1-1-2003

Pathways of elementary school mathematics teachers seeking to improve their instruction through professional development.

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PATHWAYS OF ELEMENTARY SCHOOL MATHEMATICS TEACHERS SEEKING TO IMPROVE THEIR INSTRUCTION THROUGH PROFESSIONAL DEVELOPMENT

A Dissertation Presented

by

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Submitted to the Graduate School of the University of Massachusetts Amherst
in partial fulfillment of the requirements for the degree of

DOCTOR OF EDUCATION

February 2003

School of Education

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DEDICATION

To my husband, Ned, and children, Neil and Kathleen,
who inspire me to learn and grow. They are my heart, my pride, and my joy.

To mathematics teachers in transition, whose efforts must surely result in increasing proportions of young people successfully learning more and better mathematics.

ACKNOWLEDGEMENTS

I am grateful to many people who have assisted me in the completion of this dissertation.

First and foremost, I thank my advisor, Professor Robert L. Sinclair, for his unyielding interest in my successful growth as a scholar. His insights throughout this long journey of seven years have helped me to focus on the nature and quality of educational leadership which helps all children from all families in a democracy learn well. He illuminates this concept with each interaction and it is this constancy of purpose that has caused me to persist in my efforts to complete this dissertation and to grow personally and professionally.

To my doctoral committee members, Dr. Howard A. Peelle and Dr. José N. Ornelas, whose excellent suggestions for the improvement of my work were offered with utmost kindness, I wish to extend my sincere appreciation and respect.

I thank my colleagues, Virginia Bastable and Jill Lester of SummerMath for Teachers, for providing me with access to teachers' writing, for their interest in my work, and for their positive influence on my ideas about mathematics, teaching, and learning. They, along with Deborah Schifter from the Education Development Center, are leaders in mathematics education who have successfully kindled inquiry in me as well as in thousands of other teachers. They have inspired many educators to embark upon an exciting pathway toward a changing pedagogy that better serves all learners.

I also most sincerely thank the sixteen elementary school teachers who agreed to participate in this study. Their generous contributions to this dissertation are, of course, primary and will never be forgotten.

To my colleagues in the Holyoke Public Schools and the Hampden-Wilbraham Regional School District, I extend my sincere appreciation for the many examples they provide me of devotion to young people, of caring about the teaching profession, and of the relentless pursuit of excellence.

It would be impossible to think that this dissertation could have been completed without the assistance of my husband, Ned, whose attention to every detail of life that he could handle in my place was managed without a single complaint. He expressed only joy in being able to support me in this endeavor. For this and for our many shared experiences I am forever grateful. I am blessed to have such a partner on life's journey.

I also wish to thank my children, Neil and Kathleen, and my sister, Alice, for their unwavering belief that I would surely complete this dissertation, even when I was seriously doubting this possibility. Their confidence gave me the courage and the fortitude to keep moving forward.

Finally, I wish to give special thanks to my parents, Mary and Edmund Babski, who have always believed in me and who have supported me in every possible way. They overcame extreme hardships as young people in Eastern Europe during World War II. Their experiences taught me to work hard, to respect people from diverse backgrounds, and to cherish my United States citizenship and resulting opportunities. Their stories and accomplishments shape me in many ways and their love and pleasure in my success heartens my spirit.

ABSTRACT

PATHWAYS OF ELEMENTARY SCHOOL MATHEMATICS TEACHERS SEEKING TO IMPROVE THEIR INSTRUCTION THROUGH PROFESSIONAL DEVELOPMENT

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This exploratory descriptive case study aimed to identify key issues in the transformation of mathematics teaching in elementary schools. This study told the story of what happened to sixteen elementary teachers who embarked on a quest to improve their mathematics teaching by participating in at least two in-depth professional development experiences over an interval of three years. It traced their ideas about how and why to improve instruction, identified their challenges with prevailing school organizational conditions, and reported perceived changes that were made in their teaching.

The research data, comprised of quotations from teachers' writing while they were engaged in professional development experiences at SummerMath for Teachers of Mt. Holyoke College and the researcher's notes from interviews, formed the data for analysis to answer four interrelated research questions:

1) What changes in instruction do selected elementary teachers of mathematics report that they made as a result of participating in professional development for improving the teaching of mathematics?

2) What do selected elementary teachers of mathematics report are the reasons for the changes they made in instruction?
3) What changes in instruction do selected elementary teachers of mathematics report that they regard as most effective for improving student learning?
4) What organizational conditions in their local elementary schools do selected elementary teachers of mathematics report helped or hindered their changes in instruction?

The major findings based on the collected data were summarized and presented according to the four research questions that guided this study. The major findings related to changes in instruction fell under five prominent categories: increased emphasis on student thinking and understanding, increase in student-centered activities, changes in classroom discourse, increase in conceptually-based mathematics content, and a shift in the teacher's role from an authoritarian model of instruction to one that is student-centered.

Findings from this study suggested some recommendations for educational practice for institutions preparing elementary teachers of mathematics, for in-service teacher professional development programs, and for school policies and organizational structures.

One primary recommendation involved engaging pre- and in-service teachers in revisiting the mathematics content that they currently teach or will teach within an inquiry-based teaching and learning environment so that they might draw their own conclusions as to the implications for teaching mathematics.

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CHAPTER 1

NATURE OF THE STUDY

Statement of the Problem

Current economic demands in our society and persistent concerns for improving learning in mathematics call for important reforms in the teaching of mathematics. A student's math proficiency is directly tied to future income, educational opportunities, career options, and ultimately his or her ability to participate fully in the United States system of democracy (Steen, 1990; U.S. Department of Education, 1997). Across the nation, elementary mathematics teachers are engaging in the process of reinventing their teaching practice to better serve all students, including those who have traditionally been underserved.

Changing the way mathematics is taught and learned from an authoritarian model based on one-way transmission of knowledge to a student-centered practice featuring stimulation of learning is a formidable undertaking. Those seeking to change their practice typically do not have useful models from their own experiences as mathematics teachers and learners to help them develop a classroom culture of mathematical inquiry. Teachers in the United States today are grounded in many years of formative experiences that define mathematics as a body of rules and procedures, teaching as meticulous explanations, and learning as note-taking and memorization. Achieving the kind of changes called for by reform documents (National Council of Teachers of Mathematics, 1989, 1991, 2000) requires new learning on the part of

teachers, taking place over a long period of time, with ample opportunities to test ideas and engage in professional discourse (Loucks-Horsley, 1997; Nelson, 1997).

The means by which teachers accomplish the kind of transformation that is required are not yet fully understood (Goldsmith & Schifter, 1997). The disparity between their new vision toward teaching mathematics and what they are able to actualize in their classroom can become a source of frustration and dysfunction when teachers are unable to teach the way they imagine is best. Understanding the different pathways that teachers take to change their thinking about effective instruction and to improve their practice so that they may help students increase learning in mathematics is crucial for the reform of mathematics teaching. It is reasonable to suggest that this increased understanding of the pathways that teachers take will lead to better ways of helping teachers assist students to improve their learning in mathematics. The following study contributes to the accomplishment of this important end.

Purpose of the Study

The purpose of this study is to examine changes that elementary teachers of mathematics who engage in professional development for inquiry-based teaching and learning make in their ideas about effective mathematics teaching. This study tells the story of what happened to selected elementary teachers who embarked on a quest to improve their mathematics teaching. It traces their ideas about how and why to improve instruction, identifies their challenges with prevailing school organizational conditions, and reports perceived changes that were made in their teaching practices.

Teacher writings and interviews about effective mathematics instruction form the data for analysis to answer four interrelated research questions:

1) What changes in instruction do selected elementary teachers of mathematics report that they made as a result of participating in professional development for improving the teaching of mathematics?
2) What do selected elementary teachers of mathematics report are the reasons for the changes they made in instruction?
3) What changes in instruction do selected elementary teachers of mathematics report that they regard as most effective for improving student learning?
4) What organizational conditions in their local elementary schools do selected elementary teachers of mathematics report helped or hindered their changes in instruction?

Definition of Terms

Six key terms are central to this research study.

- Ideas about mathematics instruction. Ideas about mathematics instruction are the thoughts, opinions, views, or beliefs that teachers express verbally or in writing with regard to any part of the mathematics instructional process. An example of this is "…I do believe that it's extremely important to provide our students with opportunities to explore numbers and to develop their own approaches to problem solving."

- Professional development for inquiry-based teaching and learning. Professional development for inquiry-based teaching and learning is engagement in experiences that provide alternatives to teacher-centered instructional models, involve teachers in exploring mathematics content and children's thinking in new ways, and foster a stance of inquiry rather than one of answers.
- Changes in instruction. Changes in instruction are descriptions that demonstrate a shift or reconstruction of a teacher's own instructional behaviors or the behaviors that he or she elicits from students. An example of a teacher describing changes in instruction is "…I feel I am more open to children's thoughts. I find myself asking my students how they solved a problem. Before I was too interested in the answer itself."
- Reasons for changes in instruction. Reasons for changes in instruction are statements of what problems in learning teachers want to resolve, insights about teaching and learning that they gain from professional development activities, reflections about what has or has not worked in the past to help students improve their learning, or a rationale for action.
- Organizational conditions. Organizational conditions of a school include, but are not limited to, how students are grouped for learning; the physical, social, and intellectual conditions of the school environment; schedule, flexibility, responsiveness of curriculum; evaluation of instruction; evaluation of pupil progress; policies and regulations; and democratic involvement in decision-making.

- Instructional effectiveness. Instructional effectiveness in this study refers to the degree to which teachers perceive that students are learning successfully and that teachers discern they are responsive to the individual differences of all learners in their classroom.

Significance of the Study

This study is significant because it has both theoretical and practical implications. It can potentially advance the nature and quality of elementary students' learning in mathematics, institutions preparing mathematics teachers, in-service teacher professional development programs, and school policies and organizational structures.

Theoretically this study is of value because it contributes to understanding inquiry-based teaching and learning and can encourage other scholars to conduct research into inquiry-based teaching and learning. Examining the ways that individual teachers of mathematics reconstruct their ideas about teaching mathematics for improving student learning can help us to understand what groups of teachers learn and how they develop. It will add to the literature that helps teacher educators understand what teachers take from their learning opportunities and how teachers' new ideas influence their beliefs and actions.

From a practical perspective, this study is of value because it serves as a starting point to consider the conditions necessary for the successful mathematics learning of students who are not learning mathematics well. In order to improve learning we must improve teaching. If this type of professional development program has promise in changing teachers' thinking about effective mathematics instruction, then the results of

this study can help to guide the design of professional development programs. The results should also help with the clarification of programmatic goals in preparing elementary mathematics teachers.

The significance of this study also lies in its potential to recommend professional development that a school system could foster while being fully anchored in a challenge that teachers would be interested in tackling, that is, the challenge of improving mathematics learning. The results from the study may also help school administrators understand the process of teacher change and lead to their informed efforts to help teachers in transition. It should allow them to interpret what they see in teachers' practice so that they may better support appropriate systemic organizational changes.

Approach to the Study

This exploratory descriptive case study aims to identify key issues in the transformation of mathematics teaching. The research data, comprised of quotations from sixteen elementary teachers' writing while they were engaged in two in-depth professional development experiences and the researcher's notes from interviews, captures the process by which teachers reinvent their practice to create classroom cultures that promote inquiry-based learning and teaching of mathematics. The study includes all of the teachers who agree to participate in it from a pool of forty-five ethnically, racially, and geographically diverse teachers. Those who were invited participated in at least two courses or institutes between the years 1997 and 2000 at SummerMath for Teachers, a teacher education program at Mt. Holyoke College in

South Hadley, Massachusetts. All of the courses or institutes provide professional development for inquiry-based teaching and learning through exposing teachers to alternatives to teacher-centered instructional models, engaging teachers in exploring mathematics content and children's thinking in new ways, and fostering a stance of inquiry rather than one of answers. See Appendix A for a description of the SummerMath for Teachers program.

Delimitations of the Study

The data reflect the ideas of a selected group of elementary teachers who participated in a set of experiences at a single in-service teacher education program, SummerMath for Teachers at Mt. Holyoke College. Results of this study do not necessarily generalize to other professional development programs or teachers. Nor do the results imply that the professional development experiences at SummerMath for Teachers were the only source of influence over teachers' changing ideas. Professional development in their own schools, other academic work, conferences, research, and personal experiences might have also influenced participants' ideas about effective mathematics instruction.

The elementary teachers who were invited to participate in this study were unusual in the sense that they not only volunteered to participate in the professional development experiences upon which this study is based, but they chose to do so more than once. This characteristic is deliberately built into the study population since the study is designed to examine the changes of teachers who make a long-term commitment to improving their instruction. Those who agreed to participate in the study

were, perhaps, those who perceived that their instruction changed positively as a result of the experiences. Those who did not agree to participate in the study may have been reluctant to do so due to a negative or negligible perception of the impact on their instruction.

Journal writing and interviews are major data sources in this study. Though there is a high level of confidence that the data reflect what is actualized in the classroom, there is the possibility that what teachers write and say is different from what they do. The data are not cross-referenced by observation of participants' classrooms.

Elementary teachers of kindergarten through grade six were chosen as the focus of the study because as educators who structure the early learning experiences they provide the foundation for mathematics content knowledge and skills. Teachers of the elementary grades, kindergarten through grade six, also develop within students a long-lasting disposition toward mathematics, and what it means to do it and learn it. Improvements in student learning at the secondary level, which is where women and minorities begin to be underrepresented in rigorous mathematics courses, cannot be achieved without substantial improvements at the elementary level. Results of this study, however, do not generalize to other levels of schooling.

Dissertation Chapter Outline

Chapter 1 describes the research problem, the purpose of the study, its significance and delimitations. Research questions are presented, as are important definitions.

Chapter 2 presents the literature related to the process of transforming teaching for elementary teachers of mathematics in order to provide a conceptual base for the study. The literature review centers on four interrelated themes:

- Inquiry-based mathematics teaching and learning;
- Effectiveness of professional development of elementary mathematics teachers;
- Improvement of mathematics teaching;
- Organizational conditions that foster or hinder the improvements of teaching.

Chapter 3 details the design of the study, the procedures for sample selection and the methods used for the collection of data for addressing each of the four research questions. Chapter 4 presents and discusses the results of the study. It is organized into sections according to the research questions. Chapter 5 summarizes the major research findings, makes suggestions for professional development of elementary teachers of mathematics and organizational conditions that support them, and makes suggestions for further research.

CHAPTER 2

REVIEW OF THE LITERATURE

The purpose of this chapter is to provide a conceptual base to support the four research questions that guide this study. To gain insight into how elementary teachers of mathematics negotiate the path toward significant changes in teaching mathematics one must understand the complexity of the process of improving mathematics instruction for all students. This literature review will center on four interrelated themes:

- Inquiry-based mathematics teaching and learning;
- Effectiveness of professional development of elementary teachers;
- Improvement of mathematics teaching;
- Organizational conditions that foster or hinder the improvement of teaching.

Since the study is built around the professional development that shifts a teacher from an authoritarian model of instruction based on transmission of knowledge to a student-centered practice featuring stimulation of learning and inquiry into student thinking, the review includes an analysis of inquiry-based mathematics teaching and learning. Identifying elements of a perspective of inquiry rather than one of answers is fundamental to this study and should be thoroughly explained.

This review includes studies that highlight the issues that contribute to the effectiveness of professional development of elementary mathematics teachers. Teachers' development of a stance of inquiry rather than one of answers for themselves and their students is essential to this study and the process should be examined.

The review also examines the topic of improvement of mathematics teaching for better student learning, an important motivation for the study. The studies that are

highlighted share the assumption that improvement of mathematics teaching will result in advancement toward the nation's commitment to mathematics for all. We know that the improvements extend beyond the implementation of new techniques and approaches, and the needed improvements should be explicitly specified to provide a worthy conceptual framework for this study.

Finally, it is also important to provide an appropriate foundation from which to define the situation faced by many teachers in transition whose instructional practices are in the process of changing as a result of their participation in mathematics education reform activities. The review highlights research that addresses the organizational conditions that foster or hinder the improvement of teaching.

Inquiry-Based Mathematics Teaching and Learning

Inquiry is an approach to learning that is driven by an individual's own questions, an interest to understand a phenomenon, or need to solve a problem (National Science Foundation, 1999). Inquiry-based teaching and learning of mathematics implies that this process is not only structured for students by teachers, but that the process is also central to what the teacher herself does as she analyzes students' mathematical ideas and uses the information to make instructional decisions.

Inquiry-based teaching, in the context of elementary mathematics, is about a teacher's thinking about teaching, as well as about his or her own and her students' thinking about mathematics. For example, children might think about questions posed on their own or by their teacher such as what is the relationship between the perimeter and area of given two-dimensional figures? They might make observations, pose

questions, examine books and other sources of information to see what is already known, plan investigations, create models or representations of the problem, use tools to gather, analyze, and interpret data, propose answers and explanations, make predictions, and communicate reflectively, orally and in writing, throughout this process (National Research Council, 1996).

During and after the children's investigation of this question, their teacher may wrestle with dilemmas such as what are the best ways to orchestrate the whole group discussion for maximum benefit for all and still capitalize upon the individual student's ideas that have been formed. He or she may see something unanticipated in the mathematics that is brought into relief through the children's work that evokes his or her own new learning related to the problem. The teaching dilemmas that arise during the teacher's interactions with students may be a continuous and complex source of inquiry. He or she reflects on the information learned to design the next learning experiences based on a conjecture about what children should think about next, but must also be prepared to refine that hypothesis if necessary (Cobb & Steffe, 1983). This process, which is parallel to that experienced by students, is cyclical and inspires more questions.

The work of John Dewey over a fifty-year span in the first half of the twentieth century, and others, including Bruner and Piaget in the 1950s and 1960s, influenced the nature of curriculum materials, especially in the sciences, developed in those decades and into the early 1970s. Underlying many of these inquiry-based instructional materials was the commitment to involve students in doing rather than listening or reading about a topic. The National Science Foundation sponsors the development and

evaluation of such materials in mathematics and science, as well as teacher enhancement projects to fuel the implementation of these curricula as they were intended. These efforts place as much emphasis on learning the processes of inquiry as on mastering the subject matter alone. The teaching models that are integrated into these curricula are based on theories of learning that emphasize the central role of students' own ideas and concrete experiences in creating new and deepened understandings of concepts. Work is done as a learning community, with social interactions central to each individual's learning while each individual feels a responsibility and worthiness to contribute to the learning of others (Dewey, 1938).

The characteristics of an inquiry-based educational model are in line with current research on human learning from the biological, cognitive, and psychological sciences. Brain research, for example, supports the need for multiple rich and rigorous experiences with opportunities to talk, listen, read, and act in order for humans to extract meaning (Caine & Caine, 1991). Brooks and Brooks (1993) summarize learning from the constructivist perspective as a "…self-regulated process of resolving inner cognitive conflicts that often become apparent through concrete experience, collaborative discourse, and reflection" (Brooks & Brooks, 1993, p. vii).

There is not a prescriptive formula to follow when building a practice based on these theories. Instead, these theories provide a landscape for a vision of inquiry-based teaching and learning that celebrates wonder, flourishes on debate, and honors multiple perspectives (Whitin & Whitin, 1997).

Effectiveness of Professional Development of Elementary Mathematics Teachers

Unfortunately, few educators have had the chance to engage in inquiry as they learned mathematics content. Nor did most teachers have the opportunity to learn mathematics with deep understanding, a prerequisite for teaching for understanding (Ma, 1999). Teachers must rely upon professional development opportunities to build their capacity to structure inquiry-based teaching practices and learning opportunities. This study is built around the kind of professional development that shifts a teacher from an authoritarian model of instruction based on transmission of knowledge to a student-centered practice featuring stimulation of learning and inquiry into student thinking.

The studies reviewed in this chapter share the assumption that the kind of instruction that is called for extends beyond the implementation of new techniques and approaches. Instead, the new vision for mathematics education demands the creation of radically different mathematics learning opportunities built from the restructuring of teachers' fundamental beliefs about teaching, learning, and mathematics (Schifter, 1995; Weissglass, 1994). Researchers agree that teachers' pedagogical decisions are closely connected to their system of beliefs about the nature of mathematics and the learning and teaching processes (Fennema, Carpenter, & Franke, 1996; Schoenfeld, 1983; Thompson, 1992). It is obvious that the transformation of teachers' instructional practices as a result of professional development must be measured in years rather than weeks or months (Fosnot & Schifter, 1992). The National Commission on Teaching and America's Future (1996) recommends that professional development become embedded

in teachers' daily work through joint planning, research, curriculum and assessment groups, and peer coaching.

Teachers are central figures in defining how mathematics is taught and learned in schools, and understanding their transition process has been the focus of many researchers. Two different points of view are reflected in the research on teacher change patterns. One stance implies that if professional development experiences influence beliefs and knowledge, teachers' classroom instruction will shift to reflect that practice (Clark & Peterson, 1986; Pajares, 1992; Putnam, Lampert, & Peterson, 1990; Shulman, 1986; Thompson, 1992). Guskey (1986), on the other hand, suggests that as teachers engage in particular new practices, they will see and hear things that affect their beliefs. Neither perspective makes clear how teachers negotiate the path toward significant changes in how mathematics is taught and learned. Understanding the change process and the stages that teachers in transition pass through can help to put into perspective the possible effects of professional development.

Hord et al. (1987) note that research has identified seven stages of concern that typical adult learners experience before they are able to implement the ideas of an innovation. The stages are awareness, informational, personal, management, consequence, collaboration, and refocusing. For each of these stages, there is a link to the teacher's beliefs about instructional practice and student learning. Jones et al (1994) relate the following descriptor for each phase:

Awareness	"I am not concerned about any change."
Informational	"I would like to know more."
Personal	" How will using these ideas affect me?

Management	"I am overwhelmed. How do I organize?"
Consequence	"How is the innovation affecting my students."
Collaboration	"I'm concerned about sharing ideas for change."
Refocusing	"I'm confident that I can improve on the ideas learned."

Shaw, Davis, Sidani-Tabbaa and McCarty (1990) have suggested six interrelated factors necessary for change to occur: perturbation, awareness of a need to change, commitment to change, vision, and projection into that vision. Reflection throughout that change process is necessary for change to continue. This model was used when Etchberger and Shaw (1992) examined the relationship between an elementary teacher's perceptions about where mathematics knowledge resides and how that perception influences her teaching methods. They examined this relationship as the teacher transitioned toward a constructivist view of learning, where knowledge is acquired through active concept construction (Confrey, 1990; Piaget & Inhelder, 1969; von Glaserfeld, 1987a, b).

Simon and Schifter (1991a, b) used a Levels of Use (LoU) instrument developed by Hall and his colleagues (Hall, Louks, Rutherford, & Newlove, 1975) as well as their own adaptation of it to place teachers into particular levels of teaching based on the constructivist theory of learning. Their Assessment of Constructivism in Mathematics Instruction (ACMI), was developed to evaluate the effectiveness of the SummerMath for Teachers Program. The Levels of Use instrument focused on implementation of particular strategies and served as a template for the development of the ACMI. The LoU results showed that 97% of the teachers implemented strategies but only 35% demonstrated not only stable use but also the ability to respond to students' needs. The

stages of LoU allowed the authors to learn that after one intensive experience more teachers changed in implementation of a particular strategy rather than in their beliefs about learning as enacted in instruction (Schifter & Fosnot, 1993).

> The LoU and ACMI data confirm the importance of distinguishing between those whose learning was restricted to the acquisition of new teaching strategies and those whose views of mathematics learning and teaching shifted fundamentally. Not surprisingly, innovation in teaching strategy was more easily and rapidly achieved than were changed views about learning as enacted in instruction. (Schifter & Fosnot, 1993, p. 189)

Later, Schifter (1995) developed a model that interprets teacher development along the strand of changes in their conception of school mathematics. One might think about this model as a continuum upon which we place the kinds of understandings that teachers have and act upon in their classrooms.

> I characterize conceptions of mathematics teachers enact in practice as 1) an ad hoc accumulation of facts, definitions, and computational routines; 2) student-centered activity, but with little or no systematic inquiry into issues of mathematical structure or validity; 3) student-centered activity directed toward systematic inquiry into issues of mathematical structure and validity; or 4) systematic mathematical inquiry organized around investigation of "big" mathematical ideas. Each conception or stage, entails an understanding of what counts as "doing mathematics," of the extent to which mathematical results are interconnected, and where mathematical authority resides and how it is established. (Schifter, 1995, p.18)

The concept of "big" mathematical ideas, referenced in phase four is explained as "…central organizing principles of mathematics with which students must wrestle as they confront the limitations of their existing conceptions" (Schifter, 1995, p.20).

The four phases developed by Schifter might be better understood through the following examples, which are hypothetical and should be cautiously correlated to the phases. These examples are intended to illustrate the way the four phases might be applied to teachers in transition. A conversation with any of these fictitious teachers

about their rationale for the activities could put any one of these examples into a different phase.

A teacher in the first phase who believes mathematics is an accumulation of facts, definitions, and computational routines might enthusiastically present a lesson about baseball batting averages and show students how to divide the number of hits by the times at bat and round the answer to the nearest thousandth. The students would, using the taught procedures, individually find the averages for a dozen major league players. The teacher might correct their work, write the percent correct on the top and show them the right way to do a few that were incorrect.

A teacher in phase two, who believes in the importance of student-centered activity, but with little or no systematic inquiry into issues of mathematical structure or validity might ask students to work in small groups on baseball batting averages after showing students how to use their calculators to figure it out. The students might do the averages for a dozen major league players in their groups and then create and present a poster to show how they got their answers. The teacher would correct their work, use a scoring rubric for grading purposes and perhaps show them the right way to do a few that they got wrong.

In phase three, a teacher who believes in the importance of student-centered activity directed toward systematic inquiry into issues of mathematical structure and validity might ask students to work in groups to figure out how baseball batting averages are found. The students look at the hits, times at bat, and averages of a dozen players in pairs and then share what they have noticed with the whole class. They try some new problems on their own, compare answers, and discuss the validity of each.

Afterwards, they write their questions and ideas and current understandings about this situation into their math journal, which their teacher responds to regularly.

Finally, in phase four, a teacher who understands the importance of systematic mathematical inquiry organized around investigation of "big" mathematical ideas might ask students to work in groups to figure out how baseball batting averages are found. The students look at the hits, times at bat, and averages of a dozen players in pairs and then share what they have noticed with the whole class. They try some new problems on their own, compare answers, and discuss the validity of each. The students as well as the teacher raise questions of one another such as: Why is it about this situation that makes this a division problem? Why do you divide the number of hits by the number of times at bat to get the answer? What does it mean to be batting 1000? How is that like 100%? These questions are not necessarily all resolved but the teacher knows that they are important in revealing underlying structures of mathematics and will look for opportunities to press on them again. She may choose one of them to pursue in order to reach some degree of closure. Other questions might be noted for the back burner, as ideas that will again come up for students as they work on some other problem. After the whole class discussion, they spend some time reflecting upon what they learned and what they are still wondering about in their math journal, to which the teacher responds regularly. The teacher uses all that she has learned about the students' thinking to reflect upon the mathematics each student understands and is struggling with, and what the next teaching steps should be.

Other researchers have also created ways to get a perspective on where teachers are as they embark on a pathway toward a new instructional practice. Simon et al (2000)

proposes a perception-based perspective based on research conducted during the Mathematics Teacher Development Project (Simon, 2000) that is fundamental to teachers currently participating in mathematics education reform. A perception-based perspective is grounded in a view of mathematics as connected, logical, and universally accessible.

Thompson and others (1994) categorize teachers in transition by fundamental differences in their orientation toward teaching mathematics. At one end of the continuum is the teacher with a calculational orientation, while at the other end is the teacher with a conceptual orientation. A teacher with a calculational orientation views mathematics as composed of the application of calculations and procedures for deriving numerical results. A teacher with a conceptual orientation expresses mathematics teaching in ways that focus students' attention away from the thoughtless application of procedures and toward a rich conception of situations, ideas, and relationships. They tend to aim toward giving meaning to numerical meaning and suggesting numerical operations. Thomson et al. (1994) also state that to create a conceptual orientation, "…a teacher must reflect long and deeply on her goals for, and images of mathematics and mathematics teaching."

As they attempted to characterize and document the process of teacher change among teachers who participated in their Cognitively Guided Instruction (CGI) project, Franke, Fennema, and Carpenter (1997) identified four levels, each building on the previous one, with the fourth level broken into sublevels. The levels incorporate aspects of teachers beliefs and practice with teachers at Level 1 not being consistent with the premises of CGI while those at Level 4 are. Teachers at Level 4 possess knowledge

about children's thinking and use knowledge of their students to drive their instructional decisions. They used the CGI levels to examine the change of 21 teachers over a 4-year period of time. They investigated whether the teachers' beliefs and classroom practices changed at the same time and to the same degree. They found that 90 percent of the teachers were categorized at Level 3 or higher. Tharp and Gallimore (1988) used Vygotsky's theories of how learning occurs to the learning of teachers and proposed a sequence of teacher change stages that focus on the regulation of behavior.

These studies provide some insight into the widespread efforts put forth in the interest of providing effective professional development in the interest of improving teaching and learning in mathematics classrooms. It is important to note that in an inquiry-based teaching and learning model, there is not a point of completion of the transition process.

