A Note on How to Sell a Network Good

André Veiga[1]

This version: January 21, 2018

Abstract

I consider a monopolist in an industry with positive network externalities. The firm can screen heterogeneous consumers by offering multiple products. Screening captures a greater share of consumer surplus but also segregates consumers into multiple products, thereby lowering the total network surplus. Thus, screening is socially inefficient. I show screening is never profit maximizing: the monopolist offers a single product, but at an excessive price. Thus, excessive consumer segregation is unlikely to occur in industries such as online multiplayer games, financial exchanges and messaging software.

Keywords: Network Externalities, Peer Effects, Mechanism Design, Group Design, Screening
JEL Classification Codes: D82, D85, D86

1 Introduction

Airlines offer multiple products of different qualities (e.g., first class and economy) to screen consumers.[2] However, in industries with *positive network externalities*, a product's quality is endogenously determined by its overall utilization. For instance, a financial exchange, a messaging software, and an online multiplayer game are worthless unless many users also use that product. In these industries, offering a menu of products would allow a monopolist to screen consumers, thereby capturing a greater share of consumer surplus. However, such screening would be socially inefficient because consumer segregation reduces the surplus generated through consumption externalities. This note shows that such inefficient segregation is never profit maximizing.

---

[1]Imperial College Business School, South Kensington Campus, Ayrton Rd, Kensington, London SW7 2AZ, United Kingdom; a.veiga@imperial.ac.uk. I thank Jacques Crémer, Renato Gomes, Daniel Quigley, Glen Weyl, the editor, and two anonymous referees for very helpful comments. I thank the Fundação para a Ciência e a Tecnologia (www.fct.pt) for their financial support.

[2]See Mussa and Rosen [1978], Maskin and Riley [1984], Johnson and Myatt [2003].

I consider a monopolist firm, such as an operator of financial exchanges. An exchange's quality derives from the liquidity generated by all the traders who join that exchange. Some traders ("experts") may have high valuation for liquidity, whereas others ("amateurs") have low valuations. The firm considers three possible product regimes. As a "baseline," the firm can offer a single exchange, priced such that only experts join it. A "segregated" regime would feature two exchanges, as well as prices such that experts and amateurs would join different exchanges. A "merged" regime would consist of a single exchange priced so that all traders would join it.

However, the segregated regime is never profit maximizing. To see why, consider changing from "baseline" to "segregated" (i.e., offering a second exchange to amateur traders). This change increases revenue from amateurs, who were previously excluded. However, experts must pay a lower price under "segregation," to preserve incentive compatibility. A necessary condition for profit to increase is that amateur valuations are higher than the market-share-weighted valuations of experts. Now consider a further change, from "segregated" to "merged" (i.e., merging all traders into a single exchange). The firm can no longer charge experts more than it charges amateurs. However, it can charge a higher price to amateurs, who now enjoy greater quality. In this case, a sufficient condition for profit to increase is that amateur valuations for quality are higher than the market-share-weighted valuations of experts. In sum, if "segregation" is feasible and more profitable than "baseline," then "merger" will be even more profitable than "segregation."

The model makes three main assumptions. First, offering fewer products does not increase cost, holding fixed the set of consumers being served. For instance, serving all traders with two exchanges is (weakly) more costly than serving all traders with a single exchange. Second, we assume positive within-product externalities. For instance, a trader's utility increases with the quality of the exchange she joins, but does not depend on the quality of any other exchange. Third, consumer preferences are private information. If the firm can simply prohibit experts from joining the amateur exchange, it can segregate traders without losing revenue on experts. In this case, segregation can be profit maximizing.

The result holds independently of the strength of network externalities, as long as these externalities are positive. Users can make heterogeneous contributions to product quality, and a given user can make different contributions to the quality of different products. A product's quality can be convex in total product utilization (increasing returns) or concave (congestion). There can be fixed costs per product offered or heterogeneous costs per individual served.

However, the result should also be understood with some caution. It explains why network industries do not use segregation as a screening device, even when segregation is their only means of doing so. However, screening may still occur when externalities are across products or markets are multi-sided (Ambrus and Argenziano [2009]), when products are not mutually exclusive (Gomes and Pavan [2011]), when externalities are not positive (Damiano and Li [2007]), or when the firm can directly choose product quality (Csorba and Hahn [2006]).

Section 2 conveys the results and intuition using a simplified model. Section 3 generalizes the results. Section 4 discusses the robustness of the results and additional connections to the existing literature. Section 5 concludes. Longer proofs are collected in the Appendix.

2 A simple model

We consider a monopoly firm. The buyer of a product with quality $q \geq 0$ at price p obtains utility $u = \theta q - p$. The outside option yields zero utility. A mass $\lambda \in (0,1)$ of buyers have type $\theta = \theta_h$, and a mass $1-\lambda$ of buyers have type $\theta = \theta_l < \theta_h$. A product's quality is endogenously determined by that product's buyers. If a mass $x \geq 0$ of buyers buys a given product, its quality is $q = \alpha x$, with $\alpha > 0$ (*positive within-product externalities*). The marginal cost is $k \geq 0$ per buyer.

