Closed Form Discrete Unimodular MIMO Waveform Design Using Block Circulant Decomposition

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***Abstract*—This paper deals with the waveform design under the constraint of discrete multiphase unimodular sequences. It relies on Block Circulant decomposition of the slow-time transmitted waveform. The presented closed-form solution is capable of designing orthogonal signals, such that the virtual MIMO paradigm is enabled leading to enhanced angular resolution. On the other hand, the proposed method is also capable of approximating any desired radiation pattern within the physical limits of the transmitted array size. Simulation results prove the effectiveness in terms computational complexity, orthogonal signal design and the transmit beam pattern design under constant modulus constraint.**

I. Introduction

Muliple-Input-Multiple-Output (MIMO) radar exploits the waveform diversity such that a virtual array can be constructed at the receiver, where the array size with real antennas is smaller than the virtual counterpart [1]–[5]. This diversity is achieved by appropriate waveform design based on the different design metrics proposed in literature. A commonly used design metric is the Signal-to-Interference-to-Noise Ratio (SINR) [6]–[10]. Another metric is the design of a desired radiation pattern as a function of the transmitted waveform [3], [4], [11]–[20]. However, in addition to the interest in radiation pattern design, the cross-correlation properties of the transmitted signals are also considered [4], [11], [18], [20]. The cross-correlation properties of the transmitted signal is of significant interest in a virtual MIMO configuration, since signals with good cross-correlation properties, e.g., orthogonal signals, enable unique discrimination of the transmitted signals at the receiver and therefore facilitate the construction of a virtually filled array at the receiver [1], [2].

Most of the aforementioned references reduce to the design of the transmit signal covariance matrix under certain constraints. The most common constraint is the unimodular or constant modulus signal constraint since it is considered as a power efficient modulation scheme [14]. In this context, literature considers both continuous phase design constraints [14], [18] as well as discrete phase constraints [17] for unimodular sequences. Another common constraint is the uniform antenna element power constraint [11], [13], [15], [19].

Apart from the constant modulus and uniform power constraints, the algorithmic execution time or the computational complexity is of great interest. The lower the computational complexity, the faster the signal design, the better is the algorithm suitable for adaptive signal design. The most preferable solution is that of closed form as proposed in [15]–[17].

This paper considers the beam pattern design under discrete phase, unimodular and uniform element power constraint constraints. Further, the cross-correlation properties of the designed sequences are taken into account. Furthermore, the proposed method offers a closed form solution under the given constraints. The novelty over the state of the art lies is the design methodology considering all of the aforementioned critical properties, such as discrete phase, uniform power constraint, desirable cross-correlation properties and closed form solution. The authors in [17] propose a closed form solution under the discrete phase unimodular constraint, but the cross-correlation properties are not investigated; further, the length of the transmitted symbols is limited to number of transmit antennas times number of receive antennas. The work in [15] relates closely to the present work; it proposes a closed form solution under uniform power constraint. However, [15] does not consider cross-correlation properties and unimodular signal design constraint in the design. Further, authors in [15] introduce a Toeplitz structure to the signal covariance matrix, whereas in this paper a much simpler block circulant approach similar to [20] is proposed. This paper builds on the work [20] and extends the quadrature phase two channel sequence design of [20] to the generic case of unimodular sequences with multiple phase stages. The contributions of this paper can be summarized as follows:

- Extension of the block circulant decomposition approach [20] to multi-phase unimodular waveform design
- Derivation of a closed-form solution to an optimization problem which addresses the transmit beam pattern design and minimizes the transmit signal cross-correlation under finite alphabet and uniform antenna constraints
- Algorithm capable of real time implementation

In this work the operator $\|\cdot\|_p$ defines the l_p-norm. Boldface upper and lower case alphabets denote matrices and vectors respectively. The $E\{\cdot\}$ denotes the expectation operator. The set of complex, integer and real numbers are defined as $\mathbb{C}$, $\mathbb{Z}$ and $\mathbb{R}$, respectively, while $j = \sqrt{-1}$ represents the complex number. The Kronecker product is defined as $\otimes$, while $\circ$ refers to Hadamard product.

