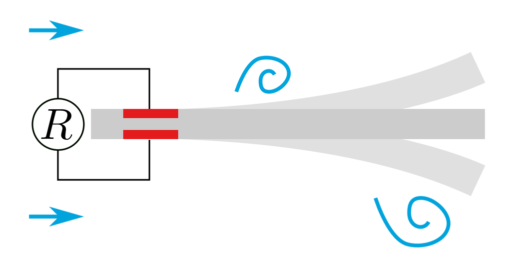
Toward fluid-structure-piezoelectric simulations applied to flow-induced energy harvesters

C. HOAREAU, L. SHANG and A. ZILIAN

Institute of Computational Engineering, FSTC, University of Luxembourg , Luxembourg, christophe.hoareau@uni.lu

UNIVERSITÉ DU LUXEMBOURG

1 Flow Induced Energy Harvesting

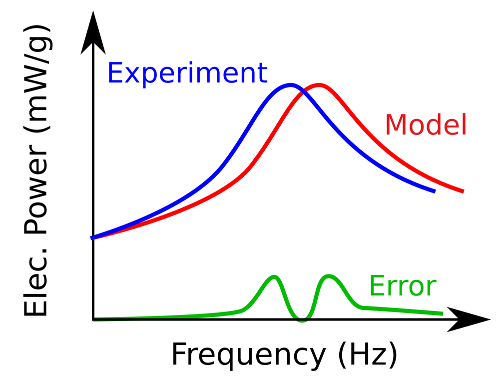


Structure — Piezoelectric patch — Fluid — Circuit

Ambient fluid flow energy → Energy Harvester → Electrical energy

- Autonomous piezo-ceramic power generators
- Nonlinear dynamic behavior
- Flutter induced vibrations (1)

2 Research objectives



Maximize the power output
Minimize the fatigue exposure

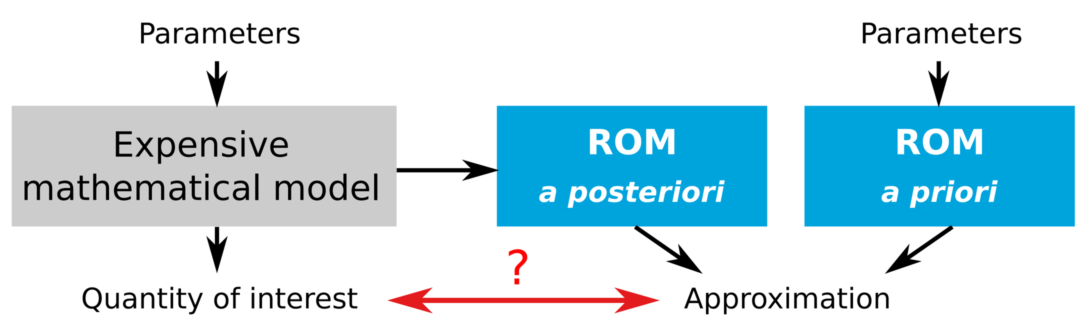
- Model and predict the nonlinear dynamic behavior (2)
- Quantify the sensibility under changing conditions
- Allow just-in-time feedback : construct on-line approximations

3 Problems

- Complex multiphysics phenomena
- Numerous fields (displacement, velocity, pressure, intensity ...)
- Incresing number of parameters
- Nonlinear behaviors (geometrical nonlinearity, fluid dynamic)

? Can we construct a Reduced Order Model to approximate a nonlinear multiphysic problem ?