The monopolist can offer one of three regimes. The *baseline* would consist of a single product, priced so that it is bought only by high-demand types θ_h. A *segregated* regime would consist of two products, priced so that each is bought by a different type. A *merged* regime would feature a single product bought by both types.[3] We will show that a segregated regime is never profit maximizing.

First, notice that "segregation" is infeasible if $\lambda \leq \frac{1}{2}$, because, in this case, the θ_l-only product would have higher quality than the θ_h-only product, so types θ_h would always prefer to join the product intended for types θ_l.[4] Moreover, "segregation" will definitely be unprofitable if $\theta_l \alpha (1-\lambda) \leq k$,because, in this case, a product bought only by types θ_l would yield negative profits.[5] Therefore, we proceed to investigate the remaining case, in which$\lambda > \frac{1}{2}$ and $\theta_l \alpha (1-\lambda) > k$.

In the baseline, types θ_h obtain quality $q_m = \alpha\lambda$. Under segregation, types θ_h obtain q_m, while types θ_l obtain $q_l = \alpha(1-\lambda)$. Under merger, both types obtain $q_h = \alpha$. Because $\lambda > \frac{1}{2}$ and externalities are positive ($\alpha > 0$), $q_h > q_m > q_l$.

In the baseline, types θ_h pay $\theta_h q_m$ because their outside option is zero. Profit is $\pi_B = \lambda(\theta_h q_m - k)$. Under segregation, types θ_l obtain q_l at price $p_l = \theta_l q_l$, because their outside option is zero. Types θ_h obtain q_m at a price p_h that leaves them indifferent between the

[3] For now, we assume the firm can choose its preferred equilibrium in case of multiplicity. At the end of the present section, we show how the firm can implement these regimes in dominant strategies.

[4] Separation of types requires that the allocation is increasing in type θ (see Salanié [1997]).

[5] Because the outside option is zero, a θ_l-only product would have price equal to $\theta_l \alpha (1-\lambda)$, which would result in negative profit.

two products: $p_h = p_l + \theta_h (q_m - q_l)$. Profit is $\pi_S = \theta_l q_l + \lambda\theta_h (q_m - q_l) - k$. Under merger, all buyers obtain q_h and pay $\theta_l q_h$. Profit is $\pi_M = \theta_l q_h - k$.

Consider a change from baseline to segregation. Profit increases by

$$\pi_S - \pi_B = \theta_l q_l - \lambda\theta_h q_l - k(1-\lambda).$$

Cost increases by $k(1-\lambda)$. The firm captures, from every buyer, the surplus gained by marginal buyers ($\theta_l q_l$). However, the firm loses, from the mass λ of types θ_h, the value of their new outside option (quality q_l), so revenue falls by $\lambda\theta_h q_l$. A necessary condition for profit to increase is that θ_l is sufficiently high, relative to the valuation and market share of experts, $\lambda\theta_h$. Formally, $\pi_S - \pi_B > 0 \Rightarrow \theta_l > \lambda\theta_h$.[6]

Now consider a change from segregation to merger. Profit increases by

$$\pi_M - \pi_S = \theta_l (q_h - q_l) - \lambda\theta_h (q_m - q_l).$$

Costs are unchanged. Again, the firm captures, from all buyers, the surplus gained by the marginal buyers, $\theta_l (q_h - q_l)$. The firm loses, from the mass λ of types θ_h, the surplus these types previously enjoyed above their outside option, $\theta_h (q_m - q_l)$.

We can now show the result. Recall that $\lambda > \frac{1}{2}$ and $\alpha > 0$ imply $q_h > q_m > q_l > 0$, so $(q_h - q_l) > (q_m - q_l)$. If this were not the case, segregation would be infeasible. By changing from segregation to merger, types θ_l experience a large increase in surplus (proportional to $q_h - q_l$), while the lost surplus from types θ_h is moderate (proportional to $q_m - q_l$). But then the condition $\theta_l > \lambda\theta_h$, which is necessary for $\pi_S > \pi_B$, is now sufficient for $\pi_M > \pi_S$. Formally,

$$\pi_S - \pi_B > 0 \Rightarrow \theta_l > \lambda\theta_h$$

$$\Rightarrow \theta_l (q_h - q_l) > \lambda\theta_h (q_m - q_l) \Leftrightarrow \pi_M - \pi_S > 0.$$

In sum, if changing from baseline to segregation is profitable and feasible, profit will increase even further by changing from segregation to merger.[7]

Social welfare is maximized by the merged regime, because this regime maximizes network surplus. Therefore, profit maximization never imposes inefficient segregation. However, it may lead to the inefficient exclusion of types θ_l, which occurs when the firm chooses the baseline regime instead of merger:

[6] Notice this condition does not depend on the strength of network effects, α. In the language of Bulow and Roberts [1989], revenue increases if types θ_l have positive "marginal revenue."

