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Estimating the 3D shape of volcanic ash to better understand sedimentation processes and improve atmospheric dispersion modelling

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Abstract

The sedimentation rate of volcanic ash through the atmosphere influences its travel distance, with important implications for aviation and health. The fall velocity of a particle depends on its size and density, but also shape, and volcanic ash is not spherical. To capture the sedimentation of ash, atmospheric dispersion models use empirical drag equations calibrated using geometric shape descriptors. However, particle shape data are scarce and there is no standard method of shape measurement. In addition, shape measurements are not always available during an eruption, when dispersion models are used operationally to forecast ash hazard. We assess the variability in the shape of volcanic ash from Icelandic eruptions using X-ray computed tomography. To consider how good different drag equations and shape descriptors are at representing the sedimentation of volcanic ash we compare calculated fall velocities to measured fall velocities of volcanic ash in air in a settling column. We then suggest the best drag equations and shape descriptors for use in atmospheric dispersion models. We find that shape-dependent drag equations produce more accurate results than a spherical approximation. However, accurate drag calculations based on the shape descriptor sphericity, which is a function of surface area, require the imaging resolution to be within the range of $10^2 - 10^5$ voxels per particle as surface area is sensitive to imaging resolution. We suggest that the large-scale form of the particle impacts sedimentation more than small-scale surface roughness. Shape descriptors based on ratios between principal axis lengths are more practical as they are less variable among particle size classes and much less sensitive to imaging resolution. Finally, we use particle shape data from this study and literature sources to make recommendations on default values for use with atmospheric dispersion models where no shape data are available.

Keywords: tephra, atmospheric dispersion, sedimentation, morphology, X-ray tomography

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9 **1. Introduction**

10 Volcanic ash (tephra particles with diameters < 2 mm) can remain in the atmosphere
11 from minutes to days or longer after a large explosive eruption (Durant et al., 2010); the
12 rate of removal from the atmosphere depends on meteorological processes and particle ter-
13 minal fall velocity. Accurate calculation of terminal velocity requires a parameterisation of
14 particle shape (e.g., Riley et al., 2003; Bagheri and Bonadonna, 2016a,b). This means that
15 forecasts of the movement of volcanic ash clouds generated using atmospheric dispersion
16 models can be sensitive to the shape parameter used in the model's sedimentation scheme
17 (Scollo et al., 2008; Beckett et al., 2015; Saxby et al., 2018). For example, Saxby et al.
18 (2018) found that a 100 μm model particle can travel 44% further from source when it
19 is modelled as non-spherical (sphericity = 0.5). The shape of volcanic ash particles can
20 also provide information on componentry (Buckland et al., 2018), eruptive style (Liu et al.,
21 2015, 2017), transport mechanisms (Rose and Chesner, 1987), emplacement conditions
22 (Dufek and Manga, 2008; Buckland et al., 2018), and tephra layer identification (Liu et al.,
23 2016; Dunbar et al., 2017).

24 The question of how to measure, and model, particle shape is relevant in many other
25 disciplines. Many atmospheric particulates are non-spherical, meaning that shape is an
26 important parameter in understanding the transport of nuclear fallout (Rolph et al., 2014),
27 ice crystals (Westbrook and Sephton, 2017), pollen (Schwendemann et al., 2007), wildfire
28 embers (Anthenien et al., 2006) and desert dust (Chou et al., 2008). The hydrodynamic
29 behaviour of particles settling in water is also a function of shape; this is important, for
30 example, in analysing assemblages of marine species such as foraminifera, which are used
31 to estimate palaeodepth (e.g., Speijer et al., 2008; Jorry et al., 2006). Particle shape also
32 affects the drag of non-spherical particles in streams (Komar and Reimers, 1978) as well
33 as particle sorting during flow (Oakey et al., 2005). Finally, shape affects crystal settling
34 velocities in magma, with implications for magma evolution and rheology (Higgins and
35 Roberge, 2003).

36 Another consideration relates to the effect of particle shape on grain size measurement.
37 For example, particles passing through a sieve mesh may have one dimension larger than
38 the mesh aperture, and so the results of sieving are dependent on the shape of the particle
39 (Arason et al., 2011); shape also affects the results of particle size distributions calculated
40 from 2D images (Higgins and Roberge, 2003). Particle volume is a shape-independent

41 measure of size, but volume is not straightforward to measure for irregular particles.

42 Volcanic eruptions produce a wide variety of particle shapes, which are related to the
43 kinetics of vesiculation and crystallization during magma ascent, and the fragmentation
44 mechanism. Ash types include vesicular pumice clasts, smooth crystals, platy bubble
45 wall shards, needle-like fragments of tube pumice, and the extremely elongate Pele's hairs
46 formed by low viscosity basaltic melt. Describing shape using one or more geometric
47 shape descriptors reduces operator bias and allows comparison between datasets; however,
48 the most useful shape descriptor will depend on the textural variability of the sample and
49 the purpose of the research, meaning there is no 'standard' way to measure shape. In ad-
50 dition, the results strongly depend on measurement parameters, such as imaging resolution
51 (Alfano et al., 2011; Liu et al., 2015; Dioguardi et al., 2017). Most shape descriptors ap-
52 plied to volcanic ash particles can be measured from projections of particle surfaces via
53 2D image analysis (e.g., Riley et al., 2003). However, recent advances in the use of optical
54 and electron microscopy (Ersoy et al., 2010; Bagheri et al., 2015; Vonlanthen et al., 2015)
55 and X-ray computed tomography (CT; Ersoy et al. (2010); Dioguardi et al. (2017); Mele
56 and Dioguardi (2018)) mean it is now easy to construct high-resolution 3D models of vol-
57 canic ash particle surfaces. We limit our analysis to 3D shape measures, by which most
58 drag equations are calibrated (e.g., Wilson and Huang, 1979; Ganser, 1993; Bagheri and
59 Bonadonna, 2016b; Dioguardi et al., 2017, 2018).

60 We assess the shape range of volcanic ash and determine how best to measure shape
61 for the purpose of calculating its terminal velocity in an atmospheric dispersion model.
62 We measure multiple 3D geometric shape descriptors using the X-ray CT method. To
63 assess the effectiveness of the shape descriptors in anticipating particle fall velocity, we also
64 measure the terminal velocity of selected mm-sized tephra particles in a settling column and
65 compare the results to calculated terminal velocities using empirical drag equations with
66 our measured shape parameters. From this we provide guidance on the best theoretical
67 approach (drag equation and shape descriptor) for use in atmospheric dispersion models to
68 represent the sedimentation of volcanic ash particles.

69 Our reference datasets include ash samples from Icelandic volcanoes (Katla, Hekla and
70 Eyjafjallajökull) spanning a wide range of composition, eruptive style, and morphology;
71 this allows us to assess the variation in the shape of volcanic ash between eruptions and
72 between size fractions, and the sensitivity of 3D shape descriptors to the CT scan parame-

73 ters. We use the resulting insights to present a database of shape descriptors for use with
74 semi-empirical drag equations ([Ganser, 1993](#); [Bagheri and Bonadonna, 2016b](#); [Dioguardi](#)
75 [et al., 2018](#)) which are valid for a wide range of flow conditions and therefore suitable for
76 modelling the sedimentation of volcanic ash in the atmosphere.

77 2. Modelling the terminal velocity of non-spherical ash

78 Particle terminal velocity is defined as the velocity reached by a falling object when the
79 drag force is equal to the gravitational force and acceleration is zero. Volcanic ash particles
80 reach terminal velocity in the atmosphere over distances which are small compared to the
81 distance required to sediment from a plume, and so it is reasonable to neglect acceleration
82 in modelling. Terminal velocity w_t can be calculated as a function of drag:

$$w_t = \left(\frac{4}{3} \frac{d}{C_D} g \frac{\rho_P - \rho}{\rho} \right)^{\frac{1}{2}} \quad (1)$$

83 where d is the particle size (for a sphere, diameter), g is gravitational acceleration, ρ is fluid
84 density, ρ_P is particle density, and C_D is the drag coefficient, a dimensionless coefficient
85 which is a function of particle shape and flow regime. Volcanic ash falling in air can be
86 subject to several flow regimes, defined by the dimensionless Reynolds number Re ; the
87 flow around a particle is classed as laminar when $Re < 0.1$ and turbulent at $Re > 1000$.
88 The drag of spheres can be calculated analytically with high accuracy for all flow regimes
89 (e.g., [White, 1974](#)). Solutions for non-spherical particles, which are characterised by higher
90 C_D than spheres of equivalent size and density, are generally empirical or semi-empirical
91 correlations which relate C_D to one or more geometric shape descriptors. Therefore, such
92 correlations are valid for finite Re and limited to particle shape ranges which are covered
93 by the experimental conditions and the formulation used. The complexity in modelling
94 volcanic ash shape means that many operational atmospheric dispersion model setups use
95 a spherical approximation by default ([Hort, 2016](#)). In addition, shape data are not always
96 available during an eruption for forecasting purposes.

