Examining Undergraduate Student Retention in Mathematics Using Network Analysis and Relative Risk

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Higher education faces challenges in retaining students who require a command of numeracy in their chosen field of study. This study applies an innovative combination of relative risk and social network analysis to enrolment data of a single cohort of commencing students from an Australian regional university. Relative risk, often used in epidemiology studies, is used to strategically investigate whether first year mathematics subjects at the university demonstrated a higher risk of attrition when compared to other subjects offered in the first year of study. The network analysis is used to illustrate the connections of those mathematics subjects, identifying service subjects through their multiple connections. The analysis revealed that attrition rates for eight of the nine subjects were within acceptable limits, and this included identified service subjects. The exception highlighted the issue of mathematics competencies in this cohort. This combined analytical technique is proposed as appropriate for use when investigating attrition and retention at faculty and institutional levels, including the determination of levels of intervention and support for any subject.

Introduction

Mathematics has become crucial to many aspects of modern life, with a wide range of tertiary-level competencies in mathematics and related areas required for the maintenance of industrialized societies [1-3]. In higher education, however, an increasing number of graduates are presenting to the workforce without the requisite mathematics knowledge and skills on which modern economies depend for sustainability and development [4]. Reports from the Office of the Chief Scientist in Australia and related bodies [3,5,6] summarize such research, and argue that industrialized nations have stagnated in producing students and teachers of mathematics.

The problem for higher education institutions, such as universities and community colleges, is related to subjects or programs[1] that have no pre-requisite or assumed quantitative skills. Thus, students from diverse backgrounds enter their tertiary mathematics studies with vastly differing competencies and struggle to successfully complete even preliminary mathematics subjects [7-9]. This means that attrition rates from mathematics subjects in some programs may be higher than from those mathematics subjects in other programs, with students who attempt mathematics subjects withdrawing, completing but failing, or changing programs. The students undertaking these programs may also have broader issues related to equity and diversity, and engagement [10,11] and these may act in combination in exacerbating mathematics difficulties.

This problem is made more complex by a growing recognition that enrolment is student driven, at least in some subjects, and related to personal agency and life choices [12]. Students entering into undergraduate degree or diploma programs that have mathematics components, are increasingly motivated by a perceived need to obtain educational experiences in mathematics that lead to a variety of educational and social goals. These goals may not necessarily align with those of employers in the industry marketplace that require particular mathematical knowledge, skills and experiences. When a student's belief that these goals are attainable conflicts with the assumed competencies in the subjects they have selected [13,14] poor retention and high attrition may result.

Some higher education institutions respond to high attrition rates in mathematics by directing effort and resources to students identified as being vulnerable to withdrawing [15,16]. Mathematics support may take differing forms, including being delivered before,

[1] The term program is used here as an umbrella term for groups of subjects that may be part of a larger course or that may form an entire course. For example, a student may undertake one of more mathematics subjects as a program as part of a Health Sciences Degree (Nursing). The term subject is used here to denote a one-semester unit of study, where students would generally undertake four subjects per semester.

in parallel to, or embedded in subjects [17]. Mentoring and other motivational or study skills programs may also be offered. Examples of this are the blend of teaching, support and pedagogic research outlined in Croft et al. [8], and the Mastery Learning Program that "endorses the belief that all students can learn and achieve the same level of content mastery when provided with the appropriate learning conditions (including time)" [18, p. 142].

A challenge for universities is how to use evidence to make wise decisions about the use of available resources in a manner that will optimize outcomes in the support of students who are considered at risk of attrition from their mathematical studies. It seems crucial that intervention occurs in a timely manner, before a student fails and with the support program provided at a time when it can be of benefit [17]. Attrition is disadvantageous to students due to costs and lost opportunities; to the university and to society more generally. In n mathematics this cost is estimated also in terms of loss of self-esteem, and student and staff satisfaction [8]. Increased retention may enhance a university's reputation, lead to beneficial cost and operating efficiency strategies and better student placement in future employment [19,20].

This paper presents an innovative approach to the strategic, high-level analysis of data captured by universities in the normal course of academic activities to allow for timely interventions to support at-risk mathematics students. The paper commences with a consideration of the importance of mathematics to a wide range of university programs and to the community at large. A short summary of how network analysis and relative risk analysis is commonly used is then presented. This is followed by an outline of how these techniques were used in the analysis of mathematics attrition data for students enrolled in the first year of undergraduate study, in this case for the 2014 commencing cohort at the study university. The paper concludes with a discussion of the opportunities presented in examining mathematics student retention and attrition by a combined approach of these two analytical techniques.

