AN EXPLORATION INTO TEACHERS' PEDAGOGICAL CONTENT

KNOWLEDGE (PCK) FOR TEACHING QUADRATIC FUNCTION IN GRADE 10

by

BALQIS OLAWUMI BANJO

(STUDENT NUMBER: 53999991)

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DECLARATION

I declare that **AN EXPLORATION INTO TEACHERS' PEDAGOGICAL CONTENT KNOWLEDGE (PCK) FOR TEACHING QUADRATIC FUNCTION IN GRADE 10** is my own work and that all the sources that I have used or quoted have been indicated and acknowledged by means of complete references.

I further declare that I have not previously submitted this work, or part of it, for examination at UNISA for another qualification or at any other higher education institution.

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       Banjo B. O.   Date

ACKNOWLEDGEMENT

LIST OF ABBREVIATIONS

PCK	Pedagogical Content Knowledge
NCS	National Curriculum Statement
CAPS	Curriculum and Assessment Policy Statements
DBE	Department of Basic Education
Hon.	Honours degree
KCT	Knowledge of Content and Teaching
KMLS	Knowledge of Mathematics Learning Standards
KC	Knowledge of Curriculum
CK	Content Knowledge
AMESA	Association for Mathematics Education of South Africa
PPK	Pedagogical-Psychological Knowledge
MPRA	Model of Pedagogical Reasoning and Action
GET	General Education and Training
TEDS-M	Teacher Education and Development Study in Mathematics
IEA	International Association for the Evaluation of Educational Achievement
FET	Further Education and Training
ANA	Annual National Assessment
SACE	South African Council for Educators
IQMS	Integrated Quality Management System
NOS	Nature of Science
SPTD	Senior Primary Teacher's Diploma
ACE	Advanced Certificate in Education

BA	Bachelor of Art
HOD	Head of Department
ATP	Annual Teaching Plan
MTSK	Mathematics Teacher's Specialized Knowledge
KCS	Knowledge of Content and Learners
SMK	Subject Matter Knowledge
SCK	Specialised Content Knowledge
CCK	Common Content Knowledge
MKT	Mathematics Knowledge For Teaching
UNISA	University of South Africa
BEd.	Bachelor of Education Degree

ABSTRACT

The study's purpose was to explore the components of pedagogical content knowledge (PCK) for teaching quadratic function in Grade 10 mathematics classrooms in Mogalakwena district, Limpopo. The study was mainly guided by Shulman's PCK and it was categorised as content knowledge, knowledge of the curriculum, knowledge of learners and knowledge of teaching strategies. The researcher employed a qualitative case study research design to explore the teaching of three purposefully selected Grade 10 mathematics teachers regarding their knowledge of quadratic function, strategies employed to teach quadratic function and knowledge of Grade 10 mathematics learners in quadratic function. In this study the researcher argued that PCK strands are interwoven, and in order to teach mathematical concepts, such as quadratic function, effectively teachers should employ components of PCK to complement one another. Data were gathered by lesson observation and interviews. Findings revealed that concepts of quadratic function are inefficiently addressed in Grade 10 due to teachers' lack or inadequacy in some aspects of PCK. Therefore, there is need to develop mathematics teachers' PCK in the Mogalakwena district to enhance their teaching of Grade 10 quadratic function.

KEY TERMS: Teaching of quadratic function; Pedagogical content knowledge; Mathematics teachers' content knowledge; Quadratic function; School mathematics curriculum; Instructional strategies; Connecting mathematics; Learners' common error; Learners' prior knowledge; Mathematics teacher.

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CHAPTER 1: INTRODUCTION TO THE STUDY

1.1 BACKGROUND TO THE STUDY

Teachers' pedagogical content knowledge (PCK) is no longer a new concept in the teaching field, but the researcher posited the need to break it down into its basic-essential-elements that can be assessed in the teaching of quadratic functions in Grade 10. The concept of PCK is clearly related to knowledge about how specific content can be interpreted in teaching situations (Cooney 1994). The PCK in the current study focused on the knowledge of content and curriculum that the teacher uses in the teaching of the quadratic function; the teacher's knowledge of a learner's thinking and how the former applies it to the teaching of the quadratic function; and the teacher's knowledge in choosing appropriate instructional strategies for teaching the quadratic function. This study is limited to the Mathematics National Curriculum Statement (NCS) of South Africa: The Curriculum and Assessment Policy Statements (CAPS) Grades R-12. Further, the researcher only explored the teaching of a unit concept in mathematics, the quadratic function. Mathematics teachers are faced with many challenges, including the use of appropriate teaching strategies for a specific lesson or class as a teacher needs to consider other factors that influence learning. It is important that teachers teach mathematics effectively to ensure that learners grasp the concept and the relevant curriculum outcomes. This is only possible if the teacher possesses the necessary knowledge required for teaching.

In South Africa, mathematics continues to demonstrate the lowest performance for the past five years among the ten key subjects: Accounting, Agricultural Sciences, Business Studies, Economics, Geography, Life Sciences, Mathematical Literacy, Mathematics and Physical Sciences (see Figure 1.1).

Table 2 below shows the performance of full time candidates at 30% and above in the 11 key subjects from 2014 and 2018.

Table 2: Candidates' performance in selected subjects, 2014 – 2018 at 30% level

Subjects	2014	2015	2016	2017	2018
Accounting	68.0	59.6	69.5	66.1	72.5
Agricultural Science	82.6	76.9	75.4	70.4	69.9
Business Studies	77.9	75.7	73.7	68.0	64.9
Economics	68.9	68.2	65.3	71.0	73.3
Geography	81.3	77.0	76.5	76.9	74.2
History	86.3	84.0	84.0	86.0	89.7
Life Sciences	73.8	70.4	70.5	74.4	76.3
Mathematical Literacy	84.1	71.4	71.3	73.9	72.5
Mathematics	53.5	49.1	51.1	51.9	58.0
Physical Science	61.5	58.6	62.0	65.1	74.2

Figure 1. 1: Performance of candidates in 11 subjects from 2014 to 2018: Extracted from NSC 2018 School Performance Report

Quadratic function is one of the topics in Grade 10 Mathematics CAPS. This topic is given maximum attention in terms of allocation of time and assessment weight. It forms the heart of mathematics because of its relevance to other main topics in mathematics such as algebra, patterns and sequences, trigonometry, and analytical geometry (DBE, 2012). A key focus of this study was exploration of how teachers make connections among the topics of mathematics, during the teaching of quadratic function in Grade 10. According to the 2017 NSC diagnostic report, the teacher should use questions on quadratic functions that involve incorporation of different topics during teaching and in class-based assessment. The issue of connecting mathematics is also evidenced in 2016 NSC November examination where Grade 12 learners failed to connect the turning point of a parabola with its range. The diagnostic report states that some candidates did not recognise that the turning point, B(2;1), had to be used to write the range. According to the NCS Diagnostic Reports (Department of Basic Education [DBE], 2015:160):

> Teachers should teach inequalities intensively and properly. They should integrate the algebra with functions, so that learners have a visual understanding of inequalities and stress the meaning of the inequality signs in the teaching of both algebra and functions...In teaching any function; teachers should expose learners to all aspects of the function. This includes sketching, interpretation of the equation and the graph, as well as finding the

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equation from given information and transformations. The teaching should also include concepts such as roots, points of intersection, intervals where graphs are relative to one another under a given condition, gradients and equations of tangents.

In Grade 10 quadratic functions, "learners should be able to recognise relationships between variables in terms of numerical, graphical, verbal and symbolic representations, and convert flexibly between these representations" (DBE, 2012: 12). Grade 10 also forms the basis of proper 'naming' of the quadratic function (otherwise known as parabolic function) and the foundation through which it advances. Teachers should endeavour to spend adequate time on teaching the parabola and other basic concepts of quadratic function to ensure that learners gain understanding of the concept before they get to Grade 12. This may improve learners' performance in functions and mathematics at large.

1.2 PROBLEM STATEMENT

The teaching of mathematics continues to be a great challenge in South Africa. Teachers do not cover curriculum content sufficiently and serious problems in the South African education system are linked to teacher knowledge, among others (Motshekga, 2012). The continuing poor learner performance in mathematics in South African public schools focuses attention on how teachers teach mathematical concepts in secondary schools. How do teachers teach mathematics in South African schools? How should they teach it?

In order to develop learners' problem-solving and cognitive skills, teaching of the quadratic function should not be limited to "**how**" but should rather feature the "**when**" and "**why**" of problem types; learning procedures and proofs without a good understanding of why they are important will leave learners ill-equipped to use their knowledge in later life (DBE, 2012).

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NSC

QUESTION 4

Given: $f(x) = -ax^2 + bx + 6$

4.1 The gradient of the tangent to the graph of f at the point $\left(-1; \frac{7}{2}\right)$ is 3.

Show that $a = \frac{1}{2}$ and $b = 2$. (5)

4.2 Calculate the x-intercepts of f. (3)

4.3 Calculate the coordinates of the turning point of f. (3)

4.4 Sketch the graph of f. Clearly indicate ALL intercepts with the axes and the turning point. (4)

4.5 Use the graph to determine the values of x for which $f(x) > 6$. (3)

4.6 Sketch the graph of $g(x) = -x - 1$ on the same set of axes as f. Clearly indicate ALL intercepts with the axes. (2)

4.7 Write down the values of x for which $f(x).g(x) \le 0$. (3)
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Figure 1. 2: Candidates performance in quadratic function: Extract from NSC 2017 Diagnostic Report

Figure 1.2 presents Question four of the Grade 12, NSC November examination paper of 2017, which deals with parabolic and linear functions, together with the analysis of candidates' performance. According to the report, questions 4.5 and 4.7, which were based on quadratic equations, are higher order questions and required learners to read off intervals from graphs. The learners performed poorly in the questions 4.5 and 4.7. However, learners' answers indicated that learners had used algebraic manipulation which resulted in making the problems more complicated and time consuming. Based on the researcher's experience and the literature, some Grade 12 learners did not possess necessary algebraic skills, which were fundamental to solving quadratic function problems. Further, some could not differentiate between various types of function based on the equations or graphs. Learners should have developed conceptual understanding of these concepts from Grade 10 quadratic function. Hence, the current researcher found it important to explore how mathematics' teachers teach quadratic function in Grade 10.

1.3 AIM OF THE STUDY

The aim of this study was to explore the concept of PCK with regard to the teaching of quadratic function in Grade 10 classrooms in the Mogalakwena district, Limpopo. The aim was to understand how mathematics teachers teach quadratic function in Grade 10 from participants' perspectives. The main objectives of this research were:

- To explore teachers' content knowledge required in the teaching of quadratic function in Grade 10;

- To explore strategies used for teaching quadratic function in Grade 10;

- To explore how teachers access learners' prior knowledge in the teaching of quadratic function; and

- To explore how teachers draw from their knowledge of the curriculum when teaching quadratic function in Grade 10.

1.4 RESEARCH QUESTIONS

The study examined the elements of PCK employed in the teaching of quadratic function. The main research question of the study is: How do mathematics teachers teach quadratic function in Grade 10? To answer this question, the following sub-questions were pursued:

1. What content knowledge do teachers possess in teaching quadratic function in Grade 10?

2. What are the teaching strategies that teachers use in the teaching of quadratic function in Grade 10?

3. How do teachers build learners' knowledge of quadratic function?

4. How do teachers draw from their knowledge of the curriculum when teaching quadratic function in Grade 10?

1.5 RATIONALE AND SIGNIFICANCE OF THE STUDY

The researcher sought to explore the concept of PCK for the teaching of quadratic function. The researcher studied the components of PCK of Grade 10 mathematics teachers and explored the interactions among these components, with regard to the teaching of quadratic function in Grade 10. The researcher believed that the information may assist to improve teachers' knowledge, and hence improve learners' performance in mathematics. In addition, this study:

- may provide guidelines for mathematics teacher development programmes;

- will increase the available literature in the area of PCK for teaching quadratic function;

- may serve as an eye-opener to mathematics teachers on the importance of PCK to the teaching of mathematics;

- may serve as a source of input to teachers in designing their teaching activities;

- may proffer solutions to challenges of teaching quadratic function.

1.6 HOW MY INTEREST IN THIS RESEARCH TOPIC DEVELOPED

The researcher's learning experiences, especially during the study of Honours Bachelor of Education Degree (Hon. B Ed) was the major source of interest which led to the current study. During that time, several activities required the learners (the researcher included) to apply/analyse educational theories or ideas to practical situations. Among others, learners were exposed to concepts such as teacher empowerment through curriculum studies, in which they learned about various implementation strategies; various models of teaching mathematics; and, most interestingly, connected mathematics. The researcher developed an interest in making connections within and across mathematics' topics, within and across the grades, as well as outside mathematics. The researcher hence adopts a belief that mathematics is connected and the teacher should facilitate learners to be aware of the connections. The researcher believes that ability to connect mathematics forms the basis of understanding of such mathematics. Also, the researcher discovered, through observable experiences that she and many other teachers do not understand what PCK entails. Through

study she understood that the term, PCK, encompasses more than just teaching method, as often used by many teachers.

Of specific importance was when the researcher had to analyse in order to defend whether the CAPS is informed by the constructivist approach. The researcher chose quadratic function as a topic of choice and then analysed connections and progression of the concepts, within quadratic function, across quadratic function and across the grades. The researcher also highlighted the areas where CAPS acknowledges the importance of exposing learners to real-life situations in order to develop mathematical and creative skills. The researcher thereby believes that the teaching of quadratic function rests on teacher's PCK, and that inadequacy in teacher's PCK about this topic may jeopardize the aims and objectives of mathematics education in South Africa at large.

1.7 DEFINITION OF KEY TERMS

In this section the key terms that are used during the study are clearly defined.

- PCK: the knowledge required of every teacher to teach effectively. It simply implies the knowledge of content and pedagogy blended together. PCK is the transformation of content (e.g., quadratic function) into pedagogically multiple representations (Fernandez, 2014). Hill, Ball and Schilling (2008) find that PCK refers to the topic-specific knowledge base; as such they review its concepts with respect to a specific topic of study. According to Cochran (1997), "PCK is a type of knowledge that is unique to teachers and is based on the way teachers relate their pedagogical knowledge (what they know about teaching) to their subject matter knowledge (what they know about what they teach)" (para. 4).

- Practical knowledge: an epistemology of teaching practice. This is the intellectual activity of a mathematics teacher that contributes to the construction of knowledge about teaching quadratic function. The teacher gains practical knowledge through the teaching of quadratic function.

- Teaching: the act of facilitating the learners' learning of quadratic function. It is an activity carried out by the mathematics teacher with the aim a of transferring mathematical knowledge and skills to his/her learners. Teaching is the process of

attending to people's needs, experiences and feelings and making specific interventions to help them learn particular things (William, 2017).

- Teacher: the secondary school mathematics teacher in the Mogalakwena district of Limpopo, South Africa.

- Learner: refers to a student in secondary school. In South African context, 'learner' should be used instead of 'student' as defined in the South African Schools Act.

- Teacher's knowledge: all the knowledge and beliefs of the mathematics teacher that influences his/her teaching of quadratic function in Grade 10.

- Functions: a mathematics topic in the mathematics curriculum of South Africa. Siyavula Mathematics Learner's Book 10 (2012) describes "functions as mathematical building blocks for designing machines, predicting natural disasters, curing diseases, understanding the world economies and or keeping airplanes in the air".

- Quadratic function: a sub-topic in Functions. It is also known as *parabolic functions*. It is one of the forms $y = ax^2 + q$ where a and q are constants.

- Grade 10: a phase/stage/class in secondary school. It is formally known as 'Form 3' in the South African education context.

1.8 ORGANISATION OF THE DISSERTATION

It is recommended that a master's dissertation should comprise five chapters [University of South Africa (UNISA), 2017]. This study was organised as follows:

- Chapter one: The Introduction

In this section the researcher discussed the background to the study; the statement of the problem; the purpose of the study; significance of study; research questions; limitation and delimitation of the study; and the definition of key terms.

- Chapter two: Literature review

This section seeks to give up-to-date information on what knowledge has already been established on the topic of research. The researcher gives a critical summary of related

resources. The researcher discusses in detail, the concept of PCK, teaching strategies and quadratics functions.

- Chapter three: Methodology

In this section the researcher discusses the research methodology; research design; population and sample; instrumentation and method of data collection; validity and reliability of the instruments; and data analysis.

- Chapter four: Presentation and analysis of data

In this chapter, the researcher discusses the findings and analysis of responses.

- Chapter five: Conclusions and recommendations

This section deals with the summary, the conclusion and recommendations of the study. It also highlights suggestions for further research.

1.9 CONCLUSION

A better South Africa depends greatly on the quality of education provided to its citizens. Quality education implies quality teaching and learning and it is a function of the teacher's knowledge. As a mathematics' teacher, the researcher is charged with responsibility of contributing to finding lasting solutions to address the massive failure in mathematics in South Africa, most especially in her own community. She takes her inspiration from the words of Andre du Plessis: "Can I improve my mathematics teaching? Teacher Action Research holds the key to how!

CHAPTER 2: LITERATURE REVIEW

2.1 INTRODUCTION

In this chapter, the researcher first discussed the theoretical framework adopted as a guide to the exploration of PCK for teaching the quadratic function in Grade 10. PCK simply refers to how teachers teach a specific topic for learners' understanding. It is concerned with what teachers do with curriculum materials, learners and instruction (Ball et al., 2005; par.1.7). The theoretical framework provides the structure to define philosophical, epistemological, methodological, and analytical approaches to a research study (Grant & Osanloo, 2014). The theoretical framework is the driver of research work. The main theoretical framework that underlies this study is PCK. Based on the experience and the study, the researcher opined that effective teaching of mathematics is influenced by several factors which fundamentally include the teacher's knowledge of the subject matter, knowledge of the school mathematics curriculum and its instructional materials, ability to understand learners' thinking, as well as good communication skills. All these attributes are embedded in the construct of PCK introduced by Shulman (1986; 1987). Secondly, in this chapter, the researcher reviewed related literature to explore the discussions on what constitutes the knowledge that mathematics teachers need to possess to teach quadratic functions in Grade 10. The discussions are based on the themes such as the background of study, components of PCK with respect to some selected scholars, the teacher's knowledge of quadratic functions, as well as PCK for teaching of quadratic functions in Grade 10. The researcher chose to focus on how each of the sources communicates and/ or contributes to the understanding of the concept of PCK, rather than its theoretical positions or the historical development surrounding it.

2.2 THE PEDAGOGICAL CONTENT KNOWLEDGE (PCK)

Different terms have been used to describe the necessary knowledge that the teacher needs to know to teach mathematics. Gaps in teachers' knowledge, among others, have been shown to impact negatively on mathematics curriculum implementation in South Africa (Motshekga, 2012). Motshekga (2012) remarked that her department (DBE) would intensify teacher development efforts on teacher PCK to improve the quality of teaching and learning of mathematics in South Africa.

Studies conducted by Carrillo, Climent, Conteras and Munoz-Catalan (2013); Charalambous (2016); and Ingvarson, Schwille, Tatto, Rowley, Peck, and Senk (2013) explored the knowledge required to teach mathematics since mathematics teachers need special knowledge that is essential to the discipline.

Fernandez (2014) defines teacher's knowledge as "all a teacher's knowledge and beliefs that influence his teaching practices. That practical knowledge is the result of what teachers know from their professional expertise". He identifies practical knowledge as knowledge that teachers gain through teaching. Thus, this notion undergirds the epistemological belief of the present researcher that mathematics teachers develop their PCK through active teaching and the mastery of their own teaching. Shulman's (1987) Model of Pedagogical Reasoning and Action (MPRA) describes the significant process of developing mathematics teachers' PCK with emphasis on the importance of reflection on the teaching process. The current researcher concurs with Akerson, Pongsanon, Rogers, Carter and Galindo (2017) who posit that reflective practice plays an important role in the development of the mathematics teacher's PCK. Carter (as cited in Fernandez, 2014) opines that practical knowledge and PCK are closely connected, yet the two are distinct. PCK forms part of professional knowledge but is broader and more formal than what characterises practical knowledge; practical knowledge is conceived to be more personal and situational. In this study, practical knowledge is understood to be a product of mathematics teachers' PCK, which together impact on teachers' professional teaching of quadratic function especially in Grade 10.

Mathematics teachers of all grades acquire pedagogical skills through years of experience: the more time the teacher spends teaching mathematics and reflecting on his/her teaching practices, the stronger the impact on his/her pedagogical knowledge (Ingvarson et al., 2013). The current study focuses on Grade 10 mathematics teachers to fill a gap in the body of

related knowledge since most extant studies on PCK have been done on teachers in the early grades (Bansilal, Brijlall & Mkhanazi, 2014; Hauk, Toney, Jackson, Nair & Tsay, 2014). The issue of PCK is crucial to mathematics teacher professional development, its management and assessment. Failing to develop or/ and assess mathematics teachers' PCK may result in poor teaching of mathematics and poor learner performance in mathematics as we currently experience in Limpopo province. The researcher believed that gathering information about PCK and the pedagogical demands of the CAPS mathematics curriculum is a necessity for quality teaching of mathematics (particularly of quadratic functions) in South Africa.

2.3 THE THEORETICAL FRAMEWORK UNDERLYING THIS INQUIRY: SHULMAN'S PCK

Shulman (1983) originally presented the construct of PCK to signify a special kind of teacher knowledge that differentiates a teacher in any given discipline from an expert of that discipline. Shulman introduced PCK at a conference at the University of Texas in 1983 in a paper titled: 'The missing paradigm in research about teaching'. His research was first published in 1987 in which he defined PCK as the intersection of content and pedagogy that belongs only to teachers (Fernandez, 2014). Shulman (1987) outlined the seven categories of the knowledge base for teaching: Content knowledge; General pedagogical knowledge; Curriculum knowledge; PCK; Knowledge of learners and their characteristics; Knowledge of educational contexts; and the Knowledge of history and philosophy of education. He (1986: 9) further placed special stress on PCK, thus defined as:

> … Knowledge, which goes beyond knowledge of subject matter per se to the dimension of subject matter knowledge for teaching. I still speak of content knowledge here, but of the form of content knowledge that embodies the aspects of content most germane to its teachability. Within the category of pedagogical content knowledge, I include, for the most regularly taught topics in one's subject area, the most useful forms of representation of those ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations—in a word, the ways of representing and formulating the subject that make it comprehensible to others. since there are no single most powerful forms of representation, the teacher must have at hand a veritable armamentarium of alternative

forms of representation, some of which derive from research whereas others originate in
the wisdom of practice.

PCK is specialised knowledge possessed by experienced teachers and used to make understanding of a topic accessible to learners. Shulman (1986) describes it as the ways of representing and formulating the subject that makes it comprehensible to others, including an understanding of what makes a specific mathematics topic easy or difficult to learn. The PCK framework has been widely proven as a model for investigating teachers' knowledge for teaching mathematics. The epistemological study of elements of PCK that a mathematics teacher should possess to teach Grade 10 quadratic functions is based on Shulman's construct of PCK, that is, the blending together of content and pedagogy into understanding of how the concepts of quadratic functions are organised, represented, and adapted to the diverse interests and abilities of Grade 10 learners, and presented for instruction. Thus, Shulman (1986) zoomed into two aspects of teacher's capacities: content knowledge and pedagogical skills. However, these two are differentiated.

2.3.1 Key components of PCK

Deconstructing PCK into possible domains is significant to the current study to help the researcher understand participants' opinions on each aspect in the discourse. Shulman (1987) "categorizes teacher's knowledge into seven domains: general pedagogical knowledge; knowledge of learners and their characteristics; knowledge of educational context; knowledge of educational ends, purposes, and values, and their philosophical and historical grounds; content knowledge; PCK; and curriculum knowledge". Some of these domains such as knowledge of learners and their characteristics, content knowledge, and curriculum knowledge, have been used to analyse data in the present study. By knowledge of learners and their characteristic, for example, the researcher looking into: how the teacher identified and used learners' prior knowledge during the teaching of quadratic function; how the teacher addressed learners' common errors and misconceptions in quadratic function; and how the teacher caters for learners' different needs while teaching quadratic function.

Shulman lays emphasis on PCK domain, which he describes as the blending of content and pedagogy into quality teaching instruction. Shulman identifies two sub-elements of PCK: knowledge of representations of specific content and instructional strategies; and understanding of learning difficulties and learners' conceptions of specific content

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(Fernandez, 2014). Conversely, Charalambous (2016) identifies the components of PCK as the tasks in which teachers engage in teaching mathematics for learner understanding. Charalambous (2016) focuses more on teaching practices while identifying five categories of knowledge that are significant to teaching of mathematics: subject matter knowledge (SMK); providing and evaluating explanations; selecting and using representation; analysing learner errors, misconceptions and non-conventional solutions; and selecting tasks. However, the current study explores discussions in the literature about the content and curriculum, teaching strategies and instructional materials, and learners' characteristics, as the researcher believed they form the key domains. Thus, the current study focuses on four categories of PCK, which are commonly discussed in the literature. These are: the content knowledge; the knowledge of curriculum; knowledge of learners; and knowledge of instructional strategies.

2.3.1.1 The content knowledge (CK)

Schulman's work (1986; 1987) elaborated subject matter knowledge (also known as content knowledge or CK) as the fundamental knowledge base a teacher must possess. He identified CK as one of the knowledge bases which is a prerequisite to PCK and this influences the development of PCK. This view is in line with some other researchers (e.g., Pournara, 2014: Ball et al., 2008) who identified CK as mathematical knowledge and hence, different from PCK. It is understood from the literature that CK can only be considered as an element of PCK, if it is discussed in relation to teaching. Therefore, the CK in the current study was conceived as an element of PCK, as the researcher explored the CK for teaching quadratic function in Grade 10. One of the research questions of the current study (cf. 1.4) is: What content knowledge do teachers possess in teaching quadratic function in Grade 10?

Krauss, Baumert and Blum (2008 as cited in Pourana, 2014) describe CK as a kind of mathematical knowledge that is more advanced than the mathematics that the curriculum requires the teacher to teach. They add that CK allows the teacher to cope with mathematical challenging situations. With regard to the level of CK required in Grade 10 quadratic functions, the current study adopted Schulman's (1986) suggestion that the CK of a teacher should be at least equal to that of his/her colleagues. According to Shulman, content knowledge includes: knowledge of concepts, theories, ideas, knowledge of proofs and evidences as well as practices and approaches to develop this knowledge (Fernandez, 2014).

This study reflects on teachers' in-depth knowledge about the concepts of quadratic function taught in Grade 10 that may positively influence their instruction and the interaction between CK and teaching (and other forms of pedagogical knowledge and skills).

2.3.1.2 The knowledge of the curriculum

Teacher's PCK also plays a significant role in the selection of curricular materials. For example, it enables the teacher to identify textbooks that the teacher finds problematic or flawed in their conception of the topic, or incomplete in their treatment or inadequate in explanation or use of examples. Knowledge of the curriculum demands that the teacher understands the structure of the subject matter. Curricular approaches in South Africa have changed from transmission-based teaching to a learner-centred approach. The curriculum requires mathematics teachers to give room for learner-initiated questions and independent thoughts and to ensure interactions among learners in the mathematics classroom environment. Sadly, the converse is the case in most South African classroom practices (Paulsen, 2009). The lack or inadequate professional knowledge at the grassroots level contributes to the poor implementation of the curriculum.

PCK is crucial for teaching of mathematics in general and quadratic function in particular. Teaching, generally, comprises intellectual activities which involve the ability to process knowledge and understanding and the ability to communicate it effectively. In other words, not only must a teacher have high competency of the subject matter but he/she must also be able to transform it into accessible forms for learner understanding. A good teacher must possess good communication skills among others.

> Teachers must not only be capable of defining for learners the accepted truths in a domain.
> They must also be able to explain why a proposition is deemed warranted, why it is worth
> knowing, and how it relates to other propositions, both within the discipline and without,
> both in theory and in practice. (Shulman, 1986, p. 9)

Planning of lesson for quadratic function is a crucial activity that sets several knowledge domains of the teacher into play. During the planning, the teacher uses knowledge of curriculum to identify the main contents of quadratic function that learners supposed to learn in grade 10. Knowledge of curriculum also affords the teacher the opportunity to prepare for teaching strategies that is suitable for the learners' characteristics.