97 A spherical particle approximation can be sufficiently accurate for dispersion mod-
98 elling purposes for small (less than $\sim 30 \mu\text{m}$ diameter) volcanic ash particles in the laminar
99 regime ([Alfano et al., 2011](#); [Saxby et al., 2018](#)), but shape begins to strongly influence
100 particle sedimentation and transport distance at particle diameters between ~ 30 and 100
101 μm , with particles $\geq 100 \mu\text{m}$ being highly sensitive to shape ([Beckett et al., 2015](#); [Saxby](#)
102 [et al., 2018, in press](#)). There is therefore a need to determine which correlations produce
103 accurate predictions of terminal fall velocity, and are valid for an Re range appropriate for
104 ash dispersion applications, and to provide a database of default shape descriptors, from
105 volcanic ash measurements, for use when no data are available.

106 **3. Tephra samples**

107 We use tephra samples from fall deposits of three Icelandic volcanoes, spanning five
108 eruptions, and collected between 10 and 242 km from vent, to investigate a range of clast
109 types, morphologies, and compositions. Sample locations, compositions and qualitative
110 morphological descriptions are given in Table 1; example ash morphologies are shown in
111 Figure 1. Two samples, KSM and KSU, are noted for their distinctive ‘needles’ (Figure 1b-
112 d), elongated tube pumice fragments containing sparse microlites and numerous elongated
113 bubbles in a glassy matrix (Larsen et al., 2001). The KVE sample is characterised by
114 numerous flat, platy bubble wall shards (Mangerud et al., 1984). We also examine two
115 more typical examples of Icelandic ash: the EYJ sample is characterised by blocky or
116 angular glassy particles with a wide range of vesicularities (Gislason et al., 2011); the HEK
117 sample is characterised by blocky vesicular particles.

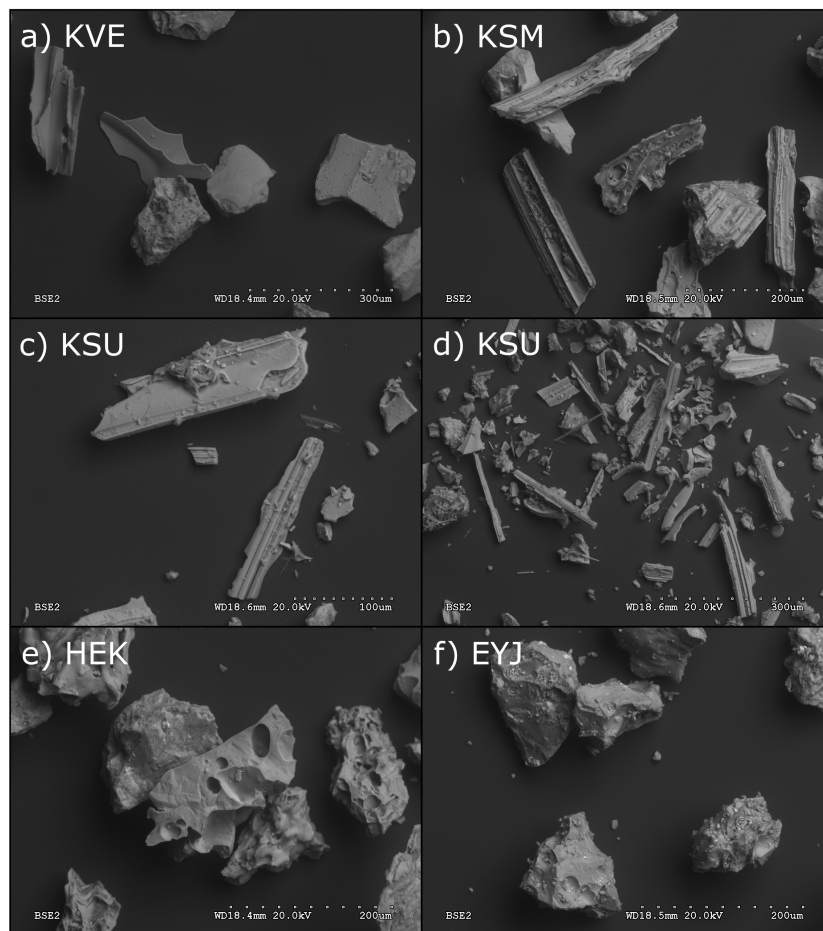


Figure 1: Images showing characteristic morphological characteristics of volcanic ash samples: a) KVE, b) KSM, c-d) KSU, e) HEK and f) EYJ. Images were taken at the University of Bristol using a Hitachi S-3500N scanning electron microscope (SEM). We obtained backscattered electron (BSE) images of particles in the 62.5 – 125 μm sieve fractions (88 – 125 μm for sample KVE).

118 4. Measurements

119 Prior to analysis, all samples were manually dry sieved at 1 or 0.5 φ intervals (where φ
120 is a measure of grain size defined as $-\log_2 \frac{D}{D_0}$, where D is the diameter of the particle in
121 millimetres and D_0 is a reference diameter of 1mm). We used sieve mesh diameters of 4 to
122 -1φ (62.5 to 2000 μm), apart from sample KVE which was sieved at half- φ intervals using
123 sieve mesh diameters of 3.5 to -0.5φ (88 to 1414 μm).

124 We measured the shape of 19557 particles from KVE, KSU, and HEK using X-ray CT
125 to provide a large volcanic ash shape database, including shape distributions for each sieve
126 fraction, as shape can vary with size (e.g., [Mele and Dioguardi, 2018](#)). For this analysis the
127 samples were analysed in bulk, including crystals and lithics. We chose only three samples
128 for this analysis due to the significant scan time needed; the samples we chose span a range
129 of qualitative shape characteristics (see Table 1).

130 To determine the effectiveness of our measured shape descriptors when used to calcu-
131 late terminal fall velocity, we also selected 46 individual particles of juvenile glass from the
132 0φ and -0.5φ sieve fractions of the KSM, KVE, HEK and EYJ samples, which were indi-
133 vidualy scanned using X-ray CT. Particles chosen ranged from 1.0 – 2.6 mm, sufficiently
134 large to image and track; again grains were selected to include a wide range of shapes. As
135 density is a crucial input in drag equations, we also measured the density of these particles.
136 We then considered how well we could calculate the fall velocity of our particles using
137 our shape measurements and different drag equations by comparing to measurements of
138 the fall velocity of the particles in a settling column. KSU particles were not used in this
139 analysis due to their fragility and the greater availability of mm-sized elongate particles
140 in the analogous KSM sample. The following sections give a detailed description of each
141 measurement method.

142

143 4.1. Particle velocities

144 Particle velocities were measured in air using a settling column and high-speed imag-
145 ing based on the method of [Bagheri and Bonadonna \(2016b\)](#). The 1.5 m high and 0.15 m
146 diameter glass settling column ensures that particles fall a great enough distance to reach
147 terminal velocity. At the top of the column is a guiding tube to ensure particles fall ap-
148 proximately in the centre to minimise errors associated with the particle's position in the

Table 1: Summary of samples used in this study.

Sample ID	Eruption	Year	Composition	Sample location (decimal degrees WGS84)	Notable qualitative shape characteristics
KSM	Katla SILK-MN	2975±12 a BP (Larsen et al., 2001)	Dacitic (~65% SiO ₂) (Larsen et al., 2001)	63.49203 -18.880946	Distinctive elongated 'needles'
KSU	Katla SILK-UN	2660±50 a BP (Larsen et al., 2001)	Dacitic (~64% SiO ₂) (Larsen et al., 2001)	63.754977 -18.49149	Distinctive elongated 'needles'
KVE	Katla Vedde	~ 12 ka BP (Wastegård et al., 1998)	Bimodal (45-58% and 72-76% SiO ₂) (Mangerud et al., 1984)	65.749955 -17.897997	Flat, platy glass shards
HEK	Hekla 1947	1947	Andesitic (~62% SiO ₂) (Larsen et al., 1999)	63.7149 -19.8311	Blocky, vesicular
EYJ	Eyjafjallajökull 2010	2010	Andesitic (~58% SiO ₂) (Gislason et al., 2011)	63.7139 -19.725	Blocky or angular glassy particles

149 column; the setup is illustrated in Figure 2. A Vision Research Phantom v9.1 high speed
150 camera was positioned 0.1 m from the bottom of the apparatus, where particles fell from the
151 settling column into a flat-sided glass box. A measure with precision of 1 mm was placed
152 at the back of the box. To focus the camera prior to the experiments, a weighted thread
153 was lowered down the guiding tube into this box, which was illuminated with two LED
154 lamps. Each grain of ash was then released individually into the guiding tube and filmed
155 at a sample rate of 1400 fps and an exposure of 711.25 μ s. Phantom 675.2 Camera Con-
156 trol software was used to output individual video frames in jpeg format, including a time
157 stamp from which terminal velocity could be calculated from 5 to 8 cm sections of each
158 particle's trajectory. The error arising from the relative positioning of the camera, particle,
159 and ruler was corrected assuming that each particle was falling in the centre of the 15 cm
160 main settling column. Each particle velocity measurement presented here represents the
161 median of 5 repeat measurements. Repeat data could not be collected for particles which
162 broke upon landing and so those experiments are not reported; the data we report are for
163 a total of 46 particles which did not break. This creates a potential bias against measuring
164 the fall velocity of particles with certain shapes and densities. Velocity data are available
165 in Supplementary Material.