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II. System Model

The system architecture comprises a local oscillator, which generates a train of Continuous Wave (CW) pulses, such as Frequency Modulated Continuous Wave (FMCW) pulses. The transmitted signal is composed of N_P pulses within one Coherent Processing Interval (CPI). Each pulse has a duration of T_P and a modulation bandwidth B with the carrier frequency f_0. The system further comprises N_R linearly mounted receive antennas with an inter-element spacing of $d_R = \frac{\lambda}{2}$, where $\lambda = \frac{f_0}{c_0}$ is the wavelength and c_0 refers to the speed of light. The transmit antenna array on the other hand consists of N_T antenna elements linearly mounted with an inter-element spacing of $d_T = N_R\frac{\lambda}{2}$, which leads to a sparse MIMO configuration. To enable simpler hardware implementation, each individual transmit antenna element is assumed capable of modulating the CW by N_T phase stages, drawn from the modulation set $\Omega = \left\{\exp(j\frac{2\pi}{N_T}1)\cdots\exp(j\frac{2\pi}{N_T}N_T)\right\}$.

A. Transmitted Signal

As mentioned above, the transmitted signals comprises a train of N_P CW-pulses. Since digital processing at receiver is considered, the transmitted signal is also modelled in the discrete domain. Therefore, each CW pulse is modelled as a pulse vector $\mathbf{u} \in \mathbb{C}^{N_S\times 1}$, where N_S is the number of discrete samples. The transmitted signal is assumed to be modulated in the slow-time domain, which further leads to an *inter-pulse modulation*. In particular, the transmitted signal during the p-th pulse interval from n-th antenna takes the form, $s_p(n)\mathbf{u}$, where $s_p(n)$ is the corresponding modulating symbol. Let $\mathbf{s}_p \in \Omega^{N_T\times 1}$ denote the stacking of the N_T transmit antenna array weightings, $\{s_p(n)\}_{n=1}^{N_T}$, for the p-th pulse. Further, define the inter-pulse modulation vector $\mathbf{s} \in \Omega^{N_T N_P\times 1}$ as,

$$\mathbf{s} = \left(\ \mathbf{s}_1^T,\ \ \cdots,\ \ \mathbf{s}_p^T,\ \ \cdots, \mathbf{s}_{N_P}^T\ \right)^T. \tag{1}$$

It has to be noted that the focus of this paper lies in the inter-pulse modulation and therefore only the inter-pulse modulation vector $\mathbf{s}$ is considered as a design variable. Further, due to system constraints, $\mathbf{s}$ contains only symbols from the set Ω.

B. System Transfer Function and Received Signal

The pulse length is assumed to be much longer than the maximum propagation delay and the target velocity is assumed slow with regards to the pulse duration. The latter assumption leads to the simplification that the target appears constant for one pulse duration, which leads in turn to a Doppler shift only in the inter-pulse domain. These assumptions result in range and Doppler information being separable; the range information appears in the fast-time signal while the slow-time signal contains the Doppler information.

The target scenario comprises K distinct targets with Swerling one model statistics for their Radar Cross Section (RCS) [21]. Therefore, the RCS of the κ-th target, $\alpha_\kappa \in \mathbb{C}$, is a statistical parameter. The receive steering vector $\mathbf{a}_{R\kappa} \in \mathbb{C}^{N_R\times 1}$ and the transmit steering vector $\mathbf{a}_{T\kappa} \in \mathbb{C}^{N_T\times 1}$ for the κ-th target, defined below, contain the MIMO channel information under the far-field and plane wave assumptions [20], [22],

$$\mathbf{a}_{R\kappa} = \left(\exp\left(jk_\kappa d_R\right), \cdots, \exp\left(jk_\kappa d_R N_R\right)\right)^T \tag{2}$$

$$\mathbf{a}_{T\kappa} = \left(\exp\left(-jk_\kappa d_T\right), \cdots, \exp\left(-jk_\kappa d_T N_T\right)\right)^T. \tag{3}$$