2.3.1.3 Knowledge of instructional strategies

The concept of appropriate instructional strategies for teaching quadratic function in grade 10 is central to this study. Shulman (1986) suggests that knowledge of theories and methods of teaching is important. An effective teaching approach is closely associated with a high quality of PCK. Yusof, Zakaria and Abdullah (n.d.) stated that PCK enables a teacher to foresee challenges that learners may experience during the teaching and therefore get equipped with appropriate methods during planning of the lessons. Also, during the presentation (the teaching) the teacher combines several knowledge domains, such as content knowledge and knowledge of instructional strategies. At this stage the teacher relates the new knowledge to what learner already know. According to Brijlall (2011), knowledge of appropriate teaching approaches such as using an improved questioning and quality explanation is primary aspect of PCK. Studies, such as Kazima et.al's (2008), categorise teacher's knowledge of instructional strategies as a core pedagogical knowledge. The researcher endorsed activity-based learning in which the teachers dominantly work with learners' ideas. The researcher opined that explanation should come after learners have explored and generated ideas. Kazima also emphasized on teacher's ability to pose higher ordered questions that are capable of developing learners' mathematical thinking and exposing their misconceptions. Researchers (such as Yusof, Zakaria & Abdullah, (n.d.); Even, 1990) identified enquiry approach, e.g. 'investigation', as a powerful instructional strategy for teaching quadratic function. The approach is an activity-based in which learners are required to check specific cases and the teacher assess learners' understanding on the spot.

In this study, the researcher explored qualitative data to provide answer to the question: What are the teaching strategies that teachers use in the teaching of quadratic function in Grade 10? The researcher gained more access to the teachers' knowledge of instructional strategies through lesson observation. This study adopted the most commonly shared characteristics of instructional strategies explored through the literature. Thus using variety of powerful representations; facilitating learners to investigate theories, explain their findings and make generalization; using an inclusive approach that ensures active participation of all the learners; and allowing the learners to work in group.

2.3.1.4 Knowledge of the learners

Bansilal (2012) stresses the importance of understanding mathematics learners. She highlights that learners conceive mathematical concepts in two ways: operationally – when learners see concepts as a process; and/ or structurally – when learners see mathematics concepts as an object. These can be related to action level and object levels discussed in another study (Bansilal et al., 2014). The two ways are crucial for deep understanding of mathematics. The above reveals the symbiotic relationship between the components of PCK: the content knowledge and the knowledge of the learners. Mathematics teachers need to be more sensitive to learners' prior knowledge and individual differences. The teacher must provide opportunities that allow learners to play an active role in assimilating new knowledge into their existing knowledge. This aspect is related to the components of PCK: the importance of learners' prior knowledge to quadratic functions and the selection of appropriate teaching strategies that afford learners active participation.

Olivier (as cited in Machaba, 2016:2) "defines misconceptions as errors or wrong answers that are systematic and are applied regularly in the same circumstances." Several scholars (e.g., Machaba, 2016; Sapire, Shalem, Wilson-Thompson & Paulsen, 2016) show the importance of error analysis, as it forms an integral aspect of teacher knowledge. Learners often hold and express incomplete mathematical knowledge. This calls upon the teacher's pedagogic skills to scrutinise, interpret, correct and extend this knowledge (Ball et al., 2005). Addressing common errors and misconceptions in quadratic function is one of the domains the present study explored. It represents an aspect of teacher's knowledge of the learners.

2.3.2   Swan's Principles of Teaching Mathematics

This study harmonizes Swan's principles of teaching mathematics with PCK framework. Swan's summaries of competent teachings of mathematics are similar to Shulman's description of PCK and are specific about teaching of mathematics. Swan (2006) highlights competent teachings as those that are capable of the following principles:

Table 2. 1: Malcolm Swan's (2006) Eight Principles of Teaching Mathematics

Teaching should build on	This means developing formative assessment techniques

the knowledge learners already have;	and adapting our teaching to accommodate individual learning needs (Black &Wiliam, 1998).
It should expose and discuss common misconceptions	Learning activities should expose current thinking, create 'tensions' by confronting learners with inconsistencies, and allow opportunities for resolution through discussion (Askew &Wiliam, 1995).
It must use higher-order questions	Questioning is more effective when it promotes explanation, application and synthesis rather than mere recall (Askew &Wiliam, 1995).
It must use cooperative small group work	Activities are more effective when they encourage critical, constructive discussion, rather than argumentation or uncritical acceptance (Mercer, 2000). Shared goals and group accountability are important (Askew &Wiliam, 1995).
It must encourage reasoning rather than 'answer getting'	Often, learners are more concerned with what they have 'done' than with what they have learned. It is better to aim for depth than for superficial 'coverage'.
It must make use of rich, collaborative tasks	The tasks we use should be accessible, extendable, encourage decision making, promote discussion, encourage creativity, encourage 'what if' and 'what if not?' questions
It must create connections between the topics	Learners often find it difficult to generalise and transfer their learning to other topics and contexts. Related concepts (such as division, fraction and ratio) remain unconnected. Effective teachers build bridges between ideas

It must make use of technology in appropriate ways	Computers and interactive whiteboards allow us to present concept in visual dynamic and exciting ways that motivate learners.

The Swan's principles summarise what characterise collaborative orientation approach. These include teacher's ability to make connections among the concepts of the topic; facilitate learners to engage in productive discussions; ensuring non-linear dialogue between the teacher and the learners; and presenting problems before explanation.

2.4 A SELECTION OF LITERATURE DRAWN FROM SHULMAN'S PCK CONSTRUCT

The following literature is considered significant for the current study as it analyses or/ and seeks to expound the special knowledge that teachers need to teach mathematical concepts. The selected works are primarily based on Shulman's construct of PCK, the theoretical framework for the current study.

2.4.1 Mathematics knowledge for teaching (MKT)

Mathematics Knowledge for Teaching (Ball et al., 2008) as presented in Figure 2.1 below, advances on the PCK construct of Schulman. Ball and colleagues continued to seek information on the sort of mathematical reasoning, insight, understanding and skills that are required for the teacher to teach mathematics. They created a test to measure teacher knowledge. The study initiated the concern about mathematics knowledge from the point of view of teaching. MKT was based on research among K to Grade 8 teachers in the United States (US). The researchers proposed two categories of teacher knowledge: Subject Matter Knowledge (SMK) and PCK. They argued that teaching requires complex management of instructional resources including teachers' SMK and PCK. Each of which was further simplified into three broad categories.
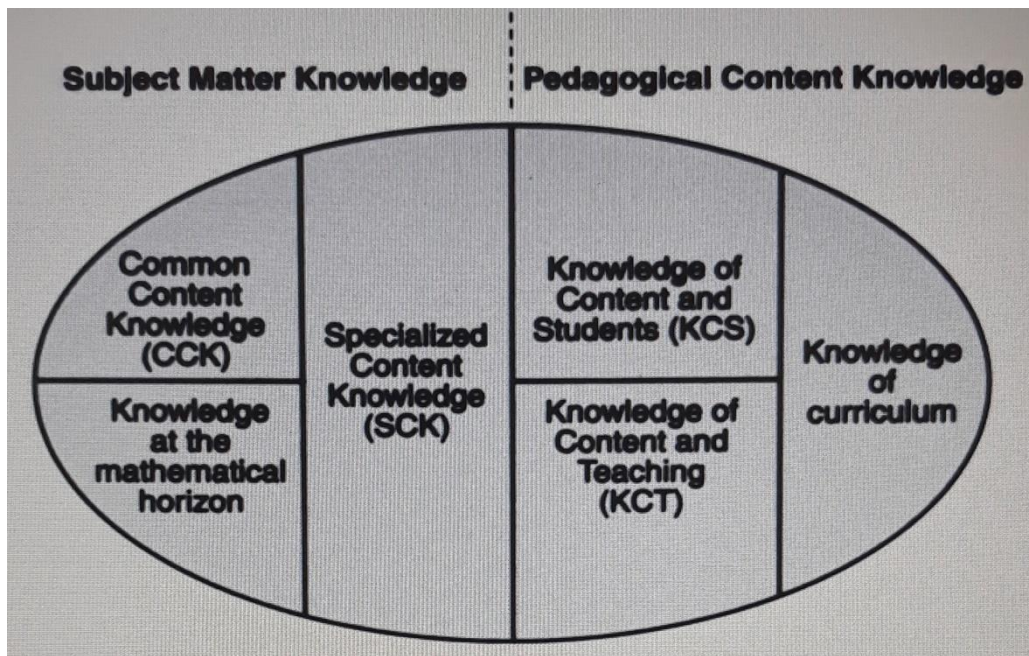
Figure 2. 1: MKT framework: Adapted from Ball's (2008) Unpacking Pedagogical Content Knowledge

The Common Content Knowledge (CCK) is associated with teaching in general. The knowledge is not particular to mathematics teachers, but also to other professions that use mathematics (Ball et al., 2008). This is synonymous with what Shulman describes as Subject Matter Knowledge, SMK. However, Specialised Content Knowledge (SCK) is a newer conceptualisation of SMK. To specialise in teaching and learning mathematics, Ball and partners see the need for mathematical knowledge that is specifically intended for mathematics teachers. This knowledge, according to the authors (2008), allows teachers to engage teaching tasks, for example, how to accurately represent mathematical ideas and provide mathematical explanations.

The CCK and SCK are both mathematical knowledge and belong to the group of SMK in the MKT model. Conceptualisation of the two does not require the knowledge of the learners or that of the teaching. This describes the boundaries between/among the components of MKT.

On the other hand, Knowledge of Content and Learners (KCS) is an aspect of PCK. Ball (2008) defines KCS as content knowledge blended with knowledge of how learners think

about, know, or learn content, for example, quadratic functions. It is believed to be the same as what Shulman originally described as 'understanding of what makes the learning of specific topic easy or difficult'. The knowledge of learners (ideas and thinking) or knowledge of content is different from KCS. KCS simply means when the teacher combines (blends together) understanding of learner thinking with a mathematical knowledge in view, and then uses it to enhance teaching of such concepts. KCS focuses mainly on how learners learn content rather than the broad mathematics' curriculum.

2.4.2   Mathematics Teacher Specialised Knowledge (MTSK)

The MTSK model was developed by Carrillo et al. (2013). Their work draws on MKT (Ball et al., 2008). The authors suggest that there is mathematical knowledge shared by mathematics' teachers and others (e.g., mathematicians) and there is a special kind of mathematical knowledge that is possessed by only the mathematics teacher. The latter, according to the authors, is referred to as PCK. It further draws a line between mathematical knowledge and the PCK for teaching mathematics. MTSK focuses exclusively on the mathematics teacher's specific knowledge with regard to effective teaching of mathematics. The authors believe that effective teaching of mathematics is a product of integration between teacher's mathematical knowledge and the PCK. They describe mathematical knowledge as that pertaining to the teacher's understanding of the content (pure mathematics). These include: knowledge of the topics, which has to do with the mathematical concepts and procedures and their underpinning theories; the knowledge of the structure of mathematics, which refers to the structure of concepts and ways of proceeding in mathematics; and lastly, the knowledge of practice of mathematics as the teacher's competency of mathematical language and ability to connect the concepts.

The current study investigated the MTSK's characterisation of PCK to get closure on what comprises the mathematics teacher's PCK with respect to teaching of quadratic functions in Grade 10. According to MTSK, PCK is the non-mathematical knowledge (but powered by mathematical knowledge) that allows the teacher to understand how the learners learn; choose appropriate materials and examples that best suit the teaching; as well as knowledge of assessment, especially by the external educational bodies. The elements of MTSK referring to PCK are built upon MKT as rightly pointed out in the beginning. The PCK elements are classified into three: (i) ***Knowledge of mathematics' teaching*** which is

synonymous to Ball's KCT. It is the knowledge that allows the teacher to choose appropriate teaching materials and examples that best illustrate the mathematical concept. (ii) ***Knowledge of features of learning mathematics***, which is concerned with knowledge of the teacher that enables him/her to understand what learner think and how they learn mathematics. It draws on the KCS of the MKT, but is particularly concerned with the teaching and learning of mathematics. (iii) ***Knowledge of mathematics learning standards***, which describes Ball's KC or curriculum knowledge with an extension that the assessment and evaluation of relevant external bodies is paramount to the successful teaching and learning of mathematics. Knowledge of mathematics learning standards captures the knowledge of the mathematics curriculum and its supporting materials (Carrillo et al., 2013).

2.4.3   Knowledge of Discourse

Hauk et al. (2014) achieved a laudable landmark in mathematics education by building a model called "Knowledge of Discourse". This model extends Ball's MKT referring to PCK, with a focus on the original framing of PCK by Shulman. This model explains the key aspects of PCK with a focus on secondary and post-secondary contexts. While comparing their work to MKT, Hauk et al. (2014) acknowledge the correlation between MKT and quality of instruction as it indicates the powerful relationship between what and how teachers know about mathematics and what occurs during classroom instruction. With regard to Ball's SMK, these researchers remarked that the nature of SMK for teachers varies according to the grades they teach, a view shared by Ingvarson et al. (2013). In other words, the SMK for secondary school teachers (specialists) is different from the SMK for primary school teachers (generalists). There is a need for mathematics teachers to possess knowledge of advanced mathematics content that includes, but is not limited to, the curricular knowledge and semiotics.

The view of PCK of Hauk et al. (2014) is generally about 'how' mathematics teachers use what they know to inform their mathematical understanding and thinking. On this note the authors endeavour to explain (and differentiate between) the three terminologies involved in the process: the knowledge, the understanding and the thinking; their pedagogic relevance to teaching; and the interplay among them. For instance, knowledge is embedded in the teacher's ability to make coherent connections of what he/she 'knows'. When teaching quadratic functions in the classroom, this is displayed in the teacher's ability to connect

learners' prior knowledge with the present situation for building learners' mathematical knowledge and skills as well as understanding every 'what' aspect of the concepts of quadratic functions. Furthermore, Hauk et al. (2014) see PCK as the teacher's dynamic understanding, that is, the vibrant co-existence of knowing and purpose. This illustrates the notion of Shulman that mathematics teachers must not only know how to define a concept, but also know why such a concept is worth knowing. Such a purpose must be stated in the lesson plan of quadratic functions in clear terms. Lastly, the authors define thinking as the mental activity of organising ideas and using knowledge. This can be viewed as an integral aspect of teaching.

For the three terms defined above, Hauk and colleagues (2014) use Ball's three components of PCK (i.e., KCS, KCT and KC) to represent the knowledge, while they focus on the relational understanding and thinking that connect the *knowledge of discourse* with these three components. They define connection between knowledge of discourse and KCS as anticipatory thinking - the teacher's thinking about strategies that guide learners to engage with content, process and concepts; knowledge of discourse and KCT as implementation thinking – the teacher's thinking about teaching strategies that best suit a particular concept and class; knowledge of discourse and KC as curricular thinking – the teacher's thinking about approaches to mathematical topics, procedures and concepts, and relationships among them.

The researchers differ with the MKT model as they posited that the relationship that exists among the components of PCK is interwoven and is more than an overlap of pedagogical and mathematical knowledge. Also, they extend the MKT referring to PCK by adding aspects of communication which the present researcher finds important to the discussion of PCK and teaching of quadratic functions in Grade 10. In the light of this communicative focus, Hauk et al. (2014) frame their work as "*Knowledge of Discourse*". The term 'discourse' here encompasses verbal and non-verbal communications that are acceptable in the mathematics classroom. They highlight the key aspects of communication as thus: vocabulary (e.g., the mathematical register); discourse practices; gestures; and the setting of norms or talking about mathematics (socio mathematical norms).

The construct, PCK, is basically about teaching. Teaching is about effective communication. Hence, the current study also examined the 'what' and 'how' of communicating the concepts

of quadratic functions to Grade 10 learners. In a nutshell, the knowledge of discourse views PCK as relational interplay among advanced mathematical understandings, teaching, and culturally mediated communication. Of great importance is discussion and relevance of teacher orientation towards intercultural differences. In the classroom where quadratic functions are taught, for example, the discourses may differ from person to person. It is important that members of the class are aware of their differences, accept and learn to adapt in this situation. This describes the teacher's knowledge of content and learners, especially the teacher's ability to consider learners' individual differences when making a decision about teaching, instructional materials and assessment. The ways that teachers and learners are aware and respond to multiple cultures is a consequence of their intercultural orientation (Hauk et al., 2014). Language and illustration aid conversation in the classroom. It is crucial to talk about the kind of teaching method that this model envisages in a mathematics classroom. A culturally mediated communication should be one that affords every individual learner a chance of participation. The teaching method envisaged here must be learner-centered, a dialogic relationship.

2.4.2   Conceptualising PCK for teaching quadratic function in grade 10

The researcher realized, through literature review, that majority of PCK studies are rooted in Shulman's notion of PCK. In this study, the researcher conceptualised the PCK for teaching quadratic function in grade 10 particular to the teaching of quadratic function in Grade 10 as thus:

- Content knowledge for teaching

- Knowledge of mathematics curriculum

- Teacher's knowledge of grade 10 learners

- Knowledge of instructional strategies

2.4.2.1 Content knowledge for teaching quadratic functions

Research into content knowledge for teaching mathematics continues to focus on the connections between teacher's content knowledge and teaching, and the kind of content knowledge needed for teaching mathematics. Hu, Son and Hodge (2018) elaborate that sound

content knowledge enables teachers to transform among different representations of quadratic function; to apply quadratic function to solve real life problems; and to give correct and complete explanations of the concepts of quadratic function.

The concept of quadratic functions is a crucial component of mathematics. This is substantiated by research. Even (1990) stated that quadratic function is fundamental and basic in the school mathematics curriculum. Hu et al. (2018) pointed to the significance of quadratic function and stated that quadratic function is a critical transition from straight line to curves. In the South African mathematics curriculum, it is suggested that a teacher should help learners develop the ability to recognise relationships between variables in terms of numerical, graphical, verbal and symbolic representations and convert flexibly between these representations (DBE, 2012). To do this, a mathematics teacher must have a good understanding of the concepts of quadratic functions and how to teach them. It affords teachers a definition of mathematics, specific aims, specific skills, focus of content areas and the weighting of content areas. Teachers should have good understanding of key characteristics that are particular to quadratic function in Grade 10. The current study gathered information about the role of the mathematics teacher's content knowledge in the teaching of Grade 10 quadratic functions, and to understand the specific details of concepts of quadratic functions that they must understand to teach the concepts successfully. The current researcher opined that deep content knowledge of mathematics broadens teachers' explanations of the concepts and their ability to choose different and relevant representations (which Shulman describes as PCK) and equips teachers with knowledge and skills that enable them to identify the level of engagement that learners are at and enables them to provide relevant progress opportunities. Ball et al. (2005) concur that the effective usage of instructional materials, knowledge of learner assessment and sound judgments about teaching and sequencing of concepts depend on the teacher's understanding of the subject matter.

In South Africa, several studies (Bansilal, 2012; Bansilal et al., 2014) have revealed poor mathematics content knowledge of mathematics teachers. In their study, Bansilal and colleagues (2014) investigated the mathematics teachers' knowledge of mathematics. The teacher participants wrote some of the questions of Grade 12, mathematics paper one. The

result shows that most teachers perform poorly in higher level questions: as the cognitive level of questions increased, teachers performed worse. Based on cognitive taxonomy, the overall average performance of the teachers is summarised as thus: level one – 84%; level two – 73%; level three – 47; and level four – 26%. The result on quadratic functions (parabola) shows that mathematics teachers find parabola more difficult than hyperbolic functions, as the average scores reveals 49% and 70% respectively. Teachers struggle with the concepts like writing the equation of the parabola; writing f(x) in the form of $y = a(x - p)^2 + q$; and the reflection of f(x). (Bansilal et al., 2014: 36-43)

Ball et al. (2005) proposed that mathematics teachers should study more mathematics. They suggested that teacher professional programmes should take the form of a practice-grounded approach that will prepare teachers in the mathematics they will use on the job. In a simplified form, Ingvarson et al. (2013) differentiate between the subject matter of a generalist teacher of mathematics and that of a specialist teacher of mathematics. They stated that the latter is likely to possess more mathematics content knowledge than the former.

The CAPS recommends that GET mathematics teachers develop learners' function and relationships competencies, among others, to prepare them for quadratic functions in Grade 10. Moreover, the Grade 10 mathematics teacher must assess the entry knowledge of learners by revising the necessary concepts with them before commencing with the concept of quadratic functions. One of the learning goals of quadratic functions in Grade 10 is that learners should be able to apply and transfer quadratic functions ideas to real life-situations (DBE, 2012). Unless a teacher possesses deep knowledge of quadratic function, he/ she may not be capable of handling the challenging situations in quadratic function in the classroom.

2.4.2.2 Knowledge of Grade 10 quadratic function in curriculum

As mentioned earlier, the teacher needs more than quadratic function's contents as stipulated in the curriculum. CAPS was the third curriculum statement to be introduced since the post-apartheid era commenced. This is to ensure that mathematics education in South Africa conforms to socio-economic transformation and promotes efficiency and productivity among the citizens (DBE, 2012). Ten topics or main content areas comprise the FET mathematics curriculum: Functions; Number Patterns, Sequences and Series; Finance, Growth and Decay; Algebra; Differential Calculus; Probability; Euclidean Geometry and Measurement;

Analytical Geometry; Trigonometry; and Statistics (DBE, 2012). The breaking down of topics ensures sequencing within the concepts and across the grades.

The CAPS for mathematics was introduced to improve teaching and learning of mathematics in South Africa. It added value to the previously amended National Curriculum Statement (NCS). The purpose was to ensure that mathematics teachers focus more on mathematical content and assist learners with better understanding, with the aim of improving their performance in mathematics (Laren, 2012). The Further Education and Training (FET) Phase of Mathematics CAPS consists of Grades 10 – 12. The key components of quadratic function are what Even (1990) labeled as 'basic repertoire'. They are usually stated in the mathematics curriculum documents. Table 2.2 presents the key components of quadratic function for Grade 10 learners.

Table 2. 2: Content of quadratic function: Extract from Limpopo Department of Basic Education 2018 Grade 10 Mathematics Annual Teaching Plan

Topic	Content
Quadratic function	1. Work with relationships between variables using tables, graphs, words and formulae and convert flexibly between these representations. 2. Point by point plotting of basic graphs defined by $y = x^2$ to discover shape, domain (input values), range (output values), axes of symmetry, turning points and intercepts on the axes. 3. Investigate the effect of a and q on the graphs defined by $y = a.f(x) + q$, where $f(x) = x^2$. 4. Sketch graphs, find the equations of given graphs and interpret graphs.

Curriculum sequencing is a feature of PCK. Shulman highlights that the teacher needs to identify the key ideas and then decide how best to organise and sequence the route to the big

ideas in a way that would make the most of the learners' understanding (Bansilal, 2012). Bansilal and her collaborators are concerned about curriculum compliance which is an important aspect of curriculum knowledge of a mathematics teacher. They are concerned that if mathematics teachers struggle with high level questions, they will be unable to design valid assessment tasks that cover the four cognitive levels (DBE, 2011). They also submitted that mathematics teachers' insufficient understanding of school mathematics is an impediment to their pedagogic content strategies.

2.4.2.3 Knowledge of instructional strategies for teaching quadratic functions in grade 10

The current study focuses on investigative teaching strategies. Even (1990) recommends that investigation is a powerful method for teaching quadratic function; he adds that teacher's understanding and wise usage of variety of representations to teach quadratic function provides learners with better and deeper understanding of the concepts of quadratic function. He (1990) gathers that learners often memorise facts about quadratic function and procedures for solving quadratic problems, thereby leaving them with poor understanding of the concepts.

The CAPS for mathematics stipulates that teachers should ensure that learners are able to investigate and communicate the effects of parameters "a" and "q" using various representations. It also suggests that teachers teach how to plot a parabola using the point-by-point plotting method and sketching based on the features of the quadratic function. Even (1990) admits that the point-by-point approach is easy for learners, but it is not sufficient to learn the quadratic function. In teaching quadratic function in Grade 10, teachers are admonished to solve problems using different methods and representations (Hu et al., 2018). The use of a variety of representations of the quadratic function is significant to the teaching of quadratic function. Teachers are expected to use algebraic formulae, language, tables and graphs to define quadratic function. In so doing, teachers provide learners the opportunities to communicate their findings about the effects of the parameters "a" and "q" ($y = ax^2 + q$).

Teaching is an aspect that describes the actions of and interactions among the teacher and the learners to achieve the goal of quadratic functions. Fernandez (2014) presents a cyclical procedure for teaching in the Model of Pedagogical Reasoning and Action (MPRA) (adapted

from Salazar, 2005; Shulman, 1987). He propounds that teaching begins by a kind of 'text' (content) that both the teacher and learners seek to understand. The process discussed in MPRA is established through five categories: comprehension, transformation, instruction, assessment, and reflection. The comprehension category describes teacher's understanding of the purposes, content structures, and ideas within and outside the discipline. For example, mathematics teachers need to know the importance of teaching quadratic functions in Grade 10, as well as its connection to other topics both within and outside mathematics. Thus, new comprehension is formed. This makes the teacher more knowledgeable/informed about the purposes of the content, the content itself, the learners' characteristics and oneself. The instruction stage of MPRA is different from its transformation, as it encompasses teacher's classroom management, presentations and interactions. The core characteristics of teaching includes the ability to process knowledge, the capacity to communicate it well and more especially, forging and maintaining effective personal interactions. In addition, a good teacher must have a good command of communication and observation skills (Dymoke, 2011).

The term 'quadratic' is introduced in Grade 10, although learners will have been introduced to its pre-requisites such as algebraic expressions and algebraic equations, factorisation of 2nd degree polynomial, graphing as well as functions and relationships in the General Education and Training (GET) phase. The CAPS requires that mathematics teachers employ more than verbal explanations to teach quadratic functions. The teacher must be able to use various representations such as diagrams, tables, formulae and graphs. This will enable the learners to grasp the concepts of quadratic functions better. The first four performance standards of Integrated Quality Management System (IQMS) (SACE, 2014) also address the way in which CAPS should be implemented. The standards help the teachers to assess their own teaching quality. Swan's principles of teaching mathematics, which form part of guideline for the current study as they highlight the important components of PCK that the mathematics teacher should demonstrate to achieve the curriculum outcomes, are embedded in IQMS principles. The aims of IQMS (IQMS Training, n.d.:25-32) relevant to this study include:

- To use teaching resources effectively;

- To ensure that each learner participates actively during the lesson;
- To make use of inclusive strategies that cater for learners' individual differences;
- To use learners' diagnostic reports to develop teaching strategies that best fit quadratic functions;
- To use learner-centred techniques to promote learners' critical thinking;
- To be creative and innovative in setting goals that help to achieve the lesson objectives of quadratic functions in Grade 10;
- To plan quadratic function lessons that are clear, logical and sequential;
- To present competent lessons that are built on learners' prior knowledge;
- To consider learners' characteristics and thinking during planning and presentation of lessons;
- To always reflect on the process of each lesson and use the findings to improve the following lessons.

2.4.2.3.1 *Selection of variety of teaching strategies and representations*

Effective teaching of quadratic functions requires diverse methods of teaching, most importantly in the ways of explaining and representing its concepts. Makgato (2012) advises constructivist methods of teaching such as problem-based learning and inquiry-based learning as they promote active participation in the classroom. Also, language and communication are great resources of instruction. Mathematics teachers should have proficiency in the use of correct mathematical language and should be able to communicate effectively using visual, symbolic and language skills in quadratic functions classrooms. Pitjeng (as cited in Leikin & Zazkis, 2010) identifies five conditions which a suitable teaching strategy for mathematics must satisfy: excellence in teaching the content; consideration of learners' prior knowledge and misconceptions; curriculum saliency; use of different symbolic representation; and highly learner-centred lessons.