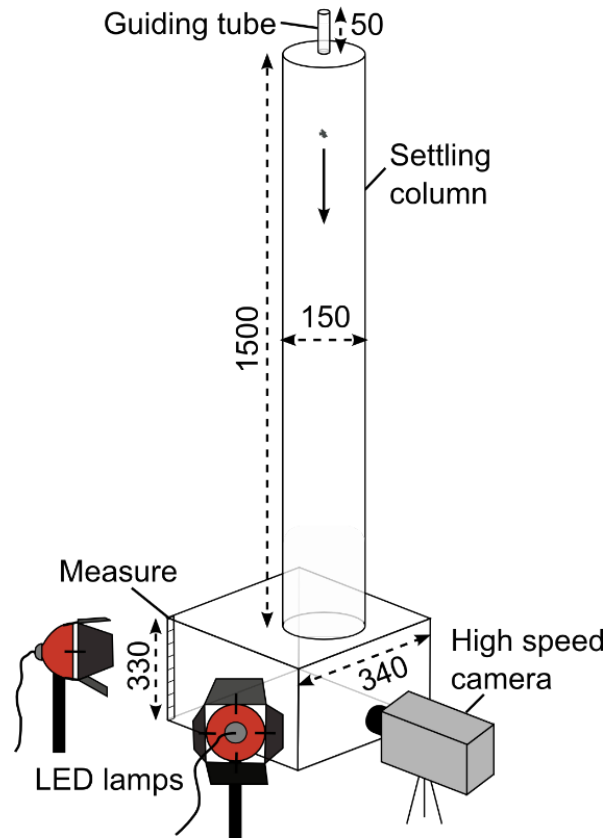


Figure 2: Schematic diagram of the settling column setup to measure terminal velocity in air. All dimensions are in mm.

166 4.2. *Density*

167 We calculated particle density using volumes from X-ray CT scans and particle mass
 168 measured on a balance with a precision of 0.0001 g. Particle dimensions calculated using
 169 Avizo CT software were checked using digital calipers, to ensure that CT data gave accurate
 170 particle size measurements. Density data are available in Supplementary Material.

171 4.3. *Particle dimensions using X-ray CT*

172 All particle scans were carried out on a Nikon XTH225ST scanner using a voltage of
 173 120 kV and a current of 58 μ A. The 46 particles for which we obtained terminal veloc-
 174 ity measurements were individually scanned by mounting in florist's foam or cotton wool
 175 within plastic pipettes. Of these, 16 were scanned at the maximum instrument resolution
 176 (voxel edge length = 3 μ m, where a voxel is a volumetric (3D) pixel). We obtained this
 177 high-resolution dataset as shape descriptors can be sensitive to imaging resolution (Liu
 178 et al., 2015); to determine the sensitivity of shape descriptors to resolution we progres-

179 sively resampled the data to give resolutions of 2, 4, 8, 16 and 32 times the original voxel
180 edge lengths. The remaining 30 particles were scanned at the lower resolution of voxel
181 edge length = 96 μm (32 times the voxel edge length of the high-resolution scans).

182 For bulk scanning of the KVE, KSU and HEK samples, we used a voltage of 100 kV
183 and a current of 70 μA . We measured all sieve fractions of these samples (62.5 to 2000
184 μm for KSU and HEK, 88 to 1414 μm for KVE). The voxel size was varied to maintain
185 a minimum resolution of between 4728 and 20,000 voxels per particle, to obtain size-
186 independent shape parameters; within this range we find no correlation between resolution
187 and shape. Samples were encased in epoxy resin to ensure good particle separation and
188 allow simultaneous imaging of up to several hundred particles, as discussed in [Saxby et al.](#)
189 [\(2018\)](#). Despite this preparation, particles from coarser sieve fractions tended to float to the
190 top of the sample container before the epoxy hardened; these particles were touching in the
191 resulting segmented 3D volumes and were separated using a watershed algorithm (Avizo
192 Separate Objects module), which first detects object centres and then simulates flooding
193 from these regions to the edges of 3D ‘catchments’ defined according to the greyscale
194 value gradient.

195 For all scans, we reconstructed 3D volumes using CT Pro 3D software and segmented
196 the volumes in Avizo. From the resulting particle surfaces, we obtained particle surface area
197 (A_{surf}), volume (V), and three orthogonal principal axis lengths: long axis L , intermediate
198 axis I and short axis S .

199 It is important to note that benchtop X-ray CT cannot be used to accurately quantify
200 the shape of the finest volcanic ash fractions relevant to aircraft hazard (< 30 to $60 \mu\text{m}$;
201 [Rose and Durant \(2009\)](#) or respiratory health (< 4 to $10 \mu\text{m}$; [Horwell et al. \(2010\)](#)), due
202 to constraints on imaging resolution. Although synchrotron X-ray CT systems can achieve
203 resolutions of $1 \mu\text{m}$ or less, in our system the minimum voxel edge length is $\sim 3 \mu\text{m}$.
204 For this reason, particles smaller than $62.5 \mu\text{m}$ (4ϕ) were not used, and our X-ray CT
205 analysis does not give a full grain shape distribution for the range of sizes typically used
206 to initiate operational dispersion models ($\sim 0.1 - 100 \mu\text{m}$; [Hort \(2016\)](#)). However, the
207 terminal velocity of very small particles ($< \sim 30 \mu\text{m}$) is low compared to atmospheric
208 turbulence and vertical advection ([Saxby et al., 2018](#)), meaning that the dispersion of these
209 particles (Class III fragments in the classification of [Koyaguchi and Ohno \(2001\)](#)) is less
210 sensitive to shape. For example, for model particles of $1 \mu\text{m}$, diffusion to ground level from

211 12 km above ground level is faster than sedimentation over the same distance (Saxby et al.,
 212 2018). Therefore we suggest it is reasonable to apply the same shape factors for particles
 213 smaller than $\sim 30 \mu\text{m}$ despite the lack of available measurements. However, we note that
 214 drag may be important for particles $< \sim 30 \mu\text{m}$ as it affects the two-way coupling of fine
 215 ash and turbulent eddies (Del Bello et al., 2017).

216 4.4. Morphological parameters

217 We compared observed terminal fall velocities with calculations based on the drag
 218 equations of Ganser (1993), Bagheri and Bonadonna (2016b) and Dioguardi et al. (2018);
 219 to calculate drag by these equations, we measured the shape descriptors used in their design.
 220 The drag equations are all applicable for the range of flow regimes expected for volcanic
 221 ash falling in air and are all calibrated using 3D geometric shape descriptors. We calculate
 222 w_t using equation 1; all the drag equations use the diameter of a volume-equivalent sphere,
 223 d_v , for the particle size parameter d . C_D is calculated as a function of one or more geomet-
 224 ric shape descriptors, given below; for the corresponding C_D equations, see Supplementary
 225 Material. Since the drag of spheres can be determined analytically to high accuracy, a
 226 popular approach in defining shape descriptors is to use a ratio of a particle parameter to
 227 that of a volume-equivalent sphere. The Ganser (1993) drag equation uses sphericity ψ_G ,
 228 the ratio of surface area of a volume-equivalent sphere to surface area of the particle being
 229 described:

$$\psi_G = \frac{\pi^{\frac{1}{3}} (6V)^{\frac{2}{3}}}{A_{\text{surf}}}, \quad (2)$$

230 where A_{surf} is a measure of 3D surface area and therefore an effective descriptor of rough-
 231 ness scales limited only by imaging resolution. Difficulty in measuring the 3D surface area
 232 of irregular particles has meant that studies have often calculated sphericity using approxi-
 233 mate surface area of a smooth scalene ellipsoid A_e (Dellino et al., 2005; Mele et al., 2011),
 234 with equal principal axes to the particle:

$$A_e = 4\pi \left(\frac{\left(\frac{L}{2}\right)^z \left(\frac{I}{2}\right)^z + \left(\frac{L}{2}\right)^z \left(\frac{S}{2}\right)^z + \left(\frac{I}{2}\right)^z \left(\frac{S}{2}\right)^z}{3} \right)^{\frac{1}{z}}, \quad (3)$$

235 where $z = 1.6075$. The Dioguardi et al. (2018) drag equation is calibrated using an ellipsoid
 236 approximation; their shape descriptor Ψ_D , which we term the Dioguardi shape factor, is the

