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Ultra High Sensitivity in Differential Coupled Micro/Nano-Resonator Sensors

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***Abstract*-The resonance frequencies (eigenvalues) and amplitudes (eigenvectors) of coupled micro/nano-resonator arrays are used in sensors to measure applied physical perturbations such as mass. A higher sensitivity is obtained by measuring amplitude changes rather than frequency variations corresponding to identical perturbations. In this paper we present a new technique by measuring the difference of displacement amplitudes of two resonators in perturbed and unperturbed conditions that can increase the sensitivity by several orders of magnitude compared with frequency/amplitude measurements. Two coupled resonators implemented in SOI technology are modeled in Coventor to provide a proof of concept for the proposed idea. In the example coupled resonator, differential sensitivity is increased by 570 times for the first mode and by 170 times for the second mode compared with absolute sensitivity.**

I. Introduction

Nano/micro resonant sensors have been used for measuring mass, acceleration, etc. in a number of applications [1-2]. However, a single sensor per chip fails to utilize the potential offered by the advances in nanofabrication. Techniques have been suggested which utilize the collective behavior of an array of coupled nano-resonators to provide sensory multiplexing on a chip [1], [3]. Two different approaches have been suggested as sensory output from these systems. In one, the resonance frequency and hence the eigenvalue of the coupled array is used for sensing [3]. Any change in any of the element would lead to a change in all eigenvalues, which can be measured from the frequency response of any of the resonators. Alternatively, one can use the amplitude ratios of the coupled resonators, and hence the eigenvectors of the system. It has been reported that the eigenvectors have significantly higher sensitivity than the eigenvalues [1]. In this paper, we report a third technique, which has significantly higher sensitivity as well as common mode noise rejection capabilities than either of these techniques. Using a set of two-coupled resonators, we show that using a differential measurement of amplitudes of the two resonators provides two order of sensitivity enhancement.

II. Coupled systems

Figure 1 demonstrates the lumped model of a set of two coupled resonators. The model can be mathematically analyzed with second order differential equations:

$$m\ddot{x}_1 + kx_1 + k_c(x_1 - x_2) = 0 \qquad (1)$$

$$m\ddot{x}_2 + kx_2 + k_c(x_2 - x_1) = 0 \qquad (2)$$

Where k denotes the spring constant of the elements and k_c denotes the strength of the coupling spring. It has been assumed that the system has weak damping, which can be ignored for mathematical analysis. The system can be analyzed using its matrix equivalent

$$M\ddot{x}_1 + Kx = 0 \qquad (3)$$

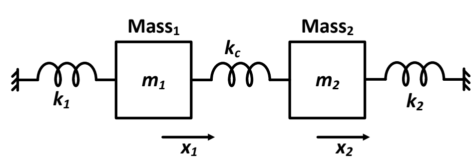


Figure 1:Lumped model of a coupled resonator

Where M is a diagonal matrix of the masses of individual elements and K is a tridiagonal matrix of spring constants.

$$K = \begin{bmatrix} k + k_c & -k_c \\ -k_c & k + k_c \end{bmatrix} \qquad (4)$$

The eigenvalue of the system can be used for finding the resonance frequencies of the system ($f_i = \sqrt{\lambda_i/(2\pi)}$). For the present system, the eigenvectors are *k/m* and *(k+2k$_c$)/m*. This means that any change in mass would lead to a change in the eigenvalues and hence the measured resonance frequencies. Furthermore, this can be recorded from the response of any one of the resonator. This, therefore provides a sensing mechanism with a well-known sensitivity and the need to measure or connect to only one resonator. In addition, the amplitudes of the two resonators at these resonance frequencies is related to the eigenvectors of the system. This in turn provides another mechanism for sensing wherein one can record the change in eigenvectors. To better appreciate this, let us consider a normalized system of $k=1$, $m=1$ and a normal weak coupling of $k_c = 0.1$, one can obtain eigenvalues to be 1 and 1.2. The eigenvectors are for this system of identical resonators are {0.7, 0.7} and {0.7, -0.7}, respectively. If one now considers small change of 1% in the first element, it would lead to a change in eigenvalues of 1.0049 and 1.2051. Similarly, eigenvectors would change to {0.69,0.72} and {0.72, -0.69}, respectively. One can observe that the change in eigenvectors have higher sensitivity for these normalized systems. This has been proposed as a technique to enhance the sensitivity of coupled resonators [1].