The receive and transmit steering vector contain the angular information of κ-th target in the wave number $k_\kappa = \frac{2\pi}{\lambda}\sin(\phi_\kappa)$, with ϕ_κ denoting the spatial angle of the κ-th target. The Doppler frequency, $f_{D\kappa}$, of the κ-th target is modelled in the diagonal Doppler matrix $\mathbf{D}_\kappa \in \mathbb{C}^{N_P\times N_P}$,

$$\mathbf{D}_\kappa = \operatorname{diag}\left(\exp\left(j2\pi f_{D\kappa}T_P\right), \cdots, \exp\left(j2\pi f_{D\kappa}T_P N_P\right)\right). \tag{4}$$

Therefore, the inter-pulse transfer function for κ-th target, $\mathbf{H}_\kappa \in \mathbb{C}^{N_P N_R\times N_P N_T}$, is defined as,

$$\mathbf{H}_\kappa = \alpha_\kappa\left[\mathbf{a}_{R\kappa}\mathbf{a}_{T\kappa}^H\right]\otimes\mathbf{D}_\kappa \tag{5}$$

Towards completing the system model, the κ-th target range information is modelled with a shift matrix $\mathbf{J}_\kappa \in \{0;1\}^{N_S\times N_S}$ [23]. Due to the superposition of the target back scatters in space, the received signal vector, $\mathbf{x} \in \mathbb{C}^{N_R N_S N_P\times 1}$ over all antennas and one CPI becomes,

$$\mathbf{x} = \sum_{\kappa=1}^{K}\left(\mathbf{H}_\kappa\mathbf{s}\right)\otimes\left(\mathbf{J}_\kappa\mathbf{u}\right) = \left(\sum_{\kappa=1}^{K}\mathbf{H}_\kappa\otimes\mathbf{J}_\kappa\right)\left(\mathbf{s}\otimes\mathbf{u}\right). \tag{6}$$

It can be seen, that the fast-time information $((\mathbf{J}_\kappa\mathbf{u}))$ is completely separable from the slow-time information $((\mathbf{H}_\kappa\mathbf{s}))$. Therefore, angular and Doppler domains are separable from any range information. Since the focus of this paper lies in the slow-time signal design, it is sufficient to only investigate the slow-time domain, such that only the slow-time received signal $\mathbf{y} \in \mathbb{C}^{N_R N_P\times 1}$ is of interest,

$$\mathbf{y} = \sum_{\kappa=1}^{K}\mathbf{H}_\kappa\mathbf{s} \tag{7}$$

The inter-pulse signal design only affects the angle-Doppler response. However, stationary targets are considered, i.e, $f_{D\kappa} = 0, \forall\ \kappa$ in this paper and Doppler shifts are considered in future works.

C. Matched Filter

The receive signal processing is based on matched filtering, where the filter coefficients take the form similar to the received signal, $\mathbf{y}_M = \mathbf{H}_M\mathbf{s}$, with every κ replaced by a generic index M, leading to a parametric version of one target response. Since the attenuation factor is unknown, the matched filter does not contain any RCS information. Further, the RCS of each target is modelled as stochastic and therefore the matched filter output is a stochastic parameter as well. Hence, a mean matched filter output, μ, is considered,

$$\mu = E\left\{\left|\mathbf{y}_M^H\mathbf{y}\right|^2\right\}. \tag{8}$$

Due to the statistical independence of RCS across targets, the expected matched filter output takes the form of a superposition of K distinct target contributions,

$$\mu = \sum_{\kappa=1}^{K} \sigma_\kappa^2 \left|\mathbf{a}_{RM}^H \mathbf{a}_{R\kappa}\right|^2 \left|P\left(\phi_M, \phi_\kappa\right)\right|^2 \text{ where,} \tag{9}$$

$$P\left(\phi_M, \phi_\kappa\right) = \mathbf{a}_{T\kappa}^H \left(\sum_{p=1}^{N_P} \mathbf{s}_p \mathbf{s}_p^H\right) \mathbf{a}_{TM}. \tag{10}$$