Representation involves the use of various powerful tools such as analogies and examples, to teach the content to the learners. Selection of instructional strategies describes using appropriate teaching methods that best fit a concept. Teacher knowledge of teaching

strategies and instructional materials include the use relevant activities, the use of real-life situations as examples, and the use of different instructional strategies and representations in instructions (Bukova-Guzel, Canturk-Gunhan, Kula, Ozgur, & Elci, 2013).

The use of alternative approaches which require a variety of representations is central to South Africa's mathematics curriculum. CAPS (2012) advocates the use of modelling, learner-centred and problem-solving approaches as opposed to transmission-based teaching (Rollnick, 2014). CAPS also recommends adopting complex verbal representation (language), which is one of challenges of effective teaching of mathematics (Laren 2012). It is important that teacher chooses teaching methods that best coordinate the interactions among the learners, the concept of quadratic functions, and the curriculum requirements. This should include activities that foster learners' active participation, teamwork and independence. Similarly, Hauk et al. (2014) agree that the teacher's view about mathematics determines the teaching style for a specific mathematics topic. They advocate the reformed-oriented approach (constructivist view) that views mathematics as a human activity and allows learners to construct mathematical understanding through independent reasoning and team work.

2.4.2.3.2 *Assessment of teaching and learning processes*

Reflective ability is a good source of PCK. Reflective practice allows mathematics teachers to gain insight into their own teaching and learning. Akerson et al. (2017) report that 'lesson study' is a reflective practice that helps teachers to develop their PCK. Akerson and colleagues investigate the translation of science teachers' understanding of the nature of science (NOS) into classroom practices. Similarly, a mathematics teacher must continually analyse his/her curriculum planning and instructional methodologies. A reflective teacher thinks back over the day-to-day situation in an attempt to analyse his/her teaching skills, the content, motivation of learners, and how he/she might improve upon the overall learning process (Henderson & Lawrence, 2011).

Equally, teachers must evaluate whether learners have achieved the learning goals of the concept. By this, teachers collect information and make judgments about learners' needs, strengths and achievement in accordance with CAPS requirements. The assessment is conceived as testing learners' understanding of the concept (during or at the end of the

lesson), as well as evaluating one's (teacher's) own performance. The reflection stage speaks about reviewing the processes involved in the teaching of quadratic functions about learners and one's own performances with the aim of adjustment. Items in the interview protocol of this study specifically treat this sub-domain because of its significance to PCK. Therein the researcher seeks to understand how mathematics teachers reflect on lesson presentation and what they do henceforth.

2.4.2.4 Knowledge of learners' thinking with regard to quadratic function

A good teacher is the one who has the knowledge of learners' mathematical ideas and thinking (Ball et al., 2008). Ball et al. (2008) described teacher knowledge of learners in relation to a specific topic in what they termed as 'knowledge of content and learners' (KCS). The teacher's knowledge of the learners with regard to quadratic functions is an important aspect of the mathematics teacher's PCK. It involves the teacher's ability to pay attention to learners as they learn the quadratic function and interpret mathematical ideas. The teacher needs to understand how learners conceive mathematical concepts.

2.4.2.4.1 Learners' prior knowledge

Learners' prior knowledge is an integral part of PCK. It enhances effective teaching and learning of quadratic functions. Understanding of quadratic functions is closely connected with learners' experiences; this implies that learners gain conceptual understanding of the concept when they can draw inferences or recognise relationships between learners' prior knowledge and the new experience (quadratic functions). Therefore, the teacher should endeavour to identify and make use of learners' prior knowledge in the teaching of quadratic functions. For example, learners are expected to come with the knowledge of the real number system and factorisation, among others. Effective teaching of quadratic function in Grade 10 will be teaching that considers the background and experiences of the learners. Laren (2012) shows that, in most cases, learners lack the necessary background to embark on new learning and this is related to the teacher's inability to complete the curriculum. It is recommended that the teacher recaps the pre-requisite knowledge and skills stipulated by the curriculum before the teaching of quadratic functions commences. Before embarking on the teaching of quadratic functions in Grade 10, the teacher should examine learners' pre-requisite knowledge and creatively expand this knowledge to link to the present lesson (quadratic

functions). Also, the teacher should endeavour to create a positive learning environment that supports inclusive mathematics teaching and learning strategies.

2.4.2.4.2 Common errors and misconceptions

The teaching of quadratic functions in Grade 10 goes beyond 'knowing the subject matter'. It encompasses the teacher's ability to recognise and analyse the learner's misconceptions of quadratic functions. Understanding Grade 10 learners in the mathematics classroom involves the teacher's ability to use learners' errors as building blocks for teaching the concepts of quadratic functions. Piaget (1995, cited by Hauk et al., 2014) suggests that mathematics teachers should seek to understand and build on learner thinking or patterns underpinning learners' errors. Error analysis forms an integral aspect of PCK. It requires of the teacher high level mathematical competency and intellectual orientation. It may facilitate learners' learning of important concepts of the quadratic function.

In South Africa, an average mathematics teacher's knowledge is weak (Sapire et al., 2016). Hence, they often shy away from engaging with learners' errors, or they only partially deal with them or accept incorrect work from learners. Sapire et al. (2016) state that the Annual National Assessment (ANA) demands that mathematics teachers should be able to interpret learners' errors and use the interpretations to develop better lessons. The study shows the impact of the relationship among the three components: error analysis, pedagogical load and cognitive load. Machaba (2016) also found that inadequate learners' prior knowledge could lead to learners' misconceptions, thereby revealing the direct relationship between the two sub-domains of knowledge of learners: learner's prior knowledge and misconceptions. Engaging with learners' errors increases the pedagogical load, which in turn places cognitive load on the teacher. In studying the problems highlighted in Machaba's (2016) article, the current researcher observes the links between the what, the how and the why. For example, during error analysis the teacher should be able to increase learner's competency in quadratic functions (what?); this is possible through a selection of appropriate pedagogic activities (how?); the last part (why?) explains the goals of teaching. In the end, learners should be able to apply knowledge acquired in quadratic functions to a real-life situation, that is, to generalise learning (why). Another problem identified by Machaba (2016) is that a teacher should reduce the extraneous load on learners. To do this, the teacher must avoid incorrect

mathematical explanations or examples when engaging with learners' errors in order not to confuse them further. A teacher can do this by observing learners as they solve quadratic problems, or/ and by asking learners how they solve the problem. Hence, the teacher will be able to understand the kind of thinking behind learners' mistakes.

2.4.2.4.3   Difficulties with quadratic function among Grade 10 learners

Knowledge of the learners involves the teacher's understanding of the concepts of quadratic function that learners often find difficult to solve and why they find them difficult. According to current researcher's experience, many teachers found it difficult to teach the graphical method of solving inequality problems; hence it also became difficult for their learners to attempt such problems. Hu et al. (2018) state that learners often struggle with generating equations to represent quadratic relationships in typical word problems. Zaslavsky (1997 cited by Hu et al., 2018) highlight some common barriers that are related to learners' understanding of quadratic function:

- Learners find it difficult to imagine the parabola as a continuous phenomenon;

- Learners often get confused about the relationship between quadratic function and quadratic equation;

- Learners tend to prefer progressing from equations to graphs, rather than from graphs to equations;

- Learners prefer the standard form, $y = ax^2 + bx + c$, to the turning-point form of quadratic function, $y = ax^2 + q$.

2.5   SUMMARY

Shulman (1987) describes the teacher as one who is not only able to define the concept, but also is able to explain **why** it is worth knowing and **how** it relates to other propositions. That is, understand why a given topic (e.g., quadratic functions) is important in mathematics. These characteristics are shared with the CAPS. Mathematics teachers develop their professional knowledge through active practices - participation or collaboration - and

continuous reflection on practices. In other words, knowledge cannot be separated from practice.

The literature review in this chapter clearly points out that a mathematics teacher needs to possess high subject competency. The teaching of quadratic functions in Grade 10 rests on the teacher's conceptual understanding of the concepts, procedural fluency and strategic competence. All these attributes sum up the mathematics teacher's PCK for effective teaching. The mathematics teacher acquires PCK through years of experience that affords that teacher the opportunity to analyse and reflect on his/her teaching processes.

Content knowledge is a subset of PCK. The PCK is a blended phenomenon that is inseparable from content knowledge. Teaching and other components of PCK are interwoven. Language and communication are drivers of good presentation (teaching). However, communication skills require mathematics teachers to have adequate understanding of what to communicate (content). Also, effective teaching of quadratic functions, for example, requires choosing appropriate multiple representations.

The amount of mathematics content knowledge required of mathematics teachers depends on the class that they teach as well as advanced content knowledge. For example, to teach quadratic functions in Grade 10, teacher needs deep understanding of functions beyond what is stated in the Grade 10 curriculum. This together with the knowledge of the curriculum can help the teacher to discover what is missing in learners' prior knowledge and misconceptions. The learner-centred method, modelling, and problem-solving are commonly considered in the available literature as suitable teaching strategies for mathematics (including quadratic functions) as they foster active participation among learners.

CHAPTER 3: RESEARCH METHODOLOGY AND RESEARCH DESIGN

3.1 INTRODUCTION

The research methodology used to explore the mathematics teachers' PCK in the teaching of quadratic function in Grade 10 is discussed in this chapter. The aim of this study was to explore how mathematics' teachers were teaching quadratic function in Grade 10. The researcher studied three mathematics teachers to explore their knowledge of quadratic function, various strategies they employed to teach quadratic function in Grade 10, and their knowledge about their Grade 10 mathematics learners about quadratic function. The methodological approach adopted was used to address the following research objectives:

- To explore teachers' content knowledge required in teaching of quadratic function in Grade 10;

- To explore strategies used for teaching quadratic function in Grade 10;

- To explore how teachers access learners' knowledge in the teaching of quadratic function; and

- To explore how teachers draw from their knowledge of the curriculum when teaching quadratic function in Grade 10.

In this chapter, the researcher discusses the research paradigm and research methodology adopted for the study. The researcher also justifies the selection of the particular research context that was adopted and provides a background to the teachers' PCK for the teaching of quadratic function in the Mogalakwena District. Furthermore, this chapter discusses the methods, with justifications, of data collection and data analysis.

This chapter is organised under sub-headings: research approach; research setting; research sampling and data sources; data collection methods; data analysis methods; issues of trustworthiness; limitation and delimitation; and summary.

3.2 RATIONALE FOR RESEARCH APPROACH

This section describes and justifies the research tradition and research methodology that was adopted in the study.

3.2.1 Qualitative Research

The research paradigm adopted in this study is qualitative research. Qualitative research methods emphasise gathering data on naturally occurring phenomena (McMillan & Schumacher, 2010:23). This research tradition is considered suitable for the current study as the researcher observed the teachers' PCK in classroom natural settings. The aim of this study is to explore the teachers' PCK for the purpose of interpretive understanding how teachers teach quadratic function in Grade 10 mathematics class. Struwig and Stead (2001) opine that a qualitative research paradigm affords the researcher the opportunity of understanding the issues being researched from the perspective of the research participants (i.e., insider perspective). This explains the epistemological viewpoint of qualitative research that stipulates that the researchers gain knowledge through participants' personal encounters; which further defines the nature of reality as multiple realities (ontology). The characteristics aforementioned justify why the researcher chose the qualitative case study approach.

3.2.2 Research Design

This study focuses on the exploration of teachers' PCK for teaching quadratic function through the participants' perspectives. As a result, the researcher adopted the case study qualitative research design. Babbie, Mouton, Vorste and Prozesky (2011) describe the features of the case study as one that: provides a detailed encounter with the participants; makes use of a small number of research participants; possesses openness to multiple sources of data; and allows the researcher to adapt and make changes to the study at any time. These features are crucial to the current study. The researcher engaged directly with the participants. The researcher observed and gathered qualitative information about how three mathematics teachers teach quadratic function in Grade 10. The study is exploratory in nature. It sought to satisfy inquisitiveness in some way and familiarised the researcher with the concept of PCK for teaching quadratic function in Grade 10, thereby leading to a better understanding of the teaching of quadratic function in grade 10 (Babbie et al., 2013). Case

study approach is essential to the current study as it allowed the researcher to explore teachers' content knowledge, knowledge of curriculum, their knowledge of the learners and various teaching strategies which they employed during the teaching of quadratic function in Grade 10. These are knowledge components that constitute the PCK. Exploration is guided by the PCK framework and the data are gathered through direct observation of the lessons and face-to-face interviews at three different secondary schools purposively selected in the Mogalakwena district.

3.3 RESEARCH SETTING

This study was conducted in three public secondary schools in Bakenberg South Circuit, Mogalakwena district, Limpopo. In Limpopo province, the organizational structure of the Department of Basic Education (DBE) can be represented as follow:

Table 3. 1: The Organisational Chart of Limpopo DBE
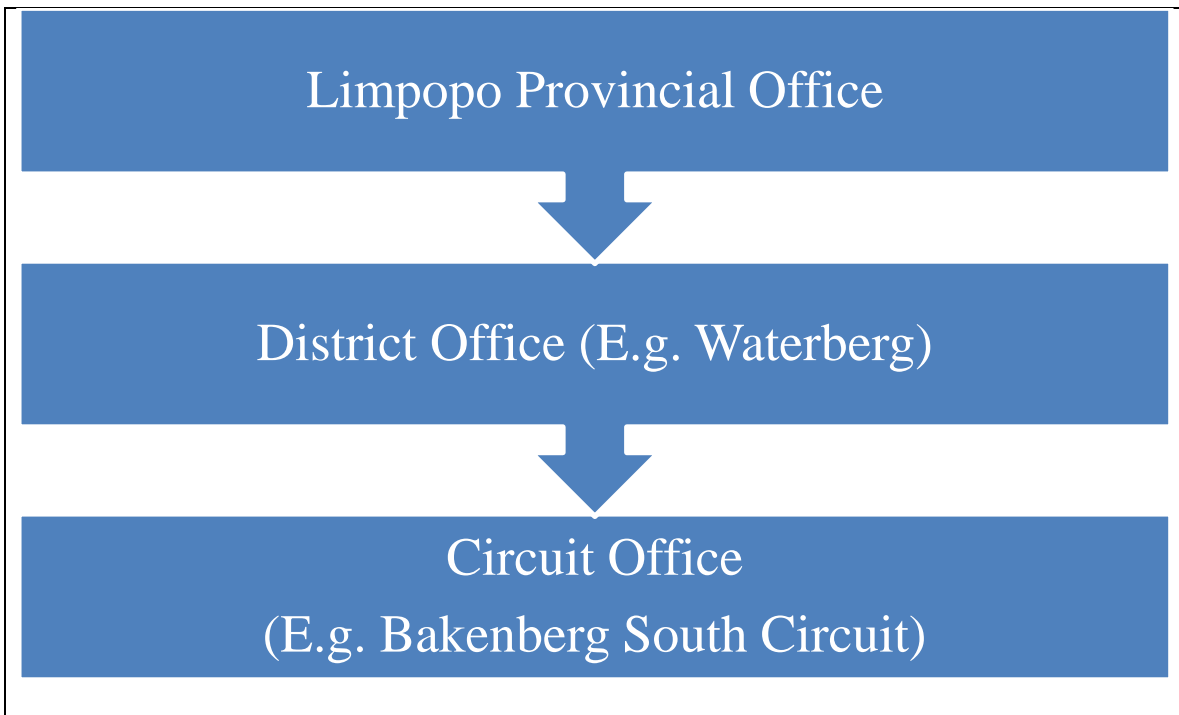
The researcher observed that in spite of a general decline in mathematics performance, including the Mogalakwena District where the current study was carried out, two of the schools selected experienced an increase in mathematics performance during 2016-2017. Table 3.2 presents the mathematics performance of schools where the participants teach mathematics. In order to protect the identity of the schools, the researcher used the code S1 for school 1; S2, for school 2; and S3 for school 3.

Table 3. 2: Selected Schools' Performance: Extracted from School Subject Reports 2016 & 2017

	2016	2017
S1	40%	60%
S2	83%	50%
S3	28%	62%

The three schools are located in three different suburbs under Bakenberg South Circuit of the Mogalakwena district in Limpopo. The context of Bakenberg South Circuit is a rural setting. The three schools belong to the category of quintile 2 as a no fee institution. None of these schools has modern facilities for teaching mathematics, except calculators. For example, there was no graph board and projector. S2 is located along the main road of Bakenberg while S1 and S3 are located in the interior parts; within their respective villages. A large percentage of mathematics teachers in this area resided in the neighbouring towns. Thus, they traveled to their workplaces on a daily and/ or weekly basis. The researcher chose the schools based on the availability and willingness of the participants. Preceding the data collection phase, the researcher approached the three participants chosen, in the same context and standard, to seek their consent and inform them of the nature and purposes of the study and the intended process of data collection. They all agreed to be part of the study.

The mathematics curriculum implemented in this area is the same as what is implemented nationwide. The data was gathered during the second quarter of the school year, 2018. Quadratic function (including some other forms of functions) in Grade 10 was scheduled in

this period from the first week to the fourth week as prescribed by the national mathematics curriculum: the Curriculum and Assessment Policy Statement (CAPS).

3.4 RESEARCH SAMPLE AND DATA SOURCES

3.4.1 Population and Sampling Procedure

The study population comprised mathematics teachers in Mogalakwena District, who were teaching quadratic function in Grade 10. The research participants were three mathematics teachers employed by the Limpopo DBE to teach in secondary schools. The participants taught Grade 10 mathematics in three separate secondary schools (S1, S2 and S3) in Bakenberg South Circuit, Mogalakwena District; Limpopo. The only factor that obviously differentiated the participants - which the researcher assume might impact on their teachings of quadratic function in Grade 10 - was the years of experience. The researcher believes that a teacher's PCK is a function of his/ her years of experience in teaching that particular subject. However, their years of experience did not pose a significant difference in their teachings. They were purposively selected among the overall population. The researcher used purposeful sampling. This method, according to Struwig and Stead (2001), is a common qualitative sampling method where researcher chooses participants who are mathematics teachers and are teaching quadratic function in Grade 10. The researcher selected participants suitable for the purpose of the study.

3.4.2 Characteristics and Size of Sample

The researcher adopted a purposive sampling technique to select three participants. This sample was chosen because the teachers were teaching quadratic function in Grade 10. A number of factors influenced the researcher when choosing the number of participants: to gain great depth, accessibility and availability of the site/ the participant as well as being able to compare realities within the study. According to Creswell (2013), choosing a sample size of not more than four or five should provide sufficient information to identify themes of the cases as well as conduct cross-case theme analysis. In all the cases, the participant was the only Grade 10 mathematics teacher for the school; in two instances, they were the only mathematics teachers for the FET phase in the school. The remaining participant shared FET mathematics with another mathematics teacher in that school. This indicates the learner population in the rural schools as compared to their counterparts in the townships or cities.

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The information was directly collected from the participants. Table 3.3 shows the profiles of the three participants:

Table 3. 3: The profile of participants of this study

	T1	T2	T3
Gender	Male	Female	Male
Years of experience	30	10	19
Highest teaching qualification	Senior Primary Teacher's Diploma (SPTD) majoring Mathematics and Physical Education	Senior Primary Teacher's Diploma (SPTD) majoring Mathematics, Technology, English and Sepedi	Advanced Certificate in Education (ACE) majoring Mathematics
Other qualification achieved	Bachelor of Art (BA) majoring in Psychology, English and Sepedi	None	Hons in Public Management and Private Sector
Another position besides the teaching	HOD	None	HOD
Number of award received for teaching mathematics	None	1 (in 2009)	None

Number of learners doing mathematics Grade 10	27	21	23

T1 is a male. He had been teaching mathematics since 1988 and was, at the time data collection, teaching only mathematics in Grades 10 to 12. T1 had the BA degree majoring in Psychology, English and Sepedi and a Senior Primary Teachers' Diploma (SPTD) majoring in Mathematics and Physical Education. He had never received any award for teaching mathematics.

T2 started his teaching career in 2008 and was, at the time of study, teaching Mathematics Grades 9 – 12 with life orientation in Grade 11. She had SPTD as the highest qualification, majoring in Mathematics, Technology, English and Sepedi. She had once received an award for teaching Mathematical literacy.

T3 had been teaching for 19 years (since 1999). He was teaching Mathematics in Grade 9 to Grade 12, at the time of this study, with Grade 8 Natural sciences. T3 had the Advanced Certificate in Education (ACE) majoring in Mathematics. He also had Honours' degree in Public Management and Private Sector. He had never received any award for teaching.

3.4.3   Ethical consideration

Prior to the data collection, the researcher approached the participants, and thereafter followed up with formal letters to the participants, the circuit manager, the school principal, the learners concerned and the parents of the learners in question. In these letters, the researcher sought for the permission to carry out the study using the chosen circuit and the schools. The letters also informed the addressee about the nature and purpose of the study, and how they would be involved. Importantly, participation in the study was voluntary and the participants had the right to end their participation at any time without risk or harm. There was no compensation for participating in the study. The name of the schools has not been identified nor recorded to protect the anonymity of the participants.

Permission for the research was obtained from the Department of Mathematics Education (through UNISA's committee for ethical issues (see appendix B)). This permission and request letter (see Appendix D) was submitted to the Bakenberg South Circuit DBE (circuit manager office) and the permission was granted. Permission from the participants was also obtained from school authorities (see Appendix E) and the participant teachers (see Appendices F and G) themselves. A letter of appreciation was later sent to all teachers who participated in the research study. Throughout this study, the participants are coded with pseudonyms T1, T2 and T3 and the names of their respective schools as ST1, ST2 and ST3.

3.5 DATA COLLECTION METHODS

Data is the foundation on which the body of study is built. In qualitative research, the relevant data are collected through interview and observation (Yin, 2016). The current researcher used mainly two data collection methods: semi-structured interview and systematic lesson observation. These are the common data collection tools in the case study approach.

The observation, in this case, described the teaching activities during the Grade 10 quadratic lessons. Rule and John (2011) mention that the observation method is suitable for a case study if the purpose is to capture and report the liveliness and situations of behaviour. The researcher gathered information by conducting an observation on the issues of PCK for teaching quadratic function in Grade 10. The researcher, as an observer, gathered field notes, made use of audiotape and took photos of activities on the chalkboard. The observational protocol was used to guide the systematic observation based on the themes of PCK and categorised in four sections: content knowledge for teaching quadratic function, knowledge of curriculum, knowledge of learners, and knowledge of instructional strategies. The researcher integrated Swan's (2006) principles of teaching mathematics to further develop the minor themes of the observational protocol. The data for lesson observation was collected over two weeks. The researcher was a complete observer, thus not participating in the teaching of quadratic function but simply recording the information (McMillan & Schumacher, 2010). Although the participants of this study are the teachers, however the

researcher observed each teacher together with his learners during the teaching of quadratic function. The researcher observed and took notes of teacher and learner behaviours, teaching and resources, and occurrence. The study used systematic observation to study three Grade 10 mathematics teachers with their learners present during the lesson observation in their various classrooms, when teaching quadratic functions. The researcher observed two lessons for each participant.

Immediately after the lesson observations, the researcher contracted flu and this impacted on her voice. Hence, the interviews which were scheduled to follow immediately were delayed.

The researcher followed up with semi-structured interview in each case. Struwig and Stead (2001) describe semi-structured interviews as the type by which a researcher systematically formulates and uses pre-determined questions in interviewing all the participants. This helped to explore participants' thinking and to justify what the researcher observed during the teaching. Yin (2016) describes the qualitative interview as a social relationship between the researcher and the participant. In qualitative interviews, the researcher uses mainly open-ended questions rather than close-ended questions to gather participants' own statements. The current researcher interviewed only the three mathematics teachers (participants). The interviews were semi-structured, face-to-face interviews. In this kind of situation, the researcher also forms part of data gathering instruments. The interviews ranged from 50 minutes to 65 minutes.

The interview instrument was used as a guideline in an interview, in which the researcher engaged the participants in one-on-one discussions. The researcher kept a journal, interviews were audio taped and later transcribed. Before that time, the researcher prepared for the task of interviewing by studying and practising the guidelines for conducting good interviews. The interview instrument consisted of four sections: personal information, knowledge of quadratic function in Grade 10, knowledge of CAPS, knowledge of instructional strategies for teaching for teaching quadratic function in Grade 10, and knowledge of the learners.

The aim of this study was to explore teachers' PCK during the teaching of quadratic function in Grade 10. The researcher did this during lesson observations and interviews under the PCK themes: content knowledge; knowledge of the curriculum; knowledge of the learners;

and knowledge of instructional strategies. The interview instrument and observational protocol were developed in accordance with the themes aforementioned.

The collection and analysis of data rely on the theory of PCK framework adapted from Shulman (1986, 1987).

3.6 DATA ANALYSIS METHODS

In qualitative study, data collection and analysis are interwoven and both influence one another (McMillan & Schumacher 2010:367). In this case, the data revealed the level of teachers' PCK employed in the teaching of quadratic function in the Mogalakwena district schools. The audiotapes were immediately transcribed, first handwritten and later typed, allowing the researcher to store the soft copy on a computer. The transcripts together with the field notes were assigned labels to reduce data into relevant categories, thereby aiding the understanding of the construct of PCK for teaching quadratic function in Grade 10. The labels were later reduced to themes that provided answers to the research questions. The researcher continuously analysed, using PCK components, data as they were being collected up until the data were completely collected. The researcher stated (sub) research questions one by one, and then stated what has been done with a number of instruments and how these are analysed.

- The researcher explored how mathematics teachers use learners' prior knowledge, symbolic representations and learners' misconception to provide quality explanation of key concepts and procedures.

- The researcher explored various ways by which mathematics teachers represent quadratic function.

- The researcher explored the types of activities that grade 10 learners engaged with during the lessons of quadratic function.

- The researcher observed how mathematics teacher structured the tasks of quadratic function.

- The researcher explored the kind of questions teacher asked during the lesson of quadratic function in grade 10. The researcher explored the instructional activities that teacher engaged with.

Images gathered during observation and tables were used to corroborate the analysis and interpretation. The researcher coded and categorised the participants as well as data collected to provide a rich description of its findings. The three teachers were coded using pseudonyms, thus: first participant (T1), the second participant (T2) and the third participant (T3).

3.7 ISSUES OF TRUSTWORTHINESS

The corrections and modifications of the project supervisor were adopted in the formulation of the data collection instruments – both the lesson observational protocol and interview instrument. All these were incorporated in the final draft of the lesson observational protocol and interview instrument in order to ensure its content credibility. The researcher also engaged the expertise of senior colleagues (teachers) to scrutinise the semi-structured questions of the interview.

As mentioned, the researcher went through the process of ethical clearance (Appendix B), which was granted. Participants' responses were coded and have been referred to in this way in the study. The researcher avoided plagiarism and followed the rules and regulation of the institution. All other persons' ideas in the study received proper acknowledgement. The study was also submitted to turnitin (Appendix A) and the report was within the acceptable standard.

3.8 LIMITATIONS AND DELIMITATIONS OF THE STUDY

In the qualitative study, the researcher is also part of the reality and hence the research may not be totally objective and value free (Struwig & Stead, 2001). The researcher was a full-time employee (mathematics teacher) of the Limpopo Department of Education. The

researcher carried out this study while she was engaged in her primary assignment as a teacher. This created time constraints to the study's scope and thereby the study was limited to only three participants. This, together with financial constraints caused the limitation of the study. The researcher experienced language barriers because she was only proficient in English and most teachers in Mogalakwena District were more comfortable with their local language (Sepedi). As a result, the researcher limited the number of the participant to three due to time and financial constraints.

3.9 SUMMARY

The aim of this chapter was to highlight the rationale for the conceptual underpinning of this qualitative research. This chapter was constructed with a focus on the research methodology: to describe the research setting and sample and describes data collection and analysis methods. It further highlighted the research paradigm and the motivation for selecting the research methodology. The population and sampling criteria were also discussed and justified, so also the motivation for using observation and interview as data collection instruments. In addition, the chapter included the issue of ethical compliance and the reliability and validity of the study.

The findings and interpretations of this research were discussed in Chapter four. Data collected from the participants are grouped on the basis of relationship or frequency and thereafter, arranged to form the themes which have links with the research questions.