The Importance of Mathematics in Higher Education

In industrialized economies, the development of the subject of mathematics has followed industrial and technical development and associated economic change and has been driven by a demand for numerate or mathematically literate people [21]. In educational institutions, such as schools and universities, this demand has led to the development of the subject of mathematics described, typically, in categories that relate the application of mathematical ideas to the solving of problems in a variety of differing real-world areas, such as taxation and commerce, land measurement and astronomy, and to measurement of change more generally [22].

Expertise in tertiary level mathematics, and the mathematical sciences[2] more generally, has become important economically. The Deloitte Report [23], for example, argued that mathematical sciences contributed to 16% of the UK gross value added, and a similar value has been given for other countries such as Australia [24]. Further, modelling by Price Waterhouse Coopers [20] suggests that shifting just one per cent of the workforce into science, technology, engineering and mathematics (STEM) roles would add $57.4 billion to GDP (net present value over 20 years). Hanushek and Woessmann [25] have suggested additionally that the cost of not having mathematics competencies could account for a significant proportion of lowered GDP.

Modern higher education requires a wide range of mathematical competencies in programs such as business, nursing and education, but the mathematics background of

[2] The term ‘mathematical sciences is used to encompass mathematics, statistics and the range of mathematics-based disciplines including teaching, teacher education and educational research.

students entering these programs indicates a lack of preparedness for the mathematics involved [8,18]. Support for mathematics teaching and student learning is a core concern in programs across higher education institutions worldwide [7,24,25]. This concern is evident not only in mathematics as a discipline, but in programs that utilize mathematics in some way, notably, for example, in the fields of science, education, engineering, nursing, psychology and business [8]. As well, there is an under-representation of women in the mathematical sciences, despite demand for women to fill positions in the workforce [24,27].

Mathematics[3] is an important and pivotal area of study for many students across a range of academic disciplines. Enrolment patterns for mathematics indicate that some subjects are offered across a number of undergraduate degrees, often from a single department. These so-called service subjects, which are usually completed in the first year of undergraduate study, are completed by students from a range of disciplines and often involve a diverse range of student backgrounds and abilities [28]. Many introductory mathematics subjects at universities and colleges are service subjects and hence are of interest in studies of support and retention. Service subjects may serve as 'flash points' in a degree that may have high failure rates. If these subjects, however, are situated in a network of degree subjects, then the pattern that emerges from the network may help in understanding enrolment and attrition.

There has been on-going international discussion regarding the need for a rethink and redesign of mathematics teaching and learning at the university and college level in order to cater for the weak mathematics foundation of some university students [8,29-32]. Many undergraduate programs of study require the completion of at least one mathematics

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[3] We recognise that the situation with statistics may similar.

subject for successful degree or program completion, even though this may only be at an introductory level. Additionally, online learning is now a standard method of instruction at colleges and universities [33]. Thus, it is important to have a means by which attrition risk across the range of subjects being studied, and the attendance modalities of subject offerings, may be appropriately compared to allow for the identification of the most appropriate points for intervention.

Analysis of Undergraduate Mathematics Enrolment and Retention

A number of different quantitative and qualitative methods have been used to examine undergraduate mathematics enrolment and retention, and related concerns. King and Cattlin [9], for example, used Nvivo to analyse semi-structured interviews to build a picture of the role of first-year coordinators and the mathematics programs at 26 Australian and New Zealand universities, successfully shedding "light on the realities of a system that in fact may be too flexible" (p. 1032). Advanced statistical techniques have been used in a number of projects. For example, Peters [26] used hierarchical linear modelling to examine relationships of undergraduate mathematics students in relation to achievement, self-efficacy and classroom climate. Two key findings were that students with high self-efficacy in mathematics also had high achievement in mathematics, and that this was correlated with teacher-centered classrooms.

Dias, Cunningham and Porte [34] also used advanced statistical analysis to examine the effect of supplemental instruction on a student cohort's results. The study used both sample versus population mean analysis and proportional analysis, with a student cohort separated into two groups: those who were enrolled; and, those who also took the exam (given high attrition rates during the course). They [34] found that supplemental instruction had a positive impact in fostering academic performance in mathematics, even though there was no significant impact of this strategy on retention in the course itself. An additional approach based on statistical techniques, has been the use of binary logistic regression models, including calculation of odds ratios. Hachey, Wladis and Conway [38], for example, used odds ratios, standard errors, and significance levels to argue for prior experience and GPA as predictors of outcomes in online STEM programs.