As mentioned above, the Doppler shift is omitted in (9) leading to (10). The cross-correlation beam pattern $P\left(\phi_M, \phi_\kappa\right)$ is a frequently used design metric, since (i) it comprises information on both the radiation pattern information (when $\phi = \phi_M = \phi_\kappa$) as well as the cross-correlation of transmitted signals (when $\phi_M \neq \phi_\kappa$) and (ii) its relevance for the underlying virtual MIMO configuration [11], [14], [18], [20].

Let the transmit signal covariance matrix $\mathbf{R}_s \in \mathbb{C}^{N_T \times N_T}$ be defined as,

$$\mathbf{R}_s = \sum_{p=1}^{N_P} \mathbf{s}_p \mathbf{s}_p^H = \mathbf{S}\mathbf{S}^H, \text{ where,} \tag{11}$$
$$\mathbf{S} = [\mathbf{s}_1, \mathbf{s}_2, \ldots, \mathbf{s}_{N_P}].$$

The inter-pulse modulation matrix $\mathbf{S} \in \Omega^{N_T \times N_P}$ contains the vector $\mathbf{s}_p$ in the p-th column. Exploiting (10) and (11), the objective pursued in the paper involves the design of the inter-pulse modulation matrix, such that a desired cross-correlation beam-pattern is well-approximated.

III. Cross-Correlation Beam-Pattern Optimization

As mentioned above, the inter-pulse modulation matrix affects the cross-correlation beam-pattern. The cross-correlation beam pattern is a two dimensional continuous function, but it is sampled in order to undertake signal design in the discrete domain. An uniform sampling of the wave numbers $k_M = \frac{2\pi}{\lambda}\sin(\phi_M)$ and $k_\kappa = \frac{2\pi}{\lambda}\sin(\phi_\kappa)$ into N points is assumed leading to non linear sampling of the angular domain ϕ_M and ϕ_κ. The transmit steering matrix $\mathbf{A}_{TM} = \left(\mathbf{a}_{TM_1} \cdots \mathbf{a}_{TM_N}\right)^T \in \mathbb{C}^{N \times N_T}$ and $\mathbf{A}_{T\kappa} = \left(\mathbf{a}_{T\kappa_1} \cdots \mathbf{a}_{T\kappa_N}\right)^T \in \mathbb{C}^{N \times N_T}$ are identical. Further, due to the Nyquist-Shannon Sampling theorem, it suffices to have number of sample points N equal to the number of transmit antennas $N = N_T$. In such a scenario, the steering matrices take the form of a scaled Discrete Fourier Transform (DFT) matrix $\mathbf{A} = \mathbf{A}_{TM} = \mathbf{A}_{T\kappa} \in \mathbb{C}^{N_T \times N_T}$. Due to the DFT structure, it is clear that an oversampling $N > N_T$ does not provide additional information as the rank of $\mathbf{A}$ remains N_T.

With the steering vectors sampled, the cross-correlation beam-pattern can be compactly represented using a matrix, $\mathbf{P} \in \mathbf{R}^{N_T \times N_T}$, as

$$\mathbf{P} = \mathbf{A}^H \mathbf{R}_s \mathbf{A}, \tag{12}$$

where (m, n)th element of $\mathbf{P}$ denotes the radiation pattern in (10) evaluated at the mth and nth sample of ϕ_M and ϕ_κ respectively.