CHAPTER 4: DATA ANALYSIS AND INTERPRETATION

4.1 INTRODUCTION

This chapter provides systematic analysis and interpretation of captured data gathered from the participants. In order to address the research question, the PCK for teaching quadratic function was described in terms of teacher's content knowledge, knowledge of curriculum,

knowledge of instructional strategies, and knowledge of learners. Lesson observation and semi-structured interview were used to collect the data. The two instruments were integrated for clear understanding of the participants' view or action with regards to the teaching of quadratic function in grade 10. The researcher draws both the lesson observation protocol and interview instrument (see Appendices I and J respectively) from the PCK framework. It was used together with the Curriculum document (Annual Teaching Plan (ATP)) to examine teacher's PCK for teaching quadratic function in Grade 10. The data of the three participants was examined concurrently based on PCK themes. Tables 4.1 serves as the PCK theoretical framework that guides the collection and analysis of the data on how teachers teach quadratic functions in Grade 10.

Table 4. 1: PCK framework used in this study: Adapted Shulman's (1986, 1987) concepts of PCK

Themes	Sub-themes
Content knowledge	- Addressing key ideas of quadratic function - Types of connections that featured in the teaching of quadratic function - Aspects of grade 10 quadratic function that teachers find challenging to teach
Knowledge of curriculum	- Concepts sequencing - Goals of quadratic function in grade 10 - Compliance with teaching strategies prescribed in CAPS
Teacher's knowledge of the learners	- Identifying and making connection with learners' prior knowledge during the teaching of quadratic function - Addressing common errors and misconceptions in quadratic function - Understanding the aspects of quadratic function that grade 10 learners usually find difficult

Knowledge of instructional strategies	• Using various representations and resources (e.g. learners' prior knowledge, mathematical languages, symbols, real-life example, diagnostic reports etc.) to explain and demonstrate quadratic function • Using various approaches (e.g. investigation, questioning, problem-solving) to assess learners understanding of quadratic function

4.2 DATA ANALYSIS PROCESS

The researcher incorporated the field notes, the photos and the audio tapes of the teaching observation and interview together. The lesson observation afforded the researcher the opportunity to gain precise details on how teachers teach quadratic function in grade 10. The researcher used interviews to explore more understanding on what was observed during the lessons. Hence, both observation and the interviews were integrated in this study. It is also important to bear in mind that some of the sub-domains used in this study may be relevant to describe more than one domain of PCK, this due to the fact that the domains of PCK are interwoven. For example the use of learners' prior knowledge was to be related to all the four domains of PCK used in this study. The researcher tried not to repeat any of the sub-domains in order to get clear and separate answer to each of the research questions.

The audio tapes are hand-written and are later typed to be stored in the computer. The researcher listened to the audio several times and examined the field notes and the photo in order to have a thorough grasp of the events. The researcher then assigned labels to the participants' activities that occurred during the teaching of quadratic function. The researcher used Microsoft Word to do the coding. The observation data of the participants were according to different documents using their pseudonyms (Participant one = T1; Participant two = T2; Participant three = T3) to save the file on the computer. The researcher used the code S1 for school 1; S2, for school 2; and S3 for school 3. The codes were sorted into categories which were informed by the themes of PCK. Thereafter, the researcher synthesised and discussed the results from the three documents (each for T1, T2, and T3)

based on the research questions, the literature review and the PCK framework. In most cases, learners responded in chorus, and thus identified as "learners". The researcher used X to code learner (for example X1T1 for learner one of Participant one) where learners were individually identified.

CAPS is a curriculum and it combines several policies such as subject policy, assessment policy, and progression. The ATP (Annual Teaching Plan) is drawn from CAPS. ATP is otherwise known as the scheme of work. The researcher used the ATP as blueprint to examine the key ideas of quadratic function that teachers are expected to teach in Grade 10 (content knowledge); the sequencing of the key ideas and the type of activities that is prescribed in the curriculum (knowledge of curriculum); as well as mathematical work of teaching that teachers employed during the teaching (knowledge of instructional strategies).

Table 4. 2: The key concepts of quadratic function in Grade 10: Adopted from Limpopo Department of Basic Education (2018) Grade 10 Mathematics ATP

Topic	Content
Quadratic function	1. Work with relationships between variables using tables, graphs, words and formulae and convert flexibly between these representations. 2. Point by point plotting of basic graphs defined by $y = x^2$ to discover shape, domain (input values), range (output values), axes of symmetry, turning points and intercepts on the axes. 3. Investigate the effect of a and q on the graphs defined by $y = a.f\left(x\right)+q$, where $f(x) = x^2$. 4. Sketch graphs, find the equations of given graphs and interpret graphs.

4.2.1 Research question 1: What knowledge of Grade 10 quadratic function do teachers possess?

This research question is focusing on teachers' content knowledge. In other words, the teachers' knowledge of quadratic function for teaching grade 10 learners. Hence, the researcher used ATP to explore the key concepts of quadratic function that teacher supposed to teach in grade 10, and explored the kinds of connections that teachers made during the teaching of quadratic function in grade 10.

4.2.1.1 The key ideas of grade 10 quadratic function

The main concepts taught by the three participants were observed during the lesson observations and were checked against the ATP. The researcher gathered that the three participants represented quadratic function in different ways. They converted the algebraic formulae into tables, and further used the tables to plot the graphs. They converted with a purpose to compare two graphs or explain the features of parabola. For example, T1 drew the Figure 4.1, which shows the representation of the relationship between the equations and the table without graphs of the three equations.
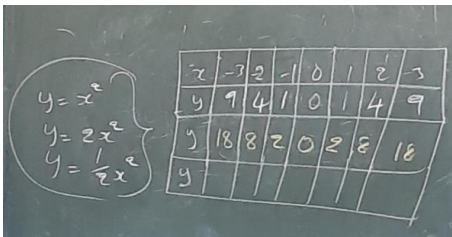
Figure 4. 1: Convert the algebraic formulae into tables: T1's case

In addition to algebraic equations, tables and graphs representation, T1 and T2 also facilitated learners to convert table values into pair-order form which is another form of representations of relationship of x and y in quadratic function.

T2 did not explain the concept of turning point as an entity. She only mentioned the turning point while discussing the effect of parameter 'q', stated that: " 'q' causes the turning point to be above or below the x-axis." Therefore, she did not teach learners how to write the coordinates of the turning point. T2 did not use basic quadratic function in her lessons.

T3:   Maximum and minimum turning point, let say the graph doesn't turn at 0 and 0.
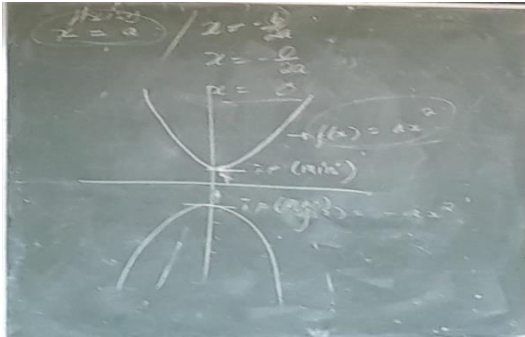
Figure 4. 2: Types of turning points: T3's case

It was not clear why T3 used the graphs that do not turn at $(0; 0)$ to explain maximum and minimum turning points before T3 had introduced parameter 'q'. Yet, T3 referred to the graph as the graph of $f(x) = ax^2$. The two are contradictory. The graph that does 'not turn at 0 and 0' cannot be $f(x) = ax^2$.

T1 led learners to complete the table for the y-values of $y = \frac{1}{2}x^2$. And also for the "negative a". $y = -x^2; y = -2x^2; and\ y = -\frac{1}{2}x^2$. T1 showed the effect of parameter 'a' on the stretch of parabola, whether the parabola will be wider or narrower.

T2:   Let's say we are given an equation $y = 2x^2$, the value of 'a' is greater than 0, which means our graph will face up. We should also have the equation $y = -2x^2$. We should have a table method, where we are going to have the x-values and the y-values.

This was the first set of algebraic examples cited by T2. However, she used these examples to demonstrate the difference between the graph where the value of 'a' is positive and the value of 'a' is negative. The researcher maintains that T2 should have started with a basic graph of quadratic function, where the value of 'a' is one, $y = x^2$, as it forms the basis by which other examples are compared. Also the teaching should be investigative whereby learners should conjecture through point-to-point plotting of graphs.

T3:   Let's look at the arms. The effect of these arms is on the shoulder of the value of 'a'.

T3 led learners to the discoveries of effect of 'a' on parabola, in terms of stretch.

T3: Let's start to substitute.

The teacher started with zero, going towards the right side (positive number) and indicated that this set of numbers are also the answers to the left side in that order. *9; 4; 1; → 0 ← 1; 4; 9.* The researcher understood T3's intention to develop learners to be methodical and more adept at filling the tables.

T3: We have given ourselves the values of x, so that we can have the values of y. And don't forget $f(x)$ is the same as?

Learners: *y!*

Figure 4. 3: Use different notations for output: T3's case

In the figure above, T3 used different notations to differentiate quadratic equations. This would also show learners that notations of the function are not limited to y and $f(x)$.

T3 did not mention the effect of 'q' with regard to vertical shifts throughout his teaching.

Content number 4 of the ATP deals with higher-order activities of quadratic function. They require high competence of the teacher in both content and pedagogical skills. T1 mentioned that this section would be discussed in the end, after other forms of function had been completed. It is important to bear in mind that the lesson observation took place over two days (one hour each); hence T1 was possibly unable to cover the whole contents of quadratic function within these short periods.

The sketching of the graphs should be based on the effects of parameters 'a' and 'q' which have been previously discussed. None of the participants taught this aspect. T3 briefly mentioned that "Sometimes people don't use table method automatically they know that if the equation is like this (Referring to $y = x^2$) then the graph will be like this (face up)". T3 did not teach this aspect in his lesson, hence only T2 was observed in this regard.

T2: Calculate the coordinates of A, B, C, D, and E.
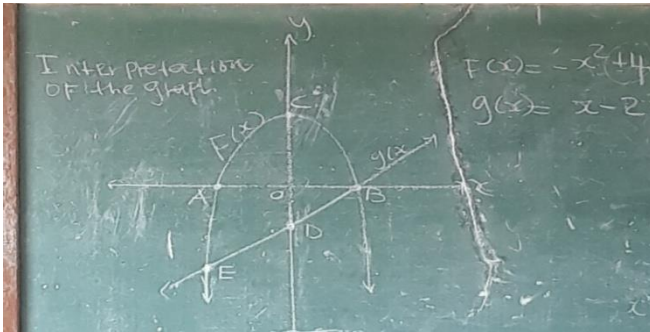
Figure 4. 4: Interpretation of the graph: T2's case

T2: How to interpret the graphs. We are having two equations where $f(x)$ is a parabola and $g(x)$ is a straight line. How are we going to calculate the coordinates of A and B?

T2 explained that A and B are x-intercepts. This is a simple exercise that learners should be able to identify from the graph.

Table 4. 3: Participants identify and organise key ideas of quadratic function

S/N	Expected	Observed
1	Point by point plotting of basic graphs defined by $y = x^2$	T1 and T3 used basic quadratic function in their teaching. T2 did not.
2	Use $y = x^2$ to discover the seven	Only T1 used basic quadratic function to explain the features of parabola.

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	features	T2 and T3 only talked about the features of parabola when they were explaining the effects of parameter 'a' and 'q' T1 mentioned all the 7 features. T2 briefly mentioned 6 with exception of axis of symmetry. T3 taught 5 features, with exception of domain and the range.
3	Effect of parameter 'a'	T1 used different examples to illustrate effect of 'a' in terms of 'smile/ sad' and stretch. T2 taught only the 'smile/ sad' aspect. T3 taught both 'smile/ sad' and stretch.
4	Effect of parameter 'q'	T1 taught effect of 'q' T2 taught effect of 'q' T3 failed to teach effect of 'q'

It was observed that T2 and T3 missed some key aspects of quadratic function they were supposed to teach grade 10 learners. The researcher perceived this may be that the teachers were failing to use the relevant curricular documents to plan their lessons, or lack the subject matter knowledge of such aspects. All the three participants failed to explain to learners that the representations mean the same relationship albeit in different forms. Also, none of them gave example of word problems that illustrate quadratic function. Hence they all failed to teach this key concept of quadratic function.

4.2.1.2 The types of connections teachers make

T1 gave the learners idea that various types of functions are related.

T1: That's all we talk about in a parabola. We move on, we talk about another graph, another graph. At the end of the day we are going to put all these graphs and combine them. We talked about the linear function and straight line graph, we talk about a parabola. We move on now, we'll talk about another type of a graph – hyperbola.

T2: Parabola faces up or faces down depending on the sign of the value of 'a'. If the value of 'a' is greater than 0 ($a > 0$), then parabola will face up; it will smile. And if the value of 'a' is less than 0 ($a < 0$), the parabola will do what?

Some learners: It will frown.

T2:   Then it will face where?

Learners: It will face down.
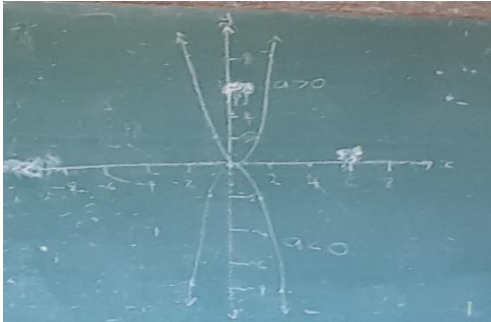
Figure 4. 5: Shapes of parabola: T2's case

T2: $a > 0$ means the value of 'a' is positive. If $a < 0$, it means the value of 'a' is how much?

Learners: Negative!
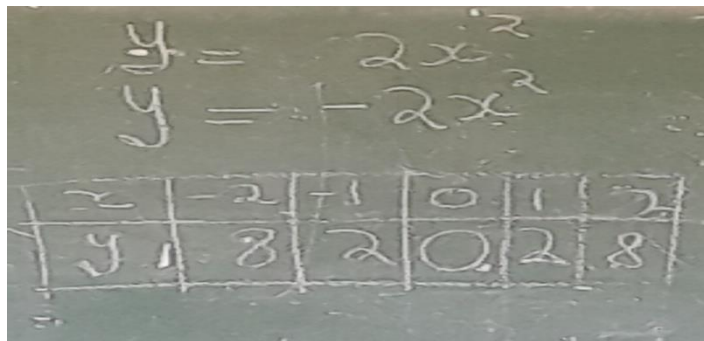
Figure 4. 6: The values of 'a' and their effects: T2's case

T2 linked the type of shape of parabola to the sign of the value of parameter 'a'.

T2:   The equation of the parabola $f(x) = ax^2$. We know that $f(x)$ represents…?

Learners: y!

As can be seen above, T2 guided learners to identify alternative ways of writing 'y'. Learners were able to identify the present topic with similar previous knowledge. Also, they could identify that 2 is an exponent as they read along with the teacher, 'f of x is equal to ax to the exponent 2'.

T2:   If we have this one ($y = ax^2$), we don't have the y-int. which is q. Again, we have got the equation of parabola which is in the form of $f(x) = ax^2 + q$. Now we are going to look at the effect of q.

T2 linked the two general formulae of parabola.
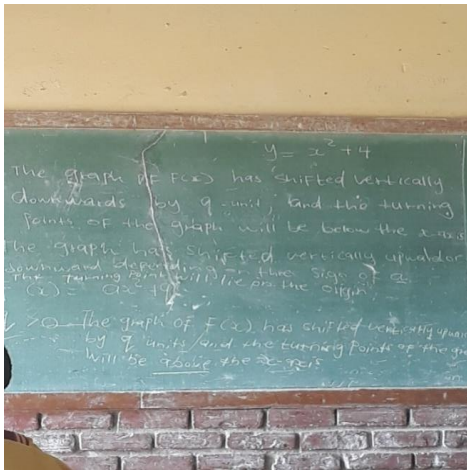
Figure 4. 7: Definition of vertical shifts: T2's case

T2 related the parameter $'q'$ with another important feature of parabola, the turning point. By this, T2 was doing what is required in content 2, which also implies making connections within the concepts of quadratic function. However, her statement indicated that she was already telling learners the outcome of their investigation. The researcher deduced that the teacher lacked adequate knowledge of the curriculum requirement especially in terms of activities/assessments.

T3:   Let's go to y-int. at that point the value of x is 0. Then we say…?

Learners: let x be 0
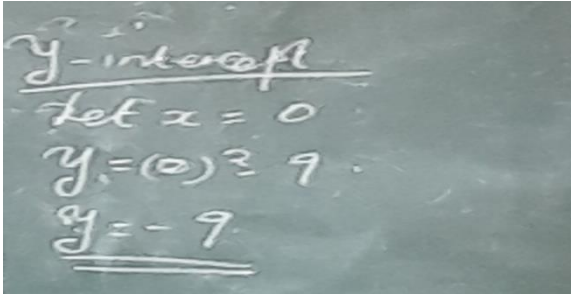
Figure 4. 8: Calculation of y-intercept: T3's case

T3 did not sketch the graph of $f(x) = x^2 - 9$. He related the axis of symmetry and the turning point. He used parameter 'q' to show how the turning point changed from previous $0 \ and \ 0$, to $0 \ and \ the \ value \ of \ q, -9$. He says, "At the turning point $x = 0 \ and \ y = -9$". He also made learners see that turning point as the same as y-intercept, $(0; -9)$.

According to Darling-Hammond (1998), connecting ideas within a topic/ or across fields and to real life situations provides better understanding to the learners; and forms a good foundation for teacher's PCK. The types of connections that the teachers made are few and limited to: connection with prior knowledge within mathematics; and connections among the features of quadratic function. T2 did more in this regards, more than her colleagues.

4.2.1.3 What teachers find challenging to teach in grade 10 quadratic function
The researcher assumed that key concepts that teachers omitted during the teaching might be challenging to the teacher. T3 was stuck when he arrived at question (e) below. T3 told the learners, 'if I were to do this, I will solve it using the inequalities method'. In other words, reading off the points from the graphs was challenging to T3.
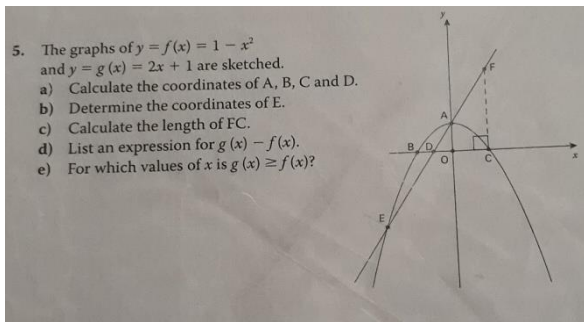
Figure 4. 9: A copy of exercises used by T3

The researcher followed up, during the interviews, on the aspects of quadratic function that are challenging for the teachers to teach.

Researcher: Which aspect of grade 10 quadratic function do you find more challenging to teach?

Below are their responses:

T1: Ehnnn. I don't really find anything challenging except in the case of learners, they confuse the shape of the graph, the maximum, the minimum.

No. I don't have any challenge.

T2: Myself? How to introduce, let say p is greater than zero, and when the graph has shifted maybe four units down, then what is the reflection of this thing. So these ones give me a little challenge.

T3: It's when they are talking about domain and the range. How to determine the range and domain. Sometimes you will find that what do we suppose to look so that we can conclude by saying this is domain; y is rational; y is greater than… sort of things. Because sometimes a learner must look at the shape of the graph whether it faces downward or upward where they have inserted the inequality. It is where I find a little bit difficult.

It was gathered in literature that to enable learners gain conceptual understanding of quadratic functions, teacher needs to possess more advanced mathematical knowledge that is beyond what the curriculum requires (Ball et al., 2005; Pourana, 2014). However, the researcher found that teachers do not have knowledge of some aspects of the quadratic functions stipulated in the CAPS. T2 and T3 indicated that some of the concepts of grade 10 quadratic function were challenging to them. This is evidence that the teachers' content

knowledge was not in the same level with their colleagues, and hence they do not posses advanced knowledge of quadratic function.

4.2.2 Research question 2: What are the teaching strategies teachers use to teach quadratic function?

In this context, the researcher explored various representations and instructional resources that participants used to facilitate grade 10 learners' learning of quadratic function, as well as how the teachers were evaluating learners' understanding of the concepts of quadratic function. In content three of the ATP, it is clearly stated that teaching activities of quadratic function should be based on "investigation" Hence, the researcher used PCK attributes of teachers' knowledge of instructional strategies to explore the approaches that teachers employed to teach quadratic function in grade 10. The researcher paid attention to teacher's pedagogical skills - examined how teachers make quadratic function accessible to all learners through different approaches such as improved questioning, team-work, explanation, demonstration, questioning and investigation. The literature indicates that the most appropriate instructional strategies for teaching quadratic function are through activation of learners' prior knowledge; paying attention to the structure of quadratic function; using multiple representations to communicate quadratic function; and most importantly, considering learners as the centre of teaching and learning of quadratic functions.
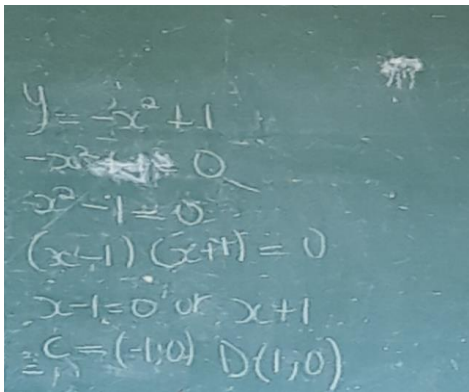
A factor that makes T1's teaching method relevant to teaching quadratic function is making connection with learner's prior knowledge. This alone does not make the method learner-centred. Actually, T1 (during observation) did not make learners apply prior knowledge; instead he used their prior knowledge to explain new concepts. The researcher saw this as one-way presentation or linear dialogue, hence termed it a demonstration method. The researcher had the sense that T1 understood that learners learned better when new experience is built on what they are familiar with. Although, the demonstration method made it difficult to differentiate learners' needs. When T1 was asked in the interview:

Researcher: Kindly describe your approach of teaching quadratic function?

T1: Is it not that I will be moving from what they know previously, step by step to what they should be knowing now. Moving from known to unknown. Like learners already know number line, horizontal line, vertical line, and quadrant. Because the first thing I will be teaching or asking them is what they already know. In that case one will be building the confidence that oh this one has been known and we are getting somewhere. They will know that from A we are moving to B, at B they will know we are moving forward.

T2 was observed to engage learners in activities. She called them to work the solutions on the chalkboard. The researcher regarded this as the participative method; it is a partly learner-centred approach. However, learners wrote solutions but they did not communicate how they arrived at those solutions, even when T2 requested.

Table 4. 4: T2's learners worked on the chalkboard

X1T2's work	T2's comment	X2T2's work	T2's comment
$D(0;1)$	How did you arrive at 0 and -1?	$y = -x^2 + 1$ $-x^2 \ldots 0$ $-x^2 - 1 = 0$ $(x-1)(x+1) = 0$ $x-1=0$ or $x+1$ $C=(-1,0)$ $D(1,0)$	Correct

According to literature, the appropriate method of teaching quadratic function should one that affords learners to express their thoughts. During the interview, the researcher asked:

Researcher: Kindly describe your approach of teaching quadratic function?

T2: Question and answer. Let's say maybe you give them question, and then the learners should be able to write the solution on the chalkboard. Question method. Example of such questions is when a teacher writes an equation on the chalkboard and asks

learners to come and show how to calculate intercepts. Even sometimes learners need to sit in groups to assist one another. Although, some learners don't participate. It allows every learner to understand. Questioning method allows more participation and teacher can easily identify those who do not understand the concept. If you group them together, most of the learners don't even do anything. They rely on other learners.

T2 compared and justified the method she used most frequently (questioning method) with the group method. She acknowledged the importance of group work but she said some learners may sit back and leave others to do the work. The reason for her preference is that it allows active learner participation. Observation indicated that she also used the method to differentiate learners so she could provide support to those who struggled to understand the concepts. However, the researcher did not gather sufficient evidence to state that T2 did not like group work because she cannot manage it effectively.

The researcher, however, observed a flaw which was raised during the interview session.

Researcher: You called a learner to work on the chalkboard,

$y = 3x^2$ and $y = -3x^2$, and then you introduced a new concept (writing equation of the graph). Both of you were working on the chalkboard at the same time, side by side. What is the reason for this?

T2: By doing things simultaneously so, we have already dealt with effects of parameters 'a' and 'q'. By giving them work on the chalkboard, I just want to see whether these learners understand the effects of 'a' and 'q'.

The researcher observes that the above case may distract the learners. The teacher's response failed to justify the action. However, the researcher decided not to probe further as she noticed (body language) that T2 was unwilling to talk more about it.

Similar to T1, T3 used the demonstration method to teach quadratic function in Grade 10 as observed during the class observation. T3's demonstration method was less effective compared to T1's.

Researcher: Kindly describe your approach of teaching quadratic function?" he answered:

T3: I can't say a specific method. I prefer to integrate different methods such as questioning and answering method, group method. Group method is where learners work in groups. It is by questioning method that I get to know their prior knowledge."

T3 could not specify a method. T3 mentioned two teaching methods he frequently used during the lesson. T3 did not use group methods during the class observation. T3 stated that he used questioning to ascertain learners' prior knowledge.

4.2.2.1 Using variety of representation and resources

The researcher also explored various instructional representations and resources that teachers used for the teaching of quadratics function in Grade 10. These include learners' prior knowledge, real-life situations, effective communication skills, improved questioning, symbolic and concrete representations, diagnostic reports etc. The use of real-life examples cannot be over-emphasised in the teaching of quadratic function.

T1 mentioned real life objects only when he was explaining the meaning of the word "features". He said, "Features of the graph. Feature is like saying the kind of a person Mr Y (name withheld) is. Is he a tall person? Is he a fat person? Is he white or is he black? That is the features".

T2 did not cite any real-life examples during the lesson. T3 used the analogy of a mirror to explain the concept of reflection. While teaching $y = -ax^2$, T3 did not do point-by-point plotting, rather he made a rough sketch on the same axes with that of $y = ax^2$. He used this to explain the concept of reflection of parabola.

T3: The only place we see the reflection is when we are visiting a mirror. The thing that we see inside the mirror is the reflection, the image. Meaning if we have $y = -ax^2$ the value of 'a' is -1, it means our graph will face…

Learners: Down.

The three participants did not give real-life examples due attention as recommended by the mathematics curriculum. This signified that teachers did not have adequate knowledge of CAPS.

Researcher: What are some relevant real-life examples that can help learners to understand quadratic function?

T1, T2 and T3 respectively said:

T1: Is it not that if you talk about the real-life examples, if you want to divide something into two equal parts, like if I say divide myself into two, I will be talking about axis of symmetry. If you are cutting an orange, an even set of twins.

T2: Real life situation in quadratic function will not work (*she meant not possible*). Eh, I'm not aware of any.

T3: It is just that sometimes I find it easier to use real life examples in one topic than another. What can I relate quadratic function to in real life situation? I fail to think where I can find quadratic function in real life, unlike when we talk about the geometry – the building of the bridge.

The use of real-life example for teaching quadratic function will provide learners with better understanding. It also defines a teacher with a high level of PCK. T1 acknowledged that he knew relevant examples but did not use them in class. T2 did not know any real-life examples that she could use in the teaching of quadratic function in Grade 10. This showed she had never thought of their importance and necessity. It is acknowledged that teacher may find real-life examples or analogies more easily accessible in one topic than another. Like T2, T3 does not have examples of real-life examples that can be used to teach quadratic function to the learners.

Of important interest was the way that each participant explained the concept of the general formula (e) of quadratic function during the lesson observation. The researcher explored how the three teachers used their content knowledge to explain the quadratic formula.

T1: This is the general formula of parabola $y = ax^2 + q$. There is 'a' there, there is also 'a' here; there is 'q' there, there's 'q' here.

T1 showed parameters 'a' and 'q' together. This may mean that the two parameters are inseparable. It is important to let learners understand that 'a' and 'q' are constants and a≠0.