[7] Merger might lead to types θ_h being charged a higher price than under segregation, but this increase in price is not necessary for $\pi_M > \pi_S$.

$$\pi_B - \pi_M > 0 \Leftrightarrow \frac{\theta_l - \lambda^2\theta_h}{1-\lambda} < \frac{k}{\alpha}.$$

Inefficient exclusion is more likely when costs are high relative to the importance of network externalities and when significant heterogeneity exists in valuations (θ_l small and θ_h large).

The result holds for any type distribution $\lambda \in (0,1)$, any level of cost $k \geq 0$, and any strength of network externalities $\alpha > 0$. However, the assumption of asymmetric information is important. If the firm could prohibit types θ_h from joining type θ_l's product, profit under segregation would be $\widetilde{\pi_S} = \lambda\theta_h q_m + (1-\lambda)\,\theta_l q_l - k$. In this case, segregation is always more profitable than the baseline ($\widetilde{\pi_S} > \pi_B$) because $\theta_l\alpha\,(1-\lambda) > k$. Segregation would also be more profitable than merger if $\widetilde{\pi_S} - \pi_M > 0 \Leftrightarrow \theta_h > \left(\frac{2}{\lambda} - 1\right)\theta_l$. Here, segregation is more profitable because it does not require the firm to satisfy the incentive compatibility of types θ_h. Finally, I emphasize that the result does not depend on assuming $q_h > q_m > q_l$. If these conditions are violated, segregation is infeasible. If the conditions are satisfied, segregation is feasible but not profitable.

How could a firm implement its preferred regime? Positive network externalities typically imply multiplicity of equilibria.[8] This multiplicity can be addressed by allowing the firm to charge contingent prices, as in Dybvig and Spatt [1983], Weyl [2010], White and Weyl [2015]. Consider the implementation of the baseline regime. If consumers believed the product would be bought by a mass $\tilde{\lambda}$, types θ_h would be willing to pay $\theta_h\alpha\tilde{\lambda}$, and types θ_l would be willing to pay $\theta_l\alpha\tilde{\lambda}$. The firm commits to charging $\theta_h q_m$ if λ buyers join, and charging $p\left(\tilde{\lambda}\right) = \frac{1}{2}\left[\theta_h + \theta_l\right]\alpha\tilde{\lambda}$ if $\tilde{\lambda} \neq \lambda$ joins. Then, it is a dominant strategy for types θ_h to join and for types θ_l not to. In this unique equilibrium, the firm obtains the desired price $\theta_h q_m$.

3 Generalization

The current section generalizes the model above and discusses technical details. Consider a unit mass of potential users. A user's type $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$ is not contractible. Assume $\overline{\theta} > 0$, but possibly $\underline{\theta} < 0$. Types θ distribute according to the continuously differentiable CDF $F(\theta)$ with associated PDF $f(\theta)$. If type θ buys a product of quality q at price P, she obtains utility $u = \theta v(q) - P$. The outside option yields zero utility. Notice that $v(\cdot)$ is common to all consumers.

A firm offers users a regime consisting of G products. Each product is also a set of

[8] Rohlfs [1974], Caillaud and Jullien [2003], Evans and Schmalensee [2010] discuss this issue. The origin of this multiplicity is that each consumer's pay-off depends on her beliefs about the behavior of other consumers. For instance, for any $p > 0$, it is an equilibrium for no consumers to purchase, if they believe all other consumers will similarly not purchase.

consumers. Each product $g \in \{1, ..., G\}$ is characterized by a price P^g and a quality q^g. I define product $G+1$ to be those users who do not purchase, and $P^{G+1} = q^{G+1} = 0$. In an equilibrium with price-quality pairs $\{P^g, q^g\}_{g=1}^{g=G}$, the set of users joining product g is $\Theta^g \equiv \left\{\theta : g \in \text{argmax}_{z\in\{1,...,G+1\}} \theta v\left(q^z\right) - P^z\right\}$.

The network technology is captured by a function $\mathcal{Q}(\cdot)$. If a set of users Θ^g joins a product, a level of quality $q^g = \mathcal{Q}(\Theta^g)$ is implied. Let $Q^g = v(q^g)$, so that Q^g captures the product quality as it is perceived by users, already taking into account the function $v(\cdot)$. Thus, utility can be re-expressed as $u = \theta Q - P$. We assume quality is invariant to zero-measure changes in Θ^g, and normalize $\mathcal{Q}(\emptyset) = 0$. This assumption, together with Assumption 1 below, implies $Q^g \geq 0$.

The firm chooses G, an incentive-compatible product regime $\mathcal{G} = \{\Theta^g\}_{g=1}^{g=G}$, and prices $\{P^g\}_{g=1}^{g=G}$. We assume the firm can use contingent pricing to implement its preferred equilibrium in the case of multiplicity. The cost of offering product Θ^g is $C^g = C(\Theta^g) \geq 0$. The number of buyers of product g is $N^g \equiv \int_{\{\theta\in\Theta^g\}} f(\theta)\, d\theta$. Profit is $\pi = \sum_{g=1}^{g=G} (N^g P^g - C^g)$. The following assumptions are discussed further in section 4.