> As a new practice emerges, teachers gradually come to view their new teaching as better and more satisfying, and they feel that it produces better outcomes. However, they also begin to realize that they will never "arrive" at a new, finished state. Rather, the quest to understand children's mathematical thinking leaves them forever in an exploratory or experimental stance toward their own teaching. There is always the question: What can I do that will help this child's thinking move forward? (Nelson, 1997, p.405)

Improvement of Mathematics Teaching

A succession of reports documents inadequacies in the mathematical performance of students in the United States (McKnight, 1987; National Research Council, 1989; Third International Mathematics and Science Study, 1998). Only 18 percent of fourth graders, 25 percent of eighth graders, and 16 percent of twelfth graders can perform at a proficient level on a national test (O'Neil, 1993). Another dilemma is

poor children and children of color are underrepresented among successful students of mathematics (Kozol, 1992). The improvement of mathematics teaching should result in advancement toward the nation's commitment to mathematics for every student. We know that the improvements extend beyond the implementation of new techniques and approaches. The needed improvements are specified below in order to provide a worthy conceptual framework for this study.

In the early nineties, out of a concurrence between changing societal needs and research in cognitive psychology, a new vision for mathematics education emerged in the United States (National Council of Teachers of Mathematics, 1991). Findings indicate that learning occurs as students actively confront new information to modify their prior conceptual understanding (Case & Bereiter, 1984; Cobb & Steffe, 1983; Lampert, 1986; Schoenfeld, 1987). Evidence suggests that mathematics that is understood is more useful and applicable to solving problems than mathematics that is learned through rote memorization (Hiebert, 1999). The seminal report, Everybody Counts: A Report to the Nation on the Future of Mathematics Education (National Research Council, 1989, pp. 58-59), describes this reformed view of teaching and learning and the resulting new roles of teachers and learners:

> In reality, no one can *teach* mathematics. Effective teachers are those who can stimulate students to *learn* mathematics. Educational research offers compelling evidence that students learn mathematics well only when they *construct* their own mathematical understanding. To understand what they learn, they must enact for themselves verbs that permeate the mathematics curriculum: "examine," "represent," "transform," "solve," "apply," "prove," communicate." This happens most readily when students work in groups, engage in discussion, make representations, and in other ways take charge of their own learning. All students engage in a great deal of invention as they learn mathematics; they impose their own interpretation on what is presented to create a theory that makes sense to them…No teaching can be effective if it does not respond to students' prior ideas.

> Teachers need to listen as much as they need to speak. They need to resist the temptation to control classroom ideas so that students can gain a sense of ownership over what they are learning. (NRC, 1989, pp. 58-59)

The Professional Standards for Teaching Mathematics (NCTM, 1990) emphasizes five major shifts that are needed in order to move from the current instructional practice to mathematics instruction that gives students ownership and power over their learning. These shifts are stated below:

1. Toward classrooms as mathematical communities—away from classrooms as simply a collection of individuals;
2. Toward logic and mathematical evidence as verification—away from the teacher as the sole authority for right answers;
3. Toward mathematical reasoning—away from memorizing procedures;
4. Toward conjecturing, inventing and problem solving—away from an emphasis on mechanistic answer-finding;
5. Toward connecting mathematics, its ideas, and its application—away from treating mathematics as a body of isolated concepts and procedures. (National Council of Teachers of Mathematics, 1991, p. 3)

As these shifts become enacted in classrooms, students share ideas, invent new procedures or justify commonly used procedures, pose questions, make models, use tools and technology, and write in order to learn mathematics. Reasoning, problem solving, making connections, and communicating are essential elements of doing mathematics. The content becomes accessible to many more types of learners as the classroom culture becomes more student-centered. The discourse, as described in the Professional Standards for Teaching Mathematics, becomes drastically different in these kinds of classrooms.

> The discourse of a classroom—the ways of representing, thinking, talking, agreeing, and disagreeing—is central to what students learn about mathematics as a domain of human inquiry with characteristic ways of knowing. Discourse is both the way ideas are exchanged and what the ideas entail. Who talks? About what? In what ways? What do people write, what do they record and why? What questions are important? How do ideas change? Whose ideas and ways of thinking are valued? Who

> determines when to end a discussion? The discourse is shaped by the tasks in which students engage and the nature of the learning environment; it also influences them. (NCTM, 1991, p. 34)

This reform vision is committed to the idea of mathematics for all while acknowledging that "...equity requires accommodating differences to help everyone learn mathematics" (National Council of Teachers of Mathematics, 2000). The idea that the study of high levels of mathematics should be accessible to all people is relatively new, and has only been gaining momentum since the 1950s (Willoughby, 2000). The level of mathematics knowledge that is expected of students to meet the demands of responsible citizenship in a democracy and participation in a technical, problem-solving workforce keep increasing. All students are expected to learn more and better mathematics and teachers are expected to ensure equity through effective instruction.

There is, however, "a huge gap" between the reform vision and "the world of classroom practice" (West, 1992, p.15). Bridging this gap is a source of inquiry among scholars since it is clear that teachers play a substantial role in affecting the learning experienced by students (Sarason, 1982; Snyder et al., 1992; Stake & Easley, 1978). In order to improve learning, it is necessary to improve teaching, and a great deal of attention is being paid to the conditions necessary for accomplishing this (Lampert, 2001; National Commission on Teaching for the 21st Century, 2000; Stigler & Hiebert, 1999). "The evidence for the positive effect of improved teaching is unequivocal" (Darling-Hammond, 1996).

Nieto (1996) identified one of the major structural barriers to the academic achievement of linguistically and culturally diverse students as dull and uninteresting pedagogy. When students aren't learning well the teaching has to become different, yet

studies show that few teachers across the United States are able to provide alternative examples for instructional practice. Teachers tend to pattern their instruction after the models they themselves have experienced as students. Sirotnik (1983) asserts the teaching and learning process

> …appears to be one of the most consistent and persistent phenomena known in social and behavioral sciences...the 'modus operandi' of the typical classroom is still didactics, practice, and little else. Teacher lecturing or total class work on written assignments continue to emerge as the primary instructional patterns. (Sirotnik, 1983, p. 21)

Schoenfeld (2002) discusses four conditions necessary for providing high quality mathematics instruction for all students: high quality curriculum, a stable, knowledgeable, and professional teaching community; high quality assessment that is aligned with curricular goals; and stability and mechanisms for the evolution of curricula, assessment, and professional development.

Schoenfeld (2002) states that for the first time in American curricular history, there is a solid curricular base from which to work, and complete curriculum packages that incorporate student-centered, inquiry-based instructional practices with standards-based content are commercially available. During the 1990s the National Science Foundation funded a number of mathematics instructional materials projects that represent a significant deviation from the typical textbook found in most classrooms. The curriculum programs that emerged align with the Standards (NCTM, 1989, 1990, 2000) and incorporate a selection of instructional models that assist in making the mathematics accessible to all students. They offer the advantages of curriculum cohesion and continuity that individual teachers would not easily be able to achieve on their own. Very importantly, Schoenfeld presents evidence that students who learn

mathematics in classrooms where they are implemented as they authors' intend, still learn their skills as well as students in conventional programs, and they perform much better than their counterparts on concepts and problem solving.

On the second condition, a professional teaching community, he promulgates that teaching is a profession more in name than in reality, that there is minimal opportunity for professional growth, and that there is a gross underestimation of the knowledge and skills required. He perhaps agrees with the National Commission on Teaching and America's Future (1996) recommendation that professional development become embedded in teachers' daily work through joint planning, research, curriculum and assessment groups, and peer coaching. Once again, it is obvious that the transformation of teachers' instructional practices as a result of professional development must be measured in years rather than weeks or months (Fosnot & Schifter, 1992).

He also notes that high quality standards-based assessments, the third condition, do exist, but are not necessarily used; that gains or losses in test scores may not reflect what the public believes to be true; and that "…high stakes testing can result both in deformation of the curriculum and loss of intrinsic motivation for students" (p. 23). There is more discussion about this organizational condition in the last section of this chapter, Conditions That Foster or Hinder the Improvement of Teaching.

On the fourth condition, stability and mechanisms for evolution of curriculum, he sites examples where long-term, consistent goals and a systemic approach improves student achievement in mathematics. The pendulum swings and provocative ideological debates about what is important to teach in mathematics and how it should be taught

have fueled the "math wars." It is time for a stability of focus and a mindset that permits evolution of beliefs and practices.

It is widely recognized that links between researchers and practitioners are weak if not non-existent. Therefore, research results tend to stay locked in professional journals and other publications rather than resulting in improvements in mathematics teaching. Efforts for bridging the gaps so that research can impact teaching need to be expanded beyond publications and conferences toward meaningful and authentic collaboration. In order to address this problem, Hiebert et al. (2002) make a case for building and sustaining a professional knowledge base for teaching with mechanisms for verification and improvement.

Organizational Conditions That Foster or Hinder the Improvement of Teaching

Organizational conditions that foster or hinder the improvement of teaching are included in the review in order to provide an appropriate foundation from which to define the situation faced by many teachers in transition whose instructional practices are in the process of changing as a result of their participation in mathematics education reform activities. Clearly, the best and most effective professional development will be wasted if organizational conditions prohibit the implementation of the newly learned practices in the classroom. For example, a school or district policy that insists that every third grader will be able to complete a certain number of multiplication facts correctly within five minutes will dampen any enthusiasm a teacher may have toward students inventing models that adequately represent 2x3x4 and comparing them to area models for 6x4. An administrator that arrives in the classroom for a yearly formal observation

while a teacher in transition is orchestrating a complex discussion about important mathematics, such as a debate about why the difference of numbers whose ones and tens digits are reversed (81-18 or 64-46) always give you an answer that is a multiple of 9. When the administrator says ,"I'll come back later when you're teaching," his response is indicative of a lack of understanding of inquiry-based teaching and learning. An evaluation system that has forms filled with indicators such as "plan book complete" or "adheres to lesson plan" is also problematic for a teacher who is primed to explore new practices. High stakes math tests that focus upon arithmetic procedures will not support an instructional approach that elicits reasoning and problem solving. A budget that does not allow for purchasing materials to engage students in the ways that a teacher envisions can also crush a teacher's initiative, as can required textbooks that deaden students' curiosity.

The Curriculum and Evaluation Standards for School Mathematics (NCTM, 1989) articulate the problem well and cause one to wonder how a teacher can be expected to over come all of the possible hurdles necessary to significantly improve instruction:

> In too many schools, teachers will find it difficult to teach the mathematical topics or create the instructional environment envisioned in these standards because of local constraints, such as directives about what chapters or pages to cover, inadequate time for instruction, and the administration of tests. In many grades, too little time is spent on mathematics instruction. Teachers and students should spend an hour a day on mathematics at all grades and take advantage of the many opportunities to connect math to other school subjects. Teachers also lack the necessary resources, the time to reflect, and the opportunities to share ideas with other teachers. Under such conditions it is difficult to create a sense of exploration, curiosity, or excitement, in the classroom. Although new standards alone cannot alter these conditions, they implicitly argue for everyone to make the work environment for teachers support professional activities. (NCTM, 1989, p. 254)

Similar ideas are further underlined and expanded upon by Sinclair and Ghory (1987), who lay out persistent organizational conditions that hinder improvement of learning for all students. They include large-group instruction, limited instructional approaches, inflexibility in school schedules, differential treatment for ability groups that leads to unequal opportunities to learn, misapplications of evaluation that reinforce a student's status as a successful or marginal learner, curriculum development and school governance that do not include teachers and parents, unionism that sets boundaries limiting teacher effort and reform, and insufficient and inequitable funding that restricts the scope of improvement to what can be managed in the current institutional organization. Any one of these alone could discourage a teacher to improve instruction in ways envisioned as a result of professional development, yet most of these conditions prevail in most schools in the United States.

The Chicago School Reform Study, which aimed to build the capacity of the school to work well as a unit and strive for a commonly shared purpose, reinforces the ideas of Sinclair and Ghory. The study identified three structural conditions that help strengthen the professional community: teaming, small school size, and school-based authority for the operation of the school and to implement a clear intellectual mission for the school. They warn that for schools without a vision and the social resources to act collectively, autonomy through deregulation is relatively ineffective (Newmann & Wehlage, 1997).

Clarke (1994) lists three major impediments to the improvement of mathematics teaching under the category of school organization: lack of structured time for individual and collaborative reflection and dialogue leading to a feeling of professional

isolation, mismatch between changes and teacher and/or student evaluation methods, and negative perceptions of the immediate community. To address the first two areas of need, the National Commission on Teaching and America's Future (1996) recommends that professional development become embedded in teachers' daily work through joint planning, research, curriculum and assessment groups, and peer coaching. Negative perceptions of the immediate community is another matter, however, and is perhaps best addressed with the help of skillful leaders. Sarason (1990), notes the importance of a leader in the organization as an active, knowledgeable participant in the improvement process, who understands the strength and depth of the belief systems and customary practices, who knows that change cannot be created by fiat and that a change in policy is not a change in practice.

Hiebert et al. (2002) make the case for efforts to more closely link researchers with practitioners' knowledge in order to create a countrywide system to steadily improve teaching in the future. They see converging efforts to change the culture of schools to places where teachers learn as well as students. "Over time, the observations and replications of teachers in the schools would become a common pathway through which promising ideas were tested and refined before they found their way into the nation's classrooms" (Hiebert et al., 2002, p. 12). Their proposal seems responsive to conditions that hinder the improvement of teaching identified by other researchers such as Clarke (1994).

Testing is sometimes cited as an impediment and sometimes an aid to improving learning. In spite of the long-term hope that when test results have significant consequences, teachers may change what and how they teach to help students respond

to the content and problems on the test, there is evidence that high stakes tests can narrow the curriculum (National Research Council, 1991; Shepard & Dougherty, 1991). Practices for the assessment of student learning can foster the improvement of teaching when student learning is made central to assessment reform. The Principles and Indicators for Student Assessment Systems developed by the National Forum on Assessment and signed by more than 80 national and local education and civil rights organizations supports a radical reconstruction of assessment practices. The seven principles endorsed by the Forum are:

1. The primary purpose of assessment is to improve student learning.
2. Assessment for other purposes supports student learning.
3. Assessment systems are fair to all students.
4. Professional collaboration and development support assessment.
5. The broad community participates in assessment development.
6. Communication about assessment is regular and clear.
7. Assessment systems are regularly reviewed and improved.

These principles are not often evident in statewide assessment systems. The recent federal legislation known as the No Child Left Behind Act, which was signed by President George W. Bush in January 2002, ensures that high stakes assessments will occur on an annual basis in reading and in mathematics beginning in 2006. Each school will be required to make "adequate yearly progress" as defined by state and federal formulas, so that by the year 2012, 100% of students will be proficient in reading and in mathematics. Schools that do not achieve the improvement expectations will offer parents the opportunity to remove their child to a "better" school in the district, that is, a

school that makes adequate yearly progress. Schools who continue to fail to make adequate yearly progress will be required to take a range of corrective actions, such as replacing staff or fully implementing a curriculum. The degree of importance upon average test scores demonstrates the value that is placed upon these scores as legitimate indicators of successful schools, without any consideration of the nature, quality, and validity of the instrument itself. The general public has great faith in test scores and does not understand that gains in test scores are not necessarily gains in learning.

There is little doubt that the testing instrument used to meet the requirements of the No Child Left Behind Act will either help or hinder the improvement of mathematics instruction at the elementary level, depending upon teachers' perceptions of its style and content. Whether the information gathered from the test results will improve student learning is another matter entirely, one that is not necessarily correlated to whether test scores improve.

Closing

This chapter developed a conceptual base to support the four research questions that guide this study. Through a review of related literature, it offered insight into the complexity of the process of improving mathematics instruction for all students. This literature review centered on four interrelated themes:

- Inquiry-based mathematics teaching and learning;
- Effectiveness of professional development of elementary mathematics teachers;
- Improvement of mathematics teaching;
- Organizational conditions that foster or hinder the improvement of teaching.

Since the study is built around the professional development that shifts a teacher from an authoritarian model of instruction based on transmission of knowledge to a student-centered practice featuring stimulation of learning and inquiry into student thinking, the review included an analysis of inquiry-based mathematics teaching and learning.

This review included studies that highlight the issues that contribute to the effectiveness of professional development of elementary mathematics teachers. Teachers' development of a stance of inquiry rather than one of answers for themselves and their students is essential to this study and this process was examined.

The review also examined the topic of improvement of mathematics teaching for better student learning, an important motivation for the study. The studies that were highlighted share the assumption that improvement of mathematics teaching will result in advancement toward the nation's commitment to mathematics for all and that the improvements extend beyond the implementation of new techniques and approaches.

The last section of this chapter provided a foundation from which to define the situation faced by many teachers in transition whose instructional practices are in the process of changing as a result of their participation in mathematics education reform activities. The review highlighted research that addresses the organizational conditions that foster or hinder the improvement of teaching.

CHAPTER 3

RESEARCH DESIGN AND PROCEDURES

This chapter reveals the research design, instruments, and the data collection and analysis procedures used in this exploratory descriptive case study. The research design and procedures are presented in three sections. First, the sample population that participated in the study is described. Second, the data sources are explained. Finally, the research design and methodology for data collection and analysis are presented.

Population

The study includes all of the teachers who agreed to participate in it from a pool of ethnically, racially, and geographically diverse elementary teachers of grades kindergarten through grade six who completed at least two intensive experiences between the years 1997 and 2000 at SummerMath for Teachers, a teacher education program at Mt. Holyoke College in South Hadley, Massachusetts. Forty-five teachers, forty-three of whom were female and two were male, were invited to participate. The invitees' participation in at least two experiences at SummerMath for Teachers distinguishes them as elementary teachers of mathematics who seem to have made a long-term commitment to improve their mathematics teaching practice. The two experiences that were common to the group were graduate level semester courses or summer institutes, each of which provided at least 48 hours of professional development for inquiry-based teaching and learning. The experiences exposed participants to alternatives to teacher-centered instructional models, involved them in exploring mathematics content and children's thinking in new ways, and engaged them

in a perspective of inquiry rather than one of answers. See Appendix A for a description of the SummerMath for Teachers program.

Eighteen elementary teachers out of the 45 who were invited to participate through a letter from the researcher agreed to participate. See Appendix B for the letter and agreement to participate form that was sent to the eligible teachers. Their agreement indicated that they allowed the researcher to read and analyze the writing that they did during their professional development experiences, that they would participate in a one-hour e-mail, telephone, or personal interview about their challenges in changing their mathematics practice and their characterization of their change process, and that they would allow the researcher to use the data collected through the writing and interview for the purpose of this research study and doctoral dissertation. Though 18 female teachers agreed to participate, the researcher was unable to reach two of them to schedule interviews. Ultimately, a total of 16 female teachers participated in the study.

The teaching experience of the participants ranges from 4 to 34 years, with the mean number of years of experience approximately fifteen, and only one teacher whose experience is less than seven years. See Table 1 for information about the distribution of years of teaching experience among participants.

Table 1

Teaching Experience of Participants

Years of Experience	0-10 Years	11-20 Years	21-30 Years	30-40 Years
Number of Participants	7	6	1	2

Although one participant is now an elementary curriculum coordinator, the remaining fifteen subjects of this study currently teach mathematics in the elementary grades, kindergarten through grade 6, in 11 different public school districts in four states: Massachusetts, Texas, New Hampshire, and New York. The districts represented are Easthampton, East Longmeadow, Holyoke, Northampton, Southwick, Springfield, West Springfield, Ware, and Westfield, Massachusetts; Exeter, New Hampshire; Fayetteville-Manlius, New York; and Houston, Texas. See Table 2 for information regarding the number of participants from each public school district.

Table 2

Districts Represented by Participants

District	Type of Community	Number of Participants
Easthampton, Massachusetts	Suburban	1
East Longmeadow, Massachusetts	Suburban	1
Exeter, New Hampshire	Rural	1
Fayetteville-Manlius, New York	Suburban	1
Holyoke, Massachusetts	Urban	2
Houston, Texas	Urban	1
Northampton, Massachusetts	Urban (Small)	3
Southwick, Massachusetts	Rural	3
Springfield, Massachusetts	Urban	1
Ware, Massachusetts	Rural	1
Westfield, Massachusetts	Urban (Small)	1

The communities represented by the districts range from small to large urban and rural to suburban. Most of the districts are small or large urban, and serve children who are racially, linguistically, and socioeconomically diverse. See Table 3 for the representation of teachers from each type of community.

Table 3

Types of Communities Represented by Participants

Type of Community	Number of Participants
Suburban	3
Urban	4
Urban (Small)	4
Rural	5

The subjects, all female, work in a variety of positions: regular education classroom teachers, special educators, multi-age primary classroom teachers, mathematics specialists, teachers of English language learners, a gifted and talented specialist, and a district elementary curriculum coordinator who taught second grade when she went through her two experiences targeted for this study. See Table 4 to learn the number of participants per type of position.

Table 4

Types of Positions Represented by Participants

Type of Position	Number of Participants
Classroom Teacher K-3	4
Classroom Teacher 4-6	4
Special Educator	2
Teacher of English Language Learners	1
Gifted and Talented Specialist	1
Elementary Mathematics Specialist	3
Elementary Curriculum Coordinator	1
Multi-age Classroom Teacher	2

Data Sources

The research data, comprised of quotations from teachers' writing while they were engaged in two in-depth professional development experiences and the researcher's notes from interviews, captures the process by which teachers reinvent their practice to create classroom cultures that promote inquiry-based learning and teaching of mathematics. One feature of this study that should be noticed is that the data represents teachers ideas over a three to five year period of time, since the writing was done between 1997 to 2000, two to five years before the interviews took place in 2002. This provides a longitudinal framework of time for the study. The description of the data consists of two parts: the first part describes the nature of the participants' writing and the second provides details about the interviews.

Participants' Writing

The writing that was used to collect data for analysis in this study was written between 1997 and 2000 in response to a range of assignments that required teachers to synthesize what they were learning and capture their thoughts in writing as they completed two or more of the following courses or institutes:

- Introductory summer institute for elementary teachers in 1997, 1998, or 1999;
- Advanced summer institute for elementary and secondary teachers in 1998, 1999 or 2000;
- Developing Mathematical Ideas (Schifter, Bastable, & Russell, 1999) academic year evening seminar for elementary teachers in 1997, 1998, 1999, or 2000.

The assignments were designed to maximize teachers' analysis of the mathematics they were learning and to bring into relief the implications of what they were learning on their mathematics instruction. At least four written responses to assignments were read and analyzed for each of the sixteen participants in the study. Only the excerpts of writing that referenced changes in instruction or reasons for those changes were utilized as data for the purpose of this study.

Interviews

The 16 interviews were conducted between May and July of 2002. Each interview took between 50 minutes and one hour. Eight participants were interviewed in person, seven were interviewed over the telephone, and one answered the researcher's questions through electronic mail. The in-person interviews took place in an assortment of venues, primarily public restaurants and cafés where it was possible to have and record conversations in a relaxed atmosphere. The first few minutes were spent getting acquainted and comfortable with one another. Participants were also provided with basic information about the study. Once the interview started, the tape recorder was activated. The researcher used the interview instrument that can be found in Appendix C as a template on which to take notes. Sometimes a question was repeated if the participant requested this, or if she seemed to lose her train of thought and not address the question in her response. On occasion, the interviewer would ask for clarification or more information. Telephone interviews took place in a parallel fashion, with virtually no difference in the pattern or length of interaction.

The interviews were taped, notes were taken, and then the researcher listened to the tapes while refining and expanding upon the notes that were taken. This process resulted in summaries of participants' responses, which are primarily transcriptions of statements that the researcher viewed as pertinent to the four interrelated questions that guide the study. All deletions were purely social interactions or references to personal circumstances. For example, interspersed throughout the interviews were brief references to children, spouses, relocation, health, recreation, the quality of the coffee they were drinking, or other topics. These were often sidebars that were important to develop a friendly conversation, but were unrelated to the research questions. See Appendix D for the notes from the interviews.

The notes were sent to the participants for their review and refinement. They were asked to make any changes necessary and confirm that the notes reflected the ideas that they intended to communicate. The revised notes were then returned to the researcher and used as the final responses to the interview questions.

Design and Methodology

The description of the design and research methodology of the study consists of two parts. The first part outlines the general aspects of the design that are anchored to each of the four research questions. The second part details the specific steps of the design that were taken to address each of the four research questions. These steps include methods for the collection, analysis, and organization of the data gathered through teachers' writing or through interviews.

General Aspects of the Design

This section outlines the aspects of the design that are related to each of the four research questions. Teachers who have engaged in long-term professional development in inquiry-based mathematics education were identified. The teachers participated in at least two of any combination of summer institutes and/or academic year courses between the years 1997 and 2000. Each of these experiences required teachers to write reflectively about what they were learning, demanded at least 48 contact hours of structured learning time in scheduled group sessions, and made them eligible to receive four graduate credits in mathematics education from Mt. Holyoke College.

All eligible teachers were invited to participate in the study. Their agreement indicated:

a) that they were willing to have the writing that they did about teaching, learning, and mathematics during their professional development experiences analyzed;
b) that they were willing to be interviewed by the researcher via telephone, in person, or via e-mail about challenges in changing their mathematics practice and their characterization of their change process;
c) that they were willing to have the interview be audio-taped (if telephone or in-person interview is conducted), summarized, and analyzed.

The researcher called all 18 teachers who signed the statement of agreement on the telephone to schedule an interview and choose whether they preferred to do the interview in person, over the telephone, or through e-mail. Two teachers could not be reached on the telephone or through e-mail even though more than one dozen efforts

were made over a two month interval of time. Sixteen teachers were successfully contacted and interviews were scheduled at their convenience.

Telephone and in-person interviews were conducted, taped, and summarized, while the e-mail response was left verbatim. Eight participants were interviewed in person, seven were interviewed over the telephone, and one answered the researcher's questions through electronic mail. The in-person and telephone interviews were taped, notes were taken, and then the researcher listened to the tapes while refining and expanding upon the notes that were taken. This process resulted in summaries of their responses, which are primarily transcriptions of statements that the researcher viewed as pertinent to the four interrelated questions that guide the study. All deletions were purely social interactions or references to personal circumstances. For example, interspersed throughout the interviews were brief references to children, spouses, relocation, health, recreation, the quality of the coffee they were drinking, or other topics. These comments were often sidebars that were important to develop a friendly conversation, but were unrelated to the research questions. See Appendix D for the notes from the interviews.

The notes were sent to the participants for their review and refinement. They were asked to make any changes necessary and confirm that the notes reflected the ideas that they intended to communicate. The revised notes were then returned to the researcher and used as the final responses to the interview questions.

The reflective writing assignments that were available from the SummerMath for Teachers program files that the teachers wrote while engaged in their professional development experiences were read through completely twice and all references to

changes in instruction and reasons for those changes, were highlighted. At least two assignments, often many more, were available from each experience. Reflective writing is a significant part of every professional development experience at SummerMath for Teachers. The writing that was used to collect data for analysis in this study was written between 1997 and 2000 in response to a range of assignments that required teachers to synthesize what they were learning and capture their thoughts in writing. The assignments were designed to maximize teachers' analysis of the mathematics they were learning and to bring into relief the implications of what they were learning on their mathematics instruction. At least four written responses to assignments were read and analyzed for each of the sixteen participants in the study. Only the excerpts of writing that referenced changes in instruction or reasons for those changes were utilized as data for the purpose of this study.

Specific Aspects of the Design

The second part of the design states each research question and delineates the research methodology, that is, the steps that were taken to address each of the specific questions. These steps delineate a plan for the collection, analysis, and organization of the data. Each question will be stated and followed by the steps that form the methodology.

Research Question 1

What changes in instruction do selected elementary teachers of mathematics report that they made as a result of participating in professional development for improving the teaching of mathematics?

Research Methodology. The following steps were taken in order to answer research question 1:

1) At least two papers that teachers wrote during each of their professional development experiences were analyzed for themes that emerge as they make sense of their experiences and consider applying what they are learning to their practice. The writing was read twice and references to changes in instruction that teachers considered making were highlighted each time. Each participant's writing over the two experiences was considered at once. That is, all sets of the first participant's writing was read and highlighted, and then all of the second participant's writing was considered, and so on. This same process took place regarding the data that was collected from the teacher interviews.

2) After the excerpts that refer to changes in instruction that each teacher considered making were highlighted, they were listed in a chart with three columns. The first column identified the teacher by number, the second column listed the relevant excerpts from the writing and the third column listed the relevant excerpts from the interview notes. See Appendix E for these excerpts.

3) The notes from the interview data were reviewed first and similar ideas were grouped together. A category or theme that describes what is the same about the ideas was chosen. Fifteen themes were identified. The theme was assigned a number that was used as coding and hand-written on the actual excerpt that fell under that category. This process continued until all of the ideas reflected in the notes were assigned a category.

4) Next the fifteen themes were condensed into five broader categories. The broader categories were successfully used to review the data for the third time. The five condensed categories were also used to code the excerpts from teachers' writing.

5) The data were then summarized in frequency tables. Each teacher, identified by number, was listed across the top of the table. The themes were in the first column. An 'x' was placed in a teacher's column in the appropriate row to capture the distribution of ideas among this cadre of teachers. Horizontal bar graphs that summarized the frequency of particular response themes were also created.

Research Question 2

What do selected elementary teachers of mathematics report are the reasons for the changes they made in instruction?

Research Methodology. The following steps were taken in order to answer research question 2:

1) At least two papers that teachers wrote during each of their professional development experiences were analyzed for themes that emerged as they made sense of their experiences and considered applying what they are learning to their practice. The writing was read twice and references to reasons for changes in instruction that teachers considered making were highlighted each time. Each teacher's writing over the two experiences was considered at once. That is, all sets of the first teacher's writing was read and highlighted, and then all of the

second teacher's writing was considered, and so on. This same process took place regarding the data that was collected from the teacher interviews.