In addition to conventional statistical methods, multifactorial analytics have been used for retention studies. As part of a study on student retention in an Australian university, Author and Farr-Wharton et al. [35-37] used multifactorial methods, including social network analysis and structural equation modelling (SEM), to analyse enrolment data of an entire undergraduate cohort. In the SEM study of undergraduate survey data across disciplines, Farr-Wharton et al. [37] showed students' levels of engagement and course satisfaction fully mediated lecturer-student relationships and intention to leave university prematurely, when demographic and socio-economic factors were controlled for. Author [35,36] used social network analysis to show that retention could be examined using a conceptualisation of student social ecologies, highlighting how factors related to risk were connected to other factors in a student's lived experience at a particular institution.

The current article focuses on utilising relative risk, in combination with a social network analysis, arguing that such combinatory analyses of statistical and multidimensional approaches may be of considerable use in examining enrolment data. The article focuses on the following two research questions.

1. *Can a combination of statistical and multifactorial techniques be used to examine enrolment patterns related to risk of attrition (failure or non-completion of a subject)?*
2. *How can such a combined analytical method be of use in determining intervention processes for mathematics student cohorts with broad competencies?*

Relative Risk

Relative risk has not been used widely in studies of mathematics education, but offers easily calculated and simple tests of statistical significance and confidence, with relative risk easier to interpret than the related odds ratios [39,40]. With regard to relative risk, Stacy and Steinle [39, p. 703] argue:

> There are several advantages, which relate to the ease of interpreting the change in risk and the way in which it provides an alternative presentation of results in possibly a more memorable form, and in a form which highlights the real meaning of differences which in absolute terms appear to be small.

Relative risk has been used extensively in epidemiological studies, since it provides an easy to understand comparison of how the risk of occurrence of a target outcome in one group compares to that of an independent group [41]. In epidemiological studies, these groups are described as the treatment group and the non-treatment group. For the purposes of the present study, relative risk was calculated using:

$$(1)\ \ Relative\ risk = \frac{p_1}{p_2}$$

where p_1 is the probability of attrition from a specific subject and p_2 is the probability of attrition for students who are not in that subject [42]. By way of example, if the attrition rate for the subject MAT01 is 12% and the attrition rate in ENG01 is 8%, then the relative risk would be calculated as:

$$(2)\ Relative\ risk = \frac{p_{MAT01}}{p_{ENG01}} = \frac{0.12}{0.08} = 1.5$$

This indicates that the risk of attrition for a student in MAT01 is 1.5 times greater than that for a student in ENG01. It should be noted that a relative risk of 1 indicates that the risk, and rate, of attrition in both subjects is the same. The approach using relative risk also allows for a single risk value to be calculated for a given subject when compared to failure across all other subjects. This provides a single value to represent the attrition risk for a subject, which in turn provides a simple way of comparing attrition risk across a wider range of subjects.

Social Network Analysis

In the past two decades social network theory (and its close companion, graph theory) has provided a useful means of examining interactions in numerous contexts, where the complex organisation within groups develops through the internal interactions of its members [43]. Studying the underlying network structure of systems has been a useful methodology as many features of complex systems arise from the basic elements and the underlying network structure that make up a system, rather than specifics of the objects and interactions [44].

Network theory provides a robust framework for interpreting the patterns of interactions within a complex system, both at the level of the individual actor and at broader levels that may include the entire system. This theory is typically applied as social network analysis (SNA, sometimes called network analysis), where the system is reduced to a set of actors (or actants) called nodes and a set of relationships called edges that link the nodes together [45]. SNA takes system elements and social network structures, and their relationships, as the fundamental unit of analysis, rather than individuals [43].

SNA has formed the basis of examinations of systems and connections in differing fields, such as in studies of transport systems and economic growth [43,44], but has only recently been applied in studies of retention and educational quality [35,36,46]. The exploration of networks and the connectivity of nodes within them based on empirical data, has developed rapidly in recent years, largely because the rules governing the relationships within such networks remain independent of the nature of the subjects being linked [45] and because of rapid advances in software for analysing large data sets [47].

Network analysis can be presented as suites of diagrams illustrating connections, based on selected weighting criteria, and accompanying metrics. Metrics can be used to measure network features, such as *Centrality*, *Density* and *Degre*e. *Centrality* is used to measure the extent to which the network activity is centred on one or a few nodes (the network core), and provides insights into where influence may be concentrated, as well as blockages and patterns of flow across the network. Common *Centrality* measures include: *Connectivity* (*Degree)*, *Betweenness* and *Closeness* [45]. *Betweenness*, for example, is related to the number of connections between two nodes, but rather than measuring how close to the centre a node is, it measures how important the node is in traversing the network.