237 ratio of 3D sphericity to 2D circularity:

$$\Psi_D = \psi_e / X \quad (4)$$

238 (Dellino et al., 2005; Dioguardi et al., 2018), where:

$$\psi_e = \frac{\pi^{\frac{1}{3}} (6V)^{\frac{2}{3}}}{A_e} \quad (5)$$

239 and

$$X = \frac{P_{proj}}{P_c}; \quad (6)$$

240 P_{proj} = maximum projected perimeter and P_c = the perimeter of a circle with equal projected
241 area to the particle being described. In this study we focus on 3D shape measurement and
242 do not measure X , as this 2D parameter is a function of particle perimeter, one of the 2D
243 parameters most sensitive to imaging resolution (Liu et al., 2015). When calculating the
244 Dioguardi shape factor we use the Dioguardi et al. (2018) best fit approximation:

$$\Psi_D \approx 0.83\psi_e. \quad (7)$$

245 Another class of particle shape descriptor, termed form factors, combines L , I and S ,
246 which measure the form of the particle but are insensitive to small-scale surface roughness
247 (Bagheri and Bonadonna, 2016b) and are therefore less sensitive to imaging resolution.
248 These include the two shape factors defined by Bagheri and Bonadonna (2016b), elongation
249 e and flatness f , where

$$e = \frac{I}{L} \quad (8)$$

250 and

$$f = \frac{S}{I}. \quad (9)$$

251 The shape descriptors we measure (sphericity, the Dioguardi shape factor, elongation
252 and flatness) are all scaled between 0 and 1, where 1 = an equant particle; this allows for
253 easy comparison between parameters.

254 **5. Results**

255 *5.1. Sensitivity of shape descriptors to CT scan and reconstruction parameters*

256 *5.1.1. Greyscale threshold*

257 Particle surfaces are reconstructed from raw CT data by separating 3D regions rep-
258 resenting particles from the surrounding epoxy; this requires the selection of a threshold
259 greyscale value. As the particle edges are characterised by a gradient (over $\sim 3 - 5$ voxels)
260 rather than a sharp boundary, the choice of threshold is subjective and so we determined
261 the sensitivity of particle volume and shape to this choice. We did this by increasing and
262 decreasing our best estimate greyscale threshold by 10, which covered the particle bound-
263 ary gradients; the results and example greyscale images are given in Supplementary Figure
264 A1. We calculate a maximum 6% error on mean d_v and 4% error on mean sphericity arising
265 from the selection of the particle boundaries.

266 *5.1.2. Voxel size*

267 As shape can be sensitive to imaging resolution, we investigated the impact of voxel
268 size on measured shape factors using X-ray CT data for 16 individual particles. Scans were
269 conducted at the scanner's maximum resolution (a voxel edge length of $\sim 3 \mu\text{m}$, giving
270 between 6.5×10^6 and 1.3×10^8 voxels per particle). We progressively resampled the scan
271 data from 2 to 32 times the original voxel edge lengths, giving a maximum voxel edge
272 length of $\sim 96 \mu\text{m}$; after each resampling we recalculated surface area, volume, sphericity,
273 the Dioguardi shape factor, elongation, and flatness.

274 The results are shown in Figure 3 and highlight the sensitivity of surface area, and the
275 surface area-based shape factor sphericity, to resolution. For a large, rough particle (KSM-
276 9, Figure 3, inset), apparent surface area decreased by between 10 and 23% each time we
277 doubled the voxel edge length; the mean surface area decrease for all particles at all scales
278 is 12%. In contrast, volume measurements are relatively insensitive to imaging resolution:
279 when halving or doubling the resolution, the mean absolute volume change is 1.4%. This
280 means the particle diameter d_v is insensitive to resolution over this range. The shape de-
281 scriptors vary in their sensitivity to resolution. Particles have higher apparent sphericity at
282 low resolution: doubling the voxel edge length resulted in sphericity increasing between
283 0.005 and 0.13 (3 – 32%, mean 14%). For most particles studied, sphericity and surface
284 area are sensitive to resolution for the whole resolution range, suggesting the particles ex-

285 hibit surface irregularities below the scale of the 3 μm resolution limit. The exceptions
 286 are the Vedde ash particles KVE-1 and KVE-2, which are smooth glass shards (Figure
 287 3, insets). For these particles, surface area and sphericity are almost constant above \sim
 288 10^5 voxels / particle. Unlike sphericity, shape descriptors which are functions of principal
 289 axis lengths (the Dioguardi shape factor, elongation, and flatness) are relatively insensitive
 290 to imaging resolution (Figure 3 d-f), and change on average by $\pm 1.2\%$, 1.6% , and 1.3%
 291 respectively when voxel edge length is doubled.

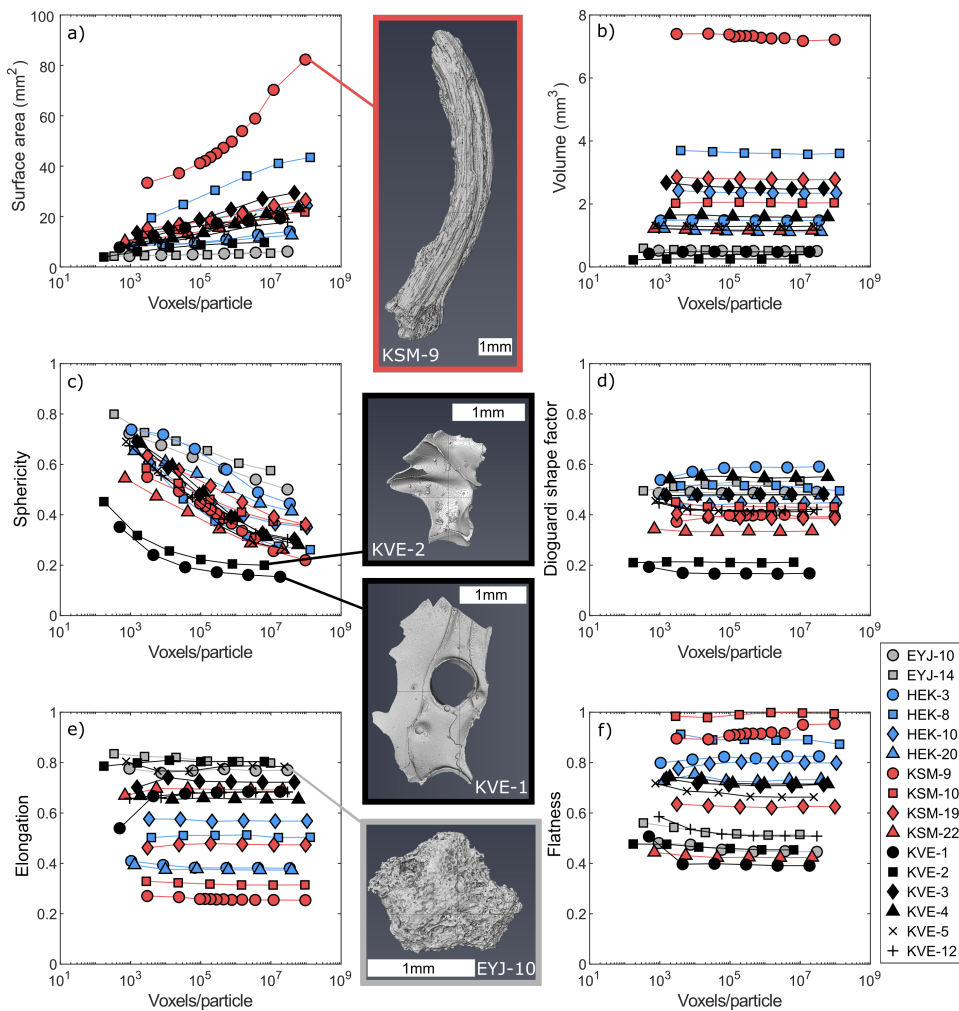


Figure 3: Sensitivity of particle measurements to CT imaging resolution. a) Surface area A_{surf} ; b) volume V ; c) sphericity sphericity (Ganser, 1993); d) shape factor the Dioguardi shape factor (Dioguardi et al., 2018); e) elongation elongation (Bagheri and Bonadonna, 2016b); f) flatness flatness (Bagheri and Bonadonna, 2016b).

292 5.2. *Assessing the effectiveness of shape descriptors in representing the aerodynamic be-*
293 *haviour of volcanic ash*

294 Recorded median terminal velocities ($w_{t, rec}$) of particles dropped in the settling column
295 range from 1.9 to 6.4 m s⁻¹. Minimum and maximum velocities deviate from the median by
296 $\leq 57\%$. We compare measured fall velocity to calculated fall velocities using shape-based
297 drag equations (Ganser, 1993; Bagheri and Bonadonna, 2016b; Dioguardi et al., 2018)
298 with our measured shape parameters, as well as a spherical particle drag equation (White,
299 1974). We calculate terminal velocity as a function of the drag coefficient defined by these
300 equations; where a shape factor is required we use our X-ray CT measurements. The full
301 drag equations are given in Supplementary Material.