T2 introduced the general formula better than T1. She broke the general formula down into two forms. She stated the first one as: "Equation of the parabola is $f(x) = ax^2$ where $f(x)$ represents y." She said, "Also we have the equation of parabola in the form of $f(x) = ax^2 + q$" By doing this, learners will understand that quadratic function does not necessarily requires 'q'. That is, 'q' can be zero.

T3 provided the general formula of quadratic function in another way. He said, "We have the general $y = ax^2 + bx + c$ but in your grade we talk about $y = ax^2 + q$. He equated the two formulae and made learners believe that c means the same q. However, this statement cannot be generalised. It may lead to misconceptions. He also forgot that the formula $y = ax^2 + bx + c$ is taught previously in the same grade during quadratic equations.

T2 did not use the graph of $y = x^2$ to illustrate the shape of the graph; instead she explained the shape in connection with parameter 'a'. She asked a leading question with the intention that the learners discover the answer. T2 said, "Parabolic graph can have a smile or it can frown; face up or face down".

T3 did not refer pertinently to the shape of quadratic function; however, he used the term, parabola very often during teaching.

All the three teachers used the 'sad and smile' analogy to represent 'face down and face up' parabolas respectively.

T1's explanation of domain was good. "'x' is an element of real numbers. $x \in R$" as he made use of graph representation and linked it to what learners had already learnt.

T2 gave a brief definition of the domain in the introductory stage of her lesson, when she explained the meaning of a function. "Input values refer to domain, where the domain represents all the values of 'x'"
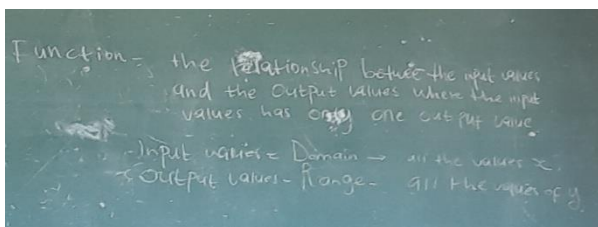
Figure 4. 10: Definition of a function: T2's case

T2 did not use any representation to illustrate the concept of domain and did not evaluate learners' understanding of the domain. T2 did not teach learners how to write the notation for the domain, although she related it to the concept that learners were familiar with, that is, input. T2 only remembered to define 'a function' after starting to explain the effect of parameter 'a' with examples $y = 2x^2$ and $y = -2x^2$. However, T2 effectively linked learners' past knowledge with new information by explaining the concept of a function in terms of input values and output values.

Writing the range of parabola is more methodological than writing the domain. Hence, teacher needs to illustrate the concept beyond verbal explanation. The question of the range often comes up during examinations. Teachers are expected to use learners' knowledge of inequalities to aid the teaching process; thus, learners will better understand and apply their prior knowledge to write the notation in different forms.

T1 used the appropriate graph to demonstrate the range of parabola. T1 gave a comprehensive explanation using the graph. T1 also guided the learners on how to write correct notations of the range. T1 used terminologies with which learners were conversant.

T1: All the numbers along the vertical line. But if you check (referring to the yellow graph in Figure 4.18), our vertical line does not include the numbers here. They start here (*0; 0*) and they are going up. The 'y' is greater than or equal to zero.

T1 explained without asking for learners' inputs. T1 then wrote the notations, "$y \geq 0$ *and* $y \in$ R". After the explanation, T1 did not assess learners' understanding of the range in any activities throughout the lesson.

Axis of symmetry of a parabola is a concept that a teacher can easily connect with real-life examples and after the explanation; learners should discover the equation of axis of symmetry. In the beginning of the lesson both T1 and T3 listed the features of parabola they wanted to discuss on the chalkboard. Axis of symmetry was included.

T1 gave a brief explanation of axis of symmetry in terms of distance, using graph representation. However, T1 did not show learners how to write the equation of axis of symmetry nor assessed learners' understanding of the concept.

T1: If the graph is up to standard, there is something called axis of symmetry. It means the distance from here to here (still using the graph of $y = x^2$) and the distance from here to here must be the same.

T2 failed to teach axis of symmetry.

T3 explained the axis of symmetry using a rough sketched parabola.

T3: Let me explain the axis of symmetry. What is axis of symmetry? Yes, X1T3?

X1T3 used his hands to demonstrate the axis of symmetry. The teacher eventually provided the answer.

T3: It is a line which divides the graph into two equal parts.

T3 repeated the explanation of axis of symmetry when he was discussing the effect of parameter 'a'.

T3: Axis of symmetry is a (vertical) line and it can be sketched. If you look at our graph, don't forget that along this line which is y-axis, the value of 'x' is zero. And along this line the value of 'y' is zero. Axis of symmetry is written in the form of $x = a$, where 'a' can be any number.

T3 demonstrated using the parabola on the chalkboard, with emphasis on the y-axis.

T3: If you look at our graph, don't forget, along this line which is y-axis, the value of x is…?

Learners: Zero!

T3: Along this line the value of y is…?

Learners: Zero!

In this instance, the teacher was referring to the coordinates of the turning point.

T3 explained the turning point in a unique way. T3's emphasis was on 'b' which he said was missing in the equation of quadratic function. The teacher related the general formula $y = x^2 + q$ to $y = ax^2 + bx + c$ and said because the value of b is zero "Our turning point is going to turn along y-axis always". T3 implied that the x-value of the turning point is the same as or depends on 'b'.
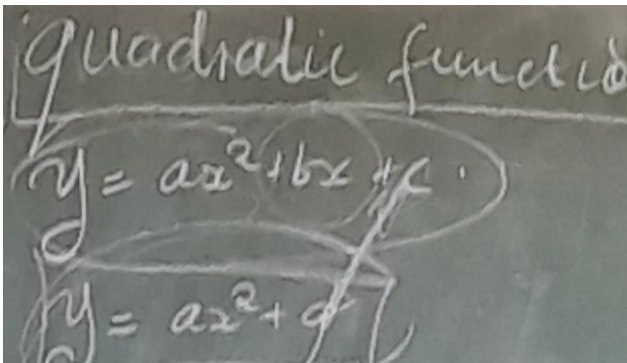
Figure 4. 11: Two forms of quadratic formulae: T3's case

T3 used a graph to demonstrate his explanations.

T3: As long as you are talking about parabola, it means somewhere it has to turn. It means the turning point is going to turn along the y-axis always, since the value of 'b' is zero. Before we can talk about this graph in full, then we have to know $y = x^2$. What about the turning point?

Learners: 0 and 0!

T3: The last thing that I want to talk about is the maximum and…?

Learners: Minimum!

This is one of the instances that indicated that T3 taught this topic in order to prepare the learners and him for the researcher's lesson observation. Hence, some of the actions the

researcher encountered in this class were not spontaneous. For example, the teacher started a definition (for the first time) and learners completed it with the correct response which the teacher expected.

T3 explained, "x-intercept is the point where our graph is going to cut x-axis. At that point, the value of y is…?" Learners responded, "*0*".

T1: If they ask for the turning point, they will be needing the two ordered-pairs, one for the x and one for the y.

T1's explanation of the turning point was brief but with the aid of graph representation and use of key terminology, ordered-pairs, learners easily grasped the turning point of the parabola. Learners stated the turning point of the parabola under discussion, '0 and 0'. T1 also talked about the turning point while discussing the effect of parameter 'q': "Because of the value of q which is one, the turning point will not be 0 and 0 anymore"

T1 used a lesson note. T1's learners used calculators and possessed mathematical sets which they used in making the tables and sketching graphs. T1 wrote exercises from Platinum Mathematics Textbook, Grade 10. T1 was the only teacher who used different coloured chalks while plotting parabolas; the others used the regular white chalk. The colour chalks enhanced the chalkboard work. The researcher asked during the interview, "I have seen you using different colour chalks in your teaching of quadratic function, especially when you were sketching the parabolas. What is the motive behind this?" He responded:

T1: Colour means a lot. Isn't it that when you use colour it is for learners to remember specially if you draw more than one graph on the same set of axes. Teacher must use different colour chalks so that learners don't confuse graphs with another. I duly recommend that teachers do it especially anything that has to do with diagram, even in geometry.

T1 used colour chalks to elicit learners' interest. He believed that using colour chalks made the plotting of parabolas more meaningful and clear. This showed that the teacher used a strategy based on his understanding of how learners learn quadratic function in Grade 10. He also used NCS exam question papers (2017) and each learner had a copy.

T2 and T3 used very few materials compared to T1. T2 used Siyavula Mathematics Textbook, Grade 10 and her learners used calculators. T3 used Classroom Mathematics Textbook, Grade 10 and learners used calculators. During the lesson, T3 made copies of an exercise (from the textbook, figure 4.16) for the learners.

T1 only used one example to explain the 'features of parabola'. There was no exercise that the learners finished on their own with regard to this content. The learners only practised how to complete the table and plot the parabola by means of point-by-point. The activities allowed partially for investigation. Learners did the activities while the teacher instructed: "Learners filled tables. Converted table values to ordered pairs. Sketched different graphs on the same axes." Learners were not prompted to complete most tasks independently. The teacher did the thinking and foretold the outcome of the investigation. Thus, learners were not facilitated to present their observations. Hence the teacher led the investigation and generalisation of parameters 'a' and 'q'.

T1 cautions learners about the shape of the graph as learners engage with plotting of the graph. "Be careful about the shape of the graph". He wrote two symbols on the chalkboard, not V but U). He was teaching learners to be skilful and methodical.

The researcher examined various examples and exercises that the teachers used in terms of algebraic representations. The aim was to see if the teacher explored all the needed values of parameters 'a' and 'q' that enabled the required investigation and generalisation. All the three teachers used both positive and negative 'a' values to explain the directions in which the arms of the parabola faced with regard to the value of 'a'. This they termed, "effect of parameter 'a'". According to the participants, when the value of parameter 'a' is positive, the parabola is 'arms up/smile' but when the value of parameter 'a' is negative, the parabola is 'arms down/sad'.

Learners were expected to carry out investigative activities and state what they observed. The teacher was supposed to facilitate the activities.

T1 explained the effects of parameter 'a': $y = x^2$

T1: Is the shape like this, or the shape is like this? It's concave up or concave down. The value of 'a', if the value of 'a' is negative, if the value of 'a' is positive. It has got impact on the graph.

T1: If you are having there *1; 4; 9*, on this side you are still going to have *1; 4; 9*. If you are having *2; 8; 18* there, you are still going to have *2; 8; 18* here.

This was an effective observation which encouraged learners to be methodical. If learners understood this explanation, it would help them to fill the table of values of quadratic function correctly and quickly.

T3 introduced parameter 'q' without giving its general formula. He wrote on the chalkboard $f(x) = x^2 - 9$. At this point he mentioned x- and y-intercepts.

T3: Don't forget, as long as the value of 'b' is zero, the axis of symmetry is going to be x = ?

Learners: 0!

T3: What's the value of 'a'?

Learners: 1

T3: What is the value of 'b'?

Learners: 0

T3: What is the value of 'c'?

Learners: *−9*

T3 only found x- and y-intercepts necessary here because of the dual intercepts method.

T3: x-int. is the point where our graph is going to cut x-axis. At that point, the value of y is …?

Learners: Zero!

T3: That's why we are saying let y be 0.

T2: We should write down the formula in the form of $y = ax^2 + q$. In order to find the value of "q" we should use (substitute) the coordinate of the turning point.
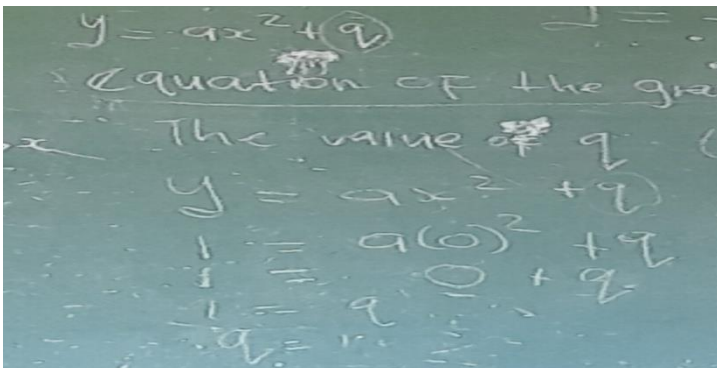
Figure 4. 12: Calculate 'q' to write equation of parabola: T2's case

The teacher explained the procedures of finding 'q' in the general formula of quadratic function.

T2: And to find the value of "a", we can either use the x-int. point on the turning point.
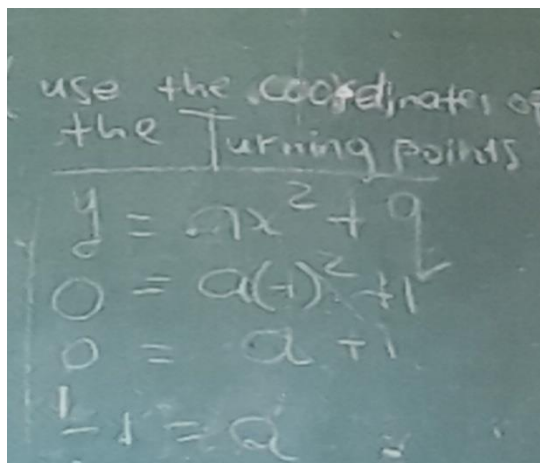
Figure 4. 13: Calculate 'a' to find equation of parabola: T2's case

T2: therefore the equation of the parabola is $-x^2$ plus how much?

She was inclined to pose leading questions.

Learners: *+1*

During the correction exercise, T2's learners provided answers of the calculation, as teacher asked at any step. This showed that the teacher allowed learners to contribute to the teaching-learning process. The researcher expected learners to make more significant contributions in terms of verbal presentations rather than answering leading questions.

T2:   Mere looking at the graph we see that the coordinate of y-int. is how much?
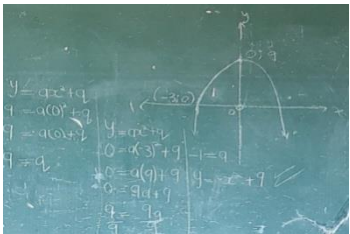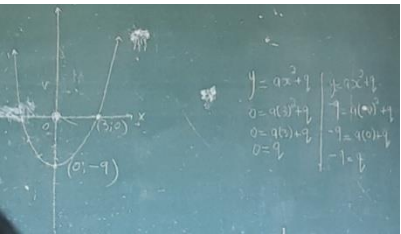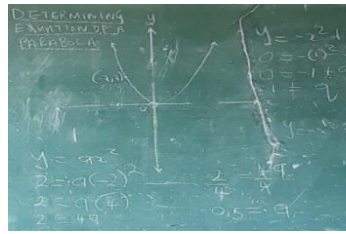
Learners: 0 and 1

T2:   Meaning that automatically q will be equal to?

Learners responded in chorus, 'one!'

T2 re-wrote learners' workings neatly and made corrections where necessary:

Table 4. 5: Activities on determining the equation of parabola: T2's case

Exercise 1	Exercise 2	Exercise 3

T2 concluded this aspect by asking learners if they understood it: "Are you satisfied?" and learners responded, "Yes." This affirmation cannot fully reveal the true situation. Thereafter, she introduced another concept of quadratic function: interpretation of the graphs.

T2 referred to Table 4.4 and explained:

T2:   In order to find A, B and C we are going to use the graph of $f(x)$. We know that $f(x)$ is the same as y.

Learners: Yes.

T2:   We can be able to factorise the equation using the intercept method. We can say that for   x-int.

Learners: Let y be equal to 0.

T2 has assisted the learners to become methodical. Whenever they came across x- and y-intercepts, they knew what it meant and what formula to use. She led learners to factorise in order to get the coordinates of A and B. In addition to identifying learners' prior knowledge (difference of two squares), they stated the factorisation out loud "$(x - 2)(x + 2) = 0$". The teacher should let the learners do this exercise on their own since learners have had previous experience in solving quadratic equations.
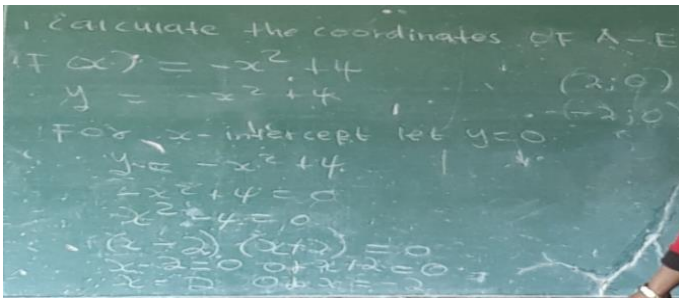
Figure 4. 14: Calculate x-intercepts: T2's case

After getting the two values of x, T2 asked learners to decide which one was for point A and which one was for point B. This simply means checking their prior knowledge of number system (number line).

T2:   Therefore the coordinate of the x-int. will be $(2; 0)$ and $(-2; 0)$. What will be the coordinates of A?

Learners made correct decisions. The teacher guided the learners to write the coordinates of A and B.

Learners: 2 and 0

T2:   What about the coordinate at B?

Learners: -2 and 0

T2 made learners focus on parabola. "Point C is also on the parabola". She further explained the position/role of point C on the parabola, "That is the turning point of the parabola. C is also the y-int., using the equation of parabola, what can you say about the coordinates of point C?" Learners were supposed to identify and justify the above explanations.

X1T2: 0 and 4

T2:   0 and …?

Learners: 0 and 4!

T2:   You can also say, for y-int

Learners: Let x be equal to 0!

The above would have been more meaningful if the teacher had allowed the learners to justify the answers.
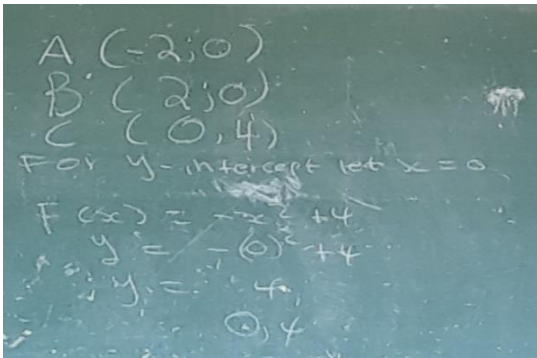
Figure 4. 15: The co-ordinates of key points on parabola: T2's case

T2:   Point E, that is the coordinate of the point of intersection of $f(x)$ and $g(x)$. If two graphs intersect, it means we are going to equate the two graphs together.

As usual, the teacher explained the calculations involved in determining the coordinates of the point of intersection.

T3 came to class with copies of a page (Figure 4.16) containing exercises numbered 4, 5 and 6. He treated only question 5 with learners, "Look at number 5". He drew the same graphs on the chalkboard.

T3:   You may be given the graph in this nature normally it's an application. You have to apply what you know about a straight line and a parabola. Don't forget that when we are saying coordinate, we are talking about point.

Learners: Yes.

T3:   You can't calculate the coordinate if you don't know the position of all the letters. For example, what's 'A'?

X1T3 responded, "A is the turning point". X2T3 said, "It's a meeting point of $g(x)$ and $f(x)$.

T3:   You understand what I mean.

T3 explained thoroughly using the graphs on the chalkboard.

T3:   It's the point of intersection between two graphs. It means the two graphs share the same point. It's (also) a turning point (of parabola). This (A) is y-intercept of both graphs. We are sure the value of x is 0, but we don't know the value of y. Because it's y-intercept, it means the value of x is 0. Substitute it in either $g(x)$ or $f(x)$

Table 4. 6: Two functions having the same y-intercept: T3's case

$g(x)$	$f(x)$

$y = 2(0) + 1$	$y = 1 - (0)^2$
$y = 0 + 1$	$y = 1 - 0$
$y = 1$	$y = 1$
$\therefore A(0; 1)$	$\therefore A(0; 1)$

T3 used axis of symmetry, turning point and y-int. to explain why x = 0 at point A.

T3: B and C, the positions are the same. What is B and C?

T3 gave learners enough time to think. The teacher then reminded them about the features of graph earlier listed. He asked learners to look at B and C along the parabola and say which of those features were related to B and C.

Learners: x-int.

T3: It means in order to get coordinates of B and C we must find the x-int. We let   x to be 0, neh?

Learners: y

T3 was doing the bulk of thinking and did not give learners any task to do independently. T3 only asked questions which could be quickly answered verbally. T3 was also fond of asking sarcastic questions: "x-int., we let x to be 0, neh?"

Table 4. 7: Learners identify 'difference of two squares': T3's case

$f(x) = 1 - x^2$
$y = 1 - x^2$
$0 = (1 - x)(1 + x)$
$x = 1 \; or \; x = -1$
$B(-1; 0) \; and \; C(1; 0)$

T3:   Is it clear?

Learners: Yes.

During this exercise, learners identified $y = 1 - x^2$ as the teacher asked, "What is this?" They answered, 'Difference of two squares'. They also identified which x-value belongs to B, and which one belongs to C. T3 learners could also factorise to solve for the values of x.

T3:   What is D?

Some learners: x-int.

T3:   x-int. of what?

Some learners: x-int. of a straight line

The teacher reminded them the formula, "for x-int. we let y be 0"

T3:   Which equation are we talking about between the two?

Learners: $g(x)$

Learners verbally stated the steps involved in solving for x, and the teacher wrote them on the chalkboard. Learners demonstrated their knowledge of the rules of solving equation.

Table 4. 8: T3's calculation of x-intercepts for linear function

$$0 = 2x + 1$$
$$-1 = 2x$$
$$2x = -1$$
$$x = -\frac{1}{2}$$
$$D\left(-\frac{1}{2}; 0\right)$$

T3: Point of intersection, where the two graphs meet. At this point the two points are equal. In order to get the point we have to equate $g(x)$ and $f(x)$.

Learners' body language showed they were surprised to hear the formula to find E, 'equate'. They murmured. The teacher led in calculation.

Table 4. 9: T3's calculation of point of intersection of g(x) and f(x)

Stage 1	Stage 2	Stage 3
$g(x) = f(x)$	$x(x + 2) = 0$	$y = 2(-2) + 1$
$2x + 1 = 1 - x^2$	$(x - 0)(x + 2) = 0$	$y = -4 + 1$
$x^2 + 2x + 1 - 1 = 0$	$x - 0 = 0 \ or \ x + 2 = 0$	$y = -3$
$x^2 + 2x = 0$	$x = 0 \ or \ x = -2$	$\therefore E(-2; -3)$

At stage 1, T3 checked on learners' prior knowledge, "Remember when we are factorising we have another method of factorising by taking out the…?"

Learners responded, "Common factor!"

Referring to stage 2's results, T3 asked, "between the two, which one can be at E?"

Learners: -2

T3: Why not 0? (an apt question by the teacher.)

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X3T3: Because the point is not along the zero.

T3: not along the y-int. It means here we have x = −2, and we have to find the value of…?

Learners: y!

T3: How can we find the value of y?

X4T3: By substituting

T3: By substituting. Where?

X4T3: In the equation

4.2.2.2 Using variety of approaches to assess learners' understanding

The teachers were supposed to facilitate learners to discover the features of parabola defined by $y = x^2$. These features include the shape, domain and range, axes of symmetry, turning point and intercepts with the axes. The features of the graph are interconnected and the teacher is expected to relate these features.

T3: Let's start to substitute.

T3 teacher started with zero. T3 asked learners the values of y, one after the other. T3 said, "If we substitute with zero" Learners responded in the same manner, "zero!" T3 continued towards the positive numbers. They filled up the table together.
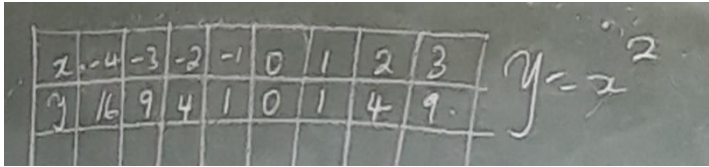
T3: If we substitute with 3?

Learners: 9!

T3: Is not 6?

Learners: No! 9!

Instead of using probing questions, T3 used sarcastic questions as demonstrated above, "Is it not 6?". This appeared to be the habit of the teacher. Learners' responses indicated that they were used to the teacher's style of questioning.

Figure 4. 16: Convert formula to table: T2's case

T3 put the points on Cartesian plane and thereafter asked the learners, "Who can join the points for me? Just join". While a learner was joining the point on the chalkboard, the teacher continued to interact with other learners.

T1:   The features of parabola: 1) the shape – is the shape like this or like this?

Table 4. 10: The shapes of parabola: T1's case
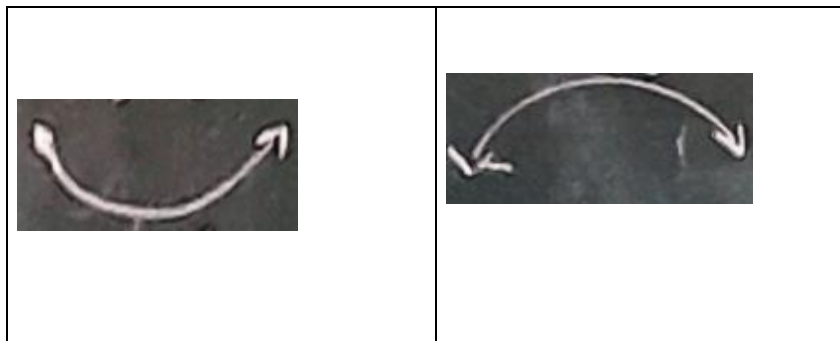
T1:   Concave up, concave down. Then we talk about 2) the value – we talk about maximum value and the minimum value. If the graph looks up like this, we say the

value of the graph is minimum. If it goes like this, we say the value of the graph is maximum.

Upon plotting the graph, learners were supposed to describe the shape of the quadratic graph in their own words (DBE, 2012). T1 did not ensure this. T1 explained the result. Although T1 provided a good explanation of the shape by using various names to indicate the two different types of parabola (concave up and concave down; minimum and maximum), T1 also used terms that learners are familiar with - a smile and a sad expression.

T1 asked, "What is the name of the graph?" T1 did not get any response and shifted to another concept: axis of symmetry. T1 asked an effective question but failed to follow it up.

T2: If the value of 'a' is greater than zero, the parabola faces up. If the value of 'a' is less than zero the parabola will do what?

Learners: It will face down!

T1: X1T1! If we talk about the domain what do we mean?

X1T1: The x-values.

At another stage, T1 used the graph of $y = x^2$, to explain 'the domain': "the number along the $x - axis$". In doing this, T1 activated learners' prior knowledge to aid their understanding of the domain.

T1: What is the meaning of this arrow? (Referring to the arrows at the terminals of x-axis line and y-axis line) It's going on and on.

This is another example of where T1 did not facilitate learners to answer the question. The teacher immediately provided the answer.

Unlike T1, T2 gave several exercises that required learners to calculate the intercepts with the axes; although it was done in connection with the effect of parameter 'q'. T2 guided the learners until almost all could correctly calculate the intercepts with the axes.

The researcher only experienced a sense of teamwork was in the class of T2, when one learner made a mistake (while working on the chalkboard) and another learner went to the chalkboard to assist. Collaboration among learners was below expectation.

There was no sign of grouping the learners in all the classes the researcher observed similar seating arrangements and this was a barrier to teamwork.

Table 4. 11: Learner seating arrangement in T1's class

L1	L2	L9	L10				L22
L3	L4	L11	L12	L13	L14	L23	L24
L5	L6	L15	L16	L17	L18	L25	L26
L7	L8	L19	L20	L21		Researcher	

All learners faced the same direction. They faced the chalkboard. They sat in twos at a table except in two cases (L21 and L22) where learners sat alone. The eight seats in the middle are joined horizontally.

Table 4. 12: Learner seating arrangement in T2's class

	L1						
L2	L3						
L4	L5						
L6	L7	L8	L9	L12	L13	L16	L17
				L14	L15	L18	L19
Researcher		L10	L11				

There were excess seats in T2's classroom. The seats were a bit disorganised and learners sat spread far apart. Similar to T1's classroom, the learners all faced the direction of the chalkboard.

Learners in T3's class also faced the direction of the chalkboard.