The virtual MIMO paradigm is satisfied, if the cross-correlation beam-pattern matrix has a diagonal structure, since it leads to the maximum array resolution [1], [2]. Therefore, desired cross-correlation beam-pattern matrix $\mathbf{P}_d \in \mathbb{R}^{N_T \times N_T}$ assumes a diagonal structure,

$$\mathbf{P}_d = \operatorname{diag}\left(\mathbf{p}_d\right), \tag{13}$$

where $\mathbf{p}_d \in \mathbf{R}^{N_T \times 1}$ contains the desired radiation pattern information. Since the cross-correlation properties are predefined, the radiation pattern $\mathbf{P}_d$ is the design requirement.

A. Objective Function

With the structure of desired $\mathbf{P}_d$ discussed and recalling the signal modulation set Ω, the objective function for designing a desired cross-correlation beam-pattern is defined as,

$$\min_{\mathbf{S} \in \Omega^{N_T \times N_P}} \left\|\mathbf{P}_d - \mathbf{A}^H \mathbf{R}_s \mathbf{A}\right\|_{\mathcal{F}}^2. \tag{14}$$

The optimization problem comprises a quartic cost function in $\mathbf{S}$ under a finite alphabet constraint. This problem is in general hard to solve [11]. In order to achieve a closed form solution, the Degrees of Freedom (DoF) has to be reduced. One possibility to reduce the DoF is to apply a block circulant construction method as presented in [20].

B. Block Circulant Decomposition

Under the assumption of N_P being an integer multiple of N_T, the Block Circulant Decomposition (BCD) decomposes the matrix $\mathbf{S}$ into N_B blocks of dimension $N_T \times N_T$

$$\mathbf{S} = \left(\mathbf{B}_1, \cdots, \mathbf{B}_b, \cdots, \mathbf{B}_{N_B}\right). \tag{15}$$

The b-th block $\mathbf{B}_b \in \Omega^{N_T \times N_T}$ is parametrized by an unique column vector $\mathbf{s}_b \in \Omega^{N_T \times 1}$, which is circular shifted across columns to obtain $\mathbf{B}_b$ [24]. This leads to a block circulant $\mathbf{B}_b$. Due to this property, the unitary DFT matrix, $\mathbf{F} \in \mathbb{C}^{N_T \times N_T}$, serves as a eigenvector matrix for all the blocks. Thus the blocks $\mathbf{B}_b$ differ only in the eigenvalue matrix $\mathbf{\Psi}_b \in \mathbb{C}^{N_T \times N_T}$, leading to the following simplification for the transmit signal covariance matrix,

$$\mathbf{R}_s = \sum_{b=1}^{N_B} \mathbf{B}_b \mathbf{B}_b^H = \mathbf{F} \left(\sum_{b=1}^{N_B} \mathbf{\Psi}_b \mathbf{\Psi}_b^H\right) \mathbf{F}^H. \tag{16}$$

As discussed earlier, the steering vector matrix is a scaled version of the unitary DFT matrix $\mathbf{A} = \sqrt{N_T}\mathbf{F}$ (scaling due to column normalization). Hence, the optimization problem in (14) simplifies as,

$$\min_{\mathbf{s}_b \in \Omega^{N_T \times 1}\ \forall\ b} \left\|\mathbf{p}_d - N_T \sum_{b=1}^{N_B} \left(\mathbf{F}^H \mathbf{s}_b\right) \circ \left(\mathbf{F}^H \mathbf{s}_b\right)^*\right\|_2^2. \tag{17}$$

The BCD ensures the actual cross-correlation beam-pattern $\mathbf{P}$ to be diagonal by construction under the assumption of a Uniform Linear Array (ULA) configuration for the transmitter. In fact, the ULA structure guarantees a DFT structure for the steering vector matrix. Thus the BCD provides a construction based framework for ULA transmit signal cross-correlation optimization. If $\mathbf{P}$ becomes diagonal, the Frobenius norm of the off-diagonal elements (14) has no contribution to the cost function. Therefore, the optimization problem (17) and (14) are equivalent.