Assumption 1. *(Positive Network Externalities)* $\Theta^g \subseteq \Theta^h \Rightarrow \mathcal{Q}(\Theta^g) \leq \mathcal{Q}\left(\Theta^h\right)$.

Assumption 2. *(Mutually Exclusive products)* $\Theta^g \neq \Theta^h \Rightarrow \Theta^g \cap \Theta^h = \emptyset$.

Assumption 3. *(Weakly Costless Mergers)* $C(\Theta^g \cup \Theta^h) \leq C(\Theta^g) + C\left(\Theta^h\right)$.

Before presenting our main result, we introduce the following Lemma.

Lemma 1. *In any feasible (incentive-compatible) regime* $\mathcal{G}$*: (1),* $t \geq \underline{\theta}$ *exists such that the set of buyers is* $\left[t, \overline{\theta}\right]$*; (2) each product* $\Theta^g = (\theta^g, \theta^{g-1}]$ *is an interval of types; (3) allocations are monotonic* ($Q^g < Q^{g-1}, \forall g \in \mathcal{G}$)*; and (4) the price of product* g *is* $P^g = P^{g+1} + \theta^g\left(Q^g - Q^{g+1}\right)$.

Proof. See Appendix A. □

Lemma 1 implies segregation is infeasible unless high types buy higher-quality products than low types $Q^g < Q^{g-1}$. This is not an assumption, but rather a necessary feature of any incentive-compatible regime.

Proposition 1. *Profit is maximized with a single product.*

Proof. See Appendix A. The proof considers a candidate profit-maximizing and feasible regime with $G \geq 1$ products. We consider two possible types of deviations that preserve the monotonicity of Q and are therefore feasible: first, merging the X products with highest quality, and second, excluding the Y products with the lowest quality. Assuming none of these deviations is profitable leads to a contradiction unless $G = 1$. □

Proposition 1 allows for a simple characterization of profit and welfare maximization, because quality and cost are defined by the marginal type t. We define quality as $Q(t) = v\left(\mathcal{Q}\left(\left[t,\overline{\theta}\right]\right)\right)$ and cost as $C(t) = C\left(\left[t,\overline{\theta}\right]\right)$. Assume these functions are differentiable. Then, $Q'(t) \leq 0$ by positive network externalities and $C'(t) \leq 0$ because exclusion does not increase cost. Because type t is marginal, $tQ(t) = P(t)$. The number of buyers is $N(t) \equiv 1 - F(t)$, with $N'(t) = -f(t)$, and profit is $\pi = P(t)N(t) - C(t)$. Because welfare maximization also prescribes a single product, welfare is $W(t) = \int_t^{\overline{\theta}} \theta Q(t) f(\theta) d\theta - C(t)$. We discuss second-order conditions in Appendix A.3. Functional arguments are omitted for clarity.

Proposition 2. *Any interior profit-maximizing threshold type $t = t^\pi$ satisfies*

$$\underbrace{P - \frac{N}{f}Q}_{\textit{marginal revenue}} = \underbrace{-\frac{C'}{f}}_{\textit{marginal cost}} + \underbrace{\frac{Q'}{f}Nt^\pi}_{\textit{Spence distortion}}.$$

Proof. The FOC is $\frac{\partial \pi}{\partial t} = P\frac{dN}{dt} + \frac{dP}{dt}N - C' = -Pf + (Q + tQ')N - C' = 0.$ □

Marginal revenue is equated to marginal cost, net of the externalities as perceived by marginal buyers. The density of marginal buyers is f. The cost of an additional marginal buyer is $-\frac{C'}{f} \geq 0$. The markup term (usually just $\frac{N}{f}$) is multiplied by Q because, in the presence of network externalities, increasing Q makes demand more inelastic.[9] The incremental quality generated by an additional marginal participant ($\frac{Q'}{f}$) allows the firm to capture, from each of the N infra-marginal users, the increase in utility experienced by the marginal user (t^π). This last point is discussed further after Proposition 3 below.

Proposition 3. *Any interior welfare-maximizing threshold type $t = t^w$ satisfies*

$$P = \underbrace{-\frac{C'}{f}}_{\textit{marginal cost}} + \underbrace{\frac{Q'}{f}NE\left[\theta \mid t^w < \theta\right]}_{\textit{externality}}.$$

Proof. The FOC is $\frac{\partial W}{\partial t} = Q'(t)\int_t^{\overline{\theta}} \theta f(\theta) d\theta - Qtf(t) - C' = Q'(t) NE\left[\theta \mid t^W < \theta\right] - Pf(t) - C' = 0.$ □

Pigouvian taxation prescribes that price be equated to marginal cost net of the total value of the externality generated by an additional user. This externality is the increase in quality due to the inclusion of an additional user ($\frac{Q'}{f} \leq 0$). This quality is enjoyed by all N infra-marginal users each according to their preferences θ; hence, $NE\left[\theta \mid t^w < \theta\right]$.