302 5.2.1. *Effect of scan resolution on velocity calculations using sphericity*

303 For the Ganser (1993) drag equation, which uses the shape factor sphericity that is very
304 sensitive to imaging resolution, we first recalculate terminal velocity using every value of
305 sphericity obtained from resampling the CT data for the 16 high resolution scans (Figure
306 3c). A mean 14% reduction in sphericity for each doubling of voxel edge length translates
307 into an average reduction of 12% in calculated velocity ($w_{t, Ganser}$). Figure 4 shows these
308 calculated velocities relative to $w_{t, rec}$. Importantly, we find that calculating sphericity with
309 high-resolution data results in velocity underestimation, as we overestimate the effect of
310 very small-scale surface irregularities on drag. For example, using the original (high reso-
311 lution) scan settings to calculate surface area predicts fall velocities ($w_{t, Ganser}$) that are 1.2
312 – 2.3 times too slow. The best agreement between measured velocities and those calculated
313 using the Ganser scheme occurs when we calculate sphericity using the largest voxel edge
314 length of 96 μm (resolutions between 10^2 and 10^5 voxels / particle). For the comparison of
315 drag equations in the following section, we use this best fit dataset, with sphericity calcu-
316 lated using a resampled 96 μm resolution, for the 16 particles scanned at high resolution, for
317 consistency with the remainder of the 46 particles which were scanned at 96 μm resolution.

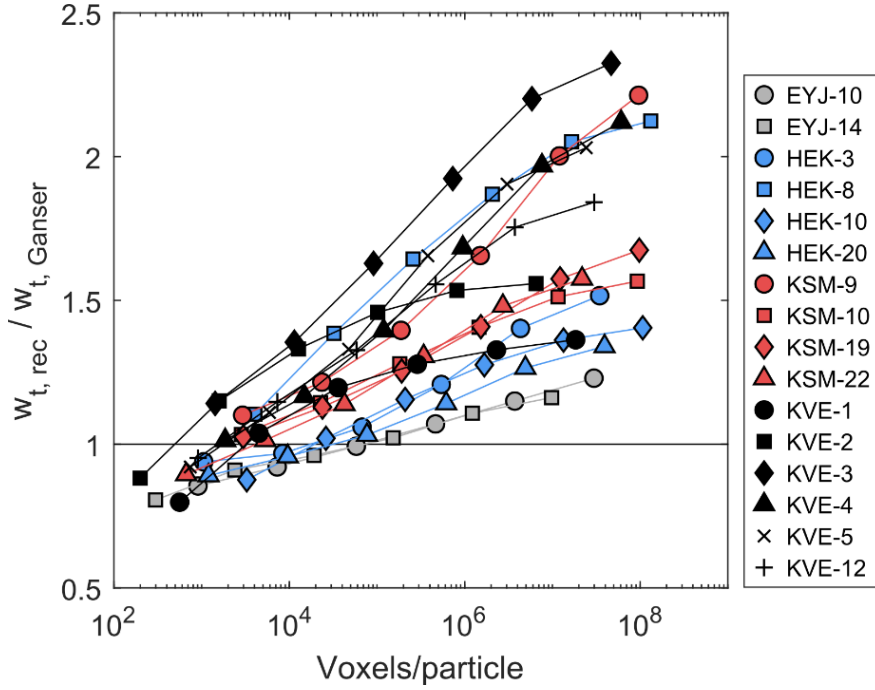


Figure 4: Measured velocity ($w_{t,rec}$) divided by velocity calculated after Ganser (1993) ($w_{t,Ganser}$) with sphericity calculated after progressive resampling of CT datasets to decrease resolution. Values of 1 indicate perfect agreement between $w_{t,rec}$ and $w_{t,Ganser}$. Higher scan resolutions result in higher surface area measurements, which give lower sphericity values and therefore calculated velocity which is too low.

318 5.2.2. Effectiveness of published drag equations

319 The results of our comparison between drag equations are shown in Figure 5. The an-
 320 nalytical drag equation of White (1974) for spheres overestimates terminal velocity with a
 321 mean absolute percentage error (MAPE) of 40% (Figure 5a); drag equations which include
 322 a shape factor (Ganser, 1993; Bagheri and Bonadonna, 2016b; Dioguardi et al., 2018) pro-
 323 duce better agreements with measured velocities. The Bagheri and Bonadonna (2016b)
 324 scheme calculates velocity as a function of elongation and flatness, which are insensitive
 325 to resolution; the scheme slightly overestimates terminal velocity, with a MAPE of 24%
 326 (Figure 5c). Velocity calculated using the Dioguardi shape factor (equation 7) and calibra-
 327 tion (Dioguardi et al., 2018) yields a MAPE of 22% (Figure 5b). Using the Ganser (1993)
 328 scheme with sphericity calculated using a voxel edge length of 96 μm results in a MAPE
 329 of 19% (Figure 5d). We show only this best fit dataset; using the sphericity data from the
 330 high resolution scans with no resampling increases the MAPE of the Ganser (1993) scheme
 331 to 69%, meaning the drag equation performs worse than a spherical approximation if the
 332 impact of surface roughness is overestimated.

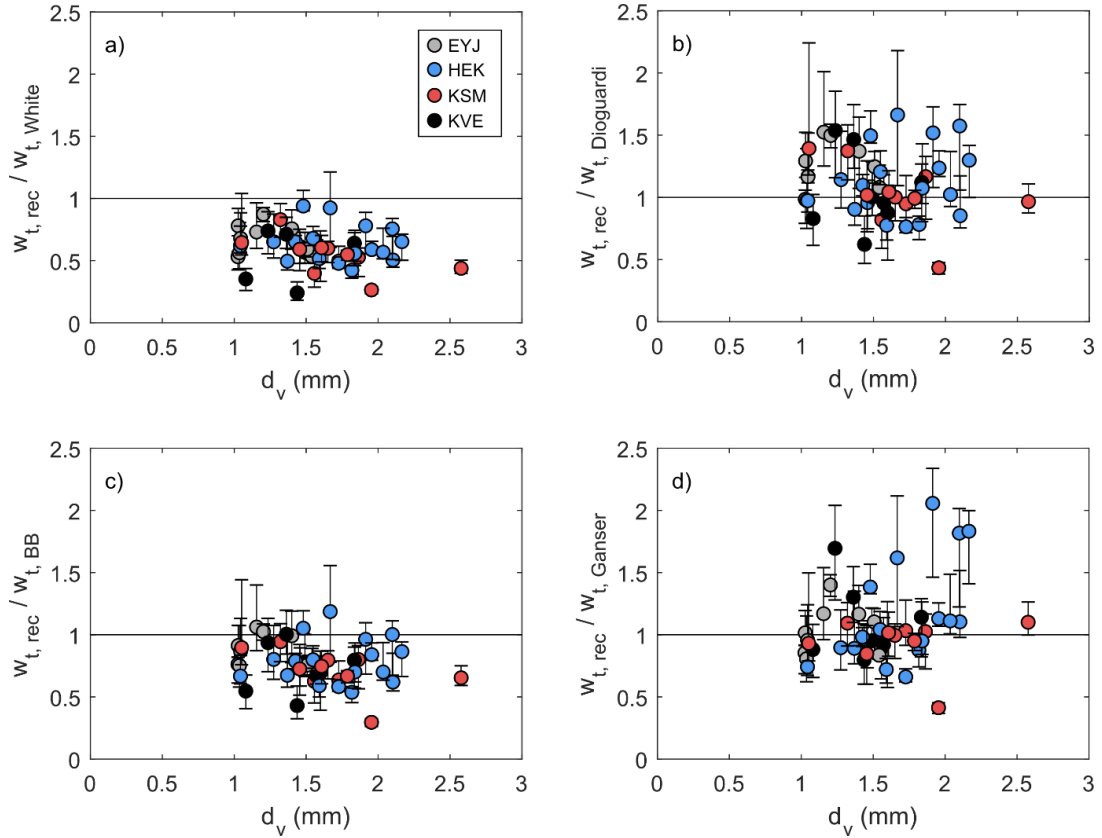


Figure 5: Measured velocity ($w_{t, rec}$) divided by velocity calculated by a) a spherical particle approximation (White, 1974), b) as a function of the Dioguardi shape factor (Dioguardi et al., 2018), c) as a function of elongation and flatness (Bagheri and Bonadonna, 2016b), and d) as a function of sphericity (Ganser, 1993) where sphericity is calculated using CT scan resolutions of $96 \mu\text{m}$ voxel edge length. Note that the Ganser (1993) scheme performs worse than even a spherical particle approximation when sphericity is calculated using much higher image resolutions.

333 5.3. A database of the shape of volcanic ash

334 We have determined that the drag equations of Ganser (1993), Bagheri and Bonadonna
 335 (2016b), and Dioguardi et al. (2018) all produce reasonable estimates of volcanic ash ter-
 336 minal velocity. In practice, when forecasting ash dispersion operationally, information on
 337 particle morphology is unlikely to be available. Therefore, the use of these schemes re-
 338 quires a database from which to choose a default ash shape.