Methodology

Data Collection

The mathematics student sample was drawn from a larger study of archived student data related to the undergraduate population of commencing domestic students at a regional Australian university [35-37]. Commencing students were considered to be those who had been offered a university placement in that calendar year and who had for the first time enrolled in at least one subject at the university, but did not exclude students who had transferred from one program to another. Students were considered as retained if they did not fail or withdraw from the subject in which they were enrolled.

A subgroup of mathematics students (n = 837, approximately 21% of the cohort, n = 4065) was drawn from a cohort comprising a single calendar year of enrolment. The cohort included both mature-aged students and students who had left high school at the end of the previous calendar year and comprised largely first-year students. The data set used for this analysis includes information on withdrawal and failure from all subjects in which students in the cohort were enrolled. The characteristics of the mathematics subgroup (those students enrolled in a mathematics subject) were compared to the overall student cohort and to the subgroup of non-mathematics students (those not enrolled in a mathematics subject) that continued their studies (n = 3228). The study did not identify those commencing students who were repeating a subject after transferring from other degree courses, although, due to the small numbers of these students, this was not expected to have any significant impact on the analysis conducted here.

Relative Risk—the Mathematics Cohort

While the relative risk of attrition is easy to calculate when comparing two specific subjects, as shown in equation (2), the calculation of the relative risk of attrition for a given subject compared to that for all students who are not in that subject is rather more complicated. In the situation described previously for MAT01, where the attrition rate was 12%, the risk of attrition was 0.12. The risk of attrition for all students who are not in MAT01 would be calculated as:

$$(3)\ \ Risk\ attrition_{Not\ in\ MAT01} = \frac{Total\ number\ of\ attritions\ for\ all\ units\ other\ than\ MAT01}{Total\ number\ of\ enrolments\ for\ all\ units\ other\ than\ MAT01}$$

The relative risk of attrition for students in MAT01, compared to all other students not enrolled in MAT01 is then:

$$(4)\ Relative\ risk\ attrition_{MAT01} = \frac{p_{MAT01}}{p_{Not\ in\ MAT01}} = \frac{0.12}{p_{Not\ in\ MAT01}}$$

A relative risk is considered significant if the 95% confidence interval does not overlap 1 [47]. This approach to the analysis of attrition data allows for comparisons that are more statistically robust than the more conventional approaches using raw percentages.

Social Network Analysis—the Mathematics Cohort

The network analysis software UCINET v6.509 [48] was used to analyse a matrix of data obtained in the larger study, with diagrams produced using Netdraw [49]. The adjacency matrix used for analysis was constructed with individual students as rows and subcategories as columns, related to factors drawn from archived data in three categories: demographic (e.g., gender, disability, socioeconomic level); academic (grade point average, attendance mode); and, engagement (mentoring program, learning management system logons) (for detail of data sourcing see [35]). The data matrix, therefore, was extremely large, with over 4000 rows and about 1500 columns after categorization and data cleaning was completed. Entries in the matrix were coded as presence=1 and absence=0 for each student intersection with a subcategory.

For the results presented in this article, the matrix included an additional coding requirement in order to differentiate individual student enrolment (by student ID) in the nine, mostly first-year or introductory mathematics subjects against the remaining 575 subjects. Enrolment in a subject was considered to be an attribute of the student. The nine mathematics subjects (numbered for convenience as MATH1 to MATH9) being considered here are each offered independently as stand-alone subjects that could be undertaken within a number of programs offered across a number of Schools. For example, the MATH5 subject could be undertaken as a component of one of several programs, such as Education and Nursing. Mathematics is offered as a single-semester subject in either online or face-to-face attendance modalities across three campus locations, with some subjects offered also using a combination of both as a blended modality.

Results and Discussion

The software used here was able to analyse enrolment data and produce a network diagram where lines (edges) between subjects (nodes) represent common student enrolments. Criteria can be used to show links, for example, only for those subjects that share more than a nominated number of students as a form of weighting, hence generating a suite of diagrams depending on weightings selected. Figure 1 shows an example of a student enrolment diagram created from the data matrix using Netdraw. This network illustrates the connections between subjects, with the mathematics subjects represented by circles (red) and non-mathematics subjects represented by squares (blue). In this diagram, connections (edges) between subjects (nodes) are only shown where a student was enrolled in one or more mathematics subjects.

[insert Figure 1 about here]

Service subjects will be characterized in the network diagram by having a high number of connections that link different areas together. In the language of network analysis these subjects are said to demonstrate a high *betweenness* value (Table 1), which indicates that the subject is important in linking together parts of the structure that are less well connected. In the current context, these subjects may be significant in 'bridging' across schools or programs and often demonstrate high enrolments with a diverse student cohort, making them a critical target for intervention strategies.