[9] Higher Q means a wider dispersion of realized user utilities, so a given price increase implies the loss of fewer marginal users. Therefore, $\frac{N}{f}Q$ is an externalities-adjusted market power term.

The welfare-maximizing price is below marginal cost, because $Q' \leq 0$ and $E[\theta \mid t^w < \theta] > 0$; otherwise, welfare would be negative.[10] Welfare maximization prescribes serving all buyers with valuations greater than their cost, but also subsidizing the participation of some additional marginal users, because this subsidy results in positive externalities for all infra-marginal participants.

Notice also that a profit maximizer considers the preference of the marginal user (t^π), whereas a welfare maximizer considers those of all infra-marginal users ($E[\theta \mid t^w < \theta]$). This distortion was originally described by Spence [1975] and can generally lead a monopolist to over- or under-provide quality. However, a sharper result obtains in this case.

Proposition 4. *A profit maximizer under-provides quality ($t^w \leq t^\pi$).*

Proof. Because $Q(t) \geq 0$, $Q' \leq 0$, and $\frac{dP}{dt} = Q(t) + tQ'$, we have

$$\frac{\partial W}{\partial t} - \frac{\partial \Pi}{\partial t} = \int_t^{\overline{\theta}} \theta Q'(t)\, dF(\theta) - \frac{dP}{dt} \int_t^{\overline{\theta}} dF(\theta) = \int_t^{\overline{\theta}} (\theta - t) Q'(t)\, dF(\theta) - Q(t) N \leq 0.$$

□

A profit maximizer always has a greater incentive to increase t, thus reducing product quality and increasing price. In this environment, infra-marginal participants have higher valuations for quality than marginal participants, so the Spence distortion puts a downward pressure on quality (upward pressure on price).

4 Discussion

We assumed utility derives exclusively from network externalities, which rules out other screening mechanisms. Doing so shows the firm does not segregate consumers even when segregation is the only means of screening. However, this assumption also implies we cannot consider a version of the model in which externalities are absent, because consumers would not purchase any product in that case.

Relaxing this assumption would require preferences of the form $u = \theta q + \zeta \tau - p$, where θ, q are as above, τ is non-network quality, and ζ are heterogeneous valuations for τ. In this case, the firm would offer a menu of (q, τ, p) to screen consumers on the basis of both ζ and θ. When $\zeta \to 0$, Proposition 1 would apply. When $\theta \to 0$, the results of Mussa and Rosen [1978], Johnson and Myatt [2003], Salant [1989] would apply. If $\theta, \zeta > 0$, the model would be one of multidimensional screening. These models are typically intractable

[10] Recall that θ can be negative for some buyers. In particular, zero marginal cost implies welfare is maximized with negative prices and therefore with the possible inclusion of some users who dislike network externalities ($\theta < 0$), if those users exist.

(Rochet and Choné [1998]) even without endogenous product quality, and such an analysis would be outside the scope of this note.

Another alternative specification might, in section 2, define the quality of a product with x users as $q = \alpha x + \gamma$, where γ could be chosen by the firm at some cost. In this case, γ would be a perfect substitute for network quality. Such a specification is close to that of Csorba and Hahn [2006], where the firm is able to physically differentiate the products. As $\alpha \to 0$, the results of Mussa and Rosen [1978], Johnson and Myatt [2003], Salant [1989] would apply.

Assumption 1 requires only that utility is monotonic in product utilization. Users can make heterogeneous contributions to product quality, and each user can contribute differently to the quality of each product. Quality can be increasing convex in a product's overall utilization to reflect increasing returns to scale, or increasing concave to reflect mild congestion. However, this assumption is violated in the presence of strong congestion effects. For instance, in section 2, if individuals derived a strong negative utility from being in a group with more than $\max[\lambda, 1-\lambda]$ individuals, the platform would clearly have an incentive to segregate buyers. Similarly, in Damiano and Li [2007], externalities are within-product but not positive.[11]

Assumption 2 states that products are mutually incompatible. For instance, the user of one messaging service derives no benefit from the utilization of a different messaging service. This assumption is violated, for instance, for providers of cell phones, because users of different cell phone networks can still communicate with one another. The assumption is also violated if consumers can multi-home (buy multiple products). If products were compatible, the firm would have an even smaller incentive to use segregation. For instance, Gomes and Pavan [2011] consider positive network externalities with multi-homing, which allows the firm to screen without wasteful segregation. Hahn [2003], Csorba and Hahn [2006], and Csorba [2008] study the telecommunications industry, where products are mutually compatible (utility increases in the overall utilization of all products). Moreover, Ambrus and Argenziano [2009] examine screening in a two-sided market where externalities are across-side (a group of consumers on side A enjoys the externalities generated by a group on side B and vice versa).

