Information Unraveling Revisited: Disclosure of Horizontal Attributes*

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Abstract

This paper analyzes in a spatial framework how much information a seller discloses about the variety he sells when he faces a buyer with a privately known taste for variety. I identify an equilibrium in which, for each possible variety, the seller's optimal strategy consists of either fully disclosing the variety or disclosing how far it is from the buyer's expected taste. The set of varieties the seller fully discloses monotonically expands as the buyer's taste for variety becomes stronger. I show that this is the unique undefeated equilibrium. From a policy perspective, mandating full disclosure is generally socially harmful.

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1 Introduction

A large literature has analyzed how much information a privately-informed seller voluntarily reveals when consumers are unable to tell the quality of a product prior to purchase. In their seminal papers, Grossman [1981], Grossman and Hart [1980] and Milgrom [1981] show that the seller fully reveals quality as long as there is a credible and costless means of conveying it. The primary force behind this finding is the fact that consumers' willingness-to-pay is strictly increasing in perceived quality. Therefore, a high-quality seller would always reveal its quality and distinguish itself from its own lower-quality images. As this reasoning applies to all seller types, if quality information is withheld, then it can only be the lowest-quality seller. Thus, information 'unravels.' Accordingly, mandatory disclosure rules are redundant because disclosure is costless and the seller voluntarily reveals quality regardless of its value.

Many goods are characterized by several attributes some of which are horizontal. However, very little attention has been paid to verifiable information disclosure when consumers are unable to observe horizontal attributes of a good. The main objective of this paper is to characterize the extent of information disclosure and the resulting social efficiency in such environments, and compare the results with those of quality disclosure. In contrast to a vertical attribute such as quality, consumers rank different varieties of a horizontal attribute differently. Geographical location of a hotel, expertise area of a researcher or sweetness of a wine are a few examples.

It is *a priori* unclear to what extent the unraveling argument works, if at all, when consumer uncertainty concerns a horizontal attribute. The seller may choose to provide only partial information, thereby bringing the perceived attribute closer to the ideal taste of the average buyer. For example, many firms these days engage in the practice of 'opaque' selling. Hotwire.com and Priceline.com are two prominent examples in the travel industry. Along with several 'transparent' options, they offer travel products whose characteristics (e.g., the airline company and the departure/arrival time in case of air tickets, or the name and the geographic location in case of hotels) are not transparent at the time of purchase.[1] Although certain characteristics of the product are revealed before

[1] People who have memberships with different airline alliances to accumulate miles will have different

purchase (e.g., the range of departure and arrival times for air tickets, or the approximate geographic location indicated by a circle on the map for hotels), participating consumers run the risk of receiving a product that they do not prefer much.[2] Similarly, a consumer can rent a compact-size car on Hotwire.com, but the supplier (e.g., Avis, Budget, Hertz or Dollar) is not revealed until after purchase.[3]

To analyze the extent of information disclosure in such environments, I consider a simple sales encounter in which there is a single seller (he) and a single buyer (she). The good is characterized by a single horizontal attribute, which I call variety. The seller is privately informed about the variety of the good while the buyer is privately informed about her ideal taste for the variety. Traditionally, markets with goods that have horizontal attributes have been analyzed using spatial models, and I continue in this tradition. Accordingly, the variety of the good and the buyer's ideal taste for it are represented by particular locations along a unit line *à la* Hotelling [1929], and the buyer strictly prefers a variety that is closer to her ideal taste. Prior to a possible transaction, the seller chooses a price and makes a report about the variety of the good. The only restriction I impose on the report is that it must be truthful. Thus, possible reports range from very precise (revealing the exact variety) to very vague (staying silent). The buyer observes the price and the report, and responds by purchasing one unit of the good or none.

I identify a class of payoff-equivalent perfect Bayesian equilibria (PBE) in which the information regarding the distance between the variety of the good and the **expected** ideal taste of the buyer (from the seller's point of view) fully unravels.[4] That is, the buyer always learns how far the variety is from her *ex ante* expected ideal taste, but not necessarily on which side. The seller fully reveals the variety if and only if the buyer's

preferences for airline companies. Similarly, two businessmen visiting a city for different business activities will have different preferences for flight times or hotel locations depending on where and when their business activities take place. So, these are horizontal attributes.

[2]Most offline travel agencies follow similar practices for all-inclusive travel packages, not revealing the airline company operating the flight or the name/location of the hotel.

[3]Another interesting example is "Fukubukuro." This is a Japanese New Year's Day tradition where retailers create 'lucky bags' with a collection of random items – not seen by buyers until after purchase – and sell them for a substantial discount.

[4]Possible PBE within this class differ only in terms of the equilibrium strategy of the seller, though all lead to the same payoffs.

preference for her ideal taste is sufficiently strong, and the set of fully revealed varieties monotonically shrinks (from all to (almost) none) as the buyer's preference for her ideal taste becomes weaker. Hence, information unraveling is still in effect, but not to the fullest extent. Moreover, this class of PBE 'defeats' all other PBE that may exist.[5]

From the seller's point of view, the probability of a purchase is higher when the variety of the good is closer to the expected ideal taste of the buyer. When it is not sufficiently close, the seller is tempted to disclose only partial information so as to bring the buyer's perceived variety (i.e., the expected variety conditional on the report received) closer to her expected ideal taste. However, such a report leaves some uncertainty regarding the true variety. The buyer dislikes uncertainty in the sense that her willingness-to-pay would be higher had the seller made a precise report indicating the same perceived variety without any uncertainty. Thus, there are two opposing factors the seller takes into account when deciding what report to make: (i) eradicating buyer uncertainty by fully revealing the variety, and (ii) bringing the perceived variety closer to the expected ideal taste of the buyer by disclosing partial information. The buyer understands that her expected ideal taste acts as a reference point for the seller. This leads her to associate a partially-revealing report with the variety that is farthest away from this reference point. Therefore, in situations when the seller discloses partial information, he never includes in his report varieties that are more distant from the expected ideal taste of the buyer than the true variety is. Since the seller employs the same strategy for all possible varieties, the distance between the true variety of the good and the expected ideal taste of the buyer fully unravels.

It may be easier to see the unraveling result with an example. As described before, the variety as well as the buyer's ideal taste are represented by locations over the unit line $[0, 1]$. Suppose that the expected ideal taste of the buyer is $\frac{1}{2}$ and that, in equilibrium, the seller fully reveals the varieties in $[0.4, 0.6]$ and makes a report in the form $[x, 1-x]$ for each other variety x. Consider the case when the seller makes a report saying that the variety belongs to $[0.3, 0.7]$. In this case, the buyer rationally infers that the true variety must be either 0.3 or 0.7 because had the variety been closer to $\frac{1}{2}$, the seller would

[5] The concept of 'undefeated' equilibria is due to Mailath, Okuno-Fujiwara and Postlewaite [1993].

have made a report that indicated a smaller maximum distance from $\frac{1}{2}$ (if, for instance, $x = 0.65$, then the seller would be better off sending $[0.35, 0.65]$ rather than $[0.3, 0.7]$). Hence, the degree of mismatch between the variety and the expected ideal taste of the buyer fully unravels.

The strength of the buyer's preference for her ideal taste plays an important role in the determination of which varieties are fully revealed. When it is weak, the buyer perceives different varieties as close substitutes, so uncertainty about the variety does not lower her willingness-to-pay too much. In this case, the seller's incentive to disclose partial information is higher. Similarly, when it is strong, the seller has a higher incentive to make a precise report since uncertainty significantly lowers the buyer's willingness-to-pay. This relationship is monotonic in the strength of the buyer's preference for her ideal taste, and therefore, the set of fully revealed varieties expands as it becomes stronger.

Whether mandatory disclosure rules are beneficial or not has been an important question. According to the literature on quality disclosure, mandatory rules are redundant because the seller voluntarily reveals the quality regardless of its value. I reach a similar finding in this paper. I find that a social planner cannot improve welfare by mandating the seller to fully reveal a particular variety that the seller voluntarily does not, while such a policy is often socially harmful. The intuition for this finding is as follows. By providing full information, the seller improves the match between the buyer and the product, thereby creating additional surplus for those buyers who have a good match. In case of partial disclosure, on the other hand, the seller faces a larger expected demand compared to full disclosure. This demand enlargement effect of disclosing partial information dominates the surplus created by providing full information, and as a result, forcing the seller to fully reveal a variety that he voluntarily does not is often socially harmful.

The basic model allows several extensions. I discuss these in section 5. Most importantly, buyer uncertainty about a vertical attribute (say, quality) can easily be incorporated. In this case, the usual unraveling story applies with respect to quality disclosure. Thus, regardless of the buyer's prior beliefs for it, quality would be fully revealed in every PBE. Accordingly, all the main results about variety disclosure remain the same.

Many authors have studied verifiable information disclosure in different contexts. However, most of them focus on vertical attributes. Examples include Jovanovic [1982] in which information disclosure is costly, Matthews and Postlewaite [1985] who allow the seller to decide whether to acquire quality information or not, Fishman and Hagerty [1990] who analyze how much discretion a seller should be allowed in choosing how much information to disclose about quality, Shin [1994] who incorporates uncertainty about the degree of information the seller possesses about quality, Board [2009], Cheong and Kim [2004], Hotz and Xiao [2010], Levin, Peck and Ye [2009], Milgrom and Roberts [1986] and Stivers [2004] who analyze quality disclosure in competitive environments,[6] Jin [2005], Jin and Leslie [2003] and Lewis [2011] who examine quality disclosure empirically, Daughety and Reinganum [2008] who incorporate the possibility of signaling quality by price into the standard disclosure framework, and Kartik [2009] who studies a unified model of verifiable disclosure and cheap talk *à la* Crawford and Sobel [1982].[7]

Three closely related papers that analyze disclosure of horizontal attributes are Sun [2011], Balestrieri and Izmalkov [2011] and Koessler and Renault [2011]. Sun [2011] considers a very similar problem in which the seller is constrained to either fully reveal all product information or stay silent. She finds that the set of seller types who reveal full information shrinks as quality increases when the buyer is uncertain only about the location. When the buyer is uncertain about both location and quality, she finds the opposite result; i.e., the set of seller types who reveal full information expands as the actual quality increases. Balestrieri and Izmalkov [2011] investigate a similar problem employing a mechanism design approach. Assuming that the product is located at either end of the unit line, they find that the optimal mechanism may involve full disclosure, no disclosure, or an option for the buyer to pay for product information prior to purchase. Koessler and Renault [2011] study a more general model that allows for both horizontal and vertical differentiation, and characterize the conditions under which a monopolist

[6] Hotz and Xiao [2010] and Levin, Peck and Ye [2009] also allow for horizontal product attributes. However, they assume that these are commonly known by consumers.

[7] See also Kamenica and Gentzkow [2011] who study how a sender can influence the decision of a rational agent by controlling her informational environment, and Rayo and Segal [2010] who examine optimal information disclosure when both the sender and the receiver possess private information. Differently from the classical disclosure literature, however, both papers assume that the sender credibly commits to a disclosure policy prior to learning his private information.

fully reveals product characteristics. They find that full revelation is always an equilibrium if product and consumer types are independently distributed. Moreover, they identify the conditions under which full revelation is the unique outcome.

Other related papers are Lewis and Sappington [1994], Anderson and Renault [2006] and Johnson and Myatt [2006]. Lewis and Sappington [1994] examine the trade-off that a seller faces in deciding how much knowledge to endow buyers of their idiosyncratic tastes for the product. While improved information facilitates price discrimination through which the seller can capture some of the extra surplus, it also leads to some buyers earning informational rents. If the buyers acquire no information, on the other hand, the seller can fully capture the surplus of the 'average' buyer. They find that the seller finds it optimal either to endow buyers with the most precise information or to provide no new information. Johnson and Myatt [2006] consider a general framework that builds upon the intuition that many economic activities – including informative advertising – influence the dispersion of consumer valuations, leading to a rotation in the demand curve. They find that profits are a U-shaped function of the dispersion of consumer valuations in many circumstances and, as a result, similar to Lewis and Sappington [1994], the seller pursues either maximal dispersion (niche-market strategy), serving high-value buyers at a high price, or minimal dispersion (mass-market strategy), serving a large fraction of buyers at a lower price. Anderson and Renault [2006] analyze the conditions under which a monopolist chooses to advertise price information and/or product match information. They introduce 'threshold match' advertising whereby a consumer learns whether her willingness-to-pay for the product is above or below a threshold. They show that a monopolist does better by advertising threshold match rather than full match. They also find that a monopolist may publicize only price, only match, or both depending on the value of the search cost consumers face.[8]

In all of the three papers above, buyers are ex-ante identical and the seller has no private information. Buyers' match value with the product is a random draw from a probability distribution that is known to both the monopolist and the consumer. There-

[8] See also Anderson and Renault [2009] which considers comparative advertising in a duopoly setting in which firms can also advertise their rival's product characteristics, and Anderson and Renault [2011] which extends Anderson and Renault [2006] by introducing quality disclosure.

fore, the particular way the seller reveals information is uninformative for buyers. Buyers make no inferences, for instance, if the seller does not reveal any information.

The remainder of the paper is organized as follows. In the next section, I introduce the basic model. In sections 3 and 4, I characterize the equilibrium level of information disclosure and investigate its social welfare properties. In section 5, I discuss possible extensions to the main model. Finally, in section 6, I present the concluding remarks.