[insert Table 1 about here]

For example, the mathematics subjects with the most connections to other subjects in Figure 1, MATH2, MATH5, MATH7 and MATH8, have high *betweenness* values (from 8138 to 1548 compared to 178 to 17 for the other five mathematics subjects) and can be identified by the network analysis as acting as service subjects. Such subjects may not necessarily be labelled as service subjects by the university (although in this case MATH5 and MATH7 are known to be service subjects)—they serve as a bridge between different parts of the network. These mathematics subjects are typical of service subjects in that they have a high enrolment, including international students, and satisfy the knowledge, skills and values requirements for a number of programs and, hence, are an important bridging subject where failing or withdrawing may have consequences for students across the broader university.

The analysis, even with links weighted for students undertaking one or more of the mathematics subjects, demonstrates that the majority of subjects (216 of 584, or 40%) form a single, continuous network. This result indicates that the schools and faculties do not operate as discrete entities, but rather share students across the cohort. It appears from this diagram that some of the highly central mathematics service subjects, those with high *betweenness* values, are linked also to highly central non-mathematics service subjects (see NON-MATH entries in Table 1, for example). This strong connection suggests that these subjects may together be critical within this university and hence be a primary target for a coordinated, strategic intervention that would have impact across the university.

Since the mathematics subjects MATH2, MATH5, MATH7 and MATH8 were highly connected across a wide range of other subjects, high attrition rates in these mathematics subjects would have ramifications across a range of undergraduate programs. As such, an effective and well-targeted intervention in these subjects, to support students and improve retention, has the potential to influence the greatest number of at-risk students. Thus, such an intervention may offer substantial rewards across the university, not just in terms of the outcomes applicable within the mathematics discipline.

Relative risk was calculated to compare the attrition in each mathematics subject against the subjects undertaken across the full commencing student cohort. There were 216 subjects from which no students withdrew, with an average enrolment of 4.16 students (standard deviation 5.65, range 1.34). These subjects were not considered in the relative risk analysis. The remaining 368 subjects had an average student enrolment of 55.89 students (standard deviation 48.48, range 1.632). Table 2 summarizes the attrition data for all of the first-year mathematics subjects at the institution. As an example of the calculation, the relative risk of attrition for students in MATH2, compared to all other students is:

$$(5)\quad Relative\ risk\ attrition_{MATH2} = \frac{p_{MATH2}}{p_{Not\ in\ MATH2}} = \frac{49/(49+124)}{1013/(1013+2879)} = 1.088$$

In total, the analysis identified 123 of the 368 subjects as presenting a relative risk greater than 1 and, of these, three were mathematics subjects, MATH2, MATH4 and, MATH6. The relative risk value of just above 1 for MATH2, however, is not statistically significant (p < .05).

<<insert Table 2 about here>>

The relative risk confidence intervals are graphically represented in Figure 2 to assist with interpretation.

<<insert Figure 2 about here>>

The relative risk value for MATH4 was the highest (compared to other subjects) and statistically significant—the risk was over twice that which would be expected, with a small confidence interval. This subject, which caters almost exclusively to students enrolled in initial teacher education programs, appears to be an appropriate target for intervention in collaboration with education staff. To assist with interpretation of the relative risk statistics, Table 3 presents a further demographic breakdown for the mathematics subjects. Table 3 shows that MATH4 had a high enrolment (315), with a greater than 50% attrition. It is of interest that this subject also had high numbers attending in face-to-face rather than online modalities.

<<insert Table 3 about here>>

The relative risk values above 1 for MATH2 and MATH6 (the next two highest values after MATH4), while not significantly above 1, indicate that these subjects had attrition rates that are comparable to attrition rates in non-mathematics subjects across the wider university. Of the two, only MATH2 had a high enrolment (173, see Table 3), although this was predominantly internal rather than distance mode, offered in the health sciences. The network analysis of enrolment (Figure 1) identified MATH2 as a service subject based on a high *betweennness* value (1554) and centrality within the network and it appears that the service requirements of the department offering this subject are being met. While the relative risk value of just above 1 was not significant in this analysis, the confidence interval overlaps the relative risk value of 1. On the basis of its identification as a service subject, MATH2 would be the most appropriate subject for a further in-depth analysis of attrition patterns and the potential for intervention. On the other hand, MATH6 has few connections outside of a localised area of the network diagram, has a low

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betweenness value and is not considered a service subject. It has only a low enrolment (15) and would not be a major target for anything other than a localised, within-subject intervention since it appears to be comparable in terms of retention to non-mathematics subjects across the university.

