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Expertise-based ranking of experts: An assessment level approach

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Abstract

The quality of a formal decision is influenced by the level of expertise of the decision makers (DMs). The composition of a team of DMs can change when new members join or old members leave, based on their ranking. In order to improve the quality of decisions, this ranking should be based on their demonstrated expertise. This paper proposes using the experts' expertise levels, in terms of 'the ability to differentiate consistently', to determine their ranking, according to the level at which they assess alternatives. The expertise level is expressed using the CWS-Index (Cochran–Weiss–Shanteau), a ratio between Discrimination and Inconsistency. The experts give their evaluations using pairwise comparisons of Fuzzy Preference Relations with an Additive Consistency property. This property can be used to generate estimators, and replaces the repetition needed to obtain the CWS-Index. Finally, a numerical example is discussed to illustrate the model for producing expertise-based ranking of experts.

Keywords: Ranking; Expertise; Fuzzy Preference Relations; Additive Consistency; Assessment level

1. Introduction

The quality of a formal decision is heavily influenced by the level of expertise of the decision maker (DM) [1]. It is presumed that a decision made by an expert is better than a decision made by a non-expert, because an expert has the ability to think differently [1–3] and the inherent ability to understand the problem in more detail and depth, so that an expert can distinguish various aspects of the situation that are usually overlooked by a non-expert [4].

When a decision is made by several decision makers (DMs), this group of experts may be responsible for making an assessment of alternatives. The group decision or group opinion is a result of the integration of the individual opinions by a mathematical aggregation [5]. One important factor that should be considered in the aggregation process is which DMs' opinions should be included in the aggregation process. This means that the composition of the DM teams can be changed, i.e. new members can join a DM team while others leave depend on their ranking [6]. To improve the decision quality, this ranking should be determined on the basis of the DM's level of expertise.

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The DM's level of expertise needs to be defined. Weiss and Shanteau [7] proposed the concept of 'the ability to differentiate consistently' to assess the expertise level, and they based this solely on the expert's level of assessing alternatives. They defined experts as those who are capable of distinguishing between cases that are similar but not exactly the same and of repeating their judgments consistently. They proposed the CWS-Index (the Cochran–Weiss–Shanteau Index), which is the ratio between discrimination and inconsistency, to assess someone's level of expertise [7,8]. The CWS-Indexes for the experts yields their ranking; the higher the CWS-Index, the higher is the DM's ranking. However, measuring inconsistency requires repetition, and accordingly the experts need to make judgments more than once. This repeated evaluation is difficult to do independently in a way that ensures that there is no influence from the previous evaluation [9]. Moreover, those whose second evaluation is similar to their first will be considered consistent, even though the first evaluation is not necessarily true [1].

In Group Decision Making research, the pairwise comparisons approach of Fuzzy Preference Relations (FPR) has the Additive Consistency (AC) property. Pairwise comparisons have the advantage of focusing the assessment on two objects at a time [10]. The AC property of FPR can be used to measure the expert's consistency level [11–16] and produces a *consistency-based experts' ranking* without considering the ability of the expert to differentiate between similar, but not identical, cases. In relation to the concept of expertise, defined as the ability to differentiate consistently, as proposed by Weiss and Shanteau [7], the methodology of determining ranking in these prior studies is not based on expertise as a whole, because the studies only consider consistency and ignore the ability to differentiate.

There have been studies to determine the ranking of experts based on their level of assessment. Among the methods used are the use of factor scores to rank the assessment result of DMs in the group decision [17], the measurement of the total deviation between the estimated value and the real value for each element of the decision matrix [18], and the measurement of the total variance of the estimated value to the actual value for each element of the decision matrix [19]. In these previous researches, the experts' ranking are determined only by the consistency of their assessments, without considering their ability to differentiate, so these studies have not used the comprehensive concept of expertise.

In this paper, we focus on a Group Decision with one criterion where the DMs are ranked based on their level of expertise, irrespective of their position in the organization. The concepts used are the combination of expertise as 'the ability to differentiate consistently' and the AC property of FPR. The experts will give their judgments in FPR, so that the repetition required in Weiss and Shanteau's methodology is replaced by an estimation using the AC property. The focus of this research is to determine the ranking of the DMs. This ranking can be used to determine which DMs' opinions should be included in the aggregation process. This ranking can also be used to determine the importance weight of the DMs and research obtaining the DMs' importance weight from their ranking has been discussed in another paper [20].

The next section of this paper discusses the concept of *expertise* and the AC property of FPR. Then a methodology to obtain an *expertise-based ranking of experts* is discussed, followed by the implementation of the proposed methodology using numerical examples. Finally, the conclusions are presented and further research associated with the development of a model of the expertise-based ranking of experts is proposed.

2. Expert's expertise level and FPR's additive consistency

This part discusses the previous methods used to identify the expertise level of experts, and FPR's AC property. These two methods will be combined to develop the proposed method called *expertise-based ranking of experts*.

2.1. Expert's expertise level

An expert is an individual who has a background in a certain area and receives recognition from his/her peers in a particular technical field [21]. If a distinction is made according to the tasks to be accomplished, there are four types of experts [7], namely: expert predictors, expert instructors, expert performers and expert judges. An expert predictor conducts an evaluation to create a scenario for the future. An expert instructor must have the ability to judge and communicate clearly to others, in the way that a football coach does to his players. An expert performer should be able to perform the task well: for instance, an expert football player can score a goal. An expert judge makes both a qualitative and a quantitative evaluation. Weiss and Shanteau [22] stated that all type of expertise are influenced by the expert's judgment, then all type of expertise can't be separated from their judgment quality and in this study, an expert means someone with expert judgment.

Previous studies to determine the *expertise* level of an *expert* have identified certain important factors to be considered:

- Consistency
  The expert's judgment should be consistent over time. Those who are inconsistent can definitely not be called experts [23,24]. Consistency is a necessary, but not a sufficient, condition for expertise [7,8].
- Discrimination
  An expert should have discriminatory ability, the ability to differentiate between cases that are similar but not exactly the same [25], therefore 'the ability to differentiate' becomes a necessary, but not a sufficient, condition [7,8].