Assumption 3 states that merging two products does not increase total cost. For instance, serving 2,000 buyers with one product is not costlier than offering two products, each bought by 1,000 individuals. This assumption holds if there is a fixed cost per product or a (possibly heterogeneous) cost per consumer served. It can also accommodate decreasing marginal costs, which are widespread in network industries. On the other hand, the firm would clearly have a mechanical incentive to segregate consumers if doing so would provide

[11] In Damiano and Li [2007], consumers care about the average type purchasing the same product.

a large cost reduction.

The article most closely related is Board [2009], which considers a general product-design problem with mutually exclusive products and externalities. Its main result is that, starting from the welfare-maximizing product regime, separating products may increase profit, but merging products does not. The present article shows that the spirit of that result is reversed under positive network externalities, because separating products never maximizes profit in this context.[12]

Salant [1989] first showed screening is feasible but unprofitable if the cost of quality is not sufficiently convex. For instance, Mussa and Rosen [1978] features price discrimination, but Stokey [1979] finds, in an apparently similar context, that price discrimination is not profitable. The reason is that Stokey [1979] considers a context where the cost is linear in the quality each consumer experiences. Such non-convex costs also occur naturally in industries with positive within-product externalities, because quality can be increased by serving the same number of consumers using fewer products. Therefore, the same "corner" solution obtains in network markets.

We have analyzed only a monopolistic model. For instance, Caillaud and Jullien [2003] and Ellison et al. [2004] consider competition between single-product firms in a two-sided market with network externalities. In our setting, we conjecture that competition would induce firms to reduce prices, but would not introduce screening. For instance, consider the model of section 2, in the limit of perfect competition (free-entry of firms). Each individual would be charged her marginal cost k. However, if all products had this price, segregating themselves could not be an equilibrium for consumers.

The assumptions of section 3 apply to a number of industries, such as financial exchanges, online multi-player gamers, and messaging services like What'sApp and SnapChat, which exhibit positive within-product externalities. The same reasoning applies to specialist discussion forums and other online communities.

5 Conclusion

We have considered the incentives of a profit-maximizing firm to price discriminate by segregating users into mutually exclusive products. Under the assumption of positive within-product network externalities, the profit-maximizing product regime always consists of a single product. Therefore, profit maximization does not result in the inefficient segregation

[12] Notice that Proposition 1 does not follow from Board [2009]'s Proposition 3. That result relies on assumptions on $\mathcal{Q}(\cdot)$, which are not satisfied in this environment, and only gives conditions for locally incomplete separation of types. These conditions might hold at every point without this implying global pooling, which is the result of this paper. Indeed, Board [2009]'s Example C and Figure 3 show just such a case.

of users. However, a profit maximizer typically charges excessively high prices. The results suggest regulation should focus on the level of prices rather than on the possibility of excessive user segregation.

These results should be interpreted with caution. They suggest an explanation for why firms in network industries do not typically use segregation as a means to price discriminate across users. However, firms may price discriminate in ways that do not involve segregation. For instance, telecommunications networks typically offer a menu of contracts that differ in their data allowance, and social networks can offer menus of premium features.

A number of open questions remain. The inclusion of other dimensions of consumer heterogeneity would constitute a significant generalization of the existing literature. An exploration of how price discrimination interacts with a firm's choice of product compatibility would also be interesting.

References

Attila Ambrus and Rossella Argenziano. Asymmetric networks in two-sided markets. *American Economic Journal: Microeconomics*, 1(1):17–52, 2009.

Simon Board. Monopolistic group design with peer effects. *Theoretical Economics*, 4(1): 89–125, 2009.

Jeremy Bulow and John Roberts. The simple economics of optimal auctions. *Journal of Political Economy*, 97(5):1060–1090, 1989.

Bernard Caillaud and Bruno Jullien. Chicken and egg: Competition among intermediation service providers. *RAND Journal of Economics*, 34(2):309–328, 2003.

Gergely Csorba. Screening contracts in the presence of positive network effects. *International Journal of Industrial Organization*, 26(1):213–226, 2008.

Gergely Csorba and Jong-Hee Hahn. Functional degradation and asymmetric network effects. *Journal of Industrial Economics*, pages 253–268, 2006.

Ettore Damiano and Hao Li. Price discrimination and efficient matching. *Economic Theory*, 30(2):243–263, 2007.

Philip H Dybvig and Chester S Spatt. Adoption externalities as public goods. *Journal of Public Economics*, 20(2):231–247, 1983.

Glenn Ellison, Drew Fudenberg, and Markus Mobius. Competing auctions. *Journal of the European Economic Association*, 2(1):30–66, 2004.

David S Evans and Richard Schmalensee. Failure to launch: Critical mass in platform businesses. *Review of Network Economics*, 9(4), 2010.

Renato Gomes and Alessandro Pavan. Price discrimination in many-to-many matching markets. http://faculty.wcas.northwestern.edu/~apa522/matching_design.pdf, 2011.