2 Model

A profit-maximizing seller (S) offers a good (G) for sale which is characterized by a location over the unit interval, denoted by $x \in [0, 1]$. The location here indicates the variety of G, such as color, sweetness, etc. S is privately informed about x. I will use masculine pronouns for S and sometimes refer to x as S's type. The production costs do not depend on x, and without loss of generality, are assumed to be zero.

On the other side of the market, there is a single potential buyer (B) who has a unit demand for G. B's ideal taste, which describes the particular variety of G that she ideally wants to consume, is described by a location $\lambda \in [0, 1]$. This is private knowledge of B. Similarly, I will use feminine pronouns for B and sometimes refer to λ as B's type. If B buys a unit of G at a price P, then her net utility is $v - t(\lambda - x)^2 - P$, where v is the gross utility B enjoys when the variety of G perfectly matches with her ideal taste (i.e., when $x = \lambda$) and t measures the degree of disutility B incurs when x and λ differ from each other.[9] Not buying G yields zero utility. If B buys a unit of G, then S's payoff is P. Otherwise, S gets zero payoff.

The timing of the game is as follows. First, Nature selects a value for $x \in [0, 1]$ from a strictly positive density function $f(x)$ which is symmetric around $\frac{1}{2}$, and a value for λ from a uniform density function defined over $[0, 1]$. Hence, the ex-ante expected value of both the location of G and the ideal taste of B is $\frac{1}{2}$. S privately observes x while B privately observes λ. After observing x, S sends a truthful and costless message $M \subset [0, 1]$, and chooses a price P to which he commits thereafter.[10] As a tie-breaking rule, I assume in case of an indifference between two or more messages that S sends the

[9] Alternatively, v can be interpreted as the quality of G. See section 5 for further discussion.

[10] A message is truthful when $x \in M$.

most precise message. B observes M and P, and then decides whether to buy G or not. Finally, the payoffs are realized. All aspects of the game are common knowledge.

It is necessary to make a few remarks about the model. First, note that B's utility function is strictly concave in $(\lambda - x)$. This means that B dislikes uncertainty about the location of G. For instance, at a given price, a precise message $M = \left\{\frac{1}{2}\right\}$ is more favorable for B than a message $M = \left[\frac{1}{2} - \varepsilon, \frac{1}{2} + \varepsilon\right]$ which implies a conditional expected value of $\frac{1}{2}$ for x. Second, although I assume a single buyer with a privately known ideal taste, the results are identical with a continuum of buyers whose ideal tastes are uniformly distributed over the unit line. These two specifications are equivalent. Third, B has a unit demand in my model. This is without loss of generality because, as it will be clear later, the probability of a purchase declines with price. In other words, despite the unit demand assumption, S faces a downward-sloping expected demand function. Fourth, I assume that S makes his reporting and pricing decisions simultaneously and that price is observed by B prior to purchase. The simultaneity assumption is not crucial; S may make his reporting and pricing decisions in any order. However, it is crucial that B observes the price prior to purchase and S commits to the price he chooses. Finally, in line with the quality disclosure literature, I focus on truthful and costless messages.

The location of the good, x, is exogenously given in this paper. However, it is possible to allow S to influence it. Consider a production process in which the choice of location is subject to an error and S chooses a target location for G (for instance, sweetness of a wine crucially depends on the climate which is difficult to predict beforehand). The realized value of the error then determines the final location of G. Assuming that B knows the distribution of the error, her prior beliefs for the final location will be defined over a subset of $[0, 1]$. In fact, if the error term has a zero-mean symmetric distribution, S chooses a target location of $\frac{1}{2}$ since this is the expected ideal taste of B from his point of view. In this case, B's prior beliefs for x will be symmetric around $\frac{1}{2}$.

I use the concept of perfect Bayesian equilibrium (PBE) to solve the model. Let $m(x)$ describe the reporting strategy of S which is a mapping from $[0, 1]$ to all subsets of $[0, 1]$ such that $x \in m$. This determines what message S sends as a function of his private information. Let $p(x \mid M)$ denote the pricing strategy of S when the message he sends is M. Similarly, let $b(\lambda, M, P)$ describe the buying strategy of B, where $b = 1$ if she buys G and $b = 0$ if she does not. Finally, let π describe how B updates her beliefs based on the message and the price chosen by S. Thus, $\pi(z \mid M, P)$ is the probability density B assigns to $x = z$ when S sends a message M and chooses a price P. A PBE for this game is then defined as follows.

Definition *A PBE for this game is a quadruple $\sigma = (b, p, m, \pi)$ which is characterized by the following four conditions:*

(D.1) *For all M and P, b is B's best buying decision:*

$$b(\lambda, M, P) = \begin{cases} 1, & \int_0^1 (v - t(\lambda - x)^2 - P)\pi(x \mid M, P)dx \geq 0 \\ 0, & \text{otherwise} \end{cases}.$$

(D.2) *Given (D.1), p is the price that maximizes S's expected revenue when he sends a message M:*

$$p(x \mid M) = \arg\max_P \int_0^1 b(\lambda, M, P)Pd\lambda.$$

(D.3) *Given (D.1) and (D.2), m is the message that maximizes S's expected revenue subject to $x \in m$:*

$$m(x) = \arg \max_{M \supseteq \{x\}} \int_0^1 b(\lambda, M, p(x \mid M))\, p(x \mid M)d\lambda.$$

(D.4) *Let Ω describe the set of locations that induce S to send a message M and choose a price P, i.e., $\Omega = \{x \mid m = M,\ p = P\}$. Then, for all M and P such that $\Omega \neq \emptyset$, B updates her beliefs in the following way:*

$$\pi(x \mid M, P) = \begin{cases} \frac{f(x)}{\int\limits_{x \in \Omega} f(x)dx}, & x \in \Omega \\ 0, & \text{otherwise} \end{cases}.$$

(D.1) states that, for any observed message M and price P, B decides to buy a unit of G only if, given her updated beliefs, her expected net utility is non-negative. S rationally anticipates B's best response to any given M and P, and chooses the best price and message that maximize his expected revenue, $\int_0^1 b(\lambda, M, P)Pd\lambda$. These are stated in (D.2) and (D.3). Finally, (D.4) states that B rationally anticipates the price and the message S chooses for each x, and updates her beliefs about x in a Bayesian way for any observed M and P.

3 Equilibrium information disclosure

In this section, I investigate the properties of equilibrium information disclosure. I start with a benchmark case in which S knows B's type while B is uncertain about the variety x. I show that full disclosure is the unique outcome in this case. This will later help me answer whether B would have any incentive to reveal her type to S if she had such an opportunity. I then move to the analysis of two-sided asymmetric information and identify a class of PBE with partial disclosure. Finally, in subsection 3.3, I discuss other PBE and then argue that the PBE I identify in subsection 3.2 is the only undefeated PBE.

3.1 A benchmark case: One-sided asymmetric information

Suppose S knows B's type. In this case, for a given message M, he will charge a price $P = v - tE\left[(\lambda - x)^2 \mid x \in \Omega\right]$ and enjoy a revenue equal to P, where Ω is the set of locations that induce S to send a message M. In other words, S will optimally choose a price that leaves no surplus to B. Obviously, S fully reveals x when it perfectly overlaps with B's taste; i.e., when $x = \lambda$. This allows him to charge a price and earn a revenue of v. When x is farther away from λ, S would ideally want to pool with the locations that are closer to λ, thus lowering the expected mismatch $E\left[(\lambda - x)^2 \mid x \in \Omega\right]$ and increasing the price. However, as this reasoning applies to all types of S, B infers from such a pooling message that x cannot be any closer to λ than the farthest location included in the message. To see this, suppose $\lambda < \frac{1}{2}$ and take a variety $x = \lambda - \varepsilon$ for some $\varepsilon \in (0, \lambda]$. Suppose that S sends a message $M = [\lambda - \varepsilon, \lambda + \varepsilon]$. If B naively interpreted this message, then S would charge a price $P = v - t\int_{\lambda-\varepsilon}^{\lambda+\varepsilon} (\lambda - x)^2 \frac{dF(x)}{F(\lambda+\varepsilon)-F(\lambda-\varepsilon)}$ and B would buy. However, a rational B would realize that the types of S with $|\lambda - x| < \varepsilon$ would never pool with $x = \lambda - \varepsilon$ or $x = \lambda + \varepsilon$. Instead, they would send a message $M = [\lambda - |\lambda - x|, \lambda + |\lambda - x|]$ which would enable them to charge a higher price and enjoy a higher revenue. Thus, B infers that x is equal to either $\lambda - \varepsilon$ or $\lambda + \varepsilon$ following a message $M = [\lambda - \varepsilon, \lambda + \varepsilon]$.

Given the above argument, S can do no better than revealing x fully because he can never induce B to believe that the variety is closer to her ideal taste than it actually is.

In other words, the distance between the variety and B's ideal taste, $|\lambda - x|$, effectively becomes a vertical attribute and it therefore fully unravels in every PBE.[11] Given the tie-breaking assumption in favor of more precise reports, S fully reveals x in the unique PBE and earns a revenue equal to $P = v - t(\lambda - x)^2$. B, on the other hand, always buys G but does not derive any consumer surplus.

3.2 Two-sided asymmetric information

In this subsection, I turn back to the analysis of two-sided asymmetric information. I first describe B's optimal behavior for a given message and price. I then describe the optimal message and the price S chooses for each x, taking B's optimal behavior given. The main result is stated in Proposition 2 which provides a description of equilibrium information disclosure.

B's optimal behavior is summarized by (D.1) and (D.4). Given a message M and a price P, she updates her beliefs about x, as described in (D.4), and buys G if and only if her net expected surplus from buying is non-negative, as described in (D.1). Thus,

$$b(\lambda, M, P) = 1 \Leftrightarrow v - tE\left[(\lambda - x)^2 \mid x \in \Omega\right] - P \geq 0, \tag{1}$$

where Ω is, as described in (D.4), the set of locations that induce S to send a message M and choose a price P. Solving expression (1) for λ yields

$$\lambda^L = \max\left\{0, E\left[x \mid x \in \Omega\right] - \sqrt{\frac{v - P}{t} - Var\left[x \mid x \in \Omega\right]}\right\}, \tag{2}$$

$$\lambda^H = \min\left\{1, E\left[x \mid x \in \Omega\right] + \sqrt{\frac{v - P}{t} - Var\left[x \mid x \in \Omega\right]}\right\}, \tag{3}$$

where λ^L (λ^H) is the lowest (highest) type of B that buys G when S sends a message M and chooses a price P.

Since S is uncertain about λ, he takes B's optimal behavior as given and chooses a message and a price that maximizes the expected revenue $E\left[bP\right]$ as described in (D.2). For notational convenience, let D denote the expected demand S faces. This is simply

[11] The same unraveling argument equally applies when B has a downward-sloping demand at a known location.

the probability that λ lies between λ^L and λ^H. Since S's priors for λ are uniform over $[0, 1]$, it is given by

$$D(P; x, v, t) = \lambda^H - \lambda^L.$$

As mentioned in the previous section, t measures how strong B's preference for her ideal taste is, and v can be interpreted as the quality of G. When t is high, a mismatch between the variety and B's ideal taste reduces B's willingness-to-pay badly. Similarly, when v is high, consumption of G offers a high utility. Therefore, the expected demand S faces at a given price is increasing in the value of $\frac{v}{t}$.

Analyzing the expected demand function, $D(P; x, v, t)$, leads to two important observations. On the one hand, S wants to bring the perceived location of G (i.e., $E[x \mid x \in \Omega]$) as close to the expected ideal taste of B (which is $\frac{1}{2}$) as possible by sending a partially-revealing message that pools the actual location of G with more central ones. This strictly raises the expected demand S faces when x is close to 0 or 1. On the other hand, S wants to keep uncertainty (captured by $Var[x \mid x \in \Omega]$) as low as possible because B dislikes it. At times λ^L and λ^H do not bind (i.e., not equal to 0 and 1, respectively), a higher uncertainty lowers the expected demand. These two factors work against each other, so S's optimal decision depends on which factor dominates.

First, consider the situation when x is commonly known (or, equivalently, when S fully reveals it). Letting a subscript 1 indicate this situation, equations (2) and (3) reduce to

$$\lambda_1^L = \max\left\{0, x - \sqrt{\frac{v-P}{t}}\right\},$$

$$\lambda_1^H = \min\left\{1, x + \sqrt{\frac{v-P}{t}}\right\}.$$

Let p_1 and R_1 denote, for a given (v, t), the optimal price S chooses and the resulting equilibrium expected revenue he makes when x is known. For a given location x, the revenue-maximizing price is[12]

$$p_1(x, v, t) = \arg\max_P PD_1(P; x, v, t),$$

which leads to equilibrium expected revenue S makes

$$R_1(x, v, t) = p_1 D_1(p_1, x, v, t).$$

[12] The equilibrium value of p_1 for all (x, v, t) can be found in section A1 of the appendix.

