Reduced Order Model
Flow induced piezoelectric energy harvester

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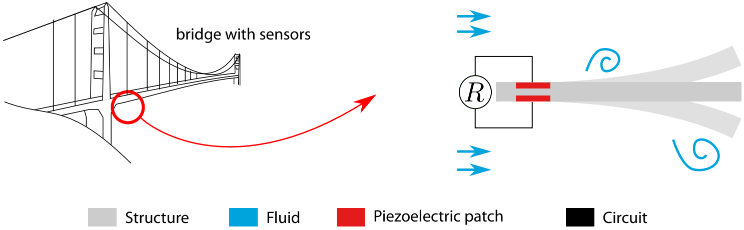
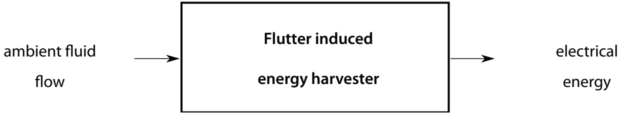
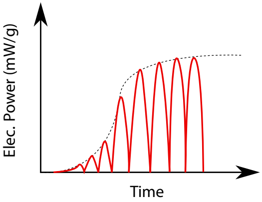
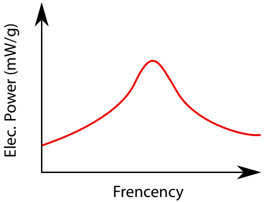
Wednesday 13 of November

Context | Energy harvesting









Maximize power output
Minimize the fatigue exposure

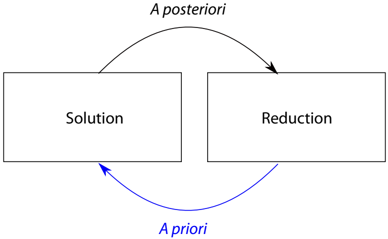
- Challenge 1 : Model and predict the nonlinear dynamic behavior
- Challenge 2 : Quantify the sensibility under changing conditions
- Challenge 3 : Allow just-in-time feedback

⇒ Reduced Order Modelling

Context | Issue and existing approaches

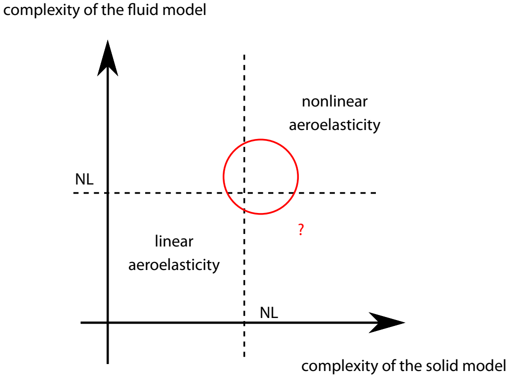
Can we construct a reduced order model of a nonlinear multiphysic problem ?

- *A posteriori* : "Data-based" reduced order model
- *A priori* : "Model-based" reduced order model



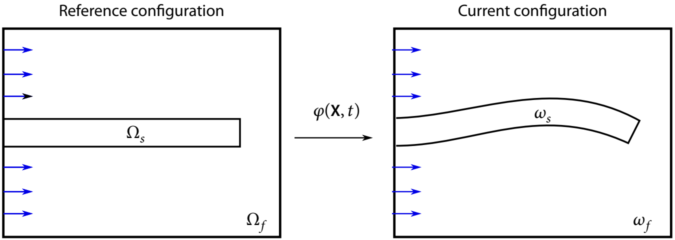
$\Rightarrow$ **POD, PGD, Modal basis, Machin learning, etc ...**

Vibrations of a structure coupled with a fluid with flow



What model is suited to capture the phenomena ?

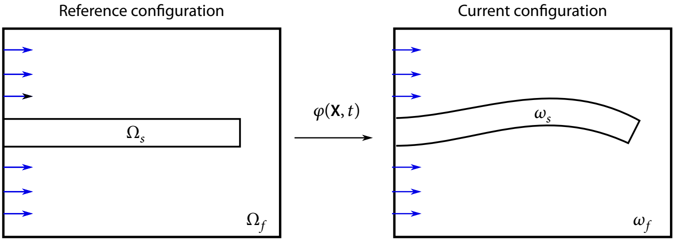
Nonlinear aeroelasticity | Complexity ?