2) After the excerpts that refer to reasons for changes in instruction that each teacher considered making were highlighted, they were then listed in a chart with three columns. The first column identified the teacher by number, the second column had the relevant excerpts from the writing and the third column had the relevant excerpts from the interview notes. See Appendix F for these excerpts regarding the reasons for changes in instruction that each teacher considered making.

3) The notes from the interview data were reviewed first and similar ideas were grouped together. A category or theme that describes what is the same about the ideas was chosen. Four themes were identified. The theme was assigned a number that was used as coding and hand-written on the actual excerpt that fell under that category. This process continued until all of the ideas reflected in the notes were assigned a category. The categories were used to repeat the process with excerpts from teachers' writing.

4) The data were then summarized in frequency tables. Each teacher, identified by number, was listed across the top of the table. The themes were in the first column. An 'x' was placed in a teacher's column and under the appropriate column whenever a teacher's data fit under a particular theme. Horizontal bar graphs that that summarized the total number of teachers whose writing quotation or interview data fit under a particular theme were created.

Research Question 3

What changes in instruction do selected elementary teachers report that they view as most effective for improving student learning?

Research Methodology. The following steps were taken in order to answer research question 3:

1) The notes from the teacher interviews were the sources of data for analysis. The researcher read and analyzed responses for themes that emerged as participants answered question 3: Which of the changes you made in instruction do you regard as most effective for improving student learning? The notes were read twice and references to the changes that teachers viewed as most effective were highlighted each time.

2) The highlighted ideas were then listed in a chart with two columns. The first column identified the teacher by number, the second column had the relevant excerpts from the interview notes. See Appendix G for these excerpts regarding the changes in instruction that each teacher perceived were most effective.

3) The notes from the interview data were reviewed and similar ideas were grouped together. A category or theme that describes what is the same about the ideas was chosen. Four themes were identified. Each theme was assigned a number that was used as coding and hand-written on the actual excerpt that fell under that category. This process continued until all of the ideas reflected in the notes were assigned a category.

4) The data were then summarized in frequency tables. Each teacher, identified by number, was listed across the top of the table. The themes were in the first column. An 'x' was placed in a teacher's column and in the appropriate. Horizontal bar graphs that summarized the total number of teachers whose interview data fit under a particular theme were created.

Research Question 4

What organizational conditions in their local elementary schools do selected elementary teachers of mathematics report helped or hindered their changes in instruction?

Research Methodology. The following steps were taken in order to answer research question 4:

1) The notes from the teacher interviews were the sources of data for analysis. The researcher read and analyzed responses for themes that emerged as participants answered questions 7 and 8: What organizational conditions in your elementary school helped you to improve your teaching in ways that you envisioned as a result of your professional development? What organizational conditions in your elementary school hindered your ability to improve your teaching in ways that you envisioned as a result of your professional development? The data from these two interview questions were dealt with separately but in a parallel fashion as described below.

2) The highlighted ideas were listed in a chart with two columns. The first column identified the teacher by number, the second column had the relevant excerpts

from the interview notes. See Appendix H for these excerpts regarding the organizational conditions that helped teachers to improve their instruction. See Appendix I for the excerpts regarding the organizational conditions that hindered teachers from improving their instruction.

3) The notes from the interview data were reviewed and similar ideas were grouped together. A category or theme that describes what is the same about the ideas was chosen. Ten themes were identified for organizational conditions that helped teachers improve their instruction. Each theme was assigned a number that was used as coding and hand-written on the actual excerpt that fell under that category. This process continued until all of the ideas reflected in the notes were assigned a category. Next the ten themes were condensed into four broader categories, which were used to code the data once again.

4) The results were then summarized in frequency tables. Each teacher, identified by number, was listed across the top of the table. The condensed themes were in the first column. An 'x' was placed in a teacher's column and in the appropriate row. Horizontal bar graphs that summarized the total number of teachers whose interview data fit under a particular theme were also created.

Closing

This chapter revealed the research design, instruments, and the data collection and analysis procedures used in this exploratory descriptive study. The research design and procedures were presented in three sections. First, the sample population that participated in the study was described. Second, the data sources were described.

Finally, the research design and methodology for data collection and analysis was presented.

CHAPTER 4

ANALYSIS AND DISCUSSION

This chapter presents and discusses the results of the study as they relate to the study's main purpose: to examine changes that the participating elementary teachers of mathematics who engaged in professional development for inquiry-based teaching and learning made in their ideas about effective mathematics teaching. The findings detailed in this chapter correspond to the four major research questions and are presented in the following order:

1) What changes in instruction do selected elementary teachers of mathematics report that they made as a result of participating in professional development for improving the teaching of mathematics?
2) What do selected elementary teachers of mathematics report are the reasons for the changes they made in instruction?
3) What changes in instruction do selected elementary teachers of mathematics report that they regard as most effective for improving student learning?
4) What organizational conditions in their local elementary schools do selected elementary teachers of mathematics report helped or hindered their changes in instruction?

The findings related to the first three research questions are presented in three separate sections. Those findings related to changes in instruction are in the first section, those related to reasons for those changes are in the second section, and those related to changes that are perceived as most effective are in the third section. The findings related to research question four are presented in two separate sections.

Findings related to organizational conditions that help changes in instruction are presented in the fourth section, while those related to conditions that hinder changes in instruction are described in the fifth section.

Findings Related to Changes in Instruction

Changes in instruction are descriptions that demonstrate a shift or reconstruction of a teacher's own instructional behaviors or the behaviors that he or she elicits from students. An example of a teacher describing changes in instruction is "…I feel I am more open to children's thoughts. I find myself asking my students how they solved a problem. Before I was too interested in the answer itself."

Fourteen teachers out of the sixteen participants expressed that their teaching was decidedly different, "like night and day," in comparison to their mathematics instruction prior to their professional development experiences as described earlier in this study. The other two out of the sixteen teachers who participated in the study explained that their instruction improved so that it more closely matched their beliefs about teaching mathematics. Their comments implied that prior to their professional development experiences at SMT they believed it best to teach in a manner similar to that modeled during their professional development experience. After their participation, however, they felt that they refined their instructional practices and strengthened their confidence and ability to respond to questioning colleagues and the parents of their students. One teacher said, "At first I had a lot of questioning from parents, but now I have the confidence to handle the questions."

All of the teachers seemed pleased to describe the nature of what they perceived were changes in their instruction as a result of their professional development experiences, and did so with a significant amount of detail and enthusiasm during the interview process.

The fifteen categories from interview responses that related to changes in instruction are shown in Table 5.

Table 5

Summary of Interview Responses Related to Changes in Instruction

Teacher #:	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Yes, instruction is different	x		x	x	x	x	x	x		x	x	x	x	x	x	x
Not different but improved		x							x							
No																
Themes (First Review)																
1. Emphasis on student thinking or understanding	x		x	x	x	x		x	x	x	x		x	x	x	x
2. Use of manipulatives	x						x	x		x		x		x	x	x
3. Multiple representations							x	x				x				x
4. Multiple approaches							x	x			x	x		x		x
5. Group work	x										x					
6. Opportunities to verbalize thinking	x				x				x		x		x	x		x
7. Increased writing	x															
8. Problem solving			x							x						
9. Increased teacher understanding of what the concepts are		x	x		x	x	x					x			x	
10. Decreased use of textbooks				x						x						x
11. Opportunity to explore				x						x			x	x	x	
12. Less teacher presentation				x			x			x	x		x	x		
13. Questioning techniques to foster inquiry (critical thinking)					x				x		x				x	
14. More purposeful mathematics objectives		x				x								x		
15. Emphasis on making connections														x		x

The fifteen themes identified in Table 5 were condensed to five prominent themes displayed in Table 6. Though interrelated to one another, each of these themes bears a different degree of emphasis and attention among the participants as they discuss and write about perceived changes in instruction, the reasons for those changes, and the changes that they regard as most effective for improving student learning.

Table 6

Original and Condensed Prominent Themes Related to Perceived Changes in Instruction

Themes After First Review	Condensed Themes After Second Review
1. Increased emphasis on student thinking or understanding	Student Thinking
2. Use of manipulatives	Student-Centered Activity
3. Multiple representations	Student-Centered Activity
4. Multiple approaches	Student-Centered Activity
5. Group work	Discourse
6. Increased opportunities to verbalize thinking	Discourse
7. Increased writing	Discourse
8. Increased problem solving	Student Thinking
9. Increased teacher understanding of the concepts	Mathematics
10. Decreased use of textbooks	Teacher's Role
11. Increased opportunity to explore	Student-Centered Activity
12. Less teacher presentation of procedures or ideas	Teacher's Role
13. Questioning techniques to foster inquiry (critical thinking)	Teacher's Role
14. More purposeful mathematics objectives	Mathematics
15. Emphasis on making connections	Student Thinking

Teachers' responses to interview questions related to changes in instruction were then coded according to these five prominent themes: Student Thinking, Student Centered Activity, Discourse, Mathematics, and Teacher's Role, as displayed in Tables 7 and 8. Table 7 indicates which theme was identified based upon the interview raw data for each teacher.

Table 7

Summary of Interview Responses Related to Perceived Changes in Instruction with Themes Condensed

Teacher #:	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Prominent Themes (Second Review – Themes Condensed)																
1. Student Thinking	x		x	x	x	x	x	x		x	x		x	x	x	x
2. Student Centered Activity	x		x	x			x	x	x	x	x	x	x	x	x	x
3. Discourse	x				x				x		x		x	x		x
4. Mathematics		x	x		x	x	x	x				x	x			x
5. Teacher's Role		x		x	x	x	x	x	x	x	x		x	x	x	x

Table 8 provides a frequency distribution by displaying the total number of teachers whose comments were categorized under each theme. Comments related to an increased emphasis on student thinking and understanding, student-centered activities, and a revised role for the teacher came up most often and equally as often during the interviews.

Table 8

Number of Teachers with Interview Responses in Each Category Related to Perceived Changes in Instruction

Number of Teachers	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1. Student Thinking																
2. Student Centered																
3. Discourse																
4. Mathematics																
5. Teacher's Role																

Table 9 summarizes the data related to perceived changes in instruction reported by the participating elementary teachers as identified through the interview data and the reflective writing that people did while they were engaged in the professional development experience.

Table 9

Summary of Changes in Instruction Reported by Elementary Teachers of Mathematics Through Interviews (I) and Reflective Writing (W) During Seminars

Teacher #:	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Prominent Themes																
Student Thinking	I W		I	I	I W	I W	I W	I W	W	I W	I W	W	I W	I W	I	I W
Student Centered Activity	I W	W	I W	I	W		I W	I	I W	I W	I W	I W	I	I W	I W	I W
Discourse	I W	W	W		I	W	W		I W		I W	W	I	I W	W	I
Mathematics		I	I		I W	I W	I W	I W		W		I W	I	W		I W
Teacher's Role	I W	W	I W	I W	I W	I W	I W	I W	I W	I W	I W		I W	I W	I W	I

Often, the perceived changes in instruction seem to evolve from the teachers' own experiences as learners of mathematics during their professional development experience as illustrated through this comment:

> My concept of doing mathematics and learning mathematics is very different especially since I have seen so many things this week. I used to believe that when I did math I knew the formula or because I could manipulate symbols and arrive at the correct answer. Because I had the right answer, I felt that I understood the concept, but now I see how far away from the truth I was...Through my own learning experience I can see how much the students need to explore, examine, play, and experiment with the new concept and they need to make up their own conclusions!

The five prominent themes, Student Thinking, Student Centered Activity, Discourse, Mathematics, and Teacher's Role, provide the organization for the presentation of the findings related to changes in instruction. It should be noted that when the acronym SMT is used, it refers to SummerMath for Teachers, the professional development program in which teachers in this study participated. Insertions by the researcher within the teacher quotations are intended to clarify what teachers are communicating and are placed within square brackets.

Student Thinking

The interest in developing reasoning and understanding through eliciting and analyzing student thinking is pervasive among the participants. The findings show that fifteen teachers were found to articulate an increased interest in the student thinking, understanding, and reasoning behind the mathematics that was being done by students in the classroom. They were no longer content with dutifully covering the curriculum objectives without the accompanying satisfaction of knowing their students understand the mathematical ideas involved. One teacher wrote,

> …so many things that teachers assume their children have learned turn out to be things that have only been taught…I need to spend more time on what is right with my students thinking rather than what I expect to find.

This indicates an interest in following students' logic and reasoning rather than looking for a correct or incorrect answer. Another wrote,

> …I used to be more limiting of the students. I had a preconceived notion of what I wanted. There was little to discuss. Now I'm more open to their thinking…The atmosphere now is more relaxed, not just drill sequence. Students need some drill, but now I am interested in developing more understanding of the math behind it.

Some of the pressures for coverage of content, as opposed to uncovering student understanding of that content are powerful, especially with current popular accountability systems. A teacher explains,

> …It just made me give more thought to what I am doing. It is really easy to be convinced by the people that you work with that coverage is most important rather than to develop understanding. Especially people who are thinking about MCAS [Massachusetts Comprehensive Assessment System] and needing to cover certain topics. Every time I go through a professional development experience it helps me affirm what I already believe. It encourages me to be more student-centered, even when there is resistance [among colleagues].

Related to the theme of student thinking, but categorized as a separate theme and addressed further in the discussion below under "Mathematics," is the idea that teachers understand that their own confidence with the mathematics content, especially the conceptual content beyond skills and procedures, is critical to the improvement of their instruction and their ability to explore student thinking. A teacher writes, "I realize that my comfort level with the math being explored in any lesson has tremendous impact on my ability to lead discussions and understand the thinking of students," and another says, "One issue of student learning that I considered during this seminar was the importance of having all the hands-on experiences to understand what is behind the convenient formulas—knowing WHY they work." They also acknowledge that their own experience learning mathematics influences how they teach:

> I just feel that the more I experience as a learner, the more I'm apt to provide meaningful experiences for my students. I'm more aware of the pitfalls, and can appreciate the different ways children attack a problem.

These findings indicate that developing student understanding through an interest in probing student thinking is a change in instruction desired by the participants. It might be inferred that this desire, which was launched through teachers' own

experiences learning mathematics within a new instructional model, must have a significant impact on the choices a teacher makes as she identifies goals, plans activities, interacts with students, and assesses their progress.

Student-Centered Activity

Student-centered activity was discussed by most of the teachers. Some teachers contrast student-centered practices such as using manipulatives, finding multiple representations for the same situation, and doing problems in more than one way, with their former dependence on following a textbook.

> I had courses around using manipulatives but the SMT courses helped me see how kids make sense of the operation and the inquiry process. I am more knowledgeable about how children think about math and how they rationalize math and math problems and the relationships between operations. There is more hands-on instruction and I use manipulatives in more meaningful ways than I did before. I am always looking for investigations for the students.

They used language like exploring and discovering when describing what they now wanted happening in their math class.

> It used to be learn the facts, drill, and memorization. There is a lot more problem solving and it's all hands-on. Kids get to make models, build things. The kids get to ask the questions. I try to spark their curiosity and not just telling them what we are doing--let them come up with it. For example if we are learning about multiplication I try to give them lots of problems to help them discover and understand the concept. I try to make learning fun by making some of the activities seem like games. I make sure the problems are relevant to the kids. I don't teach the book, the book is a tool and I use it to reinforce and practice.

Other comments, such as "…give them as many opportunities to explore, manipulate, discuss, and write, as possible…" and "… encourage students in discovering mathematical concepts through the inquiry and problem solving approach," and "…use pictures, words, numbers, and symbols to represent their thinking and why

things make sense," demonstrate the interest of these teachers to move toward a student-centered practice rather than one that centers of the one-way transmission of knowledge from teacher to student. These tactics might be considered outward signs of a changing pedagogy intended to stimulate rather than direct children's thinking: "They have a variety of ways to tackle a problem. This improves their persistence and ownership and support their own approach to thinking about a problem."

Some teachers seem to sense powerful qualities in the manipulatives as a way to achieve understanding of math ideas: "I approach each curriculum objective with an introduction using manipulatives. By using a hands-on approach students gain the knowledge and understanding needed for problem solving," and "…the learning of any concept in math needs to move from the concrete to the abstract." To further illustrate this point, a teacher writes, "…I plan to use manipulatives to model concepts and have children model their understandings as well—even those concepts that I assume most children understand because manipulatives are such a powerful tool."

Other teachers seem to see the use of manipulatives as one of many tools for providing opportunities for sense making and for identifying underlying structures within mathematics as a discipline. Some examples to illustrate this are: "I used to use manipulatives in a remedial mode; now I use them to help kids conceptualize what they are doing," and

> I used to teach procedures, now I have students develop strategies. Now I use games and multiple visual representations. For example, to teach multiplication facts I have students work with 4 by 6 arrays and other visual representations of four times six. I have them see patterns in a one to one hundred chart. I help them see the relationship between nine times one and one times nine.

This teacher is structuring opportunities for students to explore the idea of multiplication as counting quantities when one has same size groups rather than to have them simply memorize the facts in isolation from any meaningful context. She seems to be explaining that students need various contexts in order to make connections. In order to further explore this idea, see the three diagrams below for a few possible visual representations of four times six. Each representation is visually different from the next, yet each requires the computation of four times six to effectively find the total number of units.

Figure 1 displays an area model, the number of square units within a rectangle with a width of six units and a length of four units. Each of four rows constitutes a group of six square units, or, conversely, each of six columns constitutes a group of four square units.

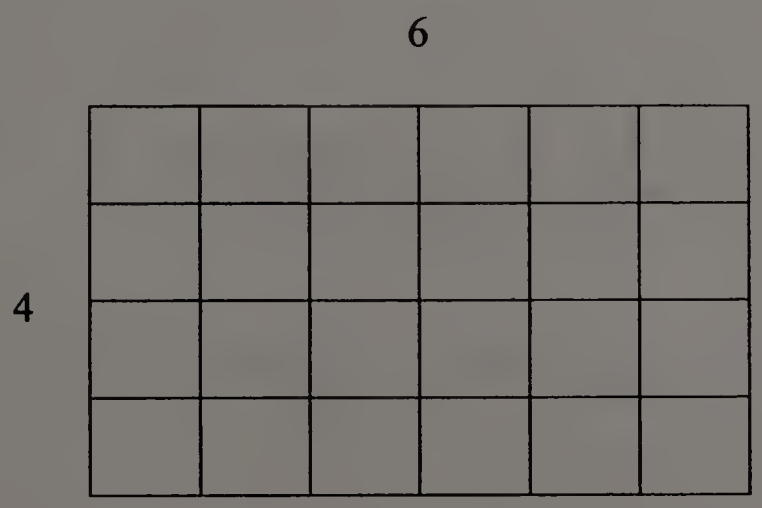

Figure 1. Four-by-Six Rectangular Array

Figure 2 shows a representation for a situation that is modeled by four groups of 6 objects.

xxxxxx
xxxxxx
xxxxxx
xxxxxx

Figure 2. Four Groups of Six Objects

Figure 3 displays the number of outfits, or permutations, that are possible when four different shirts are matched with six different skirts. Though the situation seems different from the others, it still requires counting the possibilities resulting from four groups of six. The four groups are established by the four shirts. Each of those groups has six different outfits that are created by the six different skirts.

A1	B1	C1	D1
A2	B2	C2	D2
A3	B3	C3	D3
A4	B4	C4	D4
A5	B5	C5	D5
A6	B6	C6	D6

Figure 3. Number of Outfits with Four Different Shirts (A, B, C, D) and Six Different Skirts (1, 2, 3, 4, 5, 6)

As teachers encourage children to explore multiple approaches to solving problems and to delve into the mathematical terrain, they themselves discover new connections, insights, and understandings: "…I was taught rotely and didn't understand. I had been teaching for rote learning. It helped me to learn more math when I started teaching differently." Teachers often brought up their own mathematics education as

lacking in opportunities to develop deep understandings as well as confidence in themselves as math learners:

> As the weeks have gone by I'm beginning to wish that my own geometry education during my high school years had offered the opportunities to explore mathematical ideas using manipulatives. At the time when I was a student, I was very fearful of math and often confused and lost when lessons were presented.

Another teacher underlines this point:

> Once again, I was a learner and I remembered how I wanted to understand things and how it led to more curiosity and questions. I also re-experienced the need to have tools to visualize what I was doing with numbers. The feelings of being challenged and achieving satisfaction from the problems helped me to remember how important it is for students to experience these feelings.

Student-centered activity, where children are actively involved in making sense of the mathematics they are learning in a variety of ways is a prominent theme in the findings related to changes in instruction that participants report that they made as a result of their professional development.

Discourse

The theme of discourse was identified to encompass ideas related to new kinds of interactions among students, between teachers and students, and the emphasis upon new kinds of opportunities for written and oral communication in math class. One teacher explains that in the past, when she explained everything in detail to students and then told them what to practice, "…there was little to discuss." Thirteen of the sixteen participants expressed an interest in creating a classroom culture where students talk and write much more in math class as a way of increasing their learning. This teacher seems to speak for them when she says:

> I think that when children feel safe in a classroom to share their ideas, then real learning is taking place. Children are free to share ideas, ideas which help other children gain information on a concept at a level of language that is clear to them. While direct teacher instruction is what we were brought up on, I think the kinds of instruction where learning comes from the sharing of students is far more beneficial. I think when children have the opportunity to show their class what the concept is all about in their own words, children tend to be more active listeners and more active participants in their learning.

Orchestrating discourse where students with different strengths and weaknesses share tentatively formed ideas and eventually solidify them often becomes a source of inquiry for teachers. How to elicit and value contributions as inclusively as possible is a challenge for them.

> There are a couple of areas where I would like to improve regarding my classroom instruction. One area is leading a class discussion about a problem or a situation. Very often I feel rushed or feel as though I am having to force the students to expand on their thinking verbally and participate in a classroom discussion… Also, I feel as though I am struggling with getting some upper elementary students to reflect on their thinking in their journal writing…I am wondering if there are certain prompts or lead questions that would allow for better success…My expectations are to learn not only about new teaching practices but also about the learning and understanding process of students.

Teachers own ideas about mathematics play a role in what gets discussed or reflected upon in writing: "I realize that my comfort level with the math being explored in any lesson has tremendous impact on my ability to lead discussions and understand the thinking of students." Another teacher writes,

> I learned that the way I approach, model, listen, and comment on children's work has a great impact on how freely they explore and express their findings…I just find that the more I experience as a learner, the more I'm apt to provide meaningful experiences for my students.

These findings indicate that changing the discourse in the classroom from a one-way transmission of knowledge to one where the interaction is from student to student,

student to teacher, as well as teacher to student, is another change in instruction that teachers are aspiring toward. Similarly, they are using writing to allow for personal reflection and interaction with the learners' own ideas. They seem to be challenging the notion of teaching as telling and learning as listening by acting on the principle of teaching as listening and learning as telling (Falk, 2000).

Mathematics

In addition to creating a student-centered instructional practice that fosters understanding in a setting where students discuss and write about their thinking, some teachers explained that they had different mathematics content goals for their students as a result of their professional development. Intertwined with their attention to how students were learning were new ideas about what students should learn. They attribute these new goals to their own powerful experiences revisiting the ideas of the elementary mathematics curriculum. A teacher provides a specific example of revisiting multiplication and describes relationships in mathematics that she hadn't noticed before.

> A breakthrough in my conceptualization occurred during a fourth grade array game, where small arrays were compared with larger ones by placing one on top of the other, We found you could cover a 8x5 array with a 4x5 and another 4x5 so 8x5=2(4x5). Or you can use (8x2)+(8x3), or be wild and use 3 or more: (4x4)+(4x4)+(1x8).

She is writing about the connections between geometric and algebraic representations that she had not previously explored. Some of the structures of mathematics related to the distributive property became apparent and sensible through her exploration. An eight be five array, which is covered by and therefore equivalent to

two four by five arrays, one shaded gray and one that is not shaded is shown in Figure 4 below.

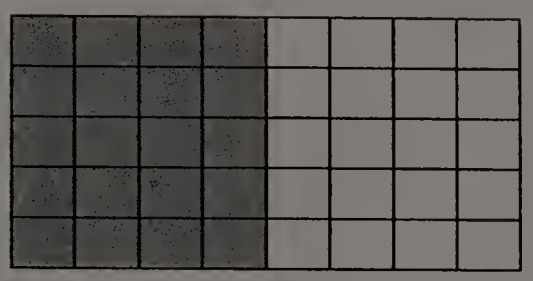

Figure 4. Eight-by-Five Array as Two Four-by-Five Arrays: 8x5=2(4x5)

An eight-by-five array can also be covered by one eight-by-two array, shaded gray, and one eight by three array, not shaded, as shown in Figure 5 below.

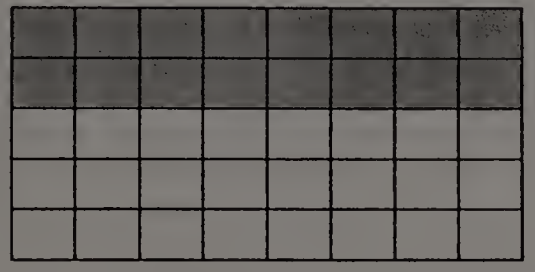

Figure 5. Eight by Five Array as One Eight by Two and One Eight by Three Array: 8x5=(8x2) + (8x3)

Another teacher talked about her excitement in creating a geometric representation with base ten blocks for the multiplication of larger quantities such as twelve times sixteen.

> I was a complete memorizer so digging into a simple problem like representing 12x16 with base ten blocks is very exciting. It allows you to see all the partial products [10x10, 10x6, 2x10, and 2x6]. This taught me so much about the meaning behind double-digit multiplication. A lot of kids really benefit from this and they end up using it more than the traditional algorithm. They see there is a lot of sense in multiplication. It

was exciting to show this type of thing to husband and family. I think a lot of people come out of high school pretty intelligent and getting by but they don't have a lot of meaning behind the math they know.

An example of the representation she describes, 12 x 16 = (10x10) + (10x6) + (2x10) + (2x6) is perhaps reminiscent of the procedures for multiplication of binomials as learned in high school and is shown below in Figure 6.

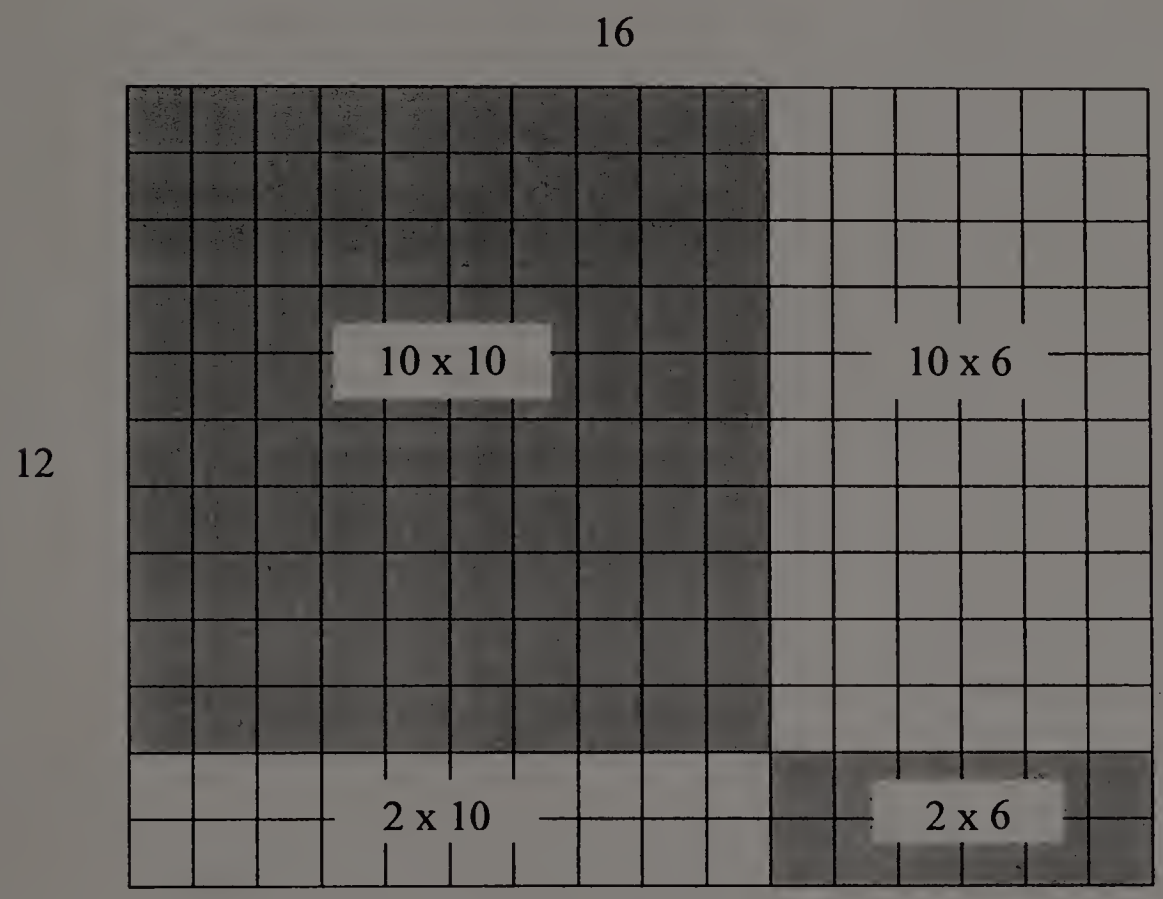

Figure 6. Twelve by Sixteen Array as Display of Partial Products:
12x16 = (10x10) + (10x6) + (2x10) + (2x6)

Opportunities to see connections between the mathematics that they are teaching and the mathematics that children will learn later in their mathematics education are important to teachers. They begin to see mathematics as patterns, relationships, and as a set of longitudinal strands of ideas that are tightly interwoven.