Proposition 1 *$R_1(x,v,t)$ is strictly increasing for $x < \min\left\{\sqrt{\frac{v}{3t}}, \frac{1}{2}\right\}$, constant for $\min\left\{\sqrt{\frac{v}{3t}}, \frac{1}{2}\right\} \leq x \leq \max\left\{\frac{1}{2}, 1-\sqrt{\frac{v}{3t}}\right\}$ and strictly decreasing for $\min\left\{\frac{1}{2}, 1-\sqrt{\frac{v}{3t}}\right\} < x \leq 1$.*

Proof. See section A2 of the appendix. ■

From S's point of view, the likelihood B buys G is higher the closer the location of G is to the expected ideal taste of B. That is why the revenue S expects under full location information increases as x gets closer to $\frac{1}{2}$. When $\frac{v}{t}$ is not too high, neither λ_1^L nor λ_1^H binds (i.e., is not equal to 0 and 1, respectively) at the optimal price S chooses for the values of x between $\min\left\{\sqrt{\frac{v}{3t}}, \frac{1}{2}\right\}$ and $\max\left\{\frac{1}{2}, 1-\sqrt{\frac{v}{3t}}\right\}$. Therefore, for these locations, S is effectively unconstrained and is able to achieve the highest revenue he can. When x is closer to the edges, on the other hand, either λ_1^L or λ_1^H becomes binding and the expected demand S faces goes down. Therefore, S earns a lower revenue as x is farther away from $\frac{1}{2}$. When $\frac{v}{t}$ is sufficiently high, either λ_1^L or λ_1^H is binding for all locations and therefore R_1 attains a unique maximum at $x = \frac{1}{2}$.

Proposition 1 has an important implication: when B is uncertain about x, S's optimal information disclosure strategy calls for fully revealing all locations $\min\left\{\sqrt{\frac{v}{3t}}, \frac{1}{2}\right\} \leq x \leq \max\left\{\frac{1}{2}, 1-\sqrt{\frac{v}{3t}}\right\}$. This is because doing so leads to a revenue of $R_1(\frac{1}{2}, v, t)$. Since neither λ_1^L nor λ_1^H is binding at the optimal price S chooses for these locations, sending a partially-revealing message cannot improve the expected demand S faces. Note that $x = \frac{1}{2}$ is fully revealed in every PBE regardless of the value of $\frac{v}{t}$.

This observation plays a key role in the characterization of equilibrium information disclosure. Suppose that S fully reveals the locations that lie in $(z, 1-z)$ in equilibrium and consider the case when $x = z$. S knows that, regardless of the message he sends, B will never assign a positive probability to the values of x between z and $1-z$ because S would normally fully reveal these locations. In other words, the usual unraveling story is at work here. In case S chooses not to fully reveal $x = z$, his problem is to choose a message that brings the perceived location as close to $\frac{1}{2}$ as possible while keeping uncertainty as low as possible. Since $f(x)$ is symmetric around $\frac{1}{2}$, S can induce a perceived location of exactly $\frac{1}{2}$ by sending, for instance, a message $M = [z, 1-z]$. This message also leads to the lowest uncertainty that S can induce. In fact, as Proposition 2 describes, S's equilibrium choice reduces to either fully revealing $x = z$ or sending a partially-revealing message that would induce B to think that x is equal to either z or $1 - z$.

Proposition 2 *There exists a class of payoff-equivalent PBE in which the value of $\left|\frac{1}{2} - x\right|$ is always revealed, whereas x is fully revealed if and only if $\frac{v}{t}$ is sufficiently low. Moreover, the set of fully revealed locations monotonically shrinks as $\frac{v}{t}$ becomes higher.*

Proof. See section A2 of the appendix. ■

Proposition 2 describes how the information unraveling result extends to markets with goods that have horizontal attributes. In equilibrium, B understands that her expected ideal taste acts as a reference point for S. This leads her to adopt a pessimistic posture in which she associates a partially-revealing message with the location that is farthest away from this reference point. Therefore, in case S chooses to send a partially-revealing message, he pools the true location only with the ones that are equally or less distant from $\frac{1}{2}$. Since S employs the same strategy for all possible locations, the distance between the true location of G and the expected ideal taste of B fully unravels.

It is important to note that there are many messages that lead to the same equilibrium outcome. For example, a message $M = [z, 1 - z]$ or simply $M = \{z, 1 - z\}$ induces B to conclude that the true location is either z or $1 - z$. Multiplicity of equilibrium messages is typical in verifiable information disclosure games. However, since all equilibria are payoff-equivalent, it does not change any of the results. It is also important to note that, in case S sends a partially-revealing message, he would choose the same price for either of the two locations that are inferred by B since, otherwise, price would signal the location. So, in equilibrium, price is not informative about location.

Similar with the earlier notation, let a subscript 0 indicate a partially-revealing message whereby p_0 and R_0 denote the optimal price S chooses and the equilibrium expected revenue he earns when he sends a partially-revealing message. If $R_0 > R_1$ for a particular x, then S chooses to send a partially-revealing message. The revenue S expects to earn in this case can be found as follows. Suppose $x = z$ and S sends a message $M = [z, 1 - z]$. B's inference is $\Omega = \{z, 1 - z\}$ where she assigns equal probability to each possibility.[13]

[13]Note that Bayes' rule does not work since both are ex-ante zero-probability events. In this case,

So,

$$E[x \mid x \in \Omega] = \frac{1}{2},$$

$$Var[x \mid x \in \Omega] = \left(\frac{1}{2} - z\right)^2,$$

and thus equations (2) and (3) reduce to

$$\lambda_0^L = \max\left\{0, \frac{1}{2} - \sqrt{\frac{v-P}{t} - \left(\frac{1}{2} - z\right)^2}\right\},$$

$$\lambda_0^H = \min\left\{1, \frac{1}{2} + \sqrt{\frac{v-P}{t} - \left(\frac{1}{2} - z\right)^2}\right\}.$$

Expressing these expressions for a generic x, the revenue-maximizing price is[14]

$$p_0(x, v, t) = \arg\max_P PD_0(P; x, v, t),$$

where $D_0(P; x, v, t) = \lambda_0^H - \lambda_0^L$. This leads to the equilibrium expected revenue S earns

$$R_0(x, v, t) = p_0 D_0(p_0, x, v, t).$$

Compared to full disclosure, sending a partially-revealing message induces S to charge a lower price. Therefore, if S chooses to send a partially-revealing message rather than a fully-revealing one, the expected demand he faces in the former case must be larger than the expected demand he faces in the latter. This is depicted in Figure 1 for $v = 0.6$, $t = 1$ and $x = 0.3$. The solid curve is the expected demand S faces when x is fully revealed, D_1, and the dashed curve is the expected demand he faces when he sends a partially-revealing message, D_0. For sufficiently high prices, neither λ_1^L nor λ_1^H is binding and therefore S can expand D_1 on both sides of x by lowering price. This is no longer

updating proceeds as follows:

$$Prob(x = z \mid x \in \{z, 1-z\}) = \lim_{\varepsilon \to 0} \frac{F(z+\varepsilon) - F(z)}{F(z+\varepsilon) - F(z) + F(1-z) - F(1-z-\varepsilon)}.$$

Using l'Hôpital's rule,

$$Prob(x = z \mid x \in \{z, 1-z\}) = \lim_{\varepsilon \to 0} \frac{f(z+\varepsilon)}{f(z+\varepsilon) + f(1-z-\varepsilon)} = \frac{f(z)}{f(z) + f(1-z)} = \frac{1}{2}.$$

[14]The equilibrium value of p_0 for all (x, v, t) can be found in section A1 of the appendix.

true for prices for which λ_1^L is binding (this happens for prices below the kink in D_1). In this region, the marginal effect of a price decrease is much smaller under full disclosure than partial disclosure. For the parameter values in Figure 1, even though $p_0^* < p_1^*$, S still chooses to send a partially-revealing message because $D_0^* - D_1^*$ is sufficiently large to ensure a higher revenue.

[Place Figure 1 approximately here]

A comparison of R_1 and R_0 yields the set of locations that are fully revealed in equilibrium. This is graphically illustrated in Figure 2 for $v = 0.6$ and $t = 1$. The horizontal axis indicates the value of x. The solid curve is the expected revenue S earns when x is fully revealed, R_1, while the dashed curve is the expected revenue he earns when he sends a partially-revealing message, R_0. As seen in the figure, a set of central locations (i.e., $x \in [x_H, 1 - x_H]$) is fully revealed because, as described earlier, S can achieve a revenue of $R_1(\frac{1}{2}, v, t)$ by fully revealing these locations. Focusing on $x \leq \frac{1}{2}$, as x gets more distant from $\frac{1}{2}$, λ_1^L becomes binding in case S fully reveals x, so S prefers sending a partially-revealing message, thereby bringing the perceived variety to $\frac{1}{2}$ and thus expanding demand. The adverse effect of uncertainty is minimal for locations close to $\frac{1}{2}$ but increases quickly as x gets closer to the edges. Therefore, the locations below x_L (symmetrically those above $1 - x_L$) are also fully revealed.

[Place Figure 2 approximately here]

Recall that for an observed message M and price P, B buys G if and only if her location is at most $\sqrt{\frac{v-P}{t} - Var\left[x \mid x \in \Omega\right]}$ units away from $E\left[x \mid x \in \Omega\right]$. For a given value of x, if $\frac{v}{t}$ is sufficiently low (i.e., when $\frac{v}{t} < Var\left[x \mid x \in \Omega\right]$), $D_0 \leq 0$ for any price, so fully revealing x is optimal. As $\frac{v}{t}$ increases, S can generate a positive demand by sending a partially-revealing message for low enough prices. Moreover, a higher $\frac{v}{t}$ lowers the negative effect of a marginal increase in price on D_0 by reducing the adverse effect of uncertainty (since the effect of $Var\left[x \mid x \in \Omega\right]$ vanishes as $\frac{v}{t}$ becomes large).

As a result, a higher $\frac{v}{t}$ makes it more likely that S sends a partially-revealing message, thereby bringing the perceived variety to $\frac{1}{2}$. When $\frac{v}{t}$ is sufficiently high, S fully reveals only the most central variety $x = \frac{1}{2}$.

Holding v constant, for lower (higher) values of t, both curves in Figure 2 shift upwards (downwards). The magnitude of the shift is higher for R_0 compared to R_1. Therefore, the set of fully revealed locations shrinks (grows). In other words, x_H increases (decreases) while x_L decreases (increases) as $\frac{v}{t}$ becomes higher. When $\frac{v}{t}$ is below a certain threshold (approximately 0.521), S fully reveals all values of x. When it is sufficiently high (higher than 0.75), S fully reveals only $x = \frac{1}{2}$, while sending a partially-revealing message for the remaining locations.

Would B reveal her ideal taste if she had such an opportunity? The answer is no since, as discussed in the preceding subsection, S would fully extract B's surplus if he knew λ. By keeping it as private information, on the other hand, certain types of B will surely enjoy a strictly positive expected utility while no type of B will ever end up with a negative utility. Thus, it is B's private information about her ideal taste that is responsible for any partial information disclosure that may arise in equilibrium.

3.3 Other PBE and equilibrium selection

The model presented in section 2 allows for other PBE. An example is a fully revealing one. A belief structure that supports this particular PBE can be described as follows: in case S sends an off-equilibrium message, B puts probability 1 on the location that is most distant from $\frac{1}{2}$, and if there are two such locations, then she puts probability 1 on the one on the left (or right).

It is well understood for models of costless information disclosure that equilibrium refinements such as 'Intuitive Criterion' or 'Universal Divinity' have no bite in selecting equilibria. This applies to the current model as well. Therefore, I turn attention to a more recent refinement introduced by Mailath et al. [1993], called 'Undefeated Equilibria.' According to this refinement, an equilibrium is 'defeated' if it fails the following test. Consider a proposed equilibrium and take a message that is off the equilibrium path. If there is an alternative equilibrium in which this message is on the equilibrium path for a non-empty set of types and it is precisely these types that obtain higher payoffs (strictly higher for at least one type) in the alternative equilibrium, then the test requires that the beliefs in the former equilibrium follow Bayes' rule for this set of types. Applying this refinement to the current model leads to the following result.

Proposition 3 *The (class of) PBE described in Proposition 2 is the only undefeated (class of) PBE.*

Proof. See section A2 of the appendix. ■

To give a sense of the proof, compare the PBE described in Proposition 2 with a fully revealing PBE. Consider the message $M = \{z, 1-z\}$ which is an off-equilibrium message for the fully revealing PBE. Suppose the parameter values are such that this is an on-equilibrium message for the PBE described in Proposition 2. Then, types z and $1-z$ must be earning a strictly higher revenue by sending $M = \{z, 1-z\}$ rather than fully revealing themselves. It thus follows that the fully-revealing PBE is defeated by the PBE described in Proposition 2 because beliefs in the former do not put probability $\frac{1}{2}$ on each location, but instead put probability 1 on $x = z$. A similar reasoning applies to all other PBE. Therefore, the class of PBE described in Proposition 2 arises as the only undefeated PBE.

4 Social Planner's Problem

In this section, I analyze the social welfare properties of equilibrium information disclosure. I focus attention on policies in which a social planner may mandate S to fully reveal a given set of locations.[15] When full disclosure is not mandatory for a particular location x, S may choose to fully reveal it or send a partially-revealing message as described in the previous section (i.e., pool it with $1-x$). Thus, if the total expected welfare (S's revenue plus B's net utility) evaluated under full disclosure is higher than the expected welfare evaluated under a partially-revealing message for a particular location x, then the social planner mandates S to fully reveal it (unless S voluntarily does so).

[15] The first-best is to set the price equal to the marginal cost of production (which is 0) and force S to fully reveal the variety at all times.

