, 25th) can reduce the charging cost by 20.5% or 19.3%, respectively, and the emissions can be reduced by 16.2% or 14.7%, respectively. In other words, approximately 92 to 95% of the cost and emissions reductions are achieved on those 4 peak days, highlighting the importance of managing electric vehicle charging on peak days. Second, although the profit maximization policy of a private utility firm distorts the electric vehicle charging schedules that would otherwise minimize the overall cost, the distortion appears to be small. Therefore, regulators should encourage both private and public utility firms to operate smart charging stations.

We now examine how the cost and emissions reductions reported in Table 2 are achieved by scheduling the charging of 2,500 electric vehicles. Figure 5 uses the same color-coding scheme as Figure 4 to identify different levels of delay-sensitivity. The brightness of the colors differentiate arrival times, with lighter (darker) colors for customers arriving earlier (later) in the day. Each color represents 100 customers as before. In Figure 5, we consider the total cost minimization policy of a public utility firm.

Figure 5 (a) shows the charging schedules for the first 1,000 customers arriving when the electricity demand from other sources is rapidly increasing. The charging schedule is close to ASAP policy, because delaying charging for these customers will only incur higher cost, but a slight delay is planned for the least delay-sensitive customers in the second group. Such a delay smooths the total demand from 1:00 to 2:00 p.m., which is reflected in Figure 5 (b). Load smoothing is particularly important when the marginal cost rapidly increases near the peak hours.

Figure 5: (Color online) Electric vehicle charging schedule on a typical peak day (August 25, 2016) under total cost minimization policy of a public utility firm