MATH7 and MATH8, both service subjects in the network analysis, demonstrated relative attrition rates well below one (< 0.5), with a relatively small confidence interval, and intervention action in these subjects, on this analysis, would not be considered as warranted. Table 3 indicates that both subjects have students enrolled in distance and blended modalities, with MATH8 also having students enrolled in internal mode.

In this analysis, the relative risk values of MATH1 indicated also a low risk of attrition, with a small confidence interval, but it was not considered a service subject. The network diagram indicates that MATH1 is connected to a small and specific group of subjects (*betweennness* value 178), in keeping with its environmental focus. MATH1 was undertaken primarily in blended or distance attendance modes (Table 3). MATH9 also has a low relative risk value, but has a large confidence interval, and this may be a sign that further investigation is required to determine how this subject is functioning, but the large interval could be argued to be a result of the low number of students enrolled (24) and their roughly equal distribution across online and blended study modes. Considering these two types of analysis together suggests that these latter two subjects may not differentiate sufficiently well at this level and may be in need of curriculum revision, or improved assessment processes.

The two remaining subjects, MATH3 and MATH5 have relative risk values that lie just below one, indicating that the relative risk of attrition is close to that of non-mathematics subjects. The confidence interval for MATH3, however, is one of the largest of the nine subjects and overlaps the relative risk value of 1. As with MATH9, however, this interval length for MATH3 may be a result of the small number of students in this subject (15). The network diagram indicates that this subject is connected to only few other subjects and this is supported by its low *betweenness* value (135). MATH5, by way of contrast, has high enrolment (230) across all three attendance modalities and is a service subject centrally located with a high *betweenness* value (8138). It has a small confidence interval and results of the two analyses together indicate that this subject may be functioning well for this cohort.

Conclusion

Based on the analysis of data it was concluded that the overall attrition across first year mathematics subjects, particularly those subjects serving programs from various departments and faculties of the university, was generally within acceptable limits for the cohort of students targeted. A significant exception was MATH4, which is offered only to initial teacher education students at this institution. This finding confirms previous research that indicates that pre-service teachers enter programs which require broad competencies, but with limited mathematics knowledge and skills [50,51]. Any generalisation of this result to teacher education programs more generally, however, may be unfounded, since this style of analysis is designed for use in examining particular cohorts at particular places. It could be argued, based on this result, that MATH4 at this particular institution may benefit from a review taking into account future examination of combinations of factors that may contribute to student success [36], or advice regarding program and subject suitability, including any difficulty in having this subject as part of their goal orientation and career objectives [35, 36].

The results indicate that potential exists for longitudinal studies that examine the relative risk of mathematics and other subjects that are connected in the enrolment pattern presented in Figure 1. Such studies may offer a perspective on variation in relative risk as it relates to subject connections across different cohorts in different years. Such longitudinal studies may also shed light on subjects that have low attrition and low risk, such as MATH1, to ensure that the quality of the offering is comparable across the university in each cohort in each year.

This paper has demonstrated the use of an innovative and statistically robust technique for analysing enrolment and attrition data suitable for consideration at the university level, for example at examination boards and curriculum reviews. One advantage of the technique includes the ease of access to data, as it is captured in the normal course of activities by universities and is readily available from databases used to record student enrolment and achievement. The technique also provides a statistically sound analysis of data at the school, faculty or university level. Relative attrition risk analysis demonstrates a statistical robustness that traditional approaches using comparisons of raw percentages do not provide, largely through the availability of confidence intervals and measures of statistical significance.

Network analysis allows for an easily interpreted visual representation of student enrolment patterns, while the relative attrition risk values present a single value for each subject that allows for a comparison of attrition in a subject with what is happening across the university. The two techniques demonstrated here for an analysis of mathematics subjects could be used for all subjects across the university and would be most suitable for use at higher administrative levels, including across disciplines within the university, providing high level managers with the sound empirical evidence required for strategic management decisions.

This combined analytical technique may prove particularly useful when investigating attrition and retention at faculty and institutional levels, including the determination of levels of support for subjects with high attrition rates and the levels of intervention in service subjects. Users of this technique, however, need to bear in mind that while it has an advantage in being student-centred and place-based, this may raise problems in any attempt to generalise across cohorts of across institutions.

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Table 1. *Betweenness* values for the nine mathematics subjects and the highest ten of the 207 non-mathematics subjects connected in Figure 1. Of the connected 216 subjects, 85 had a *betweenness* value of 0.

***Betweenness* value**	**Subject**
8138	MATH5
4142	MATH7
1918	NON-MATH
1884	NON-MATH
1554	MATH2
1548	MATH8
1482	NON-MATH
1452	NON-MATH
1031	NON-MATH
865	NON-MATH
863	NON-MATH
750	NON-MATH
670	NON-MATH
561	NON-MATH
178	MATH1
150	MATH9
138	MATH3
26	MATH6
17	MATH4

Table 2. The relative risk of attrition for each first year mathematics subject.