Even though the classical information disclosure literature typically finds excessive information disclosure, this finding critically depends on the assumption that consumers have unit demands with identical reservations prices. In this case, since disclosure does not change the equilibrium quantity purchased, it is purely redistributive. In the current model, on the other hand, the expected demand S faces is downward-sloping. Although S charges a higher price under full location information, B makes a better-informed decision. So, while it is clear that S's expected revenue goes down by mandating him to fully reveal a location which he would normally not reveal, B's net expected utility may increase. Therefore, it is *a priori* unclear whether there is any need for intervention.[16]

Denote the expected consumer surplus as CS_i and the total expected welfare as W_i, where $i = 1$ if S sends a fully revealing message and $i = 0$ if S sends a partially-revealing message. When x is fully revealed, S chooses a price p_1 and B buys G if her location is at most $\sqrt{\frac{v-p_1}{t}}$ units away from x (in other words, if $\lambda \in \left[\lambda_1^L, \lambda_1^H\right]$). Thus, for a particular value of x,

$$CS_1(x,v,t) = \int\limits_{\lambda_1^L(p_1,x,v,t)}^{\lambda_1^H(p_1,x,v,t)} (v - p_1(x,v,t) - t(\lambda - x)^2)d\lambda, \tag{4}$$

$$W_1(x,v,t) = R_1(x,v,t) + CS_1(x,v,t). \tag{5}$$

Similarly, when S sends a partially-revealing message, he chooses a price p_0 and B buys G if her location is at most $\sqrt{\frac{v-p_0}{t} - \left(\frac{1}{2} - x\right)^2}$ units away from $\frac{1}{2}$. Thus,

$$CS_0(x,v,t) = \int\limits_{\lambda_0^L(p_0,x,v,t)}^{\lambda_0^H(p_0,x,v,t)} \left(v - p_0(x,v,t) - t(\lambda - x)^2\right) d\lambda \tag{6}$$

$$W_0(x,v,t) = R_0(x,v,t) + CS_0(x,v,t). \tag{7}$$

Alternatively, consumer surplus can conveniently be expressed as the area under the corresponding demand curve and above the equilibrium price.[17]

$$CS_1(x,v,t) = \int\limits_{p_1(x,v,t)}^{v} D_1(P;x,v,t)dP,$$

[16] If disclosure is sufficiently costly, a monopoly seller may under-provide full quality information when demand is downward-sloping. See Daughety and Reinganum [2008] and Celik [2011] for further details.

[17] See section A3 of the appendix for a formal derivation.

$$CS_0(x,v,t) = \int\limits_{p_0(x,v,t)}^{v-t(\frac{1}{2}-x)^2} D_0(P;x,v,t)dP.$$

Given the set of policies available to the social planner and the welfare definitions given in equations (5) and (7), the social planner mandates S to fully reveal a particular variety x if $W_1(x,v,t) > W_0(x,v,t)$ whereas $R_1(x,v,t) < R_0(x,v,t)$ (so that S normally sends a partially-revealing message). Proposition 4 establishes that there is actually no variety x for which this is true.

Proposition 4 *Mandating S to fully reveal a location does not improve social welfare, while it is often socially harmful.*

Proof. See section A2 of the appendix. ■

For B, there are two opposing consumer surplus effects of partial disclosure. Both of these effects can be seen in Figure 1. On the one hand, being partially informed about the variety, B's expected match with G is reduced compared to full information, which leads to a decrease in consumer surplus. This is the area above p_1^* and between the two demand curves in Figure 1. On the other hand, for varieties that are close to 0 or 1, S can expand the expected demand by sending a partially-revealing message, in particular for low prices (this is because λ_1^L or λ_1^H becomes binding for prices below a threshold, so the rate at which S can expand D_1 by lowering price goes down). In such a case, S charges a lower price, which leads to a higher demand and a higher consumer surplus. This is the area between the two prices, p_0^* and p_1^*, and to the left of D_0. It is the magnitude of these two effects that determines if partial disclosure improves welfare. I show in the proof of Proposition 4 that the second effect is always larger than the first one in situations in which S normally sends a partially-revealing message (i.e., $CS_0 > CS_1$ whenever $R_0 > R_1$), so mandating S to fully reveal x harms S. In other words, the demand enlargement effect of disclosing partial information is large enough to offset the loss due to potential mismatch.[18]

[18] Under partial disclosure, some buyer types incur very high transportation costs ex post, while some

5 Discussion

In this section, I discuss several points related to possible extensions of the model. The first point regards incorporating uncertainty about quality. As noted before, the parameter v can be interpreted as the quality of the good. If the buyer is also uncertain about the quality, it can easily be shown that the seller's expected revenue is strictly increasing in the perceived quality of the buyer. This is true even when the seller is assumed to be uncertain about the buyer's taste for quality.[19] Thus, regardless of her prior beliefs for it, quality would be fully revealed in every PBE. In other words, the usual unraveling story applies with respect to quality disclosure. Accordingly, all of the main results about location disclosure remain valid.

Second, I have considered a general message technology whereby the seller could send any message that includes the true location of the good. If the seller is somehow constrained to either fully reveal the location or stay silent, then the structure of equilibrium information disclosure substantially changes. When the buyer's preference for her ideal taste is sufficiently strong, the seller fully reveals all locations. When it is weak, the seller fully reveals a set of central locations while staying silent for the remaining ones. Depending on the shape of the buyer's prior beliefs, there may be multiple PBE. In this case, each PBE is characterized with a different set of fully revealed locations. The set of fully revealed locations shrinks in every PBE as the buyer's preference for her ideal taste becomes weaker, but is always non-empty. Sun [2011] analyzes this problem when the buyer's disutility due to a mismatch increases linearly with the value of the mismatch (i.e., when the transportation cost function is linear). Her findings are very similar with one major difference. She finds that the seller stays silent for all locations if the buyer's preference for her ideal variety is sufficiently strong.

turn lucky. In section A3 of the appendix, I show that

$$\int_{\lambda_0^L}^{\lambda_0^H} (\lambda - x)^2 \, d\lambda = \int_{\lambda_0^L}^{\lambda_0^H} \frac{1}{2} \left((\lambda - x)^2 + t(1 - x - \lambda)^2 \right) d\lambda,$$

so, on average, these effects even out.

[19]Consider the utility function $\theta v - p - t(\lambda - x)^2$, where $\theta > 0$. Here, θ measures the buyer's taste for quality. It is easy to show that the seller's expected revenue is strictly increasing in the perceived quality when the seller is uncertain about θ and λ while the buyer is uncertain about v and x.

The third point is about the shape of the transportation cost function. The class of PBE described in Proposition 2 remains valid for any strictly convex transportation function. When the transportation function is linear as in Sun [2011], although the PBE described in Proposition 2 is still valid, there are many other PBE which are payoff-equivalent for the seller, but are substantially different in terms of the buyer's equilibrium inferences. To see this, suppose that $\frac{v}{t}$ is large enough so that all types of the buyer are served in equilibrium. In this case, the seller optimally charges a price equal to $v - \frac{t}{2}$ (the price that leaves the buyer types $\lambda = 0, 1$ indifferent between buying and not), but otherwise is indifferent between sending any (truthful) message that leads to a perceived location of $\frac{1}{2}$. One example is $M = \{x, 1-x\}$ for each x as in the current paper, while another example is $M = [0, 1]$ for all varieties, which is equivalent to staying fully silent. The inferences in these two examples are quite different. While the PBE described in Proposition 2 is still valid with a linear transportation function, the welfare results may change. The seller is able to capture a much higher portion of the buyer's surplus by sending a partially-revealing message since the buyer is more neutral to uncertainty. Therefore, in situations in which the seller only slightly prefers sending a partially-revealing message to fully revealing x, it may be welfare enhancing to mandate the seller to fully reveal x.[20]

The fourth point is related to the assumption of costless information disclosure. If disclosure is costly, on the contrary, then the seller may prefer staying silent when the location is close to the edges. However, provided that it is not too costly, the structure of information disclosure stays the same for more central locations. If it is too costly, the seller stays silent for all locations. In this case, it may be socially beneficial to mandate the seller to fully reveal a set of central locations. See Daughety and Reinganum [2008] and Celik [2011] for a similar result in a quality-disclosure framework when the seller faces a downward-sloping demand.

[20] As an example, take $x = 0$. If the seller sends $M = \{0, 1\}$, he chooses $p_0 = v - \frac{t}{2}$ and the resulting revenue is $R_0 = p_0$. If he fully reveals x, then the demand he faces is $D_1(P) = \frac{v-P}{t}$, so the optimal price is $p_1 = \frac{v}{2}$ and the resulting revenue is $R_1 = \frac{v^2}{4t}$. When $\frac{v}{t} = 2 - \sqrt{2}$, it is easy to verify that $R_0 = R_1$. However, $CS_0 = 0$ because all buyer types have the same willingness-to-pay when the seller sends $M = \{0, 1\}$, while $CS_1 > 0$ because the buyer types $\lambda < \frac{v-p_1}{t}$ enjoy a strictly positive surplus when $x = 0$ is fully revealed. Therefore, for $\frac{v}{t}$ slightly above $2 - \sqrt{2}$, it is better to mandate the seller to fully reveal $x = 0$.

A final point is about the prior beliefs of the buyer about the location of the good. Even though I have assumed that the prior beliefs are symmetric around $\frac{1}{2}$, the class of PBE described in Proposition 2 is valid for any prior beliefs. Consider the following off-equilibrium beliefs. When the seller includes many locations in his message, the buyer associates the good with the location that is farthest away from $\frac{1}{2}$. In case there are two such locations, the buyer assigns a positive probability to both.[21] Under these beliefs, the seller never sends a message that includes locations farther away from the center than the good's true location. Therefore, the class of PBE described in Proposition 2 remains valid. However, it is generally not the unique PBE. Unless $f(x)$ is symmetric around $\frac{1}{2}$, sending a partially-revealing message as described in Proposition 2 does not lead to a perceived location of $\frac{1}{2}$. Therefore, the seller may choose a message that brings the perceived location closer to $\frac{1}{2}$ unless the adverse effect of uncertainty is too high.

6 Conclusion

In this paper, I analyze the level of information a privately-informed monopoly seller voluntarily reveals about the horizontal attribute of the good he sells. The horizontal attribute is captured by a location over the unit line. I consider a single buyer with a privately known ideal taste which is also captured by a location (although the findings would be the same if there is a continuum of buyers with different ideal tastes). Although information unraveling does not apply to the fullest extent, it is still at work. I show that there is a unique class of undefeated PBE in which the degree of mismatch between the true location of the good and the expected ideal taste of the buyer fully unravels. The driving force for this finding is the (optimal) skepticism of the buyer; any partially-revealing message induces her to believe that the true location of the good is the one in the message that is farthest away from her expected ideal taste. The seller fully reveals the true location only when the buyer's preference for her ideal taste is sufficiently strong. As it becomes weaker, the set of fully revealed locations monotonically shrinks and when

[21] There is an exception for messages such that $M \subset \left[\sqrt{\frac{v}{3t}}, 1 - \sqrt{\frac{v}{3t}}\right]$. These are the locations for which the seller earns a revenue of $R_1\left(\frac{1}{2}, v, t\right)$. In this case, assume the buyer believes the message as it is. Since such an inference introduces a positive variance, the seller would never deviate from fully revealing $x \subset \left[\sqrt{\frac{v}{3t}}, 1 - \sqrt{\frac{v}{3t}}\right]$.

it is sufficiently weak, the seller fully reveals only the location that corresponds to the expected ideal taste of the buyer.

From a social point of view, I find that it is never welfare-improving, but is often socially harmful, to mandate the seller to fully reveal a location that he voluntarily does not. The reason for this finding is the demand enlargement effect of a partially-revealing message whereby the seller typically charges a lower price compared to what he would charge under full disclosure. This is in line with the classical information disclosure literature which also finds excessive disclosure.

I have assumed that horizontal attributes of a good can be described by a single location. Future work may consider multiple horizontal and vertical attributes and analyze the incentives of a monopoly seller to provide information on multiple dimensions. Moreover, such an extension would enable an empirical test of the model. An example is the market for real estate where there is typically a limited number of characteristics sellers may reveal in advertisements.

Appendix

A1 Equilibrium price

In this section of the appendix, I derive the equilibrium price S chooses under the two possible scenarios: when S fully reveals x and when he sends a partially-revealing message. This will later be helpful in the proofs of propositions. Note that since S's beliefs for λ are uniform over $[0,1]$, the probability B buys G at a given price is symmetric around $\frac{1}{2}$ with respect to x. So, it will be sufficient to characterize equilibrium price for $x \leq \frac{1}{2}$ only.

Case 1 *When S fully reveals x*

Since S's beliefs for λ are uniform over $[0,1]$, the probability B buys G at some given price is symmetric around $\frac{1}{2}$ with respect to x. So, it will be sufficient to characterize equilibrium price for $x \leq \frac{1}{2}$ only. By equations (2) and (3), for a given (P,v,t), if S chooses a price such that $\frac{v-P}{t} < \frac{1}{4}$, then

$$D_1(P;x,v,t) = \begin{cases} x + \sqrt{\frac{v-P}{t}} & , x < \sqrt{\frac{v-P}{t}} \\ 2\sqrt{\frac{v-P}{t}} & , \sqrt{\frac{v-P}{t}} \leq x \leq \frac{1}{2} \end{cases}.$$

If, on the other hand, $\frac{1}{4} \leq \frac{v-P}{t} < 1$, then

$$D_1(P;x,v,t) = \begin{cases} x + \sqrt{\frac{v-P}{t}} & , x < 1 - \sqrt{\frac{v-P}{t}} \\ 1 & , 1 - \sqrt{\frac{v-P}{t}} \leq x \leq \frac{1}{2} \end{cases}.$$

Finally, when $\frac{v-P}{t} \geq 1$, all types of B buy G, so $D_1(P;x,v,t) = 1$. Maximization of $P\left(x + \sqrt{\frac{v-P}{t}}\right)$ with respect to P leads to a price of $\frac{2t}{9}\left(\frac{3v}{t} - x^2 + x\sqrt{\frac{3v}{t} + x^2}\right)$, while the same for $2P\sqrt{\frac{v-P}{t}}$ leads to a price of $\frac{2v}{3}$. Checking for corner solutions leads to the following equilibrium price (tedious but otherwise straightforward algebra).