Jong-Hee Hahn. Nonlinear pricing of telecommunications with call and network externalities. *International Journal of Industrial Organization*, 21(7):949–967, 2003.

Justin P Johnson and David P Myatt. Multiproduct quality competition: Fighting brands and product line pruning. *The American Economic Review*, 93(3):748–774, 2003.

Eric Maskin and John Riley. Monopoly with incomplete information. *RAND Journal of Economics*, 15(2):171–196, 1984.

Michael Mussa and Sherwin Rosen. Monopoly and product quality. *Journal of Economic Theory*, 18(2):301–317, 1978.

Roger Myerson. Incentive compatibility and the bargaining problem. *Econometrica*, 47(1): 61–73, 1979.

Jean-Charles Rochet and Philippe Choné. Ironing, sweeping, and multidimensional screening. *Econometrica*, 66(4):783–826, 1998.

Jeffrey Rohlfs. A theory of interdependent demand for a communications service. *Bell Journal of Economics and Management Science*, 5(1):16–37, 1974.

Bernard Salanié. *The Economics of Contracts: A Primer*. MIT Press, 1997.

Stephen W. Salant. When is inducing self-selection suboptimal for a monopolist? *Quarterly Journal of Economics*, 104(2):391–397, 1989.

A. Michael Spence. Monopoly, quality, and regulation. *Bell Journal of Economics*, 6(2): 417–429, 1975.

Nancy L. Stokey. Intertemporal price discrimination. *Quarterly Journal of Economics*, 93 (3):355–371, 1979.

E. Glen Weyl. A price theory of multi-sided platforms. *American Economic Review*, 100 (4):1642–1672, 2010.

Alexander White and E. Glen Weyl. Insulated platform competition. http://alex-white.net/Home/Research_files/WWIPC.pdf, 2015.

A Appendix

A.1 Proof of Lemma 1

By the revelation principle (Myerson [1979]), we can think of the firms as choosing an allocation $Q^{\star}(\theta)$ and a price $P^{\star}(\theta)$ in an incentive-compatible and feasible way, which requires $\forall\theta,\hat{\theta} : \theta Q^{\star}(\theta) - P^{\star}(\theta) \geq \theta Q^{\star}(\hat{\theta}) - P^{\star}(\hat{\theta})$. If user θ' joins some product, all users of type $\theta > \theta'$ also join some product. Thus, the set of participants is $\theta \in \left[t, \overline{\theta}\right]$, where type t is indifferent about participating; hence, $tQ(t) - P(t) = 0$.

Incentive compatibility requires weak monotonicity in allocations (Salanié [1997]). Consequently, two overlapping products must have the same quality (if $\theta \in \Theta^g$ and $\theta', \theta'' \in \Theta^{\hat{g}}$ and $\theta' < \theta < \theta''$, then $\mathcal{Q}(\Theta^g) = \mathcal{Q}(\Theta^{\hat{g}})$). If prodcuts were overlapping, some threshold users would be indifferent between these products. In that case, profit would increase by marginally shifting the identity of the indifferent user so as to increase the quality of the product where willingnesses to pay is higher. This deviation is feasible for a sufficiently small shift in the identity of the indifferent user. Therefore, every product is an interval.

Now consider two contiguous products. First, consider product $\Theta^g = (\theta^g, \theta^{g-1}]$ with price P^g and quality Q^g. Second, consider product $\Theta^{g+1} = (\theta^{g+1}, \theta^g]$ with price P^{g+1} and quality Q^{g+1}. For any $\theta \in \Theta^g$, we must have $\theta Q^g - P^g \geq \theta Q^{g+1} - P^{g+1}$, and similarly for any user $\theta' \in \Theta^{g+1}$. Taking the limits as $\theta \to \theta^g$ and $\theta' \to \theta^g$ shows user θ^g must be indifferent between the two products. Therefore, $\theta^g Q^g - P^g = \theta^g Q^{g+1} - P^{g+1} \Leftrightarrow P^g - P^{g+1} = \theta^g \left(Q^g - Q^{g+1}\right)$.

A.2 Proof of Proposition 1

Using Lemma 1, revenue is

$$\sum_{g=1}^{G} P^g \left(F\left(\theta^{g-1}\right) - F\left(\theta^g\right)\right) = \sum_{g=1}^{G} \left(1 - F\left(\theta^g\right)\right)\left(P^g - P^{g+1}\right) = \sum_{g=1}^{G} \phi^g \Delta Q^g,$$

where $\phi^g = \left(1 - F\left(\theta^g\right)\right)\theta^g$ and $\Delta Q^g = Q^g - Q^{g+1}$. We suppose (toward a contradiction) that ta profit-maximizing feasible regime exists with $G > 1$ products, and associated profit $\Pi_0 = \sum_{g=1}^{G} \phi^g \Delta Q^g$.