However, it is worth noting that both eigenvectors related to both terms undergo a change. Furthermore, for a system of identical resonators, the amplitude for the unchanged resonators for the first eigen-modes are equal in amplitude as well as phase. This means that a differential output from these resonators will have zero output for this eigen-mode. This also means that any change introduced in the system would lead to a large relative change in the differential output. Hence, rather than measuring the amplitude of each of the two resonant system, we now measure the difference of the amplitudes. In this case, the net response of the sensor for two different resonant frequency changes from {0, 1.41} to {0.0353, -1.4138}. This, hence, suggests that a much higher sensitivity can be achieved with this sensing mechanism. Furthermore, this technique enables one to design a measurement system which essentially monitors any change of the output at the first resonant frequency from zero to a non-zero number. Such a differential measurement technique for a change from zero detector, is significantly easier to design.

III. Analysis

To further analyze the sensing approach, let us consider the eigenvalue perturbation problem. For symmetric tri-diagonal matrices such as the one observed in linear coupled arrays of resonators, one can express the eigenvalue problem in terms of the system matrix $S=M^{-1/2}KM^{1/2}$. One can then obtain the change in eigenvalue and eigenvectors for any perturbation in system matrix, ΔS to be [4]

$$\Delta \lambda_i = \boldsymbol{r}_i^T \boldsymbol{\Delta S} \boldsymbol{r}_i \qquad (5)$$

where $\boldsymbol{r}_i$ is the eigenvector corresponding to the i^{th} eigenvalue. Similarly, the change in any eigenvector can be expressed as

$$\Delta \boldsymbol{r}_i = \sum_{\substack{j=1 \\ j\neq i}}^{n} \frac{\boldsymbol{r}_j^T \Delta \boldsymbol{S} \boldsymbol{r}_i}{\lambda_i - \lambda_j} \boldsymbol{r}_j \qquad (6)$$

For an array of two identical elements, both of these simplifies to

$$\Delta \boldsymbol{r}_1 = \frac{\{0.7 \quad -0.7\}\Delta S \begin{Bmatrix} 0.7 \\ 0.7 \end{Bmatrix}}{2k_c/m} \begin{Bmatrix} -0.7 \\ 0.7 \end{Bmatrix} \qquad (7)$$

$$\Delta \boldsymbol{r}_2 = \frac{\{0.7 \quad 0.7\}\Delta S \begin{Bmatrix} 0.7 \\ -0.7 \end{Bmatrix}}{-2k_c/m} \begin{Bmatrix} 0.7 \\ 0.7 \end{Bmatrix} \qquad (8)$$

However, if one takes the difference of the response of the two resonators at each frequency (x_1-x_2), one can observe that the difference at both of these frequencies will be proportional to

$$\Delta r_1^1 - \Delta r_1^2 = 1.4\frac{\{0.7 \quad -0.7\}\Delta S\begin{Bmatrix}0.7\\0.7\end{Bmatrix}}{2k_c/m} \qquad (9)$$

$$\Delta r_2^1 - \Delta r_2^2 = 1.4\frac{\{0.7 \quad 0.7\}\Delta S\begin{Bmatrix}0.7\\-0.7\end{Bmatrix}}{2k_c/m} \qquad (10)$$

This means that the change introduced in the differential readout is at least double that of the change introduced in any one of the readouts. More importantly, the unchanged eigenvectors for the first mode have the same value and hence the difference of the response of the two resonator will be zero. Hence, the sensitivity should ideally be very high.