Fluid model complexity	Solid model complexity
Navier-Stokes	Geometrical nonlinearity
Incompressible	Material nonlinearity
Moving mesh	Load nonlinearity
Boundary layer	Homogeneous, Isotropic
Vortex	Thin thickness
Time integration	Time integration

$\Longrightarrow$ [**Ravi 2017,PhD thesis**]

Nonlinear aeroelasticity | Complexity ?



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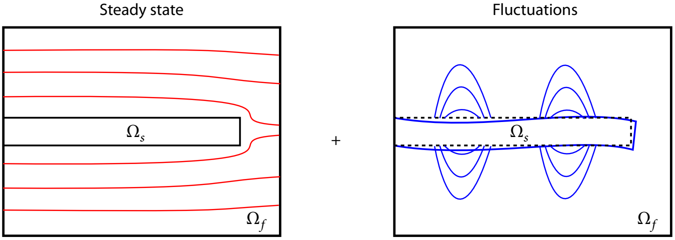
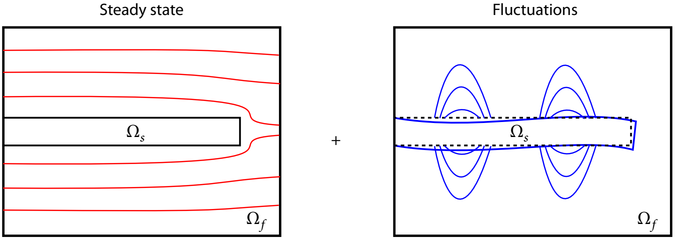
All phenomena are ”captured” but with the price of complexity

Linear aeroelasticity : | Limitations ?



Fluid model complexity	Solid model complexity
Steady + fluctuation	Linear elasticity
Potential flow	Homogeneous, Isotropic
Incompressible,Inviscid	Thin thickness
Hamonic - Time	Harmonic - Time

Linear aeroelasticity : | Limitations ?



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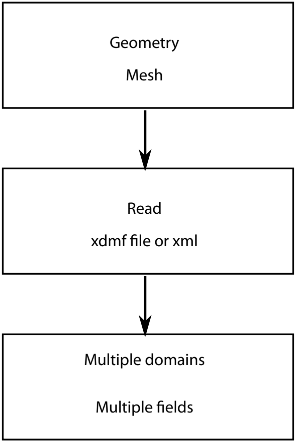
Does it capture the critical flutter velocity ?

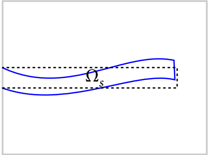

Geometry
Mesh
Read
xdmf file or xml
Multiple domains
Multiple fields

Gmsh API

+

Ex: Projection on dry modes | Modal analysis

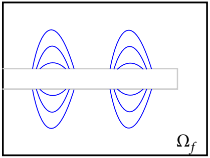
Step 1



$[\mathbf{K} - \omega^2 \mathbf{M}]\mathbf{U} = \mathbf{0}$

Compute K Dry modes $\mathbf{U}_i$

Step 2

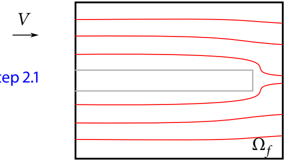


$\mathbf{H}\Phi_i = -\mathbf{C}^{\mathsf{T}}\mathbf{U}_i$

Compute K potential of displacement responses Φ_i

Now we have a base for the fluid $\{\Phi_i\}$ and the structure $\{U_i\}$

Step 2.1



$$\mathbf{H}\phi_0 = \mathbf{S}(V)$$

Compute N fluid steady velocity potential $\phi_0(V)$

Step 3 - Field pressure assumption

$$\mathbf{Ku} + \mathbf{M\ddot{u}} = \mathbf{f} \quad \text{with} \quad \mathbf{f} = \int_{\Sigma^h} \mathbf{N}^T p \mathbf{n} ds$$