339 5.3.1. Volcanic ash shape data selection criteria

340 It is important to assess the relationship between particle shape and size, as well as
 341 to obtain shape data for particles smaller than the mm-sized particles we used in settling
 342 column experiments. Operationally, Volcanic Ash Advisory Centres use volcanic ash dis-
 343 persion models with a particle size distribution (PSD); for most operational systems the

344 bulk of the modelled erupted mass is restricted to particles with diameter $< 100 \mu\text{m}$ (Hort,
345 2016).

346 As illustrated above, another important consideration is the resolution used to obtain
347 sphericity (Figure 3). Accurate calculation of terminal velocity using the Ganser (1993)
348 scheme requires sphericity to be calculated from CT data with a resolution which gives
349 between 10^2 and 10^5 voxels / particle, and so our database can only include sphericity data
350 in this resolution range. Because the Dioguardi shape factor, elongation and flatness are not
351 sensitive to imaging resolution, it is possible to directly compare our data to other studies
352 measuring the same shape factors, regardless of experimental conditions. We calculated
353 these shape factors using equations 7, 8, and 9, where only axis lengths or approximate
354 ellipsoid sphericity ψ_e were reported. We also exclude studies which use different shape
355 equations; for example, many studies calculate approximate sphericity using 2D images.
356 We prefer to limit our analysis to the exact shape descriptors by which the drag equations
357 were calibrated.

358 We obtain shape descriptors for every sieve fraction of samples KVE, KSU, and HEK,
359 adjusting voxel size, with a resolution of 4728 – 20,000 voxels / particle. The resulting par-
360 ticle shape distributions are shown in Figure 6; we compare them in Figure 7 to published
361 data from other eruptions that match our criteria. All particle shape data are available in
362 Supplementary Material.

363 Our samples, and most of the samples from literature sources in Figure 7, are bulk
364 samples of all components (glass, crystals, lithics) of an eruption; the exception is single-
365 component data from Wilson and Huang (1979) which we show for comparison but exclude
366 from analysis as for dispersion modelling purposes we are interested in all particle types.

367 5.3.2. *New ash shape results*

368 Sphericity changes significantly with particle size for all eruptions in this study (Figure
369 6). Particles of $\sim 300 \mu\text{m}$ are the least spherical, with the lowest median sphericity of 0.27
370 and 0.36 respectively for the 250 μm sieve fractions of the HEK and KSU ash, and a lowest
371 median sphericity of 0.17 for the 354 μm sieve fraction of the KVE ash (Figure 6a-c). For
372 all samples, sphericity is highly variable even within a single size fraction. The Dioguardi
373 shape factor (Dioguardi et al., 2018) shows a similar pattern, although with fewer extremely
374 high or low values (and fewer outliers). The lowest median values of the Dioguardi shape
375 factor are 0.34 (HEK, 125 μm), 0.21 (KVE, 354 μm), and 0.34 (KSU, 250 μm) (Figure 6d-

376 f). Flatness and elongation, in contrast, do not show significant variation with particle size,
377 and both shape factors are constrained to a narrower range, with median elongation and
378 flatness between 0.56 and 0.81 for all samples and size fractions in this study (Figure 6g-
379 l). The KSU sample, chosen for its 'extreme' elongated grain shapes when viewed under
380 an optical microscope, does not differ markedly from HEK, an ash sample which appears
381 more 'typical' on visual inspection, in terms of median values, although the percentage
382 of particles with lower shape factors is higher for the KSU ash. The KSU ash has more
383 elongated particles (24 % of particles have elongation < 0.5) than KVE (9 %) or HEK (6
384 %). However, the percentage of particles with flatness < 0.5 is low for all samples: 2 % for
385 HEK, 3 % for KVE, and 4 % for KSU.

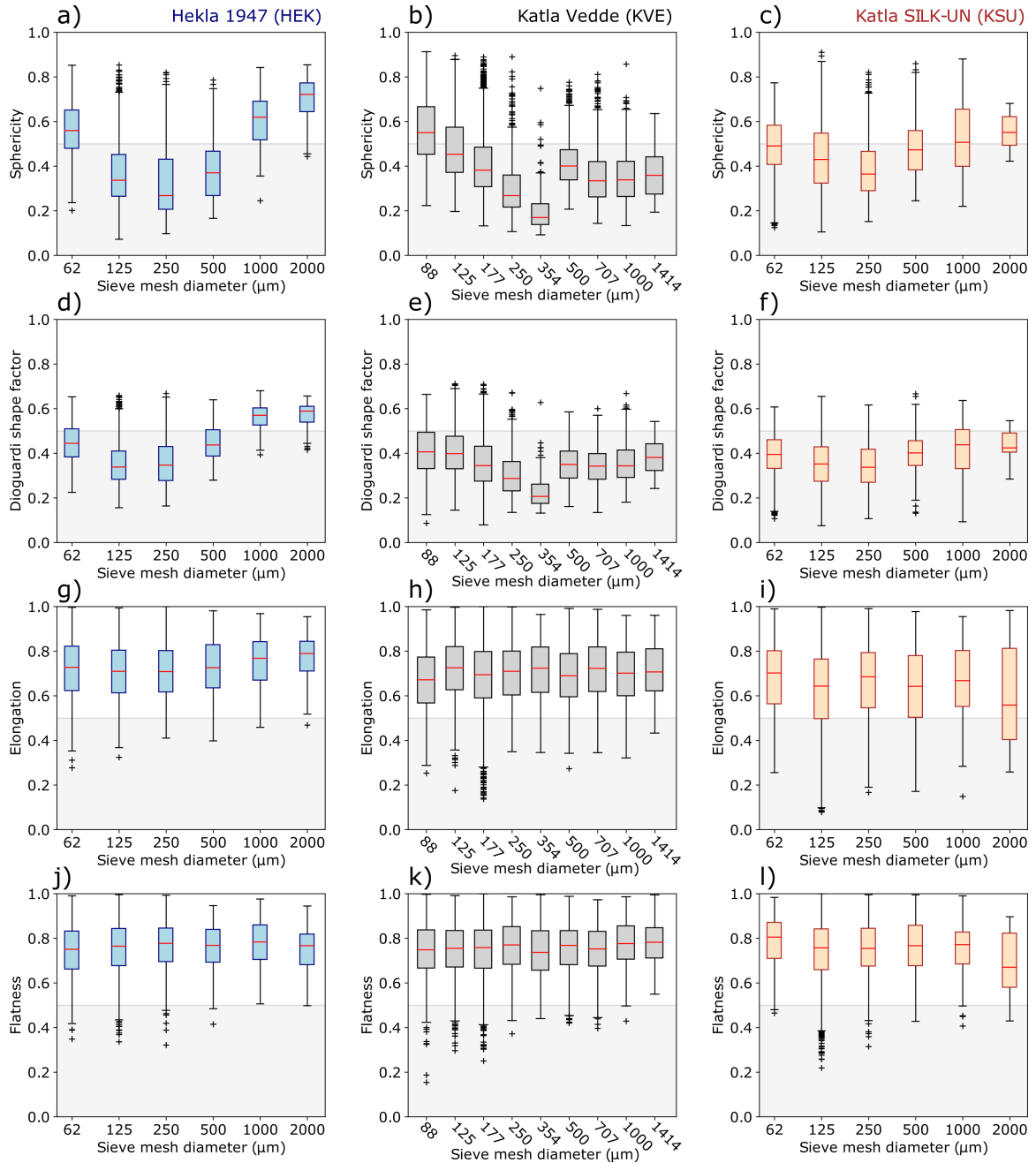


Figure 6: Shape distributions for bulk sieved samples HEK, KVE, and KSU. We chose three samples due to the scan time needed; the samples span a range of qualitative particle shape characteristics (Table 1). We shade each plot to highlight shape factors < 0.5 to aid visual comparison of the proportion of highly non-spherical particles in each sample. Red lines indicate the median; boxes show the interquartile range; crosses indicate outliers.

386 5.3.3. Volcanic ash shape database

387 We compare our shape data to data from previous studies, to expand our morphology
 388 database to include different eruptions and particle size fractions, and to determine whether

389 the eruptions studied here show a ‘typical’ range of volcanic ash shape. All ash shape data
390 are given in Figure 7. As data vary between studies, we plot only mean and standard
391 deviation of shape for each sample. Some studies report the mean shape factor for each
392 size fraction of a sample; others give the mean shape of a bulk sample; where the particles
393 vary in size, we indicate the size range using the X-axis error bars. We note that methods
394 of measuring grain size differ between studies, and so the size ranges shown here should
395 be considered approximate. For the shape descriptor sphericity, we include only data from
396 studies that use CT data with a resolution of between 10^2 and 10^5 voxels/particle, which
397 is our recommendation. Despite this limitation we still find a weak correlation between
398 image resolution and sphericity (Figure 7b).