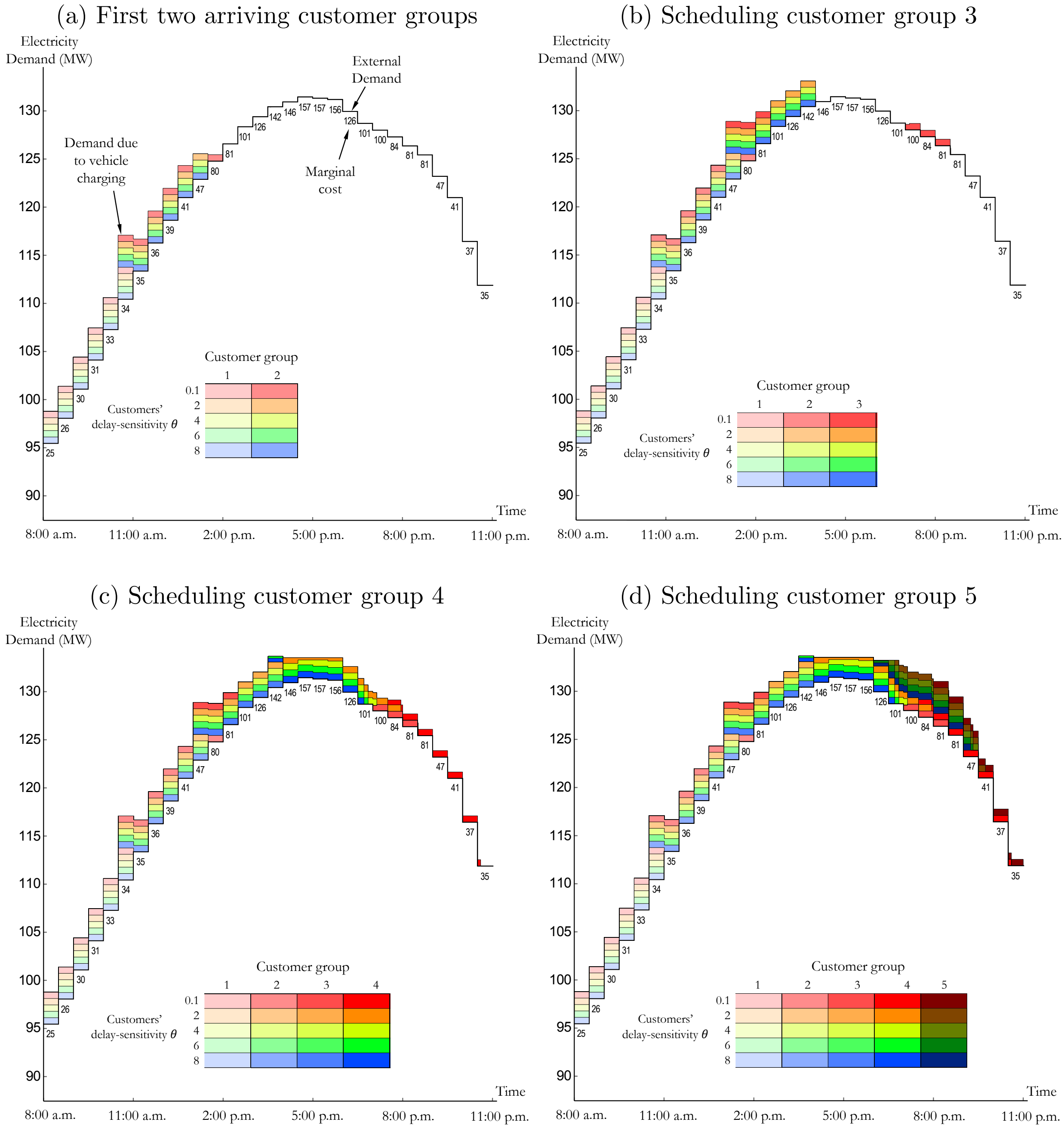


Figure 5 (b) reveals an important feature of the optimal schedule. The least delay-sensitive customers in group 3 (indicated in bright red) start charging as soon as they arrive at 1:00 p.m. The charging pauses at 2:30 p.m., resumes at 7:00 p.m., and finishes shortly after 8:30 p.m. This 4.5-hour delay allows the public utility firm to schedule the next two groups of customers.

In Figure 5 (c), the least delay sensitive customers in group 4 have their vehicles charged from

7:30 p.m. to shortly after 10:30 p.m. In addition, customers who are more sensitive to delays also have to experience some delays. All these delays greatly help reduce and flatten the total demand during the peak hours, leading to reduced charging cost and emissions. Observe that the schedule actually completes charging for group 4 customers with $\theta^{(i)}$, $i = 2, \ldots, 5$ earlier than the aforementioned group 3 customers who finish charging at 8:30 p.m.

Figure 5 (d) shows that the delayed demand during the peak hours is shifted to evening. From 7:00 to 8:30 p.m., some vehicles from groups 3, 4, and 5 are charged at the same time. Group 5 customers all experience some delays.

For brevity, we omit the illustration for the optimal charging schedule under a private utility firm's profit maximization policy, which is similar to the schedule under the public utility firm's total cost minimization policy, illustrated in Figure 5. Both policies' cost breakdowns and the incentive-compatible prices are shown in Table 3.

The profit maximization policy results in a lower inconvenience cost than the total cost minimization policy, charging customers higher prices (3.6% higher on average), yielding a revenue that is also 3.6% or $802.5 higher. The charging cost is 9.2% or $523.4 higher. Therefore, the private utility firm achieves a profit that is $279.1 or 1.7% higher than the public utility firm. However,

Table 3: Electric vehicle charging on a typical peak day (August 25, 2016)

	As soon as possible (ASAP)	Total cost minimization (TCM)	Profit maximization (PM)
Inconvenience cost ($)	0	1420.5	1010.1
Charging cost ($)	14268.6	5707.5	6230.9
Charging cost ($ per kWh)	0.285	0.114	0.125
Total cost ($)	14268.6	7128.0	7241.0
Prices ($) $[\, p_t^{(i)} \,]$	[10]	$\begin{bmatrix} 10 & 9.98 & 7.96 & 3.25 & 7.72 \\ 10 & 10 & 10 & 4.73 & 8.06 \\ 10 & 10 & 10 & 8.06 & 8.62 \\ 10 & 10 & 10 & 9.20 & 9.25 \\ 10 & 10 & 10 & 9.66 & 9.53 \end{bmatrix}$	$\begin{bmatrix} 10 & 9.98 & 7.96 & 5.53 & 8.26 \\ 10 & 10 & 9.98 & 7.05 & 8.62 \\ 10 & 10 & 10 & 8.88 & 9.18 \\ 10 & 10 & 10 & 9.52 & 9.51 \\ 10 & 10 & 10 & 9.88 & 9.70 \end{bmatrix}$
Total payment ($)	25000	22600.4	23402.9
Profit (= payment − charging cost)	10731.4	16892.9	17172.0
Total information rent ($)	0	979.1	586.9
CO_2 emissions (kilograms per kWh of EV charging)	3.48	1.65	1.88
Total CO_2 emissions from charging (metric tons)	174.0	82.7	94.0

the CO_2 emissions increase from 82.7 to 94.0 metric tons, which is a 13.7% increase. This emissions difference may be concerning, but the emissions differences are generally small for most of the days (see Table 2). For the entire August, the total CO_2 emissions under the private utility firm is 1.8% higher than that under a public utility firm. Both firms reduce emissions substantially compared to the current practice of ASAP policy.

8. Conclusion

Electric vehicles will transform urban mobility in smart cities, which requires innovative business models for charging stations. We propose smart charging of electric vehicles—delaying charging until electricity generation cost is lower during a day—as an alternative to the current practice of completing charging as soon as possible. By considering the cost minimization and profit maximization objectives of public and private utility firms, respectively, we characterize pricing schemes that motivate customers to allow smart charging.

Our results have significant managerial and policy implications. We devise the juice-filing policy which can be easily implemented to determine the smart charging schedule of electric vehicles. Through a comprehensive analysis of the largest electricity market in the U.S., we demonstrate that compared to the current practice, smart charging leads to approximately 20% and 15% cost and emissions savings, respectively. Moreover, most of these savings can be achieved by implementing smart charging only a few peak-demand days in a month, highlighting its practical relevance. From a policy perspective, our results show that allowing either public or private utility firms to own and operate charging stations can lead to significant cost and emissions savings.