- If $\frac{v}{t} < \frac{3}{4}$,

$$p_1(x,v,t) = \begin{cases} \frac{2t}{9}\left(\frac{3v}{t} - x^2 + x\sqrt{\frac{3v}{t} + x^2}\right) & , x < \sqrt{\frac{v}{5t}} \\ t\left(\frac{v}{t} - x^2\right) & , \sqrt{\frac{v}{5t}} \leq x < \sqrt{\frac{v}{3t}} \\ \frac{2v}{3} & , \sqrt{\frac{v}{3t}} \leq x \leq \frac{1}{2} \end{cases}.$$

- $\frac{3}{4} \leq \frac{v}{t} < \frac{5}{4}$,

$$p_1(x,v,t) = \begin{cases} \frac{2t}{9}\left(\frac{3v}{t} - x^2 + x\sqrt{\frac{3v}{t} + x^2}\right) & , x < \sqrt{\frac{v}{5t}} \\ t\left(\frac{v}{t} - x^2\right) & , \sqrt{\frac{v}{5t}} \leq x \leq \frac{1}{2} \end{cases}.$$

- If $\frac{5}{4} \leq \frac{v}{t} < 3$,

$$p_1(x,v,t) = \begin{cases} \frac{2t}{9}\left(\frac{3v}{t} - x^2 + x\sqrt{\frac{3v}{t} + x^2}\right) & , x < 2 - \sqrt{1 + \frac{v}{t}} \\ t\left(\frac{v}{t} - (1-x)^2\right) & , 2 - \sqrt{1 + \frac{v}{t}} \leq x \leq \frac{1}{2} \end{cases}.$$

- If $\frac{v}{t} \geq 3$,

$$p_1(x,v,t) = t\left(\frac{v}{t} - (1-x)^2\right) \text{ for all } x \leq \frac{1}{2}.$$

Note that p_1 is non-monotonic in x (as x goes from 0 to $\frac{1}{2}$). For $\frac{v}{t} < \frac{5}{4}$, when x is sufficiently close to 0, S prefers to keep the price low in order to increase the probability of a purchase, thereby leaving a positive surplus to the $\lambda = 0$ type B. So, in this region, S effectively chooses the highest type of B that he wants to serve. Therefore, as x gets closer to $\frac{1}{2}$, the price S optimally sets increases. When $\sqrt{\frac{v}{5t}} \leq x \leq \min\left\{\sqrt{\frac{v}{3t}}, \frac{1}{2}\right\}$, it becomes optimal to make $\lambda = 0$ type B indifferent between buying and not. Therefore, the equilibrium price is decreasing in this region. When $\frac{v}{t} \geq \frac{5}{4}$ on the other hand, the real question S faces is whether to sell or not to the $\lambda = 1$ type B. This particular type is willing to pay more for values of x closer to 1. Therefore, the equilibrium price is increasing over $x \in \left[0, \frac{1}{2}\right]$ when $\frac{v}{t}$ is large.

Case 2 *When S sends a partially-revealing message*

When S sends a partially-revealing message, say $M = [x, 1-x]$, B infers that the true variety must be either x or $1-x$. Hence, when S charges a price P, equations (2) and (3) reduce to

$$\lambda_0^L = \max\left\{0, \frac{1}{2} - \sqrt{\frac{v-P}{t} - \left(\frac{1}{2} - x\right)^2}\right\},$$

$$\lambda_0^H = \min\left\{1, \frac{1}{2} + \sqrt{\frac{v-P}{t} - \left(\frac{1}{2} - x\right)^2}\right\}.$$

For a given (P, v, t), if S chooses a price such that $\frac{v-P}{t} < \frac{1}{4}$,

$$D_0(P; x, v, t) = \begin{cases} 0 & , x < \frac{1}{2} - \sqrt{\frac{v-P}{t}} \\ 2\sqrt{\frac{v-P}{t} - \left(\frac{1}{2} - x\right)^2} & , \frac{1}{2} - \sqrt{\frac{v-P}{t}} \leq x \leq \frac{1}{2} \end{cases}.$$

Similarly, if $\frac{1}{4} \leq \frac{v-P}{t} < \frac{3}{4}$,

$$D_0(P; x, v, t) = \begin{cases} 2\sqrt{\frac{v-P}{t} - \left(\frac{1}{2} - x\right)^2} & , x < \frac{1}{2} - \sqrt{\frac{v-P}{t} - \frac{1}{4}} \\ 1 & , \frac{1}{2} - \sqrt{\frac{v-P}{t} - \frac{1}{4}} \leq x \leq \frac{1}{2} \end{cases}.$$

Finally, when $\frac{v-P}{t} \geq \frac{3}{4}$, all types of B buy G, so $D_0(P; x, v, t) = 1$. Maximizing $PD_0(P; x, v, t)$, with respect to P leads to the following equilibrium price (when the expected demand equals 0 for any $P \geq 0$, I assume that the equilibrium price is 0).

- If $\frac{v}{t} < \frac{1}{4}$,

$$p_0(x, v, t) = \begin{cases} 0 & , x < \frac{1}{2} - \sqrt{\frac{v}{t}} \\ \frac{2t\left(\frac{v}{t} - \left(\frac{1}{2} - x\right)^2\right)}{3} & , \frac{1}{2} - \sqrt{\frac{v}{t}} \leq x \leq \frac{1}{2} \end{cases}.$$

- If $\frac{1}{4} \leq \frac{v}{t} < \frac{3}{4}$,
$$p_0(x, v, t) = \frac{2t\left(\frac{v}{t} - \left(\frac{1}{2} - x\right)^2\right)}{3}, \text{ for all } x \leq \frac{1}{2}.$$

- If $\frac{3}{4} \leq \frac{v}{t} < 1$,
$$p_0(x, v, t) = \begin{cases} \frac{2t\left(\frac{v}{t} - \left(\frac{1}{2} - x\right)^2\right)}{3} & , x < \frac{1}{2} - \sqrt{\frac{v}{t} - \frac{3}{4}} \\ t\left(\frac{v}{t} - \left(\frac{1}{2} - x\right)^2 - \frac{1}{4}\right) & , \frac{1}{2} - \sqrt{\frac{v}{t} - \frac{3}{4}} \leq x \leq \frac{1}{2} \end{cases}.$$

- If $\frac{v}{t} \geq 1$,
$$p_0(x, v, t) = t\left(\frac{v}{t} - \left(\frac{1}{2} - x\right)^2 - \frac{1}{4}\right), \text{ for all } x \leq \frac{1}{2}.$$

In this scenario, when $\frac{v}{t}$ is small, S cannot generate any demand for G unless it is located sufficiently close to $\frac{1}{2}$. So, in this case, the choice of price is random. I assume, for simplicity, that S charges a price of 0 in such a case. In all other cases, p_0 is strictly positive and it strictly increases as x gets closer to $\frac{1}{2}$. When $\frac{v}{t}$ sufficiently large, S serves all types of B, so in this case, the equilibrium price is the one that leaves zero surplus to $\lambda = 0$ (or, equivalently, $\lambda = 1$) type B.

A2 Proofs of the Propositions

In this section, I present the proofs of the propositions stated in the main text. Since the prior beliefs for x are symmetric around $\frac{1}{2}$, I will consider only the values of x over $\left[0, \frac{1}{2}\right]$ unless otherwise noted.

Proof of Proposition 1. Using Envelope Theorem, over the values of x for which $p_1 = \frac{2t}{9}\left(\frac{3v}{t} - x^2 + x\sqrt{\frac{3v}{t} + x^2}\right)$, we have $\frac{dR_1}{dx} = P\frac{\partial D_1(P;x,v,t)}{\partial x}$ evaluated at $P = p_1$. In this range, $D_1 = x + \sqrt{\frac{v-P}{t}}$, so $\frac{\partial D_1}{\partial x} = 1$. Since $p_1 > 0$, it follows that $\frac{dR_1}{dx} = p_1 > 0$ for these values of x. For the values of x for which $\lambda = 0$ or $\lambda = 1$ type B is made indifferent between buying and not, Envelope Theorem is not applicable (because it is a corner solution). When $\frac{v}{t} < \frac{5}{4}$, this happens for $\sqrt{\frac{v}{5t}} \leq x < \min\left\{\sqrt{\frac{v}{3t}}, \frac{1}{2}\right\}$ in which case S charges a price $p_1 = t\left(\frac{v}{t} - x^2\right)$ and faces an expected demand $D_1 = x + \sqrt{\frac{v-P}{t}} = 2x$. So, the equilibrium revenue is simply $R_1 = 2t\left(\frac{v}{t} - x^2\right)x$, which is strictly increasing in x for all $x < \min\left\{\sqrt{\frac{v}{3t}}, \frac{1}{2}\right\}$. Similarly, when $\frac{v}{t} \geq \frac{5}{4}$, it happens for $\max\left\{0, 2 - \sqrt{1 + \frac{v}{t}}\right\} \leq x \leq \frac{1}{2}$

in which case S charges a price $p_1 = t\left(\frac{v}{t} - (1-x)^2\right)$ and serves all types of B (since $x + \sqrt{\frac{v-P}{t}} = 1$), so $R_1 = t\left(\frac{v}{t} - (1-x)^2\right)$. This is again strictly increasing in x. Finally, when $\frac{v}{t} < \frac{3}{4}$, S charges a price $p_1 = \frac{2v}{3}$ for $\sqrt{\frac{v}{3t}} \leq x \leq \frac{1}{2}$ and faces an expected demand $D_1 = 2\sqrt{\frac{v-P}{t}} = 2\sqrt{\frac{v}{3t}}$. The revenue $R_1 = \frac{4v}{3}\sqrt{\frac{v}{3t}}$ is constant for these values of x. ■

Proof of Proposition 2. I start with showing that there exists a PBE in which the value of $\left|\frac{1}{2} - x\right|$ fully unravels. I then proceed with showing that the set of fully revealed locations shrinks as $\frac{v}{t}$ is higher. To make the latter easier, I present two lemmas below. Finally, I argue that there are values of $\frac{v}{t}$ for which $R_0 < R_1$ for all x and for which $R_0 > R_1$ for all x. This concludes the proof.

Before proceeding, it is useful to make the following two important observations. First, if two messages lead to the same perceived variety, S strictly prefers the message associated with a lower implied variance. Formally, suppose there are two messages M and M' such that $E[x \mid x \in \Omega] = E[x \mid x \in \Omega']$ and $Var[x \mid x \in \Omega] < Var[x \mid x \in \Omega']$. Then, M leads to a strictly higher expected revenue than M'. Second, off-equilibrium beliefs cannot be randomly chosen in verifiable disclosure games. After observing an off-equilibrium message M, B will not assign a positive probability to any $x \notin M$. For example, if S unexpectedly fully reveals x, then B believes S because lying is ruled out.

First, suppose $\frac{v}{t} < \frac{3}{4}$ so that the region $\sqrt{\frac{v}{3t}} \leq x \leq 1 - \sqrt{\frac{v}{3t}}$ is non-empty. By Proposition 1, this is where the expected revenue S earns is constant and is equal to $R_1\left(\frac{1}{2}, v, t\right)$. The locations in this region must be fully revealed in every PBE because any partially-revealing PBE implies a positive variance, $Var[x \mid x \in \Omega] > 0$, and S can profitably deviate by fully revealing x thereby achieving $R_1(x, v, t) = R_1\left(\frac{1}{2}, v, t\right)$. For $x < \sqrt{\frac{v}{3t}}$, by Proposition 1, $R_1(x, v, t)$ is strictly increasing in x. Given that S fully reveals all $x \in \left[\sqrt{\frac{v}{3t}}, 1 - \sqrt{\frac{v}{3t}}\right]$, then it is best for S to either fully reveal x or reveal $\left|\frac{1}{2} - x\right|$, say by sending $M = [x, 1-x]$, for all $x < \sqrt{\frac{v}{3t}}$ (symmetrically, for $x > 1 - \sqrt{\frac{v}{3t}}$). The latter strategy is associated with the lowest variance among all possible inferences S may induce B to make. This is because pooling with locations that are farther away from $\frac{1}{2}$ than the true location simply raises $Var[x \mid x \in \Omega]$. An example of supporting off-equilibrium beliefs are as follows: after observing a partially-revealing message, B assigns a probability of 1 to the location that is farthest away from $\frac{1}{2}$ (in case there are two such locations, assume that B assigns full probability to the location on the left). Note that under these beliefs, S is indifferent between sending any message $M \subset \left[\sqrt{\frac{v}{3t}}, 1-\sqrt{\frac{v}{3t}}\right]$ for $\sqrt{\frac{v}{3t}} \leq x \leq 1-\sqrt{\frac{v}{3t}}$ because any such message leads to an expected revenue of $R_1\left(\frac{1}{2}, v, t\right)$. However, by the tie-breaking rule that S chooses the most precise message in case of indifference, he fully reveals all of these locations.