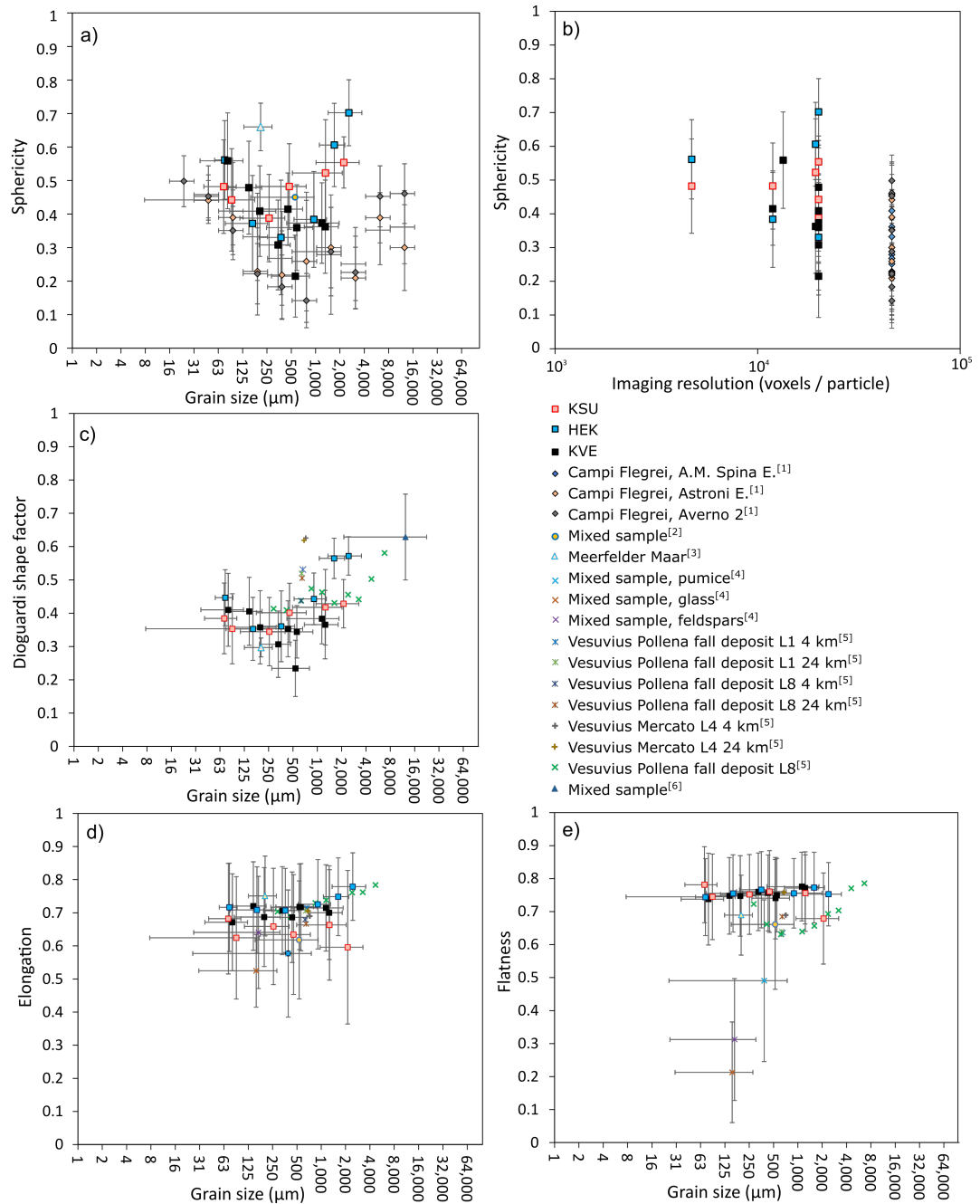


Figure 7: Shape measurements of volcanic ash particles from this study and published literature: a-b) sphericity (Ganser, 1993), c) the Dioguardi shape factor (Dioguardi et al., 2018), d) elongation and e) flatness (Bagheri and Bonadonna, 2016b). X-axis error bars indicate the grain size range, with points plotted at the middle of the range; Y-axis error bars show one standard deviation of shape for each sample, with points indicating mean shape. ‘Mixed sample’ indicates ash from multiple eruptions; see Supplementary Material for specifics. Data sources: ^[1]Mele and Dioguardi (2018); ^[2]Bagheri et al. (2015); ^[3]Vonlanthen et al. (2015); ^[4]Wilson and Huang (1979); ^[5]Mele et al. (2011); ^[6]Dioguardi et al. (2018). We exclude data from studies which use different shape equations or which calculate sphericity using data outside the range of image resolution we find to be effective.

399 Figure 7a shows that sphericity ranges from ~ 0.1 to 0.8 for particles between 10^1 and
400 10^4 μm . Where multiple size fractions of the same sample are measured, the relationship
401 between sphericity and grain size is similar to samples from this study: particles at the
402 extremes of the size range are more spherical, with a pronounced low in sphericity occur-
403 ring between 125 and 1000 μm . A similar pattern can be observed for the Dioguardi shape
404 factor (Figure 7c). There is less variation in flatness; most samples have flatness between
405 0.5 and 0.9, using the ranges given by one standard deviation for all samples. The single-
406 component (glass and feldspar) samples of [Wilson and Huang \(1979\)](#) are an exception,
407 with mean flatness between 0.2 and 0.5. Mean elongation for all samples ranges between
408 0.4 and 0.9. The KSU data are more elongated (lower elongation) than most other data
409 from the literature, as expected from visual inspection of the samples, which contain char-
410 acteristic tube pumice ‘needles’. However, neither the KSU or KVE samples differ greatly
411 from the other ash shape data in any mean shape factor.

412 6. Discussion

413 6.1. Measuring shape

414 X-ray CT is an accurate and efficient method of assessing particle size and shape in 3D,
415 and allows imaging of hundreds of particles, and multiple shape factors, relatively rapidly.
416 The analytic error resulting from manual selection of greyscale values is low ($< 4\%$ on
417 sphericity, which is insignificant compared to its sensitivity to image resolution, and < 6
418 $\%$ on diameter d_v). The high resolution makes it an invaluable tool for examining detailed
419 structures in volcanic rocks. For shape quantification, however, high resolution surface area
420 measurements result in very low sphericity and therefore underestimate terminal velocity
421 by the drag equation of Ganser (1993). We recommend using a resolution between 10^2 and
422 10^5 voxels/particle to calculate sphericity. The best agreement between measured veloc-
423 ity $w_{t, rec}$ and calculated velocity $w_{t, Ganser}$ is reached at a range of image resolutions; we
424 suggest this is due to the range of particle shapes as well as uncertainty on other param-
425 eters such as density, diameter, particle position in the settling column and the variability
426 in velocity resulting from changing orientation (e.g., Saxby et al., 2018). Using shape
427 parameters based on principal axis lengths is a more practical technique where imaging
428 resolution cannot be kept constant. We conclude that above our lower resolution limit of \sim
429 10^2 voxels/particle, imaging resolution is not a concern for calculation of these shape de-
430 scriptors, meaning that for calculation of elongation, flatness or the Dioguardi shape factor
431 it is practical to sacrifice higher resolution in favour of speed.

432 As shape parameters based on axis lengths are less sensitive to resolution, we assess
433 the accuracy of the Ganser (1993) drag equation when using the approximate sphericity of
434 a smooth ellipsoid with equivalent axes to the particle (ψ_e ; Equation 5) in place of a surface
435 area based formula (Equation 2). The results are shown in Figure 8. Calculated velocities
436 are accurate for the particles in this study, with MAPE of 23%, compared to MAPE of
437 between 19% and 69% depending on image resolution for the surface area formulation.
438 Therefore, it is valid to use the Ganser (1993) drag equation with approximate sphericity
439 ψ_e in place of sphericity for rough particles.

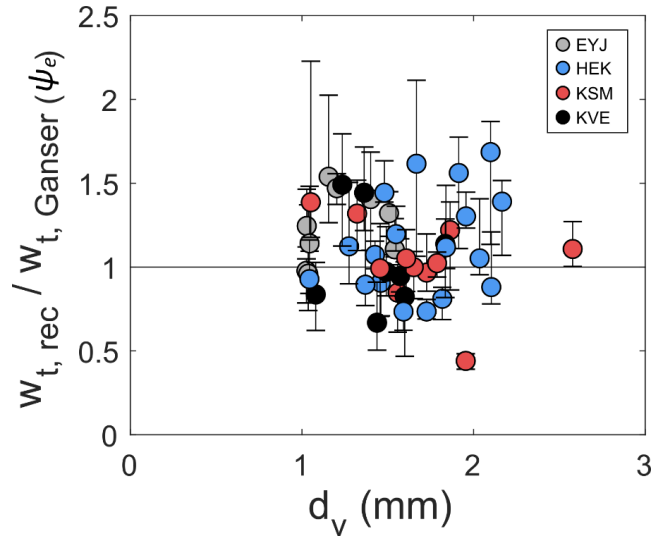


Figure 8: Measured velocity ($w_{t, rec}$) divided by velocity calculated according to the drag equation of Ganser (1993) as a function of approximate sphericity ψ_e . Values of 1 indicate perfect agreement between measured and calculated velocity. The mean absolute percentage error is 23%.