Finally, in this paper, we assume that charging stations have ample parking spots. If spots are limited, the benefits of smart charging would be less pronounced, but the fundamental tradeoff between the charging cost and the inconvenience cost that we focused on remains valid.

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A. Proofs

Proof of Proposition 1: The utility firm's second-stage problem (6)-(10) can be expressed in the standard form (Bertsekas 2007, Section 7.1) as

$$\min_{\{a_n(t),\, n=1,\ldots,N\}} h\big(x_1(\tau_1),\ldots,x_N(\tau_N)\big) + \int_0^T \tilde{c}\left(\sum_{n=1}^{N} a_n(t) + d(t)\right) \mathrm{d}t - \int_0^T \tilde{c}(d(t))\mathrm{d}t \tag{A.1}$$

$$\text{s.t. } \frac{\mathrm{d}x_n(t)}{\mathrm{d}t} = a_n(t) \qquad \forall n = 1,\ldots,N, \tag{A.2}$$

$$0 \le a_n(t) \le \overline{a} \qquad \forall n = 1,\ldots,N, \tag{A.3}$$

where $x_n(t) = \int_0^t a_n(s)\,\mathrm{d}s$ is the state variable representing the level of electricity charged by time t to electric vehicle n, $a_n(t)$ is the control variable, and

$$h\big(x_1(\tau_1),\ldots,x_N(\tau_N)\big) = \begin{cases} 0 & \text{if } x_n(\tau_n) = 1 \quad \forall n \in \{1,\ldots,N\}, \\ \infty & \text{otherwise,} \end{cases} \tag{A.4}$$

represents terminal conditions. Also, in (A.1), we substitute the cost function of (6) as $c(q(t)|d(t)) = \tilde{c}(q(t)+d(t)) - \tilde{c}(d(t))$, where $q(t) = \sum_{n=1}^{N} a_n(t)$. Given that the term $\int_0^T \tilde{c}(d(t))\mathrm{d}t$ is constant in the control variable $a_n(t)$'s, we omit it from the optimization below. A unique feature of this problem is that the terminal condition imposed on $x_n(t)$ through $h(x_1(\tau_1),\cdots,x_N(\tau_N))$ applies at different times τ_n. Accordingly, the number of control variables decreases over time.

We first show that the control policy found through the juice-filling policy is an optimal solution to the problem (A.1)-(A.3) for the special case with one electric vehicle (i.e., when $N = 1$) using Pontryagin's minimum principle (Bertsekas 2007, Section 7.3). The Hamiltonian-Jacobi-Bellman (HJB) equation is given as

$$H(x,a,p) = \tilde{c}(a+d) + pa, \tag{A.5}$$

where the adjoint equation $p(t)$ (not to be confused with charging prices $p^{(i)}$'s) is given by

$$\frac{\mathrm{d}p(t)}{\mathrm{d}t} = -\frac{\partial H\left(x^*(t), a^*(t), p(t)\right)}{\partial x} = 0, \tag{A.6}$$

where $a^*(t)$ is the optimal control trajectory and $x^*(t)$ is the corresponding state trajectory. This implies that $p(t)$ is a constant for $t \in [0,T]$, i.e., $p(t) = p$ for some p. We next minimize the HJB, subject to the terminal condition:

$$\min_{0 \le a \le \overline{a}} \{H(x,a,p) = \tilde{c}(a+d) + pa\} \tag{A.7}$$

$$s.t. \int_0^T a(t)\mathrm{d}t = 1. \tag{A.8}$$

Because $\tilde{c}(\cdot)$ is a convex function, $H(x,a,p)$ is also convex in a and the first-order condition is given as $\frac{\mathrm{d}H(x,a,p)}{\mathrm{d}a} = \tilde{c}^{'}(a+d) + p = 0$, where $\tilde{c}^{'}(\cdot)$ is the derivative of the cost function. Taking the constraint that $0 \le a(t) \le \overline{a}$ into account, the optimal action is to set $a^*(t) = \min\left((z - d(t))^+, \overline{a}\right)$, where $\tilde{c}^{'}(z) = -p$. Note that z is constant because p is a constant. To find the value of z, we substitute $a^*(t)$ in the terminal condition (A.8) so that z is implicitly given by

$$\int_0^T \min\left((z - d(t))^+, \overline{a}\right) \mathrm{d}t = 1. \tag{A.9}$$

This control policy is the same as the one found by the juice-filling policy for the special case with one electric vehicle (i.e., when $N = 1$), where z is given in Step 2 and $L(t) = d(t)$. Therefore, the juice-filling policy finds an optimal solution to the problem (A.1)-(A.3).