4 Methodology



Numerical tool (3)
Finite Element Method

FEniCS PROJECT

5 High fidelity model+FEniCS

Hypotheses

Fluid	Structure	Piezo + circuit
Incompressible Newtonian Homogeneous Isotropic Low Reynolds number Moving mesh	Elastic Homogeneous Isotropic Finite strain	Linear electromechanica Resitor circuit

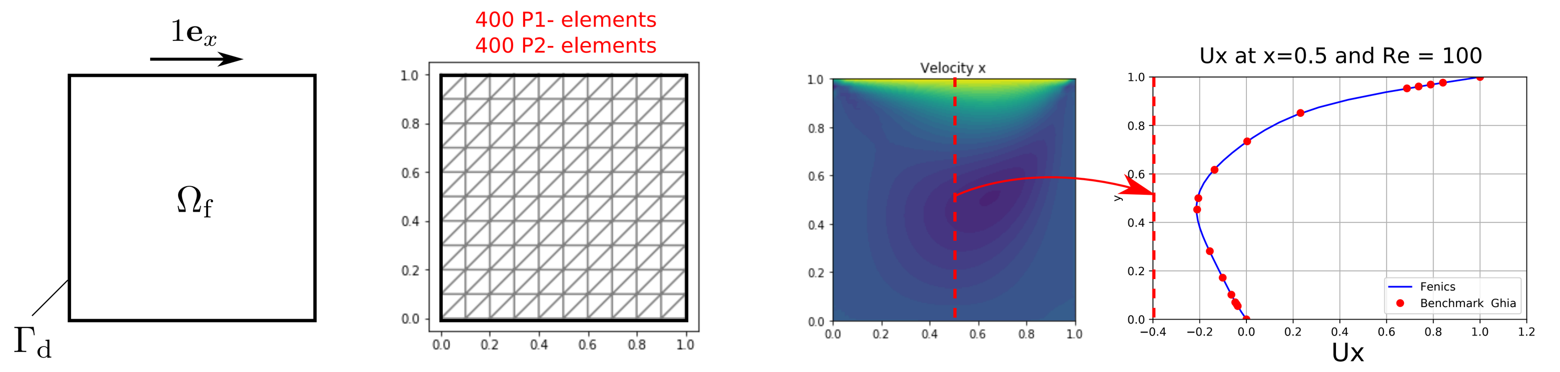
Example : Steady-Navier Stokes - 2D Lid driven cavity

- Strong form (velocity - pressure)

$$\nabla\cdot\mathbf{v}=0 \quad \text{in} \quad \Omega_f$$
$$\mathbf{v}\cdot\nabla\mathbf{v}+\nabla p-\frac{1}{\mathrm{Re}}\nabla^2\mathbf{v}=\mathbf{0} \quad \text{in} \quad \Omega_f$$
$$\mathbf{v}=\mathbf{v}_d \quad \text{on} \quad \Gamma_d$$

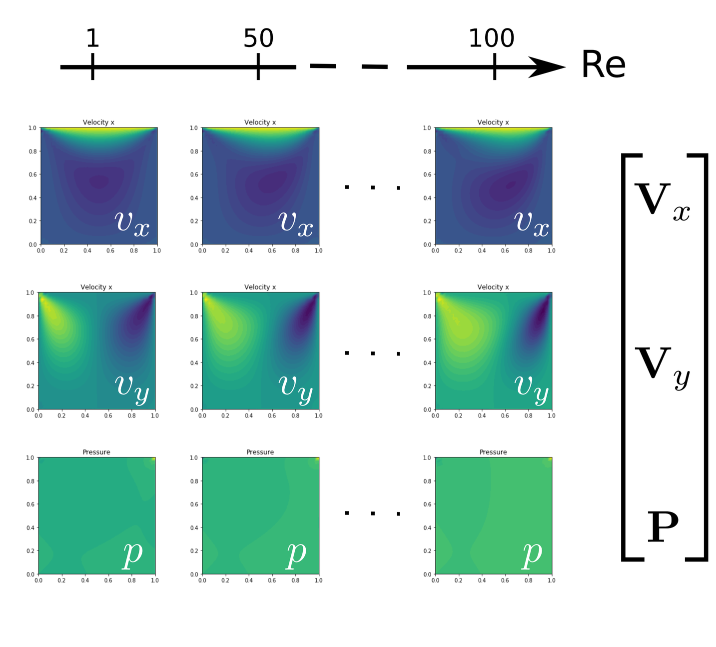
- Weak form (velocity - pressure)

$$c(\mathbf{v},\mathbf{v},\delta\mathbf{v})-b(p,\delta\mathbf{v})+\frac{1}{\mathrm{Re}}a(\mathbf{v},\delta\mathbf{v})=0\ \forall\ \delta\mathbf{v}$$
$$b(\mathbf{v},\delta p)=0\ \forall\ \delta p$$