When $\frac{v}{t} \geq \frac{3}{4}$, $R_1(x, v, t)$ is strictly increasing in x for all $x \leq \frac{1}{2}$. Again, given that S fully reveals $x = \frac{1}{2}$ in every PBE, it is best for S to either fully reveal x or reveal $\left|\frac{1}{2} - x\right|$, say by sending $M = [x, 1-x]$, for all $x \neq \frac{1}{2}$. Hence, when S sends a message $M = [x, 1-x]$, B optimally assigns a positive probability to both x and $1-x$. This completes the first part of the proof.

Next, I show that the set of fully revealed locations shrinks as $\frac{v}{t}$ becomes higher. This is substantially eased by the following two lemmas. The first one establishes that, under both strategies S may choose (i.e., either fully reveal x or reveal $\left|\frac{1}{2} - x\right|$), the derivative of equilibrium revenue divided by t with respect to $\frac{v}{t}$ is equal to the corresponding expected demand. The second lemma shows that whenever a partially-revealing message is more profitable than fully revealing x, the expected demand under the former is at least as large as the one under the latter. Before proceeding with the lemmas, note from equations (2) and (3) that, under both strategies, price enters the expected demand function as $\frac{P}{t}$. Moreover, the equilibrium prices I find in section A1 are multiples of t. Thus, both $\frac{p_j}{t}$ and $\frac{R_j}{t}$ $(j = 0, 1)$ are functions of only x and $\frac{v}{t}$.

Lemma 1 $\frac{d(R_1/t)}{d(v/t)} = D_1(p_1, x, v, t)$ *and* $\frac{d(R_0/t)}{d(v/t)} = D_0(p_0, x, v, t)$ *for all* x.

Proof of Lemma 1. I start with the case when S fully reveals x. First, take the values of x and $\frac{v}{t}$ for which $\frac{p_1}{t} = \frac{2}{9}\left(\frac{3v}{t} - x^2 + x\sqrt{\frac{3v}{t} + x^2}\right)$. In this region, $D_1 = x + \sqrt{\frac{v-P}{t}}$. By the Envelope Theorem, $\frac{d(R_1/t)}{d(v/t)} = \frac{P}{t}\frac{\partial D_1}{\partial(v/t)}$ evaluated at $P = p_1$. Since $\frac{\partial D_1}{\partial(v/t)} = -\frac{\partial D_1}{\partial(P/t)}$, and the revenue maximization problem implies $D_1 + \frac{P}{t}\frac{\partial D_1}{\partial(P/t)} = 0$ evaluated at $P = p_1$, we have $\frac{d(R_1/t)}{d(v/t)} = D_1$. When $\frac{v}{t} < \frac{5}{4}$, for $\sqrt{\frac{v}{5t}} \leq x \leq \min\left\{\sqrt{\frac{v}{3t}}, \frac{1}{2}\right\}$, S charges a price $p_1 = t\left(\frac{v}{t} - x^2\right)$ and faces an expected demand $D_1 = x + \sqrt{\frac{v-P}{t}} = 2x$. The equilibrium revenue is simply $R_1 = 2t\left(\frac{v}{t} - x^2\right)x$, and thus, $\frac{d(R_1/t)}{d(v/t)} = 2x$, which equals the equilibrium expected demand. Similarly, when $\frac{v}{t} \geq \frac{5}{4}$, for $\max\left\{0, 2 - \sqrt{1 + \frac{v}{t}}\right\} < x \leq \frac{1}{2}$, S charges a price $p_1 = t\left(\frac{v}{t} - (1-x)^2\right)$ and serves all types of B, so $R_1 = t\left(\frac{v}{t} - (1-x)^2\right)$.

Again, $\frac{d(R_1/t)}{d(v/t)} = 1$ which equals the equilibrium expected demand. Finally, when $\frac{v}{t} < \frac{3}{4}$, for $\sqrt{\frac{v}{3t}} \leq x \leq \frac{1}{2}$, S charges a price $p_1 = \frac{2v}{3}$ and faces an expected demand $D_1 = 2\sqrt{\frac{v-P}{t}} = 2\sqrt{\frac{v}{3t}}$. Hence, $\frac{R_1}{t} = 4\left(\frac{v}{3t}\right)^{3/2}$, and thus, $\frac{d(R_1/t)}{d(v/t)} = 2\sqrt{\frac{v}{3t}}$ which, again, equals the equilibrium expected demand.

When S sends a partially-revealing message, there are three prices depending on x that he can possibly charge, as given in section A1. When $x < \frac{1}{2} - \sqrt{\frac{v}{t}}$, the expected demand equals 0 for all values of the price, so the result is trivial for this case. In the range where $p_0 = \frac{2t\left(\frac{v}{t} - \left(\frac{1}{2} - x\right)^2\right)}{3}$, the expected demand is $D_0 = 2\sqrt{\frac{v-P}{t} - \left(\frac{1}{2} - x\right)^2}$. By Envelope Theorem, $\frac{d(R_0/t)}{d(v/t)} = \frac{P}{t}\frac{\partial D_0}{\partial(v/t)}$ evaluated at $P = p_0$. Since $\frac{\partial D_0}{\partial(v/t)} = -\frac{\partial D_0}{\partial(P/t)}$, and the revenue maximization problem implies $D_0 + \frac{P}{t}\frac{\partial D_0}{\partial(P/t)} = 0$ evaluated at $P = p_0$, we have $\frac{d(R_0/t)}{d(v/t)} = D_0$. Finally, in the range where $p_0 = t\left(\frac{v}{t} - \left(\frac{1}{2} - x\right)^2 - \frac{1}{4}\right)$, the expected demand is $D_0 = 1$. Hence, $\frac{R_0}{t} = \frac{p_0}{t}$, and thus, $\frac{d(R_0/t)}{d(v/t)} = 1 = D_0$. ■

Lemma 2 *If $R_0 \geq R_1$ for some x, then $D_0 \geq D_1$ for the same x.*

Proof of Lemma 2. When $x = \frac{1}{2}$, two regimes are equivalent, so the following analysis applies to $x < \frac{1}{2}$. If $R_0 \geq R_1$ for some x at which $D_0 = 1$, the result is trivial. From section A1, this happens for $\frac{1}{2} - \sqrt{\frac{v}{t} - \frac{3}{4}} \leq x \leq \frac{1}{2}$ when $\frac{3}{4} \leq \frac{v}{t} < 1$, and for all x when $\frac{v}{t} \geq 1$. For values of x for which $p_0 = 0$ or for which $p_1 = \frac{2v}{3}$, it is always true that $R_0 < R_1$, so, again, the result is trivial. For the remaining configurations, it is enough to simply compare the equilibrium values of p_0 and p_1 for the same x. When $x < \sqrt{\frac{v}{5t}}$, for all x in the range,

$$p_1 = \frac{2t}{9}\left(\frac{3v}{t} - x^2 + x\sqrt{\frac{3v}{t} + x^2}\right) \geq \frac{2v}{3} > \frac{2t\left(\frac{v}{t} - \left(\frac{1}{2} - x\right)^2\right)}{3} = p_0.$$

When $\sqrt{\frac{v}{5t}} \leq x < \min\left\{\sqrt{\frac{v}{3t}}, \frac{1}{2}\right\}$,

$$p_1\left(\sqrt{\frac{v}{3t}}, v, t\right) = \frac{2v}{3} > \frac{2t\left(\frac{v}{t} - \left(\frac{1}{2} - \sqrt{\frac{v}{3t}}\right)^2\right)}{3} = p_0\left(\sqrt{\frac{v}{3t}}, v, t\right).$$

Since p_1 is decreasing and p_0 is increasing in x in this range, it follows that $p_1 > p_0$ for all x here, too. Hence, if $R_0 \geq R_1$ for some $x < \min\left\{\sqrt{\frac{v}{3t}}, \frac{1}{2}\right\}$, then it must be that $D_0 \geq D_1$ for the same x. ■

Finally, I argue that there are values of $\frac{v}{t}$ for which $R_0 < R_1$ for all x and for which $R_0 > R_1$ for all x (except for $x = \frac{1}{2}$ where $R_0 = R_1$). Together with the two lemmas, this concludes the proof. Note that $R_0 < R_1$ for all $x \geq \sqrt{\frac{v}{3t}}$ (since $R_1 = R_1\left(\frac{1}{2}, v, t\right)$ in this region) and for all $x \leq \frac{1}{2} - \sqrt{\frac{v}{t}}$ (since $R_0 = 0$ in this region). $\sqrt{\frac{v}{3t}} = \frac{1}{2} - \sqrt{\frac{v}{t}}$ when $\frac{v}{t} = \frac{3}{8\left(2+\sqrt{3}\right)}$. Thus, when $\frac{v}{t} \leq \frac{3}{8\left(2+\sqrt{3}\right)}$, $R_0 < R_1$ for all x. Similarly, when $\frac{v}{t}$ is sufficiently high, $R_0 > R_1$ for all x. For instance, when $\frac{v}{t} > 3$, from section A1, $R_0 = t\left(\frac{v}{t} - \left(\frac{1}{2} - x\right)^2 - \frac{1}{4}\right)$ and $R_1 = t\left(\frac{v}{t} - (1-x)^2\right)$. A comparison yields that $R_0 > R_1$ when $\frac{1}{2} - x > 0$, which is true for all $x < \frac{1}{2}$.[22] Thus, for each value of x, there is a value of $\frac{v}{t}$ such that $R_0 > R_1$. Then, by Lemmas 1 and 2, if $R_0 > R_1$ at some x, then $R_0 > R_1$ at the same value of x for all higher values of $\frac{v}{t}$. $\blacksquare$

Proof of Proposition 3. I first provide a formal definition of 'defeated equilibrium.' I then provide an algorithm that establishes that the class of PBE described in Proposition 2 defeats any other class of PBE. I finally show that there exists no other PBE that defeats the class of PBE described in Proposition 2, so this class is the unique 'undefeated PBE.'

For the remainder of the proof, I restrict on-the-equilibrium-path messages to perfectly overlap with the inferences; i.e., $m(x) \equiv \Omega(m(x))$. This is without any loss of generality because it is the equilibrium inferences that distinguishes two PBEs from each other, while there are typically infinitely many messages that lead to the same inferences.

Definition *Denote $R(\sigma, x)$ as the revenue S earns for variety x in PBE σ. Then, $\sigma \equiv (b, p, m, \pi)$ defeats $\sigma' \equiv (b', p', m', \pi')$ if there exists $M \subset [0,1]$ such that*

(i) No type in σ' sends M, while the set of types in σ that send M is non-empty, i.e., $\forall x \in [0,1]$, $m'(x) \neq M$, and $T = \{x \in [0,1] \mid m(x) = M\} \neq \varnothing$;

(ii) All types that send M in σ earn higher payoffs (strictly higher for at least one) in σ than σ'; i.e., $\forall x \in T$, $R(\sigma, x) \geq R(\sigma', x)$ and $T^s = \{x \in T \mid R(\sigma, x) > R(\sigma', x)\} \neq \varnothing$;

(iii) Beliefs in σ' are inconsistent for $x \in T^s$ in the following sense: $\exists x \in T$ for which $\pi'(x \mid M) \neq \frac{f(x)\alpha(x)}{\int_{\tilde{x} \in T} \alpha(\tilde{x}) f(\tilde{x}) d\tilde{x}}$ for any $\alpha : T \to [0,1]$ satisfying $\alpha(\tilde{x}) = 1$ if $\tilde{x} \in T^s$ and $\alpha(\tilde{x}) = 0$ if $\tilde{x} \notin T$ (thus allowing for types that are indifferent to randomize).

Let the PBE described in Proposition 2 be denoted as σ^* and take some other PBE σ'. It can be established by following the algorithm below that σ^* defeats σ'.

[22]In fact, it can be shown that $R_0 < R_1$ for all x when $\frac{v}{t} \lesssim 0.521$ and $R_0 > R_1$ for all x when $\frac{v}{t} \geq 0.75$. However, the derivation is long and tedious, but otherwise straightforward algebra. Since it is not important for the results, I skip it here. It is available upon request.

(1) Locate a variety x for which $m^*(x) \neq m'(x)$.

(2) Denote the variety in $m'(x)$ that is closest to $\frac{1}{2}$ as x_1 (pick randomly if there are two such varieties).

(3) Check if $R(\sigma', 1-x_1) \leq R(\sigma', x_1)$ or not. If this holds, then it must be that both $x = x_1$ and $x = 1 - x_1$ do strictly better in σ^* than in σ'. So, beliefs in σ' are inconsistent.[23]

(4) If $R(\sigma', 1-x_1) > R(\sigma', x_1)$, then $m'(1-x_1)$ must involve at least one variety that is closer to $\frac{1}{2}$ than x_1. Locate the variety in $m'(1-x_1)$ that is closest to $\frac{1}{2}$ (pick randomly of there are two such varieties) and denote it as x_2.

(5) Repeat steps 3 and 4 until you find a variety x_n for which $R(\sigma', 1-x_n) \leq R(\sigma', x_n)$. Such a variety must exist because $R(\sigma', 1-x) = R(\sigma', x)$ for $x = \frac{1}{2}$.