Weiss and Shanteau [7] proposed to combine the concepts of 'consistency' and 'discrimination' to determine the expertise level of a person becomes 'the ability to differentiate consistently', and is expressed by the CWS-Index as shown in equations (1), (2) and (3) as follows:

$$\text{CWS-Index} = \frac{\text{Discrimination}}{\text{Inconsistency}} = \frac{\text{Variance of different alternatives' values}}{\text{Variance of the same alternative's values}} \tag{1}$$

$$\text{Discrimination} = \frac{\sum_{j=1}^{n} r(M_j - GM)^2}{n-1} \tag{2}$$

$$\text{Inconsistency} = \frac{\sum_{j=1}^{n}\sum_{i=1}^{r}(M_{ij} - M_j)^2}{n(r-1)} \tag{3}$$

where

r : The number of replications
M_j : The average of individual values for case-j
GM : The grand mean of all individual values
n : The number of different cases
M_{ij} : The individual value for replication-i of case-j

Equation (2) shows that discrimination consists of the between group variance, and equation (3) shows that inconsistency is the within group variance. This can be seen from the formula in statistics $\frac{\sum_{j=1}^{n} r(M_j - GM)^2}{n-1}$, which is the variance of the average group (M_j) to the grand mean (GM) and is better known as the between group mean of squares [26].

According to Weiss and Shanteau [7], to get the CWS-Index, the evaluated experts are asked to give their assessment twice or more. Repeating the measurements are difficult and time-consuming [9] then the method for determining the level of expertise needs to be adjusted [27]. By way of illustration, an example of the calculation of the CWS-Index in a medical field study to estimate the probability that a patient had a chronic heart failure ([28] in [7]) was reanalyzed. Several physicians were asked to rate 45 patients, and rated five of the cases twice (this repetition without their knowledge). The evaluation results for one of these experts who judged five cases twice, and the CWS-Index calculations, are shown in Table 1.

2.2. FPR's Additive Consistency

Fuzzy Preference Relations (FPR) is one of the most widely used evaluation methods for expert assessment in Group Decision Making [29,30], because FPR is a very useful tool in modeling the decision process, primarily for aggregating individual opinions into a group opinion [13].

The next model for Group Decision Making is a model proposed by Herrera-Viedma et al. [12,30]. Suppose that a group of experts $E = \{e_1, e_2, \dots, e_m\}$, $m \geq 2$ give their preferences on a finite set of alternatives $X = \{x_1, x_2, \dots, x_n\}$, $n \geq 2$ by using FPR. FPR P on a set of alternatives X, $P \subset X \times X$ having a membership function $\mu_p : X \times X \to [0, 1]$ and represented by means of the $n \times n$ matrix $P = (p_{ij})$ [31,32]. p_{ij} is the preference degree of alternative x_i over alternative x_j. $p_{ij} = 1/2$ means there is indifference between x_i and x_j, $p_{ij} \in (1/2, 1]$ means x_i is preferred to x_j with the degree of p_{ij}, and $p_{ji} \in (1/2, 1]$ means x_j is preferred to x_i with the degree of p_{ji} [33].

Table 1
Example of CWS-Index calculation.

	Case-1	Case-2	Case-3	Case-4	Case-5
Replication – 1	96	18	94	95	25
Replication – 2	96	12	91	98	27
M_j	96	15	91.5	96.5	26
$r(M_j - GM)^2$	1897.28	5040.08	1490.58	1959.38	3073.28
$\sum_{i=1}^{r}(M_{ij} - M_j)^2$	0	18	4.5	4.5	2
Discrimination	3365.15 $\sum_{j=1}^{n} r(M_j - GM)^2/(n-1) =$				
Inconsistency	$\sum_{j=1}^{n}\sum_{i=1}^{r}(M_{ij} - M_j)^2/n(r-1) = 5.80$				
CWS-Index	$3365.15/5.80 = 580.20$				

Adapted from Skånér et al. [28] in [7].

The FPR is a reciprocal relation satisfying:

$$p_{ij} + p_{ji} = 1 \tag{4}$$

Thus the matrix P has the form

$$P = \begin{bmatrix} 0.5 & p_{12} & p_{13} & p_{14} \\ 1-p_{12} & 0.5 & p_{23} & p_{24} \\ 1-p_{13} & 1-p_{23} & 0.5 & p_{34} \\ 1-p_{14} & 1-p_{24} & 1-p_{34} & 0.5 \end{bmatrix} \tag{5}$$

The FPR as a reciprocal relation has several transitivity properties, such as FG-transitivity [34,35], h – iso stochastic transitivity [36] and cycle transitivity [35,36]. For more information about the transitivity property of FPR, we refer the reader to [35,37].

Tanino [31] in [38] proposed the *Additive Consistency* (AC) property and the multiplicative transitivity among three alternatives x_i, x_j and x_k. The AC property can be expressed as follows:

$$\left(p_{ij} - \frac{1}{2}\right) + \left(p_{jk} - \frac{1}{2}\right) = \left(p_{ik} - \frac{1}{2}\right) \quad \forall i, j, k = 1, 2, \ldots, n \tag{6}$$

Suppose x_j is an intermediate alternative. Equation (6) states that the intensity of preference of alternative-x_i over alternative-x_k is the sum of the intensity of preference of alternative-x_i over the intermediate alternative-x_j and the intensity preference of intermediate alternative-x_j over the alternative-x_k.

Equation (6) can be rewritten as equation (7) [31] in [38].

$$p_{ij} + p_{jk} + p_{ki} = 3/2 \quad \forall i, j, k = 1, 2, \ldots, n \tag{7}$$

If an *expert* expressed his/her preferences as $x_i \succ x_j \succ x_k$ (he/she preferred x_i over the other alternatives x_j and x_k), it would be illogical if the intensity of preference of alternative x_i over alternative x_j is greater than the intensity of preference of alternative x_i over alternative x_k [39], and consequently we have $p_{ij} \le p_{ik}$ for this expert.

The concept of AC for FPR is parallel to the concept of consistency for the Multiplicative Preference Relations of Saaty [11,30,31]. Equation (7) can be used to obtain the following three relationships between the preferences [12]:

$$p_{ik} = p_{ij} + p_{jk} - \frac{1}{2} \quad \forall i, j, k = 1, 2, \ldots, n \tag{8}$$

$$p_{jk} = p_{ji} + p_{ik} - \frac{1}{2} \quad \forall i, j, k = 1, 2, \ldots, n \tag{9}$$

$$p_{ij} = p_{ik} + p_{kj} - \frac{1}{2} \quad \forall i, j, k = 1, 2, \ldots, n \tag{10}$$