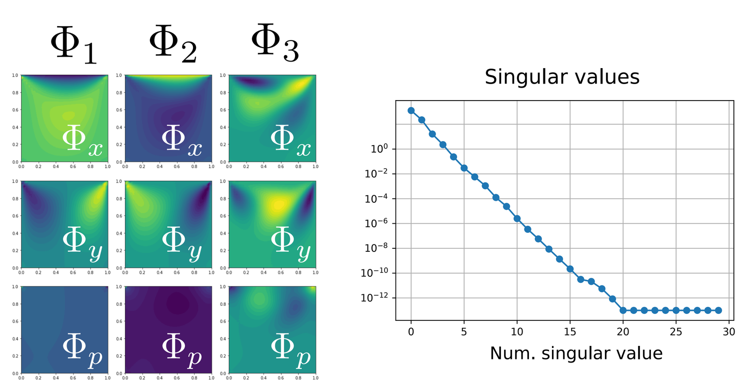


A posteriori approaches (ex: Steady Navier-Stokes)

Snapshots (velocity - pressure)



Proper Orthogonal Decomposition (4)



Discussions

- Appropriate method / weakly nonlinear problems
- Subjected to the curse of dimensionality
- Tests with moving meshes on-going

6 Reduced Order Modelling

A priori approaches

Proper Generalized Decomposition (5)

Example : Steady Navier-Stokes

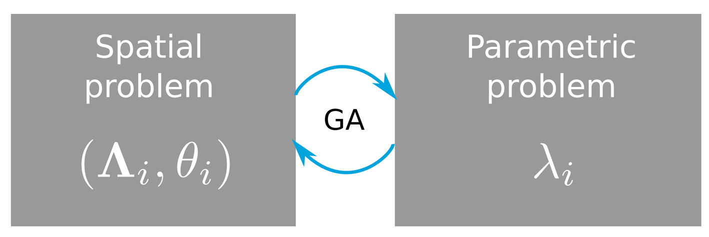
- Series of separated functions variables products

$$\mathbf{v}(\mathbf{X},\mathrm{Re})\simeq\sum_{i=1}^{N}\mathbf{\Lambda}_i(\mathbf{X})\lambda_i(\mathrm{Re})+\mathbf{v}^*$$

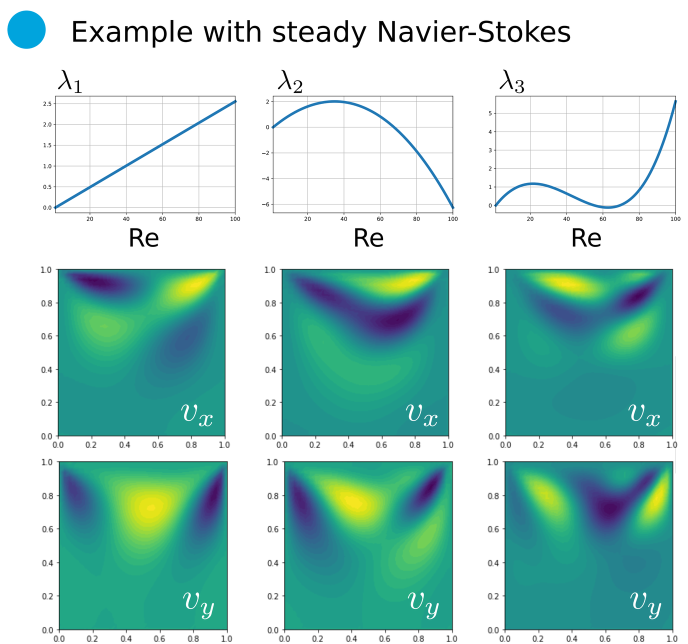
Satisfy Dirichlet BC

$$p(\mathbf{X},\mathrm{Re})\simeq\sum_{i=1}^{N}\theta_i(\mathbf{X})\lambda_i(\mathrm{Re})$$