It should be noted that the DoF is lowered by a factor of N_T due to BCD when simplifying (14) to (17). In Section III-C the optimization problem (17) is further simplified such that a closed form solution is achieved.

C. Fourier Basis Approach

1) Basis Expansion: Let the actual radiation pattern be denoted by $\mathbf{p}$, where $\mathbf{p} = N_T \sum_{b=1}^{N_B} \left(\mathbf{F}^H \mathbf{s}_b\right) \circ \left(\mathbf{F}^H \mathbf{s}_b\right)^*$. The pattern depends only on N_B vectors, $\mathbf{s}_b$, where each vector has to satisfy the modulation constraint, i.e., $\mathbf{s}_b \in \Omega^{N_T \times 1}$. In order to design the blocks $\mathbf{s}_b$, it can be observed that $\mathbf{p}$ lies in a N_T dimensional space and hence can be expressed as a superposition of N_T basis functions, $\{\mathbf{q}_i\}$,

$$\mathbf{p} = \sum_{i=1}^{N_T} v_i \mathbf{q}_i = \mathbf{Q}\mathbf{v}. \tag{18}$$

where the second term denotes the matrix-vector representation of $\sum_{i=1}^{N_T} v_i \mathbf{q}_i$. Recalling that $\mathbf{p} = N_T \sum_{b=1}^{N_B} \left(\mathbf{F}^H \mathbf{s}_b\right) \circ \left(\mathbf{F}^H \mathbf{s}_b\right)^*$, it suffices to choose, without loss of generality, the i-th basis function as $\mathbf{q}_i = \left(\mathbf{F}^H \mathbf{s}_i\right) \circ \left(\mathbf{F}^H \mathbf{s}_i\right)^*$. Therefore, the appropriate choice of basis functions $\mathbf{s}_i$ influences $\mathbf{Q}$. Further, the matrix $\mathbf{Q} \in \mathbb{R}^{N_T \times N_T}$ contains the vector $\mathbf{q}_i$ in its i-th column and is a function of the vectors $\mathbf{s}_i$. Additionally, if the basis coefficient vector $\mathbf{v} \in \mathbb{Z}_+^{N_T \times 1}$ is restricted to positive integer values, the term $\sum_{i=1}^{N_T} v_i \mathbf{q}_i$ can be rewritten as $\sum_{j=1}^{N_B} \tilde{v}_i \widetilde{\mathbf{q}}_j$, where the vectors $\{\widetilde{\mathbf{q}}_j\}$ are drawn from columns of $\mathbf{Q}$ with repetitions such that there are N_T distinct $\{\widetilde{\mathbf{q}}_j\}$ and whose number of repetitions is governed by $\{v_i\}$. In other words, the inter-pulse modulation matrix $\mathbf{S}$ comprises v_i circulant block for each vector $\mathbf{s}_i$.

2) Fourier basis as $\{\mathbf{s}_b\}$: If each vector $\mathbf{s}_i$ is chosen as a $N_T \times 1$ Discrete Fourier Transform (DFT) vector, the matrix $\mathbf{Q} = \sqrt{N_T}\mathbf{I}_{N_T}$ becomes a scaled identity matrix, as the DFT vectors are inserted in the definition of each $\mathbf{q}_i$. Further, this choice naturally renders $\mathbf{v}$ non-negative. In fact,

$$\mathbf{v} = \frac{1}{N_T^2}\mathbf{p}_d. \tag{19}$$

However, according to the earlier discussion, it is essential that the resulting $\mathbf{v}$ is an integer vector to fully utilize BCD framework. In general, this cannot be guaranteed and the modulation constraint is satisfied by rounding the real coefficient vector $\mathbf{v} \in \mathbb{R}_+^{N_T \times 1}$ to the next integer value. In view of this, following (18), $\mathbf{p}$ is approximated as $\mathbf{Q}\hat{\mathbf{v}}$ where,