Each element of the decision matrix $\boldsymbol{P}$ is estimated in three different ways. From equations (8), (9) and (10) we can obtain estimated values, using the work of Herrera-Viedma et al. [12,40], as presented in equations (11), (12) and (13):

$$\varepsilon p_{ik}^{j1} = p_{ij} + p_{jk} - \frac{1}{2}, \quad j \neq i, k \tag{11}$$

$$\varepsilon p_{ik}^{j2} = p_{jk} - p_{ji} + \frac{1}{2}, \quad j \neq i, k \tag{12}$$

$$\varepsilon p_{ik}^{j3} = p_{ij} - p_{kj} + \frac{1}{2}, \quad j \neq i, k \tag{13}$$

εp_{ik}^{j1}: Estimation of p_{ik} using the first formula, equation (8)

εp_{ik}^{j2}: Estimation of p_{ik} using the second formula, equation (9)

εp_{ik}^{j3}: Estimation of p_{ik} using the third formula, equation (10)

Due to FPR has reciprocity consistency $p_{ij} + p_{ji} = 1$, then we can prove that these formulations in equation (11), (12) and (13) yield the same result, and for every element of the FPR matrix p_{ij}, the formulations produce as many as $(n-2)$ estimators (since $j \neq i, k$). These estimators allow the AC property to be used to complete the incomplete FPR matrix [12,14–16,40–42]. Additionally, AC can be used to measure a person's level of consistency in making an assessment [11–16], based on the deviation between the values of the estimations using the AC property and the real values given by the expert. The consistency level is then used to determine the ranking of the experts and generate *consistency-based ranking of experts*.

The multiplicative transitivity proposed by Tanino [31] is expressed by equation (14):

$$\frac{p_{ik}}{p_{ki}} = \frac{p_{ij}}{p_{ji}} \cdot \frac{p_{jk}}{p_{kj}} \tag{14}$$

This transitivity is equivalent to the cycle transitivity (the iso stochastic transitivity) [35,36,43]. The cycle transitivity is considered more appropriate because in the cycle transitivity there is unlikely division by zero [35].

The use of the AC property of FPR still has a contradiction with the range of each element of the FPR matrix, i.e. $\mu_p : X \times X \to [0, 1]$ [13]. From equation (11), the maximum value of εp_{ik}^{j1} is 1.5, and this value could be obtained if the values of p_{ij} and p_{jk} are equal to 1; the minimum value of εp_{ik}^{j1} is -0.5 and this value is reached when p_{ij} and p_{jk} are 0. The same conditions occur for εp_{ik}^{j2} and εp_{ik}^{j3} in equations (12) and (13), and the range of the estimated value of the FPR matrix elements are $[-0.5, 1.5]$ or $(-0.5 \leq \varepsilon p_{ik}^{j1} \leq 1.5, -0.5 \leq \varepsilon p_{ik}^{j2} \leq 1.5, -0.5 \leq \varepsilon p_{ik}^{j3} \leq 1.5)$.

There are several ways to keep the range of each element of the FPR matrix within the interval [0, 1], as follows:

1. The range [0, 1] could be achieved directly by changing the values of the estimation that are outside the range. If $\varepsilon p_{ik}^{jr} < 0$, it is set to equal zero, and if $\varepsilon p_{ik}^{jr} > 1$, it is changed to 1 [14,27] as in equation (15).

$$p_{ik}^{jr} = \begin{cases} 0 & \text{if } \varepsilon p_{ik}^{jr} < 0 \\ \varepsilon p_{ik}^{jr} & \text{if } 0 \leq \varepsilon p_{ik}^{jr} \leq 1 \\ 1 & \text{if } \varepsilon p_{ik}^{jr} > 1 \end{cases}, \quad j \neq i, k, \; r = 1, 2, 3 \tag{15}$$

   Where p_{ik}^{jr} is the estimation of p_{ik} with formula-r.

   With the adjustment in equation (15), the range of the matrix elements for FPR now becomes $0 \leq p_{ik}^{jr} \leq 1$. The problem is how to distinguish a zero arising from a negative value and a real zero. Furthermore, the estimation matrix elements with a value greater than one will be treated in the same way as the estimation matrix elements with a value equal to 1.
2. Modified Additive Consistency [43], as described in equation (16):

$$p_{ik}^{jr} = \begin{cases} \min\{p_{ij}, p_{jk}\} & \text{if } p_{ij}, p_{jk} \in [0, 0.5] \\ \max\{p_{ij}, p_{jk}\} & \text{if } p_{ij}, p_{jk} \in [0.5, 1] \\ \varepsilon p_{ik}^{jr} & \text{otherwise} \end{cases} \tag{16}$$

   The modified Additive Consistency satisfies 'restricted max–max transitivity' and 'restricted min–min transitivity' [43]. De Baets et al. in [35] expressed this type of transitivity as the T_M Transitivity where F and G is coincide. From equation (16), the estimated value of p_{ik} for small values of p_{ij} and p_{jk} is $\min\{p_{ij}, p_{jk}\}$ (satisfies the restricted min–min transitivity [43]), so it is not likely to be negative. Similarly, the value of p_{ik} for large values of p_{ij} and p_{jk} is replaced by the value of $\max\{p_{ij}, p_{jk}\}$ (satisfies the restricted max–max transitivity [43]), so there is no possibility that the estimated value of p_{ik} is more than one. The problem that arises is that modified Additive Consistency causes many different cases to be treated equally, for small values of p_{ij} and p_{jk} or for big values of p_{ij} and p_{jk}.

3. All elements p_{ik} of the estimated matrix can be transformed using a transformation function such that the range changes from $[-a, 1+a]$, $a > 0$ to $[0, 1]$ [13].
The transformation function is presented in equation (17).

$$f(x) = \frac{x+a}{1+2a} \tag{17}$$

This transformation keeps the FPR in the range [0, 1] while maintaining some basic attributes of FPR as described below [13]:
   1. The lowest value is 0: $f(-a) = 0$
   2. The highest value is 1: $f(1+a) = 1$
   3. Additive Reciprocity: $f(x) + f(1-x) = 1\ \forall x \in [-a, 1+a]$
   4. Additive Consistency: $f(x) + f(y) + f(z) = \frac{3}{2}\ \forall x, y, z \in [-a, 1+a]$ such that $x + y + z = \frac{3}{2}$
   5. Value Indifference: $f(0.5) = 0.5$

3. The proposed method

Weiss and Shanteau [7] showed that the CWS-Index is an excellent invention for comparing the expertise level of experts; however its weakness lies in the possibility that a non-expert obtains a high CWS-Index score by giving an **incorrect** assessment **consistently** [1,8]. According to this research, we can see that the inconsistency measurements require the expert to repeat his/her evaluation. It is very difficult to conduct this repetition independently without being affected by the previous assessment [9]. For example, the medical study's experts in the illustration had to assess 45 patients (and 5 repetitions) to obtain independent judgments (actually they required only ten judgments). Furthermore, an individual whose second assessment is close to his or her first assessment will be considered to be consistent, even though the first assessment itself is not necessarily correct [1]. Therefore, we need a refinement of the method of comparing experts' expertise [27].