First, consider merging the top X products, while the price of the resulting product increases such that the marginal type remains θ^X. The compositions of the remaining products remain unchanged and therefore so do their qualities and prices. This new regime yields profit

$$\Pi_X = \phi^X \left(\mathcal{Q}\left(\left[\theta^X, \overline{\theta}\right]\right) - Q^{X+1} \right) + \sum_{g=X+1}^{G} \phi^g \Delta Q^g.$$

This deviation is not profitable if $\Pi_X \leq \Pi_0$, which implies

$$\phi^X \left(\mathcal{Q}\left(\left[\theta^X, \overline{\theta}\right]\right) - Q^{X+1} \right) \leq \sum_{g=1}^{X} \phi^g \Delta Q^g \Rightarrow \phi^X \left(\mathcal{Q}\left(\left[\theta^X, \overline{\theta}\right]\right) - Q^{X} \right) \leq \sum_{g=1}^{X-1} \phi^g \Delta Q^g.$$

Letting $X = 2$ yields $\phi^2 \left(\mathcal{Q}\left(\left[\theta^2, \overline{\theta}\right]\right) - Q^2 \right) \leq \phi^1 \left(Q^1 - Q^2 \right)$, which implies $\phi^2 \leq \phi^1$. Letting $X = i+1$, and using $\phi^i \leq \phi^1, \forall i > 1$, yields

$$\phi^{i+1} \left(\mathcal{Q}\left(\left[\theta^{i+1}, \overline{\theta}\right]\right) - Q^{i+1} \right) \leq \sum_{g=1}^{i} \phi^g \Delta Q^g \leq \phi^1 \sum_{g=1}^{i} \Delta Q^g = \phi^1 \left(Q^1 - Q^{i+1} \right).$$

Thus, $\phi^{i+1} \leq \phi^1$ and, in particular, $\phi^G \leq \phi^1$.

Now consider another feasible deviation: eliminating all except the top Y products, having P^Y adjust so that θ^Y remains the marginal type. This deviation yields profit $\Pi_Y = \sum_{g=1}^{Y-1} \phi^g \Delta Q^g + \phi^Y Q^Y$. The deviation is not profitable if $\Pi_Y \leq \Pi_0$, which implies

$$\phi^Y Q^Y \leq \sum_{g=Y}^{G} \phi^g \Delta Q^g \Rightarrow \phi^Y Q^{Y+1} \leq \sum_{g=Y+1}^{G} \phi^g \Delta Q^g.$$

Letting $Y = G-1$ yields $\phi^{G-1} Q^G \leq \phi^G Q^G$, which implies $\phi^{G-1} \leq \phi^G$. Letting $Y = i-1$ and using $\phi^i \leq \phi^G, \forall i < G$ yields

$$\phi^{i-1} Q^i \leq \sum_{g=i}^{G} \phi^g \Delta Q^g \leq \phi^G \sum_{g=i}^{G} \Delta Q^g = \phi^G Q^i.$$

Thus, $\phi^{i-1} \leq \phi^G$ and, in particular, $\phi^1 \leq \phi^G$. This is a contradiction unless $\phi^1 = \phi^G$.

A.3 Second-Order Conditions

We have welfare $W(t) = Q(t) \int_t^{\overline{\theta}} \theta df(\theta) d\theta - C(t)$ and profit $\Pi(t) = Q(t) \int_t^{\overline{\theta}} t f(\theta) d\theta - C(t)$. To tackle both problems simultaneously, define $H = Q(t) \int_t^{\overline{\theta}} h(\theta) f(\theta) d\theta - C(t)$, with $h(\theta) \in \{\theta, t\}$. Notice $h(t) = t$ and $h'(t) = 1$. Let $Q''(t) \equiv \frac{d^2 Q(t)}{dt^2}$, $f'(\theta) \equiv \frac{df(\theta)}{d\theta}$, and $C''(t) \equiv \frac{d^2 C(t)}{dt^2}$. Then

$$\frac{dH}{dt} = Q'(t) \int_t^{\overline{\theta}} h(\theta) f(\theta) d\theta - Q(t) h(t) f(t) - C'(t)$$

$$\begin{aligned}
\frac{d^2H}{dt^2} &= Q''(t)\int_t^{\overline{\theta}} h(\theta)\, f(\theta)\, d\theta - 2Q'(t)\, h(t)\, f(t) - Q\left(h'(t)\, f(t) + h(t)\, f'(t)\right) - C''(t) \\
&= Q''(t)\int_t^{\overline{\theta}} h(\theta)\, f(\theta)\, d\theta - Qtf'(t) - C''(t) - f(t)\left[2Q'(t)\, t + Q\right].
\end{aligned}$$

Concavity ($\frac{d^2H}{dt^2} \leq 0$) is assured if quality is concave ($Q''(t) \leq 0$), the type distribution is right squewed ($f'(t) \geq 0$) and cost is convex ($C''(t) \geq 0$), and the elasticity of Q is less than $\frac{1}{2}$ (because $2Q'(t)\, t + Q > 0 \Leftrightarrow \frac{1}{2} > -\frac{Q'(t)t}{Q}$).