> Through studying the meaning of operations by actually doing the mathematics with others has prepared me with a better understanding of the mathematics and I can respond to my students' mathematical thinking

> with more confidence…Perhaps the most beneficial aspect of the SummerMath [for Teachers] program has been the ability to see the total picture of how students' develop their mathematical thinking toward the big ideas and principles of algebra. As I work with students at different grade levels, I can now understand what mathematical goal I am striving for instead of a mere daily objective. With this in mind my lessons are more flexible.

There was often discussion about refined goals related to number sense and the structure of the number system as well as a new interest in relationships within mathematics:

> Even when I was doing a lot of problem solving I wasn't focused on the structure of the number system. Now I have a different purpose to the problem solving. Even with older kids doing 64-59, they trade in order to do it rather than to use their number sense. They don't think about how it is on the number line from 59 to 64 in a more meaningful way than using our traditional algorithm. Now I help them gain that sort of number sense.

This teacher seems to be relaying that there is a new and deeper level of thought about quantities and operations that she now is interested in developing among students. She is noticing that students will do the well known series of steps that involves crossing out the six and putting a five in its place as a representation of taking one group of ten from sixty; then putting a one before the four and calling it fourteen as a representation of regrouping that one group of ten with the four and combining it to get fourteen; then, moving right to left, in direct contrast with how children learn to read from left to right, nine is taken from fourteen to get five as memorized through basic math facts; finally five taken from five is zero. Thus the answer is five. This series of steps is unsatisfying to the teacher, and she is interested in having students' thoughts take a more substantive, meaningful path, such as her own example of thinking about how far these numbers are away from each other on the number line. She is now interested in having students see the quantities and relationships involved. It seems she would prefer that

students see that fifty-nine is one away from sixty and then four more away from sixty-four, for a total difference of five.

Another teacher, who is hinting at issues of equity in mathematics learning for learners with unique strengths and intelligences, shares similar sentiments below.

> Trying to get kids to not just use the traditional algorithm, but encourage alternative methods. It is amazing how kids can come up with ways that work for them and are effective. It is so important that they truly understand the number system. I remember working with this fantastic artist. Even though she is bright and talented, when she learned math she couldn't see it. She could do math very easily and quickly in her head but wasn't allowed to. I gave her an example of what kids do in my class: Why would you do the traditional algorithm when 73 + 59 is combining the sums of 70+30 and 3+9 and thus holding on to the actual quantity of these numbers. She was so excited that kids could do this instead of the traditional algorithm.

The teacher quoted above may be considering the notion that allowing for reasoning and multiple approaches to calculating answers allows more types of learners with access to the ideas being considered by the class.

There was also an interest in delving well beyond paper and pencil manipulation of numbers as explained by this first grade teacher:

> The students do the basics but they also know how the numbers fit together. They pull numbers apart and put them together. They know their addition facts but also approach multiplication, division, algebra, and geometry. It is amazing. We do stories, find math everywhere, read books with numbers, and act out math situations. There is a constant connection with everyday life--math is everywhere.

Through these excerpts teachers acknowledge that their conceptions of mathematics itself as a discipline is more than a body of rules, skills, and procedures. They seem to be discovering structures, connections, and depth that they did not previously realize existed. They also seem to have reconsidered what is important to learn in math class and realize new possibilities for developing conceptual

understandings in areas such as number sense, operations, and other strands within mathematics.

Teacher's Role

Participants in this study expressed an interest in shifting their own role away from presentation of information to stimulation of learning:

> I try to spark their curiosity and not just tell them what we are doing--let them come up with it. For example if we are learning about multiplication I try to give them lots of problems to help them discover and understand the concept. I try to make learning fun by making some of the activities seem like games. I make sure the problems are relevant to the kids. I don't teach the book, the book is a tool and I use it to reinforce and practice.

Another teachers shares, "I try to move their thinking without leading them. I try to facilitate their group work and use the strategies kids are sharing to lead in my instruction. It is less teacher directed." For another teacher, the implication is that the teacher explains less and is more deliberate about when and why she explains:

> I like to give kids a chance to grapple with a problem before I give them any directions about how they might go about solving it, share ideas during and after the process, understand that there are many doors into how you solve.

A teacher writes, "My idea is to shift the responsibility for learning from the facilitator to the learner. I see this as empowering the student." Accomplishing this is often difficult and teachers present their struggles: "I'm still stuck on the teacher's role in guiding students without stifling their thought processes…If I don't 'feed' them hints or 'jump start' their thinking they don't know where to begin."

Participants' examination of their new role often results in an interest in exploring the effectiveness of their questions and responses to students. They struggle

with the subtle but important question of whether their questions aimed toward eliciting a particular response or whether their questions communicate that they authentically interested in what their students are thinking. Four teachers write about their new attention to the subtle nuances of their questions: "For most of my teaching I have asked questions to bring children to an answer. I thought I was finding out what he or she knew but I see what limits my questions put on his or her answer," and "I am thinking more about when to listen, when to question, and when to talk in order to support students' learning" and "I am wondering if there are certain prompts or lead questions that would allow for better success," as well as "There were times when I knew I was on the right track with my questioning. I still catch myself leading my students to where I want them to be."

The teacher's role in mathematics class, perhaps more than in any other content area, has traditionally been the explainer of steps and procedures, the determiner of what is right and what is wrong. These teachers are challenging this notion as they move toward a role that embodies teaching as stimulation of learning and structuring inquiry.

Findings Related to Reasons for Changes in Instruction

Teachers' responses related to reasons for changes in instruction fell within four main themes: their own learning experiences, their students' engagement, their students' learning, and their ability to serve diverse learners. Table 10 indicates the teachers whose responses from the interviews or from the data generated by teachers' writing fell under each theme.

Table 10

Summary of Interview (I) and Written (W) Responses Related to Reasons for Changes in Instruction

Teacher #: Prominent Themes	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1. Own Learning Experiences	I							W		W			I		I	I
2. Student Engagement	W	I	I		W		I	I		I						I
3. Student Learning			W	I	I W	I	I	I	I	W	W	I		W	W	I W
4. Diverse Learners		W	I		I						I			I		I

Table 11 reflects the number of teachers whose interview responses or writing excerpts related to reasons for changes in instruction fell under each theme.

Table 11

Number of Teachers with Responses in Each Category Related to Reasons for Changes in Instruction

Category	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1. Own Learning Experiences																
2. Student Engagement																
3. Student Learning																
4. Diverse Learners																

The four main themes provide the organization for the presentation of data related to the reasons for changes in instruction as reported by the participating teachers.

Teachers' Own Learning Experiences

For six participants, their own experiences learning mathematics within a new instructional model and with deeper content goals appeared to provide them with a reason for changing their instruction. One teacher explained, "The feelings of being challenged and achieving satisfaction from the problems helped me to remember how important it is for students to experience these feelings," while another teacher wrote,

> Through my own learning experience, I can see how much the students need to explore, examine, play, and experiment with the new concept and they need to make up their own conclusions! Learning mathematics is a process. It takes time and it is hard work. We need to work on teaching the student how to internalize their knowledge. Teach them to make that knowledge their own and use it in many different ways.

Another teacher admits she changed the way she taught math

> …because I experienced learning this way firsthand myself. I had to work problems out and experience how much more meaningful the learning was. I came to know the math in a different way. I had a much deeper understanding.

Often, teachers declare that their mathematics background is weak or otherwise unsatisfactory. One teacher explained, "… Math was a very difficult subject for me as a student. I was very intimidated by math. I found that the more I understood how math operated, the better I was able to help my students make their connections." Others say that though they were successful as math students during their own schooling, their knowledge lacks the depth they need to teach in ways that they envision. They acknowledge that they simply did not know what to do to improve student learning

prior to their own experiences in a different kind of mathematics learning environment: "Before these classes I just didn't consider other approaches much. It was a really a conscious effort to make some changes. I could see an increase in student involvement, interest, and enthusiasm for math. I could also see understanding was developing."

In summary, teachers in the study acknowledged that their own experiences learning mathematics within a new instructional model provided a powerful rationale for changing their instruction.

Student Engagement

Participants declared that student engagement in mathematics was an effective motivation for changing their instruction. They could

> … see the effect it has on their thinking, problem solving, and excitement. It's exciting to see what the kids can do. It is so powerful for them to use their minds that way…I can see the difference in their enthusiasm. They seem to enjoy it. They moan and grown if they miss math. They are working so hard.

Another teacher seems to be connecting with the idea that, when taught well, mathematics learning is a natural and human endeavor: She explains her reason for making changes in her instruction

> …to make kids problem solvers. It's not just me telling them how to get the right answer or concept, but allowing them to discover the concept themselves. Let them come up with their own ideas. Solving a puzzle sparks their interest. Students need to feel like they are part of the process so that they may take ownership of their learning.

It seems logical to these teachers that students who are engaged in the learning process will, in fact, learn more.

Student Learning

Thirteen out of the sixteen teachers stated very directly that their reason for changing their instruction was that they perceived that the changes improved student learning. This was clearly the most frequently stated reason for the changes in instruction. One teacher simply states: "Learning is obviously deeper when you discover or learn it on your own rather than have someone tell or teach you how to do it." Other teachers describe their reasons for changes in instruction below:

> While direct teacher instruction is what we were brought up on, I think the kinds of instruction where learning comes from the sharing of students is far more beneficial. I think when children have the opportunity to show their classmates what the concept is all about in their own words, children tend to be more active listeners and more active participants in their learning. Though the children in the cases, videos, and my classroom are young, they do have a wealth of knowledge and experiences that, when shared, enrich our thinking, teacher and student alike.
>
> Those are the changes I feel you get best payoff for. I believe children learn best when they are doing something that they are ready to learn. Knowing where they are and what they're ready to do increases your chances that they will be successful.
>
> Before these classes I just didn't consider other approaches much. It was a really a conscious effort to make some changes. I could see an increase in student involvement, interest, and enthusiasm for math. I could also see understanding was developing.
>
> After my first experiences trying I found kids having deeper understanding and tremendous ease in math. They have more ambition to persist in finding an answer and to solve complex problems. It allows a lot of different thinkers to succeed.

In a few cases, teachers felt that they had assessment data to substantiate that their perception was correct. For example, one teacher comments:

> The state test was really important because kids need to show their reasoning and their thinking and the changes I was making in instruction were a good match for the test. And it pushed people to change what they were doing in math class. On the Iowa test [Iowa Test of Basic Skills] students performed the same or slightly less in computation but better on concepts and problem solving. On the state test our kids do very well. The school that implemented changes in instruction performed higher than the

> school that didn't. This was significant because the lower performing school has the highest average income students and they usually perform better than anyone else.

Another teacher felt she had evidence because she teaches her students for two years and could see a difference in how much children remember from year to year.

> The results are obvious. It's rewarding. What's really nice about my job is I get to see kids for more than one year. It's amazing what they will remember from year to year. They really gain an in-depth conceptual knowledge. In the conventional textbook-based lesson, you are really pushing superficial knowledge in them, which they don't remember. It's not ingrained in them. I use a lot of Investigations [an innovative standard-based complete elementary math curriculum] plus my own activities that I designed based on what I learned. These kids are the ones that really struggle and it's rewarding to see them remember what they learned in a previous year.

Teachers' perceptions and evidence of improved student learning provide them with a strong rationale for continuing their changes in instruction and inquiry into refining their practice.

Diverse Learners

Six participants noted the advantages of their changes in instruction as supportive of learners with mixed abilities: "It allows a lot of different thinkers to succeed." Another teacher explains how her former approach, which was textbook based, pointed to the need to change: "Books don't meet a range of needs…There is not enough depth for the brighter kids. And kids who are not as strong at math, do things like copy, etc. and are basically lost."

Another teacher articulates that the inclusive instructional approaches honor the backgrounds of everyone in the learning community:

> After doing the work in the courses and looking at the way I work on math in the group settings, we all bring something to the table and our prior knowledge and experience helps move the ideas forward. My kids come into class with different ideas. I want their ideas to be the basis of their learning and not my ideas.

Finally, a teacher explains the need to challenge a watered down curriculum and instructional approach for students with special needs:

> It is more fun to teach this way. For Sped kids, I like it better. The kids get more out of it. Normally Sped kids just get to do calculations. They always had trouble with the word problems but now they have a way to solve them using different approaches. Some of them are going to struggle no matter what you do. Some of them have definite math disabilities.

Since meeting the needs of diverse learners is a common concern in education, the findings indicate that participants are making inroads into this equity challenge, and consider this an important reason to continue along a pathway of inquiry into changes in instruction.

Findings Related to Changes That Were Perceived as Most Effective for Improving Student Learning

Data related to the changes that were perceived as most effective for improving student learning were generated through participant interviews and fell into the five main categories connected to changes in instruction that were identified earlier in the study. The five main categories are: Student Thinking, Student-Centered Activity, Discourse, Mathematics, and Teacher's Role. Teachers found it difficult to identify the one change that they thought was most effective, and often their responses incorporated ideas that fell under more than one of the five interrelated themes. One teacher explains it well when she says:

> The overall approach is different. I don't think it's any one thing. But I think the use of manipulatives is very powerful. Once they get the concept with the manipulatives they go to the drawing. And I think that is a logical progression. [I think it is most effective] by the responses I see from the students.

It is evident that few of the teachers were able to identify the single change in instruction that they regarded most effective for improving student learning. In fact, one teacher's response included data related to each of the five prominent categories.

> Posing good questions; giving them problems that are challenging enough to solve, coupled with discussion and writing is the key to retaining the math. They need rich problems that can be solved using multiple approaches. For example, make a 5 by 5 square and figure out how many one inch square tiles are inside. Students are learning about measurement, one and two dimensions, looking at different squares and coming up with the formula, length times width, for finding the area of a square. There is thought, discovery, and struggling during their learning process. Your role is to ask questions and not tell the answer in order for it to work, and you have to know what you are looking for. You have to know the content you are aiming for. There are lots of ways of reinforcing the important math. Like in multiplication you need equal size groups and having them think about if there is another way to figure this out. The kids are really thinking. They seem to really take what they know and when they apply it somewhere else they are developing their thinking skills. You are teaching them how to attack problems and teaching them to think, analyze, evaluate. They need to be practicing those skills. Through the questioning, you are modeling this, leading them through the critical thinking process that should eventually become automatic for them

Table 12 shows a summary of interview responses about the changes in instruction each participant regarded as most effective for improving student learning.

Table 13 displays the same information within a frequency distribution chart that displays the number of teachers with responses in each category.

Table 12

Summary of Interview Responses Related to Changes in Instruction Regarded as Most Effective for Improving Student Learning

Teacher #:	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Prominent Themes																
1. Student Thinking			x	x	x	x	x		x		x		x	x	x	x
2. Student Centered Activity	x	x	x		x	x	x	x		x	x	x	x	x	x	
3. Discourse	x	x			x			x			x		x			x
4. Mathematics					x	x										
5. Teacher's Role					x					x			x			x

Table 13

Number of Teachers with Responses in Each Category Related to Changes in Instruction Regarded as Most Effective for Improving Student Learning

Frequency of Responses in Each Category	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Student Thinking																
Student Centered Activity																
Discourse																
Mathematics																
Teacher's Role																

The five main themes, Student Thinking, Student-Centered Activity, Discourse, Mathematics, and Teacher's Role provide an organization for the presentation of findings related to changes in instruction considered most effective for improving

student learning. Since teachers usually discussed more than one change as most effective, excerpts from the interviews could have been used as examples for more than one category of change. The researcher's decision as to what category should be assigned to an excerpt for the purpose of the narrative below was subjective. Other categorical choices could be justified.

Student Thinking

The theme of increased emphasis upon student thinking as most effective for improving student learning was evident during teacher interviews. Below are three examples to illustrate how teachers express their rationale for changes in instruction related to student thinking as most effective for improving student learning.

> Teaching for understanding is the most important. That is, having kids build their own learning, not me telling them. Because they promote understanding. Students end up with some strategies to figure out a problem. They are more confident. They love math. Three of my students (GIRLS!!) scored Advanced [the highest category] on MCAS. They had their own approaches that they could use.
>
> One thing I do is allow wrong answers or misconceptions to surface. It helps generate good discussion and challenge thinking. The hardest thing is not to give an answer until a student is satisfied. But you can't go on indefinitely. Learning to think about it is more important than the right answer. Learning how to think about it is critical. If we did that at an earlier age, our students would be better thinkers. They have to be actively engaged in the process. I think the approach requires that they become independent thinkers.
>
> The assessment piece in terms of knowing where the child is and starting from there. It doesn't mean limiting or not exposing kids to things but being very aware at all times. The child has the best chance of being successful.

The development of students' thinking and understanding was an important consideration for teachers as they decided what was important to learn and how it should be taught.

Student-Centered Activity

Moving toward student-centered activities that involve manipulatives and multiple approaches to solving problems, were often described as the change in instruction that is most effective for improving student learning of mathematics.

> Using manipulative tools and having students share their thinking with each other. These changes are most effective because they allow students to work through problems in a tactile way. Sharing strategies to problem solving allows students to see more ways to find solutions. The more strategies that are shared, the more opportunity for all students to find a way that makes sense for them.

> Active involvement in problem solving and using manipulatives [is most important] because it's more developmental and matches how people learn best—through inquiry and we know that memorizing isn't effective. We are not producing people, especially women, who like math. I used to dread teaching it and now I love teaching it. It's my favorite subject to teach and the favorite subject for my kids to learn.

> Working on more story problems and the approach to story problems has helped. We always talk about the various strategies they can use. I have this really old fifth grade math book and I adapt the problems to second and third grade. I have another resource with story problems and the kids draw it to find the answer. We always review the different things they can do when they get stuck. [I think it is most effective] because it's going to give them a better understanding. They had no idea where to start before and now they have ways to think about it.

Participants who emphasized increased student centered activity as the change in instruction that is most effective for improving student learning highlighted that what students did in math class had a major effect on what they learned in math class. They

perceived the importance of the use of manipulatives, multiple representations, multiple approaches, exploration, and discovery as essential to doing and learning mathematics. At the same time, however, this change was not considered in isolation. Instead, the data also related to the goal of developing understanding, changes in classroom discourse, changes in what mathematics is emphasized, and a different role for teachers.

Discourse

New images of classroom discourse changing from a one way transmission of knowledge from teachers to students to discourse where students interact with their own ideas, those of their peers, and those of their teachers through written and oral communication often came up as most effective for improving student learning. Some of the teachers who stated this when interviewed about the changes they felt were most effective are quoted below.

> Taking math from a solitary to a community activity. Using the varied grouping: partners, small groups, discussions and allowing children to invent their own strategies out of the realm of magic and into something kids can really understand. I fight to hold off the presentation of the conventional algorithm, because then they stop thinking, they shut down. It's kind of guiding kids so that they construct the knowledge themselves, rather than me being the source and showing them how to do everything. That is the biggest change. When you communicate it forces you to deeper understanding because of the thought process when you have to explain something to someone else. When you allow kids to invent strategies then they stop thinking about it as something you have to be shown how to do and they are not afraid to tackle any new problem.

> The most effective tool is discussion about what you are thinking and how you came to that answer. I carry this over to all my subjects.

> Memorization comes easier as necessary in the upper grades because the students have a better understanding about the numbers. I am not just stuffing information into their heads, I am watering their ideas and letting

them grow. In this program I can allow for their own research and help them construct meaning.

Allowing time to explore and to express and compare their thinking. When they need to express themselves they are more cognizant of what they are doing, They can also learn more from one another as they see different children's approaches, which are also correct. I see more ownership. They all become involved even if they don't have the language to say what they mean. They are sharing, representing themselves.

When I would watch videos at SMT seminars the students would say things like "I did my problem like John did," and I would say to myself that my kids would never listen to one another's ideas that way or say anything like that. But when I changed my instruction I was amazed that they actually did say stuff like that and refer to stuff that happened in past classes. It's been really fun to watch. It gives kids the opportunity to take ownership of their learning. Once they own it their confidence is built and their understanding does too. They have a deeper understanding when they build from what they already know. Something about sharing your thoughts and thinking clarifies it and also validates it.

Biggest thing is working in groups. I am now better at dealing with that. It is really hard for even adults to work in groups. Kids working with a partner is very powerful, talking to each other really helps them along, working together. I group in different ways, partners or threes, ability or different styles. Kids can learn a lot from one another but also trying to figure out a problem on your own, hearing your own words, having someone react to your comments and using manipulatives is so helpful. Having enough manipulatives instantly available so kids can make sense of the problem situation and make it meaningful to them. I know where every kid is now and what they need to work on. I can write a problem just for that child. Kids are as involved as I am in the learning process. I get really excited and they get really excited if we find a new way of doing something. The more concrete you can get at this age, the better they can retain it. On MCAS if a child is stuck, they can find a way to figure it out even if they forgot how to do it—if they are accustomed to reasoning.

In summary, improved quality and range of classroom discourse is considered by effective for improving student learning. Nevertheless, this change is not reported in isolation, but as one part of a comprehensive paradigm shift.

Mathematics

For some teachers, changes in instruction related to mathematics content were effective for improving student learning. As in the other categories, however, changes in mathematics are intertwined with other instructional changes as described in the quotations above. Teachers who believe changes in mathematics are most important for improving student learning have a view of mathematics as a system of tightly woven, sensible, interconnected ideas.

Teacher's Role

Teachers often communicated that an important change in instruction was that related to the teacher's role.

> Not telling them but getting them to discover the concept. Letting students find their own approaches to solving the problems. Before, kids were doing it because the teacher was telling them. Now, they are in charge of their learning process. It sparks their interest; they internalize it and make it their own. Once they have ownership, they can learn more easily. Our third grade math scores went from 70% proficient to 93% proficient. So I know it works.

By considering the data reported under the other categories, however, it is clear that the teacher's role cannot be separated from the other changes in instruction.

Findings Related to Organizational Conditions that Helped Changes in Instruction

Findings related to organizational conditions that helped teachers make changes in instruction in ways that they envisioned as a result of their professional development fell into five main categories that provide the organization for this section of Chapter Four. The two categories that were most frequently mentioned were principal support

and like-minded colleagues. The next most frequently mentioned was district level support followed by curriculum materials that matched their new approaches to teaching mathematics. One teacher mentioned the state testing system, which was in line with the changes in instruction she was making, as helpful. Table 14 displays a summary of data based on interview responses.

Table 14

Organizational Conditions in That Helped Improve Teaching in the Ways That Teachers Envisioned as a Result of Their Professional Development

Teacher #:	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Prominent Themes																
1. District Level Support			x	x		x			x		x			x		x
2. Principal Support	x		x	x	x		x		x	x	x		x		x	x
3. Like-minded Colleagues	x	x		x		x	x	x		x		x		x	x	x
4. Curriculum Materials	x		x					x				x	x		x	
5. Testing Systems						x										

Table 15 displays the same information within a frequency distribution chart that displays the number of teachers with responses in each category.

The data related to organizational conditions that helped teachers improve their instruction is presented in four sections: Administrative Support, Like-minded Colleagues, Curriculum Materials, and Testing Systems.

Table 15

Number of Teachers with Responses in Each Category Related to Organizational Conditions in That Helped Improve Teaching

Frequency of Responses in Each Category	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1. District Level Support																
2. Principal Support																
3. Like-minded Colleagues																
4. Curriculum Materials																
5. Testing Systems																

Administrative Support

The support of the school principal seems to be a key organizational condition for the teachers in the study. It is reasonable to assume that this is at least in part due to the principal's role as evaluator of teachers, and as such, has an opportunity to affect how the faculty teaches and what resources they are provided. Eleven teachers stated that this condition was important to their implementation of changes in instruction.

One teacher talked about the support of a principal who "…encouraged all teachers in our building to order and use manipulative tools, and often comes in to work with and listen to the thinking of our students in math classes." She also discussed the fact that many other teachers in her building have participated in similar professional development experiences because of the principal's encouragement and allocation of resources. Another teacher said,

> I worked with a principal who gave teachers a chance to really try research-based things. There was a lot of positive feedback from administration. There was availability of a lot of different ongoing professional development and for a while we had in-classroom support.

Another teacher wrote "I had a principal who was supportive of what I was doing. She gave me a lot of freedom in developing a math lab in what I wanted to do. I felt the freedom and support to try new things." Another expressed similar sentiments:

> I had a very supportive principal at my school. She encouraged me and helped me with innovative curriculum exploration. I also had 2 team members in 3rd grade who were also willing and eager to improve math instruction.

Teachers emphasized the importance of school-based administrative support in helping them have the funding for additional professional development, purchase of needed materials, and for taking risks as they tried out and reflected upon new instructional practices. District support, on the other hand, became more critical after they successfully implemented changes in instruction and were seeing the advantages for students to have continuity and consistency from one grade to the next.

Like-Minded Colleagues

Having a collegial system of support and inquiry was highly valued yet infrequently found among the participants. Having at least one colleague with which to share successes and challenges was noted as critical. One teacher explained the importance of

> …In-classroom support so you can process with someone after a lesson. Curriculum days for grade level support meetings…You need a resource teacher to go to when you have needs or questions about the program. Teacher leaders and leadership training is helpful so people are available to think through the unit with you. There has to be somebody to help

people stay on track. Release time with good professional development to revisit the math content is also paramount.

Curriculum Materials

Curriculum materials that were in line with the intended pedagogy were noted by six teachers as helpful to the improvement of teaching. Some teachers talked about things like manipulatives and resources, while others talked about the usefulness of the availability of a cohesive, interesting, and well-designed curriculum package that is built on student-centered activities, teaching for understanding, exposing student thinking, increasing communication, and connecting the big ideas of the mathematics curriculum. Teachers appreciated that they themselves could never design such a set of high-quality experiences and questioned whether they should be expected to do so given the complexity of teaching and the many decisions a teacher had to make over the course of a day. One teacher writes, "I eventually had access to good curriculum materials that supported me in teaching the way I wanted to teach. It was hard to develop good activities on my own." Another stated the importance of what seems obvious but is not always the case: "having all the materials—you have got to have what you need!"

Testing Systems

Externally designed tests, especially those administered by the state, were discussed by teachers from three different states: Massachusetts, New York, and Texas. Participants from Texas and New York found the state tests helpful, while those from Massachusetts who mentioned them felt they were a hindrance to improving their

instruction. One teacher discussed the advantages of a state testing system that she felt supported the way she was now teaching and helped to influence her colleagues to join her in her transition:

> The state test was really important because kids need to show their reasoning and their thinking and the changes I was making in instruction were a good match for the test. And it pushed people to change what they were doing in math class.

Findings that describe testing systems as a hindrance to improving instruction are discussed below.

Findings Related to Organizational Conditions that Hindered Changes in Instruction

Data related to organizational conditions that hindered teachers from making changes in instruction in ways that they envisioned as a result of their professional development fell into five main categories. Hesitant leadership from administration and lack of funding were most frequently and most intensively discussed, followed by lack of time, and scheduling problems. Finally, testing pressures for coverage of topics/skills and parental expectations were each brought up by two teachers. Table 15 displays a summary of this information, which is based on data collected from interview responses.

Hesitant Leadership

Although there was no evidence that the teachers in this study were prohibited from testing out their new instructional approaches in their classrooms, a frequent

Table 16

Organizational Conditions in That Hindered the Improvement of Teaching in the Ways That Teachers Envisioned as a Result of Their Professional Development

Teacher #:	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Prominent Themes																
1. Hesitant leadership		x		x		x	x	x	x		x			x		x
2. Testing pressures									x		x					
3. Lack of time/scheduling		x	x					x					x			
4. Lack of funding			x	x	x							x		x	x	
5. Parental expectations						x					x					

frustration was leadership that lacked conviction about the needed changes. This was associated with a resulting lack of consistency and continuity in mathematics curriculum and instruction from grade to grade and between feeding and receiving schools. Teachers surmised that administrators might have been reacting to an undercurrent of fear of the possibility of negative test results or political backlash from parents and community members. The following excerpts from interview notes reflect the way teachers discussed the impact of weak or inconsistent administrative direction.

> But there is a lot of lip service given and there is a lot of fear and coverage keeps winning out over approaching mathematics in a constructivist way. So if you have one teacher in a grade doing constructivist math, but the other five teachers in that grade aren't, you really aren't making any progress because the students' experiences are so inconsistent year after year.

> There was a good deal of support and yet a bit of hesitancy by top administration.

> It's frustrating when the whole district is not on board with this. Then you have kids who don't have the background that they need.

> There is little input from administration about the expectations for math instruction. I don't get the sense that there is any support from the principal for the way I teach math.
>
> …my school system was wishy washy about whether they were mandating the program or not. Teachers need clear direction from the administration, but it can't be totally top-down.
>
> The district has been lacking in a focus on mathematics. Math has been ignored. We had our first district wide PD in math this past January for the first time. There has not been a strong commitment toward math except to improve MCAS scores.

It should be noted that administrators' backgrounds in mathematics are, of course, similar to those of teachers. They also experienced mathematics learning through an authoritarian model based on one-way transmission of knowledge rather than a student-centered practice featuring stimulation of learning. Moreover, administrators are less likely than teachers to have engaged in professional development that would help them build a revised vision of what should be happening in math class. The hesitant leadership that teachers witness may be the result of the personal turmoil experienced by administrators who might believe in the new pedagogy on an intellectual level, yet in practice they struggle with how best to support the development of what they consider is uncharted territory.

Lack of Funding

Lack of access to adequate funds can quickly make implementation of instructional changes a colossal task. Teachers need the resources necessary to implement the changes. One teacher said, "Money [lack of it] is the big thing. It would

have helped to be able to buy all the materials I needed. It is very time consuming to make the games and activities."