We next prove that the juice-filling policy is optimal for $N > 1$. We index electric vehicles with n such that $\tau_N \le \cdots \le \tau_1$. Let $a_n^*(t)$ for $n \in \{1, \ldots, N\}$ be an optimal solution to the charging problem (A.1)-(A.3). Using the juice-filling policy, we construct another solution $\hat{a}_n(t)$ for $n \in \{1, \ldots, N\}$ such that this solution is also optimal. We next describe the construction of this solution in the interval $t \in [0, \tau_N]$, where τ_N is the earliest completion time. In essence, we find the charging policy $\hat{a}_n(t)$ for $n \in \{1, \ldots, N\}$ for N vehicles with simultaneous arrival (i.e., $t = 0$) and completion times (i.e., $t = \tau_N$) such that each vehicle is charged by time τ_N to the same level as that under the optimal policy, i.e., $\hat{x}_n(\tau_N) = x_n^*(\tau_N) = \int_0^{\tau_N} a_n^*(t)\mathrm{d}t$ for $n \in \{1, \ldots, N\}$. Then, we show that this charging policy achieves the same cost as the optimal policy.

The charging policy $\hat{a}_n(t)$ for $n \in \{1, \ldots, N\}$ in the interval $t \in [0, \tau_N]$ can be constructed by applying the juice-filling policy with two modifications: Let $T = \tau_N$ and in Step 2, find z_n such that $\int_0^{\tau_N} \min\left\{(z_n - L(t))^+, \overline{a}\right\} \mathrm{d}t = x_n^*(\tau_N)$. Note that, these modifications do not affect the vehicle $n = N$ because $x_N^*(\tau_N) = 1$. That is, the charging schedule of vehicle N, i.e., $\hat{a}_N(t)$, is still found through the juice-filling policy.

We next verify that the resulting charging cost under the modified policy is the same as that under the juice-filling policy for charging one electric vehicle. Specifically, consider the juice-filling policy for one vehicle (which was shown to be optimal using Pontryagin's minimum principle) such that the vehicle's completion time is τ_N, its maximum charging speed is $\overline{a}N$, and the vehicle needs to be charged with $\sum_{n=1}^{N} x_n^*(\tau_N)$ units of energy. Following similar steps as above, we can show that there exists z such that

$$\int_0^{\tau_N} \min\left((z - d(t))^+, \overline{a}N\right) \mathrm{d}t = \sum_{n=1}^{N} x_n^*(\tau_N). \tag{A.10}$$

Moreover, it is straightforward to verify that z above is the same as z_N of the modified policy such that the two policies have the same cost. Given the optimality of the juice-filling policy for one vehicle, charging policy $\hat{a}_n(t)$ for $n \in \{1, \ldots, I\}$ is an optimal policy for $t \in [0, \tau_N]$. That is, $\int_0^{\tau_N} \tilde{c}\left(\sum_{n=1}^N \hat{a}_n(t) + d(t)\right) \mathrm{d}t = \int_0^{\tau_N} \tilde{c}\left(\sum_{n=1}^N a_n^*(t) + d(t)\right) \mathrm{d}t$.

Notice that in constructing $\hat{a}_N(t)$ for $t \in [0, \tau_N]$, we have used the juice-filling policy, where $\int_0^{\tau_N} \min\left\{(z_N - d(t))^+, \overline{a}\right\} \mathrm{d}t = 1$ so that $\hat{a}_N(t) = \min\left\{(z_N - d(t))^+, \overline{a}\right\}$ for $t \in [0, \tau_N]$. We then update $L(t)$ with $L(t) + \hat{a}_N(t)$, proceed to the time interval $t \in [0, \tau_{N-1}]$, where τ_{N-1} is the completion time of vehicle $N - 1$. By the same arguments as above, we can construct $\hat{a}_n(t)$ for all $n \in \{1, \ldots, N-1\}$ in $t \in [0, \tau_{N-1}]$ and show that it achieves the same cost as the optimal policy. This procedure can be repeated for all vehicles and time intervals. Therefore, the control policy $\hat{a}_n(t)$ for $n \in \{1, \ldots, N\}$ found through the juice-filling policy identifies an optimal solution to the charging problem (A.1)-(A.3). ∎

Proof of Lemma 1: (i) For given $(\tau_1, \ldots, \tau_N)$, let the optimal control be $\{a_1^*(t), \ldots, a_N^*(t)\}$. Because $\{b_1, \ldots, b_N\}$ is a permutation of $\{1, \ldots, N\}$, the control $\{a_{b_1}^*(t), \ldots, a_{b_N}^*(t)\}$ must be feasible under completion time $(\tau_{b_1}, \ldots, \tau_{b_N})$ because $\displaystyle\int_0^{\tau_{b_n}} a_{b_n}(t)\,\mathrm{d}t = 1$, $\forall n = 1, \ldots, N$. Furthermore, this feasible control yields the same objective value in (6) because $q(t) = \sum_{n=1}^{N} a_{b_n}(t) = \sum_{n=1}^{N} a_n(t)$. If, however, there exists another control $\{\widetilde{a}_{b_1}^*(t), \ldots, \widetilde{a}_{b_N}^*(t)\}$ that is strictly better than $\{a_{b_1}^*(t), \ldots, a_{b_N}^*(t)\}$, then $\{\widetilde{a}_1(t)^*, \ldots, \widetilde{a}_N^*(t)\}$ must be feasible under $(\tau_1, \ldots, \tau_N)$ and strictly better than $\{a_1^*(t), \ldots, a_N^*(t)\}$, leading to a contradiction. Hence, we have $C(\tau_{b_1}, \ldots, \tau_{b_N}) = C(\tau_1, \ldots, \tau_N)$.