It is helpful to articulate a bit more on the algorithm. First, suppose that x_1 is the only location in $m'(x)$, so $R(\sigma', x_1) = R_1(x_1)$. Then, by the assertion in step 1, it must be that $m^*(x_1) = \{x_1, 1-x_1\}$, and by the definition of σ^*,

$$R(\sigma^*, x_1) = R(\sigma^*, 1-x_1) = R_0(x_1) > R_1(x_1).$$

Assuming $R(\sigma', 1-x_1) \leq R(\sigma', x_1)$, it follows that

$$R(\sigma', 1-x_1) \leq R(\sigma', x_1) = R_1(x_1) < R(\sigma^*, x_1) = R(\sigma^*, 1-x_1).$$

This means that upon observing a message $m = \{x_1, 1-x_1\}$, beliefs in σ' should assign a probability of $\frac{1}{2}$ to both $x = x_1$ and $x = 1-x_1$. However, in such a case, S would deviate from σ' and instead send $m = \{x_1, 1-x_1\}$ for both $x = x_1$ and $x = 1-x_1$. Thus, σ^* defeats σ'.

Next, suppose that $m'(x_1) = m'(1-x_1) = \{x_1, 1-x_1\}$, so beliefs in σ' assign equal probability to $x = x_1$ and $x = 1-x_1$ upon observing this message. Due to the tie-breaking condition, this means that $R_1(x_1) < R_0(x_1)$. However, by step 1, it must be

[23] This step is satisfied for any symmetric PBE for which $R(\sigma, x) = R(\sigma, 1-x)$ for all $x \in [0,1]$. In principle, there may exist asymmetric PBE, too. As an example, suppose $f(\cdot)$ is uniform and $\frac{v}{t}$ is large enough. Then the following is a PBE: all x except for $x = 0.4$ and $x = 0.59$ are fully revealed while S sends $m = \{0.4, 0.59\}$ for $x = 0.4$ and 0.59. Possible beliefs that support this PBE involve taking the message $m = \{0.4, 0.59\}$ literally, so assigning $\Pr(x = 0.4) = \Pr(x = 0.59) = 0.5$ after seeing $m = \{0.4, 0.59\}$, and assigning probability one to the variety that is farthest away from $\frac{1}{2}$ (on the one on the left if there are two such varieties) for any other message. The forth and fifth steps of the algorithm apply only for asymmetric PBE.

that $m^* (x_1) = x_1$, which means $R_1 (x_1) \geq R_0 (x_1)$. Thus, if $m^* (x_1) = x_1$, σ' cannot involve $m' (x_1) = m' (1 - x_1) = \{x_1, 1 - x_1\}$.

Finally, suppose that there are at least two distinct asymmetric locations in $m' (x)$. Denote the variety in $m' (x)$ that is closest to $\frac{1}{2}$ as x_1 and assume that $R (\sigma', 1 - x_1) \leq R (\sigma', x_1)$. By the asymmetry of $m' (x)$, it must be that

$$R_0 (x_1) > R (\sigma', x_1) = R (\sigma', x) .$$

In other words, S would have strictly preferred sending $m = \{x_1, 1 - x_1\}$ if B assigned a probability of $\frac{1}{2}$ to both $x = x_1$ and $x = 1 - x_1$ upon observing this message. Now, suppose $m^* (x_1) = \{x_1, 1 - x_1\}$ in σ^*. Then, beliefs in σ' must be inconsistent because otherwise S would deviate from σ' by sending $m = \{x_1, 1 - x_1\}$ for both x_1 and $1 - x_1$, and earn a higher payoff. If S fully reveals x_1 in σ^*, i.e., if $m^* (x_1) = x_1$, then by the construction of σ^*,

$$R_1 (x_1) \geq R_0 (x_1) ,$$

so it follows that

$$R_1 (x_1) \geq R_0 (x_1) > R (\sigma', x_1) .$$

In such a case, S would again deviate from σ' by sending $m = x_1$ for $x = x_1$, so beliefs in σ' are inconsistent.

It is possible that σ' is asymmetric and that $R (\sigma', 1 - x_1) > R (\sigma', x_1)$. Then, as stated in step 4, $m' (1 - x_1)$ must involve at least one variety that is closer to $\frac{1}{2}$ than x_1 because otherwise $R (\sigma', 1 - x_1)$ could not be strictly larger than $R (\sigma', x_1)$. Denoting the variety in $m' (1 - x_1)$ that is closest to $\frac{1}{2}$ as x_2, one can proceed as in the previous paragraph. If it is again the case that $R (\sigma', 1 - x_2) > R (\sigma', x_2)$, the algorithm calls for repeating the same process until a variety x_n is located for which $R (\sigma', 1 - x_n) \leq R (\sigma', x_n)$. Since the algorithm alternates between the two sides of $\frac{1}{2}$ and gets strictly closer to $\frac{1}{2}$ at each new iteration, such a critical variety $x_n \neq \frac{1}{2}$ exists.

As the final step of the proof, I show that there are no other PBE that defeat the class of PBE described in Proposition 2. The approach is very similar to the first part of the proof, so I will be more brief. Suppose, on the contrary, that there is another PBE σ' that defeats σ^*. Then, there exists at least one message M such that no type in σ^*

sends M while the set of types in σ' that send M is non-empty, and all types that send M in σ' earn higher payoffs (strictly higher for at least one type) in σ' than σ^*. By this latter observation, we can immediately rule out the scenarios in which only one type in σ' sends M and in which only two types in σ' that are equally distanced from $\frac{1}{2}$ send M. Hence, there must be at least two asymmetric types in σ' that send M. Denote the variety in M that is closest to $\frac{1}{2}$ as x_1 (pick randomly if there are two such varieties). If all variaties in M are on either the left-hand side or the right-hand side of $\frac{1}{2}$, then it is obvious that type x_1 would be strictly better off by sending a fully revealing message and earning $R_1(x_1)$. By the construction of σ^*, we know that $R(\sigma^*, x_1) \geq R_1(x_1)$, so it cannot be true that all types that send M in σ' earn higher payoffs in σ' than σ^*. Similarly, if there are some variaties in M that are on the left-hand side of $\frac{1}{2}$ and some on the right-hand side, then type x_1 would be strictly better off by sending $\{x_1, 1-x_1\}$ and earning $R_0(x_1)$ since this implies a perceived variety $\frac{1}{2}$ and has a strictly lower variance. Since it must be that $R(\sigma^*, x_1) \geq R_0(x_1)$, it again cannot be true that all types that send M in σ' earn higher payoffs in σ' than σ^*. Hence, we reach a contradiction. ■

Proof of Proposition 4. From section A1, when $\frac{v}{t} \geq 1$, all types of B are served for any x in case S sends a partially-revealing message, and $v - t(\lambda - x)^2 \geq 0$ for each type of B. Thus, when $\frac{v}{t} \geq 1$, $W_0 \geq W_1$ for all x (with equality when all types of B are served under full disclosure, too). So, the proof is trivial in this case; mandating full disclosure brings no extra gain while it may be harmful. Similarly, for parameter values where the expected demand is zero under the partially-revealing strategy (this happens for $x \leq \frac{1}{2} - \sqrt{\frac{v}{t}}$ when $\frac{v}{t} \leq \frac{1}{4}$), full disclosure is welfare superior to sending a partially-revealing message. However, since S voluntarily reveals all x for these parameter values, there is no need for mandating full disclosure. For the remainder of the proof, I focus on the remaining situations (i.e., $\frac{v}{t} < 1$ and $D_0 > 0$) and show that $CS_0 > CS_1$ whenever $R_0 > R_1$, so the result follows.

Case 1: $x < \sqrt{\frac{v}{5t}}$

In this case, if S sends a partially-revealing message for x, the resulting consumer surplus is expressed as

$$CS_0 = \int_{p_0}^{v-t\left(\frac{1}{2}-x\right)^2} 2\sqrt{\frac{v-P}{t} - \left(\frac{1}{2}-x\right)^2}\, dP = \frac{4t}{3}\left(\frac{v-p_0}{t} - \left(\frac{1}{2}-x\right)^2\right)^{3/2}.$$

In the region where $p_0 = \frac{2t\left(\frac{v}{t}-\left(\frac{1}{2}-x\right)^2\right)}{3}$, this is equal to $CS_0 = \frac{4t}{3}\left(\frac{\frac{v}{t}-\left(\frac{1}{2}-x\right)^2}{3}\right)^{3/2}$. In the same region, $D_0 = 2\sqrt{\frac{v-p_0}{t} - \left(\frac{1}{2}-x\right)^2} = 2\sqrt{\frac{\frac{v}{t}-\left(\frac{1}{2}-x\right)^2}{3}}$, so $R_0 = p_0 D_0 = 4t\left(\frac{\frac{v}{t}-\left(\frac{1}{2}-x\right)^2}{3}\right)^{3/2}$. Thus, it follows that $CS_0 = \frac{1}{3}R_0$ in this region. In the region where $p_0 = t\left(\frac{v}{t} - \left(\frac{1}{2}-x\right)^2 - \frac{1}{4}\right)$, on the other hand, $CS_0 = \frac{4t}{3}\left(\frac{1}{4}\right)^{3/2} = \frac{t}{6}$.

When S fully reveals x, the resulting consumer surplus is

$$CS_1 = \int_{p_1}^{v-tx^2}\left(x + \sqrt{\frac{v-P}{t}}\right) dP + \int_{v-tx^2}^{v} 2\sqrt{\frac{v-P}{t}} dP.$$

Evaluated at $x = 0$, $p_1 = \frac{2v}{3}$, and $CS_1 = \int_{2v/3}^{v} \sqrt{\frac{v-P}{t}} dP = \frac{2t}{3}\left(\frac{v}{3t}\right)^{3/2}$. Demand evaluated at $x = 0$ is given by $D_1 = \sqrt{\frac{v-p_1}{t}} = \sqrt{\frac{v}{3t}}$, so $R_1(0, v, t) = 2t\left(\frac{v}{3t}\right)^{3/2}$. Thus, it follows that $CS_1(0, v, t) = \frac{1}{3}R_1(0, v, t)$.

Next, observe that

$$\frac{dCS_1(x, v, t)}{dx} = (v - tx^2 - p_1) - \left(x + \sqrt{\frac{v-p_1}{t}}\right)\frac{dp_1}{dx},$$

$$\frac{dR_1(x, v, t)}{dx} = p_1,$$

where the second line follows from the Envelope Theorem. The equilibrium price in this region can be rewritten as $p_1 = \frac{2t}{9}\left(\sqrt{\frac{3v}{t} + x^2} + 2x\right)\left(\sqrt{\frac{3v}{t} + x^2} - x\right)$ and the resulting demand as $x + \sqrt{\frac{v-p_1}{t}} = \frac{1}{3}\left(\sqrt{\frac{3v}{t} + x^2} + 2x\right)$. Taking the derivative of p_1 with respect to x and then multiplying the result with the expected demand gives

$$\left(x + \sqrt{\frac{v-p_1}{t}}\right)\frac{dp_1}{dx} = \frac{1}{3}\left(1 - \frac{x}{\sqrt{\frac{3v}{t} + x^2}}\right)p_1.$$

Thus,

$$
\begin{aligned}
\frac{d\left(CS_1 - \frac{R_1}{3}\right)}{dx} &= (v - tx^2 - p_1) - \left(x + \sqrt{\frac{v - p_1}{t}}\right)\frac{dp_1}{dx} - \frac{p_1}{3} \\
&= v - tx^2 - \left(\frac{5}{3} - \frac{1}{3}\frac{x}{\sqrt{\frac{3v}{t} + x^2}}\right)p_1.
\end{aligned}
$$

Note that $\frac{x}{\sqrt{\frac{3v}{t}+x^2}}$ is increasing in x, so the maximum value it can take in this region is $\frac{\sqrt{\frac{v}{5t}}}{\sqrt{\frac{3v}{t}+\frac{v}{5t}}} = \frac{1}{4}$. Similarly, $tx^2 \geq 0$ and $p_1 \geq \frac{2v}{3}$. Thus,

$$
\frac{d\left(CS_1 - \frac{R_1}{3}\right)}{dx} \leq v - \left(\frac{5}{3} - \frac{1}{12}\right)\frac{2v}{3} = -\frac{v}{18} < 0.
$$

So, $\frac{R_1}{3}$ rises more quickly than CS_1 as x increases, which means that $CS_1 < \frac{1}{3}R_1$ for all x in this region. Thus, in the region where $p_0 = \frac{2t\left(\frac{v}{t} - \left(\frac{1}{2} - x\right)^2\right)}{3}$, we have

$$
CS_0 - CS_1 > \frac{1}{3}(R_0 - R_1).
$$

This condition means that if $R_0 \geq R_1$ for a particular x, then $CS_0 \geq CS_1$ and, in turn, $W_0 \geq W_1$ for the same x. Hence, mandating full disclosure is harmful.