In Group Decision Making research, the AC property of FPR is used in research where the expert can give scores in the *incomplete* decision matrix *FPR*. The AC property can be used to supplement an incomplete FPR with an estimation that uses the existing matrix elements. In this study, the AC of FPR will be used to replicate every element of the decision matrix FPR P, so that the difficulty in measuring independent repetition can be overcome. Experts are asked to provide an expert evaluation through the pairwise comparison approach of FPR. In order to keep the range of the estimated value of the FPR matrix elements within the limits [0, 1], and because the use of both equation (15) and equation (16) leads to a large number of different cases being treated equally, this study uses the transformation of equation (17) so that the range changes from $[-a, 1+a]$ to [0, 1].

Previous studies have produced not only the expert rankings, but also the importance weight of each expert that can be used in the aggregation process of individual opinions into a group opinion. However, these studies have not covered the whole expertise assessment proposed by Weiss and Shanteau [7], because they are based on consistency without considering discrimination. Discrimination or 'the ability to differentiate' should be considered in determining the ranking of the experts, because determining ranking based only on consistency could produce less appropriate results. As an extreme example, suppose an expert judges four alternatives and gives the same value for each of them. Consequently each element of the decision matrix has the value 0.5 ($p_{ij} = 0.5$ means indifference between alternative-X_i and alternative-X_j), as stated in matrix P_e in equation (18):

$$P_e = \begin{bmatrix} 0.5 & 0.5 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0.5 & 0.5 \end{bmatrix} \tag{18}$$

All estimations using the formulae in equations (11), (12) or (13) give the same value 0.5. There is no deviation between the values replicated using AC and the actual values given by this expert. This zero deviation means that

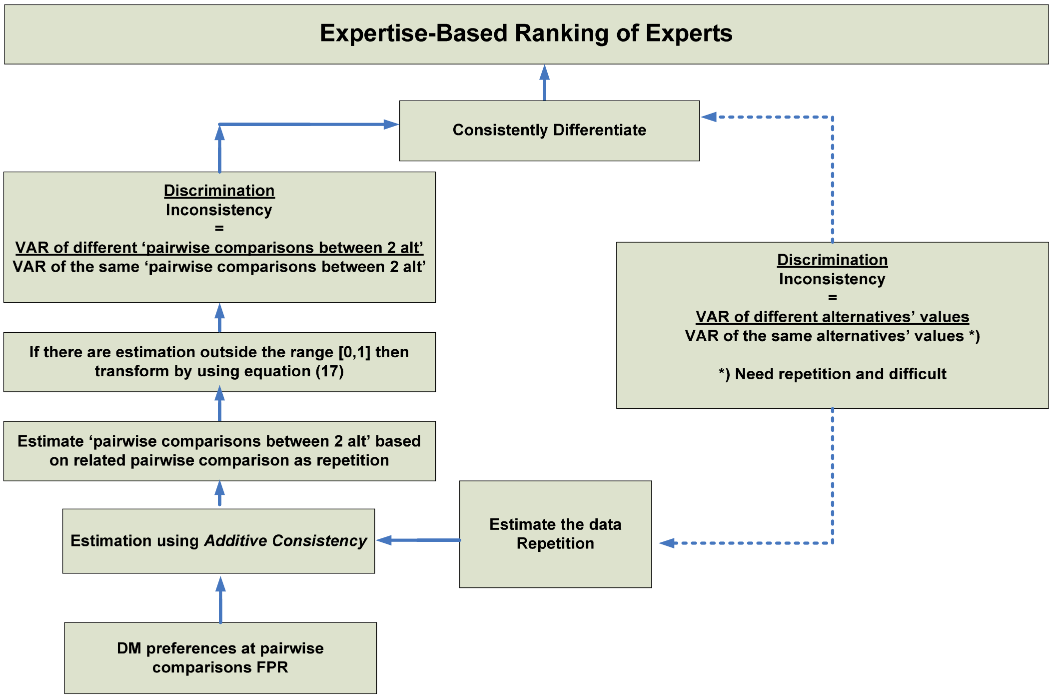


Fig. 1. Framework for *Expertise-based Ranking of Experts*.

this expert is a completely consistent expert and would have ranked first if we used the consistency-based ranking of experts as in previous studies. In our opinion, this result is not appropriate, because, if the decision matrix P_e is analyzed further, the equal value of all of the alternatives means that this expert cannot actually differentiate between the qualities of the alternatives.

This paper proposes combining the concept of *expertise* (the ability to differentiate consistently) with the AC property of FPR in the *expertise-based ranking of experts*. The determination of the level of expertise as 'the ability to differentiate consistently' is a very good concept, but it is difficult to use this to measure inconsistency because independent repetitions between the first and the following evaluations are needed. Additionally, a high CWS-Index can be obtained by giving an incorrect judgment consistently [1].

A framework for the expertise-based ranking of experts is depicted in Fig. 1. The expertise-based ranking of experts uses the concept of expertise proposed by Weiss and Shanteau [7] in form of the experts' capability to differentiate a set of alternatives consistently. The repetitions needed for the inconsistency measurement are replaced with estimation by using the AC property of FPR. There are $(n-1)$ values for each matrix element p_{ij} consist of one actual value from real data and $(n-2)$ estimated values. If there are elements from the estimation that lie outside the range [0, 1], then transform all of the values by using equation (17). The CWS-Index for the pairwise comparisons is adapted from the CWS-Index from equation (1) as follows:

$$\text{CWS-Index} = \frac{\text{Variance of different 'pairwise comparisons between two alternatives'}}{\text{Variance of the same 'pairwise comparisons between two alternative's'}} \tag{19}$$

The variance of different 'pairwise comparisons between two alternatives' is considered as the variance of different alternatives' values and the variance of the same 'pairwise comparisons between two alternatives' is considered as the variance of the same alternative's values. The expertise-based ranking of experts can be determined based on the CWS-Index values. The higher the CWS-Index of an expert is, the higher his/her ranking is.

The steps used to rank the experts using the method of *Expertise-based Ranking of Experts* are as follows:

1. Elicit each expert's opinion using the pairwise comparisons approach of *Fuzzy Preference Relations* in decision matrices.

Table 2
The evaluation form.

	X1	X2	X3	X4
X1	0.50			
X2		0.50		
X3			0.50	
X4				0.50

Table 3
Expert-1's judgment.