Two points are worth noting. First, these derivations are only true for small changes in resonators, bound by the limits of the perturbation analysis [4]. Such changes are known to be nonlinear and hence any eigenvector based sensing has a small dynamic range. Furthermore, that it is very difficult, if not impossible, to make two identical resonators. Hence, the actual sensitivity obtained will always be lower than this. Nevertheless, one can show that for reasonable mismatch between the resonators, this differential sensing will still provide significant improvement over direct eigenvalue or eigenvector based sensing.

IV. Differential Configuration of Coupled Arrays

To verify the new sensing method, we undertook a simulation study of a typical cantilever based system in a standard Silicon on Insulator (SOI) technology. Figure 2 illustrates a three dimensional (3D) model of two coupled resonators. Cantilever dimensions are 400um×35um×10um, which are coupled by a 90um×35um×10um under-etched silicon. The mechanical coupling strength can be adjusted in design by changing the length of silicon between two cantilevers. The resonance behavior of the beams was characterized by undertaking harmonic balance analysis on finite elements of the meshed model in tools provided by Coventorware. The simulation settings included nonlinear mechanical physics accompanied by harmonic balance analysis, which is setup to apply Lanczos algorithms to the meshed model of cantilever. The displacement of free ends of two cantilever tips are measured in response to a very small mechanical load applied to one of the cantilevers in perpendicular (z) direction as shown in Figure 3.

In addition, a perturbation is modeled by changing one of the cantilever arrowheads to a flat head,

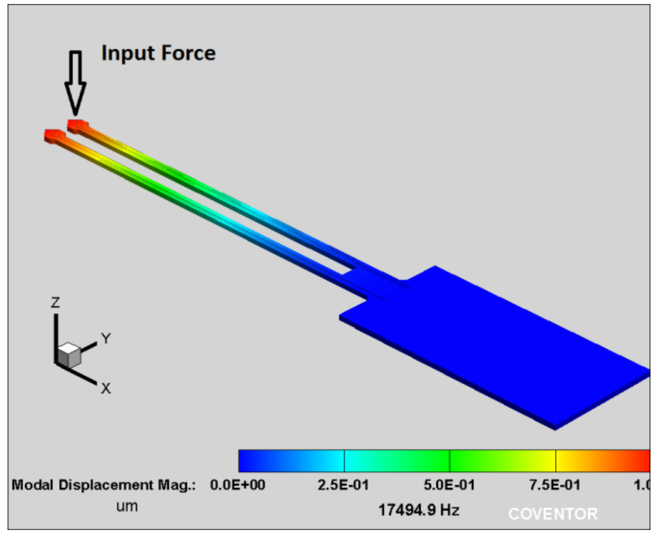


Figure 2: Fixed-free coupled microcantilever model in standard Silicon-on-Insulator (SOI) process.