Some teachers talked about their interest in continuing their professional development and having access to math resource teachers when they needed someone to help them advance their skills within their own teaching environment. One teacher explained, "Money is a big concern because I know other people who are interested but were not able to pay the tuition. I wanted to take a three-day course and it was denied." Another teacher, whose district supported a number of quality professional development initiatives, including the development of teacher leaders, for a period of a few years with grant funds, talked about the unfortunate way this support and the accompanying dialogue abruptly ended after the grant concluded. The retreat from ongoing support of teacher learning seems to inhibit the progress that teachers are interested in making as they seek to improve their instruction through professional development.

Lack of Time

Teachers found that the forty-five minute period of time typically allocated for mathematics was inadequate. In order to have time for solving problems, exploring ideas, working with materials, discussing in small and large groups, and writing reflectively about the experience, a minimum of one hour is required. "We need an hour to do this and some years/days I really can't find an hour. I don't know if the administration really understands this."

> Scheduling is a big hindrance. We have short blocks of time. Specials, assemblies, practices, chorus, pull-outs. I get totally frustrated. For example, one ESL student gets pulled out of math three times per week. I would hate to actually count how much teaching time I lose over the

> course of a year. I have tried a few innovations, but it would be easier if administration would help out.
>
> Although elementary teachers potentially have more control over their schedule than teachers at other levels, their efforts can be hindered by a lack of administrative commitment to protecting and enabling an appropriate allocation of time to mathematics instruction.

Testing Pressures

Externally designed tests, which are commonly utilized as supposed evidence of student learning, impact teachers' and possibly administrators' mindset about mathematics curriculum and instruction. One teacher described the mismatch between a "…traditional timed mid-year assessment instrument that tests basic computation…" which was chosen by her district's administration and the learning goals she had for her students. Though some teachers found state tests helpful in fueling school or district-wide changes in mathematics instruction, others identified state or other externally designed tests as a hindrance to improving mathematics instruction. One teacher communicates the conflicting ways that tests influence teachers:

> People are in a panic about MCAS and the accountability system. They perceive MCAS really promotes coverage so teachers are not likely to take risks. But I think kids have a better chance at figuring out the problems if they are confident thinkers and used to reasoning things through for themselves. I think a lot of good teachers have allowed themselves to be intimidated by MCAS. Actually, the open-ended questions require reasoning. It saddens me that more math teachers don't understand the MCAS tests were designed to encourage development of thinking skills.

Testing pressures clearly influence how teachers think about what is important to learn and how it should be taught.

Parental Expectations

Some teachers found that parental expectations that were in conflict with their changes in instruction hindered their progress. When "…parents were looking for worksheets on long division…" and instead might be seeing that their children are spending time creating a word problem for 37 divided by 5 where the correct answer is 8. (For example, how many cars will be needed to transport 37 children to a movie if 5 children fit in each car?) One teacher expressed the difficulty of dealing with emotional parents who "were questioning why math can't just stay the way it always was," especially when she was early in her own transition process. She found her confidence increased in direct proportion to her level of professional development and success with student learning.

Closing

Chapter 4 presented and discussed the results of the study as they relate to the its main purpose: to examine changes that the participating elementary teachers of mathematics who engaged in professional development for inquiry-based teaching and learning made in their ideas about effective mathematics teaching. The findings detailed in this chapter correspond to the four major research questions and are presented in five sections, findings related to changes in instruction, the reasons for those changes, the changes they perceive as most effective, organizational conditions that help the desired changes in instruction and the organizational conditions that hinder implementation of changes in instruction.

CHAPTER 5

SUMMARY AND SUGGESTIONS FOR FURTHER RESEARCH AND EDUCATIONAL PRACTICE

Introduction

This chapter has three main parts. First, a summary of the study is presented. Second, the major findings are highlighted. The chapter concludes with suggestions for further research and for educational practice.

Summary of Study

Changing the way mathematics is taught and learned from an authoritarian model based on one-way transmission of knowledge to a student-centered practice that accentuates stimulation of learning is a formidable undertaking. Teachers seeking to change their instruction typically do not have useful models from their own experiences as mathematics learners to help them develop a classroom culture of mathematical inquiry. Teachers in the United States today are grounded in many years of formative experiences that define mathematics as a body of rules and procedures, teaching as meticulous explanations, and learning as note-taking and memorization. Achieving the kind of changes called for by reform documents (National Council of Teachers of Mathematics, 1989, 1991, 2000) requires new learning on the part of teachers, taking place over a long period of time, with ample opportunities to test ideas and engage in professional discourse (Loucks-Horsley, 1997; Nelson, 1997).

The means by which teachers accomplish the kind of transformation that is required are not yet fully understood (Goldsmith and Schifter, 1997). The incongruity

between their new vision toward teaching mathematics and what they are able to actualize in their classroom can become a source of frustration when teachers are unable to teach the way they imagine is best. Understanding the different pathways that teachers take to change their instruction so that they may help students increase learning in mathematics is crucial for the reform of mathematics teaching. It is reasonable to suggest that this increased understanding of the pathways that teachers take will lead to better ways of helping teachers assist students to improve their learning in mathematics. This study contributes to the accomplishment of this important end.

The purpose of this study was to examine changes that elementary teachers of mathematics who engage in professional development for inquiry-based teaching and learning make in their ideas about effective mathematics teaching. This study told the story of what happened to sixteen elementary teachers who embarked on a quest to improve their mathematics teaching. It traced their ideas about how and why to improve instruction, identified their challenges with prevailing school organizational conditions, and reported perceived changes that were made in their teaching practices.

This exploratory descriptive case study aimed to identify key issues in the transformation of mathematics teaching. The research data, comprised of quotations from sixteen elementary teachers' writing while they were engaged in two in-depth professional development experiences and the researcher's notes from interviews, captured the process by which teachers reinvented their practice to create classroom cultures that promote inquiry-based learning and teaching of mathematics. The study included all of the teachers who agreed to participate in it from a pool of forty-five ethnically, racially, and geographically diverse teachers. Those who were invited

participated in at least two courses or institutes between the years 1997 and 2000 at SummerMath for Teachers, a teacher education program at Mt. Holyoke College in South Hadley, Massachusetts. All of the courses or institutes provide professional development for inquiry-based teaching and learning through exposing teachers to alternatives to teacher-centered instructional models, engaging teachers in exploring mathematics content and children's thinking in new ways, and fostering a stance of inquiry rather than one of answers. See Appendix A for a description of the SummerMath for Teachers program.

Teacher writings and interviews about effective mathematics instruction formed the data for analysis to answer four interrelated research questions:

1) What changes in instruction do selected elementary teachers of mathematics report that they made as a result of participating in professional development for improving the teaching of mathematics?
2) What do selected elementary teachers of mathematics report are the reasons for the changes they made in instruction?
3) What changes in instruction do selected elementary teachers of mathematics report that they regard as most effective for improving student learning?
4) What organizational conditions in their local elementary schools do selected elementary teachers of mathematics report helped or hindered their changes in instruction?

The major findings based on the collected data are summarized and presented according to the four research questions that guided this study.

Summary of Major Findings

The major findings summarized below correspond to the four major research questions and are presented in the following order: findings related to changes in instruction, the reasons for those changes, the changes they perceive as most effective, and organizational conditions that helped and hindered the desired changes in instruction. The intention is not to detail every finding, but to highlight those findings that seem compelling for gaining insight into professional development for elementary mathematics teachers.

Research Question #1

What changes in instruction do selected elementary teachers of mathematics report that they made as a result of participating in professional development for improving the teaching of mathematics?

After careful analysis of the data related to question 1, it became clear that the changes that teachers reported that they made fell into five interrelated categories: an increased emphasis on student thinking and understanding, increased opportunities for hands-on student centered activity, changes in classroom discourse, a new emphasis on the underlying structures of mathematics, and a revised role for the teachers themselves. It became apparent that a change in one category often affected another category. For example, teachers who change the nature of the classroom discourse (category #3) find that they increase their ability to gain insight into student thinking and to probe students' understanding (category #1). Another example might be that an increased emphasis on developing understanding (category #1) leads to new mathematics content

goals (category #4) and the provision of increased opportunities to write explanations and rationales for solutions to problems (category #3). Some teachers emphasized one category more than another in their writing and during their interviews.

Pervasive among the teachers was a new emphasis on developing mathematical understanding through probing student thinking. The findings showed that nearly all of the teachers articulated an increased interest in the student thinking, understanding, and reasoning behind the mathematics that was being done by students in the classroom. This indicates an interest in following students' logic and reasoning rather than simply looking for and responding to correct or incorrect answers.

Another area of reported change was an increase in student-centered activities. Participants explained that they wanted their students actively involved in learning mathematics rather than to be learning by passively listening, studying, and practicing. They expected their students to be using manipulatives, drawing diagrams, and exploring a variety of approaches to solving problems.

Classroom discourse, the ways that ideas are exchanged in participants' classrooms, is another area of reported change. Teachers said that there is much more writing and discussion in their mathematics classrooms and that there is much more to write and talk about than there had been in the past. There was an acknowledgement that children have their own ideas and methods to solve problems and that this is highly valued, both in and of itself, and for the purpose of increasing the depth and quality of what is learned. Teachers expressed inquiry into orchestrating discussions so that everyone's thinking is stretched and expanded. They also talked about the value of

children's interactions among themselves, and the importance of writing in the process of solidifying knowledge.

In addition to creating a student-centered instructional practice that fostered understanding in a setting where students discuss and write about their thinking, some teachers explained that they had different mathematics content goals for their students as a result of their professional development. Intertwined with their attention to how students were learning were new ideas about what students should learn. They attributed these new goals to their own powerful experiences revisiting the ideas of the elementary mathematics curriculum. The data show that participants discovered structures, connections, and depth in the mathematics content that they teach that they did not previously realize existed. They seemed to have reconsidered what is important to learn in math class and realized new possibilities for developing conceptual understandings in areas such as number sense, operations, and other strands within mathematics.

Finally, the data demonstrated that participants saw their own role as teachers in a new light. They expressed an interest in shifting their own role away from presentation of information to stimulation of learning, and often discussed their struggle with this shift. The teacher's role in mathematics class, perhaps more than in any other content area, has traditionally been the explainer of steps and procedures, the determiner of what is right and what is wrong. These teachers challenged this notion as they moved toward a role that embodies teaching as stimulation of learning, listening to student thinking, and using the information that they gather to structure further inquiry.

Research Question #2

What do selected elementary teachers of mathematics report are the reasons for the changes they made in instruction?

The data related to reasons for changes in instruction fell within four main themes: participants' own experiences relearning mathematics, their observation of increased student engagement in mathematics, their belief that student learning of mathematics improved with these changes, and the ability to meet the needs of diverse learners.

Teachers in the study acknowledged that their own experiences learning mathematics within a new instructional model provided a powerful rationale for changing their instruction. Revisiting the mathematics that they teach within a highly effective instructional environment that is drastically different from what they experienced during their own mathematics education created new images of what could be happening in their own mathematics class. Teachers, some of whom considered themselves weak students of mathematics, learned that they, in fact, had mathematical ideas of their own even when not shown or led to a particular method or procedure. As their own thinking became empowered, they realized that they could do the same for their students.

Another rationale for the changes in instruction was the increased engagement students demonstrated in learning mathematics as the changes were implemented. Teachers noted that more children were more fully engaged in the learning process and that learning mathematics seemed a natural, fun, and even exciting human endeavor.

Participants' perceptions and evidence of improved student learning provided them with a strong rationale for continuing their changes in instruction and inquiry into refining their practice. Some teachers noted evidence of improvements in students' ability to reason, solve problems, and remember the content that they learned from one year to the next.

Finally, the data showed that another reason for the changes in instruction was to more successfully serve children with mixed abilities, including children who excel, those who are in the process of learning English, and those with disabilities. The changes in instruction seemed to more appropriately address the unique strengths and needs of individual students. Providing access to the mathematics content to all of the learners in a teacher's charge is an important challenge for every educator. Making inroads in this area is obviously an essential goal at every level of the educational process in a democracy.

Research Question #3

What changes in instruction do selected elementary teachers of mathematics report that they regard as most effective for improving student learning?

Data related to the changes that were perceived as most effective for improving student learning were generated through participant interviews and fell into the five main categories connected to changes in instruction that were identified earlier in the study. The five main categories are: student thinking, student-centered activity, discourse, mathematics, and teacher's role. Teachers found it difficult to identify the one change that they thought was most effective, and often their responses incorporated

ideas that fell under more than one of the five interrelated themes. Seven out of the sixteen teachers had interview responses related to student centered activity and discourse as most important, six discussed that the emphasis on student thinking and understanding was most important, two teachers expressed that a new focus on the mathematics content was most important, while one teacher talked about the shift in the teacher's role as most effective. From the perspective of the researcher, however, it is not possible to extract from the data a single category perceived by the study participants as most effective for improving student learning. It might be inferred that the relationship, a smooth melding, among all of these categories is critical if not essential. It might be that a positive change in just one category without positive movement in the others, could, in fact, be problematic. A case in point would be instruction that increases student-centered activity while the teacher's role remains that of explainer and dispenser of information. Students might be using manipulatives while waiting for the teacher to show them how to get the answer rather than be authentically engaged in the process of inquiry.

Research Question #4

What organizational conditions in their local elementary schools do selected elementary teachers of mathematics report helped or hindered their changes in instruction?

Findings related to organizational conditions that helped teachers make changes in instruction in ways that they envisioned as a result of their professional development fell into five main categories. The two categories that were most frequently mentioned

were principal support and like-minded colleagues. The next most frequently mentioned was district level support followed by curriculum materials. One teacher mentioned the state testing system, which was in line with the changes in instruction she was making, as helpful.

During the interviews there was active reflection upon the need for district, school-based, and peer support of the changes in instruction. Teachers were adamant about the need for a shared vision for mathematics instruction among all constituencies. Curriculum materials and assessments that matched the changes they were making in instruction were also identified as organizational conditions that helped them improve their instruction.

Data related to organizational conditions that hindered teachers from making changes in instruction in ways that they envisioned as a result of their professional development fell into five main categories. Hesitant leadership from administration and lack of money were most frequently mentioned, followed by lack of time and scheduling problems. Finally, testing pressures for coverage of topics/skills and parental expectations were discussed.

Suggestions for Further Research

Consideration of the present study suggests changes that could possibly make similar studies more effective. In addition, several prospects for further research emerge from the findings of this study.

This study was designed to explore the ideas of teachers who have made a commitment to improve their instruction by participating in at least two in-depth

experiences over an interval of three years. Since these teachers agreed to participate in the study one could say that it is likely that they were experiencing satisfaction with their changes. Perhaps the teachers who were eligible to be part of the sample population but did not agree to participate experienced frustration or tension with their attempts to change. It would be interesting to compare or combine the results of this study with those of a study to explore the ideas of all of the teachers who may or may not have made a similar commitment to improve their instruction and who have engaged in only one or more experiences. The data from such a study would represent the ideas of teachers in general who perhaps try but do not necessarily persist in changing their approach to teaching mathematics.

Another possibility would be to replicate the study with teachers of grades seven through twelve or through generating and comparing separate data from teachers of primary, intermediate, middle, or high school levels.

In the future, this study could be replicated with a larger, more geographically diverse sample of teachers to acquire more generalizable results. It would be interesting to see if the same pathways emerged for many more elementary teachers of mathematics seeking to improve their instruction. This study could also be replicated with a broad range of professional development programs that were similar to one another, rather than just one program, as represented in this study. Criteria for program selection could be generated through the various national professional organizations.

This study might have been more effective if more teachers who were eligible could have been represented. Although only sixteen out of fifty-four subjects agreed to participate in both submitting their writing and interview data, many more might have

allowed only their writing to be reviewed. This would not require any contribution of interview time on their part. This change could provide data to substantiate or dispute the results of the study, especially with regard to questions one and two which focused upon the changes in instruction that teachers made and reasons why they made those changes.

The present study utilized one unproved interview instrument for collecting data. It would be helpful to see if a well-developed written questionnaire or an in-depth multi-session interview process, would generate similar data around the same four research questions.

The findings related to changes in instruction, the pathways of teachers seeking to improve student learning, provide ample opportunities for further inquiry. What are the common features of the classrooms of teachers such as those in the study? What is the perspective of their students? Does teachers' actual instruction match their written or verbal descriptions? Does student learning actually improve in classrooms such as those described in this study? What kinds of measures would provide evidence about student learning? How do the changes in mathematics instruction impact the teaching of other subject areas? What happens when a change in one of the reported categories does not result in a change in another?

The findings related to organizational conditions that help and hinder teachers who seek to improve their instruction also provide a fertile set of ideas to explore further in a related study. For example, instructional leadership at the school and district level that demonstrates understanding and public support of the instructional changes is clearly desirable. What kinds of experiences do administrators need in order to provide

the needed leadership? How do leaders gain the vision needed for effective leadership in mathematics education while other areas of instruction compete for their attention and expertise? Consensus among colleagues at the same grades and in feeding and receiving grade levels is also a critical condition. How does this kind of consensus become created? What conditions are necessary for this to occur? Parental understanding is also a concern. What type of communication and opportunities help parents to support the changes?

Another major finding that could be further explored is the surprising help provided by access to unconventional complete curriculum packages that are cohesive and continuous and designed for an inquiry-based instructional paradigm. This is surprising because it challenges the notion that curriculum is best developed by teachers, those who are closest to the learner. Why are such commercial curriculum packages helpful? What decisions must teachers still make as they use such curricula? How do teachers' beliefs about effective instruction affect the degree to which they implement these curricula as they are intended? When do packaged, innovative curricula get in the way of effective instruction and when are they helpful? What are the features of curriculum programs that are helpful to teachers who are in the process of changing their instruction?

Finally, an area fertile for further research is the mathematics content that teachers examine as a result of professional development. What is the mathematics content that teachers need to learn? How do we know what they have learned? How does it influence their instruction?

Suggestions for Educational Practice

Findings from this study suggest some recommendations for educational practice for institutions preparing elementary teachers of mathematics, for in-service teacher professional development programs, and for school policies and organizational structures.

The findings indicate the power of involving pre and inservice teachers in revisiting the mathematics content that they currently teach or will teach within an inquiry-based teaching and learning environment. Telling teachers how to teach mathematics within a prescriptive format is an ineffectual substitute for engaging them in learning mathematics content within an inquiry-based instructional model so that they might draw their own conclusions as to the implications for teaching mathematics. The findings suggest that improved mathematics learning for students with diverse strengths, needs, and abilities is the result of improvements in teaching. This highly desirable result provides a powerful motivation for ongoing efforts on all fronts to improve mathematics teaching.

The results of the study also suggest that the role of clear and forthright administrative vision, support, and leadership and consensus among colleagues are rare but highly desirable. Teachers understand that tackling the challenge of improving mathematics learning as individuals is exciting and rewarding yet insufficient to positively make a difference. They acknowledge a certain sense of futility for attempting to improve the overall mathematics education of all students without the explicit involvement at all levels of the educational system. Simple support, or letting it happen for those individuals interested in pursuing improvements in instruction, is not

enough for real advancement in mathematics learning. Structures for widespread and ongoing inquiry into the nature of the desired changes, a commitment to challenging conventional wisdom, assessment systems that are in-line with the changes, evidence of improvements in student learning, continuity and cohesiveness of curriculum, and a teacher evaluation system that supports the changes are conditions that are necessary.

The results from the study may also help school administrators understand the long-term nature of teachers' change process and its complexities and lead to their informed efforts to help teachers in transition. It is unrealistic to think that teachers could overhaul their instruction of elementary mathematics by attending a workshop series or even one two-week summer institute. Even though many teachers had been working on improving their instruction for five years, none of the teachers in the study indicated that they were finished learning about teaching mathematics or that all of their questions were resolved. Administrators must realize the importance of structuring an organizational climate that allows elementary school teachers to engage in professional dialogue around the challenge of improving mathematics learning.

The results of this study should also allow administrators to interpret what they see in teachers' practice so that they may better support them. Take, for example, an administrator that arrives in the classroom for a yearly formal observation. A teacher in transition is orchestrating a complex discussion about important mathematics, such as a debate about why the difference of numbers whose ones and tens digits are reversed (81-18 or 64-46) always give you an answer that is a multiple of 9. When the administrator says, "I'll come back later when you're teaching," his response is indicative of a lack of understanding of inquiry-based teaching and learning. The

teacher interprets this comment as a call for a lesson where she explains an idea or procedure within an authoritarian model of instruction. An evaluation system that has forms filled with indicators such as "plan book complete" or "adheres to lesson plan" is also problematic for a teacher who is primed to explore new practices. Finally, a budget that does not allow for purchasing materials to engage students in the ways that a teacher envisions can also crush a teacher's initiative, as can required textbooks that deaden students' curiosity.

Clearly, the best and most effective professional development will be wasted if organizational conditions prohibit the implementation of the newly learned practices in the classroom. High stakes math tests that focus upon arithmetic procedures will not support an instructional approach that elicits reasoning and problem solving. A school or district policy that insists that every third grader will be able to complete a certain number of multiplication facts correctly within five minutes will dampen any enthusiasm a teacher may have toward refining an inquiry-based model of teaching and learning. Each prevailing condition must be examined to determine the degree to which it helps or hinders the improvement of mathematics teaching.

Closing

This study has examined perceived changes that elementary teachers of mathematics who engage in professional development for inquiry-based teaching and learning make in their mathematics teaching. It told the story of what happened to selected elementary teachers who embarked on a quest to improve their instruction of mathematics. It identified their ideas about how and why to improve instruction,

described the help and hindrance of prevailing school organizational conditions, and reported perceived changes that were made in their teaching practices. By exploring the pathways of elementary teachers of mathematics teachers seeking to improve their instruction, the researcher hoped that the nature and quality of elementary students' learning in mathematics could be advanced. The researcher also attempted to impact institutions preparing mathematics teachers, in-service teacher professional development programs, and school policies and organizational structures so that they could more effectively prepare and support elementary teachers of mathematics.

This study has both theoretical and practical implications. Theoretically this study contributes to understanding inquiry-based teaching and learning and can encourage other scholars to conduct research into inquiry-based teaching and learning. It adds to the literature that helps teacher educators understand what teachers take from their learning opportunities and how teachers' new ideas influence their beliefs and actions. From a practical perspective, this study is of value because it serves as a starting point to consider the conditions necessary for the successful mathematics learning of student. Since the type of professional development program that forms the basis for this study has promise in changing teachers' thinking about effective mathematics instruction, then the results of this study can help to guide the design of professional development programs. The results should also help with the clarification of programmatic goals in preparing elementary mathematics teachers.

The study also recommends professional development that a school system could foster while being fully anchored in a challenge that teachers would be interested in tackling, that is, the challenge of improving mathematics learning. The results from

the study help school administrators understand the process of teacher change and lead to their informed efforts to help teachers in transition. It should allow them to interpret what they see in teachers' practice so that they may better support appropriate systemic organizational changes.

Understanding the different pathways that teachers take to change their thinking about effective instruction and to improve their practice so that they may help students increase learning in mathematics is crucial for the reform of mathematics teaching. It is reasonable to suggest that increased understanding of the pathways that teachers take to improve their instruction will lead to better ways of helping teachers assist students to improve their learning in mathematics. Providing all learners in a teacher's charge access to mathematics content and skills is an important challenge for every educator. Making inroads in helping all children learn mathematics well is obviously an essential goal at every level of the educational process in a democracy. It is hoped that this study contributes to the accomplishment of this important end.

APPENDIX A

DESCRIPTION OF SUMMERMATH FOR TEACHERS AT MT. HOLYOKE COLLEGE

DESCRIPTION OF SUMMERMATH FOR TEACHERS

AT MT. HOLYOKE COLLEGE

SummerMath for Teachers is an in-service teacher education program committed to the principles of constructivism. Since 1983, hundreds of teachers from across the United States have reexamined their definition of what it means to do mathematics and their system of beliefs about teaching and learning through this program. They often leave the experience with a renewed spirit of inquiry about their profession and an eagerness to deepen their understanding of the content that they teach. It is common for teachers to return to the program to continue to pose questions and pursue growth toward the kind of mathematics instruction that they envision is best for their students.

On the SummerMath for Teachers website, http://www.mtholyoke.edu/proj/SMT/, Director, Virginia Bastable and Assistant Director, Jill Lester, describe the goals of the program as:

> …to give teachers the opportunity to investigate the mathematical ideas that are embedded in the curriculum that they currently teach, to model the kind of classroom instruction and assessment that is espoused in the Curriculum Frameworks and NCTM Standards, to engage teachers in a process of reflection on the nature of learning so that their experiences as students in our SMT classrooms will inform their practice as teachers. The power of SMT programs is contained in the way these goals are totally integrated. Teachers are not told how to teach. They have the opportunity to experience learning in a classroom which supports the development of conceptual understanding, then reflect on their own experiences in the course, and finally consider the implications of their experiences for their own classrooms.

The participants of the study *Pathways of Elementary School Mathematics Teachers Seeking to Improve Their Instruction through Professional Development* completed two or more of the following in-depth professional development experiences:

- Introductory summer institute for elementary teachers in 1997, 1998, or 1999;
- Advanced summer institute for elementary and secondary teachers in 1998, 1999 or 2000;
- Developing Mathematical Ideas (Schifter, Bastable, Russell, 1999) academic year evening course seminar for elementary teachers in 1997, 1998, 1999, or 2000.

Typically a two-week summer institute or sixteen-week academic year course yields 4 graduate credits in mathematics education from Mt. Holyoke College. Though these experiences may be focused on different content strands within mathematics, they consistently provide alternatives to conventional instructional models by engaging teachers in exploring mathematics and children's thinking.

APPENDIX B

LETTER AND AGREEMENT TO PARTICIPATE STATEMENT SENT TO ELIGIBLE TEACHERS

LETTER TO ELIGIBLE TEACHERS

April 25, 2002

Dear Colleague,

I am writing to invite you to participate in a research study, *Pathways of Elementary School Mathematics Teachers Seeking to Improve Their Instruction through Professional Development*, which aims to identify issues related to the transformation of mathematics teaching. The research data will capture the process by which teachers reinvent their practice to create classroom cultures that promote learning mathematics for understanding. The study will include ethnically, racially, and geographically diverse teachers from a pool of forty-five who participated in two or more SummerMath for Teachers program experiences since 1997. Individual participants and their schools will not be named, but geographic regions and the types of community in which participants teach will be established.

Your participation in at least two experiences at SummerMath for Teachers distinguishes you as a teacher who actively seeks to change your mathematics teaching practice. I am requesting your agreement to

i) allow me to read and analyze the writing that is on file and which reflects your ideas about teaching during your professional development experiences;

ii) participate in a one-hour e-mail, telephone, or personal interview about your challenges in changing your mathematics practice and your characterization of your change process;

iii) allow me to use the data collected through the writing and interview for the purpose of this research study and doctoral dissertation.

Your contributions to this study should result in increased understanding of the different pathways that teachers take in helping students improve learning in mathematics. This could lead to informed efforts to reduce the tension for teachers in transition. I hope that you are willing to participate, which you signify by signing on the attached form. I will be pleased to answer any questions you may have about the project at any time. I have included a self-addressed, stamped envelope for your convenience. Your response to this request by April 30, 2002 will be most appreciated. If you agree to participate, I will contact you within two weeks to set up an appointment at your convenience.

Sincerely,

Donna M. Scanlon

AGREEMENT TO PARTICIPATE STATEMENT

I hereby agree to participate in the research study, *Pathways of Elementary School Mathematics Teachers Seeking to Change Their Instruction.* I understand that my participation will entail an e-mail interview, personal meeting, or telephone interview with the researcher during a previously agreed upon time and that the researcher will read and analyze a copy of my papers that I wrote as a participant in SummerMath for Teachers programs. I understand that my identity will be protected in all reporting, that I may withdraw from part or all of this study prior to its publication, and that I have a right to review the material.

Signature_____ Date _____

Printed Name _____ E-mail address _____

Address _____

Telephone number _____

APPENDIX C

INTERVIEW INSTRUMENT

INTERVIEW INSTRUMENT

Identifying Information ✓ Name ✓ District ✓ What grade level do you teach? ✓ What is your position? (ESL, etc.) ✓ How long have you been teaching? ✓ What type of community do you teach in? (Rural, urban, suburban)	
1. Why did you decide to pursue changing the way you were teaching math?	
2. Do you think your instruction is different now in comparison to your instruction prior to your professional development experiences at SMT? In what ways?	
3. Why did you make those changes?	
4. Which of the changes you made in instruction do you regard as most effective for improving student learning?	
5. Why do you think these changes are most effective?	
6. How would you characterize your change process? What was it like for you?	
7. What organizational conditions in your elementary school helped you to improve your teaching in the ways that you envisioned as a result of your professional development?	
8. What organizational conditions in your elementary school hindered your ability to improve your teaching in the ways that you envisioned as a result of your professional development?	