	X1	X2	X3	X4
X1	0.50	0.40	0.40	0.40
X2		0.50	0.40	0.70
X3			0.50	0.70
X4				0.50

2. Replace the repetition needed for the measurement of inconsistency with the estimation arising from the *Additive Consistency* of *Fuzzy Preference Relations* by using one of the formulations in equations (11), (12) or (13).
3. Transform, using equation (17), if there are elements from the estimation in step 2 that lie outside the range [0, 1].
4. Modify the CWS-Index for the expertise level in equation (1) for the pairwise comparisons approach of FPR as follows:

$$\text{CWS-Index} = \frac{\text{Variance of different `pairwise comparison between two alternatives'}}{\text{Variance of the same `pairwise comparison between two alternatives'}} \tag{20}$$

5. Determine *the Expertise-based Ranking of Experts* according to the CWS-Index values: the higher the CWS-Index of an expert, the higher his or her ranking.

4. Illustrative example

In order to show whether the proposed method is workable or not, we provide a numerical example to illustrate it. Suppose there are five people who are expert at judging the beauty of a painting. These experts are expressed as $E = \{e_1, e_2, e_3, e_4, e_5\}$. They were asked to provide an assessment of four paintings in the form of pairwise comparisons approach of FPR. These paintings form a set of four alternatives $X = \{x_1, x_2, x_3, x_4\}$.

The experts were asked to fill the evaluation form in Table 2, with the FPRs $0 \leq p_{ij} \leq 1$.

In every judgment, the experts have to focus on the assessment on one pair of alternatives to answer how much does he/she prefer alternative-x_i to alternative-x_j and fill in the blank space of Table 2.

- If there are indifferent between alternative-x_i and alternative-x_j, then $p_{ij} = 0.5$.
- If alternative-x_i is preferable than alternative-x_j, then $0.5 < p_{ij} < 1.0$.
- If alternative-x_i is absolutely preferable than alternative-x_j, then $p_{ij} = 1.0$.
- If alternative-x_i is not preferable than alternative-x_j, then $0.0 < p_{ij} < 0.5$.
- If alternative-x_i is absolutely not preferable than alternative-x_j, then $p_{ij} = 0$.

For instance, for Expert-1, the first painting, alternative-x_1 is slightly not preferable than the second painting, alternative-x_2, then Expert-1 should fill in $0.0 < p_{ij} < 0.5$, for example $p_{12} = 0.40$ as in Table 3. The complete judgment for Expert-1 is presented in Table 3 and by using the additive reciprocity property, $p_{ij} + p_{ji} = 1$, the whole cells in Table 3 can be completed as in the decision matrix P_1.

Step 1

Elicit each expert's opinion in FPR pairwise comparison of alternatives **X**. Expert-1 provides an evaluation in the decision matrix P_1, Expert-2 in the decision matrix P_2, and so on. The data for the experts' assessments are as follows:

$$P_1 = \begin{bmatrix} 0.50 & 0.40 & 0.40 & 0.40 \\ 0.60 & 0.50 & 0.40 & 0.70 \\ 0.60 & 0.60 & 0.50 & 0.70 \\ 0.60 & 0.30 & 0.30 & 0.50 \end{bmatrix}, \qquad P_2 = \begin{bmatrix} 0.50 & 0.30 & 0.40 & 0.45 \\ 0.70 & 0.50 & 0.65 & 0.60 \\ 0.60 & 0.35 & 0.50 & 0.45 \\ 0.55 & 0.40 & 0.55 & 0.50 \end{bmatrix},$$

$$P_3 = \begin{bmatrix} 0.50 & 0.90 & 0.60 & 0.70 \\ 0.10 & 0.50 & 0.30 & 0.40 \\ 0.40 & 0.70 & 0.50 & 0.60 \\ 0.30 & 0.60 & 0.40 & 0.50 \end{bmatrix},$$

$$P_4 = \begin{bmatrix} 0.50 & 0.40 & 0.80 & 0.70 \\ 0.60 & 0.50 & 0.60 & 0.35 \\ 0.20 & 0.40 & 0.50 & 0.10 \\ 0.30 & 0.65 & 0.90 & 0.50 \end{bmatrix}, \qquad P_5 = \begin{bmatrix} 0.50 & 0.70 & 0.80 & 0.60 \\ 0.30 & 0.50 & 0.65 & 0.55 \\ 0.20 & 0.35 & 0.50 & 0.40 \\ 0.40 & 0.45 & 0.60 & 0.50 \end{bmatrix}$$

Step 2

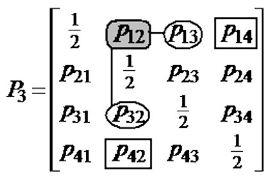
Replace the repetition used to measure inconsistency by estimations using the AC properties of Fuzzy Preference Relations. The estimations are conducted using one of the formulae in equations (11), (12) or (13). For example, Expert-4 gives the opinions in the decision matrix-P_4.

$$P_4 = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \\ p_{41} & p_{42} & p_{43} & p_{44} \end{bmatrix}$$

The estimation of each element in the matrix P_4 using formulae 1, 2 or 3 will generate two estimated values. The example below shows how to determine the estimated value of one matrix element P: P_{12} of Expert-4.

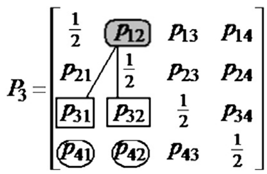
Formula 1:

$$\varepsilon p_{ik}^{j1} = p_{ij} + p_{jk} - \tfrac{1}{2}, \quad j \neq i, k$$
$$\varepsilon p_{12}^{j1} = p_{1j} + p_{j2} - \tfrac{1}{2}, \quad j \neq 1, 2$$
$$j = 3 \rightarrow \varepsilon p_{12}^{31} = p_{13} + p_{32} - \tfrac{1}{2} = 0.80 + 0.40 - 0.5 = 0.70$$
$$j = 4 \rightarrow \varepsilon p_{12}^{41} = p_{14} + p_{42} - \tfrac{1}{2} = 0.70 + 0.65 - 0.5 = 0.85$$