$$\begin{aligned} \hat{\mathbf{v}} &= \Gamma\left(\frac{\mathbf{v}}{\left\|\hat{\mathbf{v}}\right\|_1} N_B\right) = \Gamma\left(\frac{\frac{1}{N_T^2}\mathbf{p}_d}{\left\|\frac{1}{N_T^2}\mathbf{p}_d\right\|_1} N_B\right) \\ &= \Gamma\left(\frac{\mathbf{p}_d}{\left\|\mathbf{p}_d\right\|_1} N_B\right), \end{aligned} \tag{20}$$

where $\Gamma(x) = \max\{k \in \mathbb{Z} | k \leq x + 0.5\}$ denotes the rounding function. With $\hat{\mathbf{v}}$ determined, the matrix $\mathbf{S}$ is constructed such that it contains $\hat{\mathbf{v}}(i)$ circulant blocks for the i-th DFT basis, $i \in [1, N_T]$.

Thus the Fourier basis approach enables a closed form solution for the transmission waveform, although it reduces the DoF. The quality of the closed form solution depends only on the signal length, namely the number of blocks N_B. It is obvious from (20) that the precision of the rounding operation by $\Gamma(\cdot)$ improves with larger number of blocks. Table I depicts the algorithm, which provides the closed form solution of the inter-pulse signal modulation matrix $\mathbf{S}$ for a given desired radiation pattern $\mathbf{p}_d$.

TABLE I: Algorithm for discrete phase unimodular sequence design

Step 0: Initialize desired beam pattern $\mathbf{p}_d$
Step 1: Discretization of optimization output equation (20)
Step 2: Map rounded coefficients v_i to the related basis vector $\mathbf{s}_i$.
Step 3: The matrix $\mathbf{S}$ is constructed by circular shifting each vector $\mathbf{s}_i$, such that v_i circulant blocks occur in $\mathbf{S}$ for each individual index i.

IV. Simulation

Simulations are carried out with $N_T = 10$ transmit antennas and $N_R = 4$ receive antenna elements in a virtual MIMO configuration as discussed in Section II. The number of blocks is set to $N_B = 500$.

As discussed in Section III, the error in the proposed approach is largely determined by the quantization error. The quantization error is a function of the number of blocks. As can be seen in Figure 1, the quantization error becomes negligible for a sufficiently large number of circulant blocks. Further, the error fits the $\frac{1}{N_B}$ curve. This result shows the effectiveness of the presented approach. A drawback of a large number of N_B is that the CPI becomes large and at beyond certain values, the assumptions of slow and fast time separation do not hold anymore. Also, the processing time increases with the number of blocks.

A second source of error arises from the physical limitation of the array size. The finer the beam pattern resolution, the bigger the transmit antenna array size. However, the proposed algorithm is capable of designing any desired radiation pattern within the physical resolution limits, if the number of blocks is sufficiently large. Figure 2 and 3 illustrate the design capability of the proposed approach and compare it to [20]. In particular, two different radiations patterns are considered

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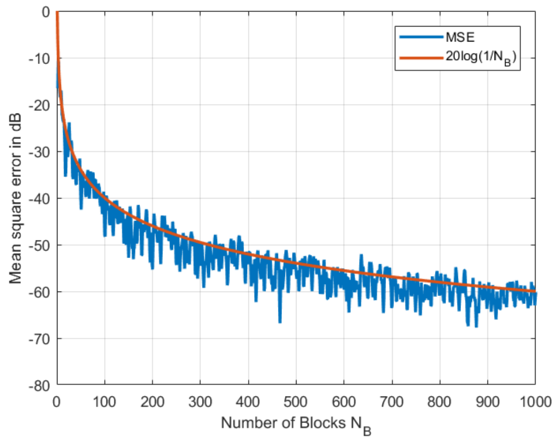

Fig. 1: Quantization Error as a function of circulant blocks.