APPENDIX D

NOTES FROM INTERVIEWS

NOTES FROM INTERVIEW OF PARTICIPANT #1

1. Why did you decide to pursue changing the way you were teaching math?	Through team teaching with [Teacher X] many years ago--she encouraged me to participate in the SummerMath for Teachers [SMT] institute at Mount Holyoke College. I have been taking SummerMath seminars over the past 11 years. It has helped me to stop being a math phobic, thus giving me the opportunity to actually learn what makes math work.
2. Do you think your instruction is different now in comparison to your instruction prior to your professional development experiences at SMT? In what ways?	Yes, I feel that my math instruction focuses more on helping students make connections with their math thinking through the use of manipulative tools, group work, class discussions and journaling.
3. Why did you make those changes?	Because these changes have helped me to understand math better. Math was a very difficult subject for me as a student. I was very intimidated by math. I found that the more I understood how math operated, the better I was able to help my students make their connections.
4. Which of the changes you made in instruction do you regard as most effective for improving student learning?	Using manipulative tools and having students share their thinking with each other.
5. Why do you think these changes are most effective?	These changes are most effective because they allow students to work through problems in a tactile way. Sharing strategies to problem solving allows students to see more ways to find solutions. The more strategies that are shared, the more opportunity for all students to find a way that makes sense for them.
6. How would you characterize your change process? What was it like for you?	The first SummerMath seminar back in 1990 was painful for me. This first experience at times was frustrating, but I think it was because of the lack of feedback I was receiving from the instructors. I think I began to change my attitude about math when I decided to try the SummerMath seminar a second time under different instructors. This second seminar was a refresher course of the first, but the experience of better feedback for questions I had has helped me see the importance of talking about math. That is why I think class discussions are so beneficial for students.
7. What organizational conditions in your elementary school helped	Although the Addison-Wesley Mathematics series has been adopted by our school, my principal has been very supportive of my using Investigations. The

	you to improve your teaching in the ways that you envisioned as a result of your professional development?	Investigations series has allowed me to present math concepts in ways that encourage group work and discussion. She has also encouraged all teachers in our building to order and use manipulative tools, and often comes in to work with and listen to the thinking of our students in math classes. Also, many other teachers in our building have been encouraged to participate in SummerMath Institute seminars.
8.	What organizational conditions in your elementary school hindered your ability to improve your teaching in the ways that you envisioned as a result of your professional development?	I do not feel that our school hinders our professional growth in any areas of learning. Rather, our school system encourages our growth to be professionally developed and encourages us to share our knowledge with staff members.

NOTES FROM INTERVIEW OF PARTICIPANT #2

1.	Why did you decide to pursue changing the way you were teaching math?	I didn't feel comfortable using old textbooks. It was very boring. I used Marilyn Burns resources and self-developed curriculum units. I used the textbook as a guide to show me what kids were supposed to be learning at that grade level. I always liked math as an elementary student but as I got to Algebra I don't know what happened. Math was one of the only things I was really good at until that point. When I was getting my master's degree I learned there was a lot more to math than just being quick with figures and numbers and that there was a lot I was missing. I began to understand what multiplication was and what division was in an entirely different way. I began to understand where the algorithm comes from. I really haven't changed the way I teach math, I've just developed the way I think about math.
2.	Do you think your instruction is different now in comparison to your instruction prior to your professional development experiences at SMT? In what ways?	I'm constantly learning and am now much more confident. I use Investigations now [an innovative curriculum program]. I follow that pretty much but it's my 6th year of using it so I can come up with my own examples. If I'm doing something in a unit and the kids aren't really getting it I can add to that curriculum because I've gotten better at expanding the math and I know what the kids are supposed to get out of it. Kids don't always need to master it because it may come back

	around again, later in the unit or in another later unit.
3. Why did you make those changes?	I see the effect it has on their thinking, problem solving, and excitement. It's exciting to see what the kids can do. It is so powerful for them to use their minds that way. When I first started doing this hard mathematical work with fifth graders who were not exposed to it in 3rd and 4th they would ask me for ditto sheets. It was really hard. I put a lot of energy into it every single day. Now I have 4th graders. Teaching 3rd and 4th graders I can see the difference in the younger kids. I can see the difference in their enthusiasm. They seem to enjoy it. They moan and grown if they miss math. They are working so hard. But maybe now I have had more experience, too.
4. Which of the changes you made in instruction do you regard as most effective for improving student learning?	The biggest thing is working in groups. I am now better at dealing with that. It is really hard for even adults to work in groups. Kids working with a partner is very powerful, talking to each other really helps them along, working together. I group in different ways, partners or threes, ability or different styles.
5. Why do you think these changes are most effective?	Kids can learn a lot from one another but also trying to figure out a problem on your own, hearing your own words, having someone react to your comments and using manipulatives is so helpful.
6. How would you characterize your change process? What was it like for you?	Taking these courses is always helpful but I do complain a lot to myself because it is so much work. When I do these papers, when I write about what happened, I can't believe the insights I gain. It's so satisfying. It's so important to be involved with other teachers; it must be ongoing. It's ideal to go through this with the people that you work with rather than those that you don't know. You need a professional community. You have to find time to be collegial. You need like-minded teachers in math. We would meet as a math group once a month. These things really worked for me in my development. It has to be ongoing. Not a one-shot workshop where you are then left on your own.
7. What organizational conditions in your elementary school helped you to improve your teaching in the ways that you envisioned as a result of your professional development?	A good deal of collegiality is helpful.
8. What organizational conditions in your	It's frustrating when the whole district is not on board with this. Then you have kids who don't have the

elementary school hindered your ability to improve your teaching in the ways that you envisioned as a result of your professional development?	background that they need. We give the Iowa test early in the year. Some kids were really mad that I gave them a test on what I didn't teach them. It was all calculations and it doesn't match what we do. The last MCAS test was really long and hard, but the kids put in so much effort. They worked on it for hours and it was supposed to be a one-hour test. Where does their persistence come from? It doesn't come from the textbook math. The district is inconsistent in implementation of this even though it the official math curriculum. But teachers need to want to do it. They need to see and understand the many benefits of a constructivist math approach. It is a lot more time-consuming to prepare. We need an hour to do this and some years/days I really can't find an hour. I don't know if the administration really understands this.

NOTES FROM INTERVIEW OF PARTICIPANT #3

1. Why did you decide to pursue changing the way you were teaching math?	I didn't like the way I was taught math though I did well in it. I became really curious when a teacher from my school became involved with SMT, and an SMT staff member used to visit that teacher's classroom. I decided to see what it was all about, and at first perceived that there was never closure, very open-ended, and frustrating. There was never a conversation, no closure, and only questions in return to the children's questions. No conventions were ever taught. Years later, I was encouraged to try it again, and decided to do so and now see it as more balanced and reasonable.
2. Do you think your instruction is different now in comparison to your instruction prior to your professional development experiences at SMT? In what ways?	Yes, but it's different every year anyway. SMT helped me understand how many different ways people can understand something. Textbooks are deadly so we make up our own problems that are relevant and meaningful to the children. Kids make up their own problems now too. It is much more meaningful and experiential. We also do much more with geometry than before. Geometry used to just be what they could get out of logo.
3. Why did you make those changes?	Books don't meet a range of needs I was really glad to teach a different way, a way that is not boring. There is not enough depth for the brighter kids. And kids who are not as strong at math, do things like copy, etc. and are basically lost.
4. Which of the changes you	Having enough manipulatives instantly available so

	made in instruction do you regard as most effective for improving student learning?	kids can make sense of the problem situation and make it meaningful to them.
5.	Why do you think these changes are most effective?	I know where every kid is now and what they need to work on. I can write a problem just for that child. Kids are as involved as I am in the learning process. I get really excited and they get really excited if we find a new way of doing something. The more concrete you can get at this age, the better they can retain it. On MCAS is a child is stuck, they can find a way to figure it out even if they forgot how to do it—if they are accustomed to reasoning.
6.	How would you characterize your change process? What was it like for you?	It was fun actually. I didn't feel intimidated by the math. I was a strong math student. I wanted to understand it more deeply and enjoyed using graph paper and other visual aids to figure out how things worked and why.
7.	What organizational conditions in your elementary school helped you to improve your teaching in the ways that you envisioned as a result of your professional development?	Administrators encouraged us, paid for the courses, a group of us was going and this was very supportive. We started to get lots of manipulatives for our classroom.
8.	What organizational conditions in your elementary school hindered your ability to improve your teaching in the ways that you envisioned as a result of your professional development?	Scheduling is a big hindrance. We have short blocks of time. Specials, assemblies, practices, chorus, pull-outs. I get totally frustrated. For example, one ESL student gets pulled out of math three times per week. I would hate to actually count how much teaching time I lose over the course of a year. I have tried a few innovations, but it would be easier if administration would help out. I always get good evaluations but they don't even mention math.

NOTES FROM INTERVIEW OF PARTICIPANT #4

1.	Why did you decide to pursue changing the way you were teaching math?	A long time ago I knew I needed to work on some things. I had a really good teacher in college so I was exposed to using manipulatives and immediately started using manipulatives when I started teaching. Then the NCTM Standards came along and there were lots of workshops. I was starting to change bits and pieces at that point. It probably all came together about

	the time I started using Investigations and attended classes at SMT. My real reason for change was children were not able to understand the math the way we were teaching it.
2. Do you think your instruction is different now in comparison to your instruction prior to your professional development experiences at SMT? In what ways?	It's based more on assessment of where the kids are. Not so much showing them how to do things but giving the, the opportunity to explore and then teach as necessary on an individual or small group basis. Before it was more like following the book in terms of the lessons that needed to be done and then doing some remediation for the kids that didn't get it.
3. Why did you make those changes?	Those are the changes I feel you get best payoff for. I believe children learn best when they are doing something that they are ready to learn. Knowing where they are and what they're ready to do increases your chances that they will be successful.
4. Which of the changes you made in instruction do you regard as most effective for improving student learning?	The assessment piece in terms of knowing where the child is and starting from there. It doesn't mean limiting or not exposing kids to things but being very aware at all times.
5. Why do you think these changes are most effective?	The child has the best chance of being successful. I think success breeds success. One girl wrote me a note at the end of the year. She said you got me to like math. I always liked reading but now I like math.
6. How would you characterize your change process? What was it like for you?	Like anything, there were ups and downs. The success keeps you going but it's a lot of work. Sometimes you think it's easier to just go back to the other ways. But you go back to how kids are responding. I think about what I really see on the kids faces as they work with math in meaningful ways, how much they are learning, how much they are getting out of it. It's not just about getting it done.
7. What organizational conditions in your elementary school helped you to improve your teaching in the ways that you envisioned as a result of your professional development?	I worked with a principal who gave teachers a chance to really try research-based things. There was a lot of positive feedback from administration. There was availability of a lot of different ongoing professional development and for a while we had in-classroom support.
8. What organizational conditions in your elementary school hindered your ability to improve your teaching in the ways that you envisioned as a result of	Naysayers, complainers; Teachers at the middle school trying to dictate what kids should know, what the product should be. Money was an issues and certain things couldn't continue after grant money ran out. There is a lack of continuity. District did not mandate the change so kids had a very uneven experience. That

your professional development?	was frustrating for kids and teachers.

NOTES FROM INTERVIEW OF PARTICIPANT #5

1. Why did you decide to pursue changing the way you were teaching math?	I realized when I was teaching what I saw in the manual was not working and I was encouraged by what I saw happening when I tried different hands-on lessons. That interested me in learning more.
2. Do you think your instruction is different now in comparison to your instruction prior to your professional development experiences at SMT? In what ways?	Definitely different. SMT helped me bring out better questions, really get students talking, include more critical thinking, pose better questions. The quality of the lessons (what I had the kids do) is better. SMT helped me develop my math content background, because I was taught rotely and didn't always understand. I had been teaching for rote learning. It helped me to learn more math when I started teaching differently. I took the summer institute but it was especially helpful when I took a yearlong seminar because I was able to apply what I learned directly into my classroom. We tried lessons, read cases and saw videotapes.
3. Why did you make those changes?	The results are obvious. It's rewarding. What's really nice about my job is I get to see kids for more than one year. It's amazing what they will remember from year to year. They really gain an in-depth conceptual knowledge. In the conventional textbook-based lesson, you are really pushing superficial knowledge in them, which they don't remember. It's not ingrained in them. I use a lot of Investigations [an innovative standard-based complete elementary math curriculum] plus my own activities that I designed based on what I learned. These kids are the ones that really struggle and it's rewarding to see them remember what they learned in a previous year.
4. Which of the changes you made in instruction do you regard as most effective for improving student learning?	Posing good questions; giving them problems that are challenging enough to solve, coupled with discussion and writing is the key to retaining the math. They need rich problems that can be solved using multiple approaches. For example, make a 5 by 5 square and figure out how many one-inch square tiles are inside. Students are learning about measurement, one and two dimensions, looking at different squares and coming up with the formula. Length times width, for finding the area of a square. There is thought,

	discovery, and struggling during their learning process. Your role is to ask questions and not tell the answer in order for it to work, and you have to know what you are looking for. You have to know the content you are aiming for. There are lots of ways of reinforcing the important math. Like in multiplication you need equal size groups and having them think about if there Is another way to figure this out?
5. Why do you think these changes are most effective?	The kids are really thinking. They seem to really take what they know and when they apply it somewhere else they are developing their thinking skills. You are teaching them how to attack problems and teaching them to think, analyze, evaluate. They need to be practicing those skills. Through the questioning, you are modeling this, leading them through the critical thinking process that should eventually become automatic for them.
6. How would you characterize your change process? What was it like for you?	It's really been a work in progress [4 years] but I was very excited by what I was learning. The process was really good [meeting every two weeks for a year] because we had to make ourselves reflect. The case studies we read and discussed were really good and we could try some if the things with our won students. A lot of different aspects of the process helped you to refine your ideas and become better at using the methodology. I was not happy with what I had seen as far as materials and I was not so entrenched in my teaching so the process was a positive one for me.
7. What organizational conditions in your elementary school helped you to improve your teaching in the ways that you envisioned as a result of your professional development?	I had a principal who was supportive of what I was doing. She gave me a lot of freedom in developing a math lab in what I wanted to do. I felt the freedom and support to try new things.
8. What organizational conditions in your elementary school hindered your ability to improve your teaching in the ways that you envisioned as a result of your professional development?	Money (lack of it) is the big thing. It would have helped to be able to buy all the materials I needed. It is very time consuming to make the games and activities. Our system buys a conventional program and it is not helpful to me, although what the system buys for K and 1 is really good and teachers like it.

NOTES FROM INTERVIEW OF PARTICIPANT #6

1. Why did you decide to pursue changing the way you were teaching math?	I graduated form college in '94. I was an elementary education major and math and Sped minor. I had one forward-thinking professor. She really influenced my approach to teaching math. My first year I was a sped teacher and did a lot of traditional textbook stuff but tried to do a lot of problem solving. My second year I taught 6th grade using a textbook. When I look back I realized that, even with a traditional program, I always started with a problem of the day. Then I moved to Syracuse and they sent me to a SMT institute, and that helped me discover how important it was for our kids to understand our number system. I realized that I never understood it very deeply. Decimals are a perfect example. They memorize the places but don't really know what it means.
2. Do you think your instruction is different now in comparison to your instruction prior to your professional development experiences at SMT? In what ways	It's definitely different. Even when I was doing a lot of problem solving I wasn't focused on the structure of the number system [the organizing principles of mathematics]. Now I have a different purpose to the problem solving. Even with older kids doing 64-59, they trade in order to do it rather than to use their number sense. They don't think about how far it is on the number line from 59 to 64 (mental image) or to count up from 59 to 64 in a more meaningful way than using our traditional algorithm. Now I help them gain that sort of number sense.
3. Why did you make those changes?	I have had a lot of opportunities to co-plan with other teachers. It is so easy to see the value compared to the way I used to do it. It is so important to have number sense. I never did enough estimation. Like for example when we do tipping. You can multiply by .15 but in your head you can easily take 10% and then take half of that and add it together.
4. Which of the changes you made in instruction do you regard as most effective for improving student learning?	Trying to get kids to not just use the traditional algorithm, but encourage alternative methods. It is amazing how kids can come up with ways that work for them and are effective.
5. Why do you think these changes are most effective?	It is so important that they truly understand the number system. I remember working with this fantastic artist. Even though she is bright and talented, when she learned math she couldn't see it.

	She could do math very easily and quickly in her head but wasn't allowed to. I gave her an example of what kids do in my class: Why would you do the traditional algorithm when like73 + 59 as combing the sums of 70+30 and 3+9 and thus holding on to the actual quantity of these numbers. She was so excited that kids could do this instead of the traditional algorithm.
6. How would you characterize your change process? What was it like for you?	It was definitely a positive experience. I was a complete memorizer so digging into a simple problem like representing 12x16 with base ten blocks is very exciting. It allows you to see all the partial products [10x10, 10x6, 2x10, and 2x6]. This taught me so much about the meaning behind double-digit multiplication. A lot of kids really benefit from this and they end up using it more than the traditional algorithm. They see there is a lot of sense in multiplication. It was exciting to show this type of thing to husband and family. I think a lot of people come out of high school pretty intelligent and getting by but they don't have a lot of meaning behind the math they know.
7. What organizational conditions in your elementary school helped you to improve your teaching in the ways that you envisioned as a result of your professional development?	My job was to help people break out of the box so they sent a few other teachers and me to SMT. The state test was really important because kids need to show their reasoning and their thinking and the changes I was making in instruction were a good match for the test. And it pushed people to change what they were doing in math class. On the Iowa test [ITBS] students performed the same or slightly less in computation but better on concepts and problem solving. On the state test our kids do very well. The school that implemented changes in instruction performed higher than the school that didn't. This was significant because the lower performing school has the highest average income students and they usually perform better than anyone else.
8. What organizational conditions in your elementary school hindered your ability to improve your teaching in the ways that you envisioned as a result of your professional development?	The hardest thing was parents who were questioning why math can't just stay the way it always was, even though many parents were also very supportive. There was a good deal of support and yet a bit of hesitancy by top administration.

NOTES FROM INTERVIEW OF PARTICIPANT #7

1. Why did you decide to pursue changing the way you were teaching math?	A professor in college (MHC) exposed me to constructivist theory of learning and people like Ginsberg. I bought books by other leaders in math education on my own, to learn about pedagogy and content that allows more students to succeed in math. I heard about SMT (MHC) through 5-College newsletter, took courses in fall and spring and the following summer. I perceived I was weak in math during K-12 schooling. I had little or no understanding behind what I was doing. When I started teaching I wanted to be a good math teacher. I had a students who couldn't do multiplication but could do division. This sparked my interest: what is going on? A guidance counselor told me most elementary teachers are women and not good in math. I was looking for better ways to serve all of my students.
2. Do you think your instruction is different now in comparison to your instruction prior to your professional development experiences at SMT? In what ways?	Yes. It used to be rote, flash cards, not teaching the concept. I used to teach procedures, now I have students develop strategies. Now I use games and multiple visual representations. For example, to teach multiplication facts I have students work with 4 by 6 rectangular arrays and other visual representations of 4 times 6. I have them see the patterns in a giant 1-100 chart. I help them see the relationship between 9x1 and 1x9. I have the confidence to use more hands-on. At first I had a lot of questioning from parents, but now I have the confidence to handle the questions.
3. Why did you make those changes?	I felt they could understand mathematics better. This is much more lasting and important than rote learning. It also helped the students to enjoy math.
4. Which of the changes you made in instruction do you regard as most effective for improving student learning?	Teaching for understanding is the most important. That is, having kids build their own learning, not me telling them.
5. Why do you think these changes are most effective?	Because they promote understanding. Students end up with some strategies to figure out a problem. They are more confident. They love math. Three of my students, GIRLS, scored Advanced [the highest category] on MCAS. They had their own approaches that they could use.
6. How would you characterize your change process? What	Gradual. I eventually got more curriculum materials, and became more involved in

was it like for you?	professional development. But some parents hated it and some loved it. Some needed convincing. It was difficult when people were questioning. Without all the coursework I wouldn't have had the confidence to withstand the questions. I might have reverted back to the old practices.
7. What organizational conditions in your elementary school helped you to improve your teaching in the ways that you envisioned as a result of your professional development?	I had a very supportive principal at my school. She encouraged me and helped me with innovative curriculum exploration. I also had 2 team members in 3rd grade who were also willing and eager to improve math instruction.
8. What organizational conditions in your elementary school hindered your ability to improve your teaching in the ways that you envisioned as a result of your professional development?	The district has been lacking in a focus on mathematics. Math has been ignored. We had our first district wide PD in math this past January for the first time. There has not been a strong commitment toward math except to improve MCAS scores.

NOTES FROM INTERVIEW OF PARTICIPANT #8

1. Why did you decide to pursue changing the way you were teaching math?	The prime reason was dissatisfaction with drill and worksheets for math. It didn't address the needs of all students. I knew all students were not getting it, wondered why, and was interested in how students viewed math.
2. Do you think your instruction is different now in comparison to your instruction prior to your professional development experiences at SMT? In what ways?	Absolutely. I used to be more limiting of the students (had a preconceived notion of what I wanted). There was little to discuss. Now I'm more open to their thinking; I learned a lot too, found I had some misconceptions myself about geometric solids. The atmosphere now is more relaxed, not just drill sequence. Students need some drill, but now I am interested in developing more understanding of the math behind it. It is now more hands-on, with a variety of manipulatives. Children use pictures, words, numbers, and symbols to represent their thinking and why things make sense. They have a variety of ways to tackle a problem. This improves their persistence and ownership and supports their own approach to thinking about a problem.

3. Why did you make those changes?	Enlightenment. Before these classes I juts didn't consider other approaches much. It was a really a conscious effort to make some changes. I could see an increase in student involvement, interest, and enthusiasm for math. I could also see understanding was developing.
4. Which of the changes you made in instruction do you regard as most effective for improving student learning?	Allowing time to explore and to express and to compare their thinking.
5. Why do you think these changes are most effective?	When they need to express themselves they are more cognizant of what they are doing, They can also learn more from one another as they see different children's approaches, which are also correct. I see more ownership. They all become involved even if they don't have the language to say what they mean. They are sharing, representing themselves.
6. How would you characterize your change process? What was it like for you?	Very gradual. I knew I wanted to change but I wasn't sure what steps to take first, how to initiate more activity, what questions were better than others. I began sitting back and letting them discover; letting them really mess up or take the wrong track. I needed to build up a sense of security before I could let them fail (not jump in and fix it). For the longest time I would preach and teach in a lecture style, but now I pose questions and let children discuss possible solutions.
7. What organizational conditions in your elementary school helped you to improve your teaching in the ways that you envisioned as a result of your professional development?	I eventually had access to good curriculum materials that supported me in teaching the way I wanted to teach. It was hard to develop good activities on my own. The games are wonderful. We have a well-organized math center and system for storing and displaying the manipulatives and tools kids choose to help them figure things out. This is where kids choose things to help them figure things out. Biggest help has been contact with other like-minded teachers. It is understood that we must teach math for 1 hour per day. We have one fairly successful math night per year mainly due to the efforts of one teacher. Parents in general don't question our instructional approaches.
8. What organizational conditions in your elementary school hindered your ability to	There is not a school-wide explicit commitment to mathematics. There is little input from administration about the expectations for math

improve your teaching in the ways that you envisioned as a result of your professional development?	instruction. I don't get the sense that there is any support from the principal for the way I teach math. Literacy instruction is the priority. I was placed in a classroom with established partners who had scheduled math right after recess. Time is lost settling students down. I am not sure how other people in the school teach math, but I hope to make changes for the next school year.

NOTES FROM INTERVIEW OF PARTICIPANT #9

1. Why did you decide to pursue changing the way you were teaching math?	I was fortunate enough to already start with a constructivist mind-set right from the beginning, so it wasn't really about changing my approach to teaching math. My professional development was more about bolstering my confidence so I could better deal with other teachers who did not believe as I did.
2. Do you think your instruction is different now in comparison to your instruction prior to your professional development experiences at SMT? In what ways?	It just made me give more thought to what I am doing. It is really easy to be convinced by the people that you work with that coverage is most important rather than to develop understanding. Especially people who are thinking about MCAS and needing to cover certain topics. Every time I go through a professional development experience it helps me affirm what I already believe. It encourages me to be more student-centered, even when there is resistance. The most important thing I improved was my questioning—how to ask the right questions. This helps a person voice what they are thinking so you can figure out what they need to think about next. I also now do a lot of pair/share to congress type of activities to help kids verbalize.
3. Why did you make those changes?	I don't think education is about instruction or the teacher. I think the learner has to be at the center. They can't just be trying to please the teacher because that does not really work. These techniques put the learner at the center of the learning process. They reason things through for themselves. This helps them be a learner.
4. Which of the changes you made in instruction do you regard as most effective for improving student learning?	One thing I do is allow wrong answers or misconceptions to surface. It helps generate good discussions and challenges thinking. The hardest thing is not to give an answer until a student is satisfied. But you can't go on indefinitely. Learning

	to think about it is more important than the right answer. Learning how to think about it is critical. If we did that at an earlier age, our students would be better thinkers.
5. Why do you think these changes are most effective?	They have to be actively engaged in the process. I think the approach requires that they become independent thinkers.
6. How would you characterize your change process? What was it like for you?	Change isn't painful for me. I don't think it was a big change as much as helping me know what I really believe. My professional development at SMT and other places bolsters me and gives me confidence so when I am challenged by other teachers I have a lot of experiences to help me know ways to respond.
7. What organizational conditions in your elementary school helped you to improve your teaching in the ways that you envisioned as a result of your professional development?	At the administrative level there is a lot of support for innovative teaching. Our principal even went through an experience with us.
8. What organizational conditions in your elementary school hindered your ability to improve your teaching in the ways that you envisioned as a result of your professional development?	But there is a lot of lip service given and there is a lot of fear and coverage keeps winning out over approaching mathematics in a constructivist way. So if you have one teacher in a grade doing constructivist math, but the other five teachers in that grade aren't, you really aren't making any progress because the students' experiences are so inconsistent year after year. People are in a panic about MCAS and the accountability system. They perceive MCAS really promotes coverage so teachers are not likely to take risks. But I think kids have a better chance at figuring out the problems if they are confident thinkers and used to reasoning things through for themselves. I think a lot of good teachers have allowed themselves to be intimidated by MCAS. Actually, the open-ended questions require reasoning. It saddens me that more math teachers don't understand the MCAS tests were designed to encourage development of thinking skills.

NOTES FROM INTERVIEW OF PARTICIPANT #10

1. Why did you decide to pursue changing the way you were teaching math?	I was teaching the way I was taught with a lot of memorization and practice. For some students this seemed to work ok but many kids didn't understand what they were doing and why. They could manipulate the numbers and get the right answers. When I went to SMT I realized I needed to rethink the way I was teaching mathematics. When I worked in groups, other people seemed to know more content, but I had to see the why behind it. By being a learner I saw that my students also want to know why but I wasn't giving them a chance to find out. Also, on the Grade 3 state and district assessments it became clear that children were weak at problem solving.
2. Do you think your instruction is different now in comparison to your instruction prior to your professional development experiences at SMT? In what ways?	Definitely very different. It used to be: learn the facts, drill and memorization. There is a lot more problem solving and it's all hands-on. Kids get to make models, build things. The kids get to ask the questions. I try to spark their curiosity and not just telling them what we are doing-let them come up with it. For example if we are learning about multiplication I try to give them lots of problems to help them discover and understand the concept. I try to make learning fun by making some of the activities seem like games. I make sure the problems are relevant to the kids. I don't teach the book, the book is a tool and I use it to reinforce and practice.
3. Why did you make those changes?	To make kids problem solvers. It's not just me telling them how to get the right answer or concept, but allowing them to discover the concept themselves. Let them come up with their own ideas. Solving a puzzle sparks their interest. Students need to feel like they are part of the process so that they may take ownership of their learning.
4. Which of the changes you made in instruction do you regard as most effective for improving student learning?	Not telling them but getting them to discover the concept. Letting students find their own approaches to solving the problems.
5. Why do you think these changes are most effective?	Before, kids were doing it because the teacher was telling them. Now, they are in charge of their learning process. It sparks their interest; they internalize it and make it their own. Once they have ownership, they can learn more easily. Our third grade math scores went from 70% proficient to 93%

	proficient. So I know it works.
6. How would you characterize your change process? What was it like for you?	When I went to SMT I had a really hard time working in groups with others who seemed to have more math background. I felt like I was behind or slow at grasping the concept because I am a pattern person; this means that I tend to look for patterns in mathematics to better understand the concept and unfortunately for me at that time everyone else seemed to know formulas and already knew how to work the problems. After SMT I could see that as a learner I needed more. Since I was relatively new to teaching it was not very difficult to make changes in my teaching. I have always seen myself as very creative and that helped. It was a challenge but if you give me a challenge I work hard to raise up to expectations.
7. What organizational conditions in your elementary school helped you to improve your teaching in the ways that you envisioned as a result of your professional development?	In my school we are all working in the same direction. I have a very supportive principal. She is the one who suggested we do SMT. Two of us went through SMT, each for two years. If I need something, she'll get it. We have the freedom to work our own schedule with our grade level. We are departmentalized and I teach math for the three third grade classes in my school. I teach math for 1 hour and 15 minutes. We have common planning time. Everyone is very supportive in our school. I have nothing but help everywhere including from parents. We even do math month in our school with all kinds of problems and special events. We do the Marvels of Math which are school wide math activities and also put up a math bulletin board center for the whole school. It is a wonderful place.
8. What organizational conditions in your elementary school hindered your ability to improve your teaching in the ways that you envisioned as a result of your professional development?	I don't think so. I feel really lucky and blessed.