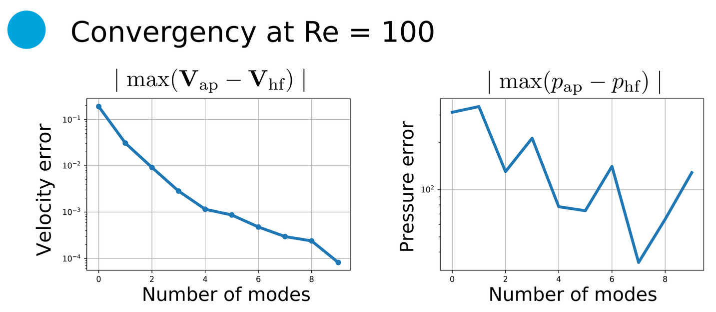
- For each i-th modes : Greedy Algorithm (GA)



- Example with steady Navier-Stokes



- Convergency at Re = 100



Discussions

- Overcome the curse of dimensionality
- No convergency of the pressure for now
- Tests with time intergation on-going

Modal Basis (6)

Example : Linearized hydroelasticity

Incompressible Potential flow Homogeneous Isotropic	Elastic Homogeneous Isotropic Linearized

- *In vacuo* eigenvalue problem

$$\mathbf{KU}+\mathbf{M}\ddot{\mathbf{U}}=\mathbf{0}$$

- Induced fluid load approximation

$$\mathbf{KU}+\mathbf{M}\ddot{\mathbf{U}}=\mathbf{f}$$

$$\mathbf{f}=\sum_{i=1}^{N}\mathbf{f}_k(\mathbf{U})+\mathbf{f}_d(\dot{\mathbf{U}})+\mathbf{f}_a(\ddot{\mathbf{U}})$$

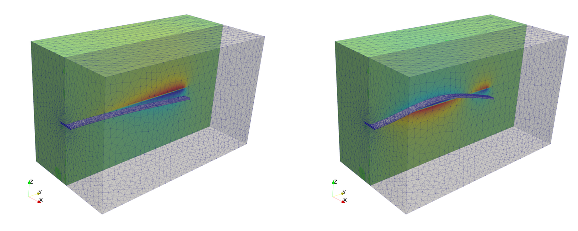
displacement → added stiffness; velocity → added damping; acceleration → added mass

- Reduced problem with reduced added operators

$$\mathbf{U}\simeq\sum_{i=1}^{N}\kappa_i\mathbf{U}_i$$

$$\mathbf{K}_{fs}\kappa+\mathbf{D}_{fs}\dot{\kappa}+\mathbf{M}_{fs}\ddot{\kappa}=\mathbf{0}$$

- Example with hydroelasticity (beam in water)



f1 = 185 Hz, f1a = 95 Hz — f2 = 1163 Hz, f2a = 605 Hz

Discussions

- Give valuable physical information
- Necessitates restriction of hypotheses
- Linearization around flow on-going

7 Conclusion - Work in Progress

Work done

- Learn and use FEniCS
- Solve Navier-Stokes problems
- Linearized hydroelasticity
- *A posteriori* reduction : POD
- *A priori* reduction : PGD

Work in progress

- Fluid - Time integration
- Fluid - Structure : ALE formulation
- Fluid - Structure : Aeroelasticity
- Structure : Velocity formulation
- Comparison between POD-PGD-MB

Outlook

- Add piezoelectricity equations
- Investigate structural models (beam - plates)
- Design an experimental set-up (7)
- Generate a data-base of F-S-P simulations
- Machin Learning

References

(1) A. Kornecki & al., JSV, 1976
(2) S. Ravi & A. Zilian, IJNME 2017
(3) M. S. Alnaes & al., ANS, 2015
(4) G. Berkooz & al., ARFM, 1993
(5) F. Chinesta & al., ACME, 2011
(6) C. Hoareau, PhD thesis, 2019
(7) C. De Marqui & al., JFS, 2018