When $\frac{3}{4} \leq \frac{v}{t} < 1$, if S sends a partially-revealing message, he charges a price $p_0 = t\left(\frac{v}{t} - \left(\frac{1}{2} - x\right)^2 - \frac{1}{4}\right)$ for $\frac{1}{2} - \sqrt{\frac{v}{t} - \frac{3}{4}} < x \leq \frac{1}{2}$. As argued before, in this region, $\frac{CS_0}{t} = \frac{4}{3}\left(\frac{1}{4}\right)^{3/2} = \frac{1}{6}$. In the following, I first show for the same region that $\frac{CS_1}{t}$ is increasing in $\frac{v}{t}$, and then show that $\max_x \frac{CS_1}{t}$ evaluated at $\frac{v}{t} = 1$ is less than $\frac{1}{6}$. First, note that CS_1 can be rewritten as

$$
\begin{aligned}
CS_1 &= (v - tx^2 - p_1)x - \frac{2t}{3}x^3 + \frac{2t}{3}\left(\frac{v - p_1}{t}\right)^{3/2} + \frac{4t}{3}x^3 \\
&= t\left(\frac{v - p_1}{t}\right)x + \frac{2t}{3}\left(\frac{v - p_1}{t}\right)^{3/2} - \frac{t}{3}x^3.
\end{aligned}
$$

Next, observe that

$$
\frac{d\left(\frac{v-p_1}{t}\right)}{d\left(\frac{v}{t}\right)} = 1 - \frac{2}{9}\left(3 + \frac{3x}{2\sqrt{\frac{3v}{t} + x^2}}\right) = \frac{1}{3} - \frac{x}{3\sqrt{\frac{3v}{t} + x^2}} > 0.
$$

Since $\frac{CS_1}{t}$ is increasing in $\frac{v-p_1}{t}$, it is also increasing in $\frac{v}{t}$. When $\frac{v}{t} = 1$, it is easy to show that $\frac{v-p_1}{t} = \frac{1}{3}\left(\sqrt{3 + x^2} - x\right)$. Plugging this back into CS_1 and maximizing it with respect

x leads to $\arg\max_x CS_1 \approx 0.2256$ and $\max_x \frac{CS_1}{t} \approx 0.141$. This is less than $\frac{1}{6}$, which means that $CS_0 > CS_1$ for all parameter values for which $p_0 = t\left(\frac{v}{t} - \left(\frac{1}{2} - x\right)^2 - \frac{1}{4}\right)$.

Case 2: $\sqrt{\frac{v}{5t}} \leq x < \min\left\{\sqrt{\frac{v}{3t}}, \frac{1}{2}\right\}$

In this case, $p_1 = v - tx^2$ and thus,

$$CS_1 = \int_{v-tx^2}^{v} 2\sqrt{\frac{v-P}{t}}dP = \frac{4t}{3}x^3.$$

The revenue S earns when he fully reveals x is $R_1 = 2t\left(\frac{v}{t} - x^2\right)x$. The condition $\sqrt{\frac{v}{5t}} \leq x < \min\left\{\sqrt{\frac{v}{3t}}, \frac{1}{2}\right\}$ can equally be represented as $3x^2 < \frac{v}{t} \leq 5x^2$, so $4tx^3 < R_1 \leq 8tx^3$. Hence, $CS_1 < \frac{1}{3}R_1$, which implies that

$$CS_0 - CS_1 > \frac{1}{3}(R_0 - R_1).$$

As argued before, in the region where $p_0 = t\left(\frac{v}{t} - \left(\frac{1}{2} - x\right)^2 - \frac{1}{4}\right)$, $CS_0 = \frac{t}{6}$. Evaluated at $x = \frac{1}{2}$, $CS_1 = \frac{4t}{3}x^3 = \frac{t}{6}$. Hence, $CS_0 > CS_1$ for all $x < \frac{1}{2}$ in this region. Again, mandating S to fully reveal x in situations when he voluntarily does not is socially harmful.

Case 3: $\min\left\{\sqrt{\frac{v}{3t}}, \frac{1}{2}\right\} \leq x \leq \frac{1}{2}$

This case is relevant only when $\frac{v}{t} < \frac{3}{4}$. In this region, $R_1 > R_0$ for all x (except for $x = \frac{1}{2}$ where two regimes are equivalent). So, mandatory disclosure rules are unnecessary. ■

A3 Consumer surplus

In this section, I show that the consumer surplus expressions given in equations (4) and (6) can alternatively be expressed in the following form:

$$CS_1(x, v, t) = \int_{p_1(x,v,t)}^{v} D_1(P; x, v, t)dP,$$

$$CS_0(x, v, t) = \int_{p_0(x,v,t)}^{v-t(\frac{1}{2}-x)^2} D_0(P; x, v, t)dP.$$

Starting with CS_1, for a given $p < v$,

$$v - p - t(\lambda - x)^2 \geq 0 \text{ for all } \lambda \in \left[\lambda_1^L(p, x, v, t), \lambda_1^H(p, x, v, t)\right].$$

We can then write $v - p_1 - t(\lambda - x)^2$ as

$$v - p_1 - t(\lambda - x)^2 = \int_{p_1}^{v} \mathbf{1}\left[v - P - t(\lambda - x)^2\right] dP,$$

for any $\lambda \in \left[\lambda_1^L(p_1, x, v, t), \lambda_1^H(p_1, x, v, t)\right]$, where $\mathbf{1}[\cdot]$ is the indicator function. Hence,

$$\begin{aligned}
CS_1(x, v, t) &= \int_{\lambda_1^L(p_1,x,v,t)}^{\lambda_1^H(p_1,x,v,t)} (v - p_1 - t(\lambda - x)^2) d\lambda \\
&= \int_{\lambda_1^L(p_1,x,v,t)}^{\lambda_1^H(p_1,x,v,t)} \int_{p_1(x,v,t)}^{v} \mathbf{1}\left[v - P - t(\lambda - x)^2\right] dP d\lambda \\
&= \int_{p_1(x,v,t)}^{v} \int_{\lambda_1^L(p_1,x,v,t)}^{\lambda_1^H(p_1,x,v,t)} \mathbf{1}\left[v - P - t(\lambda - x)^2\right] d\lambda dP \\
&= \int_{p_1(x,v,t)}^{v} \left[\lambda_1^H(P, x, v, t) - \lambda_1^L(P, x, v, t)\right] dP \\
&= \int_{p_1(x,v,t)}^{v} D_1(P; x, v, t) dP,
\end{aligned}$$

where the forth line follows from the following three observations:

$$v - P - t(\lambda - x)^2 \geq 0 \text{ for } \lambda \in \left[\lambda_1^L(P, x, v, t), \lambda_1^H(P, x, v, t)\right],$$

$$\lambda_1^L(p_1, x, v, t) \leq \lambda_1^L(P, x, v, t) \text{ for all } P \in [p_1(x, v, t), v],$$

$$\lambda_1^H(p_1, x, v, t) \geq \lambda_1^H(P, x, v, t) \text{ for all } P \in [p_1(x, v, t), v].$$

To establish the equivalence for CS_0, first note that due to the symmetry of the uniform distribution around $\frac{1}{2}$ and the fact that $\left|\lambda_0^L - \frac{1}{2}\right| = \left|\lambda_0^H - \frac{1}{2}\right|$, it follows that $\int_{\lambda_0^L}^{\lambda_0^H} \lambda \frac{d\lambda}{\lambda_0^H - \lambda_0^L} = \frac{1}{2}$. Thus, we have the following equality:

$$
\begin{aligned}
\int_{\lambda_0^L}^{\lambda_0^H} (\lambda - x)^2 \frac{d\lambda}{\lambda_0^H - \lambda_0^L} &= \int_{\lambda_0^L}^{\lambda_0^H} \left(\lambda^2 - 2\lambda x + x^2\right) \frac{d\lambda}{\lambda_0^H - \lambda_0^L} \\
&= \int_{\lambda_0^L}^{\lambda_0^H} \left(\lambda^2 - x + x^2 + \left(\frac{1}{2} - \lambda\right)\right) \frac{d\lambda}{\lambda_0^H - \lambda_0^L} \\
&= \int_{\lambda_0^L}^{\lambda_0^H} \frac{1}{2} \left((\lambda^2 - 2\lambda x + x^2) + (\lambda^2 - 2\lambda (1-x) + (1-x)^2)\right) \frac{d\lambda}{\lambda_0^H - \lambda_0^L} \\
&= \int_{\lambda_0^L}^{\lambda_0^H} \frac{1}{2} \left((\lambda - x)^2 + t(1 - x - \lambda)^2\right) \frac{d\lambda}{\lambda_0^H - \lambda_0^L}.
\end{aligned}
$$

Now, the equivalence of the two consumer surplus expressions can be shown as follows.

$$
\begin{aligned}
CS_0(x, v, t) &= \int_{\lambda_0^L(p_0,x,v,t)}^{\lambda_0^H(p_0,x,v,t)} (v - p_0 - t(\lambda - x)^2) d\lambda \\
&= \int_{\lambda_0^L(p_0,x,v,t)}^{\lambda_0^H(p_0,x,v,t)} \left(v - p_0 - \frac{t}{2}\left((\lambda - x)^2 + (1 - x - \lambda)^2\right)\right) d\lambda \\
&= \int_{\lambda_0^L(p_0,x,v,t)}^{\lambda_0^H(p_0,x,v,t)} \int_{p_0(x,v,t)}^{v} \mathbf{1}\left[v - P - \frac{t}{2}\left((\lambda - x)^2 + (1 - x - \lambda)^2\right)\right] dP d\lambda \\
&= \int_{p_0(x,v,t)}^{v} \int_{\lambda_0^L(p_0,x,v,t)}^{\lambda_0^H(p_0,x,v,t)} \mathbf{1}\left[v - P - \frac{t}{2}\left((\lambda - x)^2 + (1 - x - \lambda)^2\right)\right] d\lambda dP \\
&= \int_{p_0(x,v,t)}^{v - t(\frac{1}{2} - x)^2} D_0(P; x, v, t) dP.
\end{aligned}
$$

The upper bound of the integral in the last line above becomes $v - t(\frac{1}{2} - x)^2$ because $D_0 = 0$ for all λ for prices above $v - t(\frac{1}{2} - x)^2$.

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Figure 1. Expected demand curves S faces for x = 0.3
when v = 0.6 and t = 1.

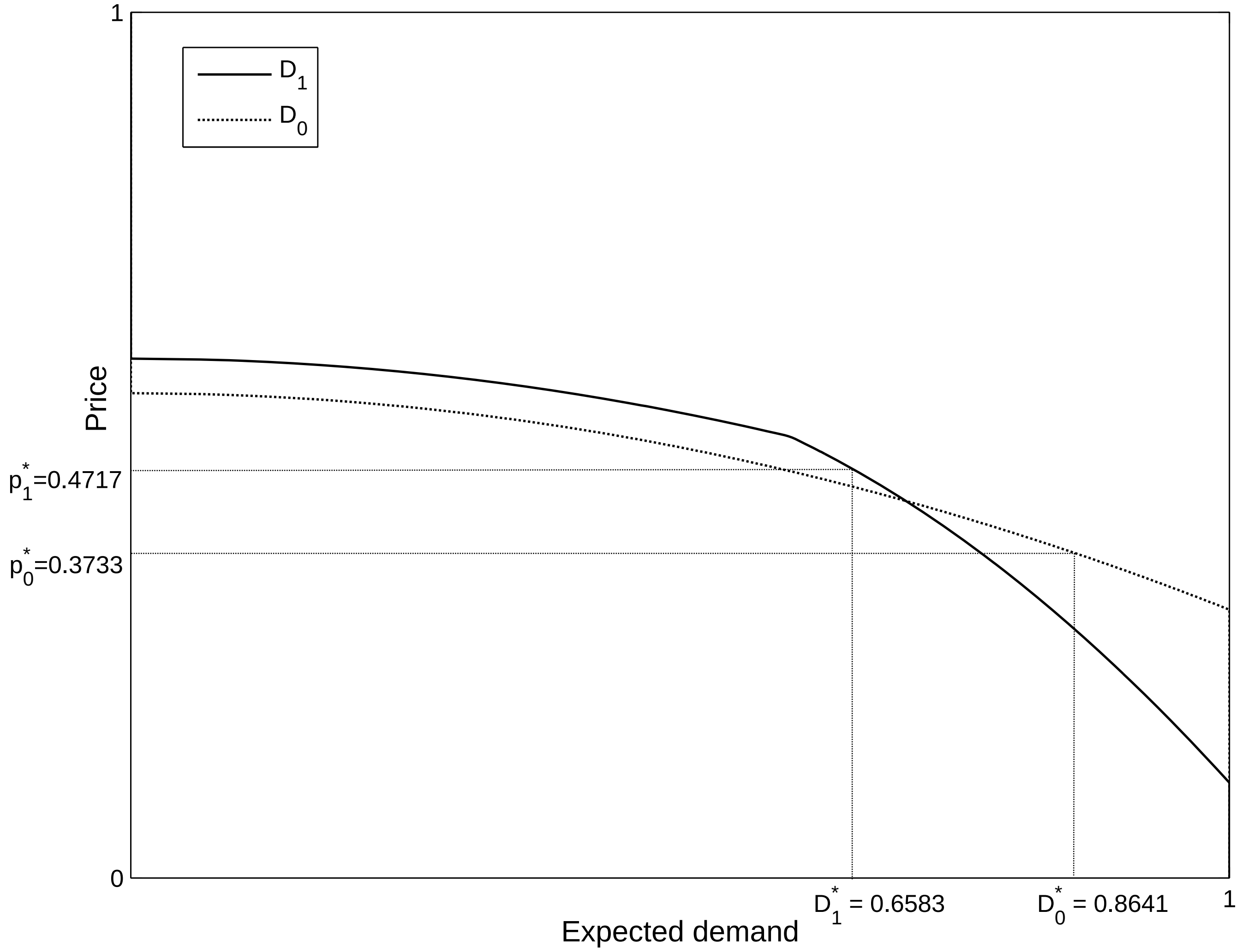

Figure 2. The set of fully revealed locations (indicated by double arrows) when v = 0.6 and t = 1.