$$p = p_k(\mathbf{u}) + p_d(\dot{\mathbf{u}}, \phi_0) + p_a(\ddot{\mathbf{u}})$$

Step 4 - Projection on dry modes

$$\mathbf{u} = \sum_{i=1}^{K} \mathbf{U}_i \kappa_i \quad \text{or} \quad \mathbf{u} = \mathbf{B}\kappa$$

and

$$\mathbf{B}^{\mathsf{T}}(\mathbf{K}\mathbf{u} + \mathbf{M}\ddot{\mathbf{u}} - \mathbf{f}_{\mathrm{k}} - \mathbf{f}_{\mathrm{d}} - \mathbf{f}_{\mathrm{a}})$$

Step 4 - Projection on dry modes

$$\mathbf{u} = \sum_{i=1}^{K} \mathbf{U}_i \kappa_i \quad \text{or} \quad \mathbf{u} = \mathbf{B}\kappa$$

and

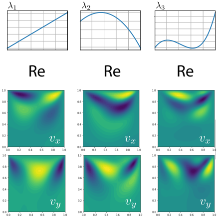
$$\mathbf{B}^{\mathsf{T}}(\mathbf{Ku} + \mathbf{M\ddot{u}} - \mathbf{f}_{\mathsf{k}} - \mathbf{f}_{\mathsf{d}} - \mathbf{f}_{\mathsf{a}})$$

$$(\mathbf{k} + \mathbf{k}_{\mathsf{add}})\kappa + \mathbf{d}_{\mathsf{add}}(\phi_0)\dot{\kappa} + (\mathbf{m} + \mathbf{m}_{\mathsf{add}})\ddot{\kappa} = \mathbf{0}$$

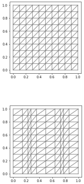
Reduced system with FSI operator which gives valuable information on the problem

Work in progress

- Proper Generalized Decomposition : Steady Navier Stokes
- Arbitrary Euler Lagrange : moving meshes
- Hydroelasticity + sloshing (3D)



a priori PGD



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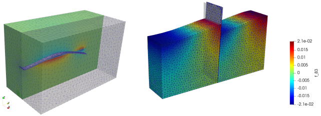

Hydroelasticity + gravity

Still learning and still adapting the way I code ... GIT on going

Future work

- Time integration very soon ...
- Nonlinear beam theory (with Lan)
- Piezoelectric equations
- Experimental studies

Thank | you all

Love FEniCS

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*** -------------------------------------------------------------------------
*** DOLFIN encountered an error. If you are not able to resolve this issue
*** using the information listed below, you can ask for help at
***
***     fenics-support@googlegroups.com
***
*** Remember to include the error message listed below and, if possible,
*** include a *minimal* running example to reproduce the error.
***
*** -------------------------------------------------------------------------
*** Error:   Unable to extract mesh from form.
*** Reason:  Non-matching meshes for function spaces and/or measures.
*** Where:   This error was encountered inside Form.cpp.
*** Process: 0
***
*** DOLFIN version: 2019.1.0
*** Git changeset:  74d7efe1e84d65e9433fd96c50f1d278fa3e3f3f
*** -------------------------------------------------------------------------
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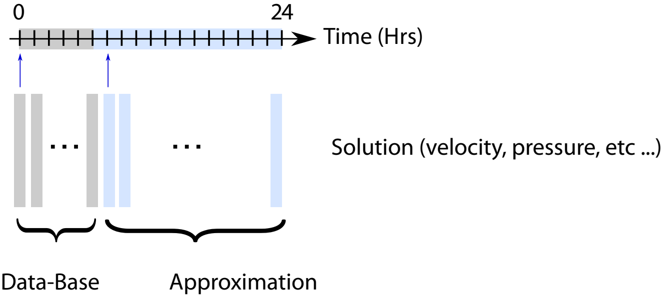
Ex 1 : Weather estimation

- Objective : estimate 24 hours of weather condition
- Problem : It would take 1 week of computation ...



Ex 2: Curse of dimensionality

- Compute reaction force for N parameters in real-time
- Problem : K^N solutions to pre-compute