440 6.2. *The shape range of volcanic ash*

441 Particle sphericity and the Dioguardi shape factor are size-dependent: in our samples,
 442 and others from the published literature, particles of ~ 125 to $1000 \mu\text{m}$ have the lowest
 443 shape factors (i.e. are the most extreme-shaped). Although sphericity is sensitive to image
 444 resolution, a similar trend seen in values of the Dioguardi shape factor, which is insensi-
 445 tive to image resolution, suggests that the size dependence is not an artefact of CT scan
 446 parameters. Instead, the correlation between size and shape is likely to be linked to bubble
 447 size. Although we do not measure bubble size in this study, the size of bubbles influences
 448 fragmentation (Liu et al., 2017); modal values of bubble size are typically $100 - 1000$
 449 μm (Rust and Cashman, 2011), corresponding to the lowest-sphericity particles. Bubbles,
 450 along with crystals, control the surface irregularities of volcanic ash particles, meaning that
 451 similarity between the particle size and the bubble size can produce highly irregular particle
 452 surfaces. Indeed, a recent X-ray CT study of volcanic ash particles (Mele and Dioguardi,
 453 2018) found that surface irregularity increased as the particle size neared the bubble size;
 454 bubble size could similarly be a primary control on shape for our samples. If the 125 to
 455 $1000 \mu\text{m}$ size fraction of a volcanic ash sample included a significant proportion of phe-
 456 nocrysts with higher sphericity than rough vesicular pumice fragments, this could increase
 457 bulk sphericity; however the samples in this study do not contain abundant phenocrysts.

458 Shape factors based on principal axis lengths (elongation and flatness) do not change
459 significantly with particle size or between eruptions (Figures 6 and 7). The exceptions are
460 three samples measured by [Wilson and Huang \(1979\)](#), with mean flatness as low as 0.2,
461 whereas most other data are 0.6 – 0.8 (Figure 7e). This difference may be partly because
462 they separate the ash into components, with the lowest flatness from a sample of purely
463 glass shards, whereas most of the other studies listed use bulk samples. Glass shards form
464 from bubble wall fragments and have a characteristically flat morphology when particle
465 size is similar to the bubble size. We do not include the [Wilson and Huang \(1979\)](#) data
466 when assessing a default range of flatness to use in dispersion modelling; however, we note
467 that some components of a volcanic ash sample can have much more ‘extreme’ shapes than
468 the bulk.

469 Some of our samples were chosen for the presence of ‘extreme’ shaped grains (KSU,
470 KVE); these do in fact contain a higher proportion of particles with shape descriptors < 0.5
471 (more extreme shapes) than the more ‘normal’ HEK sample (Figure 6). However, despite
472 the presence of unusually shaped grains, mean shape values do not differ significantly
473 from bulk ash samples in other studies, although they do differ from hand-picked juvenile
474 samples of [Wilson and Huang \(1979\)](#) (Figure 7). This suggests that the range of values
475 we observe for elongation, flatness, sphericity and the Dioguardi shape factor are valid
476 as ‘average’ values for modelling purposes even for eruptions which produce unusual ash
477 shapes. However, we note that we do not consider the most extreme-shaped pyroclasts such
478 as Pele’s hair.

479 6.3. *Using shape in dispersion models*

480 The drag equations of [Ganser \(1993\)](#), [Bagheri and Bonadonna \(2016b\)](#) and [Dioguardi
481 et al. \(2018\)](#) produce reasonable estimates of terminal velocity, which are more accurate
482 than a spherical assumption, for volcanic ash and lapilli particles in the range 1.0 – 2.6
483 mm. The schemes are all valid for a wide range of flow regimes, and the [Ganser \(1993\)](#)
484 and [Bagheri and Bonadonna \(2016b\)](#) drag equations are accurate for low-*Re* analogue par-
485 ticles (equivalent to volcanic ash from 1 μm to 1 mm in diameter; [Saxby et al. \(2018\)](#)). All
486 are therefore suitable for use in atmospheric dispersion models used to produce operational
487 forecasts of distal volcanic ash dispersion, which are usually initiated using a range of par-
488 ticle size classes. Of the three drag equations, the one chosen for inclusion in a dispersion
489 model will most likely depend on the shape data available, as they are functions of different

490 geometric shape descriptors. Although the [Ganser \(1993\)](#) law produces the lowest average
491 error on terminal velocity for our dataset (19%), it requires a specific imaging resolution
492 range for surface area measurement. We consider the [Dioguardi et al. \(2018\)](#) law equally
493 accurate given its similar error (22%). In addition, if we calculate the Dioguardi shape
494 factor using the approximation given in Equation 7, it is solely a function of the three prin-
495 cipal axis lengths of a particle and so results are independent of imaging resolution for the
496 range investigated. The [Bagheri and Bonadonna \(2016b\)](#) drag equation produces a similar
497 error (24%) for the data in this study; it uses the shape descriptors elongation and flatness,
498 which vary less between eruptions and size classes than the shape descriptors of [Ganser](#)
499 [\(1993\)](#) and [Dioguardi et al. \(2018\)](#). Therefore, we recommend the use of the [Bagheri and](#)
500 [Bonadonna \(2016b\)](#) drag equation where it is convenient to assume a constant shape value
501 across all size fractions of the PSD.

502 For volcanic ash we found that an imaging resolution of $10^2 - 10^5$ voxels per parti-
503 cle is required for determining surface-area-dependent shape parameters for accurate drag
504 calculation. This range may extend to higher resolutions if particles are smoother.

505 We suggest that shape-dependent drag equations should also be evaluated for mod-
506 elling the transport of other non-spherical atmospheric particulates of a similar size range
507 to volcanic ash, including desert dust ([Chou et al., 2008](#)), wildfire embers ([Anthenien et al.,](#)
508 [2006](#)), pollen and spores ([Schwendemann et al., 2007](#)).

509 **7. Recommendations for including ash shape in dispersion models**

510 We present a table of default shapes to be used with shape-dependent drag equations for
511 modelling atmospheric ash concentrations and travel distance (Table 2). The shape ranges
512 given are based on the mean and standard deviation of ash shape (Figure 7). [Bagheri and](#)
513 [Bonadonna \(2016a\)](#) give the extremes of a shape range for volcanic ash: flatness = 0.07 –
514 1.0 and elongation = 0.24 – 1.0. Although we do not consider the full shape ranges, mini-
515 mum shape in a sample will affect the maximum travel distance of the ash particles (for a
516 given size). In the case of very far-travelled ash, including tephra preserved as non-visible
517 horizons in sediment sequences (cryptotephra), unusual shapes can allow grains to travel
518 significantly further than spherical equivalents (e.g., [Stevenson et al., 2015](#)), creating a dis-
519 crepancy between the measured and modelled travel distance of cryptotephra grains ([Saxby](#)
520 [et al., in press](#)). Although extreme shapes may be relevant in considering the transport of
521 individual grains, our recommendations based on mean shape are aimed at forecasting ash
522 cloud location and concentration. The drag equations given all produce more accurate ve-
523 locity estimates for volcanic ash particles than a spherical assumption; they are suitable for
524 forecasting the dispersion of ash in particle size ranges typically modelled by VAACs; and
525 our shape data are calculated from axis lengths, which are insensitive to image resolution,
526 or surface area at resolutions we find to be effective for drag calculation. We recommend
527 the use of these shape values as defaults in place of a spherical approximation in volcanic
528 ash dispersion models.

529

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Table 2: Recommended default shape descriptors to be used with shape-dependent drag equations in volcanic ash dispersion models.

Drag equation	Shape descriptor	Shape equation	Particle size fraction	Shape range
Ganser (1993)	Sphericity	Sphericity: $\Psi_G = \frac{\pi^{\frac{1}{3}}(6V)^{\frac{2}{3}}}{A_{\text{surf}}}$, or sphericity of an approximate ellipsoid: $\Psi_e = \frac{\pi^{\frac{1}{3}}(6V)^{\frac{2}{3}}}{A_e}$	$< 10^2 \mu\text{m}$	0.3 – 0.8
Bagheri and Bonadonna (2016b)	Elongation e and flatness f	$e = \frac{l}{L}, f = \frac{S}{l}$	$10^2 \mu\text{m} - 10^4 \mu\text{m}$ $< 10^4 \mu\text{m}$	0.1 – 0.8 Elongation: 0.4 – 0.9 Flatness: 0.6 – 0.9
Dioguardi et al. (2018)	Dioguardi shape factor Ψ_D	$\Psi_D \approx 0.83\Psi_e$ where $\Psi_e = \frac{\pi^{\frac{1}{3}}(6V)^{\frac{2}{3}}}{A_e}$	$< 10^2 \mu\text{m}$ $10^2 \mu\text{m} - 10^4 \mu\text{m}$	0.2 – 0.5 0.1 – 0.7

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