Table 4. 13: Learner seatin arrangement in T3's class

L1	L2							
L3				L13	14			
L4	L5		L8					L19
L6	L7		L9	L10	L15	L16	L20	L21
Researcher			L11	L12	L17	L18	L22	L23

T2's method allowed more learners to participate than in T1 and T3's classes. As the learners approached the chalkboard to do exercises individually, none explained what he/she did to other learners. The researcher maintains that if the teacher had further facilitated learners to explain their solutions, the method would have been more productive. T2 admitted during the interviews that it was wrong not to explain the solution.

T2: I think it is totally wrong not to explain their findings. They have to share. This is a mistake. I believe learners also learn from learners. Some learners don't understand from the teacher but understand better from their peers.

The researcher did not clearly understand if the teacher meant she was wrong or if the learners were wrong. T2 acknowledged the benefit of peer teaching if properly done.

T1: Let's compare the shapes of $y = 2x^2$ and $y = \frac{1}{2}x^2$. So that we can reach a conclusion

$$y = 2x^2;$$ it means the value of 'a' is what…?

Learners: (in chorus) 2!

T1: $y = \frac{1}{2}x^2$ It means the value of 'a' is what…?

Learners: $\frac{1}{2}$!

T1: Can you make this table, $y = 2x^2$, on the same table? Can you do that?

Learners: Yes!

Learners completed the exercise in their notebooks.

T1: You got the numbers?

Learners: yes!

T1: OK. Let's put the numbers. (Referring to Figure 4.1)

T1: What did you observe on the table?

X1T1: Multiples.

This was a good question. But T1 failed to probe further. When a learner provided the wrong answer, T1 did not address the error. However, the provided a good demonstration using the table of values.

T1: Can you plot the graph, on the same axes?

Learners: Yes!

Learners plotted the graphs of $y = 2x^2$ and $y = \frac{1}{2}x^2$ on the same axes with $y = x^2$. T1 then asked them to observe the difference and similarity.

T1: We are about to get something. Then, we can say something about the value of 'a' because that's where our interest is.
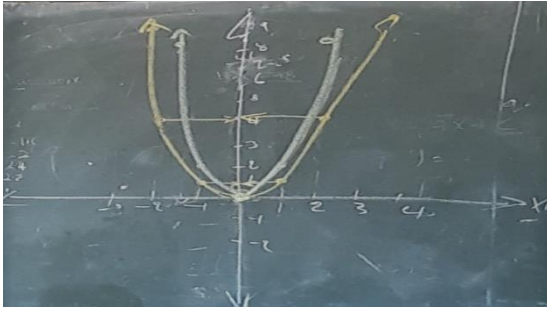
Figure 4. 17: Stretch-effect of parameter 'a': T1's case

T1: What happens if the value of 'a' gets bigger? What can you say about the value of 'a'? If the value of 'a' gets bigger and bigger, the graph gets like this (using hands to demonstrate 'narrower'). If the value of 'a' gets smaller and smaller, the graph gets wider.

In above extract, as in all cases, T1 explained instead of encouraging learners to investigate and discover the effect of 'a'. At another stage T1 used the sentence, "how the graph opens, and how the graph closes".

T1 explained the effect of parameter 'q': $y = ax^2 + q$

T1: Let's do something now, $y = ax^2 + q$: given that $y = x^2 + 1$, we are increasing the graph. The value of 'a' there is 1, the value of 'q' there is 1. Can you plot that on a table? Steve, can you plot that on a table? Then we'll be talking about shifting. Is it going up or is it going down? What can you say about the graph?

At this stage, learners were engaged in drawing the tables in their notebooks.

T1: Are you struggling?

Few learners: No.

T1: There's no one who must struggle with that. What are you supposed to do? All the number you have got previously, you just add 1.

T1 encouraged learners to be methodical.

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Learners: Ehnnnn! (Confirming they are following the instruction.)

Table 4. 14: Methodical skill for filling the tables: T1's case

X	-3	-2	-1	0	1	2	3
$y = x^2$	9	4	1	0	1	4	9
$y = x^2 + 1$	*9 + 1* *= 10*	*4 + 1* *= 5*	*1 + 1* *= 2*	*0 + 1 = 1*	*1 + 1 = 2*	*4 + 1 = 5*	*9 + 1* *= 10*

T1:   There's going to be something that we call shifting. Can you do that shape? Then in mathematics we say, the graph has shifted one unit up because of that value of 'q' which is 1. Can you draw that graph very fast?

Learners plotted the graph in their notebooks while the teacher moved around. The teacher interacted with learners one-by-one using their names. As learners continued the plotting, T1 kept pointing out the change in the turning point, thus forecasting the end result. This is one way presentation, which is a typically teacher-centred approach.

T1:   I'm not (0;0) anymore. The turning point will never be (0;0) anymore because of that *1 + 1 + 1 + 1* ... Did you get that?

Learners: Ehhhh!

T1:   If I say *+3*, the graph moves 3 units up! If I say *+4*, it means 4units up! What if I'm having minus, minus, minus? If I say *−1*?

X2T1: Down.

T1:   It will go 1unit down... 'q' plays a very important role as it tells us 'Is it moving up, is it moving down'. Do you understand that?

Learners: Yes!

The teacher explained what learners should have discovered thus allowing learners to be passive and unable to contribute to the learning process.

T1 used parameter 'q' to explain the vertical shifts (up and down) and how these shifts impact the turning point. T1 used methods which learners already know during the linear function to sketch the parabolas.

T1:   Can we check the exam question paper, 2017? Question 6.

All the learners had the 2017 National Senior Certificate (NSC) exam paper.

T1:   In this case the graph is like this, the equation says $y = 2x^2 - 8$. If you check there, it means 8 units down.

T1:   It means the value of 'q' is -8, that is why the graph is 8 units down.

While T1 was doing the effect of parameter 'q', the teacher only gave one exercise for practice (which later turned into an example), in which the value of 'q' is positive. However, T1 wrote a few others on the chalkboard and verbally explained their outcomes. Learners were not facilitated to complete most tasks on their own. T1 did the thinking and foretold the outcome of the investigation. Thus, learners were not facilitated to present their independent thoughts. Hence the teacher led the investigation and generalisation of parameters 'a' and 'q'.

T2 asked learners to see if the graph will face up, or face down. T2 drew the Cartesian plane on the chalkboard. The teacher showed how to make the scale and how to locate the point.

T2:   And here the point is 0 and 0, our origin… from there now we can join our points.
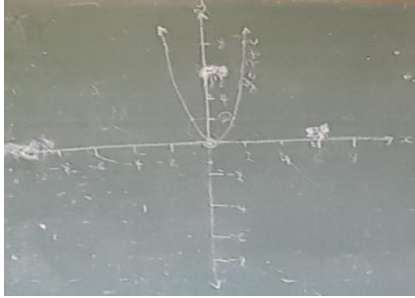
Figure 4. 18: The graph of $a > 0$: T2's case

T2: When we are looking for the effect of 'a', we normally say that if $a > 0$ or 'a' is positive, the graph will face up and it will have minimum turning point. We've got the x-values. We want the y-values, using this equation $y = -2x^2$.

T2 requested X2T2 to complete the table on the chalkboard. X2T2 used a calculator and correctly filled the y-values. However, the learner did not communicate her calculations with the class even when T2 demanded, "You have to show us how you arrive at the final answers".

Table 4. 15: T2's learner filled the table for y= $-2x^2$

X	-2	-1	0	1	2
Y	-8	-2	0	-2	-8

Thereafter, the teacher asked learners, "Do you understand how X2T2 arrived at those y-values?" Some learners responded, "No!"

T2: Where do you have problem?

The learners did not respond. T2 wrote another equation on the chalkboard $y = -x^2$ and requested for any learner to come and do it.

T2: Can you come and do this one where the input value is -1

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Possibly T2 meant to say 'a' value is -1. T2 explained the substitution process thoroughly using $y = -x^2$. T2 explained more than questioning and where the teacher did use questions, she asked leading questions. In these examples, T2 moved from complex to simple. Possibly if T2 started with basic quadratic function, learners would not have difficulty to comprehend complex examples.

In the end, learners indicated they had grasped how to fill the table/do substitution correctly. T2 confirmed their statement by requesting a learner to work with a similar problem, before moving to the next stage.

T2: Can you come and do this $y = -3x^2$, X2T2?

X2T2 filled in the table correctly.

Moving on to the next stage, T2 asked, "Who can come and plot the graph for $y = -2x^2$". Many learners raised their hands. X3T2 made the points and correctly joined the points to create a maximum parabola. Up to this point the teacher called on learners randomly and the learners carried out the activities without discussing their solutions.

The graphs of $y = -3x^2$ and $y = 3x^2$ are compared to show the effect of parameter 'a'.

T2: Can you see the difference?

Learners: Yes!

T2: This is the effect of 'a'.

This type of question is not appropriate in this kind of activity, where learners should investigate and generalise their findings. The teacher foretold the outcome of the investigation and failed to facilitate learners' independent thoughts of the graphs. Further, T2 did not talk about the effect of 'a' in terms of stretch (i.e., wide and narrow).

T3 called learners to plot only the points while he joined the points to make the parabolas on the same axes.
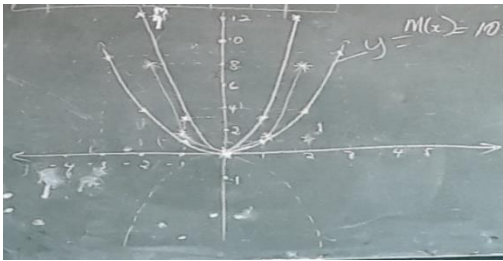
Figure 4. 19: Stretch-effect of parameter 'a': T3's case

T3:   What can we conclude about the values of 'a' if it is increasing? How does it affect the shape of the graph?

X2T3: It's getting closer

T3:   It's getting closer and closer to the y-axis. That's the effect of 'a'. What happen when the value of 'a' decreases?

X5T3: The graph will get wider

T3:   The arms get wider.

Table 4. 16: T2's calculation of point of intersection of f(x) and g(x)

Stage 1	Stage 2	Stage 3
$f(x) = g(x)$ $-x^2 + 4 = x - 2$ $x^2 + x - 2 - 4 = 0$ $x^2 + x - 6 = 0$	$(x - 2)(x + 3) = 0$ $x - 2 = 0 \ or \ x + 3 = 0$ $\therefore x = 2 \ or \ x = -3$	$y = -x^2 + 4$ $y = -(-3)^2 + 4$ $y = -(9) + 4$ $y = -9 + 4$ $y = -5$

T2 referred to the table above, the end result in stage 1, T2 asked, "Can you factorise this? Factorise the equation."

X4T2:   "$x - 2, x + 3$"

T2:   So what will be the coordinate of E? Are we going to take 2 or -3?

Learners: -3

T2 wrote the coordinate of E as $(-3; 0)$. T2 realised this is wrong and rubs it out immediately. T2 thereafter wrote: "$E(-3; y)$"

T2:   In order to find the y-coordinate (of E), we are going to substitute the coordinate of x (at E) on the original equation, either on $f(x)\ or\ g(x)$. We agree that $f(x)$ is y?

Learners: Yes,

T2:   Therefore the coordinate of E will be?

Learners: $-3\ and - 5$ (i.e. $E(-3;\ -5)$)

At five minutes before the end of the period, T2 quickly put another exercise on the chalkboard: Determine A – E. T2 differentiated the equation by assigning $g(x)$ to quadratic function and $f(x)$ to linear function. T2 called learners, per usual practice, to come and show their workings on the chalkboard.

4.2.3   Research question 3: How do teachers build learners knowledge of quadratic function?

The researcher believes that the teacher's knowledge of Grade 10 learners' characteristics influence instructional style, which in turn determine the success of teaching of quadratic function or lack thereof. In this study, knowledge about Grade 10 learners are categorised as: identifying and building on learners' prior knowledge; addressing learners' misconceptions during the teaching of quadratic function; teachers' awareness of aspects of quadratic function that learners often find difficult; and how teachers cater for learners' different needs.

4.2.3.1 Identifying and making connections with learners' prior knowledge

In this section the researcher examined how the teacher identified learners' prior knowledge and how that prior knowledge were used to tackle Grade 10 quadratic function problems. The researcher believes that learners do better when they can draw conjectures from past experiences into the new learning. Most of the learners' prior knowledge encountered in all three observations was rooted in learners' understanding of the number system.

The learners' prior knowledge observed similar to the three cases was 'Stating the formulae for finding the intercepts with the axes'. Learners chorused it: "for x-intercept, let y be equal to 0. For y-intercept, let x be equal to 0". Learners also used their knowledge of equation in quadratic function. The teachers also used learners' prior knowledge when they requested learners to fill in the table values. They understood that most learners could do 'substitution' correctly.

Learners had done factorisation in the precious grades and earlier in the year. T2 and T3 used this knowledge. They identified the knowledge and facilitated learners to use it to solve quadratic function problems.

T3:   We know this, what is this?

Some learners: Difference of two squares! (Referring to Figure 4.20)

T2 and T3 also identified and put to test the learners' understanding of the number line. T3 referred to the Table 4.8: "then we look at E, between the two, which one can be at E?" Learners responded, "-2".

Also when T1 presented quadratic function in ordered-pairs, he asked, "You remember ordered-pairs?" Learners confirmed they knew ordered-pair representation, "Yes."

Learners were familiar with the drawing of the Cartesian plane from previous grades. This made plotting of points on the Cartesian plane to make parabola easier for learners. In some cases, teacher needed to probe in order to systematically remind learners what they had learned in previous grades or lesson.

T3 identified learners' prior knowledge when he wrote $0 = x^2 - 9$. The teacher said, "We know this, what is this?" Some learners responded, "Difference of two squares". Then learners said the factors along with the teacher. They solved for x together. The teacher also showed them another way to solve for x. 'Also we can say'

$$x^2 = 9$$
$$x = \pm\sqrt{9}$$
$$x = \pm 3$$

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T1:   So if you take all the positive and all the negative number, together with zero and put them together, what type of number…? We have talked about the numbers in the beginning of the year.

T1 also showed how parameter 'a' determined the shape of the parabola using the analogy that learners were familiar with – smile and sad.

T1:   For the positive, our lecturer used to say smile; for the negative, he used to say that is sad.
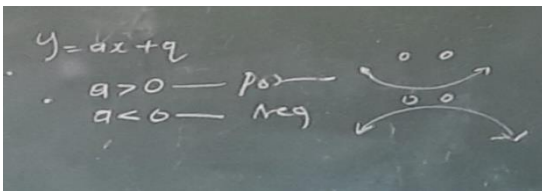
Figure 4. 20: 'Sad' and 'Smile' analogies: T1's case

All the three teachers used learners' prior knowledge to aid the teaching of quadratic function in Grade 10. During the interviews, the researcher gathered the opinions of the participants about the significance of learners' prior knowledge to the teaching of quadratic function.

Researcher: Teaching should build on the knowledge learners already have. What is your opinion about this?

They responded as follows:

T1:   Prior knowledge will enable me as a teacher to know where to start. Because if ever what I try to find out, like if most of the learners cannot remember the thing that they have done previously, then I will have to go back to remind them.

T2:   It is important. Before they can do a topic, we have to give them something related to that topic which they have done in previous years. This can happen through short class work. (How can you relate this to the teaching of quadratic function in Grade 10?). Uhnn, equations, factorisation of trinomial or taking out the common factors.

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T3:   I think learners should have a prior knowledge because they can start something with the foundation or a base. So before you can start a new topic make sure that there is another previous topic that is linking with the new topic that you want to present. So that it doesn't have a gap in between. (How can you relate this to the teaching of quadratic function in Grade 10?). Yes. I find it necessary that they must be able to plot the graph using points and then that is the learner's prior knowledge of a straight line which they have done in Grade 9 and early in Grade 10.

T1 uses learner's prior knowledge to understand learners and ascertain their readiness for the quadratic function. He also said that he provided support through revision to fill gaps.

T2 believed it is better to check learners' prior knowledge before embarking on the teaching of quadratic function. However, she did not state that the new knowledge would be based on what learners know. This may mean that the teacher sees quadratic function in Grade 10 as a separate topic. She stated that the best way to check learners' prior knowledge was to give learners short assessments and class work (not giving explanations as opined by T1). However, T2 did not observe this during the teaching of quadratic function. This implies that sometimes teachers know the best ways of teaching a concept, but they sometimes do not put them into practice. Knowledge of content came into play here too, as the teacher stated the concepts that one may link up to the quadratic function in Grade 10.

T3 was of the opinion that learner's prior knowledge serves as foundation on which new knowledge should be built. T3 specifically signified the learners' prior knowledge that could be directly linked to the new information. T3 believed in connecting mathematics. The teacher gave example of point-by-point plotting of linear graphs which learners are already familiar with. In addition to having the knowledge of Grade 10 learners, the teacher showed understanding of chronological development of plotting the graphs across the grades and across the topic. This implies knowledge of curriculum, although this in isolation cannot be count as deep knowledge of curriculum that is required to teach quadratic function.

4.2.3.2 Addressing learner's misconceptions

The teacher's ability to address learners' errors and misconceptions is significant to the teaching of quadratic function in Grade 10. The teacher should blend this knowledge with

knowledge of the content to facilitate learners' understanding of quadratic function. The researcher examined what the participants were doing with learners' error and misconceptions during their teachings.

T1:   So if you take all the positive and all the negative number, together with zero and put them together, what type of number…?

T1 paused for the answer. When there was no response, T1 simplified the question, "We have talked about the numbers in the beginning of the year."

Some learners: Integers!

Other learners: Real number!

T1:   'x' is an element of real numbers. $x \in R$

T1 was supposed to address learners' error or misunderstanding with regard to number system but instead the teacher accepted the right answer and moved on.

T1 addressed learners' error in some other instances. T1 mentioned, "I have seen some learners struggling, $(-1)^2 = -1$" In another example T1 warned the learners, "Be careful of negative sign. For example, in $y = -2x^2$, what is y when x $= -3$?" (T1 explained the solution of the chalkboard). During the interview, T1 provided a rational reason why learners cannot perform this task; the teacher indicated that the concept was not properly learned in the previous grades or topics. The researcher interpreted this as not properly taught by the teacher in a way that allows learners to develop conceptual understanding. The researcher believes this corresponded with the way T1 was observed teaching. This shows learners possess incomplete mathematics knowledge (leading to misconceptions transmitted from one class to another class) and the pattern may definitely continue unless there is an intervention. The researcher followed up during the interview to explore T1's opinion on the learners' use of the calculators:

Researcher: During your teaching, you moved around to check learners' works and you saw (mentioned) that some of them wrote:

$$(-1)^2 = -1$$

$$(-2)^2 = -4$$

$$(-3)^2 = -9$$

Then you said the error was due to the use of calculators. Do you think your learners would have done differently, perhaps correctly, if they did not use calculator?

T1: The case would be minimal because these learners are used to calculators that is my problem. Even if you say a negative multiplied by a negative, they go to the calculator of which if they do it manually, they will get it correctly. You know the concept of $- \times -= +$ (How do you think calculator is misleading your learners?). The problem is the manner in which learners use it. Misuse of calculator. Learner supposed to put the negative number inside the bracket, then square it to get the correct answer.

In all the three cases observed, the researcher gathered that some learners struggle with 'squaring of negative number' even when they use calculators. For example:

T2: I am very disappointed yesterday because when you say that, ehn, I have given negative two to the exponent two,$(-2)^2$, then you said the answer is -4. Looking at two (Referring to the exponent), two is an even number. If you raise negative number to an exponent even number, it means the answer should always be...?

Some learners: Positive!

In the above extract, T2 revised the concept of squaring of negative integers. The researcher believes that T2 was disappointed at learners' error because they have learned this concept previously. T2 further guided learners on how to use the calculator to do this concept correctly.

T2: On your calculator you have to use brackets, $(-2)^2$

The same way T3 warned his learners.

T3: I am appealing to those who are addicted to calculators; you have to put the value inside the bracket (he uses example $-2^2 = $ -4 and $(-2)^2 = 4$, to explain)

Mathematics teachers need knowledge of content and curriculum to be able to identify the cause of the problems and to proffer effective solutions. T3 mentioned that he would teach learners the concepts again in order to prepare them for the mid-year examination. However, the key idea T3 mentioned he would use to teach the learners was incorrectly stated, because the value of 'q' in quadratic formula does not always determine the coordinates of the 'turning point'. This again indicated that the teacher did not possess sufficient content knowledge required to teach Grade 10 quadratic function. The teacher provided learners with incomplete and/or wrong knowledge of the concepts of quadratic function, thus created more misconceptions for the learners.

The three teachers used their limited content knowledge to interpret what learners were thinking that brought about the error. In this regard, it was easier to correct the error and extend learners' knowledge of quadratic function. It is also important that teacher addresses learners' errors immediately.

In another example, T2 advised learners to always write fractions in a simplified form and to avoid decimals when writing the equation of parabola. Learners should write the values of the constants in fraction.

T2: Leave answer in simplified form $\frac{2}{4} = \frac{1}{2}$; let us also avoid decimal.

It requires teaching experience to gather and address common misconceptions of Grade 10 learners in quadratic function. T1 did better than T2 and T3 in this regard. He did not wait for learners to make mistakes; rather, he addressed the misconceptions in his explanation.

T1: Table method must not have big numbers. It may give you (learners) problems, e.g. if $x = 8$ in $y = x^2$, y = 64

T1: Be careful about the shape of the graph. (He wrote two symbols on the chalkboard, not V but U.)

T1 was developing learners to become methodical through the above statements and also used misconceptions as building blocks for teaching. The teacher forewarned the learners not to use big numbers while choosing domains for the table method as this may pose problems during point-to-point plotting. In addition, this statement is an indication that T1 understood his learners' thinking and that of his belief about his learners (i.e., they cannot manipulate the graph). This reflected in his instructional style.

Instead of correcting misconceptions, in some cases, teachers used incorrect and incomplete explanations that may result in creating misconceptions for the learners. For example, similar to what was previously pointed out T3 said, "We have the general formula $y = ax^2 + bx + c$ but in your grade we talk about $y = ax^2 + q$". Also, T2 said, "range refers to **all** the values of y".

T3: When we are talking about the axis of symmetry, it's written in the form of $x = a$. 'a' stands for any number, isn't it?

Learners: Yes.

In the above explanation, the researcher posited that T3 last sentence may create confusion thereby leading to a misconception. The use of 'a' in this equation may suggest to the learners that it means the same as 'a' in the $y = ax^2$. The explanation might be more meaningful if the teacher had introduced a new letter of the alphabet and followed it up with various exercises where learners could discover that the given letter of the alphabet can be any number. The teacher assured the learners that they would grasp this in the future (grade 11).

T3: You will learn in Grade 11 because the value of 'b' is zero, it means that we are going to have $x = \frac{0}{2a} = 0$. The axis of symmetry is zero, x = 0.

T2 said, "Range refers to **all** the values of y". Meanwhile, the range of a parabola is a set of selected values of y which is determined by the y-value of turning point. Thus, T2 may be creating misconceptions among the learners. Again, T3 does not teach the range of a parabola.

The researcher believes that teachers' understanding of learners, or belief about the learners plays a significant role in the method they adopt during the teaching of quadratic function. The researcher probed the participants whether their learners showed interest in learning quadratic function by asking questions.

Researcher: The teacher often utilises the question(s) that learners ask during the lesson for various purposes. Did your learners ask questions during your teaching of quadratic function in Grade 10?

The interview responses of T1, T2 and T3 respectively follow:

T1: Questions from learners to the teacher is very important. That will open the minds of the learners. In most cases, the most gifted ones are those who ask question. Learners are usually afraid to ask question. Never! they did not. The questioning was one flow; only from me to them. I must give them chance.

T2: They don't ask question, which is very wrong. If they ask question, they can tell me something they really don't understand, then I can change the style of teaching. Because, if they just keep quiet then there is something wrong. So I have to change the method of teaching.

T3: (shaking his head) They are very passive.

T1's belief about his learners is that they are passive, "afraid to ask question". The researcher wondered why he did not endeavour to encourage them. The type of strategy that T1 used (explanation) contributed to learners' behaviour. This also confirmed his teacher-centred method when he said, "Only from me to them". T1 acknowledged that he did not give learners opportunities to ask questions.

T2 also admitted that it is wrong when learners fail to ask questions during the lesson. The researcher ascribes this to T2 since she is the one who must create opportunities for learners to ask questions. The teacher acknowledged that this could be changed through instructional strategies. T2 recognised the impact on teaching of quadratic function. The teacher also saw her learners (like T1) as passive. Perceiving learners to be passive is a common belief among

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the three participants; hence they use instructional strategies that are predominantly (teacher-centred methods) to teach learners the quadratic function.

T3 explicitly said his learners are passive and never ask questions. The researcher identified the need to change teachers' views that hinder effective teaching of quadratic function in Grade 10.

4.2.3.3 Concepts of quadratic function that learners often find difficult

It is also important that teachers know the aspects of quadratic function that Grade 10 learners usually find difficult to understand and then give such concepts proper attention in their teaching. For example, learners easily determined the y-intercept while some learners struggled to find the x-intercept.
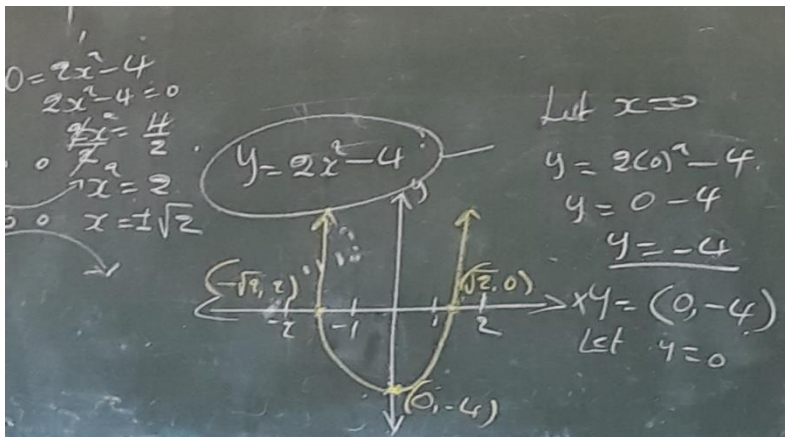
Figure 4. 21: Dual method for finding intercepts with axes: T1's case

T1 referred learners to the previous lesson of the function.

T1: Do you still remember when we were doing straight line graph, I said there is a fastest method to draw the graph? Not the table method, do you still remember?

Learners: Yes!

T1: What did we say?

Learners: (in chorus) "Let x be 0, let y be 0."

T1: Even here (in quadratic function) we can use the same method. $y = 2x^2 - 4$. Let x be 0, and then what will be the value of y?

Learners: *−4*
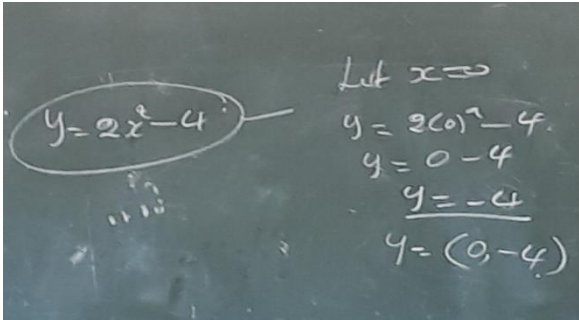
Figure 4. 22: The use of "let x be 0": T1's case

T1: What will be the second step?

Learners: Let y be 0!

T1: Then you say let y be 0 that is when you are going to involve algebra. Can you change y to 0 and work it out? That's where I need you most.

T1's learners did the exercise in their notebooks. T1 led learners to do the exercise on the chalkboard. Learners responded well, showing that they knew the steps for solving algebraic equation.

T2: Let's say for example we are having a graph of $y = x^2 + 4$ (this is an equation, not a graph). We see that the value of q is greater than 0, it means the graph has shifted vertically upward by how many units?

Learners: By 4 units!

T2: Let us have the two equations $y = x^2 + 4$ and $y = x^2 - 4$

Still on the completing the table of values, five learners were keen to write y-values on the chalkboard. The teacher called a learner, "X5T2! Come and find the y-values". The table below represents X5T2's answers for $y = x^2 + 4$:

Table 4. 17: X5T2 worked on table values (y= $x^2 + 4$)

X	-2	-1	0	1	2
Y	0	3	4	5	8

This showed that one or more learners still had difficulty with this activity.