(ii) The optimal control for (6)-(10) under any given $(\tau_1, \ldots, \tau_N)$ remains a feasible control under $(\hat{\tau}_1, \ldots, \hat{\tau}_N) \geq (\tau_1, \ldots, \tau_N)$. It follows that $C(\tau_1, \ldots, \tau_N)$ decreases in τ_n for any n. ∎

Proof of Lemma 2: The public utility firm's objective function is given as

$$\begin{aligned}&\mathbb{E}_{\{\theta_n, n=1,\ldots,N\}}\left[\sum_{n=1}^{N} \theta_n \delta(\tau_n) + C(\tau_1, \ldots, \tau_N)\right] \\ &\quad = \sum_{i_1=1}^{I}\sum_{i_2=1}^{I}\cdots\sum_{i_N=1}^{I}\left\{\prod_{n=1}^{N}\beta^{(i_n)}\left[\sum_{n=1}^{N}\left(\theta^{(i_n)}\delta(\tau^{(i_n)})\right) + C(\tau^{(i_1)}, \tau^{(i_2)}, \cdots, \tau^{(i_N)})\right]\right\}. \qquad \text{(A.11)}\end{aligned}$$

For given $\beta^{(i_n)}$'s, any solution with $\tau^{(n)} < \tau^{(m)}$ for $n < m$, we can (weakly) reduce the objective value by swapping $\tau^{(n)}$ and $\tau^{(m)}$. This swap does not affect the charging cost $C(\tau^{(i_1)}, \tau^{(i_2)}, \cdots, \tau^{(i_N)})$ due to Lemma 1 (i), but it (weakly) reduces the inconvenience cost $\sum_{n=1}^{N}\left(\theta^{(i_n)}\delta(\tau^{(i_n)})\right)$ because

$$\theta^{(n)}\delta(\tau^{(m)}) + \theta^{(m)}\delta(\tau^{(n)}) \leq \theta^{(n)}\delta(\tau^{(n)}) + \theta^{(m)}\delta(\tau^{(m)}) \qquad \text{(A.12)}$$

or equivalently, $\theta^{(n)}\left(\delta(\tau^{(m)}) - \delta(\tau^{(n)})\right) \leq \theta^{(m)}\left(\delta(\tau^{(m)}) - \delta(\tau^{(n)})\right)$. This inequality follows from

$\theta^{(m)} > \theta^{(n)}$ and $\delta(\tau)$ increases in τ. Because swapping $\tau^{(n)}$ and $\tau^{(m)}$ does not increase the objective value, there exists an optimal solution satisfying $\tau^{(1)*} \geq \tau^{(2)*} \geq \cdots \geq \tau^{(I)*}$. ∎

Proof of Proposition 2: We first show the existence of a set of prices $\{p^{(1)}, \ldots, p^{(I)}\}$ which satisfies individual rationality constraints (12) and incentive-compatibility constraints

$$\theta^{(i)}\delta(\tau^{(i)}) + p^{(i)} \leq \theta^{(i)}\delta(\tau^{(j)}) + p^{(j)}, \quad \forall i = 1, \ldots, I,\ j = 1, \ldots, I, \tag{A.13}$$

which are equivalent to the definition given in (2)-(3) such that a customer with class $\theta^{(i)}$ finds it optimal to choose the pair $(p^{(i)}, \tau^{(i)})$. Following Fudenberg and Tirole (1991, Section 7.3), a necessary and sufficient condition for an incentive-compatible pricing scheme to exist is that $\tau^{(i)}$ is decreasing in $\theta^{(i)}$. This condition holds because $\tau^{(1)} \geq \tau^{(2)} \geq \cdots \geq \tau^{(I)}$, where $\theta^{(1)} < \theta^{(2)} < \cdots < \theta^{(I)}$ by definition. Therefore, an incentive-compatible pricing scheme exists and this set of prices can be adjusted with a constant to ensure that the individual rationality constraints are also met.

We next construct this incentive-compatible and individually rational pricing scheme. For simplicity, let $\delta^{(i)} \doteq \delta(\tau^{(i)})$ for $i = 1, \ldots, I$. Because the disutility $\delta(\tau)$ increases in τ and $\tau^{(1)} \geq \tau^{(2)} \geq \cdots \geq \tau^{(I)}$, we have $\delta^{(1)} \geq \delta^{(2)} \geq \cdots \geq \delta^{(I)}$.