Subject	No. of students continuing in subject	No. of students withdraw in subject	No. of students continuing not in subject	No. of students withdraw not in subject	Relative Risk Attrition	95% Lower Bound	95% Upper Bound	Significance
MATH1	58	4	2945	1058	**0.244***	0.094	0.631	0.0036
MATH2[su]	124	49	2879	1013	1.088	0.853	1.387	0.495
MATH3	12	3	2991	1059	0.765	0.278	2.101	0.604
MATH4	146	169	2852	898	**2.240***	1.992	2.520	<0.0001
MATH5[su]	183	47	2820	1015	0.772	0.595	1.002	0.052
MATH6	11	4	2992	1058	1.021	0.440	2.367	0.962
MATH7	43	5	2960	1057	**0.396***	0.122	0.909	0.029
MATH8[su]	47	4	2956	1058	**0.298***	0.116	0.763	0.012
MATH9	24	2	2979	1060	0.293	0.077	1.111	0.071

[su] Mathematics service subject * Statistically significant $p < .05$

Table 3. Demographic information about each mathematics subject presented as Continuing [Withdrawn].

Mathematics Subject	**Demographic factor**										
	Cont [With]	Under 21	Female	NESB	ATSI	Disabled	Mid SES	Low SES	Distance mode	Blended mode	Internal mode
TOTAL	561 [276]	218 [129]	303 [193]	8 [4]	23 [13]	38 [16]	375 [196]	137 [63]	101 [19]	191 [79]	269 [178]
MATH1	58 [4]	20 [2]	31 [1]	1 [0]	1 [0]	4 [0]	36 [4]	11 [0]	22 [0]	33 [4]	3 [0]
MATH2	124 [49]	49 [11]	70 [35]	2 [1]	6 [2]	9 [5]	82 [31]	36 [12]	0 [0]	31 [15]	93 [34]
MATH3	12 [3]	4 [2]	0 [0]	0 [0]	2 [0]	2 [0]	10 [2]	0 [0]	0 [1]	7 [2]	5 [0]
MATH4	146 [169]	91 [76]	109 [136]	2 [1]	7 [8]	9 [10]	102 [121]	37 [42]	2 [4]	45 [50]	99 [115]
MATH5	183 [47]	65 [21]	71 [17]	2 [2]	7 [2]	12 [1]	125 [35]	41 [7]	62 [11]	55 [8]	66 [28]
MATH6	11 [4]	1 [0]	7 [4]	0 [0]	0 [0]	1 [0]	5 [2]	4 [2]	4 [2]	1 [1]	6 [1]
MATH7	43 [5]	10 [2]	19 [3]	0 [0]	1 [1]	2 [1]	29 [5]	8 [0]	11 [1]	32 [4]	0 [0]
MATH8	47 [4]	17 [4]	14 [0]	1 [0]	2 [0]	2 [0]	33 [3]	8 [1]	11 [0]	20 [2]	16 [2]
MATH9	24 [2]	1 [1]	14 [0]	1 [0]	0 [0]	0 [0]	15 [2]	4 [0]	12 [1]	12 [1]	0 [0]