NOTES FROM INTERVIEW OF PARTICIPANT #11

1. Why did you decide to pursue changing the way you were teaching math?	Once I was exposed to courses at SMT it made me think I wanted to change the way I was teaching math. My change started when I delved into my

	own thinking about math and my kids' thinking about math.
2. Do you think your instruction is different now in comparison to your instruction prior to your professional development experiences at SMT? In what ways?	I do think my instruction has changed. I am more focused on my questions. I ask my students questions, I try to move their thinking without leading them. I try to facilitate their group work and use the strategies kids are sharing to lead in my instruction. It is less teacher directed.
3. Why did you make those changes?	After doing the work in the courses and looking at the way I work on math in the group settings, we all bring something to the table and our prior knowledge and experience helps move the ideas forward. My kids come into class with different ideas. I want their ideas to be the basis of their learning and not my ideas. There is something to be said when you work in small groups and share thinking. When we share our ideas it seems to create a more positive learning environment.
4. Which of the changes you made in instruction do you regard as most effective for improving student learning?	Giving kids time to explore their ideas, share their strategies and compare their strategies. When I would watch videos at SMT seminars the students would say things like "I did my problem like John did," and I would say to myself that my kids would never listen to one another's ideas that way or say anything like that. But when I changed my instruction I was amazed that they actually did say stuff like that and refer to stuff that happened in past classes. It's been really fun to watch.
5. Why do you think these changes are most effective?	It gives kids the opportunity to take ownership of their learning. Once they own it their confidence is built and their understanding does too. They have a deeper understanding when they build from what they already know. Something about sharing your thoughts and thinking clarifies and also validates it.
6. How would you characterize your change process? What was it like for you?	My change process has been a challenge for me. The teachers in my building are traditional math teachers who teach math the way they learned it because that is what is comfortable for them. I am pretty much alone in this process. It is helpful that I have the connections I made with people at SMT institutes. I have been involved there for four years.
7. What organizational conditions in your elementary school helped you to improve your teaching in the ways that you envisioned as a result of	I had a heterogeneously grouped class this year for my pilot of an innovative math curriculum. I received a half-year sabbatical to study more about math reform, and I have had the chance to go to national math conferences. Principal was impressed

	your professional development?	with what the kids were doing in my class, but his plate is full. There is too much on his plate. He is always supportive of what I do, but I have to be careful when I approach him to talk things over because there is always so much going on.
8.	What organizational conditions in your elementary school hindered your ability to improve your teaching in the ways that you envisioned as a result of your professional development?	A traditional timed mid-year assessment instrument that tests basic computation; Parents were looking for worksheets on long division; District is going to adopt a program that is less of a match to what I think should be going on; Isolation has been difficult—everyone else in the building is a traditional math teacher.

NOTES FROM INTERVIEW OF PARTICIPANT #12

1.	Why did you decide to pursue changing the way you were teaching math?	I was told during an observation of my teaching that I didn't understand the concept. I am trying to teach an abstract concept in a concrete way. My students have disabilities in learning math and in memory recall. I knew I needed to figure out a way to teach math that was different than the way I learned. I have some kids who know the facts and procedures and others who are completely lost. This year it was clear that the area that needed attention was fractions, decimals, and percents. They had no introduction in previous sped classes where the emphasis is on the 4 operations with whole numbers. Kids do that year after year. I did a lot of fraction concept activities to help them all understand what was happening. They came with a lot of practical experience but couldn't connect it to the math they were learning in school.
2.	Do you think your instruction is different now in comparison to your instruction prior to your professional development experiences at SMT? In what ways?	Much different. I always keep that philosophy of trying to make something concrete that is abstract. Like if four people share 7 cookies, how much does each person get? I have them show three different ways to solve the problem. I just never thought about math very much before. I just did what I was taught.
3.	Why did you make those changes?	Because I wasn't successful. I didn't feel successful. The truth is that I don't know if I am any more successful now. However, I do have feedback from parents who are very happy with what their kids are learning in math.
4.	Which of the changes you	The overall approach is different. I don't think it's

	made in instruction do you regard as most effective for improving student learning?	any one thing. But I think the use of manipulatives is very powerful. Once they get the concept with the manipulatives they go to the drawing. And I think that is a logical progression.
5.	Why do you think these changes are most effective?	By the responses I see from the students.
6.	How would you characterize your change process? What was it like for you?	The first weekend my husband knew something happened. It is just amazing what you don't understand, and I had a strong math background. The depth of the math was the big eye opener, and that's why I went back the next year. The first year I just tackled the number system. As I tackled that I realized that I had students who didn't know how to count. I had a student who knew her multiplication tables, but couldn't count.
7.	What organizational conditions in your elementary school helped you to improve your teaching in the ways that you envisioned as a result of your professional development?	I have one other teacher in my building who is like-minded. I have been able to order anything that I have needed. My new principal is very supportive, believes in the use of manipulatives.
8.	What organizational conditions in your elementary school hindered your ability to improve your teaching in the ways that you envisioned as a result of your professional development?	I paid my own tuition, Money is a big concern because I know other people who are interested but were not able to pay the tuition. I wanted to take a three-day course and it was denied, so I knew she didn't really know what was going on. Now I have a more supportive principal.

NOTES FROM INTERVIEW OF PARTICIPANT #13

1.	Why did you decide to pursue changing the way you were teaching math?	My son was precocious in math—it has always been his passion—and I was concerned about how his second grade teacher was going to challenge him in math. So I went in to observe and I was taken with what she was doing. She has been involved in a number of SMT programs. Instead of holding my son back, the program was allowing him to explore the math and take it as far as he could. When I was teaching Sped I felt frustrated teaching math the traditional way. It never felt like I was hitting the issues. I had a student who could solve math problems but not do procedures. I knew there was math understanding there that I was not tapping

	into. I was using manipulatives, but just to reinforce the traditional algorithm not the way I use them now.
2. Do you think your instruction is different now in comparison to your instruction prior to your professional development experiences at SMT? In what ways?	Very different. I like to give kids a chance to grapple with a problem before I give them direction about how they might go about solving it. I ask them to share ideas during and after the process. I want them to understand that there are many doors into how you solve a problem. I used to use manipulatives for remediation. Now I use them to help all kids conceptualize what they are doing. I spend more time helping students make connections. Even with something like multiplication facts, we relate facts to one another. how does knowing form 6x6 help with 6x7. We connect fraction work to measuring with the ruler.
3. Why did you make those changes?	I personally experienced the power of these approaches when modeled at SMT training. I had also been into lots of classrooms where teachers were adept at the constructivist approach. I became fascinated with the idea of people explaining their thinking. I never say 'that's right." I say "Are you sure?" or "Why do you think that's the right answer?" and some of the pedagogical questioning techniques. By explaining, children solidify their own understandings.
4. Which of the changes you made in instruction do you regard as most effective for improving student learning?	1) Taking math from a solitary to a community activity. 2) Using varied grouping: partners, small groups, leading math discussions. 3) Allowing children to invent their own strategies. I hold off the presentation of conventional algorithms, because they stop thinking. They shut it down. I try to guide kids so that they construct the knowledge themselves, rather than my being the source and showing them how to do everything. That is the biggest change.
5. Why do you think these changes are most effective?	When you communicate it forces you to deeper understanding. When you allow kids to invent strategies, they stop thinking about it as something you have to be shown how to do. They are not afraid to tackle any new problems.
6. How would you characterize your change process? What was it like for you?	Revisiting the math myself was the first step. Having a chance to try to invent strategies myself. It was important for me to have teacher models at training. I always watched how we were taught. The reflective work, like writing, was really important

	for me. Changing my role from teach/show to listen/watch. A lot of my change process was like bombs exploding. Sometimes I would get too excited talking about it to my husband. It was totally rediscovering math. It got me re-interested in teaching. It revitalized my career. I was really lucky because I was able to make the change gradually.
7. What organizational conditions in your elementary school helped you to improve your teaching in the ways that you envisioned as a result of your professional development?	Class support so you can process with someone after a lesson. Curriculum days for grade level support meetings. Having all the materials—you have got to have what you need. You need a resource teacher to go to when you have needs or questions about the program. Teacher leaders and leadership training is helpful so people are available to think through the unit with you. There has to be somebody to help people stay on track. Release time with good professional development to revisit the math content is also paramount.
8. What organizational conditions in your elementary school hindered your ability to improve your teaching in the ways that you envisioned as a result of your professional development?	When my school system was wishy washy about whether they were mandating the MT program or not. Teachers need clear direction from the administration, but it can't be totally top-down. It's hard if there are too many materials you have to make. Everybody needs to be on the same page. It was not helpful when they changed the math framework. MCAS has had too much of an influence on curriculum and instruction. You need consistency and continuity. You also have to train specialists and paraprofessionals and sped teachers.

NOTES FROM INTERVIEW OF PARTICIPANT #14

1. Why did you decide to pursue changing the way you were teaching math?	I was curious as to what it was about. I wanted to refresh my ideas especially with special education. I like to try to stay current with any new ideas or strategies that come along with regard to teaching and learning.
2. Do you think your instruction is different now in comparison to your instruction prior to your professional development experiences at SMT? In what ways?	Yes. I do more investigations and problems of the day. I was one of those dictators before. This is how you do it. Now sharing with each other is a big deal. Prior to this new way of thinking I would have thought that spending a few days on one problem was horrendous. Now I take as many days as it takes for the students to show understanding.
3. Why did you make those	It is more fun to teach this way. For Sped kids, I like

changes?	it better. The kids get more out of it. Normally Sped kids just get to do calculations. They always had trouble with the word problems but now that they have a way to solve them using different approaches. Some of them are going to struggle no matter what you do. Some of them have definite math disabilities.
4. Which of the changes you made in instruction do you regard as most effective for improving student learning?	Working on more story problems and the approach to story problems has helped. We always talk about the various strategies they can use. I have this really old fifth grade math book and I adapt the problems to second and third grade. I have another resource with story problems and the kids draw it to find the answer. We always review the different things they can do when they get stuck.
5. Why do you think these changes are most effective?	Because it's going to give them a better understanding. They had no idea where to start before and now they have ways to think about it.
6. How would you characterize your change process? What was it like for you?	It was a lot of trial and error. I felt safe taking risks. I knew that if it didn't turn out well I could just drop it. I'm comfortable with having students share and talk. Some changes I made in instruction are in spite of what was modeled at SMT. I hated it. There was a lack of closure. You would spend all this time working on a math problem and not know if you were right or wrong.
7. What organizational conditions in your elementary school helped you to improve your teaching in the ways that you envisioned as a result of your professional development?	Having all the materials we need: notebooks, manipulatives. In Sped you're kind of left on your own. I had principal support. I borrowed a lot from the mainstream teachers—like the teaching manuals. We also have a nice staff that allows for easily sharing of ideas regarding the teaching of math. We all get very excited when things work!
8. What organizational conditions in your elementary school hindered your ability to improve your teaching in the ways that you envisioned as a result of your professional development?	Having just ½ hour for math is a hindrance. But next year we are going to do it differently. I won't be pulling out kids. We will be using an inclusion model. Also, my math group was a mix of grades 2 and 3 and it was rather large (12 students). Hopefully, that will change next year too.

NOTES FROM INTERVIEW OF PARTICIPANT #15

1. Why did you decide to pursue changing the way you were teaching math?	I didn't feel I had a good math background. Saw math as an area that needed shoring up. I had A's in math in school but was afraid of calculus.
2. Do you think your instruction is different now in comparison to your instruction prior to your professional development experiences at SMT? In what ways?	Absolutely, because I'm more knowledgeable. I had courses around using manipulatives but the SMT courses helped me see how kids make sense of the operations and the inquiry process. I am more knowledgeable about how children think about math and how they rationalize math and math problems and the relationships between operations. There is more hands-on instruction and use manipulatives in more meaningful ways than I did before. I am always looking for investigations for the students. When using the math text provided by the district I am able to look at the lessons critically and evaluate them for effectiveness with my students' learning styles and make appropriate changes.
3. Why did you make those changes?	Because I experienced learning this way firsthand myself. I had to work problems out and experience how much more meaningful the learning was. I came to know the math in a different way. I had a much deeper understanding. I see math as patterns, how the formulas came to be, it's not a process of memorization to me any more.
4. Which of the changes you made in instruction do you regard as most effective for improving student learning?	Active involvement in problem solving and using manipulatives.
5. Why do you think these changes are most effective?	Because it's more developmental and matches how people learn best—through inquiry and we know that memorizing isn't effective. We are not producing people, especially women, who like math. I used to dread teaching it and now I love teaching it. It's my favorite subject to teach and the favorite subject for my kids to learn.
6. How would you characterize your change process? What was it like for you?	It was fun and exciting. I had a lot of eurekas and light bulbs.
7. What organizational conditions in your elementary school helped you to improve your teaching in the ways that you envisioned as a result of your professional	We had plenty of manipulatives and a supportive principal. I did some team teaching, which was good.

development?	
8. What organizational conditions in your elementary school hindered your ability to improve your teaching in the ways that you envisioned as a result of your professional development?	We did need better curriculum materials, not text-book based, to support our changes in instruction. I had some conflict with a young teacher who was not very supportive as I changed the way I was teaching.

NOTES FROM INTERVIEW OF PARTICIPANT #16

1. Why did you decide to pursue changing the way you were teaching math?	I was initially a physical education teacher so I felt I needed training for teaching math. I was just following workbooks but it felt meaningless. I knew what I was doing was a hodge-podge, what everyone else said, and our curriculum was not as strong as it is now with the frameworks. SMT became available to me at no cost, just the right price, so I went. I just needed a focus, I didn't really care exactly what it was.
2. Do you think your instruction is different now in comparison to your instruction prior to your professional development experiences at SMT? In what ways?	Night and day. It was awful—paper and pencil—didn't know why—goal was to get through the book. When TERC Investigations came out with a book for first grade it was very helpful. Now, students verbalize, use words, numbers, or pictures, make connections, communicate what they are thinking. They know there is more than one approach, maybe more than one right answer. They love it when three people can be right. It is very empowering. Sometimes what they do is very high level math—I even have been able to talk about how their work is algebraic expression. The students do the basics but they also know how the numbers fit together. They pull numbers apart and put them together. They know their addition facts but also approach multiplication, division, algebra, and geometry. It is amazing. We do stories, find math everywhere, read books with numbers, and act out math situations. There is a constant connection with everyday life--math is everywhere. What I have done at SMT supercedes what I have done anywhere else, even the work for my master's degree, where I am going through the motions, but there is no comparison to what I have really learned at SMT.
3. Why did you make those	After my first experiences trying I found kids

changes?	having deeper understanding and tremendous ease in math. They have more ambition to persist in finding an answer and to solve complex problems. It allows a lot of different thinkers to succeed. For myself, personally, I finally realized I could do math. This was a huge revelation for me. Now I understood what I was supposed to ask and why. I understood. It seems obvious but it really nice when you understand it.
4. Which of the changes you made in instruction do you regard as most effective for improving student learning?	Getting away from the memorization and the rote memorization. I just don't find it important at all at this age. The most effective tool is discussion about what you are thinking and how you came to that answer. I carry this over to all my subjects. Memorization comes easier as necessary in the upper grades because the students have a better understanding about the numbers.
5. Why do you think these changes are most effective?	I am not just stuffing information into their heads, I am watering their ideas and letting them grow. In this program I can allow for their own research and help them construct meaning.
6. How would you characterize your change process? What was it like for you?	I was highly emotional. I was so afraid to go to SMT and I am not by any means a timid person. I learned I wasn't slow, but that I was a deep thinker. It was a total change in the way I thought about myself. I surprised myself. I had a lot of revelations. After that I didn't feel bad about myself wanting to think a problem through. I didn't feel uncomfortable if I didn't know what the formula was. I would just go off and think.
7. What organizational conditions helped you to improve your teaching in the ways that you envisioned as a result of your professional development?	Superintendent's office provided opportunity to take SMT free. Superintendent's office provided support groups at grade level on curriculum days during the first two years of implementation of TERC program. Principal's support was there. Scheduling at the same time each day. I preferred early AM.
8. What organizational conditions in your elementary school hindered your ability to improve your teaching in the ways that you envisioned as a result of your professional development?	We weren't hindered at all. We were encouraged to come up with a format that worked for us. The only hindrance was from the upper grade teachers, because in the first few years the upper grade teachers didn't feel the children were as well prepared for their curriculum. By the third year I laugh at the fact that I am now the requested teacher if a child was having difficulty in math. By the third year all grades were at least 50% on board, so the continuity of learning was there. At this point our

	school is K-5 with Investigations as our primary math tool. At the middle school the program is not continued. However, 6th grade teachers are finding more consistent understanding coming from the 4 elementary schools with only one math program. Of course there is never enough time. The program requires more time than there is in a school day. I'd like to do math all day but I do need to teach the children to read!

APPENDIX E

EXCERPTS RELATED TO CHANGES IN INSTRUCTION BASED UPON PARTICIPANTS' WRITING AND NOTES FROM INTERVIEWS

EXCERPTS RELATED TO CHANGES IN INSTRUCTION BASED UPON PARTICIPANTS' WRITING AND NOTES FROM INTERVIEWS

	Quotations from Writing	Excerpts of Interview Notes
1.	How can I provide experiences that will help to deepen understanding of measurement? So often we teach a concept for a short period of time and then move on to another concept. How can student make connections so that the ideas that they have investigated are applied when new concepts are introduced? (May, 2000) I think that when children feel safe in a classroom to share their ideas, then real learning is taking place. Children are free to share their ideas, ideas which help other children gain information on a concept at a level of language that is clear to them. While direct teacher instruction is what we were brought up on, I think the kinds of instruction where learning comes from the sharing of students is far more beneficial. I think when children have the opportunity to show their classmates what the concept is all about in their own words, children tend to be more active listeners and more active participants in their learning. Though the children in the cases, videos, and my classroom are young, they do have a wealth of knowledge and experiences that, when shared, enrich our thinking, teacher and student alike. (November, 1999) For example, I notice the struggles children have with making sense of place value, a concept that is the foundation for mathematical understanding of number sense and operations. I think to myself that if more teachers invited students to experience math through hands-on activities, the understanding of concepts might be stronger. (May 2000)	I feel that my math instruction focuses more on helping students make connections with their math thinking through the use of manipulative tools, group work, class discussions and journaling. (May, 2002)

	Quotations from Writing	Excerpts of Interview Notes
2.	I'm interested in learning how to successfully reach all students. I don't always have a good sense of how to modify a particular activity to help struggling students. One thing that stood out for me this week was that the learning of any concept in math needs to move from the concrete to the abstract. My belief in cooperative group work has been reinforced as well. I attribute what I learned this week to working with others. I discovered that group work at this level is as exhausting as it is powerful. I also learned that working in a group has to do with much more than the problem to be solved. Group dynamics need to be talked about. People need to be reminded to be considerate. Learners need to take risks and know how to ask for what they need. Though group work is complex it is worth the effort. It can teach us how to extend ourselves and learn more than we can individually. These are some of the reasonable and realistic ways in which I plan to implement what I learned in SummerMath for Teachers: continue to work on developing good work skills by talking about what works well, good/bad levels of frustration; making sure that kids get to work independently before getting together to share strategies and ideas; may set up one of the old Apples in my classroom with Logo for a math center for kids who often complete work early; I plan to use manipulatives to model concepts and have children model their understanding as well—even those concepts that I assume most children understand because manipulatives are such a powerful tool; work on my ability to pose "good questions."(July, 1998)	I'm constantly learning and am now much more confident. I use Investigations now [an innovative curriculum program]. I follow that pretty much but it's my 6th year of using it so I can come up with my own examples. If I'm doing something in a unit and the kids aren't really getting it I can add to that curriculum because I've gotten better at expanding the math and I know what the kids are supposed to get out of it. Kids don't always need to master it because it may come back around again, later in the unit or in a later unit. (June, 2002)

	Quotations from Writing	Excerpts of Interview Notes
3.	…give them as many opportunities to explore, manipulate, speculate, discuss, and write…as possible I learned that a relaxed, curious, and cooperative hands-on experiential approach will produce (as I had expected) more useful, ingrained, and open-minded knowledge…(December, 1999) As I go through this seminar, I will be looking for places where my thinking is "stuck" so that I can grow to see the math ideas more clearly…(March, 2000) I found I got what I needed through listening carefully during lectures and whole group discussions in seminar, through discussions in small groups with fellow participants, through the hands on challenges in class, through talks with L and C, by asking my students to investigate with me and through searching through math books and math dictionaries to try to find definitive answers. (December, 1999) From our experiences, it seems to be very beneficial to give academic investigations some time to gel, but again this approach does not lend itself well to being able to explain exactly when, where, why, and how specific knowledge was acquired. We learn in complex ways and only search for sources of specific information when a question or challenge arises. (December, 1999)	SMT helped me understand how many different ways people can understand something. Textbooks are deadly so we make up our own problems that are relevant and meaningful to the children. Kids make up their own problems now too. It is much more meaningful and experiential. We also do much more with geometry than before. Geometry used to just be what they could get out of logo. (June, 2002)
4.	For most of my teaching I have asked questions to bring children to an answer. I thought I was finding out what he or she knew but I see what limits my questions put on his or her answer…I am thinking more about when to listen, when to question, and when to talk in order to support students' learning. (March, 1998)	It's based more on assessment of where the kids are. Not so much showing them how to do things but giving the, the opportunity to explore and then teach as necessary on an individual or small group basis. Before it was more like following the book in terms of the lessons that needed to be done and then doing some remediation for the kids that didn't get it. (June, 2002)

	Quotations from Writing	Excerpts of Interview Notes
5.	Working through the mathematical exercises in this seminar with a variety of materials has impacted my view on how mathematics should be taught. I believed in the constructivist philosophy prior to this particular seminar, but this seminar has helped me to question how certain manipulatives can inhibit or foster the growth of student ideas. (December, 1997) Once again, I was a learner and I remembered how I wanted to understand things and how it led to more curiosity and questions. I also re-experienced the need to have tools to visualize what I was doing with numbers. The feelings of being challenged and achieving satisfaction from the problems helped me to remember how important it is for students to experience these feelings. (July, 1998) I have learned to encourage students in discovering mathematical concepts through the inquiry and problem solving approach. This is the approach I use in my math classes and students seem to enjoy getting deeply involved in a mathematics problem each day…Through studying the meaning of operations by actually doing the mathematics with others has prepared me with a better understanding of the mathematics and I can respond to my students' mathematical thinking with more confidence…Perhaps the most beneficial aspect of the Summer Math program has been the ability to see the total picture of how students' develop their mathematical thinking toward the big ideas and principles of algebra. As I work with students at different grade levels, I can now understand what mathematical goal I am striving for instead of a mere daily objective. With this in mind my lessons are more flexible. (April, 1998)	Definitely different. SMT helped me bring out better questions, really get students talking, include more critical thinking, pose better questions. The quality of the lessons (what I had the kids do) is better. SMT helped me develop my math content background, because I was taught rotely and didn't always understand. I had been teaching for rote learning. It helped me to learn more math when I started teaching differently. I took the summer institute but it was especially helpful when I took a year-long seminar because I was able to apply what I learned directly into my classroom. We tried lessons, read cases and saw videotapes. (May, 2002)

	Quotations from Writing	Excerpts of Interview Notes
6.	There are a couple of areas where I would like to improve regarding my classroom instruction. One area is leading a class discussion about a problem or a situation. Very often I feel rushed or feel as though I am having to force the students to expand on their thinking verbally and participate in a classroom discussion... Also, I feel as though I am struggling with getting some upper elementary students to reflect on their thinking in their journal writing...I am wondering if there are certain prompts or lead questions that would allow for better success...My expectations are to learn not only about new teaching practices but also about the learning and understanding process of students. (March, 1997)	Even when I was doing a lot of problem solving I wasn't focused on the structure of the number system [the organizing principles of mathematics]. Now I have a different purpose to the problem solving. Even with older kids doing 64-59, they trade in order to do it rather than to use their number sense. They don't think about how far it is on the number line from 59 to 64 (mental image) or to count up from 59 to 64 in a more meaningful way than using our traditional algorithm. Now I help them gain that sort of number sense. It was definitely a positive experience. I was a complete memorizer so digging into a simple problem like representing 12x16 with base ten blocks is very exciting. It allows you to see all the partial products [10x10, 10x6, 2x10, and 2x6]. This taught me so much about the meaning behind double-digit multiplication. A lot of kids really benefit from this and they end up using it more than the traditional algorithm. They see there is a lot of sense in multiplication. It was exciting to show this type of thing to husband and family. I think a lot of people come out of high school pretty intelligent and getting by but they don't have a lot of meaning behind the math they know. (June, 2002)

	Quotations from Writing	Excerpts of Interview Notes
7.	It is important to give a lot of variety of groups to students. SMT has taught me to listen to my students. I find I take more time to listen and try to understand their thinking. I have learned much from them. SMT has helped me to become a better teacher. Every time I take a course I improve my own mathematical understanding. When I improve I can be a better teacher. (January, 1999) I need to spend more time on what is right with my students thinking rather than what I expect to find…So many things that teachers assume their children have learned turn out to be things that have only been taught. (March, 2000) There are many issues still alive for me. One is how to find time to allow children to discover their own truths. With all that is expected of the classroom teacher, how do we allow enough time to discover the truths in mathematics? What should be our focus? What are the most important concepts to be developed over the years. (May, 2000)	Used to be rote, flash cards, not teaching the concept. I used to teach procedures, now I have student s develop strategies. Now I use games and multiple visual representations. For example, to teach multiplication facts I have students work with 4 by 6 rectangular arrays and other visual representations of 4 times 6. I have them see the patterns in a giant 1-100 chart. I help them see the relationship between 9x1 and 1x9. I have the confidence to use more hands-on. At first I had a lot of questioning from parents, but now I have the confidence to handle the questions. (May, 2002)

	Quotations from Writing	Excerpts of Interview Notes
8.	I need to move away from simplified definitions or word choices and help the children develop vocabulary that is more specific/uniform…I need to be a better observer and listener, to record student thinking, and to ask questions that encourage more dialogue and exploration. (October, 1999) I've found that if I simply ask a student, "why did you do it that way?" or "How did you get that number?" he/she is likely to revise the answer because my question intimidates the student or causes the child to feel she/he is wrong. Whereas, if I say, "That's interesting. Would you mind explaining to me how you did that?" or "Could you show me/us how you did that?" the student is more likely to replicate or extend what she/he did. (November, 1999) I learned that the way I approach, model, listen, and comment on children's work has a great impact on how freely they explore and express their findings. I learned that I really can't make assumptions as to what my students know, and that language skill can be an inhibiting factor in understanding my students' thinking. (December, 1999) I just feel that the more I experince as a learner, the more I'm apt to provide meaningful experiences for my students. I'm more aware of the peaks and pitfalls, and can appreciate the different ways children attack a problem. (February, 2000) One issue of student learning that I considered during this seminar was the importance of having all the hands-on experiences to understand what is behind the convenient formulas…knowing WHY they work. (May, 2000)	I used to be more limiting of the students (had a preconceived notion of what I wanted). There was little to discuss. Now I'm more open to their thinking; I learned a lot too, found I had some misconceptions myself about geometric solids. The atmosphere now is more relaxed, not just drill sequence. Students need some drill, but now I am interested in developing more understanding of the math behind it. It is now more hands-on, with a variety of manipulatives. Children use pictures, words, numbers, and symbols to represent their thinking and why things make sense. They have a variety of ways to tackle a problem. This improves their persistence and ownership and support their own approach to thinking about a problem. (June, 2002)

	Quotations from Writing	Excerpts of Interview Notes
9.	I now understand that there is a big difference between how to play the game and really understanding the meaning of the outcome or the processes involved in arriving at the "right" answer. (July, 1998) While first hand experience might be the more effective way for most people to learn, it is not always possible. Time may be a limiting factor. I plan to use constructivism often and with joy, but not exclusively. Sometimes you just have to get the job done…I will try harder not to be compromised by the constraints of institutionalized educational practices and make a better effort to make time for exploration while paying attention to the demands of the establishment. Somehow, this conflict needs to be resolved. (August, 1998) I have been using the LOGO notebook to encourage students to write and draw what they are thinking before we enter into a complete discussion. I do not think I used this strategy before this course. It is working well for my students and myself. I am making the time to reflect and I am hoping it will have a long-term payback. (December, 1999)	It just made me give more thought to what I am doing. It is really easy to be convinced by the people that you work with that coverage is most important rather than to develop understanding. Especially people who are thinking about MCAS and needing to cover certain topics. Every time I go through a professional development experience it helps me affirm what I already believe. It encourages me to be more student-centered, even when there is resistance. The most important thing I improved was my questioning—how to ask the right questions. This helps a person voice what they are thinking so you can figure out what they need to think about next. I also now do a lot of pair/share to congress type of activities to help kids verbalize. (July, 2020)

	Quotations from Writing	Excerpts of Interview Notes
10.	My concept of doing mathematics and learning mathematics is very different, especially since I have seen so many things this week. I used to believe that when I did math I knew what I was doing because I knew the formula or because I could manipulate symbols and arrive at the correct answer. Because I had the right answer, I felt that I understood the concept, but now I see how far away from the truth I was…Through my own learning experience, I can see how much the students need to explore, examine, play, and experiment with the new concept and they need to make up their own conclusions! Learning mathematics is a process. It takes time and it is hard work. We need to work on teaching the student how to internalize their knowledge. Teach them to make that knowledge their own and use it in many different ways. (August, 1998) I believe that understanding that there are steps we take in our learning process is important. I have discovered it is ok for me to "allow" myself to be frustrated. It helps me to know it is ok for me to make mistakes. It explains the fact that I have choices. I can either reorganize my ideas or assimilate them into my own frame of thinking. Most importantly it showed me that there is always a light at the end of the tunnel. As a teacher, it helps me to understand my students and reminds me that I, too, go through the same process, and that it is ok to be frustrated at times. (July, 1999)	It used to be: learn the facts, drill and memorization. There is a lot more problem solving and it's all hands-on. Kids get to make models, build things. The kids get to ask the questions. I try to spark their curiosity and not just telling them what we are doing--let them come up with it. For example if we are learning about multiplication I try to give them lots of problems to help them discover and understand the concept. I try to make learning fun by making some of the activities seem like games. I make sure the problems are relevant to the kids. I don't teach the book, the book is a tool and I use it to reinforce and practice. (July, 2002)