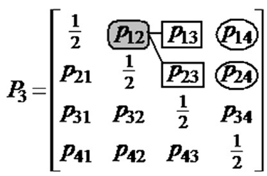
Formula 2:

$$\varepsilon p_{ik}^{j2} = p_{jk} - p_{ji} + \tfrac{1}{2}, \quad j \neq i, k$$
$$\varepsilon p_{12}^{j2} = p_{j2} - p_{j1} + \tfrac{1}{2}, \quad j \neq 1, 2$$
$$j = 3 \rightarrow \varepsilon p_{12}^{32} = p_{32} - p_{31} + \tfrac{1}{2} = 0.40 - 0.20 + 0.5 = 0.70$$
$$j = 4 \rightarrow \varepsilon p_{12}^{42} = p_{42} - p_{41} + \tfrac{1}{2} = 0.65 - 0.20 + 0.5 = 0.85$$



Formula 3:

$$\varepsilon p_{ik}^{j3} = p_{ij} - p_{kj} + \tfrac{1}{2}, \quad j \neq i, k$$
$$\varepsilon p_{12}^{j3} = p_{1j} - p_{2j} + \tfrac{1}{2}, \quad j \neq 1, 2$$
$$j = 3 \rightarrow \varepsilon p_{12}^{33} = p_{13} - p_{23} + \tfrac{1}{2} = 0.80 - 0.60 + 0.5 = 0.70$$
$$j = 4 \rightarrow \varepsilon p_{12}^{43} = p_{14} - p_{24} + \tfrac{1}{2} = 0.70 - 0.35 + 0.5 = 0.85$$



The estimated values of all elements of the matrix P_4 are presented in Table 4.

Table 4
Estimated values of matrix element P_4.

Element matrix	Actual judgment	Estimated values before transformation	
p_{12}	0.40	0.70	0.85
p_{13}	0.80	0.50	**1.10**
p_{14}	0.70	0.25	0.40
p_{21}	0.60	0.15	0.30
p_{23}	0.60	0.75	0.90
p_{24}	0.35	0.80	0.20
p_{31}	0.20	**−0.10**	0.50
p_{32}	0.40	0.10	0.25
p_{34}	0.10	0.25	0.40
p_{41}	0.30	0.60	0.75
p_{42}	0.65	0.80	0.20
p_{43}	0.90	0.75	0.60

Table 5
Transformed value calculation of CWS-Index for Expert-4.

Element matrix	Actual judgment	Estimated values after transformation		M_j	$r(M_j - GM)^2$	$\sum_{i=1}^{r}(M_{ij} - M_j)^2$
p_{12}	0.4167	0.6667	0.7917	0.6250	0.04688	0.07292
p_{13}	0.7500	0.5000	1.0000	0.7500	0.18750	0.12500
p_{14}	0.6667	0.2917	0.4167	0.4583	0.00521	0.07292
p_{21}	0.5833	0.2083	0.3333	0.3750	0.04688	0.07292
p_{23}	0.5833	0.7083	0.8333	0.7083	0.13021	0.03125
p_{24}	0.3750	0.7500	0.2500	0.4583	0.00521	0.13542
p_{31}	0.2500	0	0.5000	0.2500	0.18750	0.12500
p_{32}	0.4167	0.2917	0.1667	0.2917	0.13021	0.03125
p_{34}	0.1667	0.2917	0.4167	0.2917	0.13021	0.03125
p_{41}	0.3333	0.5833	0.7083	0.5417	0.00521	0.07292
p_{42}	0.6250	0.7500	9,2500	0.5417	0.00521	0.13542
p_{43}	0.8333	0.7083	0.5833	0.7083	0.13021	0.03125
Total					1.01042	0.93750

Step 3
Transform all the estimated values using equation (17). Table 4 shows that some estimated values are outside the range [0, 1]. This indicates the need for transformation. The estimated values are in the range [−0.1, 1.1], based on equation (17), the transformation function used is:

$$f(x) = \frac{x + 0.1}{1 + 2 \times 0.1}$$

The transformations of the estimated values are presented in Table 5.

Step 4
Calculate the CWS-Index for the experts by using equation (19). The calculation of the CWS-Index for Expert-4 is shown in Table 5. For each element of the matrix P_4 there are three values ($r = 3$), i.e. two estimated values and one real value.

$$\text{Discrimination} = \frac{\sum_{j=1}^{n} r(M_j - GM)^2}{n - 1} = \frac{1.01042}{(12 - 1)} = 0.09186$$

$$\text{Inconsistency} = \frac{\sum_{j=1}^{n} \sum_{i=1}^{r} (M_{ij} - M_j)^2}{n(r - 1)} = \frac{0.93750}{12 \times (3 - 1)} = 0.03906$$

$$\text{CWS-Index for Expert-4} = \frac{0.09186}{0.03906} = 2.351$$

Table 6
Discrimination, inconsistency, CWS-Index and ranking of experts.

	Expert-1	Expert-2	Expert-3	Expert-4	Expert-5
Discrimination	0.05333	0.04591	0.144808	0.09186	0.09118
Inconsistency	0.01389	0.00208	0.002778	0.03906	0.00486
CWS-Index	3.840	22.039	52.127	2.351	18.757
Rank	4	2	1	5	3

Step 5
Determine *the Expertise-based Ranking of Experts* according to the CWS-Index values.

The CWS-Index calculation and the results are presented in Table 6. The CWS-Indexes for Expert-1, Expert-2, Expert-3, Expert-4 and Expert-5 are, respectively, 3.840, 22.039, 52.127, 2.351 and 18.757. Based on these CWS-Indexes, the Expertise-based ranking of the experts obtained is Expert 3–Expert 2–Expert 5–Expert 1–Expert 4.

Table 6 ranks Expert-3 first because Expert-3 has the highest 'ability to differentiate' and a low 'inconsistency value' (which means that Expert-3 is very consistent) so that he/she has the highest CWS-Index and is ranked first. Relating this to the concept of expertise (the ability to differentiate consistently), Expert-3 has a very high 'ability to differentiate consistently', the highest of all the experts, so Expert-3 is put in first place.

In terms of the value of inconsistency, Expert-2 is the most consistent expert because he or she has the lowest inconsistency value. However, he or she has a low discrimination value. This low discrimination value means that Expert-2 cannot differentiate well between alternatives. An expert should have the ability to differentiate between cases that are similar but not exactly the same [7,8,25] and the ability to differentiate is a necessary condition for an expert [7,8]. The low discrimination value makes the CWS-Index of Expert-2 lower than that of Expert-3, so Expert-2 cannot be ranked first. If, instead, the ranking had been based solely on consistency values (as in *consistency-based ranking of experts*), Expert-2 would definitely has been ranked first.

Expert-5 has the second highest discrimination value, but because Expert-5 does not have a very good inconsistency value, he/she is not ranked second. Those who are not consistent are certainly not experts [23]. This is reflected in the CWS-Index which is lower for Expert-5 than for Expert-2. Based on the value of CWS-Index, Expert-2 is ranked second and Expert-5 third.