First, we set $p^{(I)} \leq u_0 - \theta^{(I)}\delta^{(I)} - \underline{u}$, so that a customer with $\theta^{(I)}$ receives at least the reservation utility if the customer chooses completion time $\tau^{(I)}$. Then, for $i = I-1, I-2, \ldots, 1$, we sequentially set $p^{(i)}$, such that the following $2(I-i)$ incentive compatibility constraints are satisfied:

$$\theta^{(j)}\delta^{(j)} + p^{(j)} \leq \theta^{(j)}\delta^{(i)} + p^{(i)}, \qquad \theta^{(i)}\delta^{(i)} + p^{(i)} \leq \theta^{(i)}\delta^{(j)} + p^{(j)}, \qquad \forall j = i+1, \ldots, I. \tag{A.14}$$

At the end of the above procedure, all of $2 + 4 + \cdots + 2(I-1) = I(I-1)$ incentive compatibility constraints for all I customer classes are satisfied.

The constraints in (A.14) can be equivalently written as

$$p^{(i)} \in \big[\max_{j>i}\{p^{(j)} - \theta^{(j)}(\delta^{(i)} - \delta^{(j)})\},\ \min_{j>i}\{p^{(j)} - \theta^{(i)}(\delta^{(i)} - \delta^{(j)})\}\big]. \tag{A.15}$$

The lower (upper) bound ensures that no customer strictly prefers to choose a lower (higher) class than his/her true class.

We next simplify the bounds for $p^{(i)}$ in (A.15) to obtain the bounds given in Proposition 2. The incentive compatibility for any two $\theta^{(j)}$ and $\theta^{(k)}$ customers requires $\theta^{(k)}\delta^{(k)} + p^{(k)} \leq \theta^{(k)}\delta^{(j)} + p^{(j)}$ and $\theta^{(j)}\delta^{(j)} + p^{(j)} \leq \theta^{(j)}\delta^{(k)} + p^{(k)}$, which are equivalent to

$$\theta^{(k)}(\delta^{(k)} - \delta^{(j)}) \leq p^{(j)} - p^{(k)} \leq \theta^{(j)}(\delta^{(k)} - \delta^{(j)}). \tag{A.16}$$

Consider the ordering of the terms, $\{p^{(j)} - \theta^{(j)}(\delta^{(i)} - \delta^{(j)}), j = i+1, \ldots, I\}$. Let $i < j < k$. Then,

$$
\begin{aligned}
p^{(j)} - p^{(k)} - \theta^{(j)}(\delta^{(i)} - \delta^{(j)}) + \theta^{(k)}(\delta^{(i)} - \delta^{(k)}) &\geq \theta^{(k)}(\delta^{(k)} - \delta^{(j)}) - \theta^{(j)}(\delta^{(i)} - \delta^{(j)}) + \theta^{(k)}(\delta^{(i)} - \delta^{(k)}) \\
&= (\theta^{(k)} - \theta^{(j)})(\delta^{(i)} - \delta^{(j)}) \geq 0,
\end{aligned}
$$

where the first inequality is due to the first inequality in (A.16), and the last inequality is due to $\theta^{(j)} \leq \theta^{(k)}$ and $\delta^{(i)} \geq \delta^{(j)}$. Therefore, $j = i + 1$ maximizes the lower bound in (A.15), i.e., $\max_{j>i}\{p^{(j)} - \theta^{(j)}(\delta^{(i)} - \delta^{(j)})\} = p^{(i+1)} - \theta^{(i+1)}(\delta^{(i)} - \delta^{(i+1)})$.

Consider the ordering of the terms $\{p^{(j)} - \theta^{(i)}(\delta^{(i)} - \delta^{(j)}), j = i+1, \ldots, I\}$. Let $i < j < k$. Then,

$$
\begin{aligned}
p^{(j)} - p^{(k)} - \theta^{(i)}(\delta^{(i)} - \delta^{(j)}) + \theta^{(i)}(\delta^{(i)} - \delta^{(k)}) &\leq \theta^{(j)}(\delta^{(k)} - \delta^{(j)}) - \theta^{(i)}(\delta^{(i)} - \delta^{(j)}) + \theta^{(i)}(\delta^{(i)} - \delta^{(k)}) \\
&= (\theta^{(j)} - \theta^{(i)})(\delta^{(k)} - \delta^{(j)}) \leq 0,
\end{aligned}
$$

where the first inequality is due to the second inequality in (A.16), and the last inequality follows from $\theta^{(i)} \leq \theta^{(j)}$ and $\delta^{(j)} \geq \delta^{(k)}$. Therefore, $j = i + 1$ minimizes the upper bound in (A.15), i.e., $\min_{j>i}\{p^{(j)} - \theta^{(i)}(\delta^{(i)} - \delta^{(j)})\} = p^{(i+1)} - \theta^{(i)}(\delta^{(i)} - \delta^{(i+1)})$.

Therefore, the feasible interval for p_i in (A.15) becomes:

$$
p^{(i)} \in \left[p^{(i+1)} - \theta^{(i+1)}(\delta^{(i)} - \delta^{(i+1)}),\ p^{(i+1)} - \theta^{(i)}(\delta^{(i)} - \delta^{(i+1)})\right], \tag{A.17}
$$

which is clearly nonempty, because $\delta^{(i)} \geq \delta^{(i+1)}$ and $\theta^{(i)} \leq \theta^{(i+1)}$.

To see that the pricing scheme is individually rational (i.e., reservation utility is met), note that

$$
(u_0 - \theta^{(i)}\delta^{(i)} - p^{(i)} - \underline{u}) - (u_0 - \theta^{(i+1)}\delta^{(i+1)} - p^{(i+1)} - \underline{u}) \geq \theta^{(i)}(\delta^{(i+1)} - \delta^{(i)}) - p^{(i)} + p^{(i+1)} \geq 0,
$$

where the first inequality is due to $\theta^{(i+1)} > \theta^{(i)}$ and the second inequality follows from (A.17). That is, the utility of a customer in class i is higher than that of a customer in class $i + 1$. Given that the pricing scheme is individually rational for a customer in class I by construction, then it is also individually rational for all classes. ∎

Proof of Proposition 3: Note that the given prices in the proposition correspond to the upper bounds of the prices in Proposition 2. Therefore, these prices continue to satisfy individual rationality constraints (12) and incentive compatibility constraints (A.13). Moreover, given that they are the upper bounds, they maximize the utility firm's profit. ∎

Proof of Proposition 4: The private utility firm's objective function in (13) is given as

$$
\mathbb{E}_{\{\theta_n, n=1,\ldots,N\}}\left[\sum_{n=1}^{N} p_n - C(\tau_1, \ldots, \tau_N)\right] = N\mathbb{E}\left[p_n\right] - \mathbb{E}\left[C(\tau_1, \ldots, \tau_N)\right]. \tag{A.18}
$$

Consider the expected revenue $N\mathbb{E}[p_n]$. Based on the iterative procedure given in (17) and letting $\delta^{(i)} \doteq \delta(\tau^{(i)})$ for $i = 1, \dots, I$ we can write

$$\begin{aligned} p^{(i)} &= u_0 - \theta^{(I)}\delta^{(I)} - \theta^{(I-1)}(\delta^{(I-1)} - \delta^{(I)}) - \cdots - \theta^{(i)}(\delta^{(i)} - \delta^{(i+1)}) - \underline{u} \\ &= u_0 - (\theta^{(I)} - \theta^{(I-1)})\delta^{(I)} - \cdots - (\theta^{(i+1)} - \theta^{(i)})\delta^{(i+1)} - \theta^{(i)}\delta^{(i)} - \underline{u} \\ &= u_0 - \theta^{(i)}\delta^{(i)} - \underline{u} - \sum_{j=i}^{I-1}(\theta^{(j+1)} - \theta^{(j)})\delta^{(j+1)}. \end{aligned}$$

Therefore, the expected revenue is

$$\begin{aligned} N\mathbb{E}[p_n] = N\sum_{i=1}^{I}\beta^{(i)}p^{(i)} &= N\sum_{i=1}^{I}\beta^{(i)}\left(u_0 - \theta^{(i)}\delta^{(i)} - \underline{u} - \sum_{j=i}^{I-1}(\theta^{(j+1)} - \theta^{(j)})\delta^{(j+1)}\right) \\ &= N(u_0 - \underline{u}) - N\sum_{i=1}^{I}\beta^{(i)}\theta^{(i)}\delta^{(i)} - N\sum_{i=1}^{I}\beta^{(i)}\sum_{j=i}^{I-1}(\theta^{(j+1)} - \theta^{(j)})\delta^{(j+1)} \\ &= N(u_0 - \underline{u}) - \mathbb{E}\left[\sum_{n=1}^{N}\theta_n\delta_n\right] - N\sum_{i=1}^{I}\beta^{(i)}\sum_{j=i}^{I-1}(\theta^{(j+1)} - \theta^{(j)})\delta^{(j+1)} \end{aligned}$$

By substituting the above expected revenue in (A.18), the private utility firm's objective in (13) can be rewritten as

$$\max_{\{\tau^{(i)}\in[\underline{w},T],\, i=1,\dots,I\}} N(u_0 - \underline{u}) - \mathbb{E}\left[\sum_{n=1}^{N}\theta_n\delta_n\right] - \mathbb{E}\left[C(\tau_1,\dots,\tau_I)\right] - N\sum_{i=1}^{I}\beta^{(i)}\sum_{j=i}^{I-1}(\theta^{(j+1)} - \theta^{(j)})\delta^{(j+1)},$$