Referring to Figure 4.15, T3 called on X5T3 and X6T3 to say the position of E. The teacher asked one learner after the other.

X5T3: Axis of symmetry

X6T3: y-intercept of both graphs

X3T3: The meeting point

X2T3: x-intercept

This showed that some learners still could not identify the axis of symmetry, x- and y-intercept despite the teacher's explanations. X3T3 had an idea, but did not use correct mathematical language.

Learners could not identify the value of 'b' in $f(x) = 1 - x^2$, until the teacher wrote it in standard form $f(x) = -x^2 + 1$.

T3:   According to the equation, it's clear that the value of 'b' is…? (No response from learners.)

T3:   Ehhh. Okay, let's try to put it as $f(x) = -x^2 + 1$. What is the value of 'b' here?

Learners: (respond reluctantly) Zero

T3:   If the value of 'b' is zero, it means the axis of symmetry is…?

X5T3: 0 and 1

Axis of symmetry is supposed to be in the form of equation. However, the teacher did not address this error; he responded in support of this wrong answer, "0 and we don't know".
In all the three cases observed, the researcher gathered that some learners struggle with 'squaring of negative number' even when they use calculators. However, no teacher specifically mentioned this during the interviews.

Researcher: Which aspect of quadratic function do learners usually struggle to understand?
 Below are their responses:

T1:   To determine the range and the domain, they just say y is greater than or equal to…, and then forget about the other thing.

Researcher: Do you know why?

T1:   I really don't know why. But I think the problem is about the concept of inequality.

T2:   Effect of parameter 'q'.

Researcher: Do you know why?

T2:   I also do ask myself that question.

Researcher: Have you ever asked your learners why?

T2:   I have never asked them. I want to re-teach them before they get to Grade 11.

T3: They struggle on range and domain. And they struggle with this question of the turning point.

Researcher: What are you going to do about it?

T3: I will be returning to them to prepare them for exam, I will explain to them that the value of 'q' is the turning point.

The researcher specifically asked T1:

Researcher: During your teaching of quadratic function, learners easily found the y-intercept by letting x be zero in $y = 2x^2 - 4$. Then when you asked them to find the x-intercepts, you said, "This is going to be a problem for some." What do you mean? $0 = 2x^2 - 4$"

T1: The question of factorisation. I find that some learners have the problem of taking out the common factor and decide that the other thing that is inside the bracket they must apply 'difference of two squares'.

Researcher: Does that mean your learners cannot factorise?

T1: Some, not all.

In the above interviews, T1 used his content knowledge to analyse the possible reasons why learners struggled to write domain and range correctly and to find the x-intercept of a parabola. T1 attributed domain and range problems to poor foundation of the concept of inequalities. The mentioned that learners struggled with the concept of inequalities and this became a barrier in learning the concepts of domain and range of a parabola. The lack of learners' prior knowledge was also the problem in finding the x-intercept of parabola. This is a concept of factorisation.

During the teaching observations, T2 explained the effect of parameter 'q' on the parabola; the teacher said that it caused the parabola to shift vertically, either upward or downward depending on the sign of the value of 'q' (i.e., positive or negative respectively). T2 added that such shift would also change the turning point. T2 demonstrated good knowledge of the

content. Since no exercise was given to learners during the class observation in this regard, the researcher has no evidence to justify T2's interview response.

T3 mentioned that learners found it difficult to find the domain and the range of a parabolic function. Meanwhile, T3 did not teach the concepts of domain and range during the teaching observation. The researcher concluded that the teacher did not teach these aspects based on his (previous) experience that learners often found them difficult to understand. The researcher failed to determine if T3 would eventually teach the concepts after the observations.

4.2.4 Research question 4: How do teachers draw from their knowledge of curriculum when teaching quadratic function?

The researcher explored teacher's curriculum compliance in terms of Concepts sequencing; goals of quadratic function in grade 10; and compliance with teaching strategies prescribed in CAPS. Knowledge of school curriculum is displayed through teacher's curriculum compliance. The researcher explored teacher's compliance to South African teaching approach as prescribed in the CAPS. As gathered in the literature, in chapter two, teachers are expected to use learner-centre approach that gives room for learner-initiated questions and independent thought, as well as interactions among the learners. Therefore, if a teacher teaches quadratic function in another way different from what the curriculum prescribed, such teacher do not comply, hence he or she lacks knowledge of the curriculum.
The approaches employed by the participants of this study were analysed in detail under the knowledge of instructional strategies.

4.2.4.1 Sequencing the concepts of quadratic function

The researcher explored how the teachers structure the lessons, using ATP as guideline. Content number one of the ATP was not sufficiently explored by the teachers. The three teachers basically started their teaching from content number two: Point by point plotting of basic graphs defined by $y = x^2$ to discover (1) shape, (2) domain, (3) range, (4) axes of symmetry, (5) turning points, (6) x-intercept, and (7) y-intercept. The researcher expected these be followed by the effect of parameter 'a', then followed by the effect of parameter 'q'.

The teachers were expected to use a particular example/ activity where 'a' is one and 'q' is zero. This is called the basic quadratic function, $y = x^2$. This is the starting point for plotting. All the participants observed point-by-point plotting of parabola.

T1 was observed teaching mainly contents two and three of the ATP (Table 4.2) and the features therein were sequentially taught.

T2 was seen consistently checking Siyavula mathematics textbook. The researcher observed that this has not really helped T2 to structure her lessons because despite this, T2 still missed out on some of the key concepts of quadratic function and also failed to use appropriate example ($y=x^2$) to investigate the features of parabola. The researcher discovered here that textbook structure may not be the same as curriculum structure.

T2 drew a parabola on the chalkboard taken from Siyavula Mathematics Grade 10 as shown below:
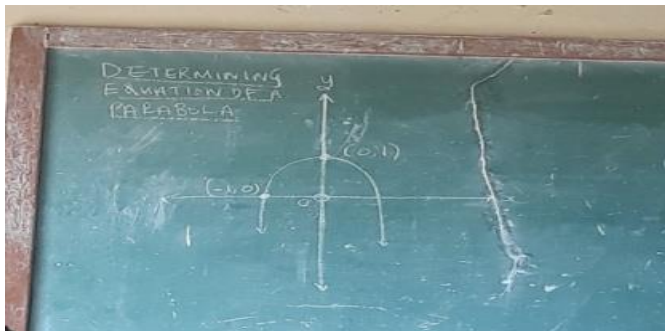
Figure 4. 23: Exercise from Siyavula Mathematics Learners' book: T2's case

T2:   We are still on parabolic graph. If we look at point B, that is the coordinate of the turning point. Where the x-value of the turning point is 0, and the y-value is 1. And then have the coordinate of the x-int., whereby the x-coordinate is -1 and the y-coordinate is how much?

Few learners: Zero.

This was the first place where T2 stated the values of the coordinates of the turning point. She also mentioned x-intercept for the first time here which she should have done as per

content two of the ATP (Table 4.2). Although T2's explanations of the contents of quadratic function are good, she failed in terms of curriculum sequencing.

T3 used the basic quadratic function together with another quadratic function to compare the graphs in order to show learners the effect of 'a' during which he talked about the feature of a parabola.

4.2.4.2 The goals of teaching quadratic function in grade 10

The specific aims of mathematics listed in CAPS (2012) include:

- Identifying a function in different representations

- Using real-life problems to teach mathematics topics, e.g. quadratic function

- Providing opportunity to develop learners to investigate, for examples parameters a and q in quadratic function defined by y=$ax^2 + q$

- Developing learners in problem solving and cognitive skill through improved questioning

The researcher studied how teachers facilitated their learners to understand and/ or identify various ways in which quadratic function can be represented.

T1:	Then you will have order-pairs, you still remember?

T1's learners responded, "Yes!"

Table 4. 18: Deduce ordered-pairs from table of values: T1's case

X	-3	-2	-1	0	1	2	3
Y	9	4	1	0	1	4	9

T1 writes on the chalkboard as the learners call out the order-pairs, "$(-3;9),(-2;4),(-1;1),(0;0),(1;1),(2;4),(3;9)$".

Researcher: In what different ways can you represent quadratic function, say for example $y = 2x^2 + 4$? How can you relate this to the teaching of quadratic function in Grade 10?

T1, T2 and T3 respectively said:

T1: Two x squared plus four… what can I say? (He gave a gesture that he does not know. In quadratic function, they should already know the flow diagram which can be translated to a straight line graph; that straight line should be drawn in a Cartesian plane which shows positive and negative numbers; also factorisation.)

T2: You will find the additive inverse of four... (She explained how to solve for x. I further explained what I intended by the term "different ways"). I think it will allow the teacher to cater for different type of learners because they differ in their understanding.

T3: I do not understand the question. I do not understand the question. Maybe because I have never done it.

This showed that the teachers often engage in teaching activities without understanding the purpose. It was observed that the three teachers were not aware of the curriculum demand in terms of aims of teaching quadratic function in Grade 10, hence the information they provided to the learners was incomplete. The researcher did not see any of the teacher solving words problems pertaining to quadratic function.

T2 was seen engaging learners with more exercises, more than T1 and T3. Their questioning approaches are not enhanced. None of the teachers actually used real-life problems to teach the concepts of quadratic function during the lesson observation. They did not provide satisfactory opportunities to develop learners to investigate (this was explained in details under instructional strategies).

The researcher used interviews to explore the teachers' understanding of what to be achieved in teaching quadratic function in grade 10.

Researcher: What is your understanding about the goals of teaching quadratic function in Grade 10?

T1, T2 and T3 replied respectively:

T1: Uhnnn. If they move gradually in grades, you'll find out that in most cases concerning our learners nowadays they are not conversant with these graphs. And you'll find out that in grades 11 and 12 they are always there on the paper having lots of marks. So teaching them in grade 10 will open their minds even if I am not around some questions in papers in grade 11, I think they can try them.
I'm preparing them for the future.

T2: I think it serves as the introduction or preparation of the learners for grade 12. It will help them to go smoothly when they are (doing quadratic function) in grade 12.

T3: The main objective is for learners to be able to identify the x-intercept, y-intercept, axis of symmetry, and the turning point, the range and the domain. And at the end of the day the learners must be able to sketch the graph. The learners must be able to determine the equation given the graph.

T1 and T2 shared the same opinion, which is to prepare learners to progress. T3 identified what learners should be able to do in the end of his teachings. The three teachers did not mention how these could be achieved. The researcher believed that having understanding how quadratic function can be used to solve real life problems will afford learners interests and conceptual understanding of its concepts.

4.2.4.3 Compliance to strategies prescribed in the CAPS

T3 mentioned that he preferred to use the table method, instead of the dual method because the table method "will show us the effect of 'a'". The teacher intended that the table method allowed point-to-point plotting in comparison with the dual method. T3 did not know that the

curriculum prescribes both methods and the reason he gave for using the table method is incorrect. T3 did not mention the domain in his teaching at all. This implied that T3 was not conversant with the curriculum content.

T2 explained the position of q on the graph. T2 appeared to think her learners already knew the meaning or position of q with regard to functions.

T2: We know that the q is the y-int. is the point where the graph cuts the y-axis. Let us look at the value of q if $q > 0$ it means that the parabola has shifted vertically upward by q units, and the turning point will be above the x-axis.

Learners should have discovered the above explanation – plot the graph and express the observed change. The teacher could then guide learners to use appropriate mathematical language to generalise the observations. In all the three cases, teachers were found using explanations and demonstrations where they were supposed to facilitate learners to discover (enquiry approach) the features of parabola, as prescribed in the CAPS. Hence, the researcher posited this action as non-compliance to curriculum practice.

4.3 SUMMARY

In this chapter, the researcher presents detailed analysis of lesson observation and interviews with the three participants. The analysis reveals how mathematics teachers teach quadratic function in Grade 10. The researcher looks into aspects of PCK that the teachers employ during their teaching. These aspects are termed 'themes' that make up the construct of PCK: knowledge of content and curriculum; teachers knowledge about learners; and knowledge of instructional strategy.

Findings indicated that teachers sometimes miss important aspects of quadratic function, thereby leading to non-completion of the curriculum. Also teachers demonstrated poor content knowledge of some aspects of quadratic function and provided learners with wrong or incomplete information. Teachers were able to connect quadratic function with learners' prior knowledge, but failed to make connections within the quadratic function. They also failed to use real life analogies or examples to facilitate the teaching of quadratic function in Grade 10. The findings revealed the connectedness of the components of PCK. It was inseparable to talk about the sub-domains of knowledge of the content without including

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some sub-domains of the knowledge of the curriculum or/ and knowledge of learners. Also, it was significantly noted, how learners responded to teacher in chorus, and there was no learner, in any of the three cases, that asked question during the lessons, making it difficult to identify learners' thinking.

In the next chapter, the researcher gave a comprehensive summary of the findings and made recommendations.

CHAPTER 5: CONCLUSIONS AND RECOMMENDATIONS

5.1 INTRODUCTION

Several studies (Bansilal, 2012; Bansilal, Brijlall and Mkhwanazi, 2014; & Sapire et al., 2016) revealed insufficient teacher knowledge of teaching mathematics as a contributing factor to learners' poor performance in mathematics in South Africa. In order to understand this better, this study explored how mathematics teachers teach mathematics with a focus on quadratic function in Grade 10. Simultaneously, the researcher believes that good teaching of quadratic function (like any other mathematics) depends on the teacher's PCK, which comprises content knowledge, knowledge of how to teach, knowledge of how to elicit learners thinking, and the knowledge of the curriculum. Hence, the researcher sought to understand mathematics teachers' PCK for teaching quadratic function in Grade 10. The main purpose was to explore the elements of PCK that mathematics teachers of Mogalakwena district in Limpopo used during the teaching of quadratic function in Grade 10. This chapter provided answers to the main research question: How do the teachers teach quadratic function in Grade 10? With reference to the following sub-questions:

1. What content knowledge do teachers possess in teaching quadratic function in Grade 10?

2. What are the teaching strategies that teachers use in the teaching of quadratic function in Grade 10?

3. How do teachers build learners' knowledge of quadratic function?

4. How do teachers draw from their knowledge of the curriculum when teaching quadratic function in Grade 10?

5.2 SUMMARY OF THE FINDINGS

The conclusions are drawn from the integration of findings, analysis and interpretation of qualitative data collected from the participants (Dale & Volpe, 2016). The researcher aimed to answer the main research question of the study: How do the teachers teach quadratic function in Grade 10? The conclusions are presented based on researcher's exploration of data that provided answers to the following:

5.2.1 Content knowledge that teachers use for teaching quadratic function on Grade 10

The content knowledge as a component of PCK refers to teacher's sound understanding of a particular topic under discussion. The current study characterised teacher's content knowledge for teaching quadratic function as thus:

- Addressing key ideas of quadratic function

- Types of connections that featured in the teaching of quadratic function

- Aspects of grade 10 quadratic function that teachers find challenging to teach

Teaching of quadratic function in Grade 10 begins with understanding its content. This study revealed that teachers do not possess in-depth content knowledge that embodies the aspects of quadratic function most connected to its instruction, for example, giving explanations without the use of visual representation (graphs).

5.2.1.1 Addressing key ideas of quadratic function

The researcher used the Limpopo Department of Basic Education 2018 Grade 10 Mathematics Annual Teaching Plan (ATP) to examine the teachers in this regard. The teachers were supposed to discuss the contents of quadratic function in Grade 10 as thus:

- Work with relationships between variables using tables, graphs, words and formulae and convert flexibly between these representations;

- Point by point plotting of basic graphs defined by $y = x^2$ to discover shape, domain (input values), range (output values), axes of symmetry, turning points and intercepts on the axes;

- Investigate the effect of a and q on the graphs defined by $y = a.f(x) + q$, where $f(x) = x^2$.

- Sketch graphs, find the equations of given graphs and interpret graphs.

Some of these key ideas were not properly addressed and, in some cases, were totally omitted during the lessons. This might have been due to teacher's lack of content knowledge. T2 did not use or talk about the basic quadratic function ($y = x^2$) during her teaching. T3 did not teach Grade 10 learners the effect of parameter 'q'. None of the teachers taught the concepts of reading off intervals from the graphs. This higher order concept was mentioned in 2018 NSC diagnostic reports that learners are found using algebraic manipulation, which resulted in making the problems more complicated and time consuming. The present study discovered a similar pattern as Bansilal et al. (2014).

5.2.1.2 Types of connections that featured in the teaching of quadratic function

The current researcher draws a line between: the amount and kinds of connections teacher makes; and how the teacher uses the connection. Teacher's ability to connect a concept with more different other concepts is themed as content knowledge. While 'making connections' to facilitate learners learning is categorised as teaching resources - appropriate usage of connection to facilitate teaching of a concept is hence categorised as teachers' knowledge of instructional strategies, in this study. The present researcher explored different kinds of connections that teachers made during the teaching of quadratic function in grade 10. The

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2017 NSC diagnostic report suggests that teachers should use quadratic questions that involve incorporation of different topics during teaching and in class-based assessment. In this study, the teachers only made few connections among the concepts of quadratic function and with learners' prior knowledge. None of them related quadratic function to any other topic outside mathematics. Hence, the researcher perceived this as insufficient connections. All the three teachers showed the link between the shape of parabola and the value of parameter 'a'. They all engaged learners with activities that featured writing algebraic formula in table form, and ordered pairs, and transforming the values into a graph; however, none put two or more of these representations side by side to make learners understand that they mean the same thing. In terms of connecting the concepts of quadratic function, T2 and T3 did more than T1. All the three teachers were observed making connections with learners' prior knowledge, such as factorisation and solving equations.

5.2.1.3 Aspects of grade 10 quadratic function that teachers find challenging to teach

It is important that to note that some concepts of quadratic function may be challenging to the teacher, either as lack of its content knowledge or lack of knowledge of appropriate strategy to teach it. During the class observation, the researcher witnessed that T3 lack content knowledge to read off intervals from the graph. This was the same aspect of quadratic function where diagnostic report (2017) found that "learners performed poorly". This study gathered that the concept of inequalities is challenging to most teachers, as revealed by T2 and T3 during the interviews. T2 also indicated that re-writing the equation of parabola after shifts is challenging. In addition to inequalities, T3 said he needed to understand how to/ what determines the domain and range. This is similar to what was stated in diagnostic report (2015) that learners could not relate the turning point to the range – did not know that turning point had to be used to write the range. The content knowledge of T2 and T3 is below that of their colleagues, such as T1.

This is evidence that learners poor performance may be due to their teachers' lack or insufficient knowledge of the subject matter.

5.2.2   The teaching strategies that teachers use in the teaching of quadratic function in Grade 10

This researcher found and has continued to indicate that components of PCK are interwoven that is, one sub-component may serve as a factor of more than one component. Although the ATP was used as a guide for the contents that the teacher should teach in Grade 10 quadratic function, the researcher argued that when a teacher made use of ATP and followed the instructions therein (e.g., teaching strategies), the teacher complied with the curriculum. CAPS advocates the teaching of quadratic function in Grade 10 using investigative approaches, whereby learners are the centre of activities and the teacher facilitates, for example, "Investigate the effect of a and q". Observation, multiple representations and investigation were insufficiently found in the teaching of quadratic function in grade 10, as in most cases teachers foretold the outcomes of what learners supposed to discover by themselves. All the teachers were observed teaching procedurally. They explained everything learners should have investigated independently. They used explanation more than questioning. This is direct teaching. It may not yield conceptual understanding of the concepts of quadratic function.

Shulman (1987) found that choosing appropriate teaching strategies for teaching a concept (e.g., quadratic functions) is based on teacher's knowledge of the subject matter and of the learners' level of understanding. This statement shows the kind of symbiotic relation among the components of PCK. The researcher explored the following with regard to suitable strategies for teaching quadratic function in Grade 10:

- Using various representations and resources (e.g. learners' prior knowledge, mathematical languages, symbols, real-life example, diagnostic reports etc.) to explain and demonstrate quadratic function

- Using various approaches (e.g. investigation, questioning, problem-solving) to assess learners understanding of quadratic function

5.2.2.1 Using various representations and resources to explain and demonstrate quadratic function

None of the participants of this study used real-life problems to teach quadratic function in Grade 10. T2 even mentioned that there was no any real life representation that could be used

to teach quadratic function. The researcher did not agree as there are many concrete or real-life illustrations that teachers can use to teach the concepts of quadratic function. For example, a teacher may talk about the height of a golf shot when a golfer hits a ball – the relationship between the time (x) and the height (y). Hence, it was observed that teachers did not employ a variety of representations as prescribed by the curriculum. This is also a lack of curriculum knowledge. In addition, none of the teachers used concrete teaching aids to facilitate the teaching (no visual aids were displayed on the wall). The teachers used teaching strategies that lack multiple representations.

The teachers did not use verbal representations to facilitate problem solving in quadratic function. The researcher expected to see teachers exposing their learners to words problems involving quadratic function, but this did not happen. In some cases, the teachers constructed good questions with appropriate mathematical language, however, they did not give learners enough time to respond or discover the solutions; and thereby answered the questions (verbally) for their learners. For example, T1 wrote a quadratic equation on the chalkboard, $y = x^2 + 1$. This was his first activity for the "effect of parameter 'q'". He instructed the learners to make a table of values and draw the graph point-by-point. But shortly after the learners started, T1 began to state the outcomes. Teachers were expected to use improved questioning – use questions such as 'why?' and 'what if'? which could enhance learners' creative thinking but instead the teachers limited learners to simple activities such as filling in the table values and point-by-point plotting of parabola.

There were few instructional resources used during the teaching of quadratic function. All the three teachers have mathematics textbooks and they all referred to the textbooks appropriately. They used different textbooks: T1 used Platinum Mathematics; T2 used Siyavula Mathematics; and T3 used Classroom mathematics. In all the classes, a large number of learners had and made use of their calculators. T1 was the only teacher who used a lesson note and coloured chalks for sketching parabolas. He provided all his learners with the 2017 NSC examination question paper and referred to it during the lessons.

5.2.2.2 Using appropriate approaches to communicate and assess learners' understanding of quadratic function

Both PCK and CAPS advocate the learner-centred approach for teaching quadratic function in Grade 10. This strategy encourages learner-initiated questions and teamwork. During class observations of this study, no proper investigation was observed in any of the classes, although there were several activities in T1 and T2 classes: learners made tables of values and plotted graphs (in some cases learners did not complete the tasks on their own). Instead of giving learners opportunity to investigate, the teachers explained. For example, T2 fore-mentioned the effects of parameters 'a' and 'q' before the learners plotted the graphs.

But in some cases, teachers were observed to develop learners' methodical skills. For examples:

T1 demonstrated a quicker way to fill the table $y = 2x^2$ and $y = x^2 + 4$, using the outputs already determined in $y = x^2$.

T2 informed the learners that if two graphs intersect, it means learners should equate the equations of the two graphs in order to determine the x-coordinate of the point of intersection.

T3 explained that whenever the value of 'b' (in $y = ax^2 + bx + c$) is zero, the axis of symmetry will be $x = 0$; and the x-coordinate of the turning point will be 0.

The researcher observed that teachers' questioning was one way, that is, only from the teachers to the learners. The participants were seen explaining what learners supposed to be investigating. Learners of T1 and T2 showed active participations in working out the solutions. Apart from these two, other activities were not observed in the Grade 10 classes. The teachers used direct teaching strategies which were devoid of learner initiative and team work.

5.2.3 How teachers build learners' knowledge of quadratic function

This simply implies how teachers use their knowledge of learners to facilitate the teaching of quadratic function in grade 10. The researcher found this sub-domain as a strategy for teaching quadratic function, as the teacher engages in pedagogical activities such as:

- Identifying and making connection with learners' prior knowledge during the teaching of quadratic function

- Addressing common errors and misconceptions in quadratic function

- Understanding the aspects of quadratic function that grade 10 learners usually find difficult

5.2.3.1 Identifying and making connection with learners' prior knowledge during the teaching of quadratic function

The researcher still emphasised the importance of connecting mathematics in this section (like in content knowledge and knowledge of instructional strategies), but this is limited to a special kind of connection, learners' prior knowledge. The PCK framework emphasises the need for the mathematics teacher to become more sensitive to their learners' prior knowledge (a constructivist approach) and how these learners make sense of mathematics concepts. Building learners' knowledge is only possible if the teacher allows learners to express themselves. To do this depends on a number of factors: teacher's use of appropriate strategies, understanding how learners learn specific aspects of quadratic function and sound knowledge of quadratic function. This is another indication that elements of PCK are interwoven.

Based on the content knowledge and curriculum knowledge, the teacher needs to understand the chronological structure of quadratic function as well as other topics in mathematics that are directly related to it. Having a sound understanding of what learners ought to know before embarking on quadratic function in Grade 10 and making proper connections therein may enhance learners' conceptual understanding of quadratic function.

In the current study, teachers made connections with learners' prior knowledge, some in the form of revision during the introduction stage and some as a form of reminder where/when the need arose. Learners were also seen using their prior knowledge to tackle quadratic function problems. In all the three cases, learners easily stated the formulae for finding x- and y-intercepts: let y equals zero, let x equals zero.

5.2.3.2 Addressing common errors and misconceptions in quadratic function

Teachers' capacity to address common errors and misconceptions is a skill that differentiates a mathematics teacher from a mathematician. In this case, the researcher observed that teachers only analysed a few simple common errors that learners often make in the learning of quadratic function, for example, the 'squaring of negative integers'. This was done appropriately in all classes. Furthermore, T2 addressed a common error of leaving fractional answers un-simplified. This was after a learner had committed the error. The researcher observed T1 as he forewarned his learners, based on his experience, to carefully draw a parabola curve. T1 emphasised that parabola should not be 'V'.

The researcher could not gather much of learners' misconceptions as learners were not given opportunities to communicate their answers or thoughts. It is also important to note that in most cases, learners responded in chorus. This happened in all the three classes.

5.2.3.3 Understanding the aspects of quadratic function that grade 10 learners usually find difficult

Shulman (1986) specifically included understanding of concepts that makes a topic difficult to learn by learners as a core description of teacher's PCK. Hence the researcher found it significant to explore what Mogalakwena teachers gathered as aspects of quadratic function that their grade 10 learners often find difficult to learn.

During lesson observations, the teachers did not evaluate learners' understanding in most of the aspects of quadratic function. They only focused on converting formulae to table forms, and then point-by-point plotting of parabola. Except T2 who engaged learners further in solving equations to get x- and y-intercepts. Basically there was insufficient assessment to discover which areas of quadratic function were challenging to learners. However, based on the little exercises that learners were exposed to, the researcher gathered that learners struggled to identify or recognise the key points (e.g. points of intersection of two graphs) on the graphs. Some learners also found it challenging to locate the co-ordinates of points on the Cartesian plane. Some learners could not find the x-intercepts of parabola: they could not solve quadratic equation. The researcher perceived this may be due to teachers' lack of content knowledge or lack of knowledge of appropriate instructional approaches to teach the learners effectively.

5.2.4   How do teachers draw from their knowledge of the curriculum when teaching quadratic function in Grade 10?

The present research explored how teachers drew from their understanding of curriculum demands to facilitate teaching of quadratic function in grade 10. The following were observed during the teaching of quadratic function in grade 10:

- Concepts sequencing

- Goals of quadratic function in grade 10

- Compliance with teaching strategies prescribed in CAPS

These attributes are similar to what was discussed in Shulman's MPRA (1987) process of teaching. It was stated that comprehension, as a theme of process of teaching, captures the purpose of teaching, the structure of ideas, and possession of ideas both from within and outside the subject.

5.2.4.1 Concepts sequencing

As earlier indicated, the participants did not teach some concepts of quadratic function. They were totally omitted. Based on what the teachers discussed in their classes, the researcher observed the following sequences:

T1 started with the general formula as y=$ax^2 + q$. The teacher stated the features of parabola to be learned on the chalkboard. He then did point-by-point plotting of y=x^2, via table of values. He moved on to teach effect of parameter 'a', during which he made his learners to recognise quadratic function in another representation – ordered pair. T1 thereafter taught the effect of parameter 'q'.