Expert-4 has high ability to differentiate, but he/she has the worst inconsistency score, so he/she is ranked last. Although the discrimination ability of Expert-4 is better than that of Expert-1, Expert-4 is the least consistent expert, having the lowest CWS-Index, so is placed in the lowest rank.

Based on the CWS-Indexes in Table 6, the expertise-based ranking of experts is Expert 3–Expert 2–Expert 5–Expert 1–Expert 4. The ranking of these same experts may be different in another case, or even in the same case using different criteria, because each expert has different expertise in different fields [44,45]. For example, if the experts assess four paintings using a different criterion, such as the economic value of the paintings, then the ranking of the experts may be different because a person could be an expert in art and but not an expert in the economic value of a painting.

This research proposed an alternative way to estimate inconsistency without the necessity to do repetitions. The price is that the experts should judge two alternatives at once using pairwise comparisons approach of FPR that seems to be more difficult judgment than merely evaluating an individual alternative. Fortunately, although judgment using pairwise comparison seems to be rather difficult, but pairwise comparison has the advantage of focusing the assessment on two objects at a time [10].

The proposed model requires nC2 pairwise comparisons (n is the number of alternatives). In this example, there are 4 alternatives and the proposed model requires $n(n-1)/2$, 6 judgments for each expert. As the number of alternatives increase, this method calls for increasingly more judgments, for example 5 alternatives needs 10 judgments, 6 alternatives needs 15 judgments, 7 alternatives needs 21 judgments etc. This will be inconvenient for the experts to do so many judgments. Another reason to limit the number of alternatives is the limitation of human capacity, as the human can differentiate up to 7 ± 2 alternatives [46–48]. The proposed model should be used when there are only a small number of different alternatives and the replication is difficult to do independently. Based on the work of Ozdemir [48], the authors suggest a maximum of 7 ± 2 alternatives. If we only have a small number of alternatives, this method has the advantage of obtaining independent replications (it is difficult to do with the previous method by Weiss and Shanteau [7,8]).

5. Conclusion

This paper proposed an *expertise-based ranking of experts* method to rank experts; in this method every expert gives his/her judgment in the pairwise comparison approach of Fuzzy Preference Relations.

Expertise-based ranking of experts in this study identifies expertise using the methodology of Weiss and Shanteau [7]; here expertise is considered to be 'the ability to differentiate consistently' and is expressed as the CWS-Index, a ratio between the discrimination and the inconsistency values. The difficulties in measuring inconsistency using independent repetition are solved by using the Additive Consistency property of Fuzzy Preference Relations.

The proposed model enables us to obtain the expertise-based ranking of experts based on their assessment level, and the result should be that the higher the expertise level of an expert, the higher his/her CWS-Index and rank.

This method has two advantages:

(1) It uses the whole expertise concept.
In previous research, the ranking of experts is only determined by the consistency of the experts' assessment. In this study, the ranking of the experts is determined based on the consistency and the ability to distinguish, so the determination of the ranking in this study uses the whole concept of expertise.

(2) It solves the difficulty of measuring independent repetition.
In previous research related to expertise, the consistency measurement required repeated measurements, but an assessment will be influenced by the previous evaluation. In this study, repetition is not necessary because it has been replaced with the estimations obtained from the additive consistency property.

6. Future work

There is room for further research based on the developments in this study, namely:

- The study of *Expertise-based ranking of experts* when the experts give their evaluations using *incomplete* FPR.
- The analysis of *Expertise-based ranking of experts* when the experts give their evaluation in a *format* that is *different* from pairwise comparisons FPR.
- *Expertise-based ranking of experts* can be developed into *Expertise-based experts' importance weights* that specify the importance weights of the experts in Group Decision Making, and can be continued with the use of these importance weights in the process of aggregating the individual opinions into a group opinion.

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References

[1] V. Malhotra, M.D. Lee, A. Khurana, Domain experts influence decision quality: towards a robust method for their identification, J. Pet. Sci. Eng. 57 (1–2) (2007) 181–194.
[2] M.T.H. Chi, R. Glaser, M.J. Farr, The Nature of Expertise, Lawrence Erlbaum Associates, New Jersey, 1988.
[3] J.R. Anderson, Cognitive Psychology and Its Implications, vol. 5, Worth Publishing, New York, 2000.
[4] C. Horn, How experience affects perception in expert decision-making, Perception 05-814 (2) (2006) 1–10.
[5] R.M. Cooke, Experts in Uncertainty. Opinion and Subjective Probability in Science, Oxford University Press, New York, Oxford, 1991.
[6] J.S. Gero, U. Kannengiesser, Modelling expertise of temporary design teams, J. Des. Res. 4 (3) (2004) 1–13.
[7] D.J. Weiss, J. Shanteau, Empirical assessment of expertise, Hum. Factors 45 (1) (2003) 104–116.
[8] J. Shanteau, D.J. Weiss, R.P. Thomas, J.C. Pounds, Performance-based assessment of expertise: how to decide if someone is an expert or not?, Eur. J. Oper. Res. 136 (2) (2002) 253–263.
[9] Michael D. Lee, Mark Steyvers, M.d. Young, B. Miller, Inferring expertise in knowledge and prediction ranking tasks, Top. Cogn. Sci. 4 (1) (2012) 151–163.
[10] T.L. Saaty, M. Sodenkamp, Making decisions in hierarchic and network systems, Int. J. Appl. Decis. Sci. 1 (1) (2008) 24–79.