This is equivalent to the private utility firm's objective in Proposition 4 because $N(u_0 - \underline{u})$ is constant in $\tau^{(i)}$'s. Finally, note that for $\beta^{(i)} = \frac{1}{I}$, the information rent term $N\sum_{i=1}^{I}\beta^{(i)}\sum_{j=i}^{I-1}(\theta^{(j+1)} - \theta^{(j)})\delta^{(j+1)} = \frac{N}{I}\sum_{i=1}^{I}\sum_{j=i}^{I-1}(\theta^{(j+1)} - \theta^{(j)})\delta^{(j+1)} = \sum_{i=2}^{I}(i-1)\left(\theta^{(i)} - \theta^{(i-1)}\right)\delta^{(i)}$. ∎

Proof of Proposition 5: Threshold $z_{n,j}$'s can be computed via the following problem:

$$\min_{\{z_{n,j},\, n=1,\dots,N,\, j=1,\dots,2N\}} \int_0^T c\big(q(t)|d(t)\big)\mathrm{d}t \tag{A.19}$$

$$\text{s.t. (20), (21), (23),} \qquad z_{n,j} = 0 \quad \forall j = 1,\dots,2N, \forall n \notin \mathcal{I}(t_j) \text{ and} \tag{A.20}$$

$$a_n(t) = \min\left\{\left(z_{n,j} - d(t) - \sum_{k<n}a_k(t)\right)^+, \overline{a}\right\} \; \forall t \in [t_j, t_j+1], \forall j = 1,\dots,2N, \forall n \in \mathcal{I}(t_j), \tag{A.21}$$

where the decision variables of the original objective function (19), i.e., $\{a_n(t),\, n = 1,\dots,N\}$, are replaced with the threshold $z_{n,j}$'s in (A.19). Given that $a_n(t)$'s are functions but $z_{n,j}$'s are scalars, solving the above problem is simpler than solving the original problem. In this formulation, (A.21) implicitly define the charging schedule as a function of the thresholds.

Similar to the simultaneous arrivals case, the charging problem (19)-(23) can be expressed in the standard form (A.1)-(A.3), with the additional constraint that vehicle n cannot be charged if it is not at the station at time t, i.e., $a_n(t) = 0, \forall t \notin [s_n, \tau_n]$.

We prove the optimality of the charging policy for non-simultaneous arrivals (given $z_{n,j}$'s) by induction. First consider the time interval $[t_{2N-1}, t_{2N}]$ in which there is only one electric vehicle at the station. Let its index be k. Consistent with the standard form (A.1)-(A.3), denote the level of electricity charged by time t_{2N-1} to vehicle k by $x_k(t_{2N-1})$. Given that vehicle k is the only vehicle in the station in this time interval, we can use Pontryagin's minimum principle to identify the optimal charging schedule as in the proof of Proposition 1: There exists a threshold $z_{k,2N-1}$, where $\int_{t_{2N-1}}^{t_{2N}} \min\left\{(z_{k,2N-1} - d(t))^+, \overline{a}\right\} \mathrm{d}t = 1 - x_k(t_{2N-1})$ such that it is optimal to set $a_k^*(t) = \min\left\{\left(z_{k,2N-1} - d(t)\right)^+, \overline{a}\right\}$ for $t \in [t_{2N-1}, t_{2N}]$. It is straightforward to verify that the charging policy for non-simultaneous arrivals essentially leads to the same charging decision. Therefore, the policy is optimal for $t \in [t_{2N-1}, t_{2N}]$.

Suppose the policy is optimal for intervals $[t_{j+1}, t_{j+2}], \ldots, [t_{2N-2}, t_{2N-1}]$ for some $j \in \{1, \ldots, 2N-1\}$. We next prove the policy is also optimal in $[t_j, t_{j+1}]$. For this time interval, let $a_n^*(t)$ be an optimal solution for $n \in \mathcal{I}(t_j)$. The charging policy for non-simultaneous arrivals (given $z_{n,j}$'s) produces a charging schedule for vehicles $k \in \mathcal{I}(t_j)$ such that each vehicle is charged $\int_{t_j}^{t_{j+1}} a_k^*(t)\mathrm{d}t$ units of electricity by time t_{j+1}. As discussed in the proof of Propositon 1, this schedule is also optimal for $t \in [t_j, t_{j+1}]$ because it is equivalent to charging one electric vehicle with $\sum\limits_{k \in \mathcal{I}(t)} \int_{t_j}^{t_{j+1}} a_k^*(t)\mathrm{d}t$ units of electricity by time t_{j+1}. Given that the policy for non-simultaneous arrivals is also optimal for $[t_j, t_{j+1}]$, by induction, it is optimal for all intervals.

Finally, in the above analysis, without loss of generality, we assume that when customer n arrives at the charging station there is at least one other customer whose vehicle is being charged. Otherwise, the utility firm's problem can be decoupled into multiple problems, which can be solved separately. ∎