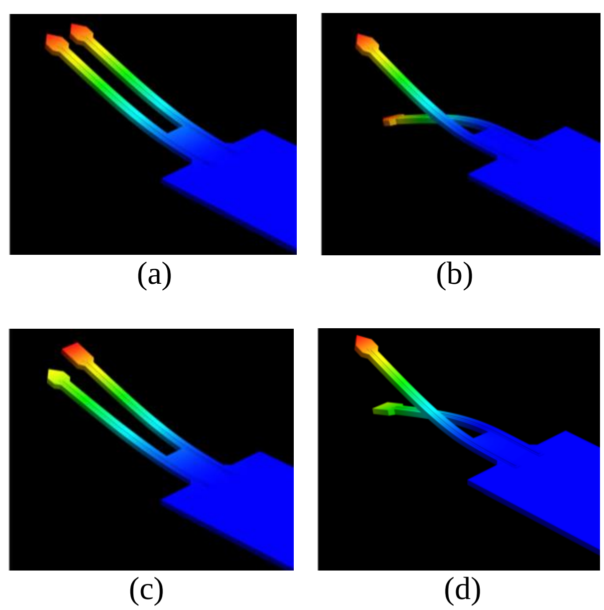
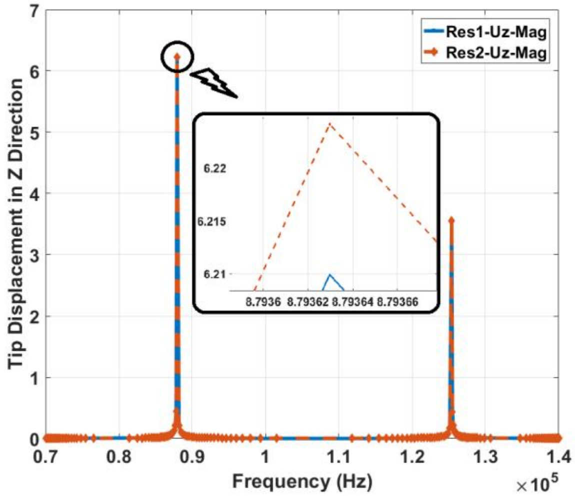
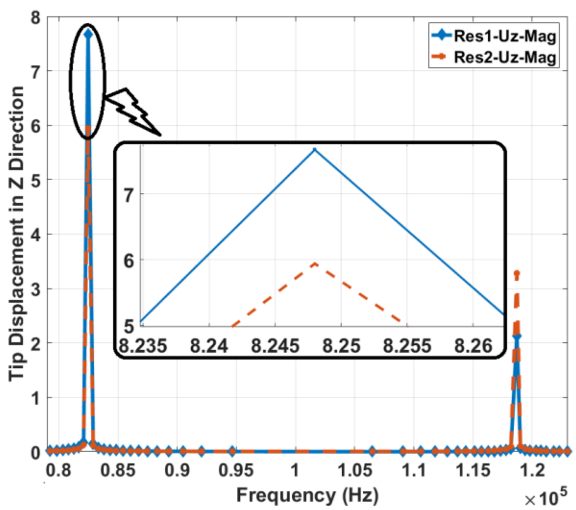


Figure 3: First two shape modes of coupled resonators, (a) in phase unperturbed, (b) out of phase unperturbed, (c) in phase perturbed, and out of phase perturbed.



(a)



(b)

Figure 4 Resonance displacement magnitude of coupled resonators, (a) Unperturbed, and (b) perturbed system.

which can model physical inputs for sensing applications such as mass changes. Simulation setup for harmonic balance analysis include 50 points over the 70KHz ~ 140KHz frequency range with 0.01 modal damping ratio. The first two modes of perturbed and unperturbed resonators are illustrated in Figure 3. The z-displacement amplitude of two cantilevers over a frequency range that covers these two modes is sketched in Figure 4. The two mode shapes are detected in identical frequencies according to the limits we had set on the simulation accuracy. Slightly asymmetric cantilever structures, to consider fabrication imperfections, leads to marginal differences in resonance amplitude of the two cantilever tips for both mode shapes. This marginal difference is magnified in an inset of Figure 3a. The difference in displacement amplitude of two resonators in perturbed and unperturbed mode shapes demonstrates a much larger variation compared to their absolute values as illustrated in Table 1. This phenomenon leads to around 660 time higher differential sensitivity (1st mode) compared to absolute measurements. The measured values in Table 1 are extracted from Figure 3 that approves the proposed methodology.

V. CONCLUSION

Coupled micro/nano resonators have been investigated for their sensing capabilities in applications such as mass and acceleration measurement. The amplitude and resonance frequency of a resonator changes as a function of applied physical input such as mass. We propose to measure the difference in the amplitude of two resonators displacement in perturbed and unperturbed conditions for an increased sensitivity. Simulation results are in absolute agreement with the presented method.

Table 1: The resonance frequency and displacment magnitude and sensitivity of coupled resonators.

	Resonance Freq (KHz)	Resonator 1 Tip displacement (um)	Resonator 2 Tip displacement (um)	Difference	Absolute Sensitivity	Differential Sensitivity
1st mode unperturbed	87.936	6.224	6.21	0.013	0.23	133
1st mode perturbed	87.479	7.67	5.94	1.73		
2nd mode unperturbed	125.39	3.555	3.538	0.017	0.4	67.5
2nd mode perturbed	118.67	2.132	3.2795	-1.15		

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