	Quotations from Writing	Excerpts of Interview Notes
11.	I approach each curriculum objective with an introduction using manipulatives. By using a hands on approach, students gain the knowledge and understanding needed for problem solving. (February, 1999) My fourth grade team teaches math in a very traditional setting. I felt I wanted to break free from that after the work I had done at Mt. Holyoke. I did so by using journals in my class for the first time…I also used group work to discuss strategies that the students were creating for problem solving. There were times when I knew I was on the right track with my questioning. I still catch myself leading my students to where I want them to be. I know that practice will only help me to improve that skill.	I am more focused on my questions. I ask my students questions, I try to move their thinking without leading them. I try to facilitate their group work and use the strategies kids are sharing to lead in my instruction. It is less teacher directed. (July, 2002)
12.	Specifically, I would like to look at the way math concepts are constructed, the development of mathematical thinking and its relationship to language…In addition I am interested in learning more ways to use collaborative groups for math inquiry, as well as writing as a tool for learning and using manipulative materials and diagrams as a way to represent thinking. (March, 1999)	Much different. I always keep that philosophy of trying to make something concrete that is abstract. Like if four people share 7 cookies, how much does each person get? I have them show three different ways to solve the problem. I just never thought about math very much before. I just did what I was taught. (June, 2002)
13.	Continually emphasizing the BIG IDEAS in math and helping students to make connections between and across topics has had a positive impact on my own practice and my supervisory role with student teachers. I am finding that teaching the BIG IDEAS is a powerful tool for any curriculum planning, be it integrated units for a classroom or structuring a teacher workshop…I realize that my comfort level with the math being explored in any lesson has tremendous impact on my ability to lead discussions and understand the thinking of students. (May, 1999)	I like to give kids a chance to grapple with a problem before I give them direction about how they might go about solving it. I ask them to share ideas during and after the process. I want them to understand that there are many doors into how you solve a problem. I used to use manipulatives for remediation. Now I use them to help all kids conceptualize what they are doing. I spend more time helping students make connections. Even with something like multiplication facts, we relate facts to one another. How does knowing form 6x6 help with 6x7? We connect fraction work to measuring with the ruler. (June, 2002)

	Quotations from Writing	Excerpts of Interview Notes
	I'd like to describe several experiences that contributed to my deeper understanding of multiplication, matrices, and the distributive property…A breakthrough in my conceptualization occurred during a fourth grade array game, where small arrays were compared with larger ones by placing one on top of the other, We found you could cover a 8x5 array with a 4x5 and another 4x5 so 8x5=2(4x5). Or you can use (8x2)+(8x3), or be wild and use 3 or more: (4x4)+(4x4)+(1x8).	
14.	I will really try to guide/model/teach students to verbalize their thinking when they are executing math topics …will use manipulatives to make the math experience more concrete and therefore more understandable...focus more on how I phrase questions. I'm still stuck on the teacher's role in guiding students without stifling their thought processes. ..If I don't "feed" them hints or "jump start" their thinking they don't know where to begin…The children need to see and hear many different types of problems in order to understand what is asked of them…and students need modeling and practice in order to fully understand and be successful.	I do more investigations and problems of the day. I was one of those dictators before. This is how you do it. Now sharing with each other is a big deal. My job is hard with mixed second and third graders and only ½ hour per day. Prior to this new way of thinking I would have thought that spending a few days on one problem was horrendous. I let them struggle for a little while so that they can make sense of things. (June, 2002)

	Quotations from Writing	Excerpts of Interview Notes
15.	Learning is obviously deeper when you discover or learn it on your own rather than have someone tell or teach you how to do it. The other element is higher level thinking—synthesis. When we have to struggle and stretch to learn our knowledge has new value. (July, 1998) The biggest change I intend to make in my teaching is to return to discovery learning and the use of manipulatives within the context of investigation…I want my students to construct their own concept of number. I don't want to just tell them how to do it…I plan to do more math writing and talking…Maybe make some posters… I want my students to be able to work in groups. I think I am able to be more sensitive to their social issues as a result of my own group interactions. (August, 1998)	Absolutely, because I'm more knowledgeable. I had courses around using manipulatives but the SMT courses helped me see how kids make sense of the operation and the inquiry process. I am more knowledgeable about how children think about math and how they rationalize math and math problems and the relationships between operations. There is more hands-on instruction and use manipulatives in more meaningful ways than I did before. I am always looking for investigations for the students. (June, 2002)

	Quotations from Writing	Excerpts of Interview Notes
16.	I believe that my method of teaching was always on an inquiry based concept. However, I never had total confidence in it or more so myself because it seemed to have holes in it…I am seeing this as a great release from the guilt that I am not going to worry that I do not follow conventional teaching (or drilling) strategies…Many times I would revert back to traditional techniques or whole language complete freedom approach, knowing I would achieve results but not depth of understanding. I am beginning to think I have not yet begun to fully trust my own knowledge, which now I may. I have had a weight lifted by changing my thinking on that I have always been a mathematical thinker and problem solver but not an arithmetic practitioner. (July, 1997) I know I've been floundering around in my math curriculum doing hodgepodge lessons. The TERC program has given me a sense of continuance. The institute has helped me with understanding the big picture. I know that I will approach the math curriculum with deeper understanding of what I am to provide for the students. (July, 1998)	Night and day. It was awful—paper and pencil—didn't know why—goal was to get through the book. When TERC Investigations came out with a book for first grade it was very helpful. Now, students verbalize, use words, numbers, or pictures, make connections, communicate what they are thinking. They know there is more than one approach, maybe more than one right answer. They love it when three people can be right. It is very empowering. Sometimes what they do is very high level math—I even have been able to talk about how their work is algebraic expression. The students do the basics but they also know how the numbers fit together. They pull numbers apart and put them together. They know their addition facts but also approach multiplication, division, algebra, and geometry. It is amazing. We do stories, find math everywhere, read books with numbers, and act out math situations. There is a constant connection with everyday life--math is everywhere. What I have done at SMT supercedes what I have done anywhere else, even the work for my master's degree, where I am going through the motions, but there is no comparison to what I have really learned at SMT. (June, 2002)

APPENDIX F

EXCERPTS RELATED TO REASONS FOR CHANGES IN INSTRUCTION BASED UPON PARTICIPANTS' WRITING AND NOTES FROM INTERVIEWS

EXCERPTS RELATED TO REASONS FOR CHANGES IN INSTRUCTION BASED UPON PARTICIPANTS' WRITING AND NOTES FROM INTERVIEWS

	Quotations from Writing	Excerpts of Interview Notes
1.	While direct teacher instruction is what we were brought up on, I think the kinds of instruction where learning comes from the sharing of students is far more beneficial. I think when children have the opportunity to show their classmates what the concept is all about in their own words, children tend to be more active listeners and more active participants in their learning. Though the children in the cases, videos, and my classroom are young, they do have a wealth of knowledge and experiences that, when shared, enrich our thinking, teacher and student alike.	Because these changes have helped me to understand math better. Math was a very difficult subject for me as a student. I was very intimidated by math. I found that the more I understood how math operated, the better I was able to help my students make their connections.
2.	I'm interested in learning how to successfully reach all students. I don't always have a good sense of how to modify a particular activity to help struggling students.	I see the effect it has on their thinking, problem solving, and excitement. It's exciting to see what the kids can do. It is so powerful for them to use their minds that way. When I first started doing this hard mathematical work with fifth graders who were not exposed to it in 3rd and 4th they would ask me for ditto sheets. It was really hard. I put a lot of energy into it every single day. Now I have 4th graders. Teaching 3rd and 4th graders I can see the difference in the younger kids. I can see the difference in their enthusiasm. They seem to enjoy it. They moan and grown if they miss math. They are working so hard. But maybe now I have had more experience, too.
3.	I learned that a relaxed, curious, and cooperative hands-on experiential approach will produce (as I had expected) more useful, ingrained, and open-minded knowledge.	Books don't meet a range of needs I was really glad to teach a different way, a way that is not boring. There is not enough depth for the brighter kids. And kids who are not as strong at math, do things like copy, etc. and are basically lost.

	Quotations from Writing	Excerpts of Interview Notes
4.		Those are the changes I feel you get best payoff for. I believe children learn best when they are doing something that they are ready to learn. Knowing where they are and what they're ready to do increases your chances that they will be successful.
5.	The feelings of being challenged and achieving satisfaction from the problems helped me to remember how important it is for students to experience these feelings…. This is the approach I use in my math classes and students seem to enjoy getting deeply involved in a mathematics problem each day… Perhaps the most beneficial aspect of the Summer Math program has been the ability to see the total picture of how students' develop their mathematical thinking toward the big ideas and principles of algebra.	The results are obvious. It's rewarding. What's really nice about my job is I get to see kids for more than one year. It's amazing what they will remember from year to year. They really gain an in-depth conceptual knowledge. In the conventional textbook-based lesson, you are really pushing superficial knowledge in them, which they don't remember. It's not ingrained in them. I use a lot of Investigations [an innovative standard-based complete elementary math curriculum] plus my own activities that I designed based on what I learned. These kids are the ones that really struggle and it's rewarding to see them remember what they learned in a previous year.
6.		I have had a lot of opportunities to co-plan with other teachers. It is so easy to see the value compared to the way I used to do it. It is so important to have number sense. I never did enough estimation like for example when we do tipping. You can multiply by .15 but in your head you can easily take 10% and then take half of that and add it together.
7.		I felt they could understand mathematics better. This is much more lasting and important than rote learning. It also helped the students to enjoy math.

	Quotations from Writing	Excerpts of Interview Notes
8.	I learned that the way I approach, model, listen, and comment on children's work has a great impact on how freely they explore and express their findings…One issue of student learning that I considered during this seminar was the importance of having all the hands-on experiences to understand what is behind the convenient formulas…knowing WHY they work.	Enlightenment. Before these classes I juts didn't consider other approaches much. It was a really a conscious effort to make some changes. I could see an increase in student involvement, interest, and enthusiasm for math. I could also see understanding was developing.
9.	While first hand experience might be the more effective way for most people to learn, it is not always possible…Anxiety over classroom practices is motivating me to seek out a better way of dealing with the entire range of needs in a classroom, especially those with exceptional capabilities in math…I hope to gain some insight in better assessing students' abilities or level in math in order to understand how best to proceed.	I don't think education is about instruction or the teacher. I think the learner has to be at the center. They can't just be trying to please the teacher because that does not really work. These techniques put the learner at the center of the learning process. They reason things through for themselves. This helps them be a learner.
10.	Through my own learning experience, I can see how much the students need to explore, examine, play, and experiment with the new concept and they need to make up their own conclusions! Learning mathematics is a process. It takes time and it is hard work. We need to work on teaching the student how to internalize their knowledge. Teach them to make that knowledge their own and use it in many different ways.	To make kids problem solvers. It's not just me telling them how to get the right answer or concept, but allowing them to discover the concept themselves. Let them come up with their own ideas. Solving a puzzle sparks their interest. Students need to feel like they are part of the process so that they may take ownership of their learning.
11.	By using a hands on approach, students gain the knowledge and understanding needed for problem solving.	After doing the work in the courses and looking at the way I work on math in the group settings, we all bring something to the table and our prior knowledge and experience helps move the ideas forward. My kids come into class with different ideas. I want their ideas to be the basis of their learning and not my ideas.

	Quotations from Writing	Excerpts of Interview Notes
12.		Because I wasn't successful. I didn't feel successful. The truth is that I don't know if I am any more successful now. However, I do have feedback from parents who are very happy with what their kids are learning in math.
13.		First, I personally experienced the power of these approaches when modeled at SMT training. I had also been into lots of classrooms where teachers were adept at the constructivist approach. I became fascinated with the idea of people explaining their thinking. I never say "that's right." I say "Are you sure?" or "Why do you think that's the right answer. Second, by explaining, children solidify their own understandings.
14.	The children need to see and hear many different types of problems in order to understand what is asked of them…and students need modeling and practice in order to fully understand and be successful.	It is more fun to teach this way. For Sped kids, I like it better. The kids get more out of it. Normally Sped kids just get to do calculations. They always had trouble with the word problems but now that they have a way to solve them using different approaches. Some of them are going to struggle no matter what you do. Some of them have definite math disabilities.
15.	Learning is obviously deeper when you discover or learn it on your own rather than have someone tell or teach you how to do it.	Because I experienced learning this way firsthand myself. I had to work problems out and experience how much more meaningful the learning was. I came to know the math in a different way. I had a much deeper understanding.
16.	Many times I would revert back to traditional techniques or whole language complete freedom approach, knowing I would achieve results but not depth of understanding.	After my first experiences trying I found kids having deeper understanding and tremendous ease in math. They have more ambition to persist in finding an answer and to solve complex problems. It allows a lot of different thinkers to succeed.

APPENDIX G

EXCERPTS RELATED TO CHANGES IN INSTRUCTION REGARDED AS MOST EFFECTIVE BASED UPON NOTES FROM INTERVIEWS OF SELECTED ELEMENTARY TEACHERS

EXCERPTS RELATED TO CHANGES IN INSTRUCTION REGARDED AS MOST EFFECTIVE BASED UPON NOTES FROM INTERVIEWS OF SELECTED ELEMENTARY TEACHERS

	Excerpts from Interview Notes
1.	Using manipulative tools and having students share their thinking with each other. These changes are most effective because they allow students to work through problems in a tactile way. Sharing strategies to problem solving allows students to see more ways to find solutions. The more strategies that are shared, the more opportunity for all students to find a way that makes sense for them.
2.	Biggest thing is working in groups. I am now better at dealing with that. It is really hard for even adults to work in groups. Kids working with a partner is very powerful, talking to each other really helps them along, working together. I group in different ways, partners or threes, ability or different styles. Kids can learn a lot from one another but also trying to figure out a problem on your own, hearing your own words, having someone react to your comments and using manipulatives is so helpful.
3.	Having enough manipulatives instantly available so kids can make sense of the problem situation and make it meaningful to them. I know where every kid is now and what they need to work on. I can write a problem just for that child. Kids are as involved as I am in the learning process. I get really excited and they get really excited if we find a new way of doing something. The more concrete you can get at this age, the better they can retain it. On MCAS is a child is stuck, they can find a way to figure it out even if they forgot how to do it—if they are accustomed to reasoning.
4.	The assessment piece in terms of knowing where the child and starting from there. It doesn't mean limiting or not exposing kids to things but being very aware at all times. The child has the best chance of being successful. I think success breeds success. One girl wrote me a note at the end of the year. She said you got me to like math. I always liked reading but now I like math.
5.	Posing good questions; giving them problems that are challenging enough to solve, coupled with discussion and writing is the key to retaining the math. They need rich problems that can be solved using multiple approaches. For example, make a 5 by 5 square and figure out how many one inch square tiles are inside. Students are learning about measurement, one and two dimensions, looking at different squares and coming up with the formula. Length times width, for finding the area of a square. There is thought, discovery, and struggling during their learning process. Your role is to ask questions and not tell the answer in order for it to work, and you have to know what you are looking for. You have to know the content you are aiming for. There are lots of ways of reinforcing the important math. Like in multiplication you need equal size groups and having them think about if there is another way to figure this out. The kids are really thinking. They seem to really take what they know and when they apply it

	somewhere else they are developing their thinking skills. You are teaching them how to attack problems and teaching them to think, analyze, evaluate. They need to be practicing those skills. Through the questioning, you are modeling this, leading them through the critical thinking process that should eventually become automatic for them.
6.	Trying to get kids to not just use the traditional algorithm, but encourage alternative methods. It is amazing how kids can come up with ways that work for them and are effective. It is so important that they truly understand the number system. I remember working with this fantastic artist. Even though she is bright and talented, when she learned math she couldn't see it. She could do math very easily and quickly in her head but wasn't allowed to. I gave her an example of what kids do in my class: Why would you do the traditional algorithm when like73 + 59 as combining the sums of 70+30 and 3+9 and thus holding on to the actual quantity of these numbers. She was so excited that kids could do this instead of the traditional algorithm.
7.	Teaching for understanding is the most important. That is, having kids build their own learning, not me telling them. Because they promote understanding. Students end up with some strategies to figure out a problem. They are more confident. They love math. Three of my students (GIRLS!!) scored Advanced [the highest category] on MCAS. They had their own approaches that they could use.
8.	Allowing time to explore and to express and compare their thinking. When they need to express themselves they are more cognizant of what they are doing, They can also learn more from one another as they see different children's approaches, which are also correct. I see more ownership. They all become involved even if they don't have the language to say what they mean. They are sharing, representing themselves.
9.	One thing I now do is put out wrong answer or a misconception. The hardest thing is not to give an answer until a student is satisfied. But you can't go on indefinitely. Learning to think about it is more important than the right answer. Learning how to think about it is critical. If we did that at an earlier age, our students would be better thinkers. They have to be actively engaged in the process. I think the approach requires that they become independent thinkers.
10.	Not telling them but getting them to discover the concept. Letting students find their own approaches to solving the problems. Before, kids were doing it because the teacher was telling them. Now, they are in charge of their learning process. It sparks their interest; they internalize it and make it their own. Once they have ownership, they can learn more easily. Our third grade math scores went from 70% proficient to 93% proficient. So I know it works.

11. Giving kids time to explore their ideas, share their strategies and compare their strategies. When I would watch videos at SMT seminars the students would say things like "I did my problem like John did," and I would say to myself that my kids would never listen to one another's ideas that way or say anything like that. But when I changed my instruction I was amazed that they actually did say stuff like that and refer to stuff that happened in past classes. It's been really fun to watch. It gives kids the opportunity to take ownership of their learning. Once they own it their confidence is built and their understanding does too. They have a deeper understanding when they build from what they already know. Something about sharing your thoughts and thinking clarifies it and also validates it.

12. The overall approach is different. I don't think it's any one thing. But I think the use of manipulatives is very powerful. Once they get the concept with the manipulatives they go to the drawing. And I think that is a logical progression. [I think it is most effective] by the responses I see from the students.

13. 1) Taking math from a solitary to a community activity. 2) Using varied grouping: partners, small groups, leading math discussions. 3) Allowing children to invent their own strategies. I hold off the presentation of conventional algorithms, because they stop thinking. They shut it down. I try to guide kids so that they construct the knowledge themselves, rather than my being the source and showing them how to do everything. That is the biggest change. When you communicate it forces you to deeper understanding. When you allow kids to invent strategies, they stop thinking about it as something you have to be shown how to do. They are not afraid to tackle any new problems.

14. Working on more story problems and the approach to story problems has helped. We always talk about the various strategies they can use. I have this really old fifth grade math book and I adapt the problems to second and third grade. I have another resource with story problems and the kids draw it to find the answer. We always review the different things they can do when they get stuck. [I think it is most effective] because it's going to give them a better understanding. They had no idea where to start before and now they have ways to think about it.

15. Active involvement in problem solving and using manipulatives [is most important] because it's more developmental and matches how people learn best—through inquiry and we know that memorizing isn't effective. We are not producing people, especially women, who like math. I used to dread teaching it and now I love teaching it. It's my favorite subject to teach and the favorite subject for my kids to learn.

16. Getting away from the memorization and the rote memorization. I just don't find it important at all at this age. The most effective tool is discussion about what you are thinking and how you came to that answer. I carry this over to all my subjects. Memorization comes easier as necessary in the upper grades because the students have a better understanding about the numbers. I am not just stuffing information into their heads, I am watering their ideas and letting them grow. In this program I can allow for their own research and help them construct meaning.

APPENDIX H

EXCERPTS RELATED TO ORGANIZATIONAL CONDITIONS THAT HELPED SELECTED ELEMENTARY TEACHERS TO IMPROVE THEIR INSTRUCTION BASED UPON NOTES FROM INTERVIEWS

EXCERPTS RELATED TO ORGANIZATIONAL CONDITIONS THAT HELPED SELECTED ELEMENTARY TEACHERS TO IMPROVE THEIR INSTRUCTION BASED UPON NOTES FROM INTERVIEWS

	Excerpts of Interview Notes
1.	Although the Addison-Wesley Mathematics series has been adopted by our school, my principal has been very supportive of my using Investigations. The Investigations series has allowed me to present math concepts in ways that encourage group work and discussion. She has also encouraged all teachers in our building to order and use manipulative tools, and often comes in to work with and listen to the thinking of our students in math classes. Also, many other teachers in our building have been encouraged to participate in SummerMath [for Teachers] Institute seminars. I do not feel that our school hinders our professional growth in any areas of learning. Rather, our school system encourages our growth to be professionally developed and encourages us to share our knowledge with staff members.
2.	A good deal of collegiality is helpful.
3.	Administrators encouraged us, paid for the courses, a group of us was going and this was very supportive. We started to get lots of manipulatives for our classroom.
4.	I worked with a principal who gave teachers a chance to really try research-based things. There was a lot of positive feedback from administration. There was availability of a lot of different ongoing professional development and for a while we had in-classroom support.
5.	I had a principal who was supportive of what I was doing. She gave me a lot of freedom in developing a math lab in what I wanted to do. I felt the freedom and support to try new things.
6.	My job was to help people break out of the box so they sent a few other teachers and me to SMT. The state test was really important because kids need to show their reasoning and their thinking and the changes I was making in instruction were a good match for the test. And it pushed people to change what they were doing in math class. On the Iowa test [ITBS] students performed the same or slightly less in computation but better on concepts and problem solving. On the state test our kids do very well. The school that implemented changes in instruction performed higher than the school that didn't. This was significant because the lower performing school has the highest average income students and they usually perform better than anyone else.
7.	I had a very supportive principal at my school. She encouraged me and helped me with innovative curriculum exploration. I also had 2 team members in 3rd grade who were also willing and eager to improve math instruction.

Excerpts of Interview Notes
8. I eventually had access to good curriculum materials that supported me in teaching the way I wanted to teach. It was hard to develop good activities [curriculum] on my own. The games are wonderful. We have a well-organized math center and system for storing and displaying the manipulatives and tools kids choose to help them figure things out. This is where kids choose things to help them figure things out. Biggest help has been contact with other like-minded teachers. It is understood that we must teach math for 1 hour per day. We have one fairly successful math night per year mainly due to the efforts of one teacher. Parents in general don't question our instructional approaches.
9. At the administrative level there is a lot of support for innovative teaching. Out principal even went through an experience with us.
10. In my school we are all working in the same direction. I have a very supportive principal. She is the one who suggested we do SMT. Two of us went through SMT, each for two years. If I need something, she'll get it. We have the freedom to work our own schedule with our grade level. We are departmentalized and I teach math for the three third grade classes in my school. I teach math for 1 hour and 15 minutes. We have common planning time. Everyone is very supportive in our school. I have nothing but help everywhere including from parents. We even do math month in our school with all kinds of problems and special events. We do the Marvels of Math, which are school wide math activities, and also put up a math bulletin board center for the whole school. It is a wonderful place.
11. I had a heterogeneously grouped class this year for my pilot of an innovative math curriculum. I received a half-year sabbatical to study more about math reform, and I have had the chance to go to national math conferences. The principal was impressed with what the kids were doing in my class, but his plate is full. There is too much on his plate. He is always supportive of what I do, but I have to be careful when I approach him to talk things over because there is always so much going on.
12. I have one other teacher in my building who is like-minded. I have been able to order anything that I have needed. My new principal is very supportive, believes in the use of manipulatives.
13. Class support so you can process with someone after a lesson. Curriculum days for grade level support meetings. Having all the materials—you have got to have what you need. You need a resource teacher to go to when you have needs or questions about the program. Teacher leaders and leadership training is helpful so people are available to think through the unit with you. There has to be somebody to help people stay on track. Release time with good professional development to revisit the math content is also paramount.
14. Having all the materials we need: notebooks, manipulatives. In Sped you're kind of left on your own. I had principal support. I borrowed a lot from the mainstream teachers—like the teaching manuals. We also have a nice staff that allows for easily sharing of ideas regarding the teaching of math. We all get very excited when things work!
15. We had plenty of manipulatives and a supportive principal. I did some team teaching, which was good.

Excerpts of Interview Notes
16. Superintendent's office provided opportunity to take SMT free. Superintendent's office provided support groups at grade level on curriculum days during the first two years of implementation of TERC program. Principal's support was there. Scheduling at the same time each day. I preferred early AM.

APPENDIX I

EXCERPTS RELATED TO ORGANIZATIONAL CONDITIONS THAT HINDERED SELECTED ELEMENTARY TEACHERS FROM IMPROVING THEIR INSTRUCTION BASED UPON NOTES FROM INTERVIEWS

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	Excerpts of Notes from Interviews
1.	I do not feel that our school hinders our professional growth in any areas of learning. Rather, our school system encourages our growth to be professionally developed and encourages us to share our knowledge with staff members.
2.	It's frustrating when the whole district is not on board with this. Then you have kids who don't have the background that they need. We give the Iowa test early in the year. Some kids were really mad that I gave them a test on what I didn't teach them. It was all calculations and it doesn't match what we do. The last MCAS test was really long and hard, but the kids put in so much effort. They worked on it for hours and it was supposed to be a one-hour test. Where does their persistence come from? It doesn't come from the textbook math. The district is inconsistent in implementation of this even though it's the official math curriculum. But teachers need to want to do it. They need to see and understand the many benefits of a constructivist math approach. It is a lot more time-consuming to prepare. We need an hour to do this and some years/days I really can't find an hour. I don't know if the administration really understands this.
3.	Scheduling is a big hindrance. We have short blocks of time. Specials, assemblies, practices, chorus, pull-outs. I get totally frustrated. For example, one ESL student gets pulled out of math three times per week. I would hate to actually count how much teaching time I lose over the course of a year. I have tried a few innovations, but it would be easier if administration would help out. I always get good evaluations but they don't even mention math.
4.	Naysayers, complainers; Teachers at the middle school trying to dictate what kids should know, what the product should be. Money was an issue and certain things couldn't continue after grant money ran out. There is a lack of continuity. District did not mandate the change so kids had a very uneven experience. That was frustrating for kids and teachers.
5.	Money (lack of it) is the big thing. It would have helped to be able to buy all the materials I needed. It is very time consuming to make the games and activities. Our system buys a conventional program and it is not helpful to me, although what the system buys for K and 1 is really good and teachers like it.
6.	The hardest thing was parents who were questioning why math can't just stay the way it always was, even though many parents were also very supportive. There was a good deal of support and yet a bit of hesitancy by top administration.
7.	The district has been lacking in a focus on mathematics. Math has been ignored. We had our first district wide PD in math this past January for the first time. There has not been a strong commitment toward math except to improve MCAS scores.

8.	There is not a school-wide explicit commitment to mathematics. There is little input from administration about the expectations for math instruction. I don't get the sense that there is any support from the principal for the way I teach math. Literacy instruction is the priority. I was placed in a classroom with established partners who had scheduled math right after recess. Time is lost settling students down. I am not sure how other people in the school teach math, but I hope to make changes for the next school year.
9.	But there is a lot of lip service given and there is a lot of fear and coverage keeps winning out over approaching mathematics in a constructivist way. So if you have one teacher in a grade doing constructivist math, but the other five teachers in that grade aren't, you really aren't making any progress because the students' experiences are so inconsistent year after year. People are in a panic about MCAS and the accountability system.. They perceive MCAS really promotes coverage so teachers are not likely to take risks. But I think kids have a better chance at figuring out the problems if they are confident thinkers and used to reasoning things through for themselves. I think a lot of good teachers have allowed themselves to be intimidated by MCAS. Actually, the open-ended questions require reasoning. It saddens me that more math teachers don't understand the MCAS tests were designed to encourage development of thinking skills.
10.	I don't think so. I feel really lucky and blessed.
11.	A traditional timed mid-year assessment instrument that tests basic computation; Parents were looking for worksheets on long division; District is going to adopt a program that is less of a match to what I think should be going on; Isolation has been difficult—everyone else in the building is a traditional math teacher.
12.	I paid my own tuition, Money is a big concern because I know other people who are interested but were not able to pay the tuition. I wanted to take a three-day course and it was denied, so I knew she didn't really know what was going on. Now I have a more supportive principal.
13.	When my school system was wishy washy about whether they were mandating the MT program or not. Teachers need clear direction from the administration, but it can't be totally top-down. It's hard if there are too many materials you have to make. Everybody needs to be on the same page. It was not helpful when they changed the math framework. MCAS has had too much of an influence on curriculum and instruction. You need consistency and continuity. You also have to train specialists and paraprofessionals and sped teachers.
14.	Having just ½ hour for math is a hindrance. But next year we are going to do it differently. I won't be pulling out kids. We will be using an inclusion model. Also, my math group was a mix of grades 2 and 3 and it was rather large (12 students). Hopefully, that will change next year too.
15.	We did need better curriculum materials, not textbook based, to support our changes in instruction. I had some conflict with a young teacher who was not very supportive as I changed the way I was teaching.

16. We weren't hindered at all. We were encouraged to come up with a format that worked for us. The only hindrance was from the upper grade teachers, because in the first few years the upper grade teachers didn't feel the children were as well prepared for their curriculum. By the third year I laugh at the fact that I am now the requested teacher if a child was having difficulty in math. By the third year all grades were at least 50% on board, so the continuity of learning was there. At this point our school is K-5 with Investigations as our primary math tool. At the middle school the program is not continued. However, 6th grade teachers are finding more consistent understanding coming from the 4 elementary schools with only one math program. Of course there is never enough time. The program requires more time than there is in a school day. I'd like to do math all day but I do need to teach the children to read!

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