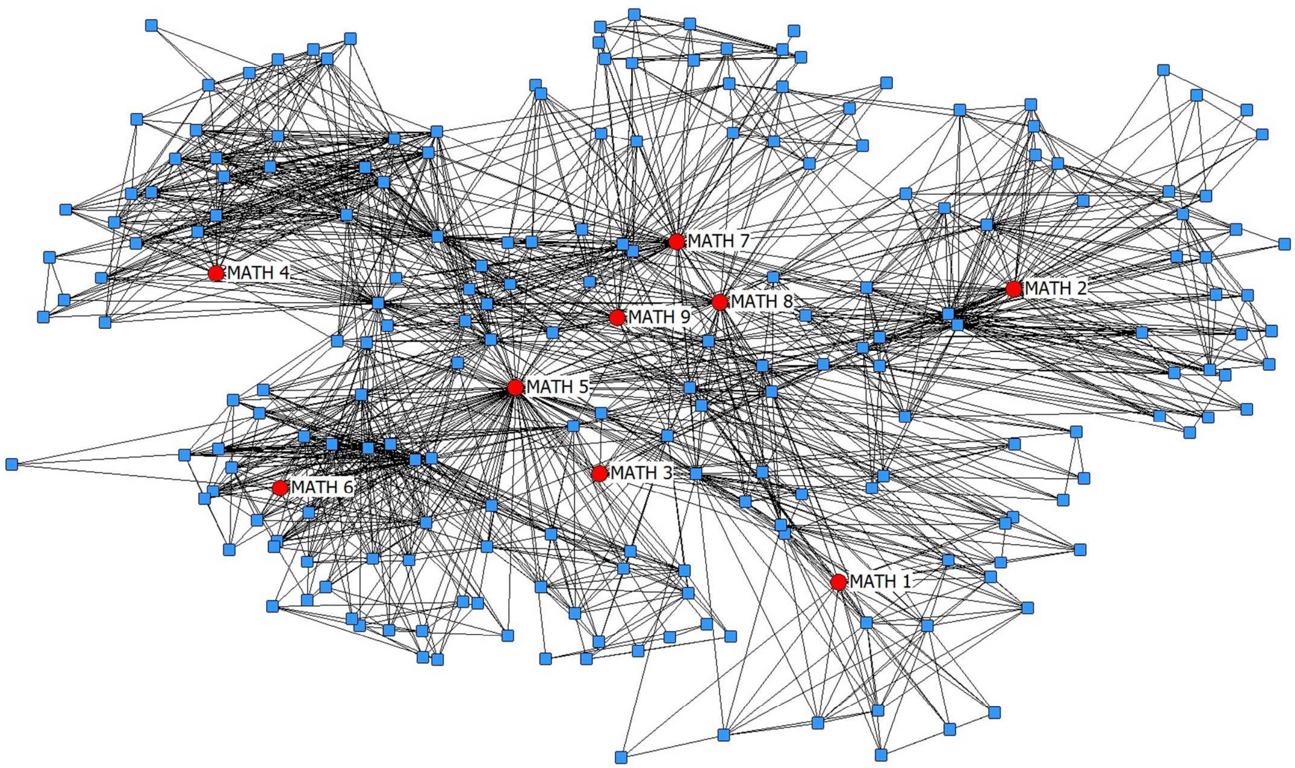

Figure 1. The mathematics student enrolment network at the study university. The circles (red) represent the first year mathematics subjects, while the squares (blue) represent non-mathematics subjects offered across the university in first year. Connections are shown only where a student was enrolled in one or more mathematics subjects.

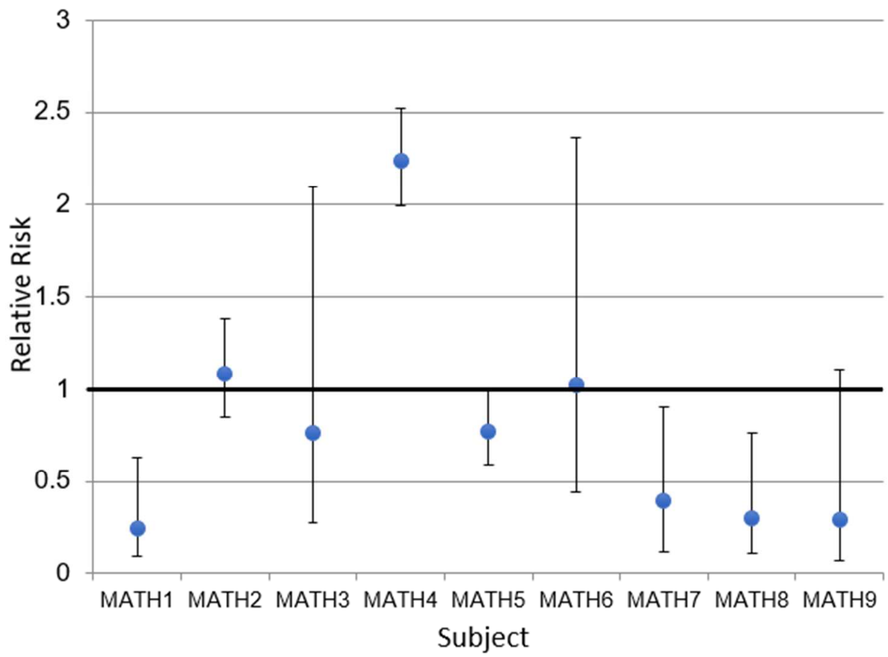

Figure 2. Graph representation of relative risk confidence intervals showing upper and lower bound.

Figure 1. The student enrolment network at the study university. The red squares represent the first year mathematics subjects, while the blue squares represent non-mathematics subjects offered across the university in first year. Connections between subjects only being shown where a minimum of 30 students was enrolled in both connected subjects.

Figure 2. Graph representation of relative risk confidence intervals showing upper and lower bound.