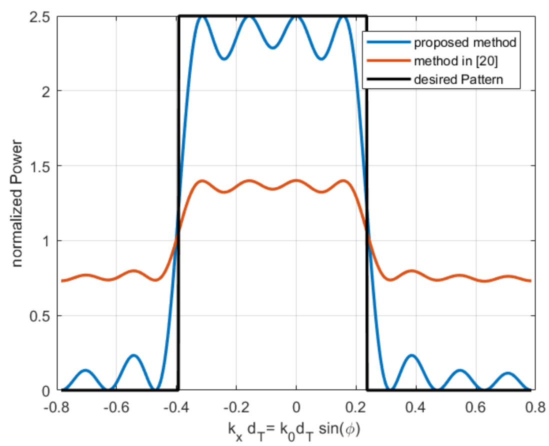

Fig. 2: Radiation pattern design example of one main lobe.

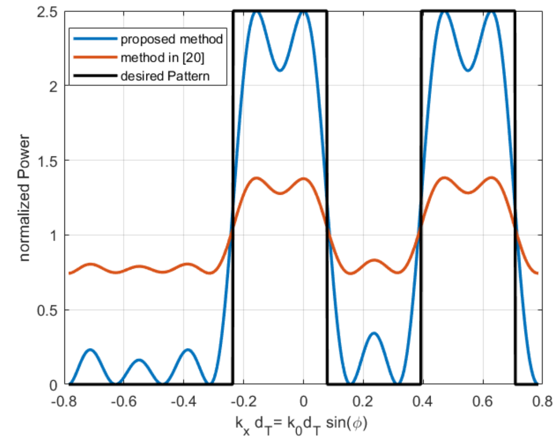

Fig. 3: Radiation pattern design example of two main lobes.

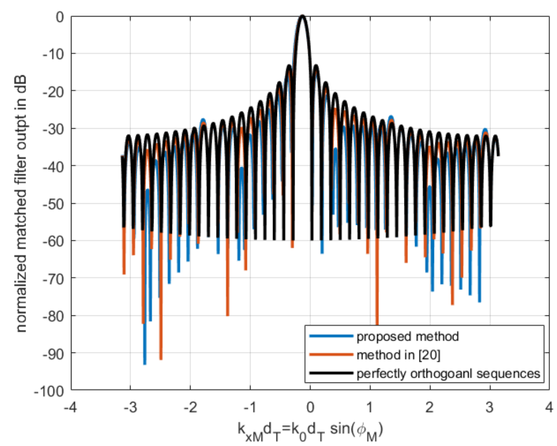

Fig. 4: Matched filter detection performance, where all curves are almost perfectly aligned with each other.

in these figures. In comparison to the method presented in [20], the proposed approach performs better in terms of beam pattern focus. This is justified by having a unimodular sequence design, instead of a multiplex approach. It can be also seen from figure 2 and 3, that the sample points perfectly match the desired beam pattern, which in turn, demonstrates that the algorithm performs optimally.

The sharp peak with low side lobes at the matched filter output in Figure 4 shows that the transmitted signals are perfectly orthogonal and the maximum virtual array resolution can be achieved, while having the capability of designing an arbitrary radiation pattern. This holds also for the approach in [20]. Therefore, the BCD provides a suitable tool for achieving good cross-correlation properties for a ULA configuration.

The ability to design a wide range of signals from perfectly orthogonal ones to those yielding desired radiation pattern at certain sample points proves the flexibility and universality of the presented method.

V. Conclusion

This paper proposes a closed form solution for transmit signal design under uniform element power and constant modulus constraints based on block circulant approach to signal design. The proposed block circulant decomposition enables design of transmit waveform with optimal cross-correlation properties while implicitly satisfying the uniform element power constraint. The proposed approach decouples the radiation pattern design from the consideration of cross-correlation properties, which further leads to an effective radiation pattern design using a discrete Fourier basis approach. While the proposed approach has lower degrees of freedom, the simulation results nonetheless corroborate the effectiveness of the proposed approach.

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Resolution in Radar Through Dynamic Beam Steering and MIMO.

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