[11] F. Chiclana, E. Herrera-Viedma, F. Herrera, S. Alonso, Some induced ordered weighted averaging operators and their use for solving group decision making problems based on fuzzy preference relations, Eur. J. Oper. Res. 182 (1) (2007) 383–399.
[12] E. Herrera-Viedma, F. Chiclana, F. Herrera, S. Alonso, Group decision-making model with incomplete fuzzy preference relations based on additive consistency, IEEE Trans. Syst. Man Cybern. B 37 (1) (2007) 176–189.
[13] E. Herrera-Viedma, F. Herrera, F. Chiclana, M. Luque, Some issues on consistency of fuzzy preference relations, Eur. J. Oper. Res. 154 (1) (2004) 98–109.
[14] S. Alonso, F. Chiclana, F. Herrera, E. Herrera-Viedma, J. Alcal'a-Fdez, C. Porcel, A consistency-based procedure to estimate missing pairwise preference values, Int. J. Intell. Syst. 23 (2) (2008) 155–175.
[15] S. Alonso, F.J. Cabrerizo, F. Chiclana, F. Herrera, E. Herrera-Viedma, Group decision making with incomplete fuzzy linguistic preference relations, Int. J. Intell. Syst. 24 (2) (2009) 201–222.
[16] G. Zhang, Y. Dong, Y. Xu, Linear optimization modeling of consistency issues in group decision making based on fuzzy preference relations, Expert Syst. Appl. 39 (3) (2012) 2415–2420.
[17] C. Xia, F. Zhi-ping, Study on assessment level of experts based on difference preference information, Syst. Eng., Theory Pract. 27 (2) (2007) 27–35.
[18] X. Zhang, G. Yue, X. Zheng, Assigning method for decision power based on linguistic 2-tuple judgment matrices, J. Softw. 6 (3) (2011) 508–515.
[19] C. Yan, L. Jing, C. Xia, L. Yuanyuan, Method on the assessment level of experts in group decision making based on multi-granularity linguistic judgment matrices, in: International Conference on Information Engineering and Computer Science, Wuhan, China, 2009, pp. 1–5.
[20] E. Herowati, U. Ciptomulyono, J. Parung, Suparno, Expertise-based experts importance weights in adverse judgment, J. Eng. Appl. Sci. 9 (9) (2014).
[21] N.R. Ortiz, T.A. Wheeler, R.J. Breeding, S. Hora, M.A. Meyer, R.L. Keeney, Use of expert judgment in Nureg-1150, Nucl. Eng. Des. 126 (3) (1991) 313–331.
[22] D.J. Weiss, J. Shanteau, Who's the best? A relativistic view of expertise, Appl. Cogn. Psychol. 28 (2014) 447–457.
[23] H.J. Einhorn, Expert judgement: some necessary conditions and an example, J. Appl. Psychol. 59 (5) (1974) 562–571.
[24] H.J. Einhorn, Expert measurement and mechanical combination, Organ. Behav. Hum. Perform. 7 (1) (1972) 86–106.
[25] K.R. Hammond, Human Judgment and Social Policy, Oxford University Press, New York, 1996.
[26] G.E.P. Box, J.S. Hunter, W.G. Hunter, Statistics for Experimenters: Design, Innovation, and Discovery, John Wiley & Sons, Hoboken, New Jersey, 2005.
[27] E. Herowati, U. Ciptomulyono, J. Parung, Suparno, Competent-based experts' ranking at fuzzy preference relations on alternatives, in: International Conference on Industrial Engineering and Service Science, Surabaya, Indonesia, 2013, pp. K7.1–K7.6.
[28] Y. Skånér, L. Strender, J. Bring, How do Gps use clinical information in the judgements of heart failure?, Scand. J. Prim. Health Care 16 (2) (1998) 95–100.
[29] J. Kacprzyk, Group decision making with a fuzzy linguistic majority, Fuzzy Sets Syst. 18 (2) (1986) 105–118.
[30] F. Chiclana, F. Herrera, E. Herrera-Viedma, Integrating three representation models in fuzzy multipurpose decision making based on fuzzy preference relations, Fuzzy Sets Syst. 97 (1) (1998) 33–48.
[31] T. Tanino, Fuzzy preference orderings in group decision making, Fuzzy Sets Syst. 12 (2) (1984) 117–131.
[32] J.L. García-Lapresta, L.C. Meneses, Modeling rationality in a linguistic framework, Fuzzy Sets Syst. 160 (22) (2009) 3211–3223.
[33] J.C. Bezdek, B. Spillman, R. Spillman, Fuzzy relation spaces for group decision theory: an application, Fuzzy Sets Syst. 2 (1) (1979) 5–14.
[34] Z. Świtalski, General transitivity conditions for fuzzy reciprocal preference matrices, Fuzzy Sets Syst. 137 (1) (2003) 85–100.
[35] B.D. Baets, H.D. Meyer, Transitivity frameworks for reciprocal relations: cycle-transitivity versus Fg-transitivity, Fuzzy Sets Syst. 152 (2) (2005) 249–270.
[36] B.D. Baets, H.D. Meyer, B.D. Schuymer, S. Jenei, Cyclic evaluation of transitivity of reciprocal relations, Soc. Choice Welf. 26 (2) (2006) 217–238.
[37] B.D. Baets, H.D. Meyer, S. Freson, Closing reciprocal relations W.R.T. stochastic transitivity, Fuzzy Sets Syst. 241 (2014) 2–26.
[38] J. Kacprzyk, M. Fedrizzi, Multiperson Decision Making Using Fuzzy Sets and Possibility Theory, Kluwer Academic Publishers, Dordrech, 1990.
[39] M. Rademaker, B.D. Baets, Aggregation of monotone reciprocal relations with application to group decision making, Fuzzy Sets Syst. 184 (1) (2011) 29–51.
[40] E. Herrera-Viedma, S. Alonso, F. Chiclana, F. Herrera, A consensus model for group decision making with incomplete fuzzy preference relations, IEEE Trans. Fuzzy Syst. 15 (5) (2007) 863–877.
[41] F. Chiclana, E. Herrera-Viedma, S. Alonso, A note on two methods for estimating missing pairwise preference values, IEEE Trans. Syst. Man Cybern. B 39 (6) (2009) 1628–1633.
[42] S. Alonso, E. Herrera-Viedma, F. Chiclana, F. Herrera, A web based consensus support system for group decision making problems and incomplete preferences, Inf. Sci. 180 (23) (2010) 4477–4495.
[43] F. Chiclana, E. Herrera-Viedma, S. Alonso, R. Alberto, M. Pereira, Preferences and consistency issues in group decision making, in: Fuzzy Sets and Their Extensions: Representation, Aggregation and Models Studies in Fuzziness and Soft Computing, vol. 220, Springer, Berlin, Heidelberg, 2008, pp. 219–237.
[44] J. Shanteau, Competence in experts: the role of task characteristics, Organ. Behav. Hum. Decis. Process. 53 (2) (1992) 252–266.
[45] D.J. Weiss, J. Shanteau, Decloaking the privileged expert, J. Manag. Organ. 18 (3) (2012) 300–310.
[46] T.L. Saaty, Why the magic number seven plus or minus two, Math. Comput. Model. 38 (3–4) (2003) 233–244.
[47] G.A. Miller, The magical number seven, plus or minus two: some limits on our capacity for processing information, Psychol. Rev. 63 (2) (1956) 81–97.
[48] M.S. Ozdemir, Validity and inconsistency in the analytic hierarchy process, Appl. Math. Comput. 161 (3) (2005) 707–720.