Making learners to recognise function in different representation is a key aim of CAP and it is embedded in content one of the ATP. During lesson observation, T2 started with general formula y=ax^2. She explained a feature of a parabola – face up and face down due to the value of 'a'. T2 used y=$2x^2$ and y=$-2x^2$ to do point-by-point plotting and to explain the effect of parameter 'a'. She moved on to effect of parameter 'q' where she briefly mentioned the turning point in connection with the value of 'q'. Turning point is a key aspect of

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quadratic function, which according to ATP the teacher supposed to have discussed in content two, with appropriate representations.

During the interview, T2 said the order should be as thus: intercepts with axes; turning point; then effects of parameters 'a' and 'q'.

T3, like T1, stated the features of parabola that learners should know on the chalkboard. He wrote two formulae of quadratic function (y= $ax^2 + bx$+c and y= $ax^2 + q$) and related the two. T3 used $y = x^2$ to do point-by-point plotting and thereafter used it to explain only axis of symmetry and the turning point. He talked about sketching (which belongs to content four of the ATP) before started to teach effect of parameter 'a' (content three).

The researcher asked, "In what order do you teach the concepts of quadratic function in grade 10?" T3 responded below:

T3:   Intercepts, axis of symmetry, turning point, the range and the domain.

As evidenced in the above, the teachings of quadratic function in grade 10 were found to be in disorder. The teachings did not follow the curriculum sequence.

5.2.4.2 Goals of quadratic function in grade 10

The current researcher explored three of the curricular goals identified in CAP (2012), thus: to make learners recognise quadratic function in different forms; to make learners identify quadratic function in real-life situations; and allow learners to investigate parameters ('a' and 'q') and thereafter generalise.

None of the participants started from content one of the ATP. They failed to give this content a special attention needed. Hence the goal of making learners recognise quadratic function in different forms was forfeited. Also, because the teachers were not conversant with the curriculum says, they could not provide satisfactory responses during the interviews.

Again, none of the teachers used real-life situations/ problems to teach quadratic function in grade 10. Also, as widely analysed in various sections of this study, teachers did not provide necessary opportunities for learners to do investigations.

5.2.4.3 Compliance with teaching strategies prescribed in CAPS

Although the teachers exposed learners to some methodical skills, however they demonstrated insufficient curriculum knowledge with regard to quadratic function in Grade

10. They showed no curriculum compliance in terms of instructional approaches for teaching quadratic function in Grade 10, content disorder, and they demonstrated poor curriculum coverage. These were not only due to lack of curriculum knowledge in itself, but also lack of other PCK elements. As already indicated, the teachers did not use a variety of representations to teach quadratic function in Grade 10. The teachers used a teacher-centred approach as opposed to a learner-centred approach prescribed in the curriculum of mathematics. The teachers explained the possible outcomes when they should have facilitated learners to investigate and make generalisations in quadratic function.

5.3 CONCLUSIONS BASED ON THE FINDINGS

The Theoretical Framework is drawn from Shulman's conception of PCK with a focus on four categories: Content knowledge; Knowledge of curriculum; knowledge of learner's mathematical thinking; and Knowledge of instructional strategies. The Theoretical Framework has helped to frame the research questions, through which both observation and interview protocols were drawn. It was also used to cross-reference the findings and thereby enabled the researcher to provide answers to the research questions.

The ATP is a curriculum document. It gave the researcher an insight into the key ideas (in order) of quadratic function that should to be addressed in grade 10. The ATP provided researcher with ideas of what the teacher should evaluate and the assessment approaches to be used (by giving learners exercises and short test that involve e.g. plotting the graphs, converting from one representation to the other, making investigation and generalisation, determine equation of parabola, and interpreting the graphs). The researcher found that teachers omitted some key aspects of quadratic function leaving learners with incomplete knowledge of the topic. Teachers did not make sufficient connection during their teaching. Teachers did not know some aspects of quadratic function they were supposed to teach in grade 10. The researcher concluded that most teachers posses insufficient knowledge of quadratic function, lower than what is prescribed in grade 10 mathematics curriculum.

The researcher used ATP to explore the teachers' data of assessment, which to this study forms part of teachers' knowledge of instructional strategies and knowledge of curriculum. It is factual that teacher may find real life examples or analogies easily accessible in one topic than another. However, there are many of such representations available, on the internet, for the teaching of quadratic function. The teaching strategies that teachers mostly used to teach quadratic function grade 10 are demonstration and explanation approaches. This is what the researcher observed during the class observation. The demonstration is considered less effective as the teachers did not explore adequate representations to do so.

Perceiving learners to be passive, is a belief common to all the three participants of this study, hence they used instructional strategies that are dominant (teacher - centred methods) to teach learners the quadratic function. The participants explicitly mentioned in the interviews that their learners were perceive, therefore they could never ask question. The researcher saw the need to change (orientate the teachers about) teachers' views that hinder effective teaching of quadratic function in grade 10. The teachers basically used learners' prior knowledge to extend learners' knowledge of quadratic function in grade 10. They did not give special attention to the aspects that are challenging to learn. They also did very little to address learners' misconceptions in quadratic function. However, they used learners' prior knowledge well, to build learners' knowledge of quadratic function.

Developing learners' methodical skills is a curriculum goal that requires teacher's knowledge of content and practice. The researcher witnessed how the teachers develop learners' mathematical skills and reasoning through activities of quadratic function. The curriculum also requires the teachers to show learners various representations that mean only one (the same) quadratic function. The researcher believes that if the teachers are up-to-date in terms of curriculum knowledge, they should be able to understand what it means to represent quadratic function in different ways. This is explicitly recommended in the mathematics curriculum of grade 10. There were evidences that the teachers did not possess sufficient knowledge of the curriculum to draw from, when teaching quadratic function in Grade 10.

It is important to know that the participants employed in this study performed better than one another at different aspects of PCK. The study also revealed the interrelationships among the

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elements of PCK: the teacher may possess adequate content knowledge of a specific concept of quadratic function but still teach it poorly because he/ she lacks ability to transform such content into pedagogically multiple representations that are accessible to learners. Unless the elements are properly integrated, the teacher may be characterised as having meager PCK. Thus, the researcher argues that the PCK strands are inseparable, that is, what gives one PCK aspect an identity cannot be divorced from the other aspects. Although these components are often viewed as separate entities, given that they are kept separate, the researcher claims that they are in fact complementing one another.

Conclusively, the findings indicated that the concepts of quadratic function are inefficiently addressed in the selected Grade 10 classrooms due to teachers' lack or inadequacy in some aspects of PCK. These findings are compatible with a similar study carried out with three teachers in Malaysia in which Yusof et.al (n.d.) concluded that teachers' PCK was at a low level and as a result, the teachers failed to deliver the teaching of function adequately and clearly.

5.4 RECOMMENDATIONS

This study revealed that teachers' PCK (curriculum knowledge; content knowledge; knowledge of learners; and knowledge of instructional strategies) influences how teachers teach quadratic functions in Grade 10. The present researcher argues that developing grade 10 learners' conceptual understanding of quadratic function can only be possible if the teacher considers the interconnectivity of the PCK components in the process of teaching. That is, the teacher needs appropriate development in all the aspects – the use of appropriate strategies for teaching quadratic function in grade 10, understanding how grade 10 learners learn specific aspects of quadratic function, sound knowledge of quadratic function and of the CAPS. Therefore, there is need to develop Mogalakwena mathematics teachers' PCK in order to enhance their teaching performance in grade 10.

Teacher should draw a mind-map of the concepts of quadratic function that he/ she intended to teach - make connection with what has been said about it in the previous grades and draw how these concepts link with other concepts of function types. Teacher should draw from

understanding of concepts in another topic that learners are familiar with. Teacher may also get ideas from other teachers from other disciplines which are related to the concept of quadratic function in grade 10 and at the same time known to the learners. This effort facilitates grade 10 learners' conceptual understanding of the concepts of quadratic function.

Teachers should make lists of good questions on quadratic function that query and expand grade 10 learners' creative thought. According to DBE (2012) teaching of quadratic function in grade 10 should not be limited to "how" but should rather feature the "when" and "why" of problem types, in order to develop learners' problem-solving and cognitive skills.

Grade 10 mathematics teachers in the same cluster should form a community of practice and meet regularly to deliberate on not only the content (e.g. Quadratic function), but also the pedagogical skills for teaching the content. In these meetings, teacher can seek clarification on difficult concepts. This will help the teacher to teach all the contents of the curriculum adequately and without omission.

Oftentimes teachers acquire new ideas and knowledge through studies and workshops but are unable to implement them due to environmental factors of the school. Frequently the knowledge is not easily transferable due to lack of certain instructional materials. It is one thing to help teachers to gain knowledge of quadratic function; it is another to support them in the teaching process. The workshop's facilitators should pay proper attention to interconnectivity of the PCK components and use it to facilitate teachers' professional development. The workshops should be organised not only to address teacher's content knowledge but also teacher's PCK such as which instructional strategies best suit a particular topic or grade. The teacher will also learn what he/she needs in order to use a particular strategy. The workshops organisers should give the latter due attention, otherwise the workshops (which mainly focus on providing new knowledge) may remain a waste of funds and time.

Teacher should develop an archive for grade 10 learners' common misconceptions in quadratic function and use this record as a teaching resource to ensure such misconceptions are not repeated. Teacher should make use of NSC diagnostic reports to address more misconceptions learners make during their learning of quadratic function.

Mathematics subject specialist and HODs of mathematics in schools should provide curriculum support programmes. They should meet with mathematics teachers regularly to discuss challenging issues related to classroom practices. Teachers need experts' advice and guidance, not only through reading but also by listening to and meeting with them. Teachers manage their work well when they know and see that the system is monitored especially by external teams.

The circuit managers' team together with mathematics experts should officially allocate professional development time in the official schedule to accommodate collaborative studies to improve teaching and learning in mathematics. They should facilitate the development of content and pedagogical ideas that best suit classrooms and find possible solutions to the challenges of non-completion of the mathematics syllabus within the period of each grade level.

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LIST OF APPENDICES

Appendix A: Turnitin Report

turnitin

Digital Receipt

This receipt acknowledges that Turnitin received your paper. Below you will find the receipt information regarding your submission.

The first page of your submissions is displayed below.

Submission author:	Balqis Olawumi Banjo
Assignment title:	Revision 2
Submission title:	Master Dissertation
File name:	Complete_chapters.docx
File size:	4.19M
Page count:	157
Word count:	40,632
Character count:	218,936
Submission date:	22-Apr-2019 02:51PM (UTC+0200)
Submission ID:	1116881819

AN EXPLORATION INTO TEACHERS' PEDAGOGICAL CONTENT KNOWLEDGE
(PCK) FOR TEACHING QUADRATIC FUNCTION IN GRADE 10

Appendix B: Ethical Clearance Certificate

UNISA
university of south africa

UNISA COLLEGE OF EDUCATION ETHICS REVIEW COMMITTEE

Date: 2019/05/15

Ref: **2017/11/15/53999991/01/MC**

Name: Mrs BO Banjo

Dear Mrs Banjo

Student: 53999991

Decision: Ethics Approval from
2017/11/15 to 2020/11/15

Researcher(s): Name: Mrs BO Banjo
E-mail address: 53999991@mylife.unisa.ac.za
Telephone: +27 73 379 1386

Supervisor(s): Name: Prof MF Machaba
E-mail address: emachamf@unisa.ac.za
Telephone: +27 12 429 8582

Title of research:
An exploration into teachers' Pedagogical Content Knowledge (PCK) for teaching quadratic function in grade 10

Qualification: M. Ed in Mathematics Education

Thank you for the application for research ethics clearance by the UNISA College of Education Ethics Review Committee for the above mentioned research. Ethics approval is granted for the period 2017/11/15 to 2020/11/15.

The **low risk** application was reviewed by the Ethics Review Committee on 2017/11/15 in compliance with the UNISA Policy on Research Ethics and the Standard Operating Procedure on Research Ethics Risk Assessment.

The proposed research may now commence with the provisions that:

1. The researcher(s) will ensure that the research project adheres to the values and principles expressed in the UNISA Policy on Research Ethics.

Open Rubric

University of South Africa
Preller Street, Muckleneuk Ridge, City of Tshwane
PO Box 392 UNISA 0003 South Africa
Telephone: +27 12 429 3111 Facsimile: +27 12 429 4150
www.unisa.ac.za

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Appendix C: Language Editor's Certificate

CONFIRMATION OF EDITING   24 April 2019

To whom it may concern:

This is to confirm that the following M Ed thesis: **AN EXPLORATION INTO TEACHERS'
PEDAGOGICAL CONTENT KNOWLEDGE (PCK) FOR TEACHING QUADRATIC
FUNCTION IN GRADE 10** by BO Banjo (STUDENT NUMBER: 53999991) has been edited for
language use and technical aspects.

Eleanor M Lemmer
864 Justice Mohamet Street
Brooklyn
Pretoria
ID 5107110118088

EM Lemmer,

Appendix D: Requesting Permission of the Circuit Manager

Nkgoru Secondary School

PO Box 1635,

Mokamole Village

0633

26 May 2017

The Circuit Manager

Department of Education

Bakenberg South Circuit Office

Marulaneng Village

Private Bag 2692, Mokopane, 0600

Limpopo

0154250900

Dear Sir/ Madam

REF: SEEKING PERMISSION TO CONDUCT RESEARCH AT BAKENBERG SOUTH CIRCUIT

I, Balqis Olawumi Banjo, am doing research under supervision of Dr M. F. Machaba, a senior lecturer in the Department of Mathematics Education towards an M Ed at the University of South Africa. We are inviting grade 10 mathematics teachers in your circuit to participate in a study entitled, 'An Insight into Teachers' Pedagogical Content Knowledge (PCK) Essential for Teaching Quadratic Functions in Grade 10'.
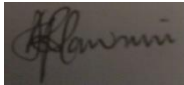
The aim of the study is to gain understanding of teachers' PCK and conceptual teaching that will assist mathematics teachers. This study will employ qualitative research method. It will use case study design in particular, to explore the main research question: 'What are the

essential components of PCK required in teaching quadratic functions in grade 10? We will be studying three grade 10 mathematics teachers through direct observation and face-to-face unstructured interviewing, in their various school environments.

This study will facilitate an improvement in Grade 10 learners' understanding and mastery of Quadratic Functions and thereby improve their performance in Mathematics.There is no potential risks involved. There will be no reimbursement or any incentives for participation in the research.

If you would like to be informed of the final research findings, please contact Banjo BO on 0733791386 or email banjo.balqis@gmail.com. Should you have concerns about the way in which the research has been conducted, you may contact Dr. M F Machaba on 012 429 8582 or email emachamf@unisa.ac.za.

Yours faithfully,

Banjo B. O. (Mrs)

Appendix E: Requesting Permission of the Principal

Title of the research: 'An Insight into Teachers' Pedagogical Content Knowledge (PCK) Essential for Teaching Quadratic Functions in Grade 10'

Date _____

The Principal

_____

_____

_____

Dear Sir/Madam,

REF: REQUEST FOR PERMISSION TO CONDUCT RESEARCH AT YOUR SCHOOL

I, Balqis Olawumi Banjo, am doing research under supervision of Dr M. F. Machaba, a senior lecturer in the Department of Mathematics Education towards an M Ed at the University of South Africa. We are inviting grade 10 mathematics teachers in your school to participate in a study entitled, 'An Insight into Teachers' Pedagogical Content Knowledge (PCK) Essential for Teaching Quadratic Functions in Grade 10'.
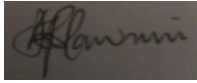
The aim of the study is to gain understanding of teachers' PCK and conceptual teaching that will assist mathematics teachers. This study will employ qualitative research method. It will use case study design in particular, to explore the main research question: 'What are the essential components of PCK required in teaching quadratic functions in grade 10? We will be studying grade 10 mathematics teachers through direct observation and face-to-face unstructured interviewing, in your school.

This study will facilitate an improvement in Grade 10 learners' understanding and mastery of Quadratic Functions and thereby improve their performance in Mathematics. There are no potential risks involved. There will be no reimbursement or any incentives for participation in the research.

If you would like to be informed of the final research findings, please contact Banjo BO on 0733791386 or email banjo.balqis@gmail.com. Should you have concerns about the way in which the research has been conducted, you may contact Dr. M F Machaba on 012 429 8582

or email emachamf@unisa.ac.za.

Yours faithfully,

Banjo B. O. (Mr

Appendix F: Informed Consent Agreement with the Participant

UNISA | college of education

Date _____

Dear _____,

I, Balqis Olawumi Banjo, am doing research under supervision of Dr M. F. Machaba, a senior lecturer in the Department of Mathematics Education towards an M Ed at the University of South Africa. You are kindly invited to participate in a research study entitled, 'An Insight into Teachers' Pedagogical Content Knowledge (PCK) Essential for Teaching Quadratic Functions in Grade 10'. It is important for you to fully understand what is entailed in the research to enable you to make an informed decision whether to participate or not. If you have any queries regarding the research study after reading this form please do not hesitate to consult me or my supervisor on the contact details given in paragraph five.

The aim of the study is to gain understanding of teachers' PCK and conceptual teaching that will assist mathematics teachers. This study will employ qualitative research method. It will use case study design in particular, to explore the main research question: 'What are the essential components of PCK required in teaching quadratic functions in grade 10?

I will like to collect data from you through direct observation of teaching during quadratic function lessons, and face-to-face unstructured interviewing at your school. Audio-recording will enable me to capture every bit of information that you will have volunteered for
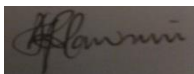
purposes of analysis and verification. Please be advised that this exercise is voluntary, you may decline to answer any of the interview questions if you so wish. Shortly after the transcription has been completed, I will send you a copy of the transcript to give you an opportunity to confirm the accuracy of our conversation and to add or to clarify any points.

All information you provide is considered completely confidential. Your name will not appear in any publication resulting from this study and any identifying information will be omitted from the report. However, with your permission, anonymous quotations may be used. Data collected during this study will be retained on a password protected computer for 5 years in my locked office.

There are no known or anticipated risks to you as a participant in this study. You will not be reimbursed or receive any incentives for your participation in the research. If you would like to be informed of the final research findings, please contact Banjo BO on 0733791386 or email banjo.balqis@gmail.com. Should you have concerns about the way in which the research has been conducted, you may contact Dr. M F Machaba on 012 429 8582 or email emachamf@unisa.ac.za.

Thank you for taking time to read this information sheet and for participating in this study.

Thank you.

Banjo B. O. (Mrs.)

Appendix G: Consent/Assent to Participate in this Study (Return Slip)

I, _____ (participant name), confirm that the person asking my consent to take part in this research has told me about the nature, procedure, potential benefits and anticipated inconvenience of participation.

I have read and understood the study as explained in the information sheet.

I have had sufficient opportunity to ask questions and am prepared to participate in the study.

I understand that my participation is voluntary and that I am free to withdraw at any time without penalty.

I am aware that the findings of this study will be processed into a research report, journal publications and/or conference proceedings, but that my participation will be kept confidential unless otherwise specified.

I agree to the recording of the interview.

I have received a signed copy of the informed consent agreement.

Participant Name & Surname (please print)   _____

_____   _____

Participant Signature   Date

Researcher's   Name   &   Surname   (please   print)
_____

_____   _____

Researcher's signature   Date

Appendix H: Lesson observation Instrument

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School		Teacher	
Grade		Date	
Topic		Sub-topic	
Class enrolment		Number of learners present	
Time:	From:	To:	

LESSON OBSERVATION SCHEDULE

1. CONTENT KNOWLEDGE FOR TEACHING QUADRATIC FUNCTION

1.1 The key concepts of quadratic function that the teacher addressed.

1.2 Examples or illustrations that the teacher use in terms of making connections: within mathematics' topics; within the contents of grade 10 quadratic function; and with other curriculum subjects.

1.3 Can the teacher's content knowledge be at least compared equal to that of his colleagues?

2. KNOWLEDGE OF CURRICULUM

2.1 The teacher's compliance with the structure of quadratic function as stipulated in CAPS; in what order are the contents addressed?

2.2 The teacher's compliance with teaching approaches: learner-centered; problem-solving; and investigative approach.

2.3 How teacher conveys the aims of learning quadratic function in grade 10.

2.4 The teacher gives valid assessment that include high cognitive levels

3. KNOWLEDGE OF LEARNERS

3.1 The teacher identifies learners' prior knowledge and pays attention to how they use it to solve problems in quadratic function.

3.2 The teacher is aware of the aspects of quadratic function that grade 10 learners usually find difficult. Identify how the teacher teaches these aspects.

3.3 The teacher identifies common misconceptions learners usually make about the concepts of quadratic function and use them as building block for the teaching.

3.4 The teacher is able to differentiate learners according to their learning needs. What does the teacher do to these categories of learners?

4. KNOWLEDGE OF INSTRUCTIONAL STRATEGIES

4.1 The teacher uses variety of powerful representations (e.g. analogies, illustrations, real-life examples, explanations and demonstrations) to teach the concepts of quadratic function.

4.2 The teacher facilitates learners to investigate theories, explain their findings and make generalization.

4.3 The teacher facilitates learners to solve problems.

4.4 The teacher flexibly groups the learners and ensures active participation of all the learners.

4.5 How the teacher uses 'what', 'how' and 'why' question type.

Appendix I: Interviewing Protocol

Interview instrument

Teacher code _____

1. **Personal Information**

I. How many years of experience do you have in teaching mathematics? How long have you been teaching in this school?

II. What is your highest degree achieved?

III. Which Grade(s) do you teach presently?

IV. Do you have another position besides your teaching role? What role is it?

V. Have you ever received any award for teaching?

2. **Knowledge of Quadratic Function**

I. What are the main concepts learners supposed to learn in quadratic function?

II. What are your experiences regarding the teaching of quadratic function in Grade 10?

III. Which aspect of grade 10 quadratic function do you find more challenging to teach?

3. **Knowledge of CAPS**

I. In what order do you teach the main concepts of quadratic function? What informs this order?

II. What is your understanding about the goals of teaching quadratic function in Grade 10?

III. What are different ways in which a single quadratic function can be represented?

IV. What are the real life situations/ problems that you can connect with quadratic

function?

4. Knowledge of Instructional Strategies for Quadratic Function

I. Kindly describe your approach of teaching quadratic function.

II. How do you evaluate learner's understanding in quadratic function?

III. Why do you think (the chosen) teaching approach will aid teaching of quadratic better than (name another one)? Based on your descriptions, I observed (during teaching) that you did not use this approach, why?

IV. What is your opinion about the use of multiple representations to teach mathematics? How did you use them to teach quadratic function in grade 10?

V. What are some relevant real-life examples or situations that can help learners to understand quadratic function?

5. Knowledge of the Learners

I. Teaching should build on the knowledge learners already have. What is your opinion about this? How do you relate this to quadratic function?

II. How do you identify weakness in a learner's preparedness for quadratic function? What do you do to support such learner(s)?

III. Teachers often utilize the questions that learners ask during the lesson for several purposes. Did your learners ask questions during the teaching of quadratic function? Can you share with me one, and your response, if you still remember? How do you facilitate that learners ask questions?

IV. Which concept of quadratic function did your learners find difficult to understand? Do you know why? I noticed most of your learners struggled to solve problems on (observed from the lesson), what are you going to do about that?

Appendix J: Requesting Parental Consent for Minors to Participate in a Research Project

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Dear Parent

Your son/daughter is invited to participate in a study entitled: An Insight into Teachers' Pedagogical Content Knowledge (PCK) Essential for Teaching Quadratic Functions in Grade 10, in Mogalakwena district. I am undertaking this study as part of my research project at the University of South Africa. The purpose of the study is to understand the concept of PCK and its basic components needed to teach quadratic functions in grade 10. The aim is to gain understanding of teachers' PCK and conceptual teaching that will assist mathematics teachers in Mogalakwena district schools to improve their teaching of quadratic functions. I am asking permission to include your child and other learners in this study because he/she is doing quadratic functions in mathematics.

If you allow your child to participate, I shall be observing him/her and other learners, together with their mathematics teacher in the classroom during the teaching of the selected topic.

Any information that is obtained in connection with this study and can be identified with your child will remain confidential and will only be disclosed with your permission. The will be used for this research purposes only.

There are no foreseeable risks to your child by participating in the study. Your child will receive no direct benefit from participating in the study; however, the possible benefits to education are teachers will improve in their practices of teaching quadratic equations. Neither your child nor you will receive any type of payment for participating in this study.

Your child's participation in this study is voluntary. Your child may decline to participate or to withdraw from participation at any time. Withdrawal or refusal to participate will not

affect him/her in any way. Similarly you can agree to allow your child to be in the study now and change your mind later without any penalty.

The study will take place during regular mathematics lesson with the prior approval of the school and your child's teacher. However, if you do not want your child to participate, an alternative activity will be available.

On top of your permission, your child must agree to participate in the study and you and your child will also be asked to sign the assent form which accompanies this letter. If your child does not wish to participate in the study, he or she will not be included and there will be no penalty. The information gathered from the study and your child's participation in the study will be stored securely on a password locked computer in my locked office for five years after the study. Thereafter, records will be erased.

If you have questions about this study please contact me, Banjo BO on 0733791386 or email banjo.balqis@gmail.com. Should you have concerns about the way in which the research has been conducted, you may contact my supervisor, Dr. M F Machaba in the Department of Mathematics Education - College of Education, University of South Africa on 012 429 8582 or email emachamf@unisa.ac.za. Permission for the study has already been given by Limpopo Department of Education (Mogalakwena district) and the Ethics Committee of the College of Education, UNISA.

You are making a decision about allowing your child to participate in this study. Your signature below indicates that you have read the information provided above and have decided to allow him or her to participate in the study. You may keep a copy of this letter.

Name of child: --------------------------------------

Sincerely

_____   _____
          _____

Parent/guardian's name (print)   Parent/guardian's signature: Date:

_____ _____
          _____

Researchers name (print)   Researcher's signature   Date:

Appendix K: A Letter Requesting Assent from Learners in a Secondary School to Participate in a Research Project

UNISA | college of education

Title of study: An Insight into Teachers' Pedagogical Content Knowledge (PCK) Essential for Teaching Quadratic Functions in Grade 10

Dear Learner

I am doing a study on teachers' pedagogical content knowledge (pck) essential for teaching quadratic functions in grade 10 in Bakenberg South circuit (Mogalakwena district) as part of my studies at the University of South Africa. Your principal has given me permission to do this study in your school. I would like to invite you to be a very special part of my study. I am doing this study so that I can find ways that your teachers can teach quadratic functions better. This will help you and many other learners of your age in different schools.

This letter is to explain to you what I would like you to do. There may be some words you do not know in this letter. You may ask me or any other adult to explain any of these words that you do not know or understand. You may take a copy of this letter home to think about my invitation and talk to your parents about this before you decide if you want to be in this study.

I would need to come and observe you and other learners, when your mathematics teacher is teaching you quadratic functions. I will write a report on the study but I will not use your name in the report or say anything that will let other people know who you are. You do not have to be part of this study if you don't want to take part. If you choose to be in the study, you may stop taking part at any time. No one will blame or criticise you for whatever reasons based on personal performance. When I am finished with my study, I shall return to your school to give a short talk about some of the helpful and interesting things I found out in my study. I shall invite you to come and listen to my talk.

There will not be any compensation or payment made to you as a result of participating in this study.

If you decide to be part of my study, you will be asked to sign below. If you have any other questions about this study, you can talk to me or you can have your parent or another adult call me, Banjo BO (Mrs.) on +27733791386 or email banjo.balqis@gmail.com. Do not sign the form until you have all your questions answered and understand what I would like you to do.

Do not sign written assent form if you have any questions. Ask your questions first and ensure that someone answers those questions.

WRITTEN ASSENT

I have read this letter which asks me to be part of a study at my school. I have understood the information about my study and I know what I will be asked to do. I am willing to be in the study.

_____

Learner's name (print):   Learner's signature: Date:

_____

Witness' name (print)   Witness' signature   Date:

(The witness is over 18 years old and present when signed.)

_____

Parent/guardian's name (print)   Parent/guardian's signature: Date:

_____

Researcher's name (print)   